

# Design Issues on Broadcast Routing Algorithms using Realistic Cost-Effective Smart Antenna Models

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**Abstract**—Wireless ad hoc or sensor networks usually operate over strictly or partially battery energy limited environment. To prolong the network operation time, energy-efficiency should be carefully considered at every layer of the network protocols and algorithms. Moreover, cross-layer effects and interactions have to be carefully analysed and utilized. While a significant amount of research on directional and smart antennas has been conducted at the physical layer and device level, a system-wide level analysis using directional antennas is still very rare especially for the broadcast routing problem over wireless ad hoc networks. In this paper we investigate the effects of various classes of directional antenna systems and consider system-level design principles for a power-efficient broadcast routing algorithm. By introducing the concept of optimal decision space, we provide various valuable insights for algorithm design.

## I. INTRODUCTION

Many heuristics and underlying theory have been investigated and proved for power-efficient broadcast routing with omnidirectional antennas over wireless ad hoc network. One of the most important results is the NP-completeness of finding a minimum power broadcast routing tree [1]. Hence, searching for more efficient heuristics became even more important. In other direction, the work on the fundamental limits on the capacity of wireless ad hoc networks proved very pessimistic results [2]. Recently, as a breakthrough for the capacity barrier inherent in the use of omnidirectional antennas, directional antennas are quickly gaining a lot of attention among research community [3]–[6].

We have seen a limited deployment of smart antenna technology in the base stations of cellular networks and RADAR applications, because the cost and size are major barriers to the penetration of consumer level devices such as mobile phones, PDAs and laptops. Ramanathan in [3] argued that there are still various usage of smart antenna technology even at the cost and regardless of the size factor as in many military applications of wireless ad hoc networks. The main thrust behind the interest in directional or smart antenna systems is the gain in capacity and increase in network operation lifetime, because mobile nodes operate solely based on limited battery resource.

Smart antenna is an array of multiple antenna elements combined with smart signal processing algorithms and a digital processor. Smart antenna systems can be roughly categorized as

(i) actively driven and (ii) parasitic. If the boresight direction of an antenna is controlled by changing the current sources of multiple elements, it is called driven. Actively driven smart antenna systems can be further classified as (i) switched beam, (ii) phased array, and (iii) adaptive array antenna. On the other hand, if the beam pattern is controlled by using passive elements around a single driven source, it is called parasitic. An example of parasitic antennas includes ESPAR antenna [7].

To deal with complex algorithms, adaptive array antenna requires powerful DSP processors. On the other hand, switched beam or phased array antenna do not require sophisticated processors, because predetermined antenna beam patterns are used either by simple switching circuit or phase allocation. If a smart antenna systems are adopted in wireless ad hoc network environment, it is highly likely that these two will be the first candidates, because they are relatively cost-effective and requires less powerful processors. This paper tries to address the following simple yet fundamental question: *is it possible or beneficial to take advantage of broadcast advantage [9] to find a broadcast routing algorithm using directional antennas?* In our previous work [4], [8], we investigated various power-efficient broadcast routing algorithms for wireless ad hoc network using directional antennas and asymptotically optimal (minimum) broadcast routing tree. We assumed an ideal flat-top adaptive antenna model, i.e., ideal angular response, such that the power gain is constant over the beamwidth of a radiation pattern. In this paper, we consider more realistic phased array and switched beam antenna models and explore their impact on broadcast advantage property and hence the effect on the design of broadcast routing algorithms.

## II. REALISTIC ANTENNA MODEL

Most of previous work on using directional antennas for ad hoc network has been relied on very simplified models of antenna radiation patterns. While this can provide first order approximations to predict the performance of interest, it is still limited in the sense that the real deployment of a network with directional antennas may not provide the expected performance. Hence, an analysis with realistic antenna models is crucial. The first work in this direction can be found in [6], where authors considered a uniform circular array (UCA) antenna model with six antenna elements [10]. UCA is particularly attractive for our purpose, because it allows steering the pencil-beam electronically over horizontal plane due to symmetry [10].

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To take into account of realistic antenna models, the following considerations are important:

- a limited number of antenna elements
- (sometimes) limited possible antenna patterns
- imperfect rotational symmetry.

In this paper, we use two realistic gain patterns. The first one called type I is adopted from [11] and modified to allow the change in beamwidth with an integer parameter  $n$ . It has the feature that while being simple it contains many characteristics of realistic antenna patterns in that the main beam has a gradual roll-off and multiple side lobes. The gain of type I is expressed as  $G_t(\phi) = \prod_{k=0}^n \cos^2(2^k \phi)$ ,  $n = 1, 2, 3, \dots$

Second one called type II is that of a UCA. Assuming  $M$  isotropic elements in the array, for a given boresight direction of elevation and azimuth angles  $(\theta_0, \phi_0)$ , the gain pattern at a direction  $(\theta, \phi)$  is determined by magnitude square of an array factor [10]. We assume that every node in the network lies in a two dimensional horizontal plane, and hence the elevation angles are set to  $\theta = \theta_0 = \frac{\pi}{2}$ . Thus, the gain of type II is a function of azimuth angle only, i.e.,  $G_t(\phi) = \left| \sum_{m=1}^M e^{-jka[\cos(\phi_0 - v_m) - \cos(\phi - v_m)]} \right|^2$ , where  $v_m = \frac{2\pi}{M}m$  is the angle of  $m$ -th element,  $a$  is the radius from the center of an array, and  $k$  is a constant phase factor of electromagnetic wave satisfying  $k = \omega/c = \lambda/2\pi$ , where  $\lambda$  is the wavelength and  $c$  is the speed of light. To obtain different patterns, we adjust either the number of elements  $M$  or the multiple  $ka$  for a given  $M$ .

Table I summarizes the parameter values and the corresponding half-power beamwidth (HPBW) and the beamwidth between first nulls (FNBW) [10] that will be used in this paper. Note that for an ideal flat-top beam pattern, HPBW and FNBW coincide.

TABLE I  
PARAMETERS AND THE CORRESPONDING HPBW AND FNBW FOR TYPE I AND TYPE II GAIN PATTERN.

Type I			Type II			
$n$	HPBW	FNBW	$M$	$ka$	HPBW	FNBW
0	$90^\circ$	$\pi$	8	1.1	$43.2354^\circ$	$81.4746^\circ$
1	$40.9853^\circ$	$\pi/2$	8	1.0	$47.6243^\circ$	$89.9544^\circ$
2	$20.0684^\circ$	$\pi/4$	8	0.9	$53.0101^\circ$	$100.6114^\circ$
3	$9.98314^\circ$	$\pi/8$	8	0.8	$59.7710^\circ$	$114.3624^\circ$
4	$4.98525^\circ$	$\pi/16$	8	0.7	$68.5601^\circ$	$132.5824^\circ$

### III. CHARACTERIZATION OF THE CAUSE OF BROADCAST ADVANTAGE

In this section, we will consider the effects of using switched beam and phased array antennas [3] on the broadcast advantage property. We will discuss that the broadcast advantage is affected due to:

- beam shape (flatness and roundness)
- (electronically) steerability of antenna (beam scanning capability)

- beamwidth expansibility

All antenna specifications include power gain or directivity as an important antenna parameter. In a wireless ad hoc network, it is customary to think of the network in terms of a geometry of point distribution in 2-dimensional plane along with antenna patterns. In this case, instead of using antenna gain patterns, it is useful to visualize them in terms of *coverage patterns*. The boundary of reachable region (coverage) is determined by the distance where the received power is larger than or equal to the receiver sensitivity threshold  $\Omega$ . From Friis transmission formula [10], the received power  $P_r(r)$  at distance  $r$  from a node transmitting with power  $P_t$  should be larger than receiver sensitivity threshold  $\Omega$ ,  $P_r = P_t \frac{G_t G_r \lambda^2}{(4\pi r)^2} \geq \Omega$  where  $G_t$  and  $G_r$  represent transmitter and receiver gains,  $\lambda$  is the wavelength and the path loss factor  $\alpha$  is assumed to be  $\alpha = 2$ . For notational simplicity, we set  $\Omega \left(\frac{4\pi}{\lambda}\right)^2 = 1$ . Then the minimum required transmit power should be  $P_t(r) = G_t^{-1} G_r^{-1} r^2$ . Assuming  $G_r = 1$ , the spatial boundary of the coverage pattern is determined by the polar plot of  $r = r(\theta, \phi) = \sqrt{P_t \left(\frac{\lambda}{4\pi}\right)^2 G_t(\theta, \phi)}$ .

#### A. Flat-top Beam Pattern + Variable Beamwidth + Steerable Adaptive Array

In our previous work [4], we considered single beam adaptive array with ideal flat-top beam pattern which can adaptively adjust its beamwidth larger than the minimum beamwidth  $\theta_{\min}$ . Let's consider a simple network comprised of only 3 nodes  $S, A$ , and  $B$ . The objective is to broadcast the same message from source node  $S$  to node  $A$  and  $B$  with minimum possible total transmit power. Let's assume the locations of node  $S$  and  $A$  are fixed at  $(0, 0)$  and  $(r_1, 0)$ , respectively, without loss of generality. The remaining node  $B$  can freely move around the two dimensional plane with two degrees of freedom and its coordinate is  $(r_2 \cos \theta_2, r_2 \sin \theta_2)$ . The distances between the nodes are  $r_1 = |\overline{SA}|$ ,  $r_2 = |\overline{SB}|$ , and  $r_{12} = r_{21} = |\overline{AB}|$ . The angles of node  $A$  and  $B$  measured from the positive  $x$ -axis direction are denoted  $\theta_1$  and  $\theta_2$ , where  $\theta_1 = 0$  and  $\theta_2 = \angle ASB$ . To broadcast from node  $S$ , there are four exhaustive cases:  $(S \rightarrow B \rightarrow A)$ ,  $(S \rightarrow A \rightarrow B)$ ,  $(S \rightarrow \{A, B\})$ ,  $(S \rightarrow A, S \rightarrow B)$ . Each case is illustrated in order in Fig. 1, where each link, beam pattern and its coverage region is shown.

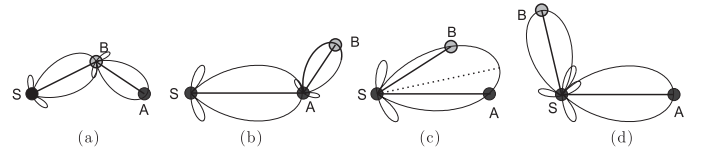


Fig. 1. (a) Multihop with relay  $B$  ( $MH_B$ ) (b) Multihop with relay  $A$  ( $MH_A$ ) (c) Broadcast Advantage ( $BA$ ) (d) Two unicast ( $2U$ ).

In the first two cases, node  $B$  and  $A$  relay the traffic for node  $S$  to reach the other node. We will call this decision as *multihop*  $MH_B$  and  $MH_A$ , respectively. The subscripts denote the relay node. In the third case, by a single transmission to node  $A$ , node  $B$  can also get the message, which we will call

*broadcast advantage BA*. Fourth example corresponds to the case when node  $S$  transmits the message to node  $A$  and node  $B$  with *two unicast* transmissions, which we will denote by  $2U$ .

We now introduce the definition of the concept of decision space that will be used throughout this paper. Depending on the location of node  $B$ , we determine which decision requires minimum total transmit power and mark the position as one of the four decision choices  $\{MH_B, MH_A, BA, 2U\}$ .

**Definition 1 (Decision space):** A decision space  $\mathcal{D}$  of an algorithm is a partition of 2-dimensional plane into decision regions each corresponding to four possible decision choices  $\{MH_B, MH_A, BA, 2U\}$  made by a broadcast routing algorithm. An *optimal decision space* is the decision space of the optimal strategy where the decision criterion made at each point is to minimize the total transmit power. A *decision boundary*  $\partial\mathcal{D}$  separates each partition with others in the decision space. Note that not all power-efficient broadcast routing algorithms give optimal decision space [4].

We assume every node has two transmission and reception modes: omnidirectional and directional. We assume all transmissions are made in directional mode with gain  $G_t$  and receptions are made in omnidirectional mode with gain  $G_r = 1$ . For a flat-top antenna pattern, the required total transmit power in each case is expressed as

$$\mathcal{P}_{MH_B} = \frac{(r_2^\alpha + r_{21}^\alpha)}{G_t}, \quad \mathcal{P}_{MH_A} = \frac{(r_1^\alpha + r_{12}^\alpha)}{G_t}, \quad \mathcal{P}_{2U} = \frac{(r_1^\alpha + r_2^\alpha)}{G_t} \quad (1)$$

$$\mathcal{P}_{BA} = \left( \frac{BW}{2\pi} \right) \max \left\{ \frac{r_1^\alpha}{G_t}, \frac{r_2^\alpha}{G_t} \right\} \quad (2)$$

where the beamwidth  $BW = \max \{\theta_{\min}, |\theta_2|\}$  and  $\theta_{\min}$  is the minimum beamwidth of an array. For further details, interested readers are referred to [4]. One of the most notable difference between omnidirectional and directional routing is that the existence of the region where  $2U$  is the best for directional case, which never happens in omnidirectional case because  $\mathcal{P}_{BA} < \mathcal{P}_{2U}$ . In Fig. 2(a), we present the optimal decision space when beamwidth can be expanded as in [4] with  $\theta_{\min} = 41.6^\circ$ . This value is chosen for a valid comparison with other cases that will be presented later in Section III-C.

### B. Flat-top Beam Pattern + Fixed Beamwidth + Steerable Phased Array

Compared to the previous scenario, we keep the beamwidth fixed at  $\theta_{\min}$ , while it is still allowed to steer the main beam to an arbitrary direction. Hence, the phased array antenna with a flat-top beam pattern is the most relevant model for this scenario. Recall that the location of node  $S$  and  $A$  are fixed at  $(0, 0)$  and  $(r_1, 0)$ , respectively, and node  $B$  can freely move around whose coordinate is  $(r_2 \cos \theta_2, r_2 \sin \theta_2)$ . Because of the fixed beamwidth constraint, while other cases  $MH_A, MH_B$  and  $2U$  are equal to previous section, the required total transmit power of  $BA$  case should be modified as:

$$\mathcal{P}_{BA} = \begin{cases} \max \left\{ \frac{r_1^\alpha}{G_t}, \frac{r_2^\alpha}{G_t} \right\} & \text{if } |\theta_1 - \theta_2| < \theta_{\min} \\ \infty & \text{otherwise.} \end{cases} \quad (3)$$

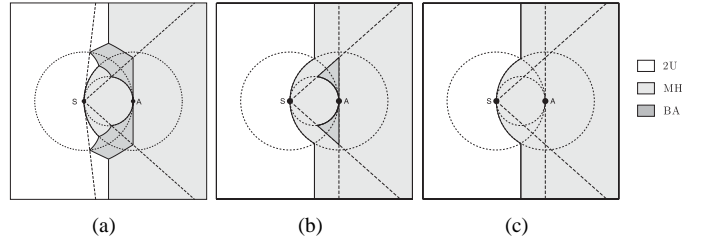


Fig. 2. Optimal decision space for (a) flat-top adaptive array antenna with  $41.6^\circ$  beamwidth, (b) flat-top phased array antenna with  $41.6^\circ$  beamwidth, and (c) realistic phased array antennas for  $0 \leq n \leq 4$ . Only one figure is shown because results are identical for different  $n$ .

Without loss of generality, we assume  $r_2 \leq r_1$  and concentrate on inside the circle of radius  $r_1$  centered at the source node location  $(0, 0)$ . Because of this assumption,  $\mathcal{P}_{MH_B} \leq \mathcal{P}_{MH_A}$ , and we don't need to consider  $MH_A$ . By comparing  $MH_B$  and  $2U$ , if  $r_{21} \leq r_1$ ,  $MH_B$  is chosen. Otherwise,  $2U$  is chosen. Because  $r_1$  is a constant, the decision boundary is determined by a circle of radius  $r_1$  centered at  $A$ . Fig. 2(b) shows the optimal decision space for  $\theta_{\min} = 41.6^\circ$ . Compared to Fig. 2(a), clearly, a wider beamwidth results in more broadcast advantage region, which is also intuitive.

### C. Realistic Beam Pattern + Fixed Beamwidth + Steerable Phased Array

Keeping all other conditions and assumptions as in previous section, we now consider a realistic beam pattern instead of the ideal flat-top pattern. Therefore, the effect of beam patterns are investigated in this section. While other cases  $MH_A, MH_B$  and  $2U$  are equal to previous section, the transmit power of  $BA$ ,  $\mathcal{P}_{BA}$ , requires a special attention. By solving a simple optimization problem, it should be modified as follows:

$$\mathcal{P}_{BA} = \begin{cases} \min_{0 \leq \phi \leq 2\pi} \mathcal{P}_{BA}(\phi) & \text{if } |\theta_1 - \theta_2| < \text{FNBW} \\ \infty & \text{otherwise,} \end{cases} \quad (4)$$

where  $\mathcal{P}_{BA}(\phi) = \max \left\{ \frac{r_1^\alpha}{G_t(|\phi - \theta_1|)}, \frac{r_2^\alpha}{G_t(|\phi - \theta_2|)} \right\}$  and  $\phi$  is the direction of boresight.

As shown in Fig. 1(c), we want to determine the minimum transmit power and the direction of a single beam which covers two nodes simultaneously. Two nodes  $A$  and  $B$  are covered if the received power at each node satisfies

$$P_{r,1} = P_t \frac{G_t(|\phi - \theta_1|)}{r_1^\alpha} \geq 1 \quad \text{and} \quad P_{r,2} = P_t \frac{G_t(|\phi - \theta_2|)}{r_2^\alpha} \geq 1.$$

Hence, the transmit power should satisfy both  $P_t \geq r_1^\alpha / G_t(|\phi - \theta_1|)$  and  $P_t \geq r_2^\alpha / G_t(|\phi - \theta_2|)$ . Namely, given the boresight direction  $\phi$ , the minimum required transmit power is  $P_t = \mathcal{P}_{BA}(\phi)$  and by minimizing over all direction, we get (4). If  $|\theta_1 - \theta_2| > \text{FNBW}$ , two nodes can never be simultaneously covered by a single main lobe, however large the transmit power is. The minimum value of  $\mathcal{P}_{BA}(\phi)$  is attained within an interval  $[\max \{\theta_1, \theta_2\} - \frac{\text{FNBW}}{2}, \min \{\theta_1, \theta_2\} + \frac{\text{FNBW}}{2}]$ .

For a meaningful comparison with other beam patterns, we define the notion of area-equivalent beam shape such that two

different beam shapes are area-equivalent as long as the area of reachable region is the same, assuming the received power at the destination node is equal. For example, for the type I gain pattern with  $n = 1$ , HPBW=40.98° and FNBW=90° as presented in Table I. The area of reachable region is computed as  $\int_{-FNBW/2}^{FNBW/2} \int_0^{\sqrt{G_t(\phi)}} r dr d\phi = \frac{1}{2} \int_{-\pi/4}^{\pi/4} G_t(\phi) d\phi = \frac{1}{2} (\frac{1}{3} + \frac{\pi}{8})$ . By making this value equal to the area of equivalent flat-top pattern  $\pi r^2 \cdot \frac{\theta}{2\pi} = \frac{1}{2} (\frac{1}{3} + \frac{\pi}{8})$  where  $r = G_t(0) = 1$ , we can find the beamwidth of the equivalent flat-top pattern as  $\theta = (\frac{1}{3} + \frac{\pi}{8}) \approx 41.6^\circ$  which is very close to HPBW of the original beam.

The optimal decision space for HPBW=40.98° case of type I pattern is presented in Fig. 2(c). In fact, for different values of  $n$  of type I pattern and for various  $ka$  values of type II pattern, we found the results are indistinguishable. Because it is analytically intractable for realistic beam patterns such as type I and type II patterns, we used Monte Carlo simulation to produce the figures of decision spaces. We randomly generated  $10^6$  points per  $r_1^2$  region, and identified the optimum decision at each point among the four choices. The area in each case is calculated as a ratio of the number of each case and the total number of points generated. For  $n \geq 1$ , it exactly matches the Fig. 2(c)—there was no single occasion when BA was best. For  $n = 0$ , there is a non-zero area where BA is best. The average area of BA region is about  $5 \times 10^{-4} r_1^2$  which is hardly recognizable in the figure.

Notice that Fig. 2(c) exactly matches the decision space of link-based MST [4]. The implication of this result is that the *beam pattern has a significant impact on broadcast advantage*. Comparing Fig. 2(c) with Fig. 2(a) and (b), the realistic beam pattern effectively “washes out” every broadcast advantage that existed in case of ideal flat-top beam pattern. *In other words, when phased arrays with a realistic directional beam pattern are used, the wireless links are effectively transformed to virtual wired links.*

#### D. Realistic Beam Pattern + Fixed Beamwidth + Non-steerable (Switched Beam)

Switched beam antenna is another class of smart antenna systems. Contrary to the phased array antennas, there are multiple predefined radiation patterns and they are simply switched on and off by a switching circuitry called beamforming network (BFN). There exists a unique characteristic of switched beam antennas unseen in other smart antenna systems known as scalloping.

*Scalloping or crossover loss* is the roll-off of the antenna pattern as a function of angle as the direction-of-arrival (DOA) varies from the boresight of each beam produced by BFN. Typically, BFNs provide beams which cross at 4 dB points. Thus a receiver’s signal strength varies as the node moves from the center of the beam to the edge of the coverage region of a particular beam [12]. Not only the scalloping is due to antenna pattern but also it is very closely related to its inability to steer the boresight to an arbitrary direction. Hence in this section, we will investigate the impact of scalloping of the switched beam

antennas. Compared to the previous section, we consider the effect of inability to steer the beam when a finite fixed number of beams are used.

We assume that the beamwidths of the predetermined antenna patterns are all equal and can not change. Because UCA can steer the boresight to arbitrary direction, it is a suitable model for phased array. To model a switched beam antenna, we use the gain pattern of type I only within the FNBW. To cover other sectors, this pattern is rotated by a proper angle. We also assume that all antennas of nodes are aligned to a specific direction (e.g., magnetic north), which is a common assumption in previous work of others [13], [14]. This can be achieved by embedding a magnetic needle in each node [13].

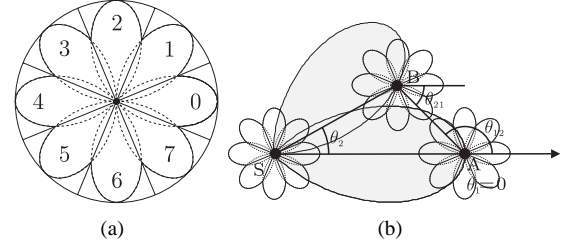


Fig. 3. (a) An illustration of a switched beam antenna with  $S = 8$  sectors. Each sector ID is numbered from 0 to 7. (b) broadcasting using switched beam antennas.

Fig. 3(a) presents an illustration of a switched beam antenna with  $S$  sectors, where  $S$  denotes the number of supported sectors in each switched beam antenna. An increasing value of sector ID  $0 \leq s \leq (S - 1)$  is assigned to each sector counter-clockwise starting from the positive  $x$ -axis direction along which the center of 0-th sector beam of all nodes are aligned. To derive the required transmit power in each case, we define  $\theta_{12}$  and  $\theta_{21}$  as drawn in Fig. 3(b). Let  $[x]$  denote the rounding function of a real value  $x$ , i.e., the function that returns the nearest integer such that  $[x] = n$ , if  $|x - n| < \frac{1}{2}$ ,  $x \in \mathbb{R}$ ,  $n \in \mathbb{Z}$ . For a node located at angle  $\theta$ , the function  $s = \lfloor \theta \frac{S}{2\pi} \rfloor \pmod{S}$  returns the ID of the sector where it belongs. Hence the  $s$ -th boresight direction corresponds to  $(\frac{2\pi}{S}s)$ . Let  $s_1 = \lfloor \theta_1 \frac{S}{2\pi} \rfloor$ ,  $s_2 = \lfloor \theta_2 \frac{S}{2\pi} \rfloor$ ,  $s_{12} = \lfloor \theta_{12} \frac{S}{2\pi} \rfloor$  and  $s_{21} = \lfloor \theta_{21} \frac{S}{2\pi} \rfloor$ . Then the required transmit power for each decision strategy is:

$$\mathcal{P}_{MHB} = \frac{r_2^\alpha}{G_t(\lfloor \frac{2\pi s_2}{S} - \theta_2 \rfloor)} + \frac{r_{21}^\alpha}{G_t(\lfloor \frac{2\pi s_{21}}{S} - \theta_{21} \rfloor)} \quad (5)$$

$$\mathcal{P}_{MHA} = \frac{r_1^\alpha}{G_t(\lfloor \frac{2\pi s_1}{S} - \theta_1 \rfloor)} + \frac{r_{12}^\alpha}{G_t(\lfloor \frac{2\pi s_{12}}{S} - \theta_{12} \rfloor)} \quad (6)$$

$$\mathcal{P}_{2U} = \frac{r_1^\alpha}{G_t(\lfloor \frac{2\pi s_1}{S} - \theta_1 \rfloor)} + \frac{r_2^\alpha}{G_t(\lfloor \frac{2\pi s_2}{S} - \theta_2 \rfloor)} \quad (7)$$

$$\mathcal{P}_{BA} = \begin{cases} \max \left\{ \frac{r_1^\alpha}{G_t(\lfloor \frac{2\pi s_1}{S} - \theta_1 \rfloor)}, \frac{r_2^\alpha}{G_t(\lfloor \frac{2\pi s_2}{S} - \theta_2 \rfloor)} \right\} \\ \infty \quad \text{if } |\frac{2\pi s_1}{S} - \theta_2| < \frac{FNBW}{2} \\ \text{otherwise.} \end{cases} \quad (8)$$

The BA case can be easily understood if we consider Fig. 3(b). Node B lies within the 1st sector of node S. However, if it lies within FNBW of 0-th sector beam, node B can be



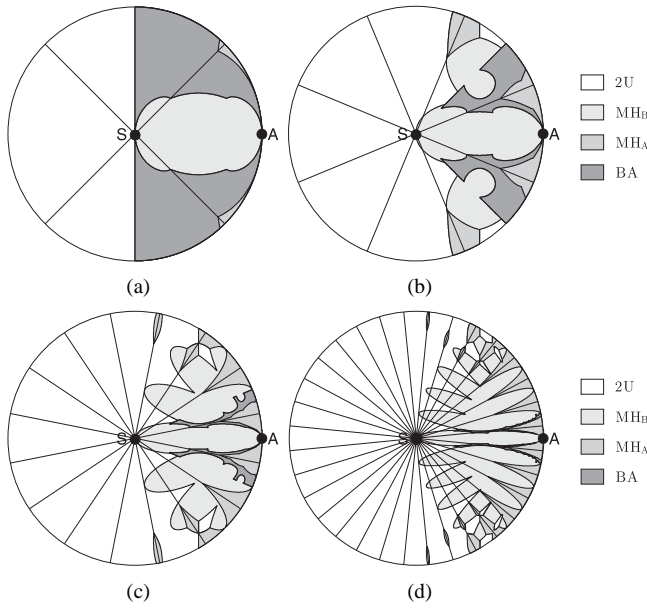


Fig. 4. Optimal decision space using switched beam antennas with HPBW (a) 90°, (b) 41°, (c) 20°, (d) 10°.

simultaneously reached by increasing the transmit power of 0-th sector.

Fig. 4 shows the optimal decision space when switched beam antennas are used to broadcast. Each subfigure corresponds to the gain pattern of type I for  $n = 0, 1, 2$  and 3. Because of duality (reciprocity) of decision space [4], we only consider the region within the circle of radius  $r_1$  centered at source  $S$ . As expected, due to scalloping, the decision space is highly irregular and complex. One fact of particular interest is the existence of  $MH_A$  region within the circle, which is due to scalloping: to reach a node near the edge of a sector, it is sometime better to take multihop route following along the center of beams where the gain is high, even if the traversed distance is longer.

To fully utilize broadcast advantage, the above mentioned results suggest that each sector beam should also consider nodes lying in adjacent sectors if the coverage angle of each sector  $2\pi/S$  is smaller than FNBW. However, it is usually against the design principle of sectored antenna systems: a node lying in  $k$ -th sector is taken care of by the  $k$ -th sector beam and power dissipation out of its sector boundaries to  $(k-1)$  and  $(k+1)$ -th sectors is considered as inter-sector interference. Therefore, it is likely that usual algorithms and protocols will not consider the nodes outside its sector boundary. If this is the case,  $BA$  region only in the 0-th sector in Fig. 4 should be counted. For example, in Fig. 4, the  $BA$  regions in sector 1 and 7 should be ignored and therefore the effective  $BA$  is almost negligible for small beamwidths ( $n = 1, 2, 3$ ).

In summary, we considered four cases including ideal adaptive array, ideal phased array, realistic phased array and realistic switched beam antennas. We presented the corresponding optimal decision spaces considering the constraints in each case. While the decision space concept is similar to a local decision

process among three nodes, the higher the directionality of an antenna, the more it becomes equivalent to the global decision.

#### IV. CONCLUSIONS

In this paper, we attempted to characterize the cause of wireless broadcast advantage with realistic antenna models. Specifically, we considered the effect of the beam patterns, steerability of a beam and adaptability of beamwidth. Some combinations of these factors correspond to different classes of a smart antenna system such as switched beam, phased array and ideal adaptive array antenna.

We defined the concept of decision space and by calculating the optimal decision spaces for different antenna classes we could derive valuable insights for the design of power-efficient broadcast routing algorithms over wireless ad hoc networks.

For example, we observed that the realistic beam pattern in conjunction with steerability (phased array) almost eliminates broadcast advantage. Therefore, any algorithm developed for link-based cost can be used without much performance degradation. In case of switched beam systems, while there exists a small amount of broadcast advantage, due to the common practice in sectored antenna system design, it is unlikely to be fully exploited.

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