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Efficient Multiple Hypothesis Tracking by Track Segment Graph*

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Abstract – *Multiple hypothesis tracking (MHT) addresses difficult tracking problems by maintaining alternative association hypotheses until enough good data, e.g., features, are collected to select the correct hypotheses. Traditional MHT's cannot track targets over long durations because they frequently generate too many hypotheses to maintain the correct ones with the available processing resources. Track segment graph provides a compact and efficient representation of the key ambiguities in long term tracking. It is used to generate the long term track hypotheses that are evaluated to select the best long term global hypothesis. Simulation examples demonstrate the efficiency and optimality of the approach.*

Keywords: Multiple hypothesis tracking, track segment graph, long term tracking, feature aided tracking.

1 Introduction

Tracking targets over a long period of time is difficult due to possible sensing gaps, high target densities, etc. The standard approach for addressing such difficult tracking problems is multiple hypothesis tracking (MHT) [1, 2]. In MHT, association decisions are deferred when there is not enough information to make a good decision. Instead, multiple association hypotheses are maintained over multiple frames of data until enough good data are collected.

There are two main approaches to MHT. The hypothesis-oriented approach, first introduced in [3] and generalized in [4, 5], recursively finds a set of (global) association hypotheses and their probabilities. On the other hand, the track-oriented approach [6–10] is based on finding the best multi-dimensional or multi-frame data association hypothesis by {0,1} integer programming [6], Lagrangian relaxation [9], or other techniques. An extension of the track-oriented approach to find the K-best hypotheses can be found in [11].

In order to avoid the combinatorial explosion of hypotheses, all MHT's use hypothesis management techniques to reduce the number of hypotheses. In particular, the tracks or track hypotheses in the track-oriented approach are pruned after a certain window or frame size is reached. The result is that outside the pruning window, only the tracks in the best association hypothesis are retained as confirmed tracks, and alternative track hypotheses may no longer be available when good data are collected.

Features can help long term track maintenance by moving the association problem into a higher dimensional space where there may be enough separation for better performance [12, 13]. Maintaining hypotheses for long durations is especially critical in feature aided tracking because feature data are not always available due to sensor coverage and the resources needed to observe or process them. Thus, it is important for a MHT algorithm to maintain enough hypotheses until feature reports are collected.

This paper presents a new multiple hypothesis tracking approach that uses an efficient representation of alternative long term track hypotheses by a track segment graph. This representation is based on three ideas. The first is using track segments instead of reports as the basic building blocks of track hypotheses. The second is representing ambiguous track segments of multiple targets in addition to pure track segments of single targets. The third is using a graph to represent and store possible associations between track segments instead of generating and storing all possible track hypotheses.

The track segment graph will be used to generate the relevant long term track hypotheses only when needed. These track hypotheses are scored using the track segment association likelihoods. From these track likelihoods, the best or K-best (global) association hypotheses are selected. The benefit is a significant reduction in the number of track hypotheses so that the alternative hypotheses can be maintained for long durations.

The rest of this paper is structured as follows. Section 2 summarizes the basic elements of multiple hypothe-

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14. ABSTRACT
Multiple hypothesis tracking (MHT) addresses difficult tracking problems by maintaining alternative association hypotheses until enough good data e.g., features, are collected to select the correct hypotheses. Traditional MHT?s cannot track targets over long durations because they frequently generate too many hypotheses to maintain the correct ones with the available processing resources. Track segment graph provides a compact and efficient representation of the key ambiguities in long term tracking. It is used to generate the long term track hypotheses that are evaluated to select the best long term global hypothesis. Simulation examples demonstrate the efficiency and optimality of the approach.

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sis tracking. Section 3 introduces the track segment graph and discusses the formation of long term track hypotheses from the track segment graph. Section 4 presents the algorithm for scoring long term track hypotheses. Section 5 describes an extended duration MHT system based upon the track segment graph approach. Section 6 presents simulation results to illustrate the approach, and Section 7 contains the conclusions.

2 Multiple hypothesis tracking

The goal of multiple target tracking is to estimate the states of moving objects or targets from sensor reports. In general, sensor reports do not contain the identity of the targets and some may be false returns. Furthermore, targets may not be observed as sensor reports when the probability of detection is less than 1. Thus a crucial component of any multiple target tracking algorithm is the association of reports to form possible target tracks.

2.1 Hypotheses formation and selection

Let $y_1, y_2, \ldots$, be a sequence of *frames* (or *scans*), where each frame $y_k = (y_{k1}, \ldots, y_{km_k})$ at t_k is a set of *measurements* or *reports* y_{kj}. Given a cumulative set of frames $(y_i)_{i=1}^k = (y_1, \ldots, y_k)$, a *track* or *track hypothesis* on $(y_i)_{i=1}^k$ is a function τ from $\{1, \ldots, k\}$ to $\{0, 1, \ldots, m_k\}$. If $j > 0$, $\tau(k) = j$ states that the *j*-th report in the *k*-th frame originates from a target hypothesized by track τ. If $j = 0$, the target is not detected in the *k*-th frame. A *data association hypothesis* or *global hypothesis* λ on $(y_i)_{i=1}^k$ is a set of non-overlapping, non-empty tracks, i.e., track hypotheses in a global hypothesis do not share reports.

Under fairly standard assumptions such as no split or merged reports, the posterior probability of each data association hypothesis λ on $(y_i)_{i=1}^k$ is given by

$$P(\lambda \mid (y_i)_{i=1}^k) = C^{-1} \left(\prod_{\tau \in \lambda} L(\tau) \right) \left(\prod_{j \notin \bigcup_{\tau \in \lambda} \{\tau(k)\}} \gamma_{FA}(k, j) \right) \quad (1)$$

where $C > 0$ is a normalizing constant, $L(\tau)$ is the likelihood of the track τ, and $\gamma_{FA}(k, j)$ is the density of false alarms contained in the *k*-th frame evaluated at the value y_{kj} of the *j*-th report.

By taking the logarithm of (1), the maximization of the a posteriori probability $P(\lambda \mid (y_i)_{i=1}^k)$ is transformed into the optimization problem.

$$\begin{array}{ll} \text{Minimize} & c^T x \\ \text{subject to} & x \in \{0,1\}^n \\ \text{and} & Ax = b \end{array} \quad (2)$$

The vector c contains element c_j's which are the logarithms of the likelihoods $L(\tau_j)$ for track τ_j. The constraint $Ax = b$ states that tracks in a single hypothesis cannot have the same reports. A is a $m \times n$ zero-one matrix, where m is the number of reports and n is the number of tracks plus slack variables for false alarms (the set of all the report y_{kj} for which $\gamma_{FA}(k, j) > 0$). The element $A_{ij} = 1$ if the *i*-th report is included in the *j*-th track or is a false alarm. The vector b is *m*-dimensional with elements that are all 1's. A feasible solution $x \in \{0,1\}^n$ is a global hypothesis λ such that $x[j] = 1$ if and only if the *j*-th track is included in hypothesis λ.

2.2 Hypothesis management

In track-oriented MHT, tracks are expanded and propagated to recursively process each frame of data. However, data association hypotheses are not maintained or propagated explicitly, and only constructed from the tracks when needed. Thus, the hypothesis space consists of tracks on the past cumulative frames. Without hypothesis management, the number of track hypotheses will grow at an exponential rate with the number of frames.

Therefore, track pruning is an essential part of any track-oriented MHT algorithm. The set of tracks and hypotheses defined on all past cumulative frames of data are naturally ordered by the predecessor-successor relationships. In this partial ordering, the tracks and hypotheses form a tree structure.

The basic hypothesis management technique in track-oriented MHT is *N-scan pruning*. Suppose hypotheses and tracks are formed on frames up to the frame K. Conventional *N*-scan pruning is based on selecting a single best hypothesis $\hat{\lambda}$ and pruning all the tracks τ that do not share the predecessor tracks at the (*K*-*N*) - th frame with the tracks in the best hypothesis λ. Equivalently, every hypothesis λ formed on frames up to frame K are pruned away if the predecessor of λ is not the predecessor of the best hypothesis $\hat{\lambda}$. Using this pruning strategy, any track initiated in frames between frames $K - N + 1$ and K are protected. However, only confirmed tracks are maintained outside the *N*-scan window.

3 Track segment graph

In the traditional track-oriented MHT with *N*-scan pruning, alternative track hypotheses are only maintained within the pruning window, and only confirmed tracks are retained outside of the window. Unless the window is very large, alternative hypotheses needed to exploit disambiguating data such as features will be lost. The track segment approach addresses this problem by representing track hypotheses in terms of track segments so that the same *N*-scan corresponds to longer time duration.

3.1 Track segments

Figure 1 shows an example of three vehicles moving through two ambiguous regions (1 and 2) where their paths almost overlap, thus making data association difficult. Measurements consist of initial video reports about vehicle color followed by GMTI measurements, and then video reports again at the end of the scenario. Because the GMTI measurements cannot be used to distinguish between the tracks in the ambiguous regions, video reports are needed to associate the tracks correctly. More specifically, alternative track hypotheses with features have to be maintained until the video reports are collected. This is difficult with standard MHT because many track hypotheses will be generated in the ambiguous regions. In fact, each possible association of measurement to track will generate a new track hypothesis. The result is that the correct track hypotheses may be pruned long before the feature reports are available.

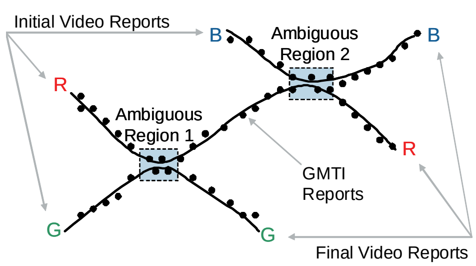

Figure 1: Three targets move through ambiguous regions

A more efficient approach is to use track segments instead of reports as the basic building blocks of track hypotheses. The track segment graph is a directed graph where the nodes are track segments. There are two types of track segments. Pure track segments contain only reports from single targets while ambiguous track segments contain reports from multiple targets. Edges connect track segments that can be associated. A directed edge connects two segments if the start time of the successor segment is larger than the end time of the predecessor segment and the two track segments can be associated. A directed path of the track segment graph corresponds to a long term track hypothesis. The track segment graph representation of the problem in Figure 1 is shown in Figures 2 and 3. The pure track segments are 1 to 7, and the ambiguous track segments are A1 and A2.

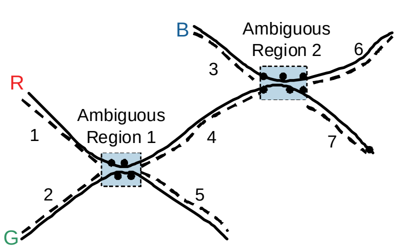

Figure 2: Sensor reports are associated into pure and ambiguous track segments

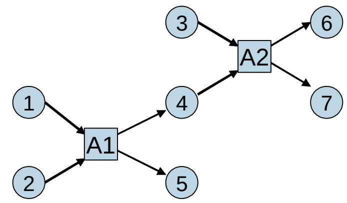

Figure 3: Track segment graph represents possible associations

3.2 Track segment representation

The representation of a pure or ambiguous track segment S_i with measurements Y_i contains the following information: start time t_i^s, end time t_i^e, start time state estimate $p(x_i^s \mid Y_i) \triangleq p(x(t_i^s) \mid Y_i)$, end time state estimate $p(x_i^e \mid Y_i) \triangleq p(x(t_i^e) \mid Y_i)$, track segment likelihood $L(S_i) \triangleq p(Y_i)$, and state estimates at various sampling times in the track segment.

All the estimates are conditioned on all measurements in the track segment. Thus the start time state estimate is a smoothed estimate while the end time estimate is a filtered estimate. Each state estimate has a kinematic and a feature component. The feature state estimate is usually given by a conditional probability distribution. For a pure track segment with a single target, a Kalman filter or smoother is used to generate the kinematic state estimate and its error covariance from the measurement reports associated with the track segment. For an impure track segment, we need to generate a state estimate for all the ambiguous targets using all the measurement reports. One approach is to merge the measurements at a single time into a single measurement and use it to update a group track with multiple targets.

There are several ways of representing merged measurements. The simplest approach is to use one of the measurements as a surrogate for the merged measurements. This is very simple but may introduce some bias depending on the measurement selected. Alternatively, a convex combination of the probability distribution of the measurements retains all the information about the measurements. If each measurement has a Gaussian distribution, then the merged measurement has a multi-modal distribution as a sum of Gaussians. This approach is not too practical because the number of modes in the track estimate grows with the number of updates with measurements. A more practical approach is to approximate the multi-modal distribution with a single Gaussian. This is similar to the approach used in probabilistic data association (PDA) filters.

Another way of representing the state of ambiguous track segments is by means of probability hypothesis density (PHD) [14]. Probability hypothesis density has been a very active area of research for the past few years because it solves the multi-target tracking problem without trying to associate the measurements to the individual targets. Instead of forming tracks for individual targets, PHD generates the a posteriori intensity measure density. It is not a probability density because its integral over the entire state space is not 1. Instead, the integral of the PHD over a region is the expected number of targets in the region. The benefit of PHD is that it may provide a more accurate state estimate for track segments than a single Gaussian.

3.3 Formation of long term track hypotheses from track segment graph

The track segment graph provides an efficient implicit representation for the long term track hypotheses. For the example in Figure 1, all the information needed to generate the long term track hypotheses is contained in the track segment graph of Figure 3. In general, only a portion of the track segment graph is used when reports are received. For example, when feature reports are received, the track segment graph is searched backwards to look for track segment nodes with the relevant features.

For the example of Figure 1, the relevant portion is the entire graph shown. Starting with the earliest nodes of the graph in Figure 3, the alternative long term track hypotheses built from track segments are (1, A1, 5), (1, A1, 4, A2, 6), (1, A1, 4, A2, 7), etc. In general, pure track segments are separated by ambiguous segments, but ambiguous track segments may be followed by other ambiguous track segments. In this case, eight long term track hypotheses are generated from the graph.

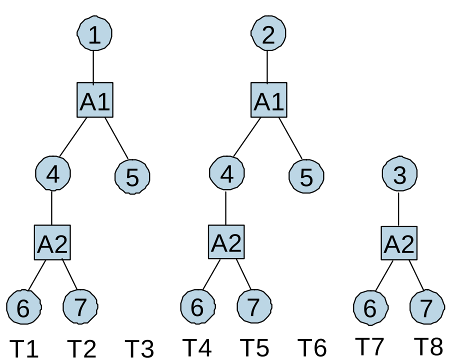

Figure 4: Long term track hypotheses are formed from track segments graph

4 Hypothesis evaluation and selection

After the long term track hypotheses are formed, their likelihoods are computed. Then the posterior probability of a global hypothesis can be evaluated from these long term track likelihoods. The best global hypothesis is found by solving an optimization problem to maximize the posterior probability.

4.1 Long term track likelihoods

For simplicity, we assume no false alarms. Consider a long term track (hypothesis) T with N track segments, S_i, $i=1,...,N$, i.e., $T=(S_1,..,S_i,...,S_N)$. The long term track likelihood is given by (see Appendix for derivation)

$$L(T)=\prod_{i=1}^{N}L(S_i)\prod_{i=2}^{N}L(i,i-1) \qquad (3)$$

where $L(S_i)$ is the likelihood of the track segment S_i and $L(i,i-1)$ is the association likelihood between the track segment S_i and the track $S^{i-1}=(S_1,..,S_{i-1})$.

The track segment likelihood $L(S_i)$ is $p(Y_i)$, the probability of the measurements in Y_i. The track segment association likelihood is given by

$$L(i,i-1)=\int\frac{p(x_i^s\,|\,Y_i)\,p(x_i^s\,|\,Y_1,..,Y_{i-1})}{p(x_i^s)}dx_i^s \qquad (4)$$

where $p(x_i^s\,|\,Y_i)$ is the smoothed estimate of the start state x_i^s given the track segment S_i, and $p(x_i^s\,|\,Y_1,..,Y_{i-1})$ is the prediction of $x(t_i^s)$ given the measurements in the long term track $S^{i-1}=(S_1,..,S_{i-1})$. The integral of the product $p(x_i^s\,|\,Y_i)\,p(x_i^s\,|\,Y_1,..Y_{i-1})$ evaluates the similarity between the smoothed estimate of the start state x_i^s given the track segment S_i and the predicted estimate of x_i^s given the track segments $S_1,..,S_{i-1}$. If x is a kinematic state for a maneuvering target, the predicted estimate depends mostly on the measurements in the most recent track segment, i.e., $p(x_i^s\,|\,Y_1,..,Y_{i-1})\simeq p(x_i^s\,|\,Y_{i-1})$. In this case, $L(i,i-1)$ depends only on the track segments S_i and S_{i-1} and measures the similarity between the start time state estimate of S_i and the end time state estimate of S_{i-1}.

4.2 Global hypothesis evaluation/selection

Just as in traditional MHT, the posterior probability of a global (long term) hypothesis λ is given by

$$P(\lambda\,|\,Y)=C^{-1}\prod_{T\in\lambda}L(T) \qquad (5)$$

where C is again a normalizing constant and $L(T)$ is the long term track likelihood given by (3). As before, the optimization problem to select the best (global) hypothesis can be converted into the 0-1 integer programming problem (2).

However, the constraint of the problem is different because while a report in traditional MHT can only belong to one track hypothesis assuming no split reports from a target, an ambiguous track segment of two targets can belong to two track hypotheses in a single global hypothesis. This new constraint can be expressed by modifying the constraint vector b in (2), which is now allowed to have values besides 1. For a two-target ambiguous segment S_i, the corresponding element in b is $b_i = 2$. Similarly, for a three-target ambiguous track segment, $b_i = 3$. Thus, hypothesis selection with both pure and ambiguous track segments is a multi-assignment problem where a single track segment may be assigned to multiple long term track hypotheses.

For the long term track hypotheses shown in Figure 4, the matrices in the constraint equation are given by equation (6). Each of the pure (single target) track segments 1 to 7 is constrained to belong to a single track in a global hypothesis but the impure (two target) track segments can belong to two tracks.

$$A=\begin{array}{c}\text{Tracks}\\ \begin{array}{cccccccc}1&2&3&4&5&6&7&8\end{array}\\ \left[\begin{array}{cccccccc}1&1&1&0&0&0&0&0\\0&0&0&1&1&1&0&0\\1&1&1&1&1&1&0&0\\0&0&0&0&0&0&1&1\\1&1&0&1&1&0&0&0\\0&0&1&0&0&1&0&0\\1&1&0&1&1&0&1&1\\1&0&0&1&0&0&1&0\\0&1&0&0&1&0&0&1\end{array}\right]\end{array}\begin{array}{c}\\ \\ 1\\2\\A1\\3\\4\\5\\A2\\6\\7\end{array}\quad b=\begin{bmatrix}1\\1\\2\\1\\1\\1\\2\\1\\1\end{bmatrix}\tag{6}$$

5 Extended duration MHT

The track segment graph representation is used to develop an extended duration MHT (EDMHT) system that can maintain hypotheses for long periods of time to incorporate disambiguating data.

5.1 Architecture

Figure 5 shows the architecture of the EDMHT system based on the track-oriented MHT approach. It consists of a standard track-oriented MHT and an extended duration addition. The standard MHT performs the hypothesis generation and management operations. The extended duration addition converts the long term track hypotheses into the efficient track segment graph representation for storage until they are needed. It also retrieves the relevant track segments and generates long term track hypotheses for the MHT to process when feature or asynchronous reports are received.

Measurements without feature information are processed in the standard MHT (bottom portion of Figure 5) The *Generate / Update Track Hypotheses* function initiates new track hypotheses and propagates the states of the current (managed) track hypotheses to the time of the input measurements. It forms possible associations of the measurements to the tracks, resulting in a set of track hypotheses. The *Manage Track Hypotheses* function computes the likelihood of each track hypothesis and finds the best global hypothesis. For a fixed *N-scan* pruning window, the tracks in the best global hypothesis above the window are the confirmed tracks. The other tracks within the window that do not share the same predecessors with the tracks in the best global hypotheses are then pruned. Since multiple track hypotheses are not maintained outside the pruning window, the standard MHT does not represent long term ambiguities.

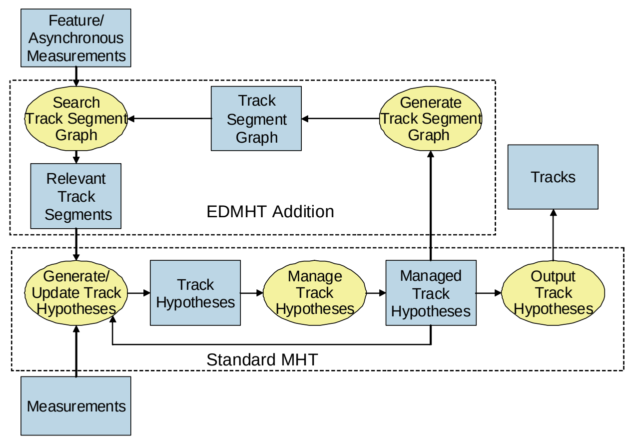

Figure 5: Extended duration MHT architecture consists of standard MHT and an extended duration addition

The *Generate Track Segment Graph* function converts the confirmed (long term) portions of the track hypotheses into a track segment graph that summarizes alternative associations of track segments. This graph allows alternative long term track hypotheses to be stored efficiently. Figures 3 and 4 show the (historical) long term track hypotheses and an equivalent track segment graph that contains the same information. Each branch of the tree is a historical track hypothesis and corresponds to one path over the graph.

Note that the extended duration addition is a separate process that runs in parallel with the standard MHT to store the long term ambiguity information in an efficient track segment graph. This ambiguity information is used to process feature or asynchronous (out of sequence) reports when they are received.

When feature reports are received, the track segment graph is searched for relevant parts of the graph that contains track segments with that feature. For the example of Figure 1, these can be the last times that feature reports are available. *Generate / Update Track Hypotheses* then forms the long term track hypotheses, each with a current portion from measurements in the pruning window and historical portion from track segments (Figure 6).

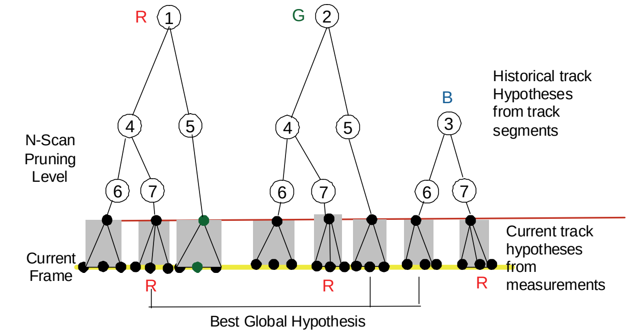

Figure 6: Long term track hypotheses consist of historical track hypotheses from track segments and current track hypotheses from measurements.

These long term track hypotheses are scored using the individual track segment likelihoods, the track segment association likelihoods, and the measurement to track association likelihoods. *Manage Track Hypotheses* then selects the best global long term hypothesis and performs pruning operations. The updated track hypotheses are converted into track segments and stored in the track segment graph and the process is repeated.

Asynchronous or time-late reports may have just kinematic information or both kinematic and feature information. When asynchronous reports are received, the *Search Track Segment Graph* function will again identify the relevant part of the track segment graph from the temporal and state (kinematic and feature) information in the report. This step is similar to the search function performed when feature reports are received. Because the time of the report may be different from the start or end times of the track segments, finding the state estimate at arbitrary times in a track segment is necessary. This is easy if the track segment graph retains the state estimates at other times between the start and end times. Otherwise, the state estimates will be interpolated from the start and end state estimates of a track segment.

Once the relevant track segments are retrieved, the *Generate / Update Track Hypotheses* function forms the long term track hypotheses that can be scored to find the best global hypothesis by *Manage Track Hypotheses*.

6 Simulation results

In this section we present simulation results to demonstrate the benefits of the track segment approach for long term MHT.

6.1 Efficiency of long term track hypotheses representation

By using track segments, the number of long term track hypotheses is reduced significantly. We considered two examples. The first example is that of Figure 1 with only three targets and two ambiguous regions. The second example consists of 400 vehicles moving in an urban environment. There are many ambiguous regions due to targets passing each other at low velocity on roads. In both examples, the measurement rate is one second.

In each example, we generate the track hypotheses directly from the reports. We also convert the reports into track segments (pure and ambiguous) and generate the track hypotheses from the track segments. Figure 7 compares the numbers of track hypotheses for the simple example.

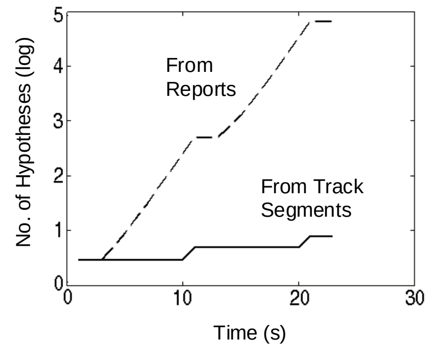

Figure 7: Comparison of numbers of track hypotheses for 3 target example

The number of track hypotheses using reports (dotted line) grows much more rapidly compared to that using track segments. In fact, because there are only two ambiguous regions, the number of tracks formed from track segments (solid line) only changes twice. On the other hand, the dotted line always increases because of association at the report level.

Figure 8 contains the comparison for the more complicated scenario. In this case, even though the long term hypotheses using the track segment graph (solid line) approach grows, its growth rate is much lower than the track hypotheses formed from reports (dotted line). The solid line always increases because targets are always moving into ambiguous regions.

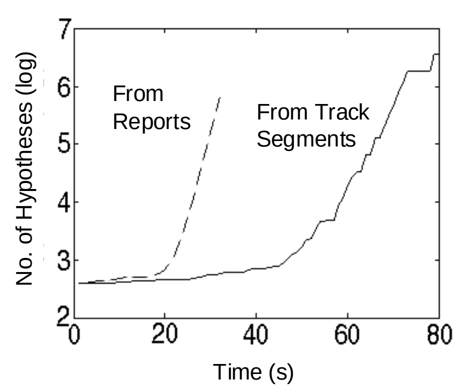

Figure 8: Comparison of numbers of track hypotheses for 400 target example

Both examples show that the EDMHT can maintain the crucial alternative hypotheses across the ambiguous regions with the track segment approach because there are far fewer track hypotheses. On the other hand, there are so many track hypotheses using reports that many alternative hypotheses cannot be maintained across the ambiguous regions.

6.2 Optimality of hypothesis scoring

We conducted experiments to demonstrate that the long term track likelihood computed from the individual track segment likelihoods and the track segment association likelihoods is identical to that computed from the individual reports. The scenario consists of a target moving in two dimensional space and observed by a GMTI radar at 2 Hz. The measurements are divided into five track segments as shown in Figure 9.

For each track segment S_i, the smoothed estimate of the start (kinematic) state and the filtered estimate of the end state are computed. The likelihood of the track segment is also calculated from the measurements in the segment to give $L(S_i)$. The track segment association likelihood $L(i,i-1)$ is calculated by assuming that $p(x_i^s \mid Y_1,..,Y_{i-1}) \simeq p(x_i^s \mid Y_{i-1})$, and then the long term track likelihood is computed.

Figure 10 compares the long term track likelihood calculated from the track segment likelihoods and that calculated directly from the measurements. The two curves are basically identical. This illustrates the optimality of the long term track likelihood algorithm.

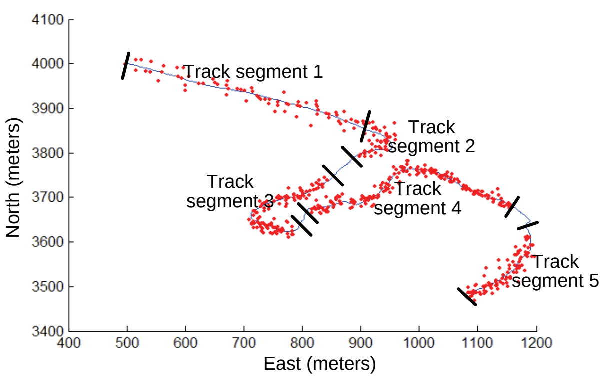

Figure 9: Long term track consists of 5 track segments

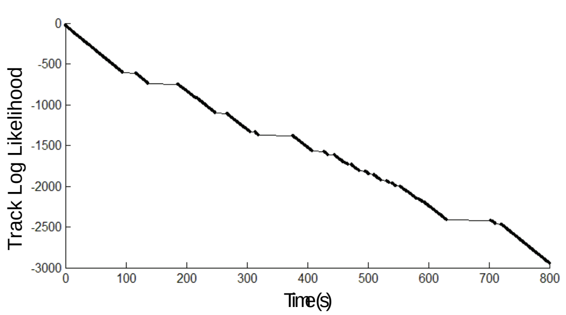

Figure 10: Long term likelihood computed from track segments is same as that computed from measurements.

Although not shown, we also computed the exact long term track likelihood without using the assumption $p(x_i^s \mid Y_1,..,Y_{i-1}) \simeq p(x_i^s \mid Y_{i-1})$. The difference of the exact likelihood from the approximation is insignificant. This shows that track segment association likelihood depends only on local measurements in the two track segments to be associated when only kinematic states are involved.

6.3 Benefit for feature aiding tracking

We used the example of Figure 1 to demonstrate the benefit of the track segment graph approach for feature aided tracking. GMTI measurements were simulated for the three targets moving through the two ambiguous regions at 10 and 20 seconds respectively. In addition, feature measurements provided the colors of the targets at the beginning and end of the scenario. The measurements were processed using the traditional MHT approach and track segment based MHT. The association results were then compared.

It is obvious that a pruning window longer than 20 seconds is needed to maintain the alternative hypotheses across the ambiguities in order to use the feature measurements. Because the traditional MHT generates track hypotheses at the report level, the number of track hypotheses grows very rapidly (Figure 7). Therefore, we had to use a pruning window smaller than 20 seconds for the MHT to run. The result is that track hypotheses could not be maintained for the required 20 seconds to exploit the features.

The number of track hypotheses using the track segment graph approach grows much slower (Figure 7). Thus we were able to use a pruning window bigger than 20 seconds to maintain all alternative hypotheses until the feature measurements were collected. Since the feature were observed with no errors, the association results were perfect.

7 Conclusions

We have developed an approach for extended duration multiple hypothesis tracking (EDMHT) that consists of the standard MHT and an extended duration addition. The core of the extended duration addition is an efficient representation of the long term track hypotheses by the track segment graph. An algorithm for evaluating the long term track likelihood has been developed. The efficiency and optimality of the representation and the likelihood calculation have been demonstrated with simple examples. We plan to conduct experiments with more complicated examples in the future.

References

[1] Y. Bar-Shalom and X-L Li, *Multitarget-Multisensor Tracking: Principles and Techniques*, YBS Publishing, Storrs, CT, 1995.

[2] S. S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems,* Artech House, Norwood, MA, 1999.

[3] D. B. Reid, "An algorithm for tracking multiple targets," *IEEE Trans. Automat. Contr.,* Vol. AC-24, No. 6, pp. 843-854, Dec., 1979.

[4] S. Mori, C. Y. Chong, E. Tse, and R. P. Wishner, "Tracking and classifying multiple targets without a priori identification," *IEEE Trans. Automat. Contr.*, Vol. AC-31, No. 5, pp. 401-409, May 1986.

[5] S. Mori and C. Y. Chong, "Evaluation of data association hypotheses: non-Poisson i.i.d. cases," *Proc. 7th Int. Conf. Information Fusion*, pp. 1133 – 1140, Stockholm, Sweden, July 2004.

[6] C. L. Morefield, "Application of 0-1 Integer programming to multi-target tracking problems," *IEEE Trans. Automat. Contr*, Vol. AC-22, No. 3, pp. 302-312, June 1977.

[7] T. Kurien, "Issues in the design of practical multi-target tracking algorithms," *Multitarget-multisensor Tracking: Advanced Applications*, ed. by Y. Bar-Shalom, Chap. 3, pp. 43 – 83, Artech House, 1990.

[8] S. S. Blackman, "Multiple hypothesis tracking for multiple target tracking," *IEEE Aerospace and Electronic Systems Magazine*, Vol. 19, No. 1, Part 2: Tutorial, pp. 5 – 18, January 2004.

[9] A. Poore and N. Rijavec, "A Lagrangian relaxation algorithm for multidimensional assignment problems arising from multitarget tracking," *SIAM Journal of Optimization*, Vol. 3, No. 3, pp. 544 – 563, August 1993.

[10] S. Coraluppi, C. Carthel, M. Luettgen, and S. Lynch, "All-Source Track and Identity Fusion," *Proc. National Symp. Sensor and Data Fusion*, San Antonio, TX, June 2000.

[11] E. Fortunato, W. Kreamer, S. Mori, C. Y. Chong, and G. Castanon, "Generalized Murty's algorithm with application to multiple hypothesis tracking," *Proc. 11th Int. Conf. on Information Fusion (Fusion 2007)*, Quebec City, 2007.

[12] P. O. Arambel, J. Silver, M. Antone, and T. Strat, "Signature-aided air-to-ground video tracking," *Proc. 9th Int. Conf. on Information Fusion (Fusion 2006)*, Florence, Italy, 10-13 July 2006.

[13] O. E. Drummond, "Feature, attribute, and classification aided target tracking," *Proc. SPIE, Conf. on Signal and Data Processing of Small Targets*, vol. 4473, 2001.

[14] R. Mahler, *Statistical Multisource-Multitarget Information Fusion*, Artech House, Norwood, MA, 2007.

Appendix – Derivation of equation (3)

The long term track likelihood is defined as $L(T) = L(S_1,..S_i,..,S_N) = p(Y_1,..Y_i,..,Y_N)$ given the measurements $Y = (Y_1,..Y_i,..,Y_N)$. This likelihood can be evaluated recursively as

$$p(Y_1,..,Y_i,Y_{i+1}) = p(Y_{i+1} \mid Y_1,..,Y_i)\, p(Y_1,..,Y_i) \qquad \text{(A1)}$$

Because

$$p(Y_{i+1} \mid Y_1,..,Y_i) = \int p(Y_{i+1} \mid x^s_{i+1})\, p(x^s_{i+1} \mid Y_1,..,Y_i) dx^s_{i+1}$$

and

$$p(Y_{i+1} \mid x^s_{i+1}) = \frac{p(x^s_{i+1} \mid Y_{i+1})\, p(Y_{i+1})}{p(x^s_{i+1})}$$

(A1) becomes

$$\begin{aligned} & p(Y_1,..,Y_i,Y_{i+1}) \\ &= p(Y_1,..,Y_i) \int \frac{p(x^s_{i+1} \mid Y_{i+1})\, p(Y_{i+1})}{p(x^s_{i+1})} p(x^s_{i+1} \mid Y_1,..Y_i) dx^s_{i+1} \\ &= p(Y_1,..,Y_i)\, p(Y_{i+1}) \int \frac{p(x^s_{i+1} \mid Y_{i+1})\, p(x^s_{i+1} \mid Y_1,..Y_i)}{p(x^s_{i+1})} dx^s_{i+1} \end{aligned} \qquad \text{(A2)}$$

Similarly,

$$p(Y_1,..,Y_i) = p(Y_1,..,Y_{i-1})\, p(Y_i) \int \frac{p(x^s_i \mid Y_i)\, p(x^s_i \mid Y_1,..,Y_{i-1})}{p(x^s_i)} dx^s_i \qquad \text{(A3)}$$

Thus,

$$p(Y_1,..,Y_N) = \prod_{i=1}^{N} p(Y_i) \prod_{i=2}^{N} \int \frac{p(x^s_i \mid Y_i)\, p(x^s_i \mid Y_1,..,Y_{i-1})}{p(x^s_i)} dx^s_i \qquad \text{(A4)}$$

which is equation (3).