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14. ABSTRACT

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On the origin of mesospheric bores

E. M. Dewan and R. H. Picard

Air Force Research Laboratory, Space Vehicles Directorate, Hanscom Air Force Base, Massachusetts

Abstract. A dramatic front of airglow radiance and wave structure reported by *Taylor et al.* [1995] was attributed, in a previous paper of ours [*Dewan and Picard*, 1998], to a manifestation of an internal undular bore in the mesosphere. In the current paper we address the question of what physical process could be responsible for generating such a bore at that altitude. While it is relatively easy to find sources of internal tropospheric bores, including the dramatic "morning glory," the same cannot be said for mesospheric bores. It will be argued here that a likely candidate for the generator of such bores is the interaction of gravity waves with the mean flow at a critical layer. This interaction could take place within an already existing inversion layer, the latter playing the role of the "channel" in which the bore propagates. As *Huang et al.* [1998] have shown, a similar wave/critical-level interaction may be responsible for the inversion layer in question. Hence we are proposing that the physical process producing the channel is the same as the one responsible for subsequently generating the bore.

1. Introduction

Taylor et al. [1995] reported a "spectacular gravity wave event" that took place during the Airborne Lidar and Observations of the Hawaiian Airglow 1993 (ALOHA-93) campaign. *Taylor et al.* [1995, Figure 1] shows the appearance of the event as imaged in both OH Meinel and atomic-oxygen green-line airglow (OI). There exists a sharp front in the image with a change of brightness and wavelike structure behind it. A striking fact is that these two images are complementary to each other in the sense that one appears to be an approximate photographic "negative" to the other. In other words, a bright feature of one corresponds to a dark feature of the other and vice versa in an approximate fashion. An explanation for these observations was offered in an earlier work of ours [*Dewan and Picard*, 1998], where it was proposed that the phenomenon was caused by an internal bore. Our explanation included a simple mathematical model, which made a reasonable quantitative fit to the observations that existed and made numerous qualitative and quantitative predictions as well.

The original idea for our bore hypothesis arose from the examination of an aerial photograph of a bore on the River Mersey published in a book by *Tricker* [1965] and duplicated by *Dewan and Picard* [1998, Figure 4]. In both the case of the river bore and the mesospheric event, there is a front followed by waves that move "locked to the front"; that is, the waves have the same velocity as the front. Such bores are called undular bores, and their physics is well known [e.g., *Lighthill*, 1979]. It would appear that a number, perhaps several tens, of further observations of borelike phenomena in the mesosphere have been made independently by a number of investigators (M. J. Taylor et al., private communications, 2000).

Tropospheric bores commonly exist when there is a reasonably strong inversion present to serve as a duct for them. *Dewan and Picard* [1998] postulated that these same physical requirements for a local inversion must also exist for mesospheric bores. Unfortunately, on October 10, 1993, when *Taylor et al.* [1995] observed the mesospheric bore, no simultaneous lidar temperature profiles were measured which could have identified the presence of such an inversion. It is, however, now known that inversions of the appropriate dimensions, strength, and altitude are seen quite commonly [e.g., *Meriwether and Gardner*, 2000].

Huang et al. [1998] studied the lidar wind and temperature records for a temperature inversion layer observed 11 days later during the same campaign [*Dao et al.*, 1995; *Tao and Gardner*, 1995] and proposed an explanation for its sudden appearance. The theory they presented was based upon gravity-wave interaction with the mean flow at a critical level. Further development of these concepts will be found in other recent studies [*Liu and Hagan*, 1998; *Liu et al.*, 1999; H. Hur and T. F. Tuan, private communication, 1998]. While other explanations for inversion layers have been published [e.g., *Meriwether and Mlynczak*, 1995], we shall, in the present paper, assume that gravity-wave–critical-level interactions are the cause.

We wish to address here the following question: How are mesospheric bores generated? In the case of tropospheric bores, including the famous "morning glory" [*Smith*, 1988], it is not difficult to find causes. It is now generally agreed (see the references by *Dewan and Picard* [1998]) that tropospheric bores are often caused by sea breezes, thunderstorm outflows, or katabatic winds that impinge upon an inversion layer. Since none of these generators exist in the mesosphere, we must begin anew to search for a mesospheric bore generator. The mechanism that we propose is closely related to the mechanism proposed for the origin of the inversion layer providing the duct, namely, gravity-wave–critical-level interaction. Our immediate objective in this paper is to derive a mathematical model for the generation of bores by critical-level interaction with gravity waves within a mesospheric duct. Our model makes use of the approach to bore generation by *Stoker* [1948, 1957], which is based on the concept of self-steepening. In section 2 we describe Stoker's theory for generation of river bores and then discuss how it applies to mesospheric-bore generation. Section 3 describes an estimate of the time required to generate a mesospheric bore and compares it to what

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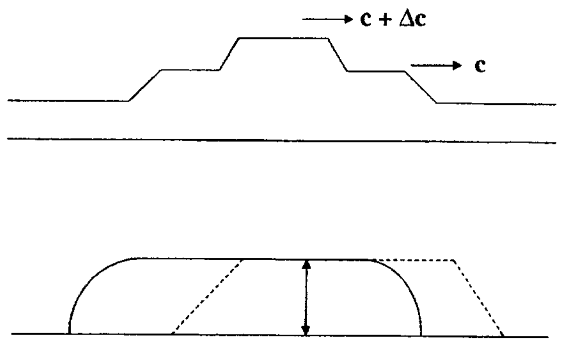

Figure 1. Self-steepening of a wave front due to nonlinear propagation effects. Wave propagation velocity c increases with amplitude Δz of disturbance. (See equation (3) which shows dependence of c on depth.) As a result, the front edge of a crest will steepen with time (dashed line), and the rear edge will flatten. In other words, because of its higher speed, the high portion of the original waveform will propagate to the front of the wave packet faster than the front of the wave packet is moving. Thus the front of the wave packet will steepen and, subsequently, become a bore, of the breaking or undular variety. Based on a figure of *Faber* [1995].

we know about bores and other mesospheric structures. The conclusions of this paper are stated in section 4, along with some discussion. In Appendix A we discuss the salient features of mesospheric bores as an aid to identifying them.

2. A Possible Bore Generation Process in the Mesosphere

2.1. How Are River Bores Generated?

Dewan and Picard [1998] used the analogy between river (or channel) bores and atmospheric bores to arrive at a model for the latter. (Since that publication, we have learned of work by *Klemp et al.* [1997] which pointed out differences between channel bores and internal bores. Their findings do not affect our conclusions, however.) We again seek guidance from this analogy to arrive at a reasonable mechanism for bore generation in the mesosphere. *Tricker* [1965] (whose image of the river bore was so illuminating) presented a theory for channel bores, based on the so called "hydraulic jump" effect, which is in direct conflict with the rest of the literature on bore formation. *Stoker* [1948, 1957] first presented the now generally accepted cause of tidal river bores, based on nonlinear shallow-fluid theory. Stoker explained both bores and breakers (as seen in waves at a beach) as the outcome of wave self-steepening brought about by the fact that surface wave speed increases with increasing wave height. (For shallow-water waves the speed is proportional to the square root of the water depth.) Figure 1, which is based on a similar figure in *Faber*'s [1995] work, illustrates how steepening takes place. In a word, a wave crest will travel faster than a wave trough, and a point will inevitably be reached when the slope of the front of the wave becomes vertical. Then what is commonly seen is that the wave breaks, resulting in a breaker or foaming bore. *Stoker* [1948, 1957] pointed out that the origins of this idea go back to *Lagrange* [1781] and more recently to Jeffreys (as cited by *Cornish* [1934, appendix]).

In principle, it is possible for wave spreading due to dispersion to compensate for nonlinear wave steepening, resulting in stable front or waveform formation. For example, this happens in the case of solitons and solitary waves (such as cnoidal waves). However, in the nonlinear shallow-fluid approximation of *Stoker* [1948, 1957], the waves are dispersionless and must steepen and break. *Ursell* [1953] addressed the paradox that there are some shallow water waves that do not break (the aforementioned solitons and cnoidal waves). Ursell showed that the nonlinear shallow-fluid approximation of *Stoker* [1948, 1957] only applies when $(a\lambda^2/h^3) \gg 1$, where a is the amplitude of the wave, λ is the horizontal wavelength, and h is the channel depth. When solitons occur, this parameter is around 15 [*Lighthill*, 1979, p. 465]. Then Stoker's theory does not apply, since it neglects effects such as vertical acceleration, which are important for large-amplitude waves. It should be kept in mind that the present calculation will be used exclusively for the purpose of studying bores and that the approximations used need not apply to gravity waves in general.

It should be mentioned that all viable theories of bore generation explicitly use nonlinear steepening. The presence of bores at any location, including the mesosphere, therefore implies that nonlinear steepening will occur there. Only in the case of an infinitely deep fluid (compared to the horizontal scale of the disturbance) will there be no increase of speed with amplitude and hence no nonlinear steepening. In this case, bore generation is impossible.

At this point we present the mathematical formulation of wave steepening and bore formation, following *Stoker*'s [1948, 1957] classical nonlinear shallow-wave approximation. There are also more recent treatments of the problem which may be more accessible, such as *Johnson* [1997, pp. 146–151], *Crapper* [1984, pp. 180–188], and *Yih* [1969, pp. 209–219]. See also *Henderson* [1966, pp. 285–304], who illustrates this technique with several examples, including the present one.

The starting point of shallow-wave theory is the pair of equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial u}{\partial x}, \tag{1}$$

$$\frac{\partial}{\partial x}[u(\eta + h)] = \frac{-\partial \eta}{\partial t}, \tag{2}$$

where $u = u\,(x, t)$ is the horizontal velocity, t is the time, and x is the horizontal coordinate down the channel. The quantity $\eta = \eta(x, t)$ is the elevation of the free surface ($\eta = 0$ for the unperturbed surface), and $h = h(x)$ is the unperturbed depth of the fluid. Equation (1) is Euler's equation of motion (Newton's second law, usually called the momentum equation). Here it is assumed that vertical accelerations can be ignored and therefore that pressure can be considered as hydrostatic. Note also that the velocity u does not depend on the vertical coordinate y, a valid approximation in the current context. Equation (2) is a form of the equation of continuity for this case. The reader can consult the references given above for further details.

Let c be the propagation velocity for waves in the channel under consideration. In general, this velocity (which applies to any disturbance) is given by [*Stoker*, 1957, p. 293]

$$c = \sqrt{g(\eta + h)}. \tag{3}$$

We now must express (1) and (2) in terms of c rather than η. Note that loci of constant c are also loci of constant η. We then have, as can be seen by differentiating (3),

$$\frac{\partial c}{\partial x} = \frac{\left(g\,\dfrac{\partial \eta}{\partial x} + g\,\dfrac{\partial h}{\partial x}\right)}{2c}, \tag{4}$$

$$\frac{\partial c}{\partial t} = \frac{g}{2c}\frac{\partial \eta}{\partial t}; \tag{5}$$

hence using $H \equiv gh$,

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + 2c\,\frac{\partial c}{\partial x} - \frac{\partial H}{\partial x} = 0, \tag{6}$$

$$2\,\frac{\partial c}{\partial t} + 2u\,\frac{\partial c}{\partial x} + c\,\frac{\partial u}{\partial x} = 0. \tag{7}$$

At this point we use the method of characteristics. We shall assume that the depth of the channel is constant or $dH/dx = 0$. Adding (6) and (7) results in

$$\left(\frac{\partial}{\partial t} + (u + c)\,\frac{\partial}{\partial x}\right)(u + 2c) = 0, \tag{8}$$

while subtracting yields

$$\left(\frac{\partial}{\partial t} + (u - c)\,\frac{\partial}{\partial x}\right)(u - 2c) = 0. \tag{9}$$

It is not difficult to find the physical interpretation of these two equations. Equation (8) says that the quantity $(u + 2c)$ remains constant in a frame of reference moving with velocity $u + c$ relative to the fluid. This frame of reference has therefore a velocity

$$\frac{dx}{dt} = u + c. \tag{10}$$

This equation defines a curve of t as a function of x. Call this curve C_1. Similarly, from (9) the quantity $(u - 2c)$ remains constant in a frame moving with velocity

$$\frac{dx}{dt} = u - c. \tag{11}$$

Call the resulting curve C_2. In short, then

$$u + 2c = \text{constant along } C_1, \tag{12}$$

$$u - 2c = \text{constant along } C_2. \tag{13}$$

The curves C_1 and C_2 are called the characteristics corresponding to (6) and (7). Different arbitrary constants in (12) and (13) will generate different members of the C_1 and C_2 families of curves. Figure 2 shows a plot of t versus x showing examples of such characteristics. *Stoker* [1948, 1957] and the other more recent references mentioned above show that (10), (11), (12), and (13) can be used to establish the following properties:

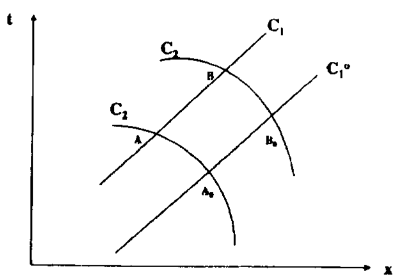

Figure 2. Characteristics C_1 and C_2 for the "simple wave" case: C_1, straight lines; C_2, curved.

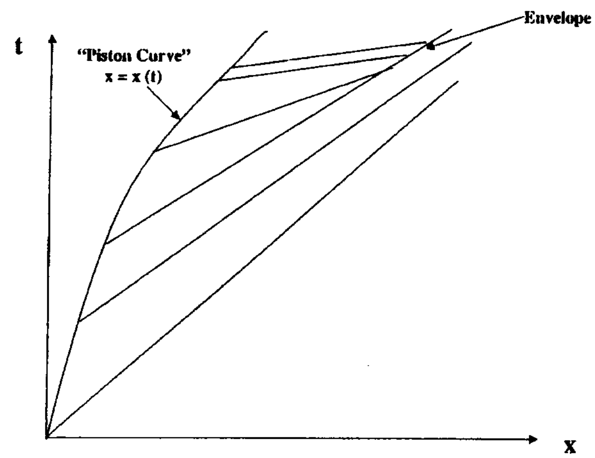

Figure 3. Characteristics C_1 for a fluid under the influence of a uniformly accelerating piston. The characteristics originating on the piston as a function of time (see piston curve) have slopes that flatten as the velocity of the piston increases. Eventually, characteristics will intersect, forming a vertical wave front, or bore. The envelope is formed by the locus of intersection points of the characteristics.

1. The C_1 characteristics for the "simple wave" problem [*Stoker*, 1957, p. 300; *Henderson*, 1966, p. 292] are all straight lines. The simple wave problem is defined by the following: h, the undisturbed depth, is constant; the fluid extends from the origin of x to infinity in the positive direction; the fluid is either at rest or has a constant velocity at $t = 0$; and the elevation of the free surface is zero at $t = 0$.
2. The C_2 characteristics all intersect a given C_1 characteristic at the same angle.
3. On any given C_1 characteristic the values of u and c are each constant.
4. Along the characteristics C_1, we have

$$\frac{dx}{dt} = \tfrac{1}{2}\,(3u(\tau) - u_0) + c_0, \tag{14}$$

where u_0 and c_0 are given initial values (in our case, we will have $u_0 = 0$) and $u(\tau)$ is the value of u at the time τ when the C_1 characteristic intersects a boundary curve (such as the "piston" curve described below) or the t axis.

We are now in position to pursue our goal, which is to show that (1) any hump of fluid in a channel ($\eta > 0$) under the above assumptions will steepen and eventually form a bore and (b) the location of the place where this happens can be calculated by the method of characteristics.

Consider the situation shown in Figure 3. We consider a fluid channel of constant depth, with the fluid initially at rest. At $x = 0$, there is a piston accelerating the fluid to the right, as shown in Figure 4. The role of the piston can be played in the river bores by forcing due to the incoming tide. As the piston accelerates, it generates a hump of fluid in front of it, which propagates faster than the piston moves. The position of the piston as a function of time is shown in Figures 3 and 5. In this case, from (14) the straight-line C_1 characteristics obey

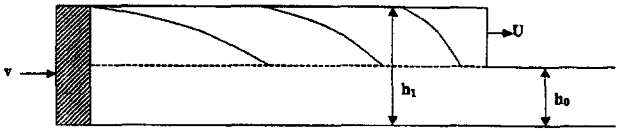

Figure 4. Formation of a bore in a channel by an accelerating piston. The piston, which is on the left side, has a velocity v. It forms a fluid disturbance in the shape of a hump, which moves away from it and steepens until a bore having velocity U is formed. The depths h_1 and h_0 refer to the fluid behind the bore and to the undisturbed fluid ahead of it, respectively. Curved lines show steepening with time of the leading edge of the transient waveform until the bore is formed.

$$\frac{dx}{dt} = \frac{3}{2} u(\tau) + c_0, \tag{15}$$

where $u(\tau)$ is the velocity of the piston at the time $t = \tau$ along the piston curve $x = x(t)$; that is,

$$u(\tau) = \left.\frac{dx(t)}{dt}\right|_{t=\tau}. \tag{16}$$

Equation (15) results from (14) when $u_0 = 0$. The important feature to notice is that as shown in the cited references (in particular, *Stoker* [1957], p. 315, Figure 10.6.1), the characteristics in Figures 3 and 5 eventually intersect. Physically, the first point of intersection represents the place and time where the hump of fluid generated by the accelerating piston has self-steepened to the point that its leading edge has become vertical. Beyond this point, the above theory starts to break down because certain nonlinear effects that are omitted from the formalism become important. It is, however, well known that the first point of intersection of the characteristics is very close to the place where the wave will break and a bore will form. *Dewan and Picard* [1998] explained that under certain circumstances (in particular, when the displacement is not very large), one can have an undular bore rather than a breaking turbulent, or foaming, bore. In the literature this first point of intersection of the characteristics marks the birthplace of the bore. (Parenthetically, it is interesting to note that this treatment of bores is in perfect mathematical analogy with the theory of shock waves in gas dynamics [*Stoker*, 1957, 1948]).

Appropriate extensions of this theory, taking into account changes in channel depth and width and effects of friction, for example [*Abbott*, 1956], have been used to correctly predict where and when tidal river bores will form. In particular, *Abbott* [1956] treated the case of the Severn River bore in detail.

2.2. Generation of Mesospheric Bores

We now turn to the question of how bores in the mesosphere are generated. *Dewan and Picard* [1998] showed that the mathematical model for channel waves and bores applied in at least in an approximate sense to internal waves and bores. Both the tropospheric and the mesospheric bores propagate on sufficiently narrow ducts that the shallow-fluid approximation applied. This model allowed us to obtain analytic solutions when the real altitude-dependent mesospheric stability can be approximated by a stability that is piecewise constant. The excitation on the internal inversion layer was regarded as oscillating in a "varicose mode," as is the case for waves in the ocean's thermocline. *Lighthill* [1979] defined the varicose mode of thermocline oscillations as "where the upper part of the thermocline is rising, the lower part is falling, and vice versa; so the thermocline region is varying in thickness." In the present situation the phrase "inversion layer" should replace the word "thermocline." Thus the internal bore, as the river bore, was regarded as depending on two spatial dimensions (x and z) and consisting of two-sided symmetrical displacements in the vertical z dimension about a central undisplaced line (or in 3-D, a plane) of nodes. The only quantitative change in the river channel equations consisted of substituting $g' \equiv (g\Delta\phi)/\phi$ for g, where ϕ is the potential temperature and $\Delta\phi$ is its change across the bore front.

In section 2.1 we showed that river bores are generated by something playing the role of an accelerating piston, usually the incoming tide, which produces a hump on the fluid surface. This hump subsequently steepens until it breaks and becomes a bore. It was also mentioned above that the role of "piston" in the case of tropospheric bores could be played by any of several different forcing mechanisms. Now we come down to the question of what could play this role in the mesosphere.

Dewan and Picard [1998] emphasized how internal atmospheric bores can be supported in a channel formed by a strong inversion layer. *Huang et al.* [1998] presented a theory in a related paper to explain how such mesospheric inversions might form. The mechanism proposed was interaction of a gravity wave with the mean flow, including the tidal component, at a critical level. Here we propose that a similar mechanism plays the role of the accelerating piston for bore generation in the duct formed by the inversion layer. Horizontal momentum would thus be imparted over a narrow range of altitudes to a local fluid element in the channel formed by the inversion layer. This is due to the presence of a wave momentum-flux divergence at a critical level [see *Lindzen*, 1990, pp. 214–215]. The momentum-flux divergence could be provided by gravity waves from a source similar to the one that was responsible for creating the inversion layer originally or from a different source. (However, see the discussion in section 4.)

A reviewer of this paper pointed out that it is not necessary to have a critical layer to impart momentum to the mean flow.

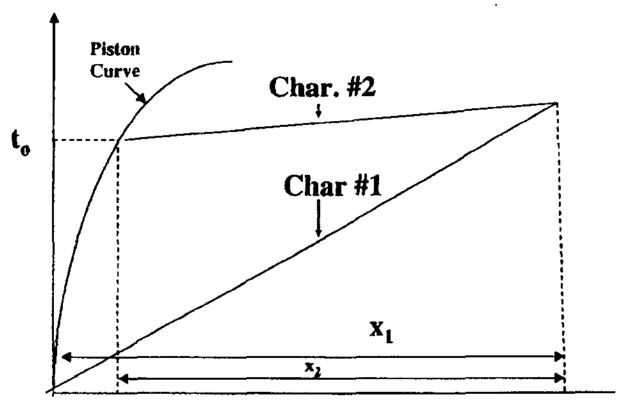

Figure 5. Detailed view of the C_1 characteristics for a fluid under the influence of a uniformly accelerating piston. Characteristic 1 originates on the piston at the moment it starts to accelerate, and characteristic 2 originates on the piston at the moment and place where the latter ceases to accelerate. The two characteristics travel distances x_1 and x_2, respectively, before meeting at a later time t_0 when the back end of the hump of fluid produced by the piston has caught up with the front end, and the bore has reached its maximum height.

In particular, any form of wave breaking would do this. Furthermore, one expects waves to break as they ascend due to the exponential decrease in the density of the atmosphere with respect to altitude [e.g., *Dewan and Good*, 1986]. This raises the question of why a critical layer is required. The answer is that we are here seeking a mechanism to play the role of a piston that will provide localized forcing only within the inversion layer. Random wave breaking does not seem to offer a practical solution to this requirement and, in any case, does not seem to explain the relative rarity of bores, accompanied by the prevalence of inversion layers. For this reason, it is very fortunate that the inversion layer is expected by the theory of *Huang et al.* [1998] to contain a critical level. The critical level therefore solves two problems at the same time. It is usually considered a positive sign when proposed explanations simultaneously solve more than one problem and when an explanation introduced to solve one problem is found to apply to other problems than originally intended. This is in line with the criterion of simplicity in theory construction, otherwise known as Occam's razor.

An important question to raise is whether or not this mechanism is quantitatively reasonable. This question will be explored in detail in section 3 below. For now, we note simply that *Fritts and Lu* [1993] have provided an estimate for the expected magnitude of the momentum-flux divergence. They found that this flux divergence produced a mean zonal acceleration in the mesosphere of the order of 100 m/s/d. Since the Fritts and Lu value is presented as a long-term global mean, we shall consider it to be a lower bound on the accelerations which could occur due to intense gravity-wave sources on a shorter timescale (such as a small fraction of a day). As a result, our calculations will be on the conservative side in that the acceleration could be significantly larger.

If, indeed, the generation of bores requires large accelerations, then this may lead to a method to predict conditions favorable to such bore formation. This is because vigorous gravity wave sources would be needed. The latter could include high winds over mountains, active thunderstorms, electrojet activity, and so on. One must also take into account the possible blockage of upward wave propagation by mean wind filtering effects (i.e., critical layers). Finally, of course, one must assume that an appropriate inversion layer, which can serve as a duct, is present prior to the bore-generating acceleration.

3. An Estimate of the Time Required to Generate a Mesospheric Bore

Consider Figure 4, which shows how to calculate the relation between the piston velocity v, the bore velocity U, and the depths h_1 and h_0 of the fluid behind the bore and of the undisturbed fluid, respectively. The equation of continuity, equation (2), can be written [*Dewan and Picard*, 1998]

$$vh_1 = U(h_1 - h_0), \tag{17}$$

so

$$v = \frac{U(h_1 - h_0)}{h_1}. \tag{18}$$

In the above reference we derived estimates for a case study motivated by data [*Taylor et al.*, 1995] taken on October 10, 1993, during the ALOHA-93 campaign. In that case, from the observations, we had $U = 76$ m/s, $h_0 = 2.7$ km, and $h_1 = 3.5$ km. Thus from (18) we can deduce that the effective "piston" velocity is $v = 17$ m/s.

The next question is as follows: How long would it take to accelerate the mesospheric piston to a velocity $v = 17$ m/s when the acceleration $a = 100$ m/s/d, the value of *Fritts and Lu* [1993]? Let that time be t_0. Then

$$t_0 = \frac{v}{a} = \frac{17 \text{ m/s}}{100 \text{ (m/s)/24 hours}} \approx 4 \text{ hours}. \tag{19}$$

In other words, with a very conservative acceleration arising from the momentum transfer to the local critical layer via the global-mean gravity-wave flux divergence, it would require about 4 hours to attain the required 17 m/s.

The next step in this calculation is to assume that after 4 hours the piston maintains the constant velocity of 17 m/s and then to apply (14) to ascertain, by the method of characteristics, how much additional time is required for the characteristics to intersect and the bore to form.

Let $x_1(t)$ be the spatial trajectory of the front of the waveform or hump produced at the initial moment of piston acceleration, and let $x_2(t)$ denote the trajectory of the "back" of the hump produced at the moment that the piston ceases to accelerate. Then using (14), the associated characteristic curves C_1 and C_2, or characteristics 1 and 2, respectively, are given by

$$x_1 = c_0 t, \tag{20}$$

$$x_2 = [\tfrac{3}{2}\, v(t_0) + c_0](t - t_0), \tag{21}$$

where $v(t_0)$ is the final piston velocity, and c_0 is given from shallow internal-wave theory by

$$c_0 = \sqrt{g' h_0}. \tag{22}$$

Since, according to *Dewan and Picard* [1998], $g' \cong 1.4$ m/s^2 and we have already taken $h_0 = 2.7$ km above, (22) gives $c_0 = 60$ m/s.

Now, since the acceleration a is constant, one has

$$x_1 = x_2 + \tfrac{1}{2} a t_0^2, \tag{23}$$

at the point where these two characteristics start to intersect. It should be noted that this point of intersection occurs subsequent to the place and time of actual bore formation, i.e., the point where any two characteristics intersect. Rather it designates the place and time where the bore has attained the height h_1 that was estimated by *Dewan and Picard* [1998]. Using (23), (21), and (20) to eliminate x_1 and x_2, we can solve for the total time required, t. Letting $v(t_0) = at_0$, we have

$$t = t_0 + \frac{c_0}{\frac{3}{2} a} - \frac{t_0}{3}. \tag{24}$$

The total time needed to create this bore is found from (24) by inserting $c_0 = 60$ m/s, $t_0 = 4$ hours, $a = (100$ m/s$)/(24$ hours$)$, giving $t = 12$ hours.

This amount of time is less than the duration of many of the observed inversions [*Hauchecorne et al.*, 1987] which can persist for several days. The proposed mechanism is therefore plausible. The fact that this mechanism is the same as the one that was hypothesized for the cause of the inversion which forms the ducting channel [*Huang et al.*, 1998] gives it even more plausibility, in our opinion.

We observe further that the total time t required to generate the previously observed bore may have been much shorter than the above estimate. Eliminating t_0 from (24) using the left member of (19), we obtain

$$t = \frac{2}{3a}(v + c_0). \tag{25}$$

This shows that t is inversely proportional to a. As was mentioned, the value of $a = 100$ m/s/d which was used above is a long-term global average. In view of the fact that mesospheric bores seem to be relatively rare phenomena, it is plausible to assume that they are most likely to occur when and where the value of a is significantly larger than this average value. Let us suppose, for example, that a is 10 times the average value. Then, from (25), t would reduce to 1.2 hours, which is much less than the duration of the inversion observed by *Dao et al.* [1995] and described by *Huang et al.* [1998].

4. Conclusions

Mesospheric bores were first hypothesized and described by *Dewan and Picard* [1998], in an attempt to explain the observations of *Taylor et al.* [1995]. Here we extend those considerations to include a possible mechanism for generating mesospheric bores. Continuing on the theme of applying insights from channel-bore theory, we have described a simplified general model of how bores can be generated in a river channel by means of a forcing mechanism visualized as a "piston" [e.g., *Stoker*, 1948, 1957]. This model has been used successfully to predict the occurrence and location of tidal bores in rivers [*Abbott*, 1956]. *Dewan and Picard* [1998] speculated that the mechanism of critical-level interaction of gravity waves with the mean flow, which produced the inversion layer responsible for the bore channel, could well also be the mechanism responsible for the fluid acceleration which initiates bore formation.

In the present paper we have explored this possibility, and our calculations show that this is a plausible hypothesis. Using a long-term global average for acceleration due to gravity-wave momentum-flux divergence, we very conservatively estimated a time of 12 hours for the generation of a bore of the type observed. This time exceeds the duration of some inversion layers. Since bores are temporally local phenomena, we also considered a modest value for a short-term local acceleration of 10 times the long-term global average, which resulted in a formation time of only 1.2 hours, shorter than the duration of most observed inversion layers.

To the extent that gravity-wave critical-level interactions depend strongly on the background winds, and the background winds include a tidal component, tides form an important element of the forcing. The importance of the nonlinear interaction between tides and gravity waves for inversion-layer formation was pointed out by *Huang et al.* [1998], based on the early work of *Walterscheid* [1981], and has been demonstrated explicitly in a 2-D numerical model [*Liu and Hagar*, 1998; *Liu et al.*, 1999]. We would, in a similar manner, expect that tides will influence the forcing responsible for generating a bore. However, unlike the situation in river bores, we do not believe that tides by themselves, without the presence of gravity waves or some other as-yet-undiscovered elements, can be responsible for the forcing resulting in mesospheric bore formation.

An interesting question, which might occur to the reader, is as follows: Why are mesospheric bores rare and mesospheric inversions ubiquitous? We offer the following considerations:

1. To date, mesospheric bores have only been observed on imagers using airglow emissions. Hence they were seen only in a certain altitude region. In contrast, inversions have been observed at altitudes with lidar or in situ measurements where no airglow layer exists.
2. In the theory adopted here [*Huang et al.*, 1998], inversions are caused by gravity-wave–critical-level interactions that can store energy in a shear layer until the latter reaches a threshold and breaks down. In contrast, bore generation seems to be caused by gravity-wave–critical-level interactions that entail large accelerations, and the latter may be a relatively rare phenomenon.
3. The bore reported by *Taylor et al.* [1995] was very spectacular. Lesser bores may be less rare but remain unnoticed since their detection may require a more sophisticated search.
4. There could be more than a single mechanism for inversion formation at mesospheric altitudes.
5. An inversion could, under the right conditions, be rendered incapable of supporting a bore, or bore proof, due to wind-shear effects. This is the opposite of Doppler ducting, which is another mechanism besides inversion layers that could conceivably form a duct for a bore, once again under the right conditions.
6. Finally, it should be noted that while we hypothesize that the mechanisms for formation of the ducting inversion layer and for bore generation are the same, the requirements for gravity-wave sources for each differ greatly. The bore duct must be spatially extensive enough to allow sufficient time (1) to form the bore and (2) for the bore to propagate far enough that there is a reasonable chance of observing it. Hence inversion-layer formation requires an equally extensive gravity-wave source that need not be particularly strong. In contrast, the forcing "piston" must be more localized. Hence the gravity-wave source must be relatively local and unusually strong. Unfortunately, there are no related observations that indicate that gravity-wave activity was either particularly strong or particularly weak on the night of the observation of the ALOHA campaign bore by *Taylor et al.* [1995].

To test our bore-generation hypothesis experimentally, one could start by estimating the time elapsed since bore formation by the method described by *Dewan and Picard* [1998] for an undular bore. This involves counting the total number of wave crests in the undulations that follow and are locked to the bore, then using the estimate in that paper of the rate of generation of crests (~2–3 per hour) to determine the time the bore was created. (The reference explains how this rate of crest generation varies with respect to measurable parameters of the bore.) The next step would require the use of imagers and Doppler wind-temperature lidars to characterize the wave field, to identify possible bore ducts, and possibly to determine momentum-flux divergence. In this way, critical-level interactions could be detected and the formation time of the bore could be verified, if the bore's time and place of creation are sufficiently close to be studied by the instruments in question. In addition, the proposed forcing mechanism for bore generation can be validated by comparing the flux-divergence measurements to the bore characteristics.

To validate the theory of *Dewan and Picard* [1998] and of the current paper, we intend, in the future, to compare it against other bore observations, some tens of which have been described since the initial observations of *Taylor et al.* [1995], by

Taylor and other observers. While it was not the purpose of this paper to discuss in detail the newer observations, we can say that some of the more recently observed bores do not display the complementarity features between radiance variations observed on different airglow layers by *Taylor et al.* [1995]. In our model this lack of complementarity is predicted to occur whenever the inversion layer occurs at an altitude either above or below all the observed airglow layers. Unfortunately, to our knowledge there are no observations yet available that tie bore occurrences to inversion layers nor to the strength of gravity wave sources at the birth of the bore.

Appendix A

In this appendix we review some of the properties of mesospheric bores in view of identifying some of the necessary conditions for their identification.

1. There must be a front that separates dark and light regions in the airglow images and which, as a guess, should travel at a speed in the range of ~20–100 m/s.

2. In the case of undular bores the front will be followed by a fixed pattern of waves which is locked to the movement of the front. Unusually strong bores may be nonundular, in which case there will be no waves behind the front.

3. There must be an inversion layer at the altitude of the bore to provide a duct for the bore, or conceivably, an alternate ducting structure formed through some combination of windshear and thermal inversions.

4. Images of different airglow layers associated with the same bore will generally vary in phase or 180° out of phase with one another, in the latter case displaying the complementarity features described by *Taylor et al.* [1995] and by *Dewan and Picard* [1998]. Whether airglow variations are in phase or out of phase with one another will depend on the relative altitudes of the inversion layer and of the various airglow layers in the manner described above in section 4.

5. The speed of the bore and the wavelength of its associated waves should obey equations (12) and (13) of *Dewan and Picard* [1998].

6. The change in temperature ΔT across the front (measurable by an associated lidar, from Doppler width by a Fabry-Pérot interferometer, or from rotational temperature by means of a Michelson interferometer, as in the work of *Taylor et al.* [1995]) should agree approximately with $(h_1 - h_0) \times (10$ K/km$) = \Delta T$ [see *Dewan and Picard*, 1998].

7. For undular bores there should be a tendency for the number of waves trailing the front to gradually increase over time. *Dewan and Picard* [1998] estimated the formation rate at 2–3 wave crests per hour, in the case when no wave energy is radiated out of the duct.

8. It might be possible to associate the presence of bores to occasions where gravity-wave sources are unusually strong.

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References

Abbott, M. R., A theory of the propagation of bores in channels and rivers, *Proc. Cambridge Philos. Soc.*, *52*, 344–362, 1956.

Cornish, V., *Ocean Waves and Kindred Geophysical Phenomena*, Cambridge Univ. Press, New York, 1934.

Crapper, G. D., *Introduction to Water Waves*, Halsted, New York, 1984.

Dao, P. D., R. Farley, X. Tao, and C. Gardner, Lidar observations of the temperature profile between 25 and 103 km: Evidence of a strong tidal perturbation, *Geophys. Res. Lett.*, *22*, 2825–2828, 1995.

Dewan, E. M., and R. E. Good, Saturation and the "universal" spectrum for vertical profiles of horizontal scalar winds in the atmosphere, *J. Geophys. Res.*, *91*, 2742–2746, 1986.

Dewan, E. M., and R. H. Picard, Mesospheric bores, *J. Geophys. Res.*, *103*, 6295–6305, 1998.

Faber, T. E., *Fluid Dynamics for Physicists*, Cambridge Univ. Press, New York, 1995.

Fritts, D. C., and W. Lu, Spectral estimates of gravity wave energy and momentum fluxes, part II, Parameterization of wave forcing and variability, *J. Atmos. Sci.*, *50*, 3695–3713, 1993.

Hauchecorne, A., M. L. Chanin, and R. Wilson, Mesospheric temperature inversion and gravity wave breaking, *Geophys. Res. Lett.*, *14*, 933–936, 1987.

Henderson, F. M., *Open Channel Flow*, Macmillan, Indianapolis, Indiana, 1966.

Huang, T. Y., T.-F. Tuan, X. Li, E. M. Dewan, and R. H. Picard, Sudden narrow temperature inversion-layer formation in ALOHA-93 as a critical-layer-interaction phenomenon, *J. Geophys. Res.*, *103*, 6323–6332, 1998.

Johnson, R. S., *A Modern Introduction to the Mathematical Theory of Water Waves*, Cambridge Univ. Press, New York, 1997.

Klemp, J. B., R. Rotunno, and W. C. Skamarock, On the propagation of internal bores, *J. Fluid Mech.*, *331*, 81–106, 1997.

Lagrange, J. L., Théorie du mouvement des fluides, *Oeuvres Lagrange*, *4*, 694, 1781.

Lighthill, J., *Waves in Fluids*, Cambridge Univ. Press, New York, 1979.

Lindzen, R. S., *Dynamics in Atmospheric Physics*, Cambridge Univ. Press, New York, 1990.

Liu, H.-L., and M. E. Hagan, Local heating/cooling of the mesosphere due to gravity wave and tidal coupling, *Geophys. Res. Lett.*, *25*, 941–944, 1998.

Liu, H.-L., P. B. Hays, and R. G. Roble, A numerical study of gravity wave and impacts on turbulence and mean state, *J. Atmos. Sci.*, *56*, 2152–2177, 1999.

Meriwether, J. W., and C. S. Gardner, A review of the mesosphere inversion layer phenomenon, *J. Geophys. Res.*, *105*, 12,405–12,416, 2000.

Meriwether, J., and M. G. Mlynczak, Is chemical heating a major cause of the mesospheric inversion layer?, *J. Geophys. Res.*, *100*, 1379, 1995.

Smith, R. K., Traveling waves and bores in the lower atmosphere: The "morning glory" and related phenomena, *Earth Sci. Rev.*, *25*, 267–290, 1988.

Stoker, J. J., The formation of breakers and bores, *Commun. Pure Appl. Math.*, *1*, 1, 1948.

Stoker, J. J., *Water Waves*, Wiley Intersci., New York, 1957.

Swenson, G. R., J. Qian, J. M. C. Plane, P. J. Espy, M. J. Taylor, D. N. Turnbull, and R. P. Lowe, Dynamical and chemical aspects of the mesospheric Na "wall" event on October 9, 1993, during the Airborne Lidar and Observations of Hawaiian Airglow (ALOHA) campaign, *J. Geophys. Res.*, *103*, 6361, 1998.

Tao, X., and C. Gardner, Heat flux observations in the mesopause region above Haleakala, *Geophys. Res. Lett.*, *22*, 2829–2832, 1995.

Taylor, M. J., D. N. Turnbull, and R. P. Lowe, Spectrometric and imaging measurements of a spectacular gravity wave event observed during the ALOHA-93 campaign, *Geophys. Res. Lett.*, *22*, 2848–2852, 1995.

Tricker, R. A. R., *Bores, Breakers, Waves, and Wakes*, Elsevier Sci., New York, 1965.

Yih, C.-S., *Fluid Mechanics*, McGraw-Hill, New York, 1969.

Ursell, F., The long-wave paradox in the theory of gravity waves, *Proc. Cambridge Philos. Soc.*, *49*, 685–694, 1953.

Walterscheid, R. L., Inertiogravity-wave-induced accelerations of mean flow having an imposed periodic component: Implications for tidal observations in the meteor region, *J. Geophys. Res.*, *86*, 9698–9706, 1981.

E. M. Dewan and R. H. Picard, Air Force Research Laboratory/VSBM, 29 Randolph Road, Hanscom AFB, MA 01731-3010. (edmond.dewan@hanscom.af.mil)

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