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Adaptive Filtering for Single Target Tracking

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ABSTRACT

Many algorithms may be applied to solve the target tracking problem, including the Kalman Filter and different types of nonlinear filters, such as the Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF) and Particle Filter (PF). This paper describes an intelligent algorithm that was developed to elegantly select the appropriate filtering technique depending on the problem and the scenario, based upon a sliding window of the Normalized Innovation Squared (NIS). This technique shows promise for the single target, single radar tracking problem domain. Future work is planned to expand the use of this technique to multiple targets and multiple sensors.

Keywords: Filter suite, tracking, consistency checks, Extended Kalman Filter, Unscented Kalman Filter, Particle Filter

1 INTRODUCTION

There are several different tracking algorithms that may be applied to a tracking problem. These tracking algorithms include the linear Kalman Filter and different types of nonlinear filters, including the Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF) and Particle Filter (PF). Each of these filters makes certain assumptions about the scenario at hand. The Kalman Filter provides a minimum mean square error (MMSE) estimate through the hypothesis that the dynamic system is Gaussian and linear, whereas the Particle Filter makes neither of these assumptions. Some algorithms have a lower computational cost but these approaches may degrade in accuracy if an algorithm's underlying assumptions are not consistently true throughout a scenario. The UKF and EKF, which are nonlinear extensions of the KF, have been shown to exhibit accurate performance under weak nonlinearity, but diverge under more highly nonlinear cases. The PF however, which requires significant computational complexity has shown better performance than the EKF and UKF in these strongly nonlinear cases [1].

Ideally one would like to employ a specific filter under the conditions that the filter's notions about a system are true and only apply more computationally complex tracking algorithms when an alternate filter does not provide sufficient accuracy. Since online knowledge of a system is limited, a measure of how nonlinear a target's motion will be in the future is not possible. In the absence of an online measure of nonlinearity, a measure of filter performance is shown to be sufficient to adaptively switch between algorithms based upon the current state of the system. This paper provides the description of, as well as example results for an Adaptive Filtering algorithm that was developed to elegantly select the appropriate filtering technique based upon a sliding window of the Normalized Innovation Squared (NIS). This technique shows promise for the single target, single radar tracking problem domain. Future work is planned to expand the use of this technique to multiple targets and multiple sensors.

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In Section 2, a brief description of the tracking algorithms is provided, as well as detail concerning the filter divergence detector based on the Normalized Innovation Squared. The results of using the divergence detector to adaptively switch between different filtering algorithms are verified by simulation runs in Section 3. Conclusions are drawn in Section 4.

2 APPROACH

2.1 Filtering techniques

It is well known that the Kalman filter is optimal when the system is linear and Gaussian. When the linear assumption is violated, there are several nonlinear algorithms that may be applied to a target tracking problem. Perhaps the simplest of these nonlinear filters is the EKF. The EKF, a version of the Kalman Filter that linearizes the nonlinear state to measurement transformation and state transition matrices, is being used in thousands of systems in the field today for tracking when nonlinearities in the system are present. The UKF is a more recent development [7] that uses a parameterized set of sample points, called "sigma points" to model the nonlinearity, and has been shown to achieve superior performance to that of the EKF. In the UKF sample points are not drawn at random, but according to a specific deterministic algorithm. Another approach that is not a derivative of the KF and is based on a slightly different approach is the Particle Filter (PF). The PF is an approach that uses Monte Carlo integration to handle nonlinearities by sampling from the posterior distribution. In the case of a PF, the initial sample points are drawn at random, from a so-called proposal distribution. In order to update the state, a set of importance weights are calculated and assigned to the samples. These weights are normalized and updated according to a re-sampling process. There are a number of different variations of particle filters, depending on the re-sampling technique used, as well as other factors. The drawback of the PF is that it requires a great deal of computational power, due to the Monte Carlo nature of the solution. Yet, this must be weighed against the degree of accuracy, for in highly nonlinear scenarios the PF has been shown to out-perform both the EKF and UKF.

2.2 Target and measurement models

The target motion is modeled as follows

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{v}_k \tag{1}$$

where $\mathbf{x}_k = [x_k \dot{x}_k y_k \dot{y}_k]'$ is the target state at time *k*, containing positions and velocities along *x* and *y* in a twodimensional Cartesian plane. **F** is the state transition matrix, modeled by constant velocity as

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(2)

in which T is the time interval between subsequent measurement samples. \mathbf{v}_k denotes the zero-mean white Gaussian process model noise with covariance matrix

$$\mathbf{Q} = q \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0\\ T^2/2 & T & 0 & 0\\ 0 & 0 & T^3/3 & T^2/2\\ 0 & 0 & T^2/2 & T \end{bmatrix}$$
(3)

where q is a constant scaling factor, which is the process noise intensity. Measurements from a radar report with rangebearing information are modeled. These measurements, \mathbf{z}_k , for a radar report are mathematically expressed as

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{w}_k \tag{4}$$

where \mathbf{w}_k represents the zero mean Gaussian measurement noise with covariance matrix \mathbf{R} . $\mathbf{h}(\mathbf{x}_k)$ is a function that transforms target positions (x_k, y_k) into measurement space and is written as

$$\mathbf{h}(\mathbf{x}_k) = \begin{bmatrix} \sqrt{(x_k - x_s)^2 + (y_k - y_s)^2} \\ \tan^{-1}[(y_k - y_s)/(x_k - x_s)] \end{bmatrix}$$
(5)

with (x_s, y_s) as the position of the sensor. Note that for the EKF, the Jacobian of the measurement is given by

$$\mathbf{H}_{k} = \begin{bmatrix} \frac{\Delta x_{k}}{\sqrt{\Delta x_{k}^{2} + \Delta y_{k}^{2}}} & 0 & \frac{\Delta y_{k}}{\sqrt{\Delta x_{k}^{2} + \Delta y_{k}^{2}}} & 0\\ \frac{-\Delta y_{k}}{\Delta x_{k}^{2} + \Delta y_{k}^{2}} & 0 & \frac{\Delta x_{k}}{\Delta x_{k}^{2} + \Delta y_{k}^{2}} & 0 \end{bmatrix}$$
(6)

where $\Delta x_k = \hat{x}_{k|k-1} - x_s$ and $\Delta y_k = \hat{y}_{k|k-1} - y_s$. Clearly the motion model is linear and Gaussian, whereas the measurements are nonlinear functions of the target state. Therefore, the variations of nonlinearity that will be addressed in the experimental portion of this paper will be due to the measurements.

2.3 Consistency checks

2.3.1 Extended and Unscented Kalman Filter consistency check

The Normalized Innovation Squared (NIS) of the EKF or UKF provides a consistency check for filter divergence. The NIS is obtained from the innovation, or residual, and the innovations covariance at each timestamp k. The innovation is the difference between the measurement and its prediction at time k, and is

$$\tilde{z}_k = \mathbf{z}_k - \hat{z}_{k|k-1} \tag{7}$$

The innovations covariance matrix S_k is given by

$$S_k = \mathbf{H}_k P_{k|k-1} \mathbf{H}_k^T + \mathbf{R}$$
(8)

where $P_{k|k-1}$ and **R** are the predicted target state and measurement covariance matrices respectively. The NIS as defined by these two equations is

$$NIS_k = \tilde{z}_k^T S_k^{-1} \tilde{z}_k \tag{9}$$

The NIS can serve as a divergence detector for each of the filters when compared to a threshold. While this threshold can be tuned, it is important to note that when a linear, Gaussian assumption is reasonably true the NIS follows a Chi-Square distribution with n_z degrees of freedom. Therefore checking if the NIS falls outside a confidence region for a Chi-Square random variable with n_z degrees of freedom has been shown to provide an indication of filter divergence [1]. It would not however be practical to adaptively switch based upon a single NIS value outside the threshold, as this may cause numerous unnecessary switches based upon a single spike in NIS value. Therefore the method employed in this work was a sliding window over the NIS values. The test statistic J_k for divergence is

$$J_k = \sum_{i=k-l}^k \frac{NIS_i}{w} \tag{10}$$

where w is the length of the sliding window and l = w - 1. The sliding window required divergence to not be a consequence of the phenomenon of a single stray measurement. For a linear and Gaussian system, the innovation

sequence is a white sequence [6]. With the linear and Gaussian assumptions, it is easy to show that wJ_k follows a Chi-Square distribution with wn_z degrees of freedom. Therefore, a test threshold for J_k can be set according to the distribution of a $\chi^2_{wn_z}$ random variable.

2.3.2 Particle filter consistency check

Just as a test variable, J_k , is needed for the EKF and UKF, a similar measure must be devised for the PF in order to determine consistency, and ultimately in the proposed algorithm to decide upon usage of the PF. A simple approach is to maintain a Gaussian assumption, and compute a measure for the Particle Filter that is very similar to the NIS [4]. This statistical measure d_k is

$$\mu_k = \sum_{i=1}^N w_k^i \hat{z}_k^i \tag{11}$$

$$\Sigma_{\hat{z}_{k}} = \mathbf{R} + \sum_{i=1}^{N} w_{k}^{i} \left[\hat{z}_{k}^{i} - \mu_{k} \right] [\hat{z}_{k}^{i} - \mu_{k}]^{T}$$
(12)

$$d_k = (z_k - \mu_k)^T \Sigma_{\hat{z}_k}^{-1} (z_k - \mu_k)$$
(13)

where w_k^i and \hat{z}_k^i are the weight and predicted measurement of the *i*th particle at time *k*. Similar to a sliding window of the NIS for the EKF and UKF, a sliding window of d_k is used to compute J_k for the Particle Filter.

The authors in [3] suggest an alternative approach based upon binning the number of particles greater than or less than for each component of the measurement vector. The technique suggests that the number of particles in each bin should be uniformly distributed over a window of time. Then a chi-square test is used to determine if the particles are in fact uniformly distributed. While this approach does not require a Gaussian assumption and may be needed in additional efforts, it was not selected due to the fact that this test requires more computations, and also more time steps, to acquire a solution.

2.4 Adaptive filtering

The proposed algorithm of this paper is to use the variable J_k as a means for deciding whether the current filter used during time k should be used for time k + 1, or to choose to switch to an alternative available filter. A switch would mean that the information, or more specifically the current state and covariance estimates, contained by the filter during timestamp k would be passed to a different filtering algorithm at timestamp k + 1. The approach by the authors is always to use the least computationally complex filter whenever possible. Therefore general practice is to start with the use of the EKF and switch to the UKF or PF only when J_k exceeded a predefined threshold. Likewise to prevent unnecessary computational burden, once J_k stabilized to a lower threshold value, a less computationally complex filter would be utilized.

3 RESULTS

3.1 A single radar with weak nonlinearity

The first example taken is one with weak nonlinearity as described by [1] with a single radar sensor taking measurements on a target, where the standard deviation (s.d.) of the range measurement is 100, and bearing s.d. is 1° . The radar geometry and target trajectory are depicted in Fig.1. In order to determine the accuracy of each algorithm, Root Mean Squared Errors (RMSEs), with respect to target positions, were averaged over 100 Monte-Carlo simulation runs. Comparing the results of these averaged RMSEs for a PF with $1X10^4$ particles, an EKF, UKF, and our adaptive approach that switches between a EKF and UKF algorithm, it is clear that this weakly linear problem yields similar results for all filters.





Fig. 1a Trajectory of the true target and position of the radar sensor

Fig. 1b RMSEs of position for different algorithms.

3.2 A single radar with moderate nonlinearity

In order to exercise the algorithms against a more difficult scenario, the s.d. of the range measurements are taken to be 10m, whereas the bearing measurements are somewhat coarse with an s.d. of 4° . Two experiments were run with these same measurement standard deviations, but with different radar geometries, number of scans, and trajectories as depicted in Fig. 2. Particularly the geometry of Fig.2a, combined with the error in the measurements causes the uncertainty region of the target position to be a thin area. As in the experiment with weak nonlinearity, there is still a single radar sensor taking measurements on the target.



Fig. 2a and 2b Trajectory of the true target and position of the radar sensor

Comparing the results of the averaged RMSEs for this more highly nonlinear scenario, it is clear from Fig. 3a that although the EKF diverges the adaptive approach converges to the results of the PF and UKF. The results from Fig. 3b,

also reveal consistent performance, where even though the EKF does not as greatly diverge, the adaptive filter performs similarly to the UKF.



Fig. 3a and 3b RMSEs of position for different algorithms.

The divergence of the EKF for trajectory in Fig. 3a is evident when looking at the NIS values in Fig.4a. Between timesteps 10 and 40 all NIS values fall outside the 99% confidence region for the EKF. The other three algorithms are able to handle the high nonlinearity producing results with similar accuracies and whose estimates remain within the confidence region for the NIS.



Fig. 4 NIS of Adaptive Filter, PF, UKF, and EKF respectively.

The main advantage to being able to switch between various algorithms is the adaptability to a scenario. Unless all available algorithms diverge, the adaptive algorithm will always have an alternative option that maintains accurate

tracks. Table 1 also shows that there is a gain in terms of computation. Computationally the adaptive algorithm is less expensive than the UKF and the PF, yet converges quickly to the results of the more powerful filters. The reason for this computational advantage is the selective nature of the approach that uses more computationally expensive filters only when necessary, specifically over all 100 Monte-Carlo simulations the EKF was on average utilized 52% of the time for trajectory 1 and 48% of the time for trajectory 2, whereas the more complex UKF was used 48% of the time for trajectory 1 and 52% of the time for trajectory 2.

| Trajectory | Number of | EKF | UKF | PF | Adaptive |
|------------|-------------------|----------|----------|----------|----------|
| | Measurement Scans | | | | |
| 1 | 100 | 0.009430 | 0.042324 | 1.857040 | 0.024967 |
| 2 | 200 | 0.017208 | 0.085778 | 2.905170 | 0.053940 |

Table 1. Average length of time in seconds for each simulation run of the different algorithms.

While the results presented here are only for an adaptive filter that switches between an EKF and UKF, a similar idea as described in our approach can be used to incorporate a PF. This approach was implemented and yielded similar results, however for the scenarios used here it was not necessary to use a PF, for there was increased computational cost with little or no gain in accuracy. However, a scenario in which both the EKF and UKF diverge would warrant the need for a PF to be incorporated into an adaptive algorithm.

3.3 A single radar with strong nonlinearity

For the final scenario, an s.d. for range measurements is taken to be 10 m, and the s.d. of the bearing is taken to be 10° . In order to further increase the nonlinearity of this scenario, the distance of closest approach is set at 7 km. The target trajectory is shown in Fig. 5. In this case, as in previous ones, one radar sensor is used to take measurements, located at x = 0, y = 0. The target starts at x = 0, y = 7, and moves away from the sensor.



Fig. 5 Trajectory of the target and position of the radar sensor

Comparing the average RMSE over 100 Monte Carlo runs in Fig. 6, this is a highly nonlinear case. The high amount of nonlinearity is an effect of the large uncertainty in angle, complemented by the closeness of the target. This high nonlinearity resulted in a large amount of error in the conventional EKF algorithm. The UKF and PF are better equipped to deal with nonlinearity, and as a result didn't suffer the same divergence as the EKF. It is important to point out that the UKF and PF, while not diverging as severely as the EKF, still caused the RMSE to reach values in excess of 1000 m.



Fig. 6 RMSEs of position for different algorithms.

Table 2 shows a comparison of the execution times for the strong nonlinearity scenario. In this case the EKF was used 51% of the time and the UKF was used 49% of the time. One can ascertain that using NIS as a switching mechanism between the two filter algorithms has prevented the divergence seen in the EKF, while achieving a faster execution time than the UKF.

| Trajectory | Number of Measurement Scans | EKF | UKF | PF | Adaptive |
|------------------------|--------------------------------|----------|----------|----------|----------|
| Strong Nonlinearity | 100 | 0.009455 | 0.047213 | 1.691609 | 0.028329 |

Table 2. Average length of time in seconds for simulation run of the different algorithms.

The ultimate goal of the filter suite is to gain improved filter performance, while still maintaining a reasonable execution time. Since the filter suite does not run the filters in parallel, the ideal worst case scenario would be to obtain the execution time of the UKF. Results have also shown the filter suite almost always outperforms either the EKF or the UKF, and sometimes outperforms both. For the majority of cases it is generally the UKF that provides more accurate

estimates than the EKF, so in the case of the filter suite, the UKF plays the part of correcting for the EKF in problematic scenarios. With some NIS threshold adjustments, the switching algorithm can be tuned to emphasize a particular filter, giving added flexibility to the suite.

4 CONCLUSION

This paper explored the concept of adaptively selecting a filtering algorithm based upon the Normalized Innovation Squared. The algorithms used in this effort were the EKF, UKF, and PF, all nonlinear filtering approaches that can vary in accuracy depending on the dynamics of the problem at hand. The NIS was implemented as a divergence detector for each filter, to obtain an online gauge of filter performance. Although the NIS seemed to be sufficient for most cases, the concept of adaptive filtering does not limit the decision process to rely upon the NIS, in fact in the future another consistency check or measure may be used to decide the appropriate algorithm to be used at a given time. For example, the credibility metrics proposed in [14] could be used to test the consistency of the filters. Clearly, from the results, being able to adaptively select a filter can provide benefits in terms of computational costs and root mean squared error.

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