# Understanding the Role of Chaos Theory in Military Decision Making

A Monograph

by


MAJ Donovan O. Fuqua

United States Army


School of Advanced Military Studies

United States Army Command and General Staff College

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Understanding chaos has several applications for military planning in real world environments. Because chaos is bounded, planners can create allowances for system noise. The existence of strange and normal chaotic attractors helps explain why system turbulence is uneven or concentrated around specific solution regions.

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| | |
|---|---|
| __________________________________<br>William J. Gregor, Ph.D. | Monograph Director |
| __________________________________<br>Alex Ryan, Ph.D. | Monograph Reader |
| __________________________________<br>Stefan J. Banach, COL, IN | Director,<br>School of Advanced<br>Military Studies |
| __________________________________<br>Robert F. Baumann, Ph.D. | Director,<br>Graduate Degree<br>Programs |

# Abstract


UNDERSTANDING THE ROLE OF CHAOS THEORY IN MILITARY DECISION MAKING by MAJOR Donovan O. Fuqua, US Army, 54 pages.

Chaos theory is a poorly understood concept in social science and in military analytical decision making systems. Military decision makers require a multidisciplinary approach of mathematical analysis, modeling and simulation, topology, and post-structural philosophy if they intend to conceptualize chaos theory and complex adaptive systems and theirs relevance to military planning. The essence of this understanding is that while chaos appears random, chaos properly understood is a deterministic series found in very simple forms. These forms exhibit sensitivity to initial conditions, bounding, and attractors. Despite various methods for detecting chaos in mechanical systems, data set size limitations and inability to separate out adaptive behaviors make these techniques of little value *in situ.*

Adaptation and complexity are phenomena that are very different from chaos. Higher order interactions and effects, self-organization, and propensity of co-evolution and novel emergence distinguish chaos from stochastic processes. The self-organization and emergence are evident when a cumulative effect is different from the additive effects of the components. These self-organizing components differ from chaos because the properties of resolution and scope are fundamentally different. The fractal nature of chaos ensures that it is scale less and, therefore, unable to produce novel emergent effects.

One way to conceptualize chaos within complexity is through the Deleuzian post-structural Philosophy of Difference regarding Smooth and Striated Spaces and Nomad versus the Sedentary agents. This conceptualization, transferred to chaos applications, links turbulence to barriers and increased gridding on the surface of open systems. These barriers inform agents on suitable terrain and options during decision-making.

Understanding chaos has several applications for military planning in real world environments. Because chaos is bounded, planners can create allowances for system noise. The existence of strange and normal chaotic attractors helps explain why system turbulence is uneven or concentrated around specific solution regions. Finally, the presence of chaos limits the effectiveness of single variable variation reduction techniques such as Lean Six Sigma because of unpredictable system behavior.

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# Introduction

Annually, the US Army Combined Arms Center prepares a list of research topics. The list is distributed to the students enrolled in the Command and General Staff College to encourage research in areas of interest to the Army. Among the topics found in the AY2008-09 research list was a question asking whether Chaos Theory was applicable for understanding Army sustainment operations. The topic may have only reflected a general curiosity that many have for Chaos Theory because many researchers have tried to apply chaos theory to understand a variety of business and academic activities. It might also reflect the interest in the Army logistics community in contemporary business methods and by extension, chaos theory. Regardless of the motive for listing the topic, it strongly suggests that military leaders need to learn more about Chaos Theory before leaping into logistic applications.

Military decision makers tend to view the real world in terms of direct causal relationships and linear effects.  One example is from personal experience as a transportation brigade operations officer (S3) working to support deployment and distribution during Operation Iraqi Freedom. During the operations, a higher headquarters attempted to build a chart called “the race track” to track the performance of the container distribution system from depots in the United States and Germany through Kuwait and into Iraq and back to the United States and Germany. The higher command wanted each decision maker to focus on decreasing delivery time. However, the casual and linear paradigm was insufficient for explaining and predicting the environment.

Despite the effort to control every transportation process, delivery times exhibited aperiodic variation. Through analysis of the port clearance times, transit times, and hold times at each stop, the planners could not attribute the variation to changes in procedures at any location.. Although the higher headquarters was pleased with an overall reduction of the mean time for transit, variation in materiel delivery time continued to be a problem since forward deployed units tended to stockpile equipment or extend deployment or redeployment timelines based upon imprecise guaranteed delivery times. These aperiodic variations were seemingly beyond the operators' control, but were definitely part of the environment.

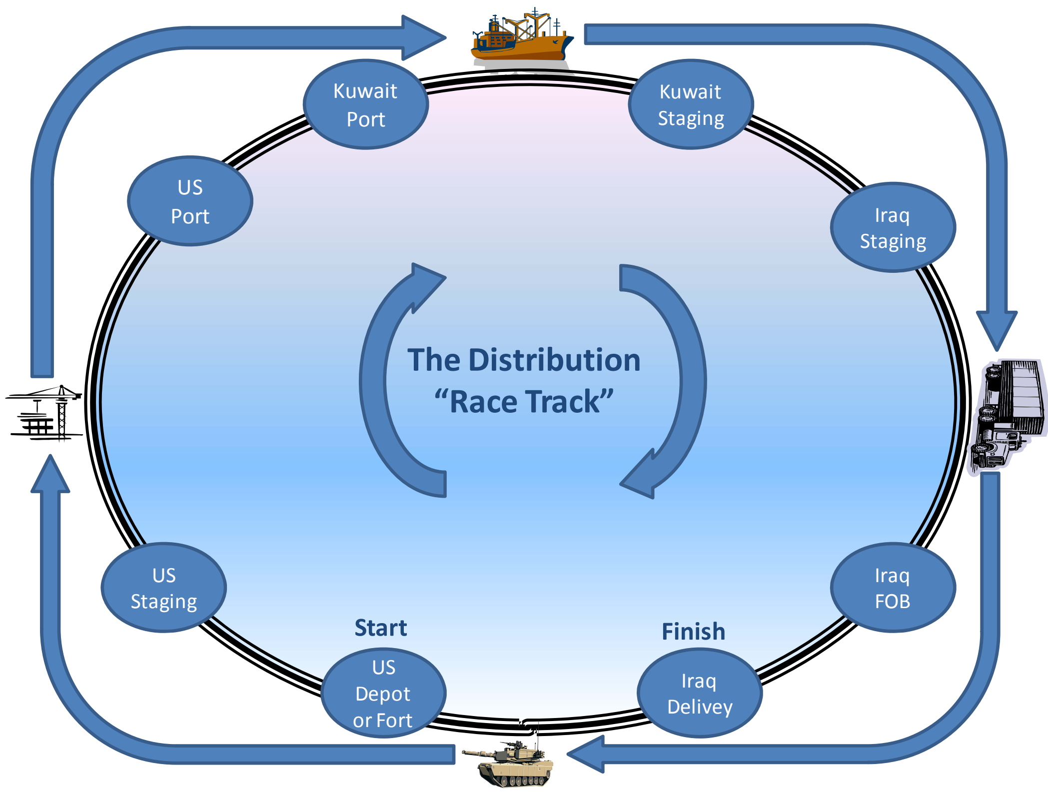


**Figure 1: Graphical Representation of "The Race Track"**

One way to view "the race track" problem is through systems theory and analysis. In systems thinking, locations, actors, and routes are conceptualized as nodes, adaptive agents, and arcs. By framing the problem as a mental model, the command could have conducted analysis on the source of the variation. However, systems framing requires choices on what to include and what to ignore to develop a predictive model of reality. A frame that includes only adaptive elements will not generate an adequate representation of reality due to the exclusion of physical phenomena with chaotic or fractal components. Decision makers must consider including fractal and chaotic components as in any model intending to mirror reality.

Unfortunately, the concept of chaos is poorly understood within most business, social science, and military literature and casual vernacular. The confusion is generated by the failure to distinguish between stochastic randomness and mathematical determinism. [1] The phenomenon of chaos and chaos theory refers to the the technical mathematics definition of a dynamic deterministic process that appears random.[2] Based on its deterministic nature however, the short term past and future behavior of chaotic systems can be known exactly through calculation, even though the output appears random.[3]

Unfortunately, the deterministic nature of chaos theory recalls the commander of US Joint Forces Command recent criticisms of Effects Based Operations (EBO).[4] If chaos were examined as the sole mechanism in complex systems, the argument would be that through perfect knowledge of the model, decision makers would precisely know the future states and behaviors of the system. By reducing the system to its components, planners would have perfect predictive abilities. This reductivist argument does not actually describe the characteristics of open systems. Open systems are comprised of multiple adaptive elements with complex interactions that result in highly ordered effects and the potential for novel emergent properties. Unfortunately, limiting a system frame to adaptive elements does not accurately describe chaos in nature, where small fluctuations in initial conditions and changes in parameters have radical changes to system behaviors.

Decision makers should consider chaos as a possible superficial element of complex adaptive systems that can create system noise and first order effects[5] but not direct self-organization or novel

[1] When a variable has an associated probability density, it is stochastic. Stochastic is derived from a Greek word meaning 'guess' and refers to randomness. While the exact value of a stochastic variable is random, it can take values within a set distribution with a mean and variance (a probability density).

[2] It is important to note that the models presented imply perfect system knowledge and infinite resolution. In infinite resolution, chaos is not random. However, when the start point is unknown and resolution is finite, chaos can be a source of actual randomness. This is an important point that separates the models from real world phenomena. The models explain the mechanics of chaos, not to predict real events.

[3] A system is an abstraction of reality – a mental model used to explain and simplify the real world. Systems are an arrangement or a set of autonomous and interrelated entities, parts, units, components and variables that form an integrated whole.

[4] GEN James Mattis, "Assessment of Effects Based Operations", Memorandum, (United States Joint Forces Command, Norfolk, Virginia, 14 August 2008).

5 Ordered effects can refer to the degree of interaction between elements to generate an effect. First order effects are caused by additive properties of the elements, where there is no interaction effect. Higher ordered effects occur when elements interact to form a nonadditive effect. For example: $Effect = ax + by$ is first order when x

emergence. Because all chaos is derived from mechanical or deterministic mechanisms, when considered in isolation, chaos theory has little relevance for military decision making. From the systems perspective, decision makers should consider chaos as a component of complex adaptive systems. Using this framework, chaos and other forms of deterministic turbulence can form the first order structure of smooth and striated spaces in open systems.[6] This, in effect, has the potential to create system noise and low resistance regions (smooth space) and high resistance regions (striated space). These regions help form the macrostructure of the system, and can guide first order adaptive behavior.

Based on a systems framework, chaos and other forms of deterministic turbulence affect complex adaptive systems through inputs and outputs that influence first order system structure. The null hypothesis is that deterministic turbulence has no effect on open complex systems because adaptive entities entirely mask non-adaptive portions. These hypotheses are labeled:

$H_0: Chaos\ has\ no\ effect\ on\ complex\ adaptive\ systems$
$H_{a:} Chaos\ can\ affect$ the behavior of $complex\ adaptive\ systems$

**Figure 2: Null and Alternative Hypotheses**

The hypotheses are tested using both qualitative and quantitative analysis. This methodology utilizes computer simulation, philosophical reasoning, and real-world case study examination. These methods compare and contrast the behaviors of adaptive systems to chaos systems both *in situ* and *in silica*[7]. While the argument contains mathematical and system analysis, the intended audience is military

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and y are variables and a and b are constants. $Effect = a(xy) + by$ is a second order effect due to the single interaction between x and y.

[6] Smooth and striated spaces refer to Gilles Deleuze and Felix Guttari's concepts of the nature of spaces in *A Thousand Plateaus.* In the book, they use smooth and striated in many different contexts. The one used in the monograph refers to freedom of movement. Smooth space is analogous to the ocean or a desert; where movement is unhindered due to lack of obstacles. Striated Space is analogous to a heavily populated city or a sharp mountain range. Heavily striated space limits maneuver or choices.

[7] *In situ* is a Latin phrase that means 'in the place'. It refers to an observation or experiment in a natural or ordinary setting. In contrast, *in silica* refers to a experiment or observation that occurs in a computer setting, such as a computer simulation. Both offer advantages and disadvantages. While systems viewed *in situ* show actual behavior, it is difficult to change conditions and experiment. *In silica* experiments allow changes and multiple iterations, but can often ignore key elements that influence system behavior.

planners who want to better understand the concept of chaos and how it fits into the decision making process.

Through an examination of the history, structure, and outcomes of chaos, the reader gains a common frame of reference for understanding chaos theory. In order to understand chaos theory, military decision makers must have a common definition and an understanding of the history and physical applications of chaos. Increasingly complex models show provide background to the reader. Rudimentary models such as the one variable logistics equation give a basic understanding while more complicated models with two and three variable differential equations better mimic real world phenomena. These models give better understanding of the implications of chaos: sensitivity to initial conditions, strange attractors, and constants of motion. By showing the routes that lead relatively simple models to chaos, planners can recognize that small shifts can cause radically different behaviors.

Once the planner has an understanding of the basic mechanics of chaos, it is possible to consider applications and detection methods. Using those methods permits investigation of a variety of cases. The cases considered include the use of autocorrelation, Fourier analysis, and Kolmorgov-Sinai entropy equations. The point of the analysis is to show the difficulties and data set limitations in predicting and controlling chaos. The discussion next teurns to fractal geometry and power laws because they are related to chaos. While these systems are deterministic, they have different behaviors, detection methodologies, and slightly different implications for planning.

As stated, chaos in isolation is worthless to a miltiary planner because it only describes deterministic and mechanistic effects. Therefore, real-world military operations are almost always conducted in open complex environments. Based upon that frame, there are definite possible characteristics of complex adaptive interactions: novel emergence, co-evolution, and self-organization. After describing these traits, it was important to use computer simulation to show *in silica* how novel behavior is generated. Based upon analysis through basic topology, systems thinking, and philosophy, the monograph shows how chaos can influence first order system behavior but cannot cause emergence and other traits associated with higher ordered behavior.

Based on this understanding, planners must conceptualize the influence of chaos on complex adaptive systems. A proposed model outlines the implications of chaos properties such as sensitivity to initial conditions and attractors. This understanding then generates possible tools for decision making and control of complex systems with adaptive and chaos components.

## Chaos Theory Overview

Chaos theory is a relatively new branch of mathematics that is expressed in completely deterministic motion equations.[8] Although chaos is a mathematical model, it is important to note that real world environments have behaviors that can be explained and predicted through the use of chaos theory. The fact that military operations occur *in situ* obligates planners to consider the periodic effects of physical phenomena. Weather patterns, waves, and other physical motion can have varied effect on operations. For example, it is impossible to analyze sea port of debarkation (SPOD) activities without considering tides, wave patterns, and storms. Therefore, decision makers must understand the history, definitions, and some basic examples of chaos to illustrate the concepts of sensitivity to initial conditions, system attractors, and bounded strange attractors.

When a decision maker looks at a random-appearing data set, the two questions that should immediately arise are: what caused this behavior and what does it mean to the operation. Chaos theory is one tool used to understand the phenomenon of system turbulence. While the theory is definitely not a perfect tool, systemic analysis offers distinct improvements over simply disregarding the behavior and making suboptimal decisions based on a lack of understanding.

Chaos is random-like behavior with certain necessary but not sufficient criteria. In order to be considered chaos, a system must be bounded, nonlinear, sensitive to small changes in the initial

[8] Edward Lorenz and other scientists developed deterministic models to explain random-like behavior over a time series starting in 1963. Although the genesis of Chaos Theory is over 40 years old, in mathematics, that constitutes a relatively new field. James Gleick, *Chaos, Making a New Science.* (Viking Penguin, New York, 1987), 5.

conditions, and dynamical.[9] Those ingredients do not guarantee chaos, but give the basic components that allow a deterministic system to be driven into random-like behavior. The important point of chaos is that it is *random-like.*

True random behavior is a stochastic mechanism that resembles Brownian movement, also called a random walk.[10] If you rolled a fair six-sided die, you would have an equal chance of hitting any side, however, there would be no way of predicting what the next throws would generate. A way to visualize this is to ask a classic statistics question: if you rolled a 'six' fifty times in a row, what would be the chance of rolling a 'six' on the fifty first roll? The answer is that you would have a one-sixth chance because each roll is an independent random event. The next event in chaos, however, is completely determined by the last event and that system is greatly influenced by initial conditions. In order to understand this phenomenon and show that chaos is not a dense, impeneratrable branch of theoretical mathematics, three simple one-variable systems simulate chaos.

**One Variable Chaos**

Chaos can be modeled using a relatively simple one-variable nonlinear difference equation.[11] In real environments these behaviors are manifest as simple birth-death rates, limited carrying capacities, or the spread of a disease. Equation 1 is a one variable difference equation where the next value of the variable 'x' is generated by an operation to the previous variable value. This system is easily simulated and graphed in Microsoft Excel by selecting different start points (0.5 and 1.0) to show sensitivity to initial conditions.[12]

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[9] Glenn E. James, *Chaos Theory: The Essentials for Military Applications*, (Naval War College, Newport, 1997), 38.

[10] Brownian movement refers to truly random behavior. It is named for Robert Brown, a Scottish Biologist, who described the movement of a particle in a liquid as a random walk, where the direction and length of a step cannot be determined by any of the previous steps.

[11] A difference equation models future discrete states given an operation applied to the current discrete state.

[12] Model adapted and simulated based on simple equations suggested in C Gregobi, *Crises*, "Sudden Changes in Chaotic Attractors, and Transient Chaos", *Physica* D7, (1984), 632-638.

$$x_{t+1} = 1.9 - x_t^2 \tag{1}$$

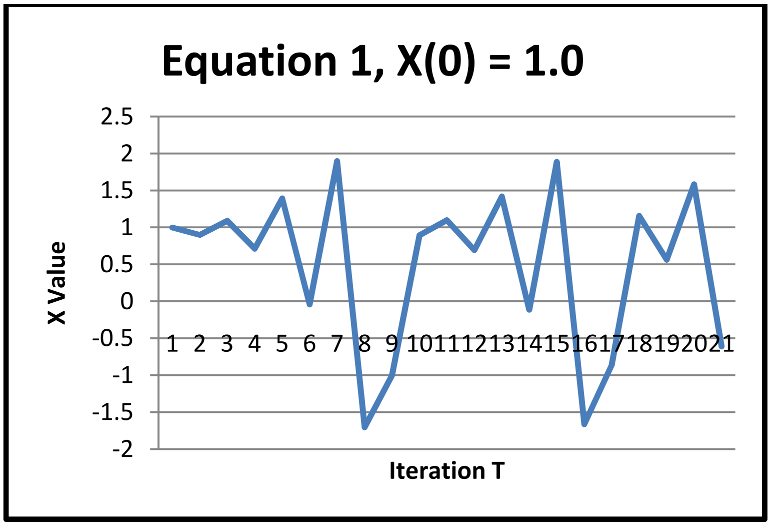


**Figure 3: One Variable Chaos - Sensitivity to Initial Conditions, x(0) = 1.0**

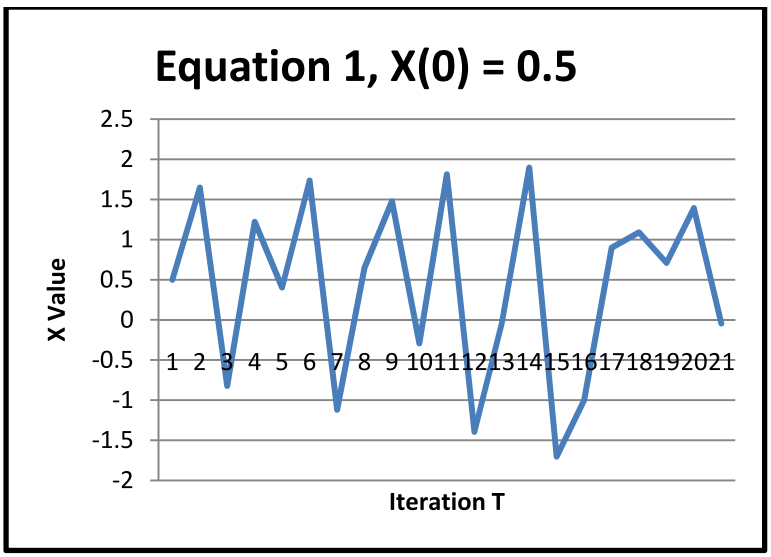


**Figure 4: One Variable Chaos – Sensitivity to Initial Conditions, x(0) = 0.5**

While the equation is the same in both graphs (Figures 3 and 4), system behavior is radically different given a change in initial conditions. This symptom of chaos also explains the idea that while short-term behavior can be predicted, long-term prediction of specific points becomes impossible. In chaotic physical systems, any perturbation can change long-term behavior.

Long-term system motion and dynamics can be relatively predictable in controlled environments. In the graphs, while the individual points are scattered, the system behavior is bounded in that no value exceeds the absolute value of the parameter 1.9. Therefore, despite the turbulence of the individual points, the general system behavior is predictable – it varies between -1.9 to 1.9 as long as x is less than or equal to 1.

The second powerful implication of chaos is the concept of the attractor. Three types of attractors deserve mention: the point attractor, the periodic attractor, and the strange attractor[13]. While there are key differences in each, they imply that chaos systems can exhibit steady states over time. By understanding this concept, it is possible to understand why a turbulent system can exhibit two or more phases.

The best way to begin to explain the concept of attractors is through an examination of the logistics equation.[14] This is a nonlinear difference equation in the general form (Equation 2) where k is the parameter and x is the variable.[15] For low parameter values where k > 1, the points converge to a single point zero. This is a point attractor of the system. When 1 < k < 3, solutions are driven to nonzero point attractors.[16] At k = 3, the equation exhibits a bifurcation and exhibits two alternating points. When k > 3, the system rapidly moves into chaos, where period doubling occurs according to an approached constant. This constant, the Feigenbaum number, approaches a limit (Equation 3 where period doubling occurs at a parameter value $k_i$ and the next period doubling occurs at $k_{i+1}$. This limit approximates the next critical parameter value where period doubling can be anticipated. Figures 5 through 8 illustrate the phenomenon of period doubling as a route to chaos.

$$x_{t+1} = kx_t(1 - x_t) \tag{2}$$

[13] An attractor is defined as a phase space point or set of points representing the various possible steady state conditions of a system. An equilibrium state or group of states to which a dynamical system converges. Garnett Williams *Chaos Theory Tamed*, (Joseph Henry Press, Washington, 1997), 447.

[14] In this usage, logistic refers to the shape of the growth curve rather than the military usage implying the control of supply and distribution. Although the continous formulation of the logistics equation is not chaotic, the equation is highly useful as an example to illustrate period doubling.

[15] Garnett Williams *Chaos*,, 162.

[16] Ibid., 164-165.

$$\lim_{n\to\infty} \frac{\Delta k_i}{\Delta k_{i+1}} = 4.6692 \tag{3}$$

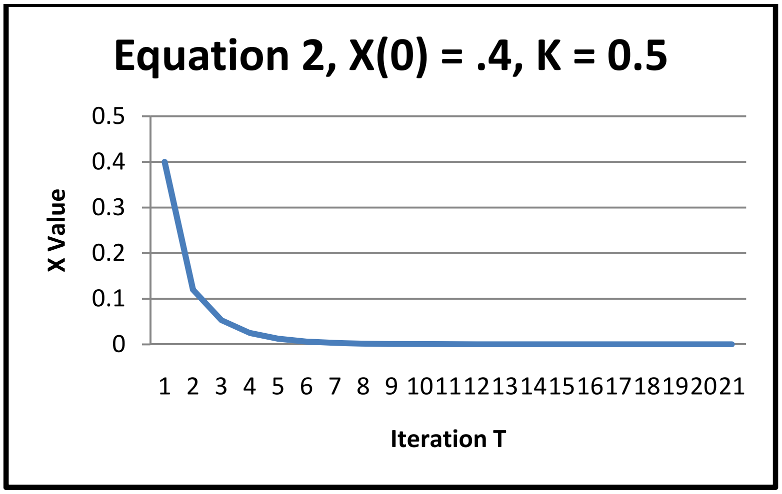


**Figure 5: The Logistics Equation: The Move to Chaos through Period Doubling, K=0.5**

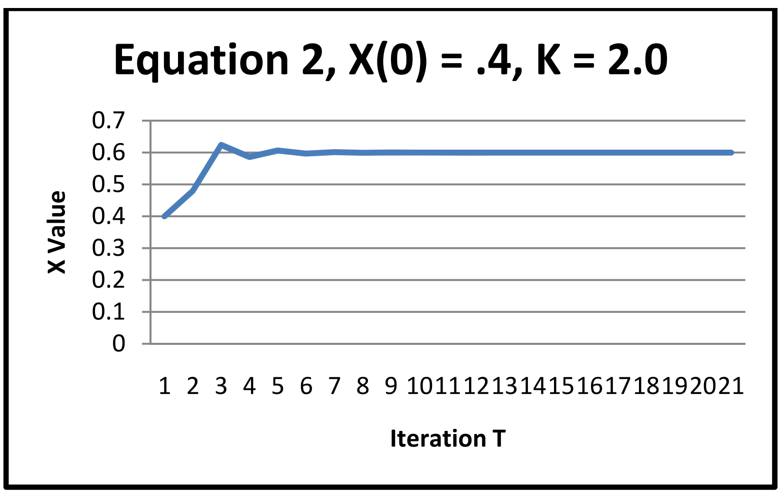


**Figure 6: The Logistics Equation: The Move to Chaos through Period Doubling, K=2.0**

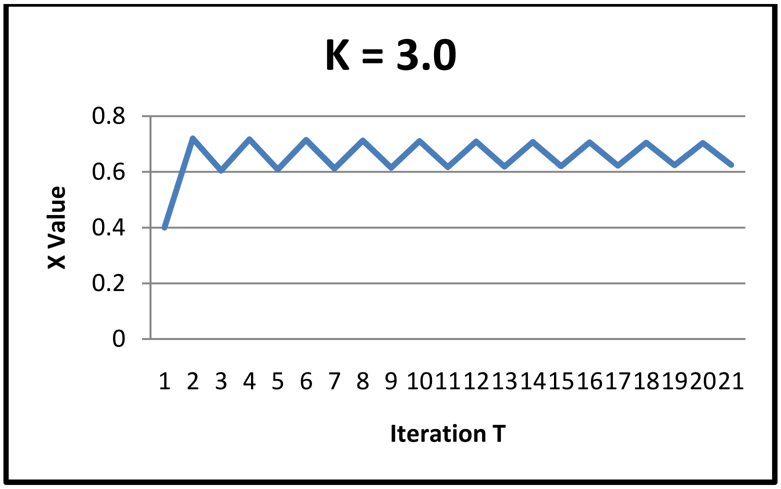


**Figure 7: The Logistics Equation: The Move to Chaos through Period Doubling, K=3.0**

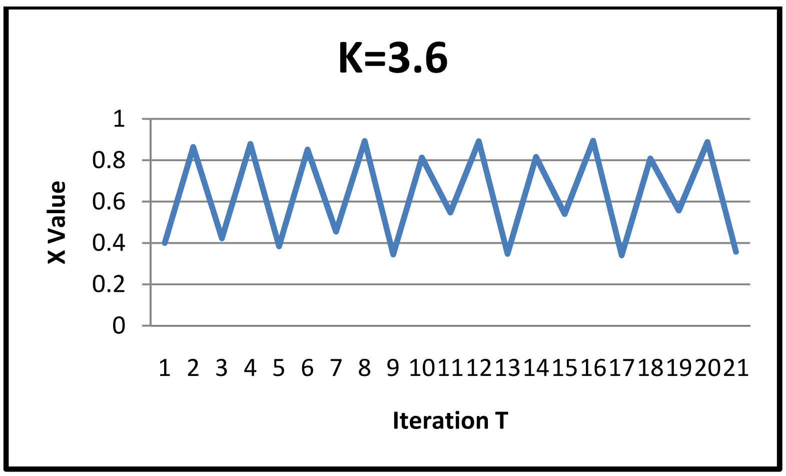


**Figure 8: The Logistics Equation: The Move to Chaos through Period Doubling, K=3.6**

In the logistics equation, the system behaves as a single trajectory that converges to a single attractor. This occurs when the system cycles between two or more points. Feigenbaum's number (Equation 3) suggests that period doubling occurs at approximately k=3.0, 3.4, 3.55. 3.6 and so on. At these values, the number of points in the series grows geometrically from 2, 4, 8, to 16. By the time k=4.0, the system has so many points within the series that the concept is no longer useful for prediction and the system has evolved into chaos.

Although there are multiple specific models for how systems move from stable states to chaos, they are generally variations on three categories: period-doubling, quasiperiodicity and intermittency.[17] In the logistic equation where k is increased from 3.0 to 4.0, chaos is reached through period doubling that is determined by universal bifurcation limits as defined by Feigenbaum's number. Quasiperiodicity is similar to period-doubling, but involves the mixing of two or more sinusoidal[18] inputs with different periods and frequencies (see Figure 9 for example of quasiperiodicity.)

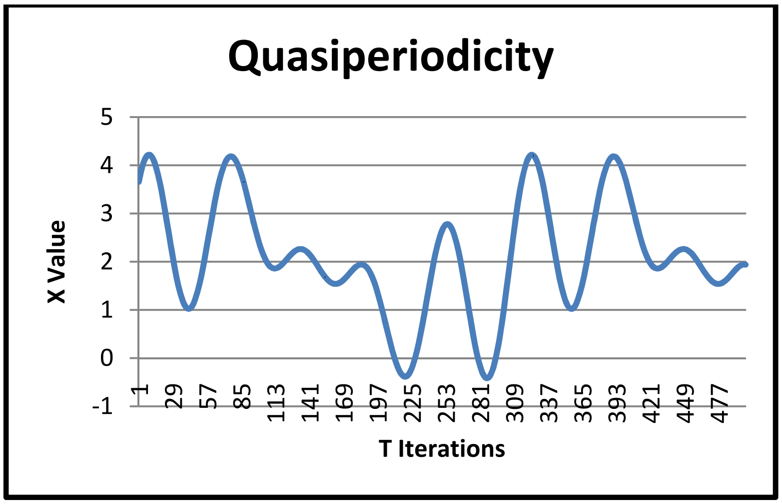


**Figure 9: Quasiperiodicity Caused by Three Sine Functions with Different Frequencies**

$$y = \sin(x) + \left(\frac{1}{2} + \sin(.2x)\right) + (1. + \sin(.8x) \qquad (4)$$

Chaos can be reached in a variety of methods that are neither complex nor difficult to understand. Quasiperiodic chaos in a real world example could be the result of two or more naturally occuring periodic inputs. One possible example is road trafficability that is influenced by seasonal precipitation, wind, and visibility. If these three periodic phenomena were out of phase, chaos-like behavior could be seen when trafficability is plotted as a variable (see Figure 9 and Equation 4 for an example.) Intermittent and period doubling routes to chaos can be the result of a parameter shift as discussed. These types of

[17] Ibid., 166.

[18] Sinusoidal waves are in the shape of a sine wave – a regular oscillation between two points.

chaos explain single variable systems that exhibit point or periodic attractors. In order to generate strange attractors, a different and more complicated model is required.[19]

### Multiple Variable Chaos

There are two types of equations that are dynamic: difference and differential equations.[20] The logistic equation is a difference equation that describes motion in discrete steps where the next value is influenced by a function of the previous value. The next two examples introduce the concept of dynamical differential equations. Differential equations model continuous change in a system or a single variable as a continuous derivative rather than looking at discrete steps. When an equation uses a dx or δx operator, it implies a moment of the variable ‘x’.[21]

The Lotka-Volterra equation is a system of two ordinary differential equations that model the populations of a predator and a prey. In the system, predators hunt and eat prey at a certain rate and reproduce at different rates. These rates are parameters within the equation. Based on parameter decisions and initial populations, the Lotka-Volterra equation shows point and periodic attractors. Through experimentation, a researcher can locate the tipping points where point attractors change to periodic attractors. The following figure shows periodic attraction when the initial conditions are 100 prey and 30 predators.[22]

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[19] A Strange Attractor is an aperiodic and bounded orbit where the attractor is not convergent.

[20] A difference equation is an equation based on changes that occur at a discrete interval (usually time) and solved by iteration. A differential equation **is a**n equation expressing a relationship between a function and one or more of its derivatives based on changes that occur continuously (usually time).

[21] The use of dx//dt implies that the equation only consists of a single variable and is considered an ordinary differential equation. δx/δt implies a partial differential equation where there are multiple variables, but the differential is only applied to x with respect to t.

[22] As shown in Figure 9, the program was intentionally written to have each variable continuous. This does not model reality as the entity count would be integers. However, by making the variables continuous, it is easier to show the concept of periodic attractors.

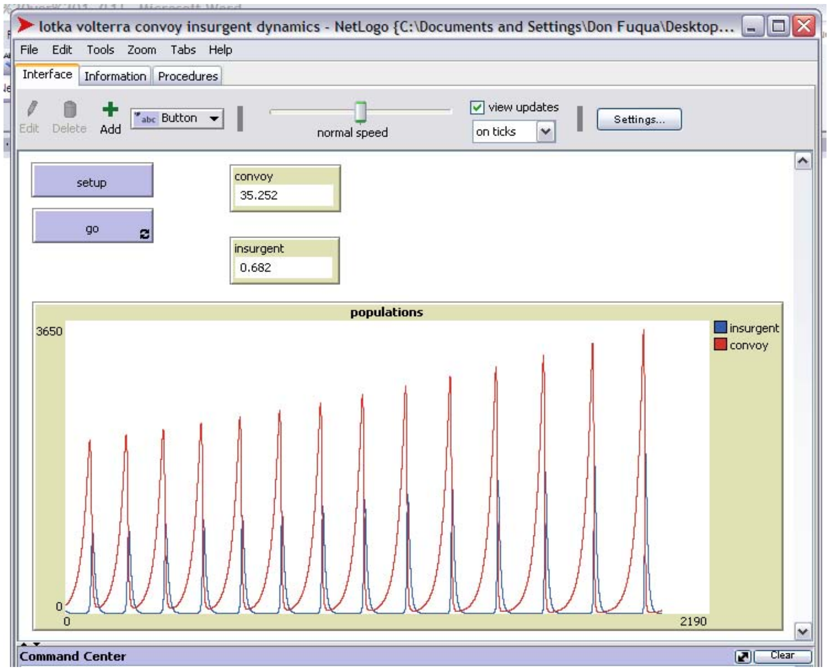


**Figure 10: NetLogo Simulation of Lotka-Volterra Equations**

$$\begin{aligned} \frac{dx}{dt} &= x(\propto - \beta y) \\ \frac{dy}{dt} &= -y(\gamma - \delta x) \end{aligned} \tag{5}$$

NetLogo, a system normally used for agent based modelling, peformed the simulation by approximating differential equations using agents using a dynamic modeling application.. Agents, modeled as Convoys and Insurgents, interact through a code written to approximate the Lotka Volterra two-variable differential equations. See Equation 5, x and y are phase variables that measure the populations of two species: convoys and insurgents. The system parameters are defined by the user: σ (sigma), β (beta), γ(gamma), and δ (delta), and represent predatation and reproductive rates of the entities. In the simulation, t represents time and is continous in Equation 4, but is discrete in the simulation. The NetLogo source code is located in Appendix 2.

One limitation of the NetLogo simulation was that true continous behavior can only be approximated. In Figure 10, the number of predators is increasing and is unbounded. It is temporarily periodic, but each population would eventually go to zero. Limited simulation choices in the parameter and starting entity values constrained the illustration of true periodic outputs (as seen inFigure 7: the logistic equation when k = 3.0).

A Matrix Laboratory (MATLAB) program (coding is shown in Appendix 2, Part 2) models movement of a system of three coupled ordinary differential equations in three dimensional space. The equations were originally described by Edward Lorenz in 1963 while attempting to generate mathematical models for predicting weather patterns.[23] In order to generate the model, he studied the phenomenon of convection in a laboratory environment. Lorenz found that the system was highly sensitive to initial conditions and had dense orbits around strange attractors. The locations of these attractors are governed by the values of the parameters applied in the equations. In the Lorenz equation, x,y, and z are phase variables that measure the flow of heated water in a closed system. The system parameters are defined by the user: σ (sigma), ρ (rho), and β (beta).

$$\begin{aligned} \frac{dx}{dt} &= \sigma x + \sigma y \\ \frac{dy}{dt} &= \rho x - y - xz \\ \frac{dz}{dt} &= -\beta z + xy \end{aligned} \tag{6}$$

[23] James Gleick, *Chaos, Making a New Science.* (Viking Penguin, New York, 1987), 5.

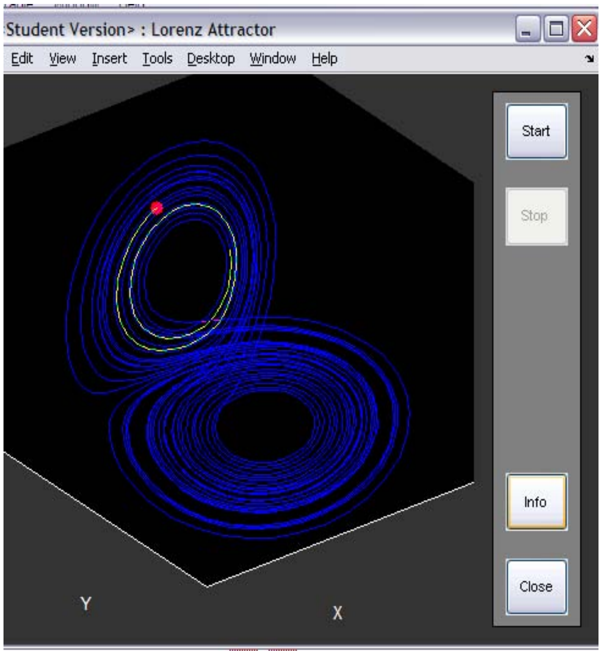


**Figure 11: MatLab Simulation of Lorenz Equations: The Butterfly Effect**

Lorenz discovered that minute changes to the initial conditions and parameters can have varied and often unpredictable consequences in the future. Based on the shape of the graph of the strange attractor with two quasi-periodic regimes, this phenomena of bifurcation was described as the "Butterfly Effect". Given a random starting point and user-set parameter values, the MATLAB coding exhibited the graph shown in Figure 11:[24] The graph shows the trace of the possible behaviors given the three Lorenz equations. Based on carefully selected parameters and starting points, the solution trace tightly orbits around two strange attractors and resembles two butterfly wings.[25] If a decision maker were trying to understand a system that was governed by similar differential equations, he would not see the strange

[24] For the MATLAB simulation, sigma = 10, rho = 28, and beta = 2.33 were used to show the best graphical result.

[25] The Butterfly Effect is an analytical tool for understanding the ideas of sensitivity to initial conditions and attractors in Chaos Theory. Williams, *Chaos, 218.* This concept has been expanded through popular media (such as the 2004 film, *The Butterfly Effect* ) to encapsulate the idea that a butterfly flapping its wings in Australia can cause a hurricane in Florida. This farfetched idea obviously ignores effects such as dampening, bounding, and rigid physical laws that might apply.

attractors and orbits by plotting points over time across one variable. The only way to see the attractors is by plotting the three variables in a three dimensional graph without a time component. This method requires both time to allow enough observations to occur to see the attractors and the ablilty to keep system parameters constant throughout the observation period.

Based on the understanding generated, the idea of the “Butterfly Effect” has a number of different implications. The idea that a butterfly flapping its wings in one part of the world and causing a hurricane in another is flawed due to the concept of bounding and the influence of perturbations occurring throughout the system. Unfortunately, a phenomenon named for the shape of a graph exhibiting the dynamics of a system of ordinary differential equations has become linked with the idea that determinism can override adaptive behavior and interaction.

Up to now, models have shown the mechanisms that make types of systems highly susceptible to chaos. Solely based on these models, the case can be made that chaos can be detected, monitored, and even manipulated given control of parameters.  Remember, however, that the one variable and multivariable examples were idealized and highly controlled mechanistic models not influenced by adaptive interaction or real world complications. Given this understanding, readers must now recognize the limited and unforecastable nature of chaos within real world adaptive environments.

## Detection and Applications of Chaos and Fractals

Given a general understanding of the concept of chaos, trying to find evidence of its existence in nature is a logical progression. So far, chaos has been explained as a process that appears totally random, but is actually deterministic. While various general mathematical models were simulated to show the evolution of chaos, the importance of initial conditions, and the three types of attractors, this information is purely academic without some connection to real-world problems. What has not been discussed is how to determine whether a behavior is truly chaotic, or if this understanding is relevent for military planning.

There are many different types of chaos detection used in a variety of disciplines with varying efficacy in analysis on dynamic adaptive systems. The main types of detection include Fourier, Lyapunov exponents trajectory, and Kolmogorov-Sinai (K-S) analysis. It is impossible to describe detection, however, without understanding the concept of fractal geometry as a way of understanding turbulence within a time series.

**Fourier Analysis**

Fourier analysis is a technique used to dissect a time series output into its constituent wave forms.[26] This type of analysis is useful in deconstructing quasiperiodic chaos. Because many simulateous waves create the appearance of randomness, Fourier separated wave forms into an algebraic equation. Each wave is compared to a basic wave on a fundamental frequency and wavelength. The resulting equation is written as a general Fourier series: where α and β are the discrete Fourier coefficients of each wave, θ is the phase angle of the constituent wave, N is the total number of observations, and h is the number of the harmonic.

$$y = \sum_{h=0}^{\frac{n}{2}} \propto_h \cos[h\theta] + \beta_h \sin[h\theta] \tag{7}$$

Fast Fourier Transforms (FFT) are a current method of using computing technology to quickly approximating, analying, and separating constituent waves using complex numbers. In this method, a discrete Fourier transform is approximated for each harmonic. Using the same identities as the Fourier the transform for each harmonic is approximated through a complex series as $\alpha_h + i\beta_h$ where i is an imaginary number ($\sqrt{-1}$).[27] Unfortunately, to gain any certainty in analysis, multiple data points are required.[28] Also, the tests are valid for waves in isolation. When adaptive elements are added to the system, Fourier analysis cannot extract non-wave patterns. Therefore, while this is a useful technique in mechanical closed systems, Fourier Analysis offers little utility for decision makers.

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[26] Fourier Analysis is named for the mathematician Jean Joseph Fourier (1768-1830), a self-taught public administrator in the French Republic, Williams, *Chaos*, 108.

[27] Ibid., 125-6.

[28] Generally, in Fourier Analysis, N is generally over 1000 points in order to gain any significance using Fisher Tests.

The second portion of quasiperiodic analysis is testing for autocorrelation. This is testing for how much a series resembles itself at a given lag. Generally for a series with N components, lag is tested for all variations from $1 \leq N \leq \frac{N}{4}$. As with Fourier Analysis, these tests assume that the system is closed and mechanical. Also, to gain any degree of fidelity, N must be a sufficiently large number.[29] There are a number of methods used to determine statistical significance such as the Chi (χ) –Square Test and the Fisher Exact Test.[30] These methods are not explicitly discussed, but should be studied for those interested in learning more about relevence testing.

$$\frac{\sum_{t=1}^{N-m}(x_t - \bar{x})(x_{t+m} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2} \tag{8}$$

**Lyapunov Trajectory Analysis**

A baseic understanding of chaos has established the importance of sensitivity to initial conditions and attractors as both conditions and powerful implications of chaos. Lyapunov Analysis generates a Lyapunov exponent, also called a characteristic exponent, which describes the average rate of convergence or divergence of two trajectories in phase space.[31] A positive exponent indicates sensitivity to initial conditions by quantifying the average rate that a perturbation diverges.

Without involving mathematical proofs and the technical descriptions of the mechanism of the exponent, decision makers should know that Lyanpunov Analysis is only one technique for quantifying chaos. Unlike Fourier Analysis, the technique can be used within non-mechanical systems because the exponent only gives a general description of the convergence or divergence of a series rather than

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[29] The size of the sample is a function of the variation within the sample and the degree of fidelity needed in the experiment.

[30] The Fisher Exact Test and the Chi-Square Test are two statistical tests used to either validate a null hypothesis or tentatively accept an alternate. While Chi-Square tests require large data samples, the Fisher Exact is normally used in samples where N < 10.

[31] Williams, Chaos, 353-354.

reducing this convergence or divergence to individual components. The second important distinction is that fewer data points are needed for analysis.

The Lyapunov Equation is a relatively simple equation that can be computed using various programs. MATLAB, Maple, and Mathmatica are popular mathematic programs used for Lyapunov Analysis.[32] Although fewer data points are needed for analysis, more observations improve the confidence intervals of the results. The general form of the Lyapunov Equation is shown in Equation 7 where b is the exponent constant that describes either convergence or divergence, δ (delta) is the finite value of either the iteration (n) or the intercept value (a). The value 'e' is the natural exponent gained from the basic identity in Equation 10, where C is a constant and e is approximately equal to 2.7181.

$$\delta_n = \delta_a e^{bn} \tag{9}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \mathrm{e} = \int \frac{1}{\mathrm{x}}\,\mathrm{dx} \tag{10}$$

**Kolmogorov-Sinai Entropy Analysis**

Entropy is a measure of disorganization or unavailable energy within a system. This concept is the focus of the second law of themodynamics – that entropy within a system will always increase on average. One way to describe the concept is by imagining a ceramic coffee cup precariously perched on a table. At the initial state, the coffee cup has organized potential energy due to its position (a few feet above a tiled floor) and through the bonds that form the porcelin into a cup. After falling, the energy within the cup can not become more organized. When the cup hits the floor, sound and heat energy are released on impact. The solid ceramic mass that made the cup is now in shattered pieces. Although still present, the total energy from the initial state has become dissapated and less available for use. Without devolving into proofs, the reason systems naturally transition from highly ordered to less ordered energy is because dissapated energy states have much greater probabilities of occuring.

[32] Maple is not an acronym or abbreviation, but rather is a tribute to the Canadian origins of the software.

There are a number of ways to visualize the idea of how probability determines future states. Consider a room full of oxygen, hydrogen, and nitrogen gas molecules. Expecting that in an average room with a normal atmosphere that oxygen molecules would occupy only one side of the room is both illogical and impossible. However, this highly organized state, would require no more energy than a room with homogeneously mixed gases. Because it is highly improbable, the organized state is never observed.[33] In this concept, organization is a statistical constrain on systems because there are many more unorganized possibilities. Therefore, organized states tend to evolve towards disorder in closed system.

In the one variable logistics equation, when the parameter increased from less than 3 to values above 3, the number of possible attractors increased at a rate estimated by the Feigenbaum number. The route to chaos can be described as increasing disorder based on the system behavior and the description of entropy.

Kolomogorov-Sinai (K-S) methods allow a researcher to examine a system for chaos based on changes in the amount of total entropy in the system. The three main features of K-S Analysis are sequencing, rate, and limiting. A series is examined through a sequence as a rate of entropy by limiting the size of the observation and increasing the time as a double limit. These observations give rise to a prediction for the next events, predictions of the decay of accuracy, and rate of information loss.[34]

K-S entropy in a system requires a computation of diminishing bin sizes (ε) over inceasing time steps (t). The entropy equation requires a large amount of data and generally only describes mechanical systems.[35]. This limitation generally removes K-S analysis from use in a social environment or *in situ* setting. The important consideration is the link between increasing disorder and detection of chaos. This may seem like an obvious observation, but this link shows that there is a turbulence scale and a route

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[33] In a room with 1 mole of oxygen molecules ($6.02 \times 10^{26}$), the probability that all the molecules would occupy the same side of the room is less than $\frac{n!}{k!(k-n)!}$ when n=0 and k=$6.02 \times 10^{26}$ (a very small number!).

[34] L.S. Young, “Entropy, Lyapunov Exponents, and Hausdorff Dimension in Differentiable Dynamical Systems, *Physical Review Letters*, (PLR, 1983) 1095-1098.

[35] Generally samples less than n=10,000 do not produce significant results. Studies in K-S Analysis use greater than $1 \times 10^{6}$ data points. Williams, *Chaos*, 397.

attached to chaos. If data sets are one detriment to using K-S analysis, another is the computing power required to accurately run the models. This is due to the large data set, and the complicated algorithm needed to measure entropy (H) as a supremum at various partitions as the batch decreases and the rate size increases.[36]

$$H_{\mu}(T)\ \sup\nolimits_{Q\, H_{\mu}(T,Q)} \tag{11}$$

**Examples of Chaos Detection in Military Operations**

There are multiple examples of physical systems exhibiting chaos. The routes to chaos that can be easily described through interacting waves and difference and differential equations seen both *in vitro*[37] as in Lorenz's micro-weather system analysis or *in silica* seen in simulation examples. While these examples are interesting, they have little relevence for decision makers who are solving complex problems that are neither in a laboratory (*in vitro*) or in a computer (*in silica*), but instead in the real world (*in situ).*

Using Lyanpunov Trajectory Analysis and K-S Entropy Analysis, certain military operations such as U.S. aircraft losses in the Vietnam War have shown chaos properties.[38] These studies are completed after the events and have not been used for prediction or control. The presence of adaptation negates any attempts for process control through Chaos. Changing techniques of aircraft flying routes based on chaos analysis guarantees that the enemy will change his tactics, and therefore change the system.

The better way of applying and understanding chaos is through considering the effects of potential physical phenomena rather than adversary decision cycles. There is a reason that examples of

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[36] A supremum is a least upper bound of a set of real numbers and is the smallest real number that is greater than every partition of the set. For the KS entropy, this requires high speed computing due to the heuristic search mechanism required to find the smallest real number across all possible partitions of the set.

[37] *In vitro* translated literally from Latin mean 'in the glass'. *In vitro* observations occur in a test tube or in a controlled laboratory environment.

[38] Glenn E. James, *Chaos Theory: The Essentials for Military Operations*, (Naval War College, Newport, 1996) 2, 75-77.

chaos in real world environments are general and done after events are completed. By attempting to change a system, self-organization of adaptive elements will react.

## Fractals

Fractals are scaleless patterns that are ubiquitous in nature and mechanical systems.[39] They were first described by Benoit Mandlebrot, an applied mathmatician who used fractals to describe fluctuations in the cotton futures market prices.[40] While chaos describes behavior within a time series, fractals describe the geometry and overall characteristics of a system. However, long term chaos behavior and the presence of attractors often has fractal characteristics. Because of this characteristic, fractal analysis has been used to detect and analyze chaos.[41]

The term fractal geometry is used because these systems have a predictable structure when analyized in a fractal dimension.[42] By examining cotton prices in terms of a fractal dimension, Mandlebrot found that a seemingly random series in an integer dimension was linear and predicatable in a fractional dimension. Since then, multiple examples of fractal strucures in nature have been found such as river branches, coastlines, the branching of veins and neurons in living beings. Many social systems have also shown fractal geometries, Instances of stock prices, commodity derivatives, the time between state conflicts, election results have all been describes as fractal in scientific research..[43] Statistician LF Richardson, used a logrithmic scale to catalog a fatal quarrels (where one death can be shown as equal to $10^0$ or a scale 1 conflict, one million deaths equals $10^6$ or a scale 6 conflict, and so on). He found that

[39] Williams, *Chaos*, 237

[40] B. Mandlebrot, "How Long is the Coast of Britain: Statistical Self-Similarity and Fractal Dimension" *Science 156*, (1967), 636-638.

[41] F. Moon, "*Chaotic and Fractal Dynamics*", (John Wiley, New York, 1992), 305.

[42] A fractal has a non-integer dimension.

[43] Williams, *Chaos*,238.

both on a global scale fron 1820 to 1945 and in various local levels, conflicts followed a power law where the log-log graphs were linear.[44]

There are two general types of fractals: deterministic and stochastic. The main difference between the two is that deterministic fractals are models without bounds where there is no variation between scales. Stochastic fractals are found in nature and have difference in scale based on a degree of randomness.[45] In nature, fractals are not scaleless. A leaf that has a fractal structure in its xylem and phloem vein structure is bounded because the leaf irself has a discrete size.

While there are many types of dimensional analysis used in detection and analysis of chaos, there are no agreed upon methods for detecting chaos within real world behaviors. This type of detection is an emerging mathmatical field. However, knowing the existing techniques for chaos analysis in controlled environments is important. Military decision makers should recognize the severe limitations of this type of analysis and be able to avoid over analysis or the danger of accepting a false hypothesis based on wide confidence intervals due to lack of data.[46]

The two points for planners are: first, that chaos detection has not been developed for practical use, and second that chaos is a component of a complex open system and the requirements for data would normally precude their use in adaptive problem solving. Each method discussed investigates behavior either *in silica* or *in vitro* and not *in situ* since military planners operate in the real world. This fact should discourage using these methods in solving problems within the inherent complexity of adaptive systems.

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[44] Lewis F Richardson, "Variation of the Frequency of Fatal Quarrels with Magnitude", *Journal of the American Statistical Association 43,* (December 1948), 523-546.

[45] Ibid., 242.

[46] Accepting a false hypothesis is referred to as a Type II or consumer error since in manufacturing a defective part not detected in the factory would affect the end-user. Douglas Montgomery, *Introduction to Statistical Quality Control.* (Hoboken, John Wiley & Sons, 2005).

## Military Operations: Adaptive, Complex, and Dynamic

Consider the following scenario: a logistics planner is attempting to minimize the average time for a single customer to order and receive materiel. On the surface, this appears to be a simple problem – the planner must ensure that either the correct item is on hand in a nearby warehouse when the customer requires it or ensure that the distribution system from the depot (where he assumes a sufficient quantity of items are available) work as quickly as possible to expedite the movement of the materiel to the customer.

In this example, the planner creates a supply chain network in order to form a mental model. The customer, warehouse, and depot are shown as nodes, one set of arcs represent roads connecting the nodes, another set of arcs represent the communications links between the nodes. The entities moving through the network are data through the communications links and materiel through the road network. See Figure 12 for a representation of the initial supply chain network.

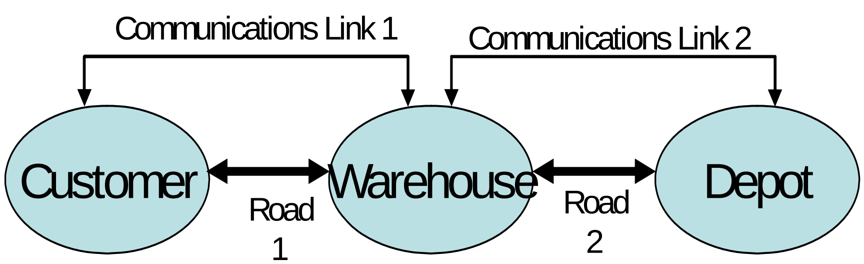


**Figure 12: Simple Supply Chain Network**

Based on his initial understanding of the network (Figure 12), the planner creates a linear mathematical program to optimize the system (Figure 12). Since the planner knows the space limitations of his warehouse, the general capacity and speed of the roads and the communications infrastructure, and a general idea about the demand history for the item, he confidently executes his plan for minimizing customer wait time.

$$
\begin{cases}
Minimize\quad SUM\ (transport\ time) + (holding\ time) + (communications\ time) \\
\quad Subject\ To\colon\ Warehouse\ Capacity\ and\ Speed\ Constraints \\
\qquad\qquad Demand\ Constraints \\
\qquad\quad Road\ Capacity\ and\ Speed\ Constraints \\
\qquad Communications\ Capacity\ and\ Speed\ Constraints \\
\quad Positive\ Integers\ (Non-negativity\ restriction)\ \text{Constraints}
\end{cases}
$$

**Figure 13: Linear Program for Simple Supply Chain Network Model**

This type of simple closed system, however, does not model reality. Real-world supply chains are open systems comprised of known and unknown relationships and assemblages acting through dependent, interdependent, and independent mechanisms. In the case of the logistician in Iraq, an insurgent group may have influenced trafficability of the road networks and the usage rate of materiel by the customer. Periodic weather phenomena can influence trafficability and communications. Political influences that are often not seen by the logistician can play a large role in the behavior of the system. Beyond those factors, multiple actors and physical phenomena that are either apparent or unknown will have varying influences and interdependencies in governing the performance of the system that he is attempting to control.

Based on all these factors, a common mistake is to declare that the system cannot be managed and give up hope of controlling the process. Deciding that the process is beyond control negates the power that the planner has in shaping the environment. A second mistake is to reason that since the system is complex and adaptive, the system can only be influenced by rational actors reacting to a changing environment. This reasoning fails to consider the influence of physical dynamics on the system (e.g. weather, sea movement, etc.). The failure of both rationales is in correctly framing real-world complex adaptive systems. Therefore, readers must be able to defines complex adaptive systems, understand the requirements for emergence and self-organization, and know the differences between adaptive entities and deterministic dynamics.

Complex adaptive systems are defined as open environments where multiple entities form interactions that influence their own behavior. These interactions cause entities to self-organize and generate emergent behavior that is extremely difficult to predict.[47] These systems are influenced through variation, selection, and interaction between agents and as a group. While there may be multiple ways of interacting and influencing these systems, there is not an accepted model or methodology in dealing with complex adaptive systems.[48]

Most complexity models either treat systems as black boxes or as mechanical process to be dissected. While the black box or holistic approach only considers inputs and behaviors, the mechanistic approach attempts to mathematically decompose the inner working of systems. While the black box method can fail to recognize why or what is necessary to cause adaptive behaviors the mathematical reduction of a system requires over-simplification and generalization of the different mechanisms of adaptation. A more mathematical approach to conceptualizing complex adaptive systems is needed, while recognizing the intricacies and general nature of emergent behavior. The research uses a systemic approach, which fits somewhere between the holistic and mechanistic models. This approach appreciates the complexity and general nature of a system while attempting some measure of prediction and control using quantitative methods.

In complex adaptive systems, there are recognized necessary but insufficient conditions for adaptation in systems. First, the system must contain some form of variation between agents. This difference moves the system from a natural equilibrium into a state of flux. Second, a feedback loop must be established where the agents attempt to deal with disequilibrium through a process of interaction.[49] The final necessary condition is selection, where past behavior is changed through adaptation to a new structure that responds to differentiation. The new behavior is referred to as emergent when the result of the system cannot be inferred through an examination of the individual components.

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[47] Robert Axelrod, *Harnessing Complexity,* (New York, Basic Books, 2000), xi

[48] Glenda Eoyang, *Conditions That Support Self-Organization in a Complex Adaptive System,* (Williamsburg, International Association of Facilitators, 1999), 2

[49] Ibid., 4., Eoyang refers to this step as creating a transforming step across the differentiation. Axelrod discusses this step in terms of interaction between entities.

There is a subtle difference between self organization and emergence. Each state can be mutually exclusive; emergence can occur without self-organization and self-organziation without emergence. Simply put, self-organization occurs when agents or entities become more organized.[50] This property is not intuitive or logical. From the discussion on entropy, systems are supposed to move from order to disorder, not the other way. Adaptive agents, which do not act chaotically, can self organize.

While there is considerable debate regarding the ability of chaos to cause general emegence, it cannot self-organize. Emergence occurs when behaviors at the macro-level cannot be predicted or deduced by observing micro-level components.[51] Unfortunately, this is a broad definition that has been liken either to a provisional construct or an epistemiology. For example, in the Lorenz equation is it possible to deduce from the variables that a strange attractor could develop? Again, this is an open debate. For the purpose of this research, emergence refers to novel characteristics formed by the self-organization of adaptive agents

Correctly framing real-world complex adaptive systems therefore cannot be accomplished through reductive techniques that examine each component individually.[52] Instead, managing the system requires a holistic understanding of all combined elements while appreciating that elements and connections that are either unknown or highly variable. This systemic understanding must consider deterministic as well as adaptive components in order to exercise some form of control or prediction. While the decision makers must consider the effect of the adaptive and deterministic assemblages, the nature of complex systems is formed through the interaction and connections between entities and assemblages.

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[50] Cosma Shalizi, "Self-Organization", *Center for the Study of Complex Systems,* 16 April 2009, Database On-Line, Http://cscs.umich.edu/~crshalizi/notebooks/self-organization.html/, Accessed 1 May 2009.

[51] Jeffery Goldstein, "Emergence as a Construct: History and Issues", *Emergence* I(1), (1999) 49-72

[52] Yaneer Bar-Yam, *Making Things Work: Solving Complex Problems in a Complex World* (Cambridge: NESCI Knowledge Press, 2004), 110.

Adaptive entities or agents are able to respond and interact with their environment and with other agents.[53] *In situ*, agents can be human beings, sentient creatures, or elements that act according to a set of rules of procedures based on group or individual need. Pseudo-adaptive agents are simulated *in silica* to approximate adaptive behavior and emergence. These agents act individually according to programmed rules. Despite the artificiality, self-organization does occur within the bounds of the computer programs.

Despite the ominous prospect of making decisions in adaptive environments, there is potential advantage to wielding emergence as a positive force.[54] Many strategies are outlined in Robert Axelrod's book, *Harnessing Complexity*. Without listing them, they center around recognizing, fostering, and steering adaptation within an organization. This methodology is counter to the notion that emergence must be controlled or eliminated. In this paradigm, success is not obtained through long-term broad goals, but instead through constant feedback mechanisms, evolving measures of success, and an understanding that short term successes can lead to long term failures as well as the reverse.

An effective way of visualizing adaptation and novel emergence is through agent based simulation. Agent based models use individual 'actors' that make choices based on a set of programmed rules. This type of simulation can show how societies of agents react to different conditions. While not an automatic response from independent acting agents, emergence can form given the right circumstances.[55]

NetLogo is one agent based modeling program used in a variety of educational and research specialties. The NetLogo program architecture was developed from StarLogo and was written by Uri Wilenski. NetLogo is currently maintained by the Computer Science Department at the Northwestern University and available for free download.[56] The program incorporates an advanced form of the Logo programming language using agents (with objects names: turtles, patches, links, and observer) to interact

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[53] Axelrod, Harnessing Complexity, 5

[54] Ibid., 156-158

[55] Emergent behavior is a process where the macro-level behavior is different than the additive properties of the components (the micro-state).

[56] For download use *ccl.northwestern.edu/**netlogo**/.*

within a grid defined by the user. Earlier, a Chaotic system was simulated in two variables using the NetLogo System Dynamics modeler. An adaptive model shows the properties and limits of emergence.

An easy model to demonstrate emergence is the Segregation Model first described by Thomas Schnelling.[57] In the model, a variety of agents are randomly identified as ‘reds’ or ‘greens’. Each agent does not compete with opposite color agents, but does desire to ‘live’ next to a certain percentage of similar colored neighbors. If the agent is happy, it does not move. If the agent is not happy, it moves in a random direction at a constant step size. The following figure shows the initial conditions for the ‘pond’ at time zero with a random set of 2500 agents.

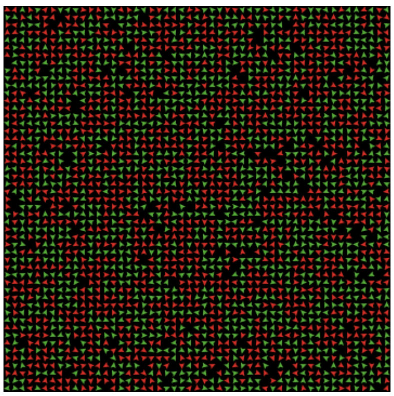
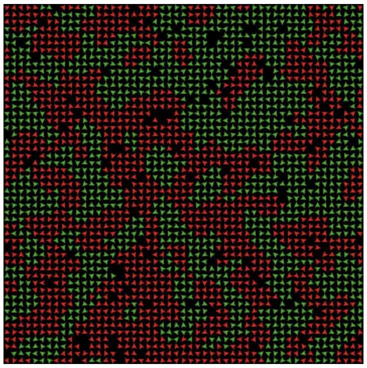


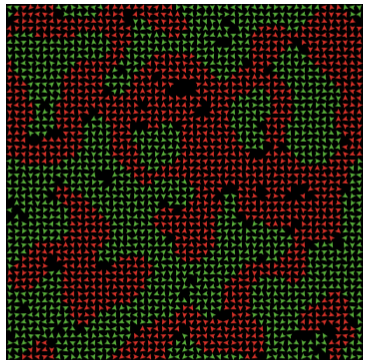
**Figure 14: Segregation Model at Time = 0**

Once the model starts, each agent moves based on the criteria that a certain percentage of neighbors should be similar. Through experimentation a variety of states are shown given a change in the parameter ‘% similar wanted’. Based on the experimentation, with 2500 agents, the tipping point is 72%. Above this number, emergence does not occur. Below that number, emergence occurs, but with less organization.

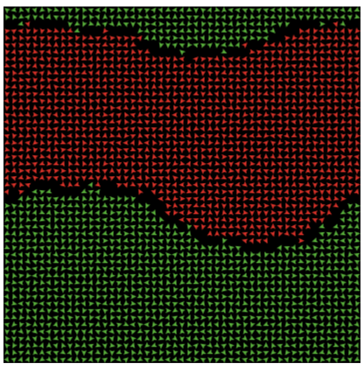
[57] Thomas Schelling, *Micromotives and Macrobehavior*, (Norton, New York, 1978).



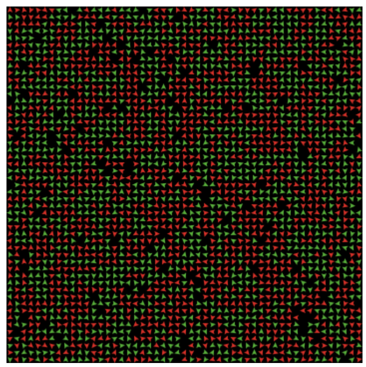
% SIM = .25



% SIM = .50



% SIM = .72



% SIM = .80

**Figure 15: Effect of Changing a Parameter in the Segregation Simulation**

As noted in this experiment, the patterns and grouping of the agents are not additive or obvious from the components. This is the essence of emergence. Also, by placing too much control on a system, there is a acute point where emergence can no longer occur.[58] The second interesting phenomenon associated with this trend is the apparent contradiction with the properties of entropy. In the Segregation model, entropy decreases and order increases when the % SIM parameter is less than .73. Obviously energy is required and becomes dissapated in the movement of the agents, but the increase of order over time shows a fundamental difference between self-organizating adaptive agents and the routes to chaos in physical effects of dyanmical systems.

[58] While this monograph does not explore the concept of the tipping points in emergence, there is an interesting linkage to Chaos. Does Chaos and turbulence push these tipping points in either direction?

## How Chaos Differs From and Influences Adaptive Systems

Chaos is fundamentally different from adaptive interaction despite the implication that chaos can have a varied number of variables and parameters. This difference can be explained through the use of the necessary topological conditions for emergence and co-evolution. Given this understanding, decision makers must understand how chaos and other forms of deterministic turbulence influence first order effects only while adaptive entities can affect first, and higher ordered effects. These higher level effects are the exclusive domain of novel emergent properties.

One cogent definition of novel emergence is where "it is present in a macrostate but it is not present in any microstate, where the microstates differ from the macrostate only in scope."[59] Using this definition, it is unlikely for objects with deterministic fractal geometry to produce novel emergence in a system.[60] This property can be carried forward to chaos given the understanding that there is a fractal geometry to deterministic dynamical systems.

Another way to describe the difference between the macro and microstates is by examining the Segregation and Logistic models. When the Segregation model is viewed at the microscale, each agent is moving according to specific set of simple rules – it either does not move if it is satisfied with its position or moves one step in a random direction. This movement is, in effect Brownian and is stochastic. Structure only appears when the agents are considered at the macroscale. By moving away and reducing the resolution, a completely separate order is revealed.

In the logistics equation, reducing resolution only decreases the apparent turbulence in the system. A system that is bounded and turbulent appears stable when viewed from a higher vantage point. This is analogous to hiking in a mountain range versus flying over it at thirty thousand feet. While it is possible to see the overall range, the physical intricacies seen at the microlevel are lost.

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[59] Alex Ryan, "Emergence is Coupled to Scope Not Level", *Complexity 13* (December 2007), 67-77.

[60] Again, this in an area of active debate whether deterministic systems can have emergent properties. See Goldstein, *et al*.

## Conceptual Model of Chaos in Complex Adaptive Systems

There are key parts of chaos that a planner should know. First, there are various, and often simple routes that can lead from chaos to stable behavior. Second, a system's sensitivity to initial conditions explains how small changes can create big differences in behavior, and also how long-term system predicatability is not guaranteed. Third, while there are powerful tools for detecting chaos, these tools are often only useful in isolated closed systems and require multiple data points in order to gain reasonable confidence intervals. While detection is useful in mechanical systems, complex analytical tools offer little utility when describing chaos within a complex adaptive system. Fourth, military operations are generally complex adaptive systems. Because military operations are conducted in open environments, chaos can influence the first order behavior of the system. Finally, unlike adaptive entities, chaos is unlikely to create novel emergence and self-organization due to its lack of differences in scope and adaptive components.

Given these understandings, the research proposes a model for understanding chaos as a potential component of complex adaptive systems. In this model, adaptive agents behave according to their multiple interactions within a system. At the same time, physical effects enter and leave the system. These physical effects, through lack of scale and interactive binding, do not create second and higher order effects but do act across the surface of the adaptive system to either dampen or amplify the system. These effects, however, do not influence either novel emergent behaviors nor create steady states. Regardless, they are a part of the system and cannot be ignored due to the methods that adaptive agents interact with the physical effects to reach their preferred states.

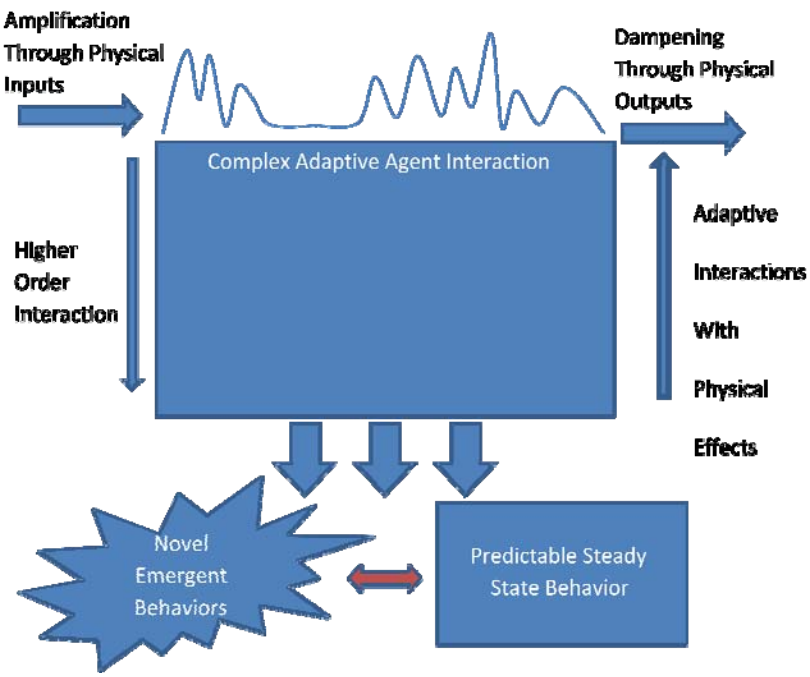


**Figure 16: Conceptual Model for Understanding Chaos as a Part of Adaptive Systems**

Novel emergent behavior and steady state behavior are the product of adaptive interaction. These behaviors are not static, but can move between states based on changes to the environment, changes to parameters, and new patterns of interaction. While physical effects create barriers and permissable spaces in the system, the presence of complex adaptation negates any ablility for prediction or control based on chaos alone.[61]

By viewing chaos as a scaleless model, decision makers can argue that chaos and other physical dynamics such as fractals cannot produce novel emergence or coevolution. Yet, these dynamics are important when considered as either system noise or roadblocks that influence where and when adaptive agents choose to interact. One conceptual method of thinking about chaos and adaptation is through the Philosophy of Difference in the post-structural writings of Felix Guattari and Gilles Deleuze, specifically in their treatment of the concepts of Smooth and Striated spaces and the concept of the Nomad.[62]

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[61] Axelrod, Harnessing Complexity, 27

In their writings,Deleuze and Guattari discuss the implications of Smooth and Striated as a concept of the physical world spaces interacting with adaptive agents[63]. In *A Thousand Plateaus,* these agents are identified as the Nomad and the Sedentary. These agents either prefer the formless event-driven space (Smooth/Nomads) or the restricted barrier-driven space (Striated/Sedentary). Based on these spaces, two models of adaptation are possible: the War Machine or the State Apparatus.

Regions where parameters drive physical effects to point attractors can be thought of as Striated Spaces. In these areas, agents are limited to moving across specific paths and grids across the system. Linear regions can be thought of as Smooth Spaces. In flat planes, agents have no impediments and can move nomadically across infinite paths analogous to sailing in a smooth pond. Using Deleuzian logic, this is the region where the War Machine acts without boundaries. Conversely, Striated Space can be throught of as turbulent space where boundaries to free action exist. In these regions, the State Apparatus is favored based on the use of highly ordered rule sets and the desire to control variation.

While Deleuzian philosophy is not a perfect fit, the Philosophy of Difference does describe some of the ways that chaos and adaptation interact in a system. Chaos constructs roadblocks and obstacles in military operations. As discussed, this could be interactive effects between cyclic weather, season, and tidal phenomena while a military planner attempts to maximize main supply route throughput. In combat chaos could be manifest as failure rates for a key component within a vehicle due to vibration or failure influenced by conditional probabilities. The key point is that these effects are part of the system. The planner can either determine the bounds of the turbulence and give allowances for them, or through trial and error, discover the Smooth Spaces within a system. Yet, when that Smooth Space is found, the dyanamic and unpredictable combined effects of the system dictates that the system can quickly transition to Striated Space. These observations are due to sensitivity to initial conditions and sensitivity to pertubation – two key components of chaos theory.

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[63] Gilles Deleuze and Felix Guattari. *A Thousand Plateaus: Capitalism and Schizophrenia.* (Minneapolis: University of Minnesota Press, 1987).

Based on this complex adaptive understanding of chaos, military decision makers must rethink other concepts dealing with control of turbulence in a system. By framing chaos as a component of complex adaptive systems, tools such as Lean Six Sigma and Total Quality Management become less relevent. Although Lean Six Sigma is a powerful tool in manufacturing and non-interacting processes, chaos, self-organization, and emergence counteract the ability to deter variation within a process.. In complex adaptive systems, military decision makers cannot wish away chaos by concentrating on reducting variation on a single variable. This macro-scale solution is a likely route to failure in managing fine-scale problems.[64]

Although controlling variation has always been a core component of any quality control program, engineers working for Motorola in the early 1980's were the first to coin the term Six-Sigma. Bill Smith, a reliability engineer, found that actual defect rates at Motorola were higher than those seen in the factory (Type I Defects), and that these defects were then found by consumers (Type II Defects). He and Mikel Henry, at the Motorola Research Institute, refined their methodology and helped establish the idea of defect-free manufacturing to all other sectors of business at Motorola. Today, Motorola claims that Six-Sigma is "part of the genetic code of [their] future leadership.[65]

Motorola, Texas Instruments, Microsoft, American Express, and General Electric have all used Six Sigma. To varying degrees, these businesses have incorporated the Six Sigma process into their culture as an overarching method of improving quality and reducing costs. Because success almost always breeds imitation, Six-Sigma has become a catch-phrase and a business upon itself, often far from its original statistical roots. A quick search on Amazon confirms the bulk of books written on Six Sigma. A roughly equal number of businesses will train managers from any paying business on the "best" practices. In the past few years, the US Army has become a voracious customer.

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[64] Dietrich Doemer, The Logic of Failure: Recognizing and Avoiding Error in Complex Situations (Basic Books, New York, 1989).

[65] Greg Brue. *Six Sigma for Managers*, (McGraw-Hill, New York, 2002).

As Six-Sigma has expanded from a statistical tool to more of a managerial tool; model instructors have expanded the program from improving manufacturing quality control to enabling cultural changes within an organization. Training is geared toward different levels in the firms and is often given Karate level titles: Green Belts, Black Belts, Master Black Belts, Champions, and Executive Leaders. The idea is that all levels of the business must be committed to instituting Six-Sigma quality as a core component of the firm. The US Army has embraced this belt system – even offering additional skill identifiers (ASIs: 1X, 1Y, and 1Z) for training completed.[66]

Lean Six Sigma focuses on controlling the variation on a single variable without considering interaction effects in the system. This is apparent when you consider that the origin of Lean Six Sigma was in manufacturing. When you are constructing a silicon wafer with specification limits within one micron, Lean Six Sigma is a powerful tool in limiting variation. When dealing with highly interdependent social systems, reducing variation in a single variable could have unintended consequences.

Lean Six Sigma fails to consider inherent process turbulence seen in dynamical systems when working to control variation. Natural process turbulence can often appear as random variation if it is not thoroughly analyzed. Turbulence (even within one variable) can result from periodicity, autocorrelation, Chaotic system behavior, or fractional geometries within time series data.

Shewhart[67] control charts, Lean Six Sigma, Total Quality Management, and other similar statistical qualtiy control methodologies are a powerful tool in controlling quality in linear systems or nonlinear systems not influenced by process interactions. However, in the Army, very few operational processes can be classified through linear causation models. Real world environments are much more likely to be dynamic, contain self-organizing and adaptive variables, and have complex variable

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[66] US Army, DA PAM 611-21, Table 4-3, (USAPA, Fort Belvoir, 22 January 2007).

[67] Walter Shewhart is often referred to as the "father of statistical quality control" for his work in standardizing and controlling manufacturing at Bell Telephone Laboratories in the 1925 through 1956. He developed a series of control charts that indicated when a process was moving out of tolerance based on process mean (a variable), numbers of parts nonconforming in a sample (an attribute) or as a exponentially weighted average of a sample (to reduce process memory). His control charts are normally built around a three standard deviation limit.

interaction. For an example, imagine a situation where you want to reduce variation of delivery time (a variable) from seaports in the US to a seaport in a deployed location. Doubtless, there are apparent and possibly unknown variables in that model are deterministic, stochastic, and self-organizing. You can work to control the speed of the vessels, but you cannot control the weather processes (which is not a constant factor) or the political processes on vessel selection (a set of self-organizing variables). While you can control the speed variables, there would be second and third order effects based on decisions. This is a relatively simple example, but is representative of what happens when you attempt to control a more complex system.

## Conclusion and Topics for Further Research

Armed with an understanding of chaos viewed through systems and complexity theories, the interpretation of reframing of the "the race track" presented at the outset of this paper can now be reframed. Based on knowledge about chaos mechanics, limitations on detection, and interaction with adaptive components, the model would look very different from a concept constructed using the linear and causal paradigm. First, the decision maker would recognize the influence of physical phenomena. Second, the concept of bounding would allow the planner to build specific tolerance limits and expectations into the model. Third, the planner would be prepared to look for attractors to appear. At these attractors, he or she would expect either a convergence or divergence of adaptive components. Finally, the planner would understand that given limited resolution of the environment, chaos generates true randomness and that long term prediction is not possible.

Holistic thinking provides a compelling argument against reductivist methods for isolating chaos out of system while considering overall effects and appreciating possible behaviors. This understanding is a clear departure from currently available literature on military operations and chaos. Much of the literature attempts to make the argument that if decision makers could isolate and understand chaos, they could possibly control a system. This, however, cannot be accomplished within an open complex system.

While adding to an understanding of systems, chaos theory is not a tool for eliminating variation or noise. This is a key difference from past studies that did not consider chaos as a potential component of complex systems. Sensitivity to initial conditions, presence of strange attractors, and Brownian motion guarantee that certain systems will either rarely or never remain fixed or steady. Rather than reductivism, the best way to conceptualize and manage chaos within complex adaptive system is through an understanding of the basic routes and forms of deterministic turbulence and heuristic learning/system interaction. This cannot be done by removing and analyizing chaos away from the system, but rather must be done through holistic visualization and experimentation.

Heuristics are analogous to a search patterns when trying to find lost individuals. Each activity has an effect , either positive, neutral, or negative. Based on those effects, a new test or pattern is established based upon what has been learned to date. In search and rescue, the discovery of a article of clothing narrows the search and focuses effort. Conversely, not finding any evidence results in reframing. Even if an optimal state is never reached, heuristic methods continually improve performance. Similarly when managing complexity, small tests expose portions of the nature and dynamics of a system. Holistic problem conceptualization and management occurs when all parts are examined with respect to interactions and connectivity. Chaos must therefore be viewed as part of the ebb and flow of a system. Understanding bounds and potential attractors assist in acknowledging and planning for expected variation. This is in stark contrast what occurs when system turbulence is not anticipated.

The proposed conception of chaos argues that chaos should be understood as an element of complex adaptive systems. Unfortunately, much more research is required to understand the exact mechanisms by which adaptive entities react to chaos. A next possible step in this research direction is to develop a simulation that introduces dynamic deterministic turbulence into an agent based model that had a propensity for novel emergence. Unfortunately, the software limitations within both MATLAB and especially within NetLogo did not allow this methodology. While MATLAB is geared toward mechanical simulations, the basic toolkit does not have an agent based model where entity interaction is possible. On the other hand, while NetLogo does have an excellent agent based architecture that can also approximate system dynamics, it can carry out only one “setup” within a run. For example, it is not possible to overlay Lotka-Volterra system dynamics on top of the Segregation model. This type of simulation is an open question within the field of complexity research.[68]

More research is also required to understand how to analyze and detect chaos in a complex adaptive system. As stated, chaos detection in the literature is limited to mechanistic models without adaptive components. This may be a consequence of the general dominance of adaptive entities in affecting long-term behavior through the mechanisms of emergence. Regardless, tools such as KS entropy, trajectory, and Fourier analysis break down in the presence of novel system characteristics. A new methodology is required for detecting chaos in real world adaptive environments.

In summary, this research has established that chaos can be an important element of complex adaptive systems and the environments where military planners are required to access and manage. Holistic and systems thinking require that active portions of the environment are considered. Chaos, and its relation to adaptive elements, generate areas that allow or restrict self-organization and emergence. Planners currently utilize a variety of methods to deal with difficult, fine-scale problems. One method is to use a macro-scale solution such as Lean Six Sigma to concentrate on a single portion or variable within a problem. Another method is to consider the problem beyond assistance and allow the system to move to its natural propensity. As argued, these two method can lead to failures. Planners and decision makers must continuously learn and reframe to gain a sense of the characteristics, depth, and possible outcomes of difficult problems

[68] Nadine Schieritz, “Emergent Structures in Supply Chains: A Study Integrating Agent Based and System Dynamics Modeling” *IEEE: International Conference on System Sciences*, (2003), 1-9

# Appendix 1: Definitions

**Adaptation** – The effect of a system using a selection process that leads to improvement using a measure of success (Axelrod, p7).

**Attractor** – The phase space point or set of points representing the various possible steady state conditions of a system. An equilibrium state or group of states to which a dynamical system converges (Williams, p447).

**Autocorrelation** – Correlation of variable at one time with itself at another time (the autocovariance at a given lag divided by the output variance.) This is equal to:

$$autocorrelation = \frac{\sum_{t=1}^{N-M}(x_t - \bar{x})(x_{t+m} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2}$$

**Bifurcation** – Generally, a branching in a system (usually from one segment to two) towards steady state attractors.

**Chaos Theory –** An interdisciplinary field based on the mathematics of nonlinear hynamics that explains that seemingly random behavior of dynamical systems with deterministic variables over a time series. Chaotic systems are highly sensitive to initial conditions and often exhibit attractors.

**Coevolution** – The effect of multiple populations of agents interacting to each other (Axelrod, p8).

**Complexity** – The presence of active nonlinear interactions between variables and constraints.

**Complex Adaptive System** – A system that contains agents or populations that seek adaptation (Axelrod, p7).

**Deterministic (mathematical)** – Future states are completely determined by the current state.

**Difference Equation** – An equation based on changes that occur at a discrete interval (usually time) and solved by iteration.

**Differential Equation –** An equation expressing a relationship between a function and one or more of its derivatives based on changes that occur continuously (usually time).

**Discrete** – Contains a countable number of elements.

**Dynamical System** – A system that changes over time

**Emergent Properties** – Properties of the system that the separate parts do not have individually (Axelrod, p15).

**Entropy** – A measure of unavailable energy; a degree of disorder or disorganization; or a probability (in inverse proportion) of uncertainty, randomness, variety of choice, surprise, or information (Williams, p 452). Such that:

$$\sum_{i=1}^{N} P_i \log\left(\frac{1}{P_i}\right)$$

**Fourier Analysis** – A technique for describing time series data in terms of the frequency-domain characteristics of its periodic constituents. The equation for each constituent harmonic has the form:

$$y = \sum_{h=0}^{\frac{n}{2}} \propto_h \cos[h\theta] + \beta_h \sin[h\theta]$$

where h is the number of the harmonic, φ is the phase angle, A is the wave amplitude, and θ is the angle.

**General Systems Theory** – An interdisciplinary field that attempts to understand the structure and behavior of complex systems. Generally considered to be the founding field for systems thinking in the West.

**Linear Programming** – A mathematical technique that optimizes a linear objective function given a set of linear constraints.

**Linearity** – The property when a mathematical function or map displays the properties of additivity and commutativity. In mathematical terms, a function is linear iff $f(x_1 + x_2 + \ldots\ x_n) = f(x_1) + f(x_2) + \cdots f(x_n)$

**Logistics** – (two definitions used in the monograph) 1) The science of the management of resources and their distribution in a system. 2) As a mathematical function, it is a sigmoid curve that can represent the size of a population with a limit (based on finite quantities of food, land, etc.).

**Network** – The representation of a system as a collection of nodes (agents) connected by a series of arcs (links) to other agents.

**Nonlinearity** – The property of a function to be nonadditive (not linear).

**Queuing Theory** – A model that predicts the behavior of waiting lines (queues). Enables the calculation of expected line length, wait time, and total time in system.

**Periodicity** – The quality of an event reoccurring at a given interval. The sine curve is an example of a periodic function.

**Perturbation –** A change (normally slight) in the value of one or more parameters in a dynamic equation.

**Propensity** – A natural inclination or tendancy within a system. Related to the concept of an attractor state.

**State Space** – An abstract space in which coordinates represent variables needed to specify the phase of a dynamical system. (Williams, p468).

**Stochastic** – Variable characterized by a probability density function.

**Supply Chain Network -** A system that deals with the distribution, warehousing, and accountability of materiel.

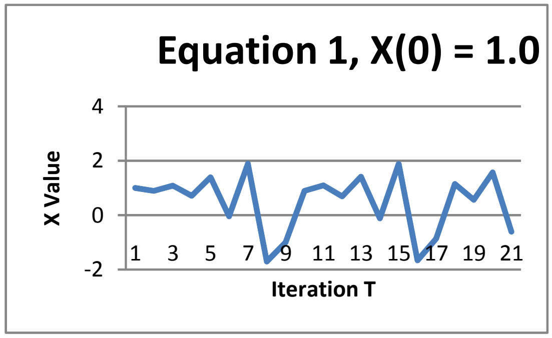
**System -** A system is an abstraction of reality – a mental model used to explain and simplify the real world. Systems are a set of independent and interrelated entities that form an integrated whole. The structure of the system is defined by its assemblages, their compositions, and behavior (inputs, outputs, and processing).

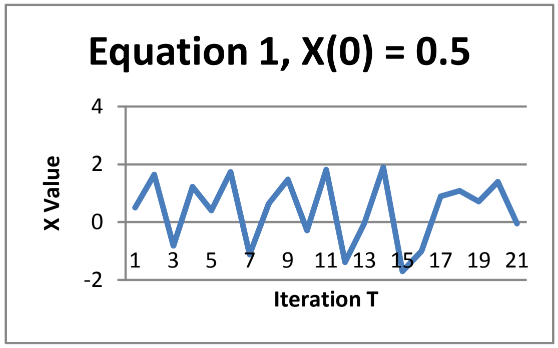
**Topology** – The branch of mathematics that studies properties of figures that are not changed by stretching, bending, and other continuous deformations.

## Appendix 2: Program Coding for Monograph

1. One variable chaos showing sensitivity to initial conditions: $x_{t+1} = 1.9 - x_t^2$

| X Value, X(0) = 1.0 | X Value, X(0) = 0.5 |
|---|---|
| 1 | 0.5 |
| 0.9 | 1.65 |
| 1.09 | -0.8225 |
| 0.7119 | 1.223494 |
| 1.393198 | 0.403063 |
| -0.041 | 1.73754 |
| 1.898319 | -1.11905 |
| -1.70361 | 0.647736 |
| -1.0023 | 1.480438 |
| 0.89539 | -0.2917 |
| 1.098277 | 1.814914 |
| 0.693789 | -1.39391 |
| 1.418657 | -0.04299 |
| -0.11259 | 1.898152 |
| 1.887324 | -1.70298 |
| -1.66199 | -1.00014 |
| -0.86221 | 0.899713 |
| 1.156587 | 1.090517 |
| 0.562308 | 0.710772 |
| 1.58381 | 1.394803 |
| -0.60845 | -0.04548 |




2. Logistics equation $x_{t+1} = kx_t(1 - x_t)$ with change of parameter settings showing the evolution of chaotic turbulence.

| | K = 0.5 | K = 2.0 | K = 3.0 | K=3.3 | K=3.6 | K=4.0 |
|---|---|---|---|---|---|---|
| x(0) | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| x(1) | 0.12 | 0.48 | 0.72 | 0.792 | 0.864 | 0.96 |
| x(2) | 0.0528 | 0.624 | 0.6048 | 0.543629 | 0.423014 | 0.1536 |
| x(3) | 0.025006 | 0.58656 | 0.717051 | 0.818719 | 0.878664 | 0.520028 |
| x(4) | 0.01219 | 0.606268 | 0.608667 | 0.489781 | 0.38381 | 0.998395 |
| x(5) | 0.006021 | 0.596768 | 0.714575 | 0.824655 | 0.8514 | 0.006408 |
| x(6) | 0.002992 | 0.60159 | 0.611873 | 0.477176 | 0.455466 | 0.025467 |
| x(7) | 0.001492 | 0.599199 | 0.712453 | 0.823281 | 0.89286 | 0.099273 |
| x(8) | 0.000745 | 0.600399 | 0.614591 | 0.480115 | 0.344379 | 0.35767 |
| x(9) | 0.000372 | 0.5998 | 0.710607 | 0.823695 | 0.812816 | 0.918969 |
| x(10) | 0.000186 | 0.6001 | 0.616934 | 0.479231 | 0.547727 | 0.29786 |
| x(10) | 9.3E-05 | 0.59995 | 0.708979 | 0.823577 | 0.8918 | 0.836557 |

| | | | | | | |
|---|---|---|---|---|---|---|
| x(12) | 4.65E-05 | 0.600025 | 0.618983 | 0.479484 | 0.347375 | 0.546917 |
| x(13) | 2.32E-05 | 0.599988 | 0.707529 | 0.823611 | 0.81614 | 0.991195 |
| x(14) | 1.16E-05 | 0.600006 | 0.620795 | 0.47941 | 0.5402 | 0.034909 |
| x(15) | 5.81E-06 | 0.599997 | 0.706226 | 0.823601 | 0.894182 | 0.134761 |
| x(16) | 2.9E-06 | 0.600002 | 0.622413 | 0.479432 | 0.340633 | 0.466403 |
| x(17) | 1.45E-06 | 0.599999 | 0.705045 | 0.823604 | 0.808568 | 0.995485 |
| x(18) | 7.26E-07 | 0.6 | 0.62387 | 0.479426 | 0.557228 | 0.017978 |
| x(19) | 3.63E-07 | 0.6 | 0.703969 | 0.823603 | 0.88821 | 0.070621 |
| x(20) | 1.82E-07 | 0.6 | 0.62519 | 0.479427 | 0.357455 | 0.262535 |

3. Lotka-Volterra system dynamics coding in NetLogo:

```
to setup
 ca
 system-dynamics-setup
 system-dynamics-do-plot
end

globals [
 convoy-birth-rate
 attack-rate
 attack-efficiency
 insurgent-death-rate
 convoy
 insurgent
 dt]

to system-dynamics-setup
 reset-ticks
 set dt 0.1
 set convoy-birth-rate .04
 set attack-rate 3.0E-4
 set attack-efficiency .8
 set insurgent-death-rate 0.15
 set convoy 100
 set insurgent 30
end

to system-dynamics-go
 let local-convoy-births convoy-births
 let local-convoy-deaths convoy-deaths
 let local-insurgent-births insurgent-births
 let local-insurgent-deaths insurgent-deaths
 let new-convoy max( list 0 ( convoy + local-convoy-births - local-convoy-deaths ) )
 let new-insurgent max( list 0 ( insurgent + local-insurgent-births - local-insurgent-deaths ) )
 set convoy new-convoy
 set insurgent new-insurgent
 tick-advance dt
```

```
end

to-report convoy-births
 report convoy-birth-rate * convoy * dt
end

to-report convoy-deaths
 report convoy * attack-rate * insurgent * dt
end

to-report insurgent-births
 report insurgent * attack-efficiency * attack-rate * convoy * dt
end

to-report insurgent-deaths
 report insurgent * insurgent-death-rate * dt
end

to system-dynamics-do-plot
 if plot-pen-exists? "convoy" [
  set-current-plot-pen "convoy"
  plotxy ticks convoy]
 if plot-pen-exists? "insurgent" [
  set-current-plot-pen "insurgent"
  plotxy ticks insurgent]
end
```

4. MATLAB Coding for the Lorenz Equations: Modified from an in-program demo

```
function lorenz(action)
global SIGMA RHO BETA
SIGMA = 10.;
RHO = 28.;
BETA = 8./3.;
play= 1;
if nargin<1,
  action='initialize';
end
switch action
  case 'initialize'
    oldFigNumber=watchon;
    figNumber=figure( ...
      'Name','Lorenz Attractor', ...
      'NumberTitle','off', ...
      'Visible','off');
    colordef(figNumber,'black')
    axes( ...
      'Units','normalized', ...
      'Position',[0.05 0.10 0.75 0.95], ...
      'Visible','off');
    text(0,0,'Press the "Start" button to see the Lorenz demo', ...
```

```
  'HorizontalAlignment','center');
 axis([-1 1 -1 1]);

 xPos=0.85;
 btnLen=0.10;
 btnWid=0.10;\
 spacing=0.05;
 frmBorder=0.02;
 yPos=0.05-frmBorder;
 frmPos=[xPos-frmBorder yPos btnLen+2*frmBorder 0.9+2*frmBorder];
 uicontrol( ...
  'Style','frame', ...
  'Units','normalized', ...
  'Position',frmPos, ...
  'BackgroundColor',[0.50 0.50 0.50]);
 btnNumber=1;
 yPos=0.90-(btnNumber-1)*(btnWid+spacing);
 labelStr='Start';
 callbackStr='lorenz("start");';
 btnPos=[xPos yPos-spacing btnLen btnWid];
 startHndl=uicontrol( ...
  'Style','pushbutton', ...
  'Units','normalized', ...
  'Position',btnPos, ...
  'String',labelStr, ...
  'Interruptible','on', ...
  'Callback',callbackStr);
 btnNumber=2;
 yPos=0.90-(btnNumber-1)*(btnWid+spacing);
 labelStr='Stop';
 callbackStr='set(gca,"Userdata",-1)';
btnPos=[xPos yPos-spacing btnLen btnWid];
 stopHndl=uicontrol( ...
  'Style','pushbutton', ...
  'Units','normalized', ...
  'Position',btnPos, ...
  'Enable','off', ...
  'String',labelStr, ...
  'Callback',callbackStr);
 labelStr='Info';
 callbackStr='lorenz("info")';
 infoHndl=uicontrol( ...
  'Style','push', ...
  'Units','normalized', ...
  'position',[xPos 0.20 btnLen 0.10], ...
  'string',labelStr, ...
  'call',callbackStr);
 labelStr='Close';
 callbackStr= 'close(gcf)';
 closeHndl=uicontrol( ...
  'Style','push', ...
```

```
      'Units','normalized', ...
      'position',[xPos 0.05 btnLen 0.10], ...
      'string',labelStr, ...
      'call',callbackStr);
    hndlList=[startHndl stopHndl infoHndl closeHndl];
    set(figNumber,'Visible','on', ...
      'UserData',hndlList);
    set(figNumber, 'CloseRequestFcn', 'clear global SIGMA RHO BETA;closereq');
    watchoff(oldFigNumber);
    figure(figNumber);
  case 'start'
    axHndl=gca;
    figNumber=gcf;
    hndlList=get(figNumber,'UserData');
    startHndl=hndlList(1);
    stopHndl=hndlList(2);
    infoHndl=hndlList(3);
    closeHndl=hndlList(4);
    set([startHndl closeHndl infoHndl],'Enable','off');
    set(stopHndl,'Enable','on');
    set(figNumber,'Backingstore','off');
    set(axHndl, ...
      'XLim',[0 40],'YLim',[-35 10],'ZLim',[-10 40], ...
      'Userdata',play, ...
      'XTick',[],'YTick',[],'ZTick',[], ...
      'Drawmode','fast', ...
      'Visible','on', ...
      'NextPlot','add', ...
      'Userdata',play, ...
      'View',[-37.5,30]);
    xlabel('X');
    ylabel('Y');
    zlabel('Z');
    FunFcn='lorenzeq';
    y0(1)=rand*30+5;
    y0(2)=rand*35-30;
    y0(3)=rand*40-5;
    t0=0;
    tfinal=100;
    pow = 1/3;
    tol = 0.001;
    t = t0;
    hmax = (tfinal - t)/5;
    hmin = (tfinal - t)/200000;
    h = (tfinal - t)/100;
    y = y0(:);
    L = 50;
    Y = y*ones(1,L);
    cla;
    head = line( ...
      'color','r', ...
```

```
      'Marker','.', ...
      'markersize',25, ...
      'erase','xor', ...
      'xdata',y(1),'ydata',y(2),'zdata',y(3));
    body = line( ...
      'color','y', ...
      'LineStyle','-', ...
      'erase','none', ...
      'xdata',[],'ydata',[],'zdata',[]);
    tail=line( ...
      'color','b', ...
      'LineStyle','-', ...
      'erase','none', ...
      'xdata',[],'ydata',[],'zdata',[]);
    while (get(axHndl,'Userdata')==play) && (h >= hmin)
      if t + h > tfinal, h = tfinal - t; end
      % Compute the slopes
      s1 = feval(FunFcn, t, y);
      s2 = feval(FunFcn, t+h, y+h*s1);
      s3 = feval(FunFcn, t+h/2, y+h*(s1+s2)/4);
      delta = norm(h*(s1 - 2*s3 + s2)/3,'inf');
      tau = tol*max(norm(y,'inf'),1.0);
      if delta <= tau
        t = t + h;
        y = y + h*(s1 + 4*s3 + s2)/6;
        Y = [y Y(:,1:L-1)];
        set(head,'xdata',Y(1,1),'ydata',Y(2,1),'zdata',Y(3,1))
      set(body,'xdata',Y(1,1:2),'ydata',Y(2,1:2),'zdata',Y(3,1:2))
        set(tail,'xdata',Y(1,L-1:L),'ydata',Y(2,L-1:L),'zdata',Y(3,L-1:L))
        drawnow;
      end
 if delta ~= 0.0
        h = min(hmax, 0.9*h*(tau/delta)^pow);
      end
      if ~ishandle(axHndl)
        return
      end
    end
    set([startHndl closeHndl infoHndl],'Enable','on');
    set(stopHndl,'Enable','off');
  case 'info'
    helpwin(mfilename);
end
function ydot = lorenzeq(t,y)
global SIGMA RHO BETA
A = [ -BETA  0   y(2)
  0 -SIGMA  SIGMA
  -y(2)  RHO  -1 ];
ydot = A*y;
```

5. NetLogo Coding for ‘Segregation’ Agent Modelling Emergence Model

```
globals [
 percent-similar
 percent-unhappy ]

turtles-own [
 happy?
 similar-nearby
 other-nearby
 total-nearby ]

to setup
 clear-all
 if number > count patches
  [ user-message (word "This pond only has room for " count patches " turtles.")
   stop ]

 ask n-of number patches
  [ sprout 1
   [ set color red ] ]
 ask n-of (number / 2) turtles
  [ set color green ]
 update-variables
 do-plots
end

to go
 if all? turtles [happy?] [ stop ]
 move-unhappy-turtles
 update-variables
 tick
 do-plots
end

to move-unhappy-turtles
 ask turtles with [ not happy? ]
  [ find-new-spot ]
end

to find-new-spot
 rt random-float 360
 fd random-float 10
 if any? other turtles-here
  [ find-new-spot ]
 move-to patch-here
end

to update-variables
 update-turtles
 update-globals
end

to update-turtles
 ask turtles [
  set similar-nearby count (turtles-on neighbors)
   with [color = [color] of myself]
```

```
  set other-nearby count (turtles-on neighbors)
    with [color != [color] of myself]
  set total-nearby similar-nearby + other-nearby
  set happy? similar-nearby >= ( %-similar-wanted * total-nearby / 100 )
 ]
end

to update-globals
 let similar-neighbors sum [similar-nearby] of turtles
 let total-neighbors sum [total-nearby] of turtles
 set percent-similar (similar-neighbors / total-neighbors) * 100
 set percent-unhappy (count turtles with [not happy?]) / (count turtles) * 100
end

to do-plots
 set-current-plot "Percent Similar"
 plot percent-similar
 set-current-plot "Percent Unhappy"
 plot percent-unhappy
end
```

# Bibliography


Argyis, John, Guntar Faust, and Maria Haase. *An Exploration of Chaos.* New York: North-Holland, 1994.

Axelrod, Robert, and Micheal D. Cohen. *Harnessing Complexity: Organizational Implications of a Scientific Frontier.* New York: Basic Books, 2000.

Ball, Philip. *Critical Mass: How One Thing Leads to Another.* London: Random House, 2000.

Bar-Yam, Yaneer. *Making Things Work: Solving Complex Problems in a Complex World.* Cambridge: NESCI Knowledge Press, 2004.

Casti, John. *Would Be Worlds: How Simulation is Changing the Frontiers of Science.* New York: John Wiley & Sons, 1999.

Current, Michael. *Chaos, Complexity, and Ethnic Conflict.* Leavenworth: Army Command and General Staff College, 2000.

Czerwinski, Thomas. *Coping With the Bounds: Speculations on Nonlinearity in Military Affairs.* Washington: National Defense University, 2003.

Deleuze, Gilles, and Felix Guattari. *A Thousand Plateaus: Capitalism and Schizophrenia.* Minneapolis: University of Minnesota Press, 1987.

Dobson, Phea. *Chaos Theory and the Effort in Afghanistan.* Carlyle Barracks: Army War College, 2008.

Dockery, John, and A. Woodcock. *The Military Landscape.* Cambridge: Woodhead, 1993.

Doerner, Dietrich. *The Logic of Failure: Recognizing and Avoiding Error in Complex Situations.* New York: Basic Books, 1989.

Eoyang, Glenda, and Doris Jane Conway. "Conditions That Support Self-Organization in Complex Adaptive Systems." *International Association of Facilitators.* Williamsburg, VA: University of Minnesota, 1999. 1-7.

Evans, John. *Structures of Discrete Event Simulation.* New York: John Wiley & Sons, 1990.

Frazier, Christopher. "Chaos Theory and Transportation Systems: An Instructive Example." *83rd Annual Meeting of the Transportation Research Board.* Washington, 2004.

Gleick, James. *Chaos, Making a New Science.* New York: Viking Penguin, 1987.

Goldstein, Jeffery. "Emergence as a Construct: History and Issues." *Emergence*, 1999: 49-72.

Gregobi, C. "Sudden Changes in Chaotic Attractors and Transcient Chaos." *Physica*, 1984: 632-638.

Hayes, Brian. "Statistics of Deadly Quarrels." *American Scientist*, 2002: 10-15.

Ikonen, T. "Search for Deterministic Chaos in ELM time Series." *IEEE Transaction on Plasma Science*, 2005: 1115-1122.

Ilachinski, Andrew. *Irreducible Semi-Autonomous Adaptive Combat (ISAAC): An Artificial-Life Approach to Land Combat.* Report, Alexandria: Center for Naval Anaylsis, 2008.

James, Glenn. *Chaos Theory: Essentials for Military Applications.* Newport: Naval War College, 1996.

Johnson, Neil F. "Complexity in Human Conflict." In *Understanding Complex Systems*, 303-320. Heidelberg: Springer, 2008.

Johnson, Steven. *Emergence: the Connected Lives of Ants, Brains, Cities, and Software.* New York: Scribner, 2001.

Katz, Richard. *The Chaos Paradigm: Developments and Applications in Engineering and Science.* New York: AIP Press, 1994.

Krasner, Saul. *The Ubiquity of Chaos.* Washington: American Association for the Advancement of Science, 1990.

Lauren, Michael. *Characterising the Difference Between Complex Adaptive and Conventional Combat Models.* Auckland: Defence Operational Support Technology Establishment, 1999.

Lauren, Michael. *Describing Rates of Interaction Between Multiple Autonomous Entities: an Example of Using Combat Modeling.* Auckland: Defence Technology Agency, New Zealamd Defence Force, 2001.

Lauren, Michael. *Fractal Methods Applied to Describe Cellular Automaton Combat Models.* Auckland: Defence Operational Technology Support Establishment, 2008.

Lauren, Michael. *How Agent Models Can Address Asymmetric Warfare: An ANZUS Collaboration.* Auckland: Defense Technology Agency, New Zealand Defence Force.

Lauren, Michael. *On the Temporal Distribution of Casualties and Determination of Medical Logistical Requirements.* Auckland: Defence Technology Agency, New Zealand Defence Force, 2003.

Lauren, Michael, and Roger Stephen. "Fractals and Combat Modeling: Using MANA to Explore the Role of Entropy in Complexity Science." *Fractals*, 2002: 481-489.

Lauren, Michael, Gregory McIntosh, Nigel Perry, and James Moffat. *Art of War Hidden in Kolmogorov's Equations.* Auckland: Defence Technology Agency, New Zealand Defence Force, 2008.

Matsumoso, T. *Bifurcations: Sights, Sounds and Mathematics.* New York: Springer-Verlag, 1993.

Mattis, James. *Assessment of Effects Based Operations.* Memorandum, Norfolk, Virginia: US Joint Forces Command, 2008.

Montgomery, Douglas C. *Introduction to Statistical Quality Control.* Hoboken: John Wiley & Sons, 2005.

Ott, Edward, and Tim Sauer. *Coping With Chaos: Analysis of Chaotic Data and the Exploitation of Chaotic Systems.* New York: John Wiley & Sons, 1994.

Richardson, Lewis F. "Variation of the Frequency of Fatal Quarrels with Magnitude." *Jounal of the American Statistical Association*, 1948: 523-546.

Ryan, Alex. "Emergence is Coupled to Scope, Not Level." *Complexity*, 2007: 67-77.

Salmon, Wesley C. *Causality and Explanation.* New York: Oxford University Press, 1998.

Schieritz, Nadine, and Andreas Groessler. "Emergent Structures in Supply Chains: A Study Integrating Agent-Based and System Dynamics Modeling." *Proceedings of the 36th Hawaii International Conference on System Sciences.* Honolulu: IEEE, 2003. 1-9.

Singh, Jaglit. *Great Ideas of Operations Research.* New York: Dover, 1968.

Strevens, Michael. *Bigger than Chaos: Understanding Complexity Through Probability.* Cambridge: Harvard University Press, 2003.

Surana, Amit, and Soundar Kumara. "Supply Chain Networks: A Complex Adaptive Systems Perspective." *International Journal of Production Research*, 2005: 4235-4265.

Thompson, J., and Stewart H. *Nonlinear Dynamics and Chaos.* New York: John Wiley & Sons, 1986.

Wass de Czege, Huba. "Systemic Operational Design: Learning and Adapting in Complex Missions." *Military Review*, 2009: 2-8.

Wheeler, Scott, Alex Ryan, and Tristan Turner. *Modelling Logistics Chains for Optimal Supply and Maximum Throughput.* Edinburgh: Land Operations Division; Defense Science and Technology Organisation, 2005.