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## 14. ABSTRACT

Calculating the reflected irradiances produced by a specularly reflecting object at many observation points is computationally intensive, the total computational load proportional to the product of the number of facets times the number of observation points. In order to capture specular glints at all observation points, it is necessary to finely discretize the surface of the object into a large number of facets. This can result in a massive number of computations.
The computational load can be reduced by approximating the surface of the object by curved triangular facets modeled as either quadric surfaces or point-normal triangles. Starting with a coarse discretation of the surface, a finer representation can be produced by subdividing the initial facets. For a single observation point, only a small fraction of the surface contributes to the specular glint; therefore only a few facets need to be significantly subdivided for accurate computations. By adaptively subdividing, the number of facets required per observation point is greatly reduced, resulting in fewer computations and thus increased overall computational speed. The speed increase is illustrated for a cylindrical object and different angular widths of the specular peak. As the width decreases, adaptive faceting increases the computational savings.

## 15. SUBJECT TERMS

Reflection modeling, Adaptive modeling, BRDF, Quadric surface, PN triangle

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# Adaptive Facet Reflection Modeling 

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#### Abstract

Calculating the reflected irradiances produced by a specularly reflecting object at many observation points is computationally intensive, the total computational load proportional to the product of the number of facets times the number of observation points. To capture specular glints at all observation points, it is necessary to finely discretize the surface of the object into a large number of facets. This can result in a massive number of computations. The computational load can be reduced by approximating the surface of the object by curved triangular facets modeled as either quadric surfaces or point-normal triangles. Starting with a coarse discretation of the surface, a finer representation can be produced by subdividing the initial facets. For a single observation point, only a small fraction of the surface contributes to the specular glint; therefore only a few facets need to be significantly subdivided for accurate computations. By adaptively subdividing, the number of facets required per observation point is greatly reduced, resulting in fewer computations and thus increased overall computational speed. The speed increase is illustrated for a cylindrical object and different angular widths of the specular peak. As the width decreases, adaptive faceting increases the computational savings.


KEYWORDS: Adaptive modeling, BRDF, PN triangle, Quadric surface, Reflection modeling

## Nomenclature

| $E_{O}$ | light intensity at the observation position from a facet |
| :--- | :--- |
| $E_{O_{n}}$ | light intensity at the observation point from subfacet $n$ |
| $f_{r}$ | bidirectional reflection distribution function (BRDF) |
| $\widehat{H}$ | unit vector bisecting the incident and reflection directions |
| $\hat{n}$ | surface unit normal |
| $P$ | variable vector |
| $Q$ | quadric coefficient matrix |
| $q_{x x}, q_{x y}, q_{y y}, \ldots$ | quadric equation coefficients |
| $\boldsymbol{x}_{C}$ | center position of the facet |

[^0]$\boldsymbol{x}_{O}$ observation position
$\beta \quad$ angle between the halfway vector $\widehat{H}$ and the incident and reflected angles
$\theta_{i} \quad$ polar angle of the incident light
$\theta_{N} \quad$ angle between the halfway vector $\widehat{H}$ and the surface normal
$\theta_{r}$ polar angle of the reflected light
$\Xi \quad$ microfacet tilt distribution function
$\sigma \quad$ specular peak width parameter
$\Phi_{A} \quad$ luminous power on a facet
$\phi_{\mathrm{i}} \quad$ azimuthal angle of the incident light
$\phi_{\mathrm{r}} \quad$ azimuthal angle of the reflected light

## 1. Introduction

Consider an object that is illuminated by a light source, such as a laser. If the object is diffusely reflecting, the light reflected toward a specific observation point will come from all parts of the object that are illuminated by the light source and observable from the point of view at the observation point. If, on the other hand, the object is shiny, most of the light at the observation point will come from the areas of the illuminated object where the direction of the specular reflection from the object is toward the observation point. The portions of the object where the specular reflection is in some other direction will contribute comparatively little light.

The most general way to model the light reflecting from an object is to subdivide its surface into a large number of small facets and compute the reflection from each facet. If a great many facets are used, the time required for numerical computations will be large. For a given observation point, the faceting does not need to be highly refined except near the areas of the surface that produce specular glints toward that observation point; except near the specular direction the reflected light is a weak function of the angle and coarse faceting suffices to give reasonable accuracy. However, near the portions of the object that produce specular glints, it will be necessary to finely tessellate the object for accurate results. Thus, by faceting most of the surface coarsely and using fine faceting only near the areas of glint, a considerable reduction in the required computation can be achieved.

If one is considering many observation points or a time-dependent situation in which the light source, illuminated object, or observers are moving, then different portions of the object will be areas of glint for different observers at different times. To ensure that all of the specular glints are accurately resolved, it will be necessary to finely tessellate much or all of the illuminated object, even though for a given observer at a given time only a small portion of the surface needs to be finely resolved.

A time-saving alternative is to use adaptive faceting: model the surface with coarse facets that can be subdivided to more accurately model the surface when needed. For a given coarse facet, the level of subdivision that is used will be different for each observation point and time. Only a small number of the coarse facets will need to be significantly subdivided, greatly reducing the computation time required.

This adaptive faceting technique has been used in the High Energy Laser Collateral Assessment Tool (HELCAT) for modeling the hazards due to laser light reflected off laser targets. The technique significantly reduces the time required for computational analysis of a scenario.


Fig. 1. A triangle can be subdivided into four smaller triangles.

## 2. Methodology

For reasons to be discussed later in this section, a triangular surface mesh has been used, though similar techniques could also be implemented using a rectangular mesh. Consider a given coarse facet reflecting light from a light source to a given observation point at a specific time. One would first determine the estimate of the reflected light using this coarse faceting. Consider a triangular facet defined by the three vertices $V_{1}, V_{2}$, and $V_{3}$. Let $\Phi_{A}$ be the luminous power on a facet and $\boldsymbol{x}_{C}$ be the center point defined as the position that is the average of the three vertex positions. The intensity of the light at the observer position $x_{O}$ reflected from the facet is

$$
\begin{equation*}
E_{O}=\frac{\Phi_{A} f_{r}\left(\theta_{i}, \phi_{i}, \theta_{r}, \phi_{r}\right) \cos \left(\theta_{i}\right) \cos \left(\theta_{r}\right)}{\left(\boldsymbol{x}_{C}-\boldsymbol{x}_{O}\right)^{2}} \tag{1}
\end{equation*}
$$

where $f_{r}$ is the bidirectional reflectance distribution function (BRDF), dependent on the incident polar $\left(\theta_{i}\right)$ and azimuthal ( $\phi_{i}$ ) angles and reflected polar $\left(\theta_{r}\right)$ and azimuthal ( $\phi_{r}$ ) angles as measured relative to the normal to the facet and some chosen direction in the facet plane indicating the material anisotropy, such as the machining grooves in rolled sheet metal. Here the facet normal is taken to be the normal to the plane defined by the three vertices of the facet, even though the actual surface may be curved.

A more accurate estimate of the intensity of the light is obtained by subdividing this initial coarse facet into four subfacets as shown in Fig. 1. The intensity can then be computed as

$$
\begin{equation*}
E_{O}^{\prime}=\sum_{n=1}^{4} E_{O_{n}}, \tag{2}
\end{equation*}
$$

where $E_{O_{n}}$ is the value for $E_{O}$ as computed for subfacet $n$. One can then compute the change in the estimated value as $\Delta E_{O}=\left|E_{O}-E_{O}^{\prime}\right|$. If this change is small, then no further refinement is needed. If it is significant, then each of the subfacets should be refined as well. The process can be continued recursively until the residual error is acceptably small (Fig. 2).

Unless the surface of the illuminated object is flat, the subdivision points $V_{12}, V_{23}$, and $V_{31}$ will not lie in the plane of the original facet and the subfacets will all have different normals. For the subdivision process to give accurate results, an interpolation scheme must


Fig. 2. The faceting can be hierarchically refined to as many levels as are needed for accurate computations.
be available to determine the values of the subdivision points. Two different schemes have been investigated: quadric surfaces and point-normal triangles.

A quadric surface is any surface defined by a general quadratic equation in $x, y, z$ :

$$
\begin{equation*}
q_{x x} x^{2}+q_{y y} y^{2}+q_{z z} z^{2}+2 q_{x y} x y+2 q_{x z} x z+2 q_{y z} y z+2 q_{x} x+2 q_{y} y+2 q_{z} z+q_{0}=0 \tag{3}
\end{equation*}
$$

or

$$
\begin{equation*}
P^{T} Q P=0 \tag{4}
\end{equation*}
$$

where

$$
Q=\left[\begin{array}{llll}
q_{x x} & q_{x y} & q_{x z} & q_{x}  \tag{5}\\
q_{x y} & q_{y y} & q_{y z} & q_{y} \\
q_{x z} & q_{y z} & q_{z z} & q_{z} \\
q_{x} & q_{y} & q_{z} & q_{o}
\end{array}\right]
$$

and

$$
P=\left[\begin{array}{l}
x  \tag{6}\\
y \\
z \\
1
\end{array}\right] .
$$

Quadric surfaces can represent many common geometrical shapes, including planes, cylinders, spheres, cones, parabolas, and ellipsoids. If a quadric surface is associated with a coarse facet, it is simple to compute the subdivision points to construct the subfacets.

Most three-dimensional model formats used by solid modeling programs represent objects as triangles in which the normals at the vertex points are separately specified and not simply defined by the normals to the flat surface defined by the three vertex points. These are referred to as point-normal triangles or PN triangles. In 2001 Vlachos et al. presented a scheme to construct a cubic Bézier patch with quadratically varying normals to interpolate the surface position and normal vectors over the triangle in a computationally efficient manner. ${ }^{2}$ In situations in which the actual surface shape is not specified for a triangular patch, this represents an excellent means of approximating the surface.

Because a quadric surface has nine independent variables and three vertices with three normals provide nine constraint equations, it should be possible to solve for a quadric
i


Fig. 3. The unit vector $\widehat{H}$ bisects the source and observer directions. The half-angle $\theta_{N}$ is the angle between $\widehat{H}$ and the surface normal $\widehat{n}$. The angle between $\widehat{H}$ and the source and observer directions is given by $\beta$.

\# of Reflection Calculations
Fig. 4. Number of reflection calculations required when using adaptive faceting (solid lines) versus nonadaptive faceting (dashed lines).
surface defined by a set of vertex points with specified normals. This has been tried, and sometimes it works well. However, frequently the solution matrix is singular and a unique solution cannot be found. PN triangles do not require a matrix solution and work better for modeling cases in which the surface shape is not known a priori.

## 3. Example Case

A test case was simulated for light reflection from a cylindrical object using the HELCAT code. The object was tessellated using quadric surface elements, allowing the surface to approach an ideal cylinder as the surface is refined. The case examined was for a fixed configuration with 186 observation points. A simple but physically reasonable microfacet

BRDF was used ${ }^{1}$ :

$$
\begin{equation*}
f_{r}\left(\theta_{i}, \theta_{r}\right)=\frac{\Xi\left(\theta_{N}\right)}{4 \cos \theta_{i} \cdot \cos \theta_{r}} \tag{7}
\end{equation*}
$$

where

$$
\begin{equation*}
\Xi\left(\theta_{N}\right)=\frac{\exp \left(-\tan ^{2} \theta_{N} / 2 \sigma^{2}\right)}{2 \pi \sigma^{2} \cos ^{4} \theta_{N}} \tag{8}
\end{equation*}
$$

is the normalized microfacet tilt distribution function and $\theta_{N}$ is the angle between the surface normal and the vector bisecting the incident and reflected directions (Fig. 3).

The tests varied the specular peak width parameter, $\sigma$, and the error tolerance. The results are shown in Fig. 4. For a narrow specular peak ( $\sigma=0.003$ ), the adaptive faceting resulted in an approximately 20 -fold savings in the number of reflection calculations required. For a broader specular peak ( $\sigma=0.03$ ), adaptive faceting gave only about a twofold savings in the required reflection calculations.

## 4. Conclusions

Adaptive faceting can considerably reduce the number of reflection calculations required for modeling the illumination at a point from the light reflected from a surface with a narrow specular peak. In the absence of a narrow peak, the benefits are small.

Similar reductions in computational requirements might be possible when the effective illumination is required over a large area or volume by using a similar scheme with an adaptive number of observation points. Hierarchical refinement would allow for refining the observer spacing only as required for specific sets of reflective facets and observation points.

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## References

[^1]
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