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14. ABSTRACT
A mobile tactical network is characterized by wireless communication nodes operating over a disperse geographical area. As tactical nodes move during an operation, the network may partition into several segregated clusters. Once the network has partitioned, mobile nodes cannot in different clusters cannot maintain connectivity due to insufficient radio transmission range. The partitioned network will have limited capability in providing seamless communication services to sensors and combat systems. To mitigate this problem, a subset of the mobile nodes can be collocated with and connected to a more powerful communication node to form a gateway node. These more powerful nodes have longer radio transmission range and are assumed to be connected with each other to form an upper tier network (e.g., satellite network). To reach its destination mobile node through nodes in the upper tier network, a regular mobile node can first connect to a gateway node. The gateway node can then forward traffic through the connected upper tier network to another gateway node.
In this scenario, communication between mobile nodes in different clusters can only occur when each cluster contains a gateway node. In this paper, we investigated the number of gateway nodes needed in order to maintain certain level, of connectivity in a mobile network. Given the node density of the mobile network, we quantified the relationship between network connectivity and the number of gateway nodes. In a densely populated mobile network, we found that only a small number of gateway nodes are needed to achieve good network connectivity. Moreover, as the node density increases, the percentage of gateway nodes can decrease at a larger rate than the node density increase rate while still achieving a good network connectivity.

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On Heterogeneous Mobile Network Connectivity: Number of Gateway Nodes[1]

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***Abstract*— A mobile tactical network is characterized by wireless communication nodes operating over a disperse geographical area. As tactical nodes move durng an operation, the network may partition into several segregated clusters. Once the network has partitioned, mobile nodes cannot in different clusters can not maintain connectivity due to insufficient radio transmission range. The partitioned network will have limited capability in providing seamless communication services to sensors and combat systems. To mitigate this problem, a subset of the mobile nodes can be collocated with and connected to a more powerful communication node to form a gateway node. These more powerful nodes have longer radio transmission range and are assumed to be connected with each other to form an upper tier network (e.g., satellite network). To reach its destination mobile node through nodes in the upper tier network, a regular mobile node can first connect to a gateway node. The gateway node can then forward traffic through the connected upper tier network to another gateway node.**

In this scenario, communication between mobile nodes in different clusters can only occur when each cluster contains a gateway node. In this paper, we investigated the number of gateway nodes needed in order to maintain certain level of connectivity in a mobile network. Given the node density of the mobile network, we quantified the relationship between network connectivity and the number of gateway nodes. In a densely populated mobile network, we found that only a small number of gateway nodes are needed to achieve good network connectivity. Moreover, as the node density increases, the percentage of gateway nodes can decrease at a larger rate than the node density increase rate while still achieving a good network connectivity.

I. Introduction

Future military tactical communication networks are envisioned to extend network services from the fixed Internet or Global Information Grid (GIG) to the forward tactical edge. The tactical communication environment is expected to be comprised of a number of interconnected heterogeneous networks across different echelons in a military unit, ranging from stationary nodes connecting to the Internet or GIG to lower echelon mobile networks. However, to integrate heterogeneous networks together seamlessly, a thorough study of network connection architecture and its impact on the individual network is required. In this paper, we address the impact of the number of gateway nodes on network connectivity when we integrate two heterogeneous mobile ad hoc networks.

We consider two types of networks in this paper: a lower tier (LT) network and an upper tier (UT) network. Both the LT network and the UT network are mobile ad hoc networks. The LT network consists of mobile communication nodes with a short radio transmission range. The UT network consists of communication nodes with a much longer radio transmission range than nodes in the LT network. Here, we assume the nodes in the UT network are always connected. For example, each UT node has access to a satellite link. Gateway nodes are those nodes which participate in both the LT and UT networks.

As tactical nodes move during an operation, the network may partition into several segregated clusters. Once the network has partitioned, mobile nodes in different clusters cannot maintain connectivity due to insufficient radio transmission range. The partitioned network will have limited capability in providing seamless communication services to sensors and combat systems. To mitigate this problem, a subset of the mobile nodes can be collocated with and connected to a more powerful communication node to form a gateway node. A set of gateway nodes will improve the connectivity of the LT network through the use of UT network. To reach its destination LT node when a LT network partition occurred, the source LT node can first sent traffic to a gateway node. The gateway node then forwards the traffic through UT network to another gateway node. Finally, the destination gateway node forwards traffic in its cluster to the destination node. In this way, LT nodes in a cluster with a gateway node can still communicate with LT nodes in other clusters with gateway nodes. In this paper, we want to investigate the number of gateway nodes needed in the LT network in order to maintain a certain level of connectivity among LT nodes. Here, the term connectivity refers to the number of different LT source and destination pair that can communicate with each other at a particular instant. The connectivity metric here does not measure how long a particular source and destination pair remain connected.

In this paper, we assume that the LT nodes are uniformly distributed in a square. Obviously, in a real tactical mobile communication environment, nodes' movement will be correlated with each other; hence, the nodes' position distribution in the area of interest cannot be uniform as well. However, the assumption of uniform node distribution provides a worst case analysis since it assumes that no information about nodal movement is known. Given more information about the node movement, we can certainly obtain a better performance by taking advantage of the available information. Since our objective here is to find out how many gateway nodes are needed to maintain certain level of connectivity, we let each node in the LT network have a fixed probability p_g to be a

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gateway node. This probability p_g is our design parameter, and it indicates the fraction of LT nodes that are gateway nodes. The connectivity of the network depends on the network's node density as well as the fraction of nodes which are the gateway nodes. Intuitively, in a densely populated LT network, only a few gateway nodes would be required to obtain a good network connectivity due the following: clusters in a dense LT network tends to have a large number of nodes, and it is more likely for a large cluster to contain a gateway node. Given the density of a LT network, our results here quantify p_g to achieve a given level of connectivity. We also show that, as the density of the LT network increases, the number of gateways nodes does not need to grow linearly with the number of LT nodes in order to achieve good connectivity. Specifically, as the density gets large, we quantify analytically the rate of decrease for p_g in the one dimensional case. In the two dimensional case, our simulation results indicate that p_g can decrease at a larger rate than the rate of increase in the node density, while still maintaining a good network connectivity.

The problem of maintaining network connectivity in a MANET has received considerable attention in various contexts [1] [5] [6]. In [5], the author investigated controlling the mobility of the backbone nodes in order to maintain network connectivity. Algorithm and heuristic that minimized the number of backbone nodes were presented and analyzed. Similarly in [6], the authors optimized the location and movement of UAVs to improve the connectivity. [1] studied the connectivity property for both a purely ad-hoc network and a hybrid network, where a *fixed* base station can be reached in multiple hops. The authors showed that the introduction of a fixed base station in sparse network significantly increase the network connectivity in one dimensional network. Our work here focus on the case where heterogeneous network nodes are collocated at the gateway nodes and are able to move together.

The rest of the paper is organized as follows: Section II states the assumptions, definitions, and the problem objective of the connectivity problem. In Section III, we provided an analysis of the LT network connectivity. Analytical solution for the one-dimensional problem was presented as well. In Section IV, computer simulation is provided for the two dimensional problem. Section V concludes the paper.

II. Definition and Problem Statement

Each node in the lower tier (LT) network is able to move in an area of interest. Each node has a transmitter with transmission range of R. A gateway node in the LT network will be connected the UT network. For example, gateway node can have a radio that is capable of satellite transmission. In the gateway node, both the LT node and the UT node are collocated and connected to each other. We use the following definitions in describing the LT network properties:

- Two nodes are said to be connected *directly* if they are within distance R of each other.
- Two nodes A and B are said to be *connected* if there exists a set of nodes $\{A_1, A_2, \cdots A_n\}$ such that the pairs $(A, A_1), (A_1, A_2), \cdots, (A_n, B)$ are directly connected pairs.
- A *cluster* is a set of nodes that are connected to each other.
- p_g denotes the probability that a LT node is also a gateway node.

We also make the following assumptions:

- The LT nodes are uniformly distributed on a square.
- Gateway nodes are always connected to each other through the UT network.

We made the assumption that LT nodes are uniformly distributed on a square. As we mentioned earlier, real tactical mobile nodes are not going to be uniformly distributed in the field. Our analysis under the uniformity assumption serves as the worst case analysis. It provides a lower bound on the network performance. The actual field connectivity performance will likely to be better than results obtained from the uniform assumption. The assumption that gateway nodes are always connected is based on the fact that UT nodes are always connected. In real implementation of the upper tier network, UT nodes are often equipped with satellite transmission capability. Hence, it is reasonable to assume that the UT nodes are always connected.

Other parameters of interest are listed as follows:

- The number of LT nodes is N.
- The area of the square is L^2.
- The density of the nodes is N/L^2.

Let the term SD pair denote source and destination pair. We define connectivity in the LT network as the following:

Definition 1: The connectivity of a network is defined as

$$C = \frac{\text{(number of unique SD pairs that are connected)}}{\text{(total number of unique SD pair)}}. \quad (1)$$

The connectivity definition C is fairly straightforward. It is simply the fraction of SD pairs that are connected. However, this definition of connectivity, albeit simple, is hard to obtain. We therefore come up with an alternative measure of the network connectivity which is given by the following:

$$C^* = \frac{\text{(total number of LT nodes connected to UT)}}{\text{(total number of LT nodes)}}. \quad (2)$$

The measure C^* can be used as a connectivity definition by itself. It is simpler to obtain than the first connectivity definition. In fact, we will use C^* to obtain C here. C^* is the fraction of nodes that are connected to the UT network and, through the UT network, are also connected to each other.

As we mentioned previously, once LT network partitioned, LT nodes in a cluster with a gateway node can still communicate with LT nodes in other clusters with gateway nodes. Our objective here is to investigate how LT network connectivity can be improved with increasing number of gateway nodes. Furthermore, as the node density increase in the LT network, we want to investigate how p_g can change while maintaining a good LT network connectivity.

III. Connectivity Analysis

Let us start the connectivity analysis by first finding a bound on C^*. When the N nodes are uniformly distributed in the square, nodes will form a certain number, say m, clusters.

Some of the clusters will contain gateway nodes, and some will not. A cluster without any gateway node is an isolated cluster. We first define the following terms:

- α_s: the probability that a nonempty cluster contains s nodes.
- N_{UT}: the total number of nodes with direct or indirect connection to the UT network.
- N_i: the total number of LT nodes in cluster i.
- I_i: an indicator random variable such that

$$I_i = \begin{cases} 1 & \text{if cluster } i \text{ is isolated;} \\ 0 & \text{otherwise.} \end{cases}$$

From the definition of C^*, as the number of clusters m gets large, we have the following:

$$\begin{aligned} 1 - C^* &= 1 - \frac{N_{UT}}{N} && (3) \\ &= \frac{\sum_{i=1}^{m} N_i \cdot I_i}{\sum_{i=1}^{m} N_i} && (4) \\ &= \frac{m \cdot P(I_1 = 1) E[N_1 \mid I_1 = 1]}{m \cdot E[N_1]} && (5) \\ &= \frac{m \cdot P(I_1 = 1) \sum_{k=1}^{\infty} k \cdot \frac{(1-p_g)^k P(N_1 = k)}{P(I_1 = 1)}}{m \cdot \sum_{k=1}^{\infty} k \cdot \alpha_k} && (6) \\ &= \frac{\sum_{k=1}^{\infty} k \cdot (1 - p_g)^k \cdot \alpha_k}{\sum_{k=1}^{\infty} k \cdot \alpha_k} && (7) \end{aligned}$$

The nodes with no connection to the UT network are isolated. These nodes can only communicate with nodes in the same cluster since there is no gateway node in the cluster. The derivation from Eq. (4) to Eq. (5) is based on the assumption that the number of nodes becomes large. To be precise, we let the number of nodes and the area increase while keeping the node density constant. From the law of large number, we have Eq. (4) approaches Eq. (5) as the number of nodes and the number of cluster increases. Since each cluster has the same statistical properties by symmetry, we use cluster 1's properties to represent a general cluster's properties.

To see Eq. (6), we have the following:

$$E[N_1 \mid I_1 = 1] = \sum_{k=1}^{\infty} k \cdot \frac{(1 - p_g)^k P(N_1 = k)}{P(I_1 = 1)}$$

To find an upper bound on $1 - C^*$, let $q_g = 1 - p_g$. Note that the function $f(x) = x \cdot q_g^x$ has the following properties:

$$\begin{aligned} f'(x) &= q_g^x + x \cdot (\ln q_g) q_g^x \\ f''(x) &= [2 \ln q_g + x (\ln q_g)^2] q_g^x \end{aligned} \qquad (8)$$

The function $f''(x)$ is negative for $x < -2/\ln q_g$, and it is positive for $x > -2/\ln q_g$. Let

$$m_o = \lfloor -\frac{2}{\ln q_g} \rfloor.$$

We know that the function $f(x) = x q_g^x$ is concave for $1 \leq x \leq m_o$ and is convex for $m_o \leq x$. We can then derive the following bounds:

$$q_g^{\bar{S}} \leq \frac{\sum_{k=1}^{\infty} k \cdot q_g^k \cdot \alpha_k}{\sum_{k=1}^{\infty} k \cdot \alpha_k} \leq q_g^{\hat{S}} \qquad (9)$$

where the terms $\bar{S}$ and $\hat{S}$ are defined as follows:

$$\begin{aligned} \bar{S} &= E[N_1 \mid N_1 > m_o] \\ \hat{S} &= E[N_1 \mid N_1 \leq m_o] \end{aligned} \qquad (10)$$

Hence, we have

$$q_g^{\bar{S}} \leq 1 - C^* \leq q_g^{\hat{S}} \quad \text{or} \quad 1 - q_g^{\bar{S}} \geq C^* \geq 1 - q_g^{\hat{S}}$$

The terms $\bar{S}$ and $\hat{S}$ cannot be obtained analytically in the two dimensional case. However, when nodes are located on a two dimensional *grid* network, the exact form of $\hat{S}$ can be obtained [4]. In the case where nodes are located on a one dimensional grid. Exact solution for C^* can be obtained.

A. One dimensional grid network

The connectivity problem in one dimensional grid can be solved analytically. Many of the characteristic features encountered in higher dimensions are presented in one dimension as well [2]. The solution in $1d$ will provide insights in understanding the problems in the two dimensional space. In the $1d$ problem, we consider a one dimensional lattice with infinite number of sites of equal spacing arranged in a line, shown here in Fig. 1. In Fig. 1, a black dot at a site implies that the site is being occupied by a node. A cross at a site indicates that the site is not being occupied. In the figure, we see four clusters of size one, two, three and four respectively. Here are assumptions made in the one dimensional network:

- Nodes are placed on a one-dimensional grid.
- Each site of the grid is independently occupied by a node with probability p.
- Nodes are said to be connected if they are adjacent to each other.
- Node is a gateway node with probability p_g.

Fig. 1. One dimensional grid network.

Following the analysis in [2] and [3], one can derive the following cluster size distribution function for the one-dimensional grid case:

$$\alpha_k = k(1 - p)^2 p^{k-1}$$

and

$$E[\text{number of points in a cluster}] = \frac{1 + p}{1 - p}$$

Using the above results, we have

$$\begin{aligned} 1 - C^* &= 1 - \frac{N_{UT}}{N} && (11) \\ &= \frac{\sum_{k=1}^{\infty} k \cdot q_g^k \cdot \alpha_k}{\sum_{k=1}^{\infty} k \cdot \alpha_k} && (12) \\ &= \frac{\frac{(1-p)^2}{p} \frac{(1-p_g)p + (1-p_g)^2 p^2}{[1-(1-p_g)p]^2}}{\frac{1+p}{1-p}} && (13) \\ &= \frac{(1 - p)^3}{(1 - q_g p)^3} \cdot \frac{q_g p + q_g^2 p^2}{p + p^2} && (14) \end{aligned}$$

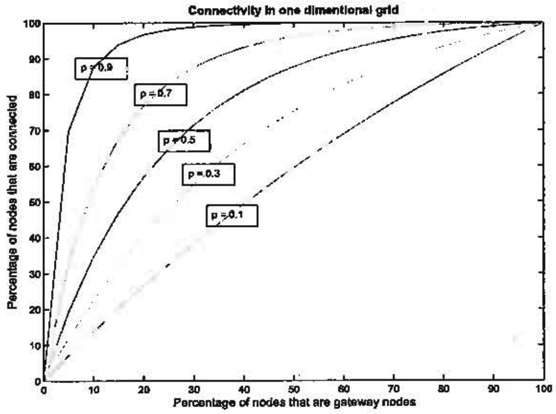

Fig. 2. Nodes connectivity in the one dimensional grid network.

From the above expression, in Fig. 2, for different value of node density (i.e., $p = 0.1, p = 0.3, \cdots, p = 0.9$), we plot the number of gateway nodes in the LT network versus the number of LT nodes that are connected. When the node density is high (i.e., $p = 0.9$), we see from Fig. 2 that a 10 percent increase in p_g from zero will result in almost 90 percent increase in node connectivity. However, when the node density is low (i.e., $p = 0.1$), the increase in the percentage of gateway nodes is almost linear with the increase in the percentage of connected LT nodes. Let

$$f(p, q_g) = \frac{(1-p)^3}{(1-q_g p)^3} \cdot \frac{q_g p + q_g^2 p^2}{p + p^2} \tag{15}$$

The partial derivatives $\frac{\partial f(p,q_g)}{\partial q_g}$ and $\frac{\partial f(p,q_g)}{\partial p}$ will give us insight in designing the network. Consider the practical question of how many gateway nodes one should use, given that we know the number of LT nodes and the area of coverage. Suppose the cost of building n gateway nodes is $V(n)$. The marginal cost of adding an additional gateway node is then $V(n) - V(n-1)$. The marginal increase in the connectivity by building additional gateway nodes is given by $\frac{\partial f(p,q_g)}{\partial q_g}$. A network designer can balance the marginal cost and marginal connectivity improvement in achieving desired connectivity and cost tradeoff.

In the discussion so far, we either chose a fixed value p_g and examined the connectivity of the network as p increased, or chose a fixed p and examined the network connectivity as p_g increased. Our intuition tells us that larger and larger clusters will form as the node density p increases to one. To achieve a good network connectivity, p_g does not need to be fixed as p goes to one. To verify this, we will now investigate how p_g can decrease as a function of p when p goes to one, while still achieving good network connectivity. The answer will help us address the practical question of whether the number of gateway nodes has to increase linearly with the total number of nodes in order to achieve good network connectivity.

Let us consider a family of functions of the form $p_g = (1-p)^a$, where $a > 0$, and study the range of a such that the following holds:

$$\lim_{p \to 1} f(p, q_g) = 0.$$

Substituting $q_g = 1 - (1-p)^a$ into Eq. (15), we have the equation below:

$$h(p) = \frac{(p-1)^3}{(1+p)} \cdot \frac{[(1-p)^a - 1 - p + 2p(1-p)^a - p(1-p)^{2a}]}{[1 - p + p(1-p)^a]^3} \tag{16}$$

To see the impact of a on $f(p)$, we first let $\Delta = 1 - p$. Then, Eq.(16) simplifies to the following:

$$\begin{aligned} h(p) &= \frac{-\Delta^3}{1+p} \cdot \frac{\Delta^a - 1 - p + 2p\Delta^a - p\Delta^{2a}}{(\Delta + p\Delta^a)^3} \\ &= \frac{-\Delta^3}{2} \cdot \frac{\Delta^a - 2 + 2\Delta^a - \Delta^{2a}}{(\Delta + \Delta^a)^3} \quad \text{as } p \to 1 \\ &= \frac{2 - 3\Delta^a + \Delta^{2a}}{2(1 + \Delta^{a-1})^3} \end{aligned} \tag{17}$$

It is straightforward to see from the above equation that the following holds:

$$\lim_{p \to 1} h(p) = \begin{cases} 1 & \text{if } a > 1\ ; \\ \frac{1}{8} & \text{if } a = 1\ ; \\ 0 & \text{if } a < 1\ ; \end{cases}$$

Hence, p_g can decrease at a rate of $(1-p)^a$, where $a < 1$, as p approaches one while still achieving a good network connectivity. Practically, as the network density increases, the number of gateway nodes does not need to be a constant fraction of the total nodes in order to achieve a good connectivity.

B. Bound on C

When the N nodes are uniformly distributed in the square, nodes will form a certain number, say m, clusters. Without loss of generality, we say that clusters $1, \cdots, l$ do not contain any gateway nodes. Clusters $l+1, \cdots, m$ each contains at least one gateway node. Let $s(i)$ denote the number of nodes in cluster i. We then have the following:

$$\begin{aligned} &\text{number of unique SD pairs that are connected} \\ &= \sum_{i=1}^{l} s(i)(s(i)-1) + (\sum_{i=l+1}^{m} s(i))(\sum_{i=l+1}^{m} s(i) - 1) \end{aligned} \tag{18}$$

To see the above equation, note that the first l clusters do not contain any gateway node. In the cluster i where $i \leq l$, the number of unique source and destination (SD) pair is $s(i)(s(i)-1)$. In clusters $l+1, \cdots, m$, all nodes are connected since each cluster contain at least one gateway nodes. Hence, the number of connected unique SD pair is

$$(\sum_{i=l+1}^{m} s(i))(\sum_{i=l+1}^{m} s(i) - 1).$$

From the analysis of C^*, we know $C^* \geq 1 - q_g^{\hat{S}}$. Hence, we have

$$\sum_{i=l+1}^{m} s(i) = N_{UT} \geq N \cdot (1 - q_g^{\hat{S}})$$
$$(\sum_{i=l+1}^{m} s(i))(\sum_{i=l+1}^{m} s(i) - 1) \geq [N(1 - q_g^{\hat{S}})][N(1 - q_g^{\hat{S}}) - 1] \quad (19)$$

From the definition of C, we will have the following:

$$\begin{aligned} C &= \frac{\text{(number of unique SD pairs that are connected)}}{\text{(total number of unique SD pair)}} \\ &= \frac{\sum_{i=1}^{l} s(i)(s(i)-1) + (\sum_{i=l+1}^{m} s(i))(\sum_{i=l+1}^{m} s(i) - 1)}{N(N-1)} \\ &= \frac{\sum_{i=1}^{l} s(i)(s(i)-1)}{N(N-1)} + \frac{(\sum_{i=l+1}^{m} s(i))(\sum_{i=l+1}^{m} s(i) - 1)}{N(N-1)} \end{aligned} \quad (20)$$

As N increases, the first term goes to zero. The second term is greater than $(1 - q_g^{\hat{S}})^2$. Hence,

$$C \geq (1 - q_g^{\hat{S}})^2$$

IV. Simulation

In the simulation environment, we let each node has a transmission range $R = 1$. Nodes are uniformly distributed in a square of size $L \times L$. We start with $L = 50$. The node density D is used to describe the average number of nodes on a unit square. The total number of nodes in the square is therefore $N = \lfloor D \cdot L^2 \rfloor$. Note that there is no grid structure in the simulation here. In Fig. 3, as in the 1 dimensional case, we plot the number of gateway nodes in the LT network versus the number of LT nodes that are connected for different node densities. When the node density is large and the percentage of gateway node small, a small increase in the number of gateway node will cause a significant increase in the number of connected nodes. However, as the number of gateway nodes increase, its marginal benefit decreases.

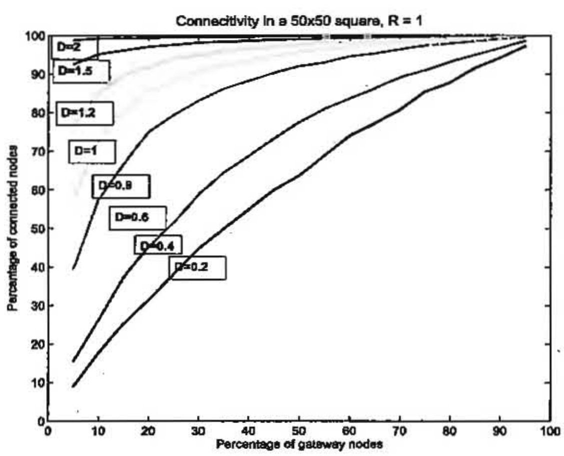

Fig. 3. Nodes connectivity in the one dimensional grid network.

In the section of 1-dimensional grid network, we investigate how p_g can decrease as a function of p while still achieving a good network connectivity. In this two-dimensional uniform distribution case, we need related the density D with the probability p in the one dimensional case. Let us divide the square under consideration ($L \times L$) into squares of size $d \times d$. We choose $d = 0.8R$. Let p denote the probability that a square ($d \times d$) has at least one point. Hence, we have the following:

$$p = 1 - (1 - 1/n)^{Dd^2 n}$$

where n is the number of small square ($d \times d$) inside a the square ($L \times L$). As L increases, the above expression approaches

$$p = 1 - e^{-Dd^2}.$$

Let us again consider the family of functions of the form: $p_g = (1-p)^a = e^{-aDd^2}$. For different values of a, we then examine how the percentage of connected nodes change as the node density increases. For $a = 2$ and $a = 3$, their relationships are plotted in Fig. 4. From the figure, for both $a = 2$ and $a = 3$,

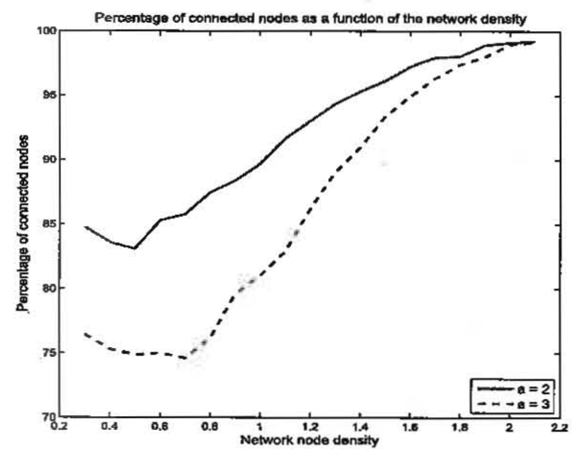

Fig. 4. Nodes connectivity in the one dimensional grid network.

we see that the percentage of the connected nodes approaches 100 percent as node density increases. Hence a given network has high node density, instead of setting a fixed percentage nodes as gateway nodes, we can set the percentage of gateway nodes as a function of the density (i.e., $p_g = e^{-aDd^2}$). If the LT network is known to have high node density, we can let $p_g = e^{-aDd^2}$, resulting in a significant saving in the number of gateway nodes.

V. Conclusion

In this paper, we investigated the number of gateway nodes needed in order to maintain certain level of connectivity in a mobile network. Given the node density of the mobile network, we quantify the relationship between network connectivity and the number of gateway nodes. In a densely populated mobile network, we found that only a small number of gateway nodes are needed to achieve good network connectivity. Moreover, the percentage of gateway nodes can decrease at a faster rate than the node density increase rate while still achieving a good network connectivity.

In our simulation study of the rate of decreasing for p_g, we are able to see that most LT nodes are connected when we let $a = 2$ and $a = 3$. In the future, we would like to provide a range of the value a such that most nodes in the network are connected.

VI. Appendix

From $1 - C^*$, we can write the following:

$$\begin{aligned}
& \frac{\sum_{k=1}^{\infty} k \cdot q_g^k \cdot \alpha_k}{\sum_{k=1}^{\infty} k \cdot \alpha_k} \\
= & \frac{\sum_{k=1}^{m_o} k q_g^k \alpha_k + \sum_{k=m_o+1}^{\infty} k q_g^k \alpha_k}{\sum_{k=1}^{m_o} k\alpha_k + \sum_{k=m_o+1}^{\infty} k\alpha_k} \qquad (21) \\
= & \frac{\frac{1}{B_1}\sum_{k=1}^{m_o} k q_g^k \alpha_k + \frac{1}{B_1}\sum_{k=m_o+1}^{\infty} k q_g^k \alpha_k}{\frac{1}{B_1}\sum_{k=1}^{m_o} k\alpha_k + \frac{1}{B_1}\sum_{k=m_o+1}^{\infty} k\alpha_k} \qquad (22) \\
\leq & \frac{\hat{S} q_g^{\hat{S}} + \frac{1}{B_1}\sum_{k=m_o+1}^{\infty} k q_g^k \alpha_k}{\hat{S} + \frac{1}{B_1}\sum_{k=m_o+1}^{\infty} k\alpha_k} \qquad (23) \\
& \text{(by Jensen's inequality)} \\
\leq & \frac{\hat{S} q_g^{\hat{S}} + \frac{1}{B_1} q_g^{m_o} \sum_{k=m_o+1}^{\infty} k\alpha_k}{\hat{S} + \frac{1}{B_1}\sum_{k=m_o+1}^{\infty} k\alpha_k} \qquad (24) \\
= & \frac{\hat{S} q_g^{\hat{S}} + \frac{1}{B_1} q_g^{m_o} B_2}{\hat{S} + \frac{1}{B_1} B_2} \qquad (25) \\
= & \frac{q_g^{\hat{S}} (\hat{S} + \frac{1}{B_1} B_2)}{\hat{S} + \frac{1}{B_1} B_2} \qquad (26) \\
= & q_g^{\hat{S}}
\end{aligned}$$

From Eq. (21) to Eq. (22), we multiply the term $1/B_1$ on the both the numerator and denominator where

$$B_1 = \sum_{k=1}^{m_o} k\alpha_k.$$

From Eq. (22) to Eq. (24), we use Jensen's inequality and the concave property of $f(x) = x q_g^x$ for $1 \leq x \leq m_o$.

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