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A New Method for Calculating the Critical Penetration Velocity (V_0)

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ABSTRACT

Research into ballistic protection often requires the determination of some velocity value that characterises a material. Very often this value is stated as the V_{50}, or the velocity at which 50 percent of the projectiles will penetrate the material. More recently, some specifications have called for the determination of a so-called 'V-Nought' or V_0 value. There are several papers that provide a means for determining this value by means of iterative schemes. It is the purpose of this paper to present a direct method for calculating V_0

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Executive Summary

The ballistic performance of materials is often specified in terms of a so-called V_{50} value that is intended to represent the velocity value at which fifty percent of the impacting projectiles will be arrested by the target. However, other measures of ballistic performance have recently been proposed. One of these is the 'V-Nought' or V_0 value of a target, intended to represent the maximum velocity at which no penetration at all is expected. One reason for proposing this new measure is an anticipated increase in confidence in the performance of the ballistic material. A number of papers have described various methods for calculating the V_0 value (or an estimate of it) from previously obtained impact vs. residual velocity data. For example, graphical methods and a matrix iteration method have been proposed for calculating V_0. In this paper a direct calculation method is presented. This direct method is easily applied to the observed data. The method uses calculus minimisation techniques to generate a simple equation that relates the V_0 value to the observed impact and residual velocities. The new method is validated against literature data and shown to provide an accurate estimate of V_0.

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1 Introduction

There are a number of techniques for calculating a 'ballistic limit' for assessing the ballistic performance of an unknown material. The most commonly used parameter is designated the 'V-fifty' or V_{50} value. This is supposed to be the velocity at which fifty percent of the impacting projectiles will penetrate the target and fifty percent of them will not. This velocity is accepted, more or less, as a norm by many countries as well as test facilities. Improvements can be made to the method of determining the V_{50} value. For example, Kneubuehl [1] presents a technique for evaluating the variance of a V_{50} test. More recently, a need has been identified for a parameter that will, supposedly, provide a higher level of confidence in the performance of a ballistic material with previously undetermined properties. Much work has been conducted in this area by personnel working at (what is now) Dstl in the United Kingdom [2] where a method for calculating what is called the 'V-Nought' or V_0 value, for a sample of ballistic material, has been developed. Tobin [3] emphasises that the V_0 value is of great importance in casualty reduction analysis. He also states that the U.K. intends to adopt the V_0 value as the standard measure for ballistic penetration performance. Tobin also points out that there are a number of objections to sole reliance on a graphical technique (these stem from the basic inaccuracy of the method) for determining V_0 and presents an outline of a method that uses convergence of successive mean values. The method, outlined by Kelly [2], relies on construction of a matrix of values and an assessment of a parameter that is then used to calculate V_0 by an iteration process. In this work a direct method for calculating V_0 is presented. This method is an improvement over earlier methods because the V_0 value is obtained from a simple calculation and it does not require graphical manipulation or large amounts of computer time.

2 Statement of the Problem

It is required to calculate the value of V_0 from a given set of data that are obtained by subjecting the target material to a range of impact velocities, V_i, and subsequently measuring the exit or residual velocity, V_r, when the projectile passes through the target (or fails to penetrate). Kelly [2] provides an explanation of the uncertainty that is necessarily associated with any attempt at determining V_0. This uncertainty has a number of sources, including experimental error and variations in the targets being tested. It is suggested that a new term be introduced to account for this statistical uncertainty. This new term is V_a and it represents a velocity that is a 'best estimate' of V_0. The objective is to deduce this number from the available data. It must be clearly understood that this is an estimate and that any estimate has a measure of uncertainty associated with it. There is a possibility that an impact below V_a could, conceivably, result in a penetration. However, it is reasonable to expect that there is a low probability of such an occurrence. The principle, used by Kelly [2] (and used in this paper), relies on the principle of conservation of energy. It is explained by Kelly [2] how the impact and residual energies are related to the value of V_a,

$$\frac{1}{2}M_i V_a^2 = \frac{1}{2}M_i V_i^2 - \frac{1}{2}M_r V_r^2. \tag{1}$$

Here, M_i and M_r are the initial and residual masses, respectively, of the impacting projectile. This can be solved for V_a to yield,

$$V_a = \sqrt{V_i^2 - kV_r^2}. \tag{2}$$

The parameter k is defined by,

$$k = \frac{M_r}{M_i} \tag{3}$$

and it will be seen to play a crucial role in the following discussion. Note that the value of k is not necessarily equal to unity because of the possibility of projectile erosion or material loss from the target [2]. However, if both the following conditions are satisfied,

1. No loss of target material
2. No change in projectile mass

then Equation (2) becomes,

$$V_a = \sqrt{V_i^2 - V_r^2}. \tag{4}$$

In this case V_a can be determined by applying a simple averaging process to the calculated V_a values. However Kelly's research has indicated that even a small change in residual mass can significantly influence the calculated V_a value. This means that the value of k is of great importance in the determination of V_a. A further point that emerges from Kelly's discussion [2] is that testing should initially be conducted at very high V_i values because this will reduce the variance in the estimated value of V_0. Kelly found that the variance at initial velocities near V_a could be very large but that it was reduced if higher impact velocities were used. Kelly's method consists of constructing a matrix where the rows represent the velocity values and the columns are calculated for different values of k. The correct value of k is the one that results in the most consistent value of V_a, as calculated from the impact and residual velocities using Equation (2). The difference between the largest and smallest V_a values is used to indicate the correct k value. Once this value is found it may be substituted back into Equation (2) and, if necessary, averaged over all the observations to yield the correct value of V_a. It must be emphasised that this work, in common with Kelly's, relies on the assumption that the energy absorbed by the target is approximately constant over the range of impact velocities employed. This is found, from experiment, to be a reasonable assumption although its accuracy does vary, depending on the projectiles and targets under investigation.

3 A Direct Solution

Assume that a number, N, of (V_i, V_r) trial values have been determined by experiment. Label the values V_i^m and V_r^m where integral m varies between 1 and N. From Equation (1) a quantity V_a^m may be calculated for each value of m using

$$(V_a^m)^2 = (V_i^m)^2 - k_m (V_r^m)^2, \tag{5}$$

where $k_m = \frac{M_r^m}{M_i^m}$ and M_i^m and M_r^m are initial and residual mass values for each trial. If we assume that M_r^m and M_i^m remain effectively constant for all the trials then we may say,

$$(V_a^m)^2 = (V_i^m)^2 - k (V_r^m)^2, \tag{6}$$

where $k = \frac{M_r^m}{M_i^m}$ is assumed to be constant for all values of m. It is easier not to deal with squared quantities, therefore we define the following,

$$A_m = (V_i^m)^2; B_m = (V_r^m)^2; C_m = (V_a^m)^2. \tag{7}$$

Substituting (7) into (6),

$$C_m = A_m - kB_m. \tag{8}$$

Taking averages,

$$\bar{C_m} = \bar{A_m} - k\bar{B_m}. \tag{9}$$

Here,

$$\bar{A_m} = \frac{1}{N} \Sigma_{m=1}^N A_m, \tag{10}$$

$$\bar{B_m} = \frac{1}{N} \Sigma_{m=1}^N B_m, \tag{11}$$

$$\bar{C_m} = \frac{1}{N} \Sigma_{m=1}^N C_m. \tag{12}$$

Our objective is to find the value of k that minimises the sum of the squared deviations from the mean value of C_m,

$$S = \Sigma_{m=1}^N (\bar{C_m} - C_m)^2. \tag{13}$$

This can be re-written,

$$S = \Sigma_{m=1}^N C_m^2 - N\bar{C_m}^2. \tag{14}$$

Now use Equations (8) and (9),

$$S = \Sigma_{m=1}^N (A_m - kB_m)^2 - N(\bar{A_m} - k\bar{B_m})^2. \tag{15}$$

This can be expanded to read,

$$S = \Sigma_{m=1}^{N}[A_m^2 - 2kA_mB_m + k^2B_m^2] - N(\bar{A_m}^2 - 2k\bar{A_m}\bar{B_m} + k^2\bar{B_m}^2). \tag{16}$$

In order to minimise Equation (16), we first calculate its partial derivative with respect to k,

$$\frac{\partial S}{\partial k} = 2k[\Sigma_{m=1}^{N}B_m^2 - N\bar{B_m}^2] + 2[N\bar{A_m}\bar{B_m} - \Sigma_{m=1}^{N}A_mB_m]. \tag{17}$$

This must equal zero, leading to an equation for k,

$$k = \frac{\Sigma_{m=1}^{N}A_mB_m - N\bar{A_m}\bar{B_m}}{\Sigma_{m=1}^{N}B_m^2 - N\bar{B_m}^2}. \tag{18}$$

This equation indicates the correct value of k to calculate either a minimum, maximum or point of inflection for the function S. Note that, from the observed impact and residual velocity data, this equation predicts a unique value for k as would be expected for a physically valid quantity. We must now prove that this value of k is indeed relevant to calculation of a minimum value of S. We therefore calculate the second derivative of S,

$$\frac{\partial^2 S}{\partial k^2} = 2[\Sigma_{m=1}^{N}B_m^2 - N\bar{B_m}^2]. \tag{19}$$

Note that Equation (19) may be re-written as,

$$\frac{\partial^2 S}{\partial k^2} = 2\Sigma_{m=1}^{N}(B_m - \bar{B_m})^2. \tag{20}$$

Note that $B_m = (V_r^m)^2$, i.e. it is the square of the residual velocity. It may be expected that, for a reasonable number of trials, at different impact velocities, the values of V_r^m will be different. This means that the sum in Equation (20) can reasonably be expected to be greater than zero. Therefore the second derivative must exceed zero (for all practical purposes). This, in turn, is the desired proof that the value of S, calculated using the value of k from Equation (18), is indeed a minimum value. After the optimum value of k has been calculated using Equation (18) it is substituted into Equation (9) to calculate the desired value of $\bar{C_m}$ and, consequently, V_a from the square root of $\bar{C_m}$, i.e. $V_a = \sqrt{\bar{C_m}}$.

This equation will now be applied to Kelly's data. The following quantities are readily calculated from the data presented in [2] (and listed in the table for reference),

$$\bar{A_m} = 885416.6667, \tag{21}$$

$$\bar{B_m} = 349806.9167, \tag{22}$$

$$\Sigma_{m=1}^{12}B_m^2 = 2.0149 \times 10^{12}, \tag{23}$$

Table 1: *Kelly's data for impact and residual velocity*

V_i (m/s)	V_r (m/s)
650	204
700	294
750	367
800	432
850	491
900	547
950	601
1000	653
1050	703
1100	752
1150	801
1200	848

$$\Sigma_{m=1}^{12} A_m B_m = 4.5372 \times 10^{12}. \tag{24}$$

Substitution of these values into Equation (18) yields a value of $k = 1.5012$. When this value of k is substituted into Equation (9) together with the values of $\bar{A_m}$ and $\bar{B_m}$ from Equations (21) and (22), we find a value of $\bar{C_m} = 360286.5234$ and by taking the square root of this number we get a value of $V_a = 600.24$. These results compare very well with the results in [2] of $k = 1.5$ and an average $V_a = 600.17$.

4 Discussion

The method presented in this paper treats the value of k as a variable. It has been shown how the requirement for minimisation of the sum of the squared deviations from the mean of V_a^2 results in an equation that permits calculation of the optimum values of k and V_a. In addition, a proof that this is indeed a minimum value has been found. The method presented here possesses several advantages over methods previously described in the literature. The most obvious advantage is that it is a direct method and it does not require any iterations to find the optimum k value. Another advantage is that there is no possibility that an optimum value of k might be overlooked by an inexperienced experimenter. The method also has the advantage that no recourse needs to be made to graphical data (this technique might lead to errors due to improperly drawn graphs) and it is relatively simple to apply to the known data values. This method (as well as the method presented by Kelly [2]) also permits a determination of the mass change of the projectile as a result of the impact. However, the method presented here still requires a reasonable number of trials so that confidence can be placed in the obtained results. It is important that the cautions presented by Kelly [2] are observed when conducting experiments for the determination of V_0.

5 Conclusions

1. A direct method has been presented to calculate an estimate of V_0 for an arbitrary data set

2. This method has been validated against experimental data from the open literature

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