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CMS-Wave Model: Part 5. Full-plane Wave Transformation and Grid Nesting

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PURPOSE: This Coastal and Hydraulics Engineering Technical Note (CHETN) describes the full-plane (FP) wave transformation and grid nesting capability in the Coastal Modeling System's wave model (CMS-Wave). The FP wave transformation of CMS-Wave is a new feature that has been added to the existing half-plane (HP) version (Lin et al. 2008, 2011). Grid nesting in the FP is an extension of the grid nesting method that existed in the HP version (Lin et al. 2010). The theoretical background and user's manual for CMS-Wave are available in previous reports and CHETNs (Lin et al. 2006; Demirbilek et al. 2007). CMS-Wave is part of the Coastal Modeling System (CMS) developed under the Coastal Inlets Research Program (CIRP) for simulating combined waves, currents, sediment transport, and morphology change at coastal inlets, estuaries, and river mouths (Demirbilek and Rosati 2011).

BACKGROUND: As affected by underlying bathymetric features, currents, and fluctuating water levels, wave characteristics nearshore are usually varying on the scale of tens of meters whereas offshore waves are generally more homogenous at the scale of kilometers. The CMS-Wave model was developed to simulate nearshore wave propagation of two-dimensional spectral waves to inlets and navigation channels that are often protected by jetties and breakwaters. CMS-Wave is a finite-difference, phase-averaged spectral wave model based on the wave action balance equation (Mase 2001). It employs a forward-marching, finite-difference method to solve the wave action conservation equation for wave transformation including wave shoaling, refraction, diffraction, reflection, transmission over structures, depth-limited breaking, dissipation, wind-wave generation, and wave-wave and wave-current-structure interactions (Lin et al. 2008, 2011). A number of wave breaking formulas included in CMS-Wave have been evaluated in laboratory and field applications (Zheng et al. 2008).

CMS-wave can be applied in FP or HP mode. CMS-Wave uses a rectangular grid that permits a variable cell spacing to save computer time. Most coastal applications are sufficient to run CMS-Wave in the HP mode that wave energy propagates only from the offshore toward the nearshore in a ± 90 deg HP sector with respect to the *x*-axis of the grid, usually shore-normal. In the HP mode, waves traveling in the negative *x*-direction (e.g. waves reflected from the structures, shoreline, bottom features, and waves generated by offshore-blowing winds) are neglected. In contrast, waves from all directions are calculated in the FP mode (e.g., waves are generated, transformed and propagated on a full 360-deg sector). While the primary waves propagate from the seaward boundary toward shore, the FP mode combines model results from running two halfplane wave transformations, with each covering the wave direction exactly opposite to the other half-plane wave transformation. It must be emphasized that the FP mode is a new feature added to the CMS-Wave and is not intended to replace the HP mode. Comparatively, the HP mode runs faster with less memory requirements than the FP mode, and is more than adequate for most nearshore coastal applications. The only exceptions are isolated surface-piercing structures and

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Standard Form 298 (Rev. 8-98) Prescribed by ANSI Std Z39-18 islands, semi-enclosed bays and lakes, where incident waves can come from different directions and local wind generation effects are important and winds can blow from any direction. In such applications, there is no distinct incident wave boundary of the modeling domain. For these types of applications, the FP mode should be used.

CMS-Wave can be coupled to CMS-Flow (Demirbilek and Rosati 2011), a hydrodynamic and sediment transport model, to calculate the sediment transport and morphology change forced by combined action of tides, currents and waves. There is a stand-alone version of CMS-Wave for waves-only types of applications, and also an inline version that combines flow and wave models in one code, which can be used for general hydrodynamic, sediment transport and morphodynamic studies requiring wave forcing.

Multiple nested grids can include many large and small grids. The most commonly applied grid nesting involves two model grids: a large grid (parent grid) and a small grid (child grid). The application of grid nesting can reduce the computational time as compared to a large grid with fine resolution for the entire model domain. A parent grid (PG) with coarser resolution may be used to simulate the regional processes such as wave generation and propagation in a large domain. A child grid (CG) with finer resolution can resolve more complex bathymetry and shoreline geometry in a smaller area. Wave spectra calculated from a large grid with coarser resolution are saved at selected locations along the offshore boundary of a smaller fine resolution grid. Conventionally, it is efficient to save a single-location spectrum from PG for wave input and apply it to the entire sea boundary of a CG. For a more inhomogeneous wave field, multiple-locations wave spectra may be saved from PG and interpolated for more realistic wave forcing along the seaward boundary of CG. The main goal of grid nesting is to minimize the computational time while maintain the wave modeling accuracy (Lin et al. 2010).

FP MODE: CMS-Wave has been fully operational within the U.S. Army Corps of Engineers' (USACE) Surface-water Modeling System (SMS). The CMS-Wave FP option is available in SMS Version 11.1 and higher. The default in CMS-Wave and SMS is the HP mode. The FP mode uses the same grid and model control settings as the HP mode and allows opposing winds generating waves moving in opposite directions. Users can specify additional wave input forcing on the opposite side of the grid. These types of situations would occur when calculating waves around an island or islands.

The HP and FP mode of CMS-Wave is controlled by the model parameter IVIEW, where IVIEW = 0 corresponds to the HP mode and IVIEW = 1, 2 correspond to the FP. For IVIEW=1, the wave input is given only on the seaward (primary) boundary and wind wave generation is permitted in any direction. For IVIEW = 2, CMS-Wave can read a second wave input file at the shoreward boundary (secondary boundary). For users who are familiar with CMS-Wave input files, the IVIEW parameter is listed in the *.std or std.dat file (in SMS Versions 10.1 and 11.0 or higher), and the second wave input file (i.e., *.spc) is specified under the SPEC2 card in the *.sim file or specified by a default file name wave.spc. Appendix A lists the CMS-Wave Version 3.2 model control parameters for both HP and FP modes.

For FP mode with IVIEW = 2, users can prepare a second wave input file (*.spc or wave.spc) in SMS with two steps: (1) switch the origin of the principle axes (x, y) to the opposite corner (imaginary origin) of the grid. This can be achieved by first right-clicking on the CMS-Wave

Grid symbol in the Data-tree (also referred to as 'Project Explorer') box, and selecting *Rotation* of 180 deg under *Create Transformed Grid*, and then clicking the new *Transformed* grid in the Data-Tree box, and (2) generate the second wave input file under the regular *Spectral Energy* routine. Users can run the CMS-Wave FP mode with IVIEW = 1 or 2 in DOS or SMS, the same way as in HP mode.

Figure 1 shows an idealized barrier island example for running CMS-Wave in FP mode with IVIEW = 2 and using the same wave input spectrum on the seaward and bayside boundaries. The primary origin is at the lower-left corner of the seaward boundary (local coordinates in x and y axes) and the imaginary origin is at the upper-right corner of the bayside boundary (local coordinates in x' and y' axes). If variable spectra are specified as input to the seaward boundary (i.e., IBND = 1 or 2 in the *.std or std.dat) in FP mode and IVIEW=2, it is also required to providing variable spectra at the bayside boundary is also required.



Figure 1. Barrier island FP settings and calculated wave field.

CMS-Wave can be run in a stand-alone model or in coupled mode with CMS-Flow. In the coupled mode, the flow and wave models are run separately in sequential order in that CMS-Flow provides the calculated water level, flow field, and bathymetry as input data to CMS-Wave, and CMS-Wave supplies calculated wave field and radiation stress components to CMS-Flow. The coupled flow-wave run can be carried out by the inline code or using the SMS *Steering Module* under the *Data* menu. When using the SMS *Steering Module* with variable spectra for the wave input boundary (IBND = 1 or 2), users are required to make a copy of the *.eng file to nest.dat, regardless of running CMS-Wave in the HP or FP mode.

FP GRID NESTING: The CMS-Wave FP grid nesting is similar to the HP as described in ERDC-CHETN-IV-76 (Lin et al. 2010). Users can run the parent or child grid separately or run both grids within SMS. Running the CMS-Wave PG in the FP non-steering mode will generate both *.nst and wav.spc for the CG boundary condition. If PG is run in the SMS *Steering module*, a "nst.dat" file in addition to *.nst and wav.spc will be generated to save calculated spectra at nesting output locations for the CG boundary condition. The parent and child grids may have different orientations, and the difference is recommended to be less than 30 deg. With closer grid orientations, a comparable amount of wave energy will be passed from the parent to the child grid.

When running CMS-Wave with CG in FP mode, both the nst.dat (or *.nst if nst.dat does not exit) and wav.spc files, saved from the PG simulation, need to be renamed to nest.dat and wave.spc, respectively, to create input files for two wave boundaries of CG. A copy of the wave.spc should also be saved as *.eng file listed in *.sim. Because nest.dat and wave.spc files include variable spectra for input to two wave boundaries of CG, it is necessary to specify IBND = 2 and IVIEW = 2 in the *.std or std.dat files for CG (required manually for SMS10.1 and SMS11.0 because the IVIEW option is available in SMS11.1 or higher). It is noted that std.dat is a temporary copy of the *.std file and can be modified to overwrite the control parameters in the *.std for the SMS steering of CMS-Wave with CMS-Flow. Figure 2 shows the idealized barrier island example for the calculated FP wave field for a CG inside a PG. The wave input to the CG is the spectral data saved from the PG at eight nesting output locations (red squares in Figure 2). A sample nesting output spectrum saved from the PG is displayed in Figure 2. Appendix B summarizes IVIEW and IBND specifications associated with the nst.dat, nest.dat, wav.spc, and wave.spc files.



Figure 2. Barrier island child grid calculated wave field.

EXAMPLE APPLICATION – MISSISSIPPI COASTAL IMPROVEMENT PROGRAM: A CMS-Wave FP application with grid nesting capability is illustrated in this section.

BACKGROUND: The Mississippi Coastal Improvements Program (MsCIP), a congress directed federal project, has maintained two nearshore directional wave gauges (USACE Gulf Gauge and Sound Gauge) at Ship Island, MS, as part of a barrier island restoration project. The details of the data collection and analysis are described elsewhere (Wamsley et al. 2011). The offshore wave data are available from NDBC directional Buoy 42040 (165-m depth), located 90 km offshore Dauphin Island, AL.

The CMS-Wave PG extended from the 15-m depth contour to the shoreline (41 km cross-shore and 93 km alongshore). The rectangular domain included variable cell spacing of 5 to 200 m in the cross-shore direction and 70 to 200 m in the alongshore direction. Figure 3 shows the CMS-Wave parent and child grids with higher resolution in the vicinity of the wave gauges (black dots in Figure 3) and East and West Ship Island. Bathymetric data were based on surveys taken by the USGS, USACE and NOAA representing the most recent conditions. The CG domain is a rectangular area inside the PG that covers Ship Island and surrounds with all water boundaries. Ship Island is actually a collective name for two barrier islands: East Ship Island and West Ship Island. Hurricane Camille split the once single island into two separate islands in 1969. East Ship Island has experienced increasing erosion in recent years under severe storms and several powerful Gulf hurricanes (Katrina, Rita and Wilma in 2005; Gustav and Ike in 2008). Ship Island is a part of a barrier island chain in the northern Gulf of Mexico and is valuable not only for storm protection but also for refuge of wildlife and the surrounding marine environment. Restoring Ship Island is a major component of Barrier Island Restoration Plan in the MsCIP.



Figure 3. Mississippi Sound regional bathymetry grid and a child nesting grid for Ship Island (black circles are local wave gauges; red circles are the nesting output locations).

CMS-Wave was set to run in standalone FP mode. Wind input was available from NOAA Coastal Station 8744707 (Gulfport Outer Range). For PG, incident wave spectra at the model seaward boundary were transformed from NDBC Buoy 42040 using a simple wave transformation by Snell's Law and shore parallel depth contour assumption.

Figure 4 shows the calculated wave field for PG in FP mode under a steady northerly wind of 24 kt, during a cold front, at 18:00 GMT on 24 April 2010. The incident wave condition is a stationary spectrum of 0.36 m (significant height) and 5 sec (spectral peak) from south on the seaward boundary (IVIEW = 1 and IBND = 0). The input spectrum was discretized in a total of 30 frequency bins and 35 directional bins. The calculated waves show a general south-bound direction as result of wave generation under a strong wind condition.



Figure 4. Calculated wave field for 18:00 GMT on April 24, 2010 under a steady northerly wind of 24 kt (black circles are local wave gauges; red circles are nesting output locations).

The CG domain size is 12 km x 19 km with a constant cell spacing of 50 m. The incident waves for CG are based on two nesting output files, *.nst and wav.spc, saved from the PG run. Figure 5 shows the CG bathymetry domain and a sample incident wave spectrum at the west-end location of the CG southern water boundary. Figure 6 shows the calculated wave field of 18:00 GMT 24 April 2010 for CG in FP mode (IVIEW = 2 and IBND = 2) using the incident waves (given in *.nst and wave.spc) supplied by the PG simulation. As generated by strong wind, large waves enter the CG through the northern boundary, refract upon local depth changes, and diffract around Ship Island. In the downwind side of Ship Island, the calculated wave field shows a mix of diffracted waves from north (Mississippi Sound) with waves shoaling and refraction from the south (Gulf). This Ship Island application example demonstrates the CMS-Wave FP and grid nesting capability to assist the numerical modeling in MsCIP and other coastal barrier island restoration projects.



Figure 5. Ship Island bathymetry grid (black circles denote local wave gauges).



Figure 6 Ship Island local grid calculated wave field for 18:00 GMT April 24, 2010 under a steady northerly wind of 24 kt (black circles mark local wave gauges).

CONCLUSIONS: This CHETN describes the FP mode and grid nesting capability of the CMS-Wave model for coastal and nearshore wave transformation applications, available within the SMS version 11.1 and higher. Grid nesting is not necessary in all applications, but is useful in applications with large grids to efficiently model regional processes and small grids to resolve details of complex bathymetry and shoreline geometry near the coast. The grid nesting is useful in modeling alternatives or incorporating structures in the small domains without re-running the large domain regional grid. The FP operation and its grid nesting capability are demonstrated in this CHETN for the Mississippi Coastal Improvements Program study. Additional information

about the CMS FP nested grids for MsCIP and other coastal island restoration study will be published in a technical report.

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Lin, L., J. Rosati, and Z. Demirbilek. 2012. CMS-Wave Model Part 5: Full-plane Wave Transformation and Grid Nesting. Coastal and Hydraulics Engineering Technical Note ERDC/CHL CHETN-IV-81, Vicksburg, MS: U.S. Army Engineer Research and Development Center. An electronic copy of this CHETN is available from: <u>http://chl.erdc.usace.army.mil/chetn</u>.

REFERENCES

- Demirbilek, Z. and J. D. Rosati. 2011. Verification and Validation of the Coastal Modeling System: Report I, Executive Summary. *Tech. Report ERDC/CHL-TR-11-xx*, U.S. Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory, Vicksburg, MS.
- Demirbilek, Z., L. Lin, and A. Zundel. 2007. WABED model in the SMS: Part 2. Graphical interface. ERDC/CHL CHETN-I-74. Vicksburg, MS: U.S. Army Engineer Research and Development Center.
- Lin, L., Z. Demirbilek, and H. Mase. 2011. Recent capabilities of CMS-Wave: A coastal wave model for inlets and navigation projects. *Journal of Coastal Research*, Special Issue 59, 7-14.
- Lin. L., I. Watts, and Z. Demirbilek. 2010. CMS-Wave Model: Part 3. Grid nesting and application example for Rhode Island south shore regional sediment management study. ERDC/CHL CHETN-IV-76. Vicksburg, MS: U.S. Army Engineer Research and Development Center.
- Lin, L., H. Mase, F. Yamada, and Z. Demirbilek. 2006. Wave-Action Balance Equation Diffraction (WABED) model: tests of wave diffraction and reflection at inlets. ERDC/CHL CHETN-III-73. Vicksburg, MS: U.S. Army Engineer Research and Development Center.
- Lin, L., Z. Demirbilek, H. Mase, J. Zheng, and F. Yamada. 2008. CMS-Wave: a nearshore spectral wave processes model for coastal inlets and navigation projects. Coastal Inlets Research Program, Coastal and Hydraulics Laboratory Technical Report ERDC/CHL TR-08-13. Vicksburg, MS: U.S. Army Engineer Research and Development Center.
- Mase, H. 2001. Multidirectional random wave transformation model based on energy balance equation. *Coastal Engineering Journal* 43(4):317-337 JSCE.
- Zheng, J., H. Mase, Z. Demirbilek, and L. Lin. 2008. Implementation and evaluation of alternative wave breaking formulas in a coastal spectral wave model. *Ocean Engineering* 35:1090-1101.
- Wamsley, T., B. W. Bunch, R. S. Chapman, M. B. Gravens, A. S. Grzegorzewski, B. D. Johnson, R. L. Permenter, and M. W. Tubman. 2011. Mississippi Coastal Improvement Program, Barrier Island Restoration Numerical Modeling. In progress. Technical Report, Vicksburg, MS: U.S. Army Engineer Research and Development Center.

Appendix A: List of CMS-Wave Version 3.2 model control parameters

The CMS-Wave V3.2 has the full-plane (FP) capability. The model control file (*.std or std.dat) can have a maximum of 25 control parameters. The first 15 parameters are the same as described in the CMS-Wave Technical Report (Lin et al. 2008) and the other 10 parameters are new.

1st 2nd 3rd 4th 5th 6th 7th 8th 9th 10th 11th 12th 13th 14th 15th kout ibnd iwet ibf iarkr akap bf iprp icur ibk irs iark ark arkr iwvbk 18th 22nd 16th 17th 19th 20th 21st 23rd 24th 25th nonln igrav irunup imud iwnd isolv ixmdf iproc iview iroll

At least the first 6 parameters above are required in the *.std (or std.dat) and the remaining parameters starting any parameter after the 6^{th} will be assigned to default values if not specified in *.std (or std.dat). The description of the 1st to 25^{th} parameters is given below.

iprp = 0 (wave propagation with wind input in *.eng)

1 (wave propagation only, neglect wind input in *.eng)

- -1 (fast mode)
- 2 (forced grid internal rotation)
- 3 (without lateral energy flux)

icur = 0 (no current input)

1 (with current input *.cur)
2 (with *.cur, use only the 1st set current data)

ibk = 0 (no wave break info output)

1 (output breaking indices *.brk) 2 (output energy dissipation rate *.brk)

irs = 0 (no wave radiation stress calc)

1 (output radiation stress *.rad) 2 (calculate/output setup/max-water-level + *.rad)

kout = number of special wave output location, output spectrum in *.obs

and parameters in selhts.out

ibnd = 0 (no input a parent spectrum *.nst)

(read *.nst, averaging input spectrum)
 (read *.nst, spatially variable spectrum input)

iwet = 0 (allow wet/dry, default)

1 (without wet/dry)

-1 (allow wet/dry, output swell and local sea files)

-2 (output combined steering wav files)

-3 (output swell, local sea, and combined wav files)

ibf = 0 (no bottom friction calc)

1 (constant Darcy-Weisbach coef, c_f)

2 (read variable c_f file, *.fric)

3 (constant Mannings *n*)

4 (read variable Mannings *n* file, *.fric)

iark = 0 (without forward reflection) 1 (with forward reflection)

iarkr = 0 (without backward reflection) 1 (with backward reflection)

akap = 0 to 4 (diffraction intensity, 0 for zero diffraction, 4 for strong diffraction, default)

bf = constant bottom friction coef c_f or Mannings *n* (typical value is 0.005 for c_f and 0.025 for Mannings *n*)

ark = 0 to 1 (constant forward reflection coef, global specification, 0 for zero reflection, 1 for fully reflection)

arkr = 0 to 1 (constant backward reflection coef, global specification, 0 for zero reflection, 1 for fully reflection)

iwvbk = 0 to 6 (option for the primary wave breaking formula: 0 for Goda-extended, 1 for Miche-extended, 2 for Battjes and Janssen, 3 for Chawla and Kirby, 4 for Miche, 5 for relaxing breaking, 6 for lifting breaking)

nonln = 0 (none, default) 1 (nonlinear wave-wave interaction)

igrav = 0 (none, default) 1 (infra-gravity wave enter inlets)

irunup = 0 (none) 1 (automatic, runup relative to absolute datum) 2 (automatic, runup relative to updated MWL)

imud = 0 (mud.dat, default; typical max kinematic viscosity is $0.04 \text{ m}^2/\text{sec}$) 1 (none)

iwnd = 0 (wind.dat, default) 1 (none)

isolv = 0 (GSR solver, default) 1 (ADI)

ixmdf = 0 (output ascii, default) 1 (output xmdf) 2 (input & output xmdf)

iproc = 0 (same as 1, default) n (n processors for isolv = 0; recommended n = (total row number)/300)

iview = 0 (half-plane, default)

- 1 (full-plane with one wave input file)
- 2 (full-plane with two wave input files at two wave boundaries)

iroll = 0 to 4 (wave roller effect, 0 for no effect, default, 4 for strong effect; valid in surf zone with grid cell < 10 m)

Appendix B: I/O specification of nst.dat, nest.dat, wav.spc, wave.spc

Table B.1 summarizes input and output specifications associated with the nst.dat, nest.dat, wav.spc, and wave.spc files for FP or HP grid nesting application. In general, nst.dat and nest.dat are required for grid nesting application in the SMS steering of CMS-Wave with CMS-Flow while wav.spc and wave.spc are involved for grid nesting in the CMS-Wave FP application regardless of using CMS-Wave standalone or SMS steering.

Table B.1. Input and output specification for nst.dat, nest.dat, wav.spc and wave.spc												
CMS-Wave Control Parameter		CMS-Wave standalone				CMS-Wave/CMS-Flow steering						
IVIEW	IBND	nst.dat	nest.dat	wav.spc	wave.spc	nst.dat	nest.dat	wav.spc	wave.spc			
0 (HP [*])	0 (PG⁺)					output						
0 (HP)	1,2 (CG⁺)						input					
1,2 (FP [*])	0 (PG)			output		output		output				
1,2 (FP)	1,2 (CG)				input		input		input			

HP and FP denote CMS-Wave half-plane and fall-plane modes, respectively.

⁺ PG and CG denote parent and child grids, respectively.

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