Conditions for the spin wave nonreciprocity in an array of dipolarly coupled magnetic nanopillars

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It is demonstrated that collective spin waves (SWs) propagating in complex periodic arrays of dipolarly coupled magnetic nanopillars existing in a saturated (single-domain) ground state in a zero bias magnetic field could be nonreciprocal. To guarantee the SW nonreciprocity two conditions should be fulfilled:(i) existence of a nonzero out-of-plane component of the pillars' static magnetization and (ii) a complex periodicity of array's ground state with at least two elements per a primitive cell, if the elements are different, and at least three elements per a primitive cell, if the elements are identical. The obtained results show that coupled arrays of magnetic nanopillars with out-of-plane shape or/and crystallographic anisotropy could be used for the development of miniature unbiased microwave isolators and circulators.

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The wave nonreciprocity can be defined as a property of waves to change their characteristics (such as frequency, group velocity, or dissipation) when the direction of the wave propagation is reversed. The wave nonreciprocity is widely used in signal transmission and information processing to design microwave isolators and circulators, which, e.g., prevent the cross-interference between a receiver and transmitter mounted on the same platform.

The majority of currently used nonreciprocal microwave devices, such as resonance valves, Y-circulators, etc., use the nonreciprocity of *electromagnetic waves* in waveguide systems with asymmetrically placed ferro- or ferrimagnetic materials (usually ferrites), in which the electromagnetic waves propagating in opposite directions have different propagation constants. In these devices the nonreciprocity exists only if the ferromagnetic materials in them are magnetized to saturation, usually, by application of an external bias magnetic field created by rather heavy and bulky permanent magnets[1–4]. The necessity to have a permanent magnet to bias the nonreciprocal devices complicates their miniaturization and affects their compatibility with conventional microelectronics. Note, also, that the wavelengths of microwave electromagnetic waves propagating in waveguides containing biased ferromagnets are rather large (millimeter to centimeter range), which also limits the minimum size of a device, even when highly anisotropic magnetic materials, which are self-biased, are used[5].

Both above mentioned problems can be solved by using *collective spin waves* (SWs) propagating in nanostructured magnetic materials, e.g. in arrays of magnetic nanopillars, the single-domain saturated state of which is supported by their shape or/and crystallographic anisotropy as well as by their size. It is well known that if the sizes of a magnetic nanopillar are comparable to the material's exchange length, the ground state of the pillar in a zero external bias field becomes single-domain (saturated), so that the elementary magnetic moments at each point inside a pillar have the same direction[6–8]. Note, that the existence of such a preferential direction of static magnetization (common to all the elements of an unbiased array) is a necessary condition for the nonreciprocity of collective SWs propagating in this array. Also, the collective SWs propagating in arrays of dipolarly coupled nanopillars have wavelengths that are substantially smaller than the wavelengths of electromagnetic waves of the same microwave frequency[1,9,10].

In this Letter we are deriving the necessary conditions for the existence of *nonreciprocal collective spin waves* in a two-dimensional array of dipolarly coupled magnetic nanopillars. The array is assumed to be in a periodic ground state. The periodicity of the ground state is described by the basis vectors $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, which form the array's lattice $\mathcal{L} = \{n_1\boldsymbol{a}_1 + n_2\boldsymbol{a}_2 \,|\, n_1, n_2 \in \mathbb{Z}\}$. The array's periodicity could be complex, i.e. there could be $P > 1$ pillars per a primitive cell of the array (see examples in Figs. 1(a), 3). In such a case each pillar belongs to a certain sublattice $p \in [1, P]$ of the array, and the position of the pillar's center is defined as $\boldsymbol{r}_{j_p} = \boldsymbol{\delta}_p + n_1\boldsymbol{a}_1 + n_2\boldsymbol{a}_2$, where $\boldsymbol{\delta}_p$ are the shift vectors that determine the mutual positions of the sublattices (for details see Refs.11 and 12). In our calculations presented below we used the macrospin approach, assuming that the distributions of both static and dynamic magnetization inside each nanopillar are uniform. This restriction is, however, not very severe, since the structure of eigenvalue problems for the collective SWs in an array under the macrospin approach and in more general case (nonuniform static

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14. ABSTRACT
It is demonstrated that collective spin waves (SWs) propagating in complex periodic arrays of dipolarly coupled magnetic nanopillars existing in a saturated (single-domain) ground state in a zero bias magnetic field could be nonreciprocal. To guarantee the SW nonreciprocity two conditions should be fulfilled:(i) existence of a nonzero out-of-plane component of the pillars? static magnetization and (ii) a complex periodicity of array?s ground state with at least two elements per a primitive cell, if the elements are different, and at least three elements per a primitive cell, if the elements are identical. The obtained results show that coupled arrays of magnetic nanopillars with out-of-plane shape or/and crystallographic anisotropy could be used for the development of miniature unbiased microwave isolators and circulators.

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or/and dynamic magnetization inside a dot) are quite similar[11,12].

The general theory of collective SW excitations in arrays of magnetic nanodots was developed in Refs. 11 and 12. Under the macrospin approximation the magnetic state of each element is fully described by one magnetization vector $\boldsymbol{M}_j$, which can be represented as a sum of static magnetization $\boldsymbol{\mu}_j$ and linear dynamic magnetization (or excitation) $\boldsymbol{m}_j$: $\boldsymbol{M}_j = M_s(\boldsymbol{\mu}_j + \boldsymbol{m}_j)$, where M_s is the saturation magnetization of the elements' magnetic material. The eigenfrequencies $\omega_{\boldsymbol{k}}$ and eigenvectors $\boldsymbol{m}_{\boldsymbol{k},p}$ (that describe the structure of the dynamic magnetization in each element) of collective SWs in an array are the solutions of the following eigenvalue problem:

$$-i\omega_{\boldsymbol{k}}\boldsymbol{m}_{\boldsymbol{k},p} = \boldsymbol{\mu}_p \times \sum_q \hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq} \cdot \boldsymbol{m}_{\boldsymbol{k},q} \,, \tag{1}$$

where the Hamiltonian tensor $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$ is given by

$$\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq} = \gamma B_p \delta_{p,q} \hat{\boldsymbol{I}} + \omega_M \hat{\boldsymbol{G}}_{\boldsymbol{k}}(\boldsymbol{\delta}_{pq}) \,. \tag{2}$$

Here γ is the gyromagnetic ratio, B_p is the static internal field in p-th sublattice, $\omega_M = \gamma\mu_0 M_s$, $\boldsymbol{\delta}_{pq} = \boldsymbol{\delta}_p - \boldsymbol{\delta}_q$ and

$$\hat{\boldsymbol{G}}_{\boldsymbol{k}}(\boldsymbol{\delta}) = \sum_{\boldsymbol{r}\in\mathcal{L}} \hat{\boldsymbol{N}}(\boldsymbol{r}+\boldsymbol{\delta}) e^{-i\boldsymbol{k}\cdot(\boldsymbol{r}+\boldsymbol{\delta})} \,, \tag{3}$$

where $\hat{\boldsymbol{N}}(\boldsymbol{r})$ is the mutual demagnetization tensor of magnetic nano-elements[13].

All the properties of the collective SWs in a nanopillar array are determined by the Hamiltonian matrix $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k}}$ and the general rules of magnetization dynamics, resulting in the structure of the eigenvalue problem in the form of Eq. (1). Thus, the conditions for the nonreciprocity of collective SWs in a nanopillar array can be obtained from the analysis of the form of Eqs. (1) and (2) even without actually solving the eigenvalue problem itself.

We note that solution of Eq. (1) yields *complex* amplitudes of the collective SW modes $\boldsymbol{m}_{\boldsymbol{k},p}$. The *real* expressions for the SW modes are obtained from the sum $\boldsymbol{m}_{j_p} = \boldsymbol{m}_{\boldsymbol{k},p} \exp\left[i(\boldsymbol{k}\cdot\boldsymbol{r}_{j_p} - \omega_k t)\right] + \text{c.c.}$, where c.c. denotes the complex conjugate terms. The vector $\boldsymbol{m}^*_{\boldsymbol{k},p}$ is a solution of the eigenvalue problem that is complex-conjugate to Eq. (1). At the same time, it follows from Eqs. (2)-(3) that the complex vector $\boldsymbol{m}^*_{\boldsymbol{k},p}$ is also an eigenvector of the eigenvalue problem corresponding to the opposite sign of SW wave vector $-\boldsymbol{k}$, since $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k}} = \hat{\boldsymbol{\Omega}}^*_{-\boldsymbol{k}}$, and the tensor $\hat{\boldsymbol{N}}(\boldsymbol{r})$ is real[11,13]. Thus, the eigenvalue problem Eq. (1) describes SWs propagating in both "positive" (respective to $\boldsymbol{k}$) direction (solutions $\omega_{\boldsymbol{k}} > 0$) and "negative" one ($\omega_{\boldsymbol{k}} < 0$). Therefore, there are two ways to demonstrate nonreciprocity of the collective SW modes described by the eigenvalue problem Eq. (1): (i) either to show that positive eigenvalues of the problem Eq. (1) and of its complex-conjugate are not equivalent, or (ii) to show that the absolute values of the positive and negative eigenvalues of the problem Eq. (1) are different.

A test for the nonreciprocity of collective SW in a dipolarly coupled nanopillar array can be started from the analysis of the general symmetric properties of the characteristic matrix $\left(\boldsymbol{\mu}_p \times \hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}\right)$ of the eigenvalue problem Eq. (1). The Hamiltonian matrix $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$ consists of the static-static (describing the influence of the static magnetization of the q-th sublattice on the static magnetization of the p-th sublattice), static-dynamic and dynamic-static, and dynamic-dynamic components. Only the dynamic-dynamic components of the Hamiltonian matrix $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$ are directly present in the characteristic matrix $\left(\boldsymbol{\mu}_p \times \hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}\right)$, which is effectively $2P \times 2P$-dimensional. If all the off-diagonal dynamic-dynamic components of the tensors $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$ vanish, the characteristic matrix contains $2P^2$ components that are identically equal to zero: $\left(\boldsymbol{\mu}_p \times \hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}\right)_{j,j+2n} = 0$, $n \in \mathbb{Z}$. As it follows from the definition of a matrix determinant[14], in this case the characteristic equation of the eigenvalue problem Eq. (1) is bipolynomial with respect to the SW eigenfrequency $\omega_{\boldsymbol{k}}$, and the solution of this characteristic equation consists of P pairs of eigenvalues $\omega_{\boldsymbol{k},\nu}$ and $(-\omega_{\boldsymbol{k},\nu})$, which means that the collective SWs in the array are reciprocal.

Such a case is realized in an array of flat dots (dots with constant height) with the same for all dots height if the directions of static magnetization of the dots lie in the array's plane. Indeed, the dynamical components of the matrix $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$ are proportional to the dynamical components of the mutual demagnetization tensor $\hat{\boldsymbol{N}}(\boldsymbol{r})$ (see Eqs. (2,3)), and the αz-components of the tensor $\hat{\boldsymbol{N}}(\boldsymbol{r})$ vanish in the case when all the dots have the same height[11,13]: $N^{\alpha z}(\boldsymbol{r}) = 0$, $\alpha = x, y$, where z-axis is perpendicular to the plane of the array's, as it is shown in Fig. 1(a). Thus, for an in-plane magnetized dot the cross-interaction between the dynamical m_z component of the dot magnetization and all the in-plane magnetization components vanishes identically. This means that in any array of flat in-plane magnetized dots the off-diagonal dynamic-dynamic components of the Hamiltonian matrix $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$ also identically vanish, independently of the lattice structure and the ground state of the array. Therefore, the collective SWs in an array of in-plane magnetized flat magnetic dots are always reciprocal, and, if a *non-reciprocity* is needed, there should exist a non-zero *out-of-plane component of the dot magnetization* $\mu_{p,z} \neq 0$. This can be achieved, for example, by using vertically elongated magnetic pillars.

Another necessary condition of the SW nonreciprocity follows from the general symmetry property of the Hamiltonian matrix: $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq} = \hat{\boldsymbol{\Omega}}^*_{-\boldsymbol{k},pq}$. If the matrix $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$ is real, the eigenvalue problem Eq. (1) does not change with the reversal of the SW wave vector $\boldsymbol{k}$, meaning that the collective SWs are reciprocal. Whether the Hamiltonian matrix $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$ is real or complex depends only on the structure of the array's lattice (see Eq. (3)). In particular, if all the dots in an array are identical (in shape and material parameters) *and* the dots are arranged in a simple lattice (having only one dot per a primitive cell), the Hamiltonian matrix is real, because there is only one sublattice with $\boldsymbol{\delta}_{pp} \equiv 0$ and, as it follows from Eq. (3), the tensor $\hat{\boldsymbol{G}}_{\boldsymbol{k}}(\boldsymbol{0})$ is real. At the same time, it follows from Ref. 11 that tensors $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$ for $p \neq q$ *can be complex* if a primitive cell of an array contains $P > 1$ elements. Therefore, collective SWs in a dipolarly coupled array of elements *having out-of-plane magnetization* could be *nonreciprocal* only if the *primitive cell of the array contains several magnetic elements*.

The simplest example of an array existing in such a complex spatially-periodic ground state is an array having $P = 2$ dots in a primitive cell. This case can be easily analyzed analytically. If an array exists in a ferromagnetic (FM) ground state, i.e. the static magnetizations of all the elements are the same $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_2$, one can easily derive an explicit characteristic equation of the eigenvalue problem Eq. (1):

$$\omega_{\boldsymbol{k}}^4 + C_2\omega_{\boldsymbol{k}}^2 + C_1\omega_{\boldsymbol{k}} + C_0 = 0 \ . \qquad (4)$$

If the coefficient $C_1 = 0$ vanishes, the characteristic equation of the problem has two pairs of solutions $[\omega_{\boldsymbol{k},\nu}, -\omega_{\boldsymbol{k},\nu}]$, which, as it has been explained above, means that collective SWs are reciprocal. Thus, the nonreciprocity appears only when the coefficient $C_1 \neq 0$. The magnitude of this coefficient can be evaluated as:

$$C_1 = \left(\Omega_{11}^{(xx)} - \Omega_{22}^{(xx)}\right)\left(\Omega_{12}^{*(xy)}\Omega_{12}^{(yy)} - \text{c.c.}\right) + \dots \ , \qquad (5)$$

where the xx-term is shown explicitly and two other terms can be obtained by a cyclic permutation of the indices $(xx) \to (xy) \to (yy)$. The direction of the z axis is chosen to be parallel to the direction of the elements' static magnetization, $\boldsymbol{\mu}_p = \boldsymbol{e}_z$.

As it is evident from Eq. (5), the necessary condition for the SW nonreciprocity in the considered case is the inequality between the diagonal elements of the Hamiltonian matrix $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},11} \neq \hat{\boldsymbol{\Omega}}_{\boldsymbol{k},22}$. This can be achieved if the elements (pillars), which belong to different sublattices, differ from one another by their geometric or/and material parameters, i.e. if there are two groups of different elements in the array.

To illustrate the case of an array with two sublattices we calculated SW spectra in an array of circular nanopillars arranged in a rectangular lattice (see Fig. 1(a)), assuming that one group of pillars has additional out-of-plane anisotropy B_{an}. Qualitatively similar result can be obtained for an array in the FM state, when the pillars differ by their radius.

As expected, SWs in such an array are nonreciprocal – the waves with opposite wave vectors have different eigenfrequencies (Fig. 1(b)). While the difference in the SW frequencies (nonreciprocal frequency splitting) is relatively small, the difference in group velocities of the oppositely directed SWs is substantially larger (Fig. 1(c)). This difference can be easily detected experimentally and, possibly, used in applications.

The effect of nonreciprocity exists for all the directions of the SW wave vector $\boldsymbol{k}$, except $\boldsymbol{k} = k_x\boldsymbol{e}_x$ (for this particular geometry). Note, also, that in all the symmetric points of the first Brillouin zone (e.g. $\boldsymbol{k} = \boldsymbol{0}$, $\boldsymbol{k} = \pi\boldsymbol{e}_y/a_y$) the SW eigenfrequency does not depend on the sign of the wave vector. This property is general and is related to the periodicity of the SW dispersion relation[11]: $\omega_{\boldsymbol{k}} = \omega_{\boldsymbol{k}+\boldsymbol{K}}$ in respect to all the characteristic vectors $\boldsymbol{K}$ of the reciprocal lattice (Bravais lattice).

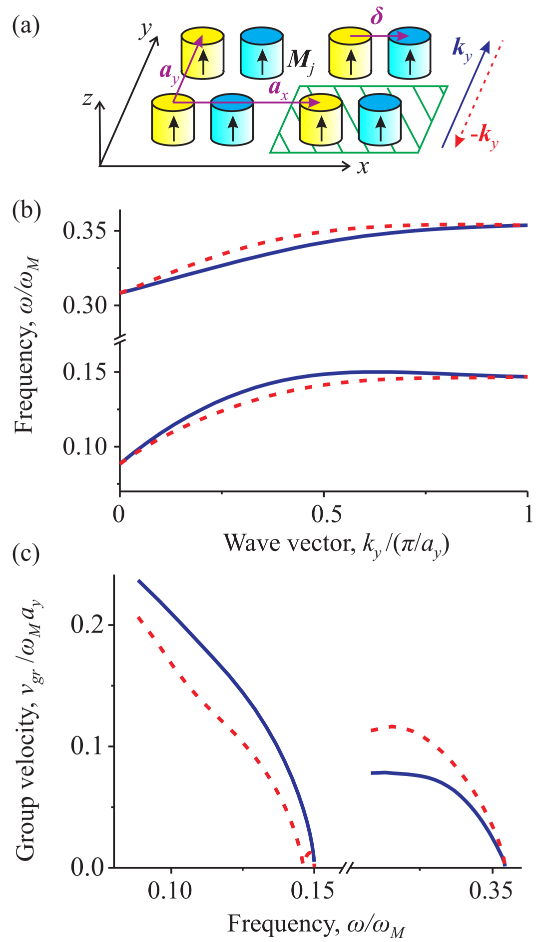

FIG. 1. (Color online) (a) Structure of an array comprised of 2 different types of pillars arranged in a complex lattice and existing in a perpendicular FM ground state. Yellow dots are isotropic, while blue dots have an additional uniaxial anisotropy $B_{an} = 0.2\mu_0 M_s$ in the z-direction. Green dashing shows a primitive cell of the lattice; (b) Spectra of collective SWs propagating along the k_y direction at a zero external field. Solid blue lines correspond to the "positive" propagation direction, dashed red lines – to the opposite ("negative") propagation direction; (c) Absolute values of the SW group velocity: blue solid lines - $v_{gr} > 0$, red dashed lines - $v_{gr} < 0$. Parameters, used for calculations: height/radius aspect ratio of the pillar $h/R = 5$, lattice constants $a_x = 10R$, $a_y = 3.3R$, shift between the sublattices $\delta = 3.3R$.

The situation becomes more interesting when the ground state of an array is not ferromagnetic. Our calculations show that in this case the condition $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},11} \neq \hat{\boldsymbol{\Omega}}_{\boldsymbol{k},22}$ is not necessary anymore for the appearance of the SW nonreciprocity. This, in particular, means that the nonreciprocal branches of SWs can appear in an array of

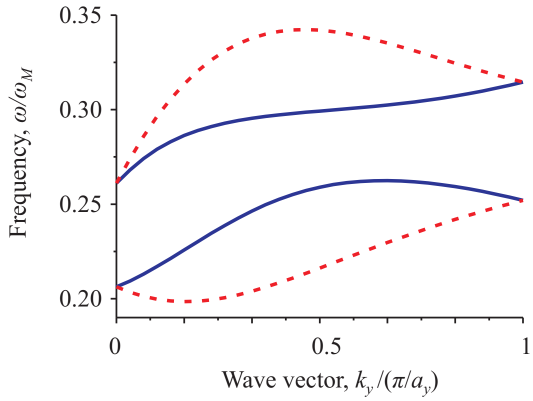

FIG. 2. (Color online) SW spectra of an array comprising identical nanopillars in the SAFM ground state. The structure of the array is shown in Fig. 1(a), where all the pillars are identical, but the blue and yellow pillars have the opposite directions of the static magnetization (perpendicular to the array's plane). Calculation parameters, except for the additional anisotropy, are the same as in Fig. 1.

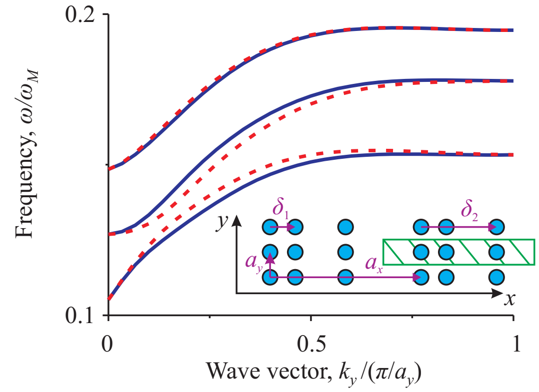

FIG. 3. (Color online) SW spectra of an array of identical pillars, arranged into a complex lattice with $P = 3$ pillars per a primitive cell in a FM ground state ($\boldsymbol{k}_y$ direction). Blue solid and red dashed lines correspond to the "positive" and "negative" SW propagation directions, respectively. The sketch of an array is shown in the inset; a primitive cell is dashed. The static magnetizations of the pillars are perpendicular to the array's plane. Calculations' parameters: $h/R = 5$, $a_x = 20R$, $a_y = 3.3R$, $\delta_1 = 3.3R$, $\delta_2 = 10R$.

identical nanopillars. For example, Fig. 2 shows the SW spectra of an array of identical pillars existing in a stripe-like antiferromagnetic (SAFM) ground state.

Comparing Figs. 1(b) and 2 one can see that the nonreciprocal frequency splitting in the SAFM state is significantly larger than in the FM state, while the geometrical parameters and, therefore, the strength of the magneto-dipolar interaction are the same.

Our calculations performed using a well-known stationary perturbation theory[15] show that nonreciprocal frequency splitting ($\omega_{\boldsymbol{k}} - \omega_{-\boldsymbol{k}}$) is proportional to the difference in polarization ellipticity of the SW eigenmodes $\boldsymbol{m}_p$ corresponding to two different sublattices. Thus, the nonreciprocal frequency splitting has a maximum magnitude when the eigenmodes of different sublattices are orthogonal, e.g. in the case when one of the modes has a clockwise and the other one – counterclockwise circular polarizations, which are realized in an antiferromagnetic out-of-plane ground state. A similar enhancement of the nonreciprocal frequency splitting can be achieved in the FM ground state by using two different types of pillars having in-plane shape or material anisotropy with different directions of anisotropy axes.

It should be noted, that the nanopillar arrays shown in Figs. 1 and 2 have a *complex lattice structure*, i.e. if one neglects the difference in geometric and material parameters of the pillars and considers only the positions of the pillar's centers, the resulting geometric lattice will still be non-simple – with 2 dots per a primitive cell. The lattice becomes simple only when $\delta = a_x/2$. As it is known from[11], in an array having a *simple* lattice structure, the tensor $\hat{\boldsymbol{G}}_{\boldsymbol{k}}$ can be represented as:

$$\hat{\boldsymbol{G}}_{\boldsymbol{k}}(\boldsymbol{\delta}) = \frac{1}{P}\sum_p \hat{\boldsymbol{F}}_{\boldsymbol{k}+\boldsymbol{\kappa}_p} e^{i\boldsymbol{\kappa}_p\cdot\boldsymbol{\delta}} \,, \tag{6}$$

where $\hat{\boldsymbol{F}}_{\boldsymbol{k}}$ is a real tensor, and the values of $e^{i\boldsymbol{\kappa}_p\cdot\boldsymbol{\delta}}$ belong to a set of P-th order roots of 1. Therefore, the collective SWs in any two-dimensional array with $P = 2$ types of different dots arranged into a *simple* lattice are *always reciprocal*, since the coefficients of the inter-dot interaction in such an array are real. Therefore, the collective SWs in arrays with a simple lattice structure could become nonreciprocal only in the array comprises of at least 3 distinct sublattices (different in geometry, material parameters, or static magnetization).

Further analysis of the eigenvalue problem Eq. (1) shows that the conditions $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pp} \neq \hat{\boldsymbol{\Omega}}_{\boldsymbol{k},qq}$ and $\boldsymbol{\mu}_p \neq \boldsymbol{\mu}_q$ for $p \neq q$ are not necessary of the SW nonreciprocity in the $P \geq 3$ case. Thus, the collective SWs in this case can be nonreciprocal even in an array consisting of *identical* nanopillars in a FM ground state. An example of such an array and corresponding nonreciprocal SW spectra are shown in Fig. 3.

In conclusion, we demonstrated that collective SWs in an array of dipolarly coupled magnetic nanopillars can be nonreciprocal. The main requirements for the SW nonreciprocity are: (i) the presence of a non-zero out-of-plane component of the elements' static magnetization and (ii) the existence of a non-zero imaginary part of the Hamiltonian matrix $\hat{\boldsymbol{\Omega}}_{\boldsymbol{k},pq}$. The last condition is satisfied in arrays existing in a complex periodic ground state with at least $P = 2$ pillars per a primitive cell, if the pillars belonging to different arrays' sublattices are different, or at least $P = 3$ pillars per a primitive cell, if all the pillars in the array are identical. We believe that arrays of coupled magnetic nanopillars having a complex lattice structure could become materials of choice for the development of miniature unbiased microwave isolators and circulators requiring the nonreciprocity of the waves propagating in opposite directions.

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