REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
28-07-2011	Conference Proceeding	-

4. TITLE AND SUBTITLE
A Comparison of the ML-PDA and the ML-PMHT Algorithms

5a. CONTRACT NUMBER
W911NF-10-1-0369

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
611102

6. AUTHORS
S. Schoenecker, P. Willett, Y. Bar-Shalom

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
University of Connecticut - Storrs
Office for Sponsored Programs
University of Connecticut
Storrs, CT 06269 -1133

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S)
ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
57823-CS.21

12. DISTRIBUTION AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT
The Maximum Likelihood Probabilistic Data Association (ML-PDA) tracker and the Maximum Likelihood Probabilistic Multi-Hypothesis (ML-PMHT) tracker were applied to five synthetic multistatic active sonar scenarios featuring multiple targets, multiple sources, and multiple receivers. For each of

15. SUBJECT TERMS
ML-PDA, ML-PMHT, maximum likelihood,multistatic, bistatic, sonar, tracking, expectation maximization,EM

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
UU	UU	UU	UU		Yaakov Bar-Shalom
					19b. TELEPHONE NUMBER 860-486-4823

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std. Z39.18

Report Title

A Comparison of the ML-PDA and the ML-PMHT Algorithms

ABSTRACT

The Maximum Likelihood Probabilistic Data Association (ML-PDA) tracker and the Maximum Likelihood Probabilistic Multi-Hypothesis (ML-PMHT) tracker were applied to five synthetic multistatic active sonar scenarios featuring multiple targets, multiple sources, and multiple receivers. For each of the scenarios, Monte Carlo testing was performed to quantify the performance differences between the two algorithms. Both trackers ended up performing well. For most scenarios, MLPMHT slightly outperformed ML-PDA in terms of in-track percentage. However, in a scenario with closely-spaced targets, ML-PDA exhibited superior performance

Conference Name: Proc. 14th Intn'l Conf. on Information Fusion

Conference Date: July 01, 2011

14th International Conference on Information Fusion
Chicago, Illinois, USA, July 5-8, 2011

A Comparison of the ML-PDA and the ML-PMHT Algorithms

Steven Schoenecker
NUWC
Division Newport
Newport, RI
Email: steven.schoenecker@navy.mil

Peter Willett
Electrical Engineering
University of Connecticut
Storrs, CT
Email: willett@engr.uconn.edu

Yaakov Bar-Shalom
Electrical Engineering
University of Connecticut
Storrs, CT
Email: ybs@engr.uconn.edu

***Abstract*—The Maximum Likelihood Probabilistic Data Association (ML-PDA) tracker and the Maximum Likelihood Probabilistic Multi-Hypothesis (ML-PMHT) tracker were applied to five synthetic multistatic active sonar scenarios featuring multiple targets, multiple sources, and multiple receivers. For each of the scenarios, Monte Carlo testing was performed to quantify the performance differences between the two algorithms. Both trackers ended up performing well. For most scenarios, ML-PMHT slightly outperformed ML-PDA in terms of in-track percentage. However, in a scenario with closely-spaced targets, ML-PDA exhibited superior performance.**

Keywords: ML-PDA, ML-PMHT, maximum likelihood, multistatic, bistatic, sonar, tracking, expectation maximization, EM.

I. Introduction

The Maximum Likelihood Probabilistic Data Association (ML-PDA) tracker and the Maximum Likelihood Probabilistic Multi-Hypothesis (ML-PMHT) tracker are both simple, straightforward algorithms that can be used in an active multistatic sonar framework. With some basic assumptions about a target (or targets) as well as the environment, likelihood ratios can be developed for both algorithms and then optimized. The main difference between the two algorithms is in the target assignment model; ML-PDA assumes that at most one measurement per scan can originate from a target, while ML-PMHT allows for any number of measurements to have originated from a target. While this assumption may reduce the appeal of the ML-PMHT, the resulting algorithm does offer advantages in both its implementation (especially fine-scale optimization) and in terms of its multitarget formulation.

The algorithms were tested with Monte Carlo trials on five different synthetic multistatic sonar scenarios. These scenarios were designed to test a range of geometries, including a single target, closely-spaced targets, crossing targets, large numbers of targets, and large numbers of sources and receivers.

II. ML-PDA and ML-PMHT Formulations

This section briefly describes the development of the ML-PDA and the ML-PMHT likelihood ratios (LR), as well as the manner in which each log-likelihood ratio is optimized. It also discusses how each algorithm is extended to work for multiple targets, and how they handle maneuvering targets.

A. ML-PDA Likelihood Ratio

The ML-PDA concept was initially developed in [13]. Subsequently, in [19], [20], and [7] it was applied in a multistatic active application, which is how we currently employ it. The assumptions used to develop the ML-PDA LR are [8], [14]:

- A single target is present in each frame with known detection probability P_d. Detections are independent across frames.
- There are zero or one measurements per frame from the target.
- The kinematics of the target are deterministic. The motion is usually parameterized as a straight line, although any other parameterization is valid.
- False detections (clutter) are uniformly distributed in the search volume and their number is Poisson distributed with known density.
- Amplitudes of target and false detections are Rayleigh distributed. The parameter of each Rayleigh distribution is known (although the SNR may be tracked [8]).
- Target measurements are corrupted with zero-mean Gaussian noise with known variance.
- Measurements at different times, conditioned on the parameterized state, are independent.

Based on these assumptions, the ML-PDA log-likelihood ratio can be constructed. Its development is extensively explained in [4], so it is only summarized here. For a single scan of data with m_i measurements, the ML-PDA LR is

$$\frac{p_1[Z(i)|\mathbf{x}]}{p_0[Z(i)|\mathbf{x}]} = 1 - P_d + \frac{P_d}{\lambda}\sum_{j=1}^{m_i} p[z_j(i)|\mathbf{x}]\rho_j(i) \qquad (1)$$

In this expression, $Z(i)$ is the set of measurements in the i^{th} scan, and $z_j(i)$ is an individual measurement (the j^{th} measurement in the i^{th} scan). P_d is the probability of detection of the target in the scan, λ is the spatial clutter density, and $\rho_j(i)$ is the amplitude likelihood ratio of an individual measurement. Since one of the assumptions for ML-PDA (and ML-PMHT) is that, conditioned on $\mathbf{x}$, measurements at different times are independent, for N_w scans/frames, the

Supported by ONR grants N00014-10-10412, N00014-10-1-0029, and ARO grant W911NF-06-1-0467.
Proc. 14th Intn'l Conf. Info. Fusion. Chicago, IL, July 2011.

likelihood ratio of a batch is just the product over i, where $i = \{1, \ldots, N_w\}$, of the right-hand side of (1). Taking the logarithm produces the log-likelihood ratio for a batch of data:

$$\Lambda(\mathbf{x}, Z) = \sum_{i=1}^{N_w} \ln \left\{ 1 - P_d + \frac{P_d}{\lambda} \sum_{j=1}^{m_i} p[z_j(i)|\mathbf{x}] \rho_j(i) \right\} \quad (2)$$

Here, Z refers to the set of all measurements in the batch. In the case where measurements include range-rate (Doppler), and information is available on the distribution of the Doppler measurements resulting from clutter, a "moving target indicator" de-weighting term can be added to the log-likelihood ratio. The MTI deweighting term $\zeta[d_j(i)]$ is given by [15]

$$\zeta[d_j(i)] = \frac{e^{d_j(i)^2/2a^2}}{\frac{1}{m_i} \sum_{l=1}^{m_i} e^{d_l(i)^2/2a^2}} \quad (3)$$

Here, $d_j(i)$ is the j^{th} Doppler measurement in the i^{th} scan, and a^2 is the variance of the clutter Doppler with an assumed $\mathcal{N}(0, a^2)$ distribution. This term is incorporated into equation (2) as

$$\begin{aligned} \Lambda(\mathbf{x}, Z) \quad = \quad & \sum_{i=1}^{N_w} \ln \left\{ 1 - P_d + \frac{P_d}{\lambda} \times \right. \quad (4) \\ & \left. \sum_{j=1}^{m_i} \; p[z_j(i), d_j(i)|\mathbf{x}] \rho_i(j) \zeta[d_j(i)] \right\} \end{aligned}$$

This equation is what is implemented in this work for ML-PDA. In order to find the vector $\mathbf{x}$ that gives the maximum likelihood, a typical nonlinear optimization scheme is used. In this application, a conjugate gradient approach was found to work both accurately and rapidly.

B. ML-PMHT Likelihood Ratio

The Maximum Likelihood Probabilistic Multi-Hypothesis tracker is very similar to the ML-PDA algorithm. The ML-PMHT likelihood ratio was first formulated for the PMHT algorithm in [3], [16], [17] and [18]. It was implemented with a maximum-likelihood formulation in a bistatic active application in [20] and [21], which is how we currently employ it. The assumptions that go into it are almost exactly the same as those listed above for ML-PDA, with one (major) exception. Instead of allowing only zero or one measurements in a scan to be assigned to the target, ML-PMHT allows any number of measurements in a scan to be assigned to the target. While this would seem to add complexity to the problem, it actually makes the log-likelihood ratio much easier to develop. For a single scan of data, the likelihood ratio is

$$\frac{p_1[Z(i)|\mathbf{x}]}{p_0[Z(i)|\mathbf{x}]} = \prod_{j=1}^{m_i} \left\{ \pi_0 + \pi_1 V p[z_j(i)|\mathbf{x}] \rho_j(i) \right\} \quad (5)$$

where π_0 is the prior probabilitity that a measurement is from clutter, π_1 is the prior probability that a measurement is from the target, and V is the search volume. Again, since measurements at different times, conditioned on the state, are independent, the likelihood ratio of a batch is just a product over i of the terms on the right-hand side of equation (5). Doing this and then taking the logarithm produces the ML-PMHT log-likelihood ratio for a batch:

$$\Lambda'(\mathbf{x}, Z) = \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} \ln \left\{ \pi_0 + \pi_1 V p[z_j(i)|\mathbf{x}] \rho_j(i) \right\} \quad (6)$$

Finally, if Doppler measurements are present, and information is available on the Doppler clutter, the MTI deweighting term described above can be incorporated into the log-likelihood ratio, in a manner similar to ML-PDA:

$$\begin{aligned} \Lambda'(\mathbf{x}, Z) \quad = \quad & \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} \ln \left\{ \pi_0 + \right. \quad (7) \\ & \left. \pi_1 V p[\mathbf{z}_j(i), d_j(i)|\mathbf{x}] \rho_j(i) m_i \zeta[d_j(i)] \right\} \end{aligned}$$

C. ML-PMHT optimization

An advantage of the ML-PMHT log-likelihood ratio formulation is that it can be optimized with a closed-form expression using expectation maximization [10]. As long as there is a linear relationship between the state $\mathbf{x}$ and the predicted measurement $\hat{\mathbf{z}}$, we can write the cost function $J(\mathbf{x}, Z)$ as

$$\begin{aligned} J(\mathbf{x}, Z) = \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} & \left\{ [\mathbf{z}_j(i) - \mathbf{H}\mathbf{x}]^T \mathbf{R}_{ij}^{-1} \times \right. \quad (8) \\ & \left. [\mathbf{z}_j(i) - \mathbf{H}\mathbf{x}] + \ln(|2\pi \mathbf{R}_{ij}|) \right\} w_j(i) \end{aligned}$$

Here, $\mathbf{H}$ is the measurement matrix, $\mathbf{R}_{ij}$ is the measurement covariance matrix for the j^{th} measurement in the i^{th} scan, and $w_j(i)$ is the association probability of the measurement. The EM algorithm for this case involves iteratively calculating $w_j(i)$ and then using this value to solve for the minimum of equation (8) [3]. The expression for $w_j(i)$ is

$$w_j(i) = \frac{\pi_1 p[\mathbf{z}_j(i)|\mathbf{x}] \rho_j(i)}{\pi_0 / V + \pi_1 p[\mathbf{z}_j(i)|\mathbf{x}] \rho_j(i)} \quad (9)$$

In equation (8), the relationship between the predicted measurement $\hat{\mathbf{z}}$ and the state is linear

$$\hat{\mathbf{z}} = \mathbf{H}\mathbf{x} \quad (10)$$

This relation is valid when the measurements are two-dimensional (typically x and y Cartesian coordinates), and the state vector is given by

$$\mathbf{x} = [x_0 \;\; \dot{x} \;\; y_0 \;\; \dot{y}]^T \quad (11)$$

Here, (x_0, y_0) is the Cartesian position of the target at the beginning of the batch, and $\dot{x}$ and $\dot{y}$ are its Cartesian velocity components. The measurement matrix is given by

$$\mathbf{H} = \begin{bmatrix} 1 & t & 0 & 0 \\ 0 & 0 & 1 & t \end{bmatrix} \quad (12)$$

In this case, the initial time $t = 0$ is at the beginning of the batch. When the relationship described by (10) holds, the minimization of the cost function in (8) is a simple vector quadratic minimization, the solution to which is easily worked out to be

$$\mathbf{x} = \Big[\sum_{i=1}^{N_w}\sum_{j=1}^{m_i} w_j(i)\mathbf{H}^T\mathbf{R}_{ij}^{-1}\mathbf{H}\Big]^{-1} \times \sum_{i=1}^{N_w}\sum_{j=1}^{m_i} w_j(i)\mathbf{H}^T\mathbf{R}_{ij}^{-1}\mathbf{z}_j(i) \tag{13}$$

When Doppler is added to the measurement, the relationship between the predicted measurement and the state vector is no longer linear, and the above solution can not be used. In order to avoid this problem, the new (three-dimensional) predicted measurement can be linearized about some initial state vector $\mathbf{x_0}$. In this case, the linearized measurement is

$$\hat{\mathbf{z}}(\mathbf{x}) \approx \begin{bmatrix} \mathbf{H} \\ \nabla\tilde{r}(\mathbf{x}_0)^T \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \tilde{r}(\mathbf{x}_0) - \nabla\tilde{r}(\mathbf{x}_0)^T\mathbf{x}_0 \end{bmatrix} \tag{14}$$

In this equation, $\tilde{r}$ is the bistatic Doppler, a function of the state vector and the positions and velocities of the source and receiver [9]. Now, the vector on the right-hand side of this equation can be shifted to the left-hand side of the equation to produce a modified predicted measurement

$$\tilde{\mathbf{z}} = \hat{\mathbf{z}} - \begin{bmatrix} 0 \\ 0 \\ \tilde{r}(\mathbf{x}_0) - \nabla\tilde{r}(\mathbf{x}_0)^T\mathbf{x}_0 \end{bmatrix} \tag{15}$$

and

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \nabla\tilde{r}(\mathbf{x}_0)^T \end{bmatrix} \tag{16}$$

so we can write

$$\hat{\mathbf{z}} = \bar{\mathbf{H}}\mathbf{x} \tag{17}$$

The linear relationship between the (modified) predicted measurement and the state again holds, so equation (13) can be used for the optimization of the cost function and the solution for the state vector $\mathbf{x}$ for ML-PMHT.

D. Extension to Multitarget Scenarios

It is very difficult to extend ML-PDA to multiple targets. While it is possible to write down the multitarget log-likelihood ratio, the number of terms increases dramatically with the number of targets, and the expression becomes essentially intractable for the multitarget case. In order to handle multiple targets, ML-PDA treats the problem like a sequence of single-target problems. For a batch of data, it optimizes the LLR, and if this value exceeds a certain threshold, a target is declared. Next, the measurement that has the highest association probability with that solution is excised from each scan, and the sequence is repeated for the next target. This method is not elegant, but it has been shown to work effectively for multitarget scenarios in [11] and [12]. The ML-PMHT LLR, on the other hand, is very easily extended to a multiple target framework. For n targets with state vectors $\mathbf{x_1}, \ldots, \mathbf{x_n}$, the multitarget LLR is expressed as

$$\Lambda'(\mathbf{x}, Z) = \sum_{i=1}^{N_w}\sum_{j=1}^{m_i} \ln\Bigg(\pi_0 + \pi_1 V\Big\{p[\mathbf{z}_j(i)|\mathbf{x_1}]\rho_{1j}(i) + \cdots + p[\mathbf{z}_j(i)|\mathbf{x_n}]\rho_{nj}(i)\Big\}\Bigg) \tag{18}$$

While this is an elegant formulation, it does lead to problems with track declaration. For a single target, a track is declared if the LLR produced by the target exceeds some threshold τ. Now consider the case where there is already an existing target, and a new target is spawned. The new (joint) likelihood would be given by equation (18) for $n = 2$. The increase in LLR that is due only to target 2 is given by

$$\Delta\Lambda'(\mathbf{x}, Z) = \sum_{i=1}^{N_w}\sum_{j=1}^{m_i} \ln\left\{1 + \frac{\pi_1 V p[\mathbf{z}_j(i)|\mathbf{x_2}]\rho_{2j}(i)}{\pi_0 + \pi_1 V p[\mathbf{z}_j(i)|\mathbf{x_1}]\rho_{1j}(i)}\right\} \tag{19}$$

This is the test statistic that must be used to determine the existence of a track for target 2; however, it is a function of the states of both targets 1 and 2. The test statistic for a target track should only be a function of that target. This problem only gets worse as the number of targets increases – the test statistic for the n^{th} target will be a function of the previous $n-1$ state vectors. For this reason, in this work, ML-PMHT is treated in the same way as ML-PDA. One target at a time is found in a batch. After each target is found, measurements associated with this target are excised from the data and the process is repeated. Future work will examine how to take better advantage of ML-PMHT's multitarget LLR formulation.

E. Target Maneuvers

One of the assumptions of ML-PMHT and ML-PDA is that target motion is deterministic and can be parameterized. For this work, all motion was parameterized as a straight line. However, this was done with a relatively short, sliding batch, with the idea that any target motion could be approximated by a series of line segments. In this case, a batch of 11 time periods was used (each time period was 60 seconds), and then at every tracker update, the batch was slid forward by two periods.

III. ML-PDA VS ML-PMHT PERFORMANCE COMPARISONS

Five multistatic sonar scenarios were created to measure performance differences between ML-PDA and ML-PMHT. For each scenario, Monte Carlo testing was performed in order to accurately quantify any of these differences. The parameters used for ML-PMHT and ML-PDA for the scenarios are listed in Table I (these parameters match the conditions in their respective scenarios). At the conclusion of each scenario, the following metrics were measured: target in-track percentage, target duplicate tracks, target root-mean square error (RMSE), and overall number of false tracks.

Table I
ML-PMHT AND ML-PDA PARAMETERS

	ML-PMHT			ML-PDA	
	π_0	π_1	V	λ	P_d
Scen. 1-4	0.95	0.05	1.26×10^9	3.7×10^{-9}	0.8
Scen. 5	0.95	0.05	1.26×10^9	6.7×10^{-10}	0.8
Scen. 1 Rayleigh clut.	0.95	0.05	1.26×10^9	2.0×10^{-7}	0.8

A. Scenarios

Five scenarios were developed for Monte Carlo testing. All scenarios are shown (with results overlaid) in Figures 1 – 12. Scenario 1 featured a single target moving past a source and a receiver (the source was a receiver as well). This was used as a simple baseline for performance comparisons. Scenario 2 was intended to test each algorithm's ability to track closely spaced maneuvering targets. It featured three targets separated by about 500 distance units moving past several source-receiver pairs. It should be noted here that none of the plots showing the scenarios have any units on them, nor are any units listed in this work. This is because distances in the data were completely arbitrary. The program that created the scenario data simulated target return signal-to-noise ratio (SNR) at the output of a matched filter by setting the SNR at a distance of 1000 units (which could be anything) from the target and then simply assuming cylindrical spreading losses, again referenced to 1000 distance units. No attempt was made to model "real-world" acoustic propagation loss.

Scenario 3 was designed to test the ability of the trackers to follow crossing targets. It featured two targets, closing each other from the east and west. As the targets passed each other, the target moving from east to west performed a 180-degree turn and followed the other target back to the east. Scenario 4 was set up with a large number of targets. It featured ten very low-speed targets and three relatively high-speed targets. Finally, scenario 5 had a large number of transmitters/receivers (20), and two targets that approached each other, paralleled each other for a period, and then separated.

Clutter for the scenarios (with one exception) was also given K-distribution clutter, as described in [2] and [1]. Thresholding for the scenarios was set so the clutter density for an individual source-receiver pair was about 4×10^{-9} false detects per unit of search volume.

B. Results

For each of the scenarios, several hundred runs were performed (the number varied from scenario to scenario, depending on the individual execution times). In terms of computational performance, ML-PMHT was slightly more expensive than ML-PDA. The time to calculate the actual likelihood ratios was almost identical; however, the EM optimization used by ML-PMHT was slower than the conjugate gradient optimization used by ML-PDA. At the conclusion of each Monte Carlo run set, the following metrics were computed: in-track percentage, RMSE, target track fragmentation, target duplicate tracks, and number of false tracks. A track was associated with a target if the average distance to the target over the length of the track was less than 2000 distance units. A duplicate track was declared if more than one track was associated with a target for a given time. In-track percentage was simply the ratio of the number of target truth points for each target that were associated with a track to the total number of truth points for that target. Fragmentation was determined by counting the number of disjoint tracks associated with a target, minus the number of duplicate tracks (essentially the number of breaks in the track for a target). Finally, the total number of false tracks was determined by simply counting the number of tracks not associated with a target. Each scenario was 60 minutes long, so the number of false tracks is equivalent to false tracks per hour. For each scenario, these metrics and a plot of a tracking example for both ML-PDA and ML-PMHT are shown below.

1) Scenario 1: Performance metrics for scenario 1 (with K-distribution clutter) are shown in Table II, and examples of tracking runs for this scenario are shown in Figures 1 and 2. In terms of RMSE, duplicate tracks, and percentage time in track, there was no dramatic performance difference in this case between ML-PDA and ML-PMHT. In terms of in-track percentage, ML-PMHT was slightly better than ML-PDA (the 95-percent confidence intervals for the two values did not overlap).

Table II
SCENARIO 1 PERFORMANCE METRICS

Target #	Fragmentation	Duplicate Tracks	Percentage Time in Track	RMSE
ML-PDA				
1	1.5	0.016	63%	77
ML-PMHT				
1	0.8	0.014	68%	85

2) Scenario 2: Performance metrics for scenario 2 are shown in Table III, and tracking run examples for this scenario are shown in Figures 3 and 4. This scenario featured three targets in close proximity to each other, and this was a case where ML-PDA clearly outperformed ML-PMHT. Both algorithms were able to track target number 2 almost 100 percent of the time. However, ML-PDA was able to track the other two targets about 1/2 of the time, whereas ML-PMHT was only able to track the other two targets 1/4 of the time. This probably was due to the target assignment model and the way that ML-PMHT handled multiple tracks. As described above, for ML-PMHT, any number of measurements in a scan may be assigned to a target. In the implementation of ML-PMHT for this work, the likelihood ratio was optimized for individual tracks one at a time while accounting for already existing tracks. If a new target created a measurement near an already existing track, the existing track could "claim" the measurement in a probabilistic sense at the expense of the new track. (This "claiming" happens because the ML-PMHT likelihood ratio does not allow for a measurement to

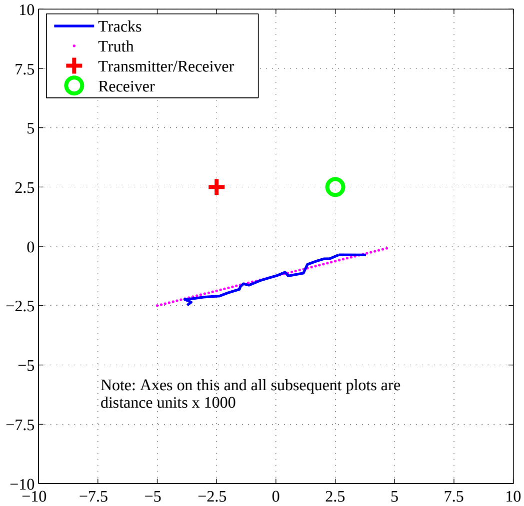

Figure 1. Scenario 1 ML-PMHT estimated tracks

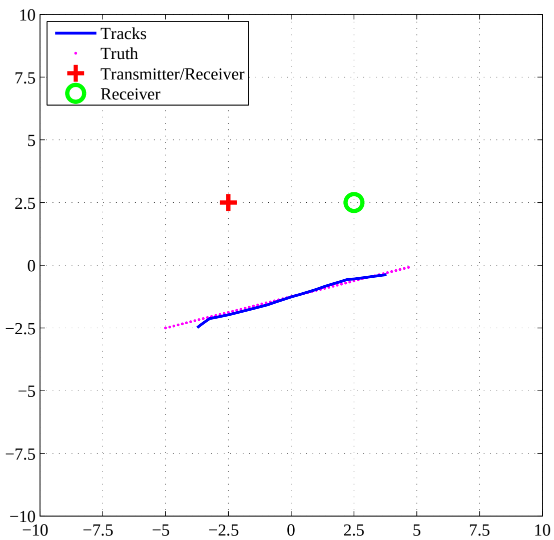

Figure 2. Scenario 1 ML-PDA estimated tracks

be assigned to more than one target simultaneously.) Future work will examine if there is a better way to implement ML-PMHT in a multitarget framework where the likelihood ratio is optimized for multiple targets all at once.

3) Scenario 3: Performance metrics for scenario 3 are shown in Table IV, and tracking run examples for this scenario are shown in Figures 5 and 6. As with scenario 1, ML-PMHT slightly outperformed ML-PDA in terms of percentage time in track (again, the confidence intervals for the values obtained did not overlap). This is interesting, because as with scenario 2, scenario 3 featured targets that were close to one another. However, the targets were moving in opposite directions when they were at their closest point to each other, so there was only a limited time where one measurement could probabilistically claim the other target's measurement as seemed to happen with

Table III
SCENARIO 2 PERFORMANCE METRICS

Target #	Fragmentation	Duplicate Tracks	Percentage Time in Track	RMSE
ML-PDA				
1	0.48	0.44	47%	124
2	0.04	1.24	91%	47
3	0.40	0.46	45%	66
ML-PMHT				
1	0.00	0.12	22%	86
2	0.00	1.51	100%	40
3	0.01	0.08	22%	57

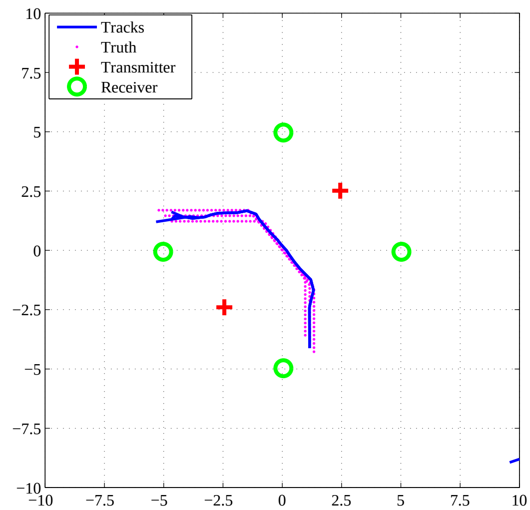

Figure 3. Scenario 2 ML-PMHT estimated tracks

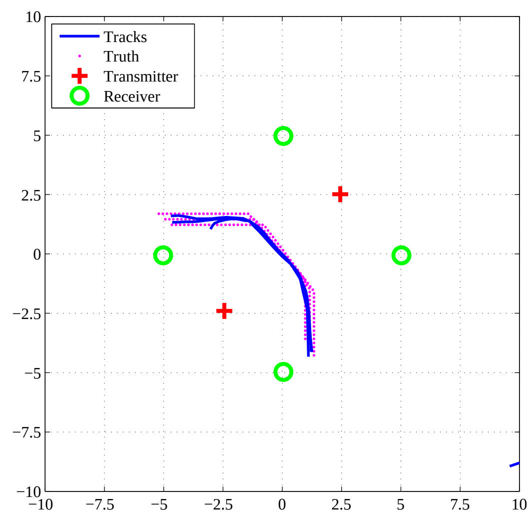

Figure 4. Scenario 2 ML-PDA estimated tracks

Table IV
SCENARIO 3 PERFORMANCE METRICS

Target #	Fragmentation	Duplicate Tracks	Percentage Time in Track	RMSE
ML-PDA				
1	0.70	0.25	59%	170
2	1.06	0.02	45%	245
ML-PMHT				
1	0.28	0.12	68%	195
2	0.46	0.01	57%	214

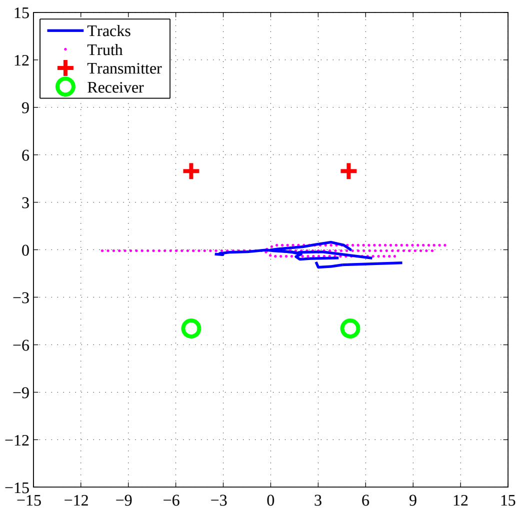

Figure 5. Scenario 3 ML-PMHT estimated tracks

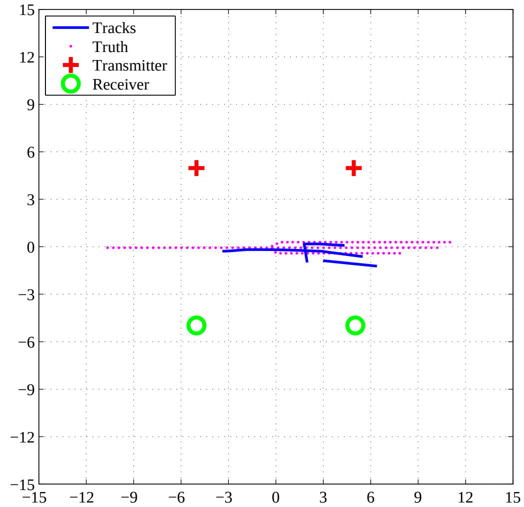

Figure 6. Scenario 3 ML-PDA estimated tracks

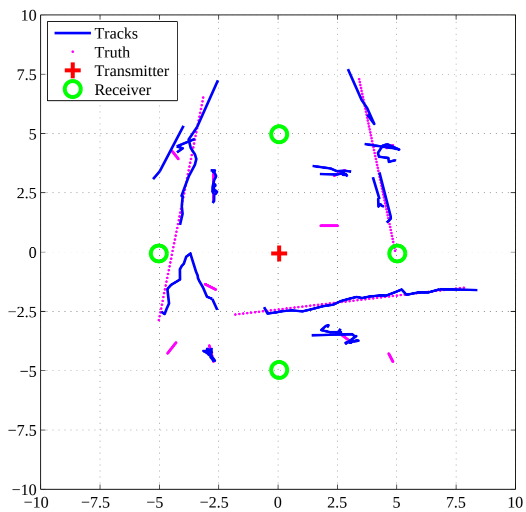

Figure 7. Scenario 4 ML-PMHT estimated tracks

scenario 2. Other metrics were for the most part consistent between the two algorithms.

4) Scenario 4: Performance metrics for scenario 4 are shown in Table V, and tracking run examples for this scenario are shown in Figures 7 and 8. For this scenario, there were 13 targets — 10 slow-speed targets and 3 higher-speed targets. In Table V one representative low-speed target and one representative high-speed target are shown. Again, as with scenario 1, ML-PMHT slightly outperformed ML-PDA in terms of in-track percentage.

5) Scenario 5: Performance metrics for scenario 5 are shown in Table VI, and tracking run examples for this scenario are shown in Figures 9 and 10. Again, ML-PMHT does better than ML-PDA in terms of in-track percentage. This is interesting, since for a portion of this scenario, the two tracks are close to one another, so as with scenario 2, we might expect ML-PDA to best ML-PMHT in terms of in-track percentage. However, the targets are only close for about 1/3 of the scenario – in scenario 2, they were close the entire time. The performance of ML-PMHT when the targets are separate may be the dominant factor in this situation.

Table V
SCENARIO 4 PERFORMANCE METRICS

Target #	Fragmentation	Duplicate Tracks	Percentage Time in Track	RMSE
ML-PDA				
7	0.46	0.023	59%	80
13	0.34	0.026	41%	95
ML-PMHT				
7	0.82	0.039	69%	191
13	0.59	0.072	64%	125

6) Scenario 1 with Rayleigh clutter: All of the above scenarios were run with K-distribution clutter and a relatively high threshold (12 dB for scenarios 1-4 and 14 dB for scenario 5). Scenario 1 was redone with Rayleigh clutter, a lower initial target SNR (8 dB down from the other scenario 1 run) and a much lower threshold (2 dB). Results from this are shown in Table VII, along with tracking run examples in Figures 11 and 12. Here, ML-PDA seems to be slighly outperforming ML-PMHT in terms of in-track percentage,

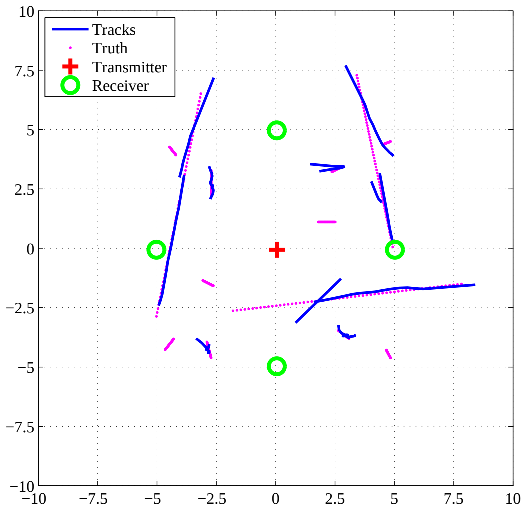

Figure 8. Scenario 4 ML-PDA estimated tracks

Table VI
SCENARIO 5 PERFORMANCE METRICS

Target #	Fragmentation	Duplicate Tracks	Percentage Time in Track	RMSE
ML-PDA				
1	0.81	0.28	78%	167
2	0.68	0.34	63%	148
ML-PMHT				
1	0.51	0.05	90%	112
2	0.51	0.03	77%	97

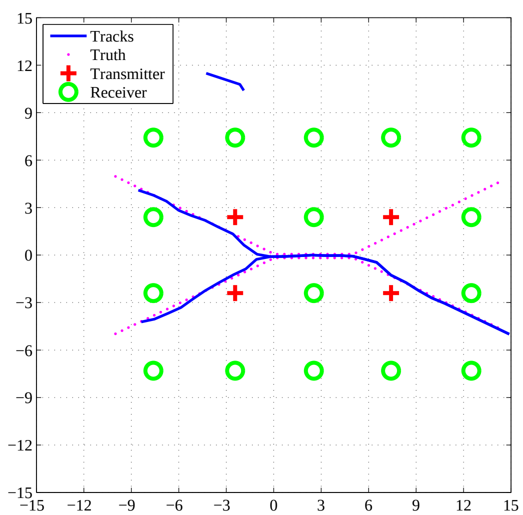

Figure 9. Scenario 5 ML-PMHT estimated tracks

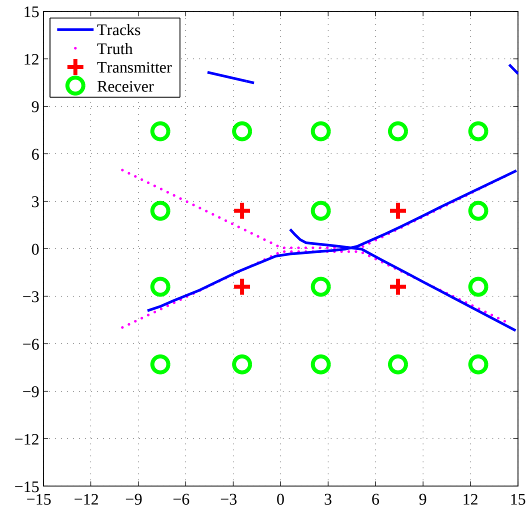

Figure 10. Scenario 5 ML-PDA estimated tracks

Table VII
SCENARIO 1 PERFORMANCE METRICS (RAYLEIGH CLUTTER)

Target #	Fragmentation	Duplicate Tracks	Percentage Time in Track	RMSE
ML-PDA				
1	0.73	3.3	76%	497
ML-PMHT				
1	0.68	1.9	71%	416

which would contradict the results from the other scenario 1 run. However, examining the false track results, shown in Table VIII, indicates why this might be the case. The false alarm rate (or average number of false tracks per run) was higher for ML-PDA than it was for ML-PMHT, indicating that the threshold used for declaring tracks for ML-PDA might have been set too low. These thresholds were set in accordance with [5] and [6] with the goal of having equivalent false track rates for the two algorithms. The thresholds were determined by simulating clutter-only enviroments, fitting the results to an Extreme-Value distribution [5], and then picking a threshold commensurate with an appropriate false track rate for each algorithm. This was done for all the scenarios; however, this scenario had a very high clutter level due to the low measurement thresholding, and therefore took a long to time to complete an individual run. As a result, a relatively small number of realizations were run (1000) for this scenario's clutter threshold determination compared with the other scenarios (5000-10000). For this level of clutter and low target SNR, even a small difference in the threshold makes a significant difference in the results, so inaccuracies in determining the threshold may be biasing the results.

IV. CONCLUSIONS

The ML-PDA and ML-PMHT tracking algorithms were applied to five different multistatic scenarios with Monte Carlo trials. Overall, both algorithms performed well. In several

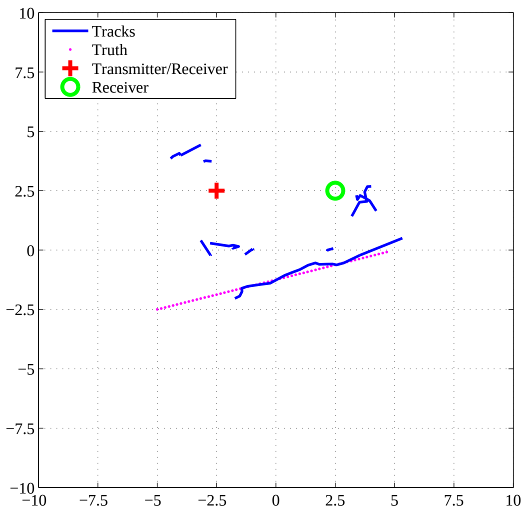

Figure 11. Scenario 1 ML-PMHT estimated tracks with Rayleigh clutter

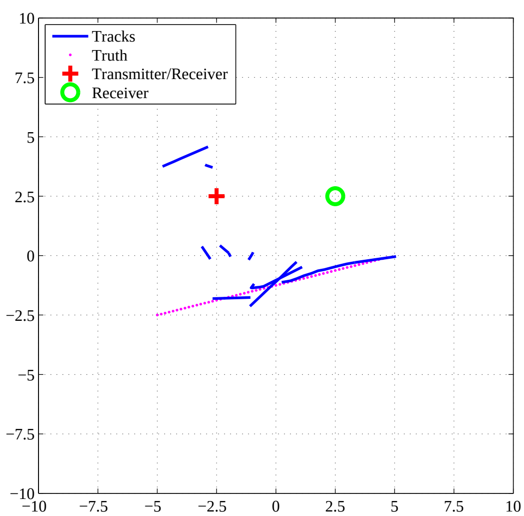

Figure 12. Scenario 1 ML-PDA estimated tracks with Rayleigh clutter

Table VIII
AVERAGE NUMBER OF FALSE TRACKS.

	ML-PDA	ML-PMHT
Scenario 1	0.07	0.045
Scenario 2	1.09	1.22
Scenario 3	0.29	0.40
Scenario 4	0.01	0.05
Scenario 5	2.49	4.26
Scenario 1 (Rayleigh)	7.62	3.73

of the scenarios where there was either only one target or the targets were separated for much of the time, ML-PMHT seemed to perform slightly better in terms of in-track percentage. However, although the ML-PMHT likelihood ratio is very naturally extended to multiple targets (while the ML-PDA likelihood ratio is not), ML-PMHT was outperformed by ML-PDA in the scenario where targets were closely spaced. This was due to the manner in which ML-PMHT was implemented, where an existing track could probabilistically claim a measurement from a new track. Future work will investigate how to use ML-PMHT as a true multitarget tracker by using the multitarget likelihood ratio to search for and declare the presence of multiple targets simultaneously.

REFERENCES

[1] D. Abraham, "Detection-threshold approximation for non-Gaussian backgrounds," *IEEE Journal of Oceanic Engineering*, vol. 35, no. 2, pp. 355–365, 2010.

[2] D. Abraham and A. Lyons, "Novel physical interpretations of K-distributed reverberation," *IEEE Journal of Oceanic Engineering*, vol. 27, no. 4, pp. 800–813, 2002.

[3] D. Avitzour, "A maximum likelihood approach to data association," *IEEE Trans. on Aerosp. and Electronic Syst.*, vol. 28, no. 2, pp. 560–566, 1996.

[4] Y. Bar-Shalom, P. Willett, and X. Tian, *Tracking and Data Fusion: A Handbook of Algorithms*. YBS Publishing, 2011.

[5] W. Blanding, P. Willett, and Y. Bar-Shalom, "Offline and real-time methods for ML-PDA track validation," *IEEE Trans. on Signal Processing*, vol. 55, no. 5, pp. 1994–2006, 2007.

[6] ——, "ML-PDA: Advances and a new multitarget approach," *EURASIP Journal on Advances in Signal Processing*, vol. 2008, pp. 1–13, 2008.

[7] W. Blanding, P. Willett, and S. Coraluppi, "Sequential ML for multistatic sonar tracking," in *Proc. Oceans 2007 Europe Conf.*, Aberdeen, Scotland, June 2007.

[8] M. R. Chummun, Y. Bar-Shalom, and T. Kirubarajan, "Adaptive early-detection ML-PDA estimator for LO targets with EO sensors," *IEEE Trans. on Aerosp. and Electronic Syst.*, vol. 38, no. 2, pp. 694–707, 2002.

[9] H. Cox, "Fundamentals of bistatic active sonar," BBN Systems and Technologies Corporation, Tech. Rep. W1038, 1988.

[10] A. Dempster, N. Laird, and D. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *Journal of the Royal Statistical Society, Series B (Methodological)*, vol. 39, no. 1, pp. 1–38, 1977.

[11] R. Georgescu, P. Willett, and S. Schoenecker, "GM-CPHD and MLPDA applied to the SEABAR07 and TNO-blind multi-static sonar data," in *Proc. of 12th Intern'l Conf. on Info. Fusion*, Seattle, WA, 2009.

[12] ——, "GM-CPHD and ML-PDA applied to the Metron multi-static sonar dataset," in *Proc. of 13th Intern'l Conf. on Info. Fusion*, Edinburgh, Scotland, 2010.

[13] C. Jauffret and Y. Bar-Shalom, "Track formation with bearing and frequency measurements in clutter," in *Proc. of the 29th Conf. on Decision and Control*, Honolulu, December 1990.

[14] T. Kirubarajan and Y. Bar-Shalom, "Low observable target motion analysis using amplitude information," *IEEE Trans. on Aerosp. and Electronic Syst.*, vol. 32, no. 4, pp. 1637–1382, 1996.

[15] S. Schoenecker, P. Willett, and Y. Bar-Shalom, "Maximum likelihood probabilistic data association tracker applied to bistatic sonar data sets," in *Proc. SPIE Conf. on Signal and Data Processing of Small Targets*, Orlando, FL, 2010.

[16] R. Streit and T. Luginbuhl, "A probabilisitic multi-hypothesis tracking algorithm without enumeration," in *Proc. 6th Joint Data Fusion Symposium*, Laurel, MD, June 1993.

[17] ——, "Maximum likelihood method for probabilistic multi-hypothesis tracking," in *Proc. SPIE Conf. on Signal and Data Processing of Small Targets*, Orlando, FL, 1994.

[18] ——, "Probabilistic multi-hypothesis tracking," Naval Undersea Warfare Center, Tech. Rep. TR 10428, 1995.

[19] P. Willett and S. Coraluppi, "Application of the MLPDA to bistatic sonar," in *Proc. of IEEE Aerosp. Conf.*, Big Sky, March 2005.

[20] ——, "MLPDA and MLPMHT applied to some MSTWG data," in *Proc. 9th Intern'l Conf. on Info. Fusion*, Florence, Italy, July 2006.

[21] P. Willett, S. Coraluppi, and W. Blanding, "Comparison of soft and hard assignment ML trackers on multistatic data," in *Proc. of IEEE Aerosp. Conf.*, Big Sky, March 2006.