Differential Game Based Guidance Law for High Angle of Attack Missiles

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Abstract

A nonlinear differential game theoretic intercept guidance law for short range missiles is derived. Differential geometric transformations are used to convert the nonlinear missile and the target models into a convenient form for the formulation and solution of the guidance problem. Guidance law is then derived using the necessary conditions for optimality. Due to the inclusion of all the significant nonlinearities in the formulation, the guidance law is useful in advanced missiles executing large angle of attack maneuvers. The guidance law performance is illustrated in air-to-surface and air-to-air intercept missions.

Introduction

Recent aeronautical research has identified high angle of attack maneuverability and direct force capabilities as the keys to future combat aircraft air superiority [1 - 3]. High angle of attack maneuvers involve sustained flight at angles of attack far beyond the conventional stall limits, while direct force capabilities enable aircraft to maneuver at arbitrary fuselage attitudes. The X-31 program at NASA Dryden Flight Research Center has investigated the development and use of these enhanced fighter maneuverability (EFM) concepts through an ambitious flight test-based program [4].

These developments have major implications on missile technologies. Superior maneuverability of high angle of attack aircraft will demand significantly better agility from tactical missiles. In the air-to-surface missions, missile launch at arbitrary aircraft attitudes will require the missile airframe to be able to maneuver effectively through the high angle of attack regime to ensure successful target intercept. Modern stealth fighters are able to approach targets at close ranges without being detected. This capability may require unusual missile launch

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geometries to ensure mission success. For instance, in order to ensure a safe egress out of hostile territory, a missile launch might be required while the aircraft is heading away from the target. Another launch scenario may involve large magnitude missile maneuvers to avoid known obstacles in order to achieve target intercept. In all these cases, the missile will encounter the high angle of attack flight regime.

Designing a satisfactory autopilot and guidance system for such missiles will require the use of multiple control strategies and techniques. This paper addresses the development of an intercept guidance law for high angle of attack missiles. The performance of the missile guidance law is demonstrated using a pitch-plane rigid-body simulation of an air-to-surface and air-to-air intercept engagements. High angle of attack missile autopilot design issues are addressed in a companion paper [5].

A block diagram of a pitch plane missile flight control system is given in Figure 1. The guidance law generates steering commands to orient the missile velocity vector in the direction of the target. The autopilot has the responsibility for tracking the normal acceleration command while stabilizing the missile airframe. The aerodynamic surfaces are used for high speed low angle of attack flight, while a combination of reaction jets and aerodynamic surfaces may be employed in the high angle of attack regime. An actuator sharing logic converts the actuator commands from the autopilot into the commands for the aerodynamic surface actuator and the reaction jets based on the flight condition. Reaction jets may be commanded whenever the aerodynamic surfaces saturate.
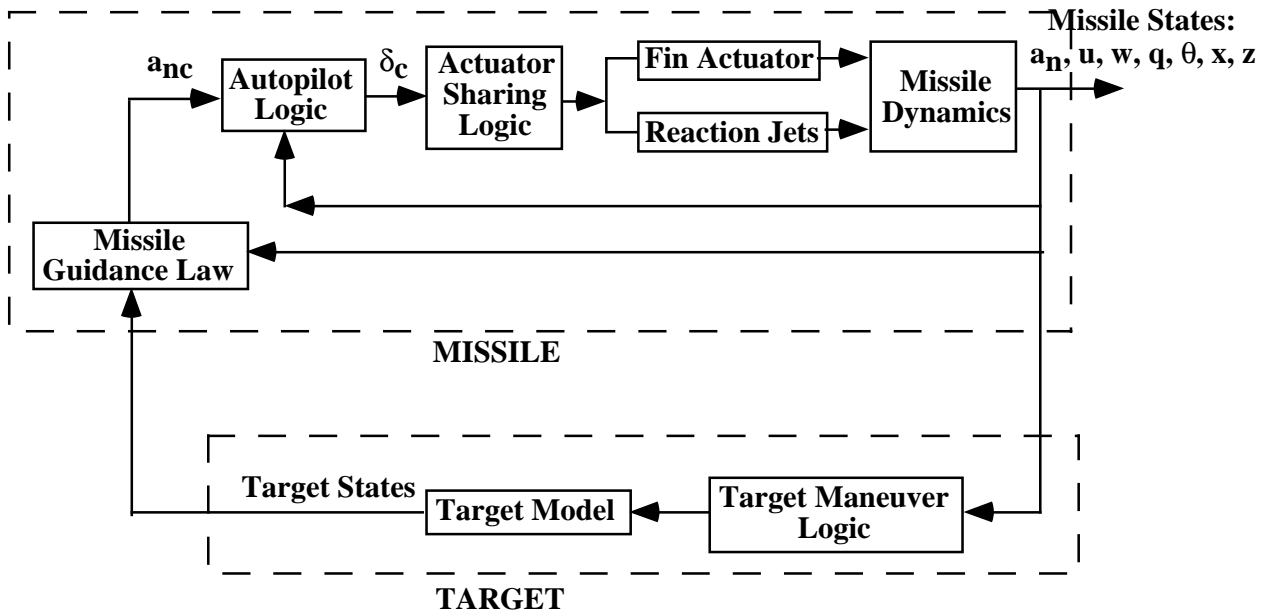
Fig. 1. Missile Flight Control System

The missile velocity vector can be oriented in a desired direction by applying a force normal to the direction of the instantaneous velocity vector. At low angles of attack, the velocity vector can be oriented by using the aerodynamic force normal to the missile body as is done in conventional missile configurations. However, in high angle of attack missiles, a force normal to the velocity vector may include both the aerodynamic normal force and a component of the main motor thrust during the initial portion of the flight.

Considering the pitch plane, The net normal acceleration is given by the expression:

$$a_n = a_z \cos \alpha - a_x \sin \alpha$$

a_z and a_x are the acceleration components along the X_B and Z_B body axes, and α is the angle of attack, see Figure 2 for a definition of the conordinate system. Note that at low angles of attack, the acceleration along the Z_B axis adequately approximates the acceleration normal to the velocity vector.
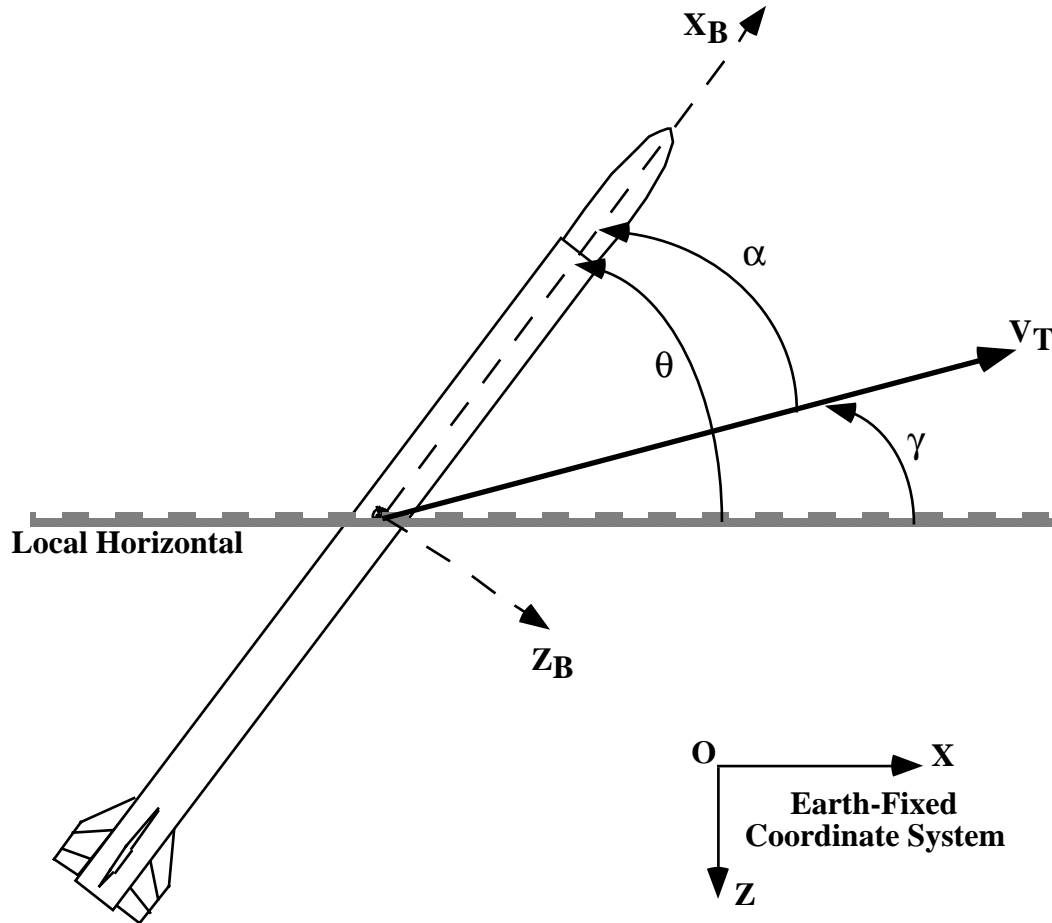
Fig.2. Missile Pitch Plane Coordinate System

Missile Model

The pitch-plane rigid body model of the missile incorporates two translational degrees of freedom and a rotational degree of freedom. Figure 2 illustrates the missile coordinate systems and the variables of interest. The missile equations of motion are expressed in terms of two coordinate systems. The coordinate system X - Z is used to define the position and attitude of the missile with respect to an earth fixed reference, while the body fixed coordinate system X_B - Z_B is used to define the missile velocity components and body rates. Aerodynamic forces and moments are also expressed in the body frame. The direction Z in the earth-fixed coordinate system points in the direction of the local gravity vector.

The missile equations of motion in the pitch plane are given by the following six nonlinear differential equations [6].

$$\dot{u} = [T - F_A]/m - g \sin \theta - q\,w$$

$$\dot{w} = q\,u + g\cos\theta - [F_N + F_r]/m$$

$$\dot{q} = \frac{M - l_r F_r}{I_{yy}}\ ,\ \dot{\theta} = q$$

$$\dot{z}_M = V_T\sin\left(\theta - \alpha\right),\ \dot{x}_M = V_T\cos\left(\theta - \alpha\right)$$

These equations assume a flat, non-rotating earth with a quiescent atmosphere. In these expressions, u is the missile velocity component along the X_B body axis, w is the velocity component along the Z_B body axis, T is the missile thrust, F_A is the axial force, F_N is the normal force, m is the vehicle mass, g the acceleration due to gravity, θ the pitch attitude, q the pitch rate, F_r is the force generated by the reaction jets, and l_r is the reaction jet lever arm. M is the aerodynamic pitching moment, I_{yy} is the pitch moment of inertia, $-z_M$ is the altitude, and x_M is the down range. The variable γ shown in Figure 2 is the flight path angle.

The form of the missile aerodynamic forces and moments, and the definition of other related variables are given by the following expressions:

$$F_A = C_A(\text{Mach},\ \alpha,\ \delta)\,\overline{q}\,s$$

$$F_N = C_N(\text{Mach},\ \alpha,\ \delta)\,\overline{q}\,s$$

$$M = C_m(\text{Mach},\alpha,\ \delta)\,\overline{q}\,s\,l_{\text{ref}}$$

C_A is the axial force coefficient, C_N is the normal force coefficient, and C_m is the pitching moment coefficient, all given as functions of Mach number, angle of attack α and fin deflection δ. The variable s is the reference area and l_{ref} is the reference length. Mach number, dynamic pressure $\overline{q}$, total velocity V_M, and angle of attack α are defined as:

$$\text{Mach} = V_M/a,\ \overline{q} = \frac{1}{2}\rho\,V_M^2,\ V_M = \sqrt{u^2 + w^2},\ \alpha = \tan^{-1}[w/u]$$

A point mass model of the missile can be obtained by assuming that the missile pitch attitude dynamics is much faster than the translational dynamics. In this case, the missile can be considered to be in moment equilibrium at each flight condition, permitting the neglection of the missile pitch attitude dynamics. Next, the forces in the body frame may be resolved along the inertial frame, leading to the missile point-mass model in the inertial frame as:

$$\ddot{x}_M = \left(\frac{T - F_A}{m}\right)\cos\theta - \left(\frac{F_N + F_R}{m}\right)\sin\theta$$

$$\ddot{z}_M = -\left(\frac{T - F_A}{m}\right)\sin\theta - \left(\frac{F_N + F_R}{m}\right)\cos\theta + g$$

Target Model

A passive ground target and an actively maneuvering aircraft target model are used in the present research. For the air-to-air case, the target is assumed to be a high performance aircraft capable of maximum 9 g normal acceleration maneuvers. Point-mass model of the target aircraft is given by the following dynamic equations:

$$\dot{V}_T = \frac{T_T - D_T}{m_T} - g \sin\gamma_T$$

$$\dot{\gamma}_T = \frac{g}{V_T}\left(\frac{L_T}{m_T g} - \cos\gamma_T\right)$$

$$\dot{x}_T = V_T \cos\gamma_T$$

$$\dot{h}_T = V_T \sin\gamma_T$$

V_T is the aircraft speed, γ_T is the flight path angle, T_T is the aircraft thrust, D_T is the aircraft drag, L_T the lift, m_T its mass, and g is the acceleration due to gravity. x_T and h_T are the target position components in an earth fixed inertial frame. This model assumes that the aircraft thrust acts along its path, a flat non-rotating earth, and a quiescent atmosphere. Figure 3 shows the definition of some of the variables.
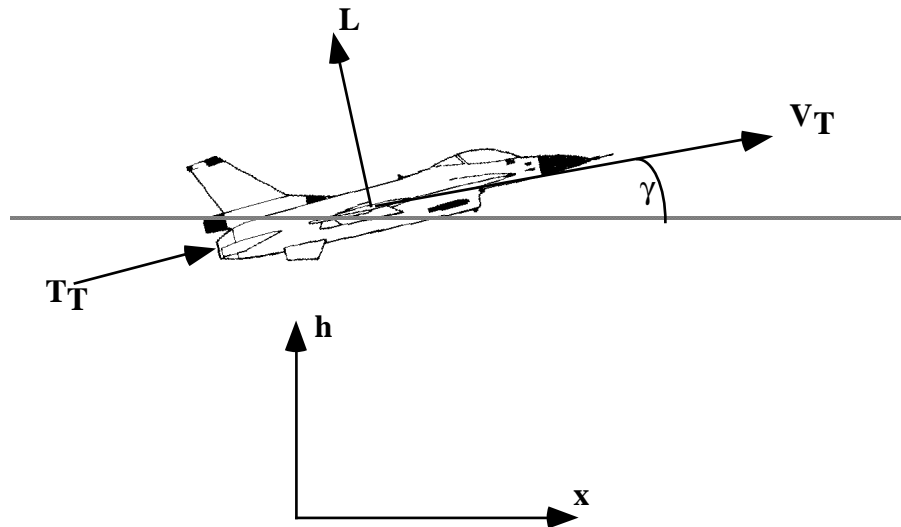

Fig. 3. Target Coordinate System

For the present analysis, the target aircraft will be assumed to fly with maximum engine thrust, while maneuvering using the lift L. Differentiating the down range and altitude equations

once with respect to time and substituting for the rate of change of airspeed, and flight path angle, the target equations of motion in the inertial frame can be as:

$$\ddot{x}_T = \left(\frac{T_T - D_T}{m_T}\right) \cos \gamma_T - \left(\frac{L_T}{m_T g}\right) g \sin \gamma_T$$

$$\ddot{h}_T = \left(\frac{T_T - D_T}{m_T}\right) \sin \gamma_T + \left(\frac{L_T}{m_T g}\right) g \cos \gamma_T - g$$

This model can be made consistent with the missile coordinate system defined in Figure 2 by defining $\ddot{z}_T = -\ddot{h}_T$. Moreover, in order to avoid having to deal with thrust and drag models, it is often convenient to model the target in terms of acceleration components along and perpendicular to the velocity vector. With this, the point-mass target model can be written in a concise form as:

$$\ddot{x}_T = a_{VT} \cos \gamma_T - a_{nT} \sin \gamma_T$$

$$\ddot{z}_T = -a_{VT} \sin \gamma_T - a_{nT} g \cos \gamma_T + g$$

During missile encounters, a high performance aircraft tends to use maximum thrust, while maneuvering using the acceleration normal to the velocity vector. Thus, it is reasonable to assume that the acceleration component a_{VT} remains constant during the engagement, while the pilot maneuvers using the normal acceleration component a_{nT}. In most manned high performance aircraft, the normal acceleration is constrained to stay within about 9 g's.

The missile and target models developed in the previous section and in this section will be used in the following to develop the missile guidance law.

Missile Guidance Law

Missile guidance law development presented in this paper is based on differential game theory [7]. If the target is capable of employing evasive maneuvers, differential game theory can generate optimal strategies for both missile and the target. It will be assumed in the guidance law derivation that the target is capable of evasive maneuvers. Restricting the results to the case of stationary or non-maneuvering target is straightforward.

Direct application of differential game theory to the nonlinear missile/target dynamics will produce an intractable two-point boundary-value problem, which will require powerful numerical algorithms for solution. An alternate approach is to employ transformations to convert the differential game into a more tractable form as in References 8 and 9. In References 8 and 9, the

nonlinear equations of motion for the flight vehicles are first transformed into a linear time-invariant form using a state feedback transformation. The state feedback transformation essentially changes the basis of the state space, enabling the equations of motion to be represented in a linear, time-invariant form. Differential game theory is then applied to the transformed models. The resulting guidance law is subsequently transformed back to the original coordinates to yield the nonlinear guidance law. The results in Reference 8 and 9 did not permit the direct imposition of hard control limits. The present research allows such control constraints, but requires more complex computations for on-board implementation.

The state transformation can be based on the performance objectives of the guidance law. Since the objective of the missile is to get as close as possible to the target at the end of the differential game, define a new variable:

$$\eta = \frac{1}{2}\left[a_1(x_T - x_M)^2 + a_2(z_T - z_M)^2\right]$$

To reiterate, the variables x_M, z_M are the missile position components in an inertial frame, and x_T, z_T are the target position components in the same inertial frame. The parameters a_1 and a_2 control the relative weight between the down-range and altitude errors. These weighting factors can be used to control the target intercept angle. Note that if the weighting factors a_1 and a_2 were unity, then η is half the square of the range to the target. This quantity will be positive at missile launch, and will be zero at the intercept. Starting from a positive value at launch, the objective of the missile guidance is to drive η to smallest possible value.

The variable η can also be considered to be the square of a *miss distance*. Differentiating the expression for η twice with respect to time, one has:

$$\dot\eta = a_1(x_T - x_M)\,(\dot x_T - \dot x_M) + a_2(z_T - z_M)\,(\dot z_T - \dot z_M)$$
$$\ddot\eta = a_1\,(\dot x_T - \dot x_M)^2 + a_2\,(\dot z_T - \dot z_M)^2 + a_1(x_T - x_M)\,(\ddot x_T - \ddot x_M) + a_2(z_T - z_M)\,(\ddot z_T - \ddot z_M)$$

The second expression can be written in a more succinct form as:

$$\ddot\eta = W - U$$

With,

$$U = a_1(x_T - x_M)\; \ddot{x}_M + a_2(z_T - z_M)\; \ddot{z}_M$$

$$W = a_1\; (\dot{x}_T - \dot{x}_M)^2 + a_2\; (\dot{z}_T - \dot{z}_M)^2 + a_1(x_T - x_M)\; \ddot{x}_T + a_2(z_T - z_M)\; \ddot{z}_T$$

U can be treated as the control variable that the missile uses to drive the miss distance towards zero, while W can be treated as the control variable that the target uses to increase the miss distance.

Substituting for the missile and target acceleration components in the inertial frame, one has:

$$U = \left[a_1(x_T - x_M)\; \cos\theta - a_2(z_T - z_M)\; \sin\theta\;\right]\left\{\frac{T - F_A}{m}\right\}$$

$$- \left[a_1(x_T - x_M)\; \sin\theta + a_2(z_T - z_M)\cos\theta\right]\left\{\frac{F_N + F_R}{m}\right\} + a_2(z_T - z_M)\; g$$

Similarly, the target model can be used to relate W to the target control variables, viz., the acceleration along and normal to the target velocity vector.

$$W = a_1\; (\dot{x}_T - \dot{x}_M)^2 + a_2\; (\dot{z}_T - \dot{z}_M)^2 + \; \left[a_1(x_T - x_M)\cos\gamma_T - a_2(z_T - z_M)\sin\gamma_T\;\right] a_{VT}$$

$$- \; \left[a_2(z_T - z_M)\cos\gamma_T + a_1(x_T - x_M)\sin\gamma_T\right] a_{nT} + a_2(z_T - z_M)\; g$$

The transformed model can next be used to design the missile guidance law.

Differential Game Based Guidance Law

It will be seen that the application of differential game theory to the guidance problem generates strategies for both the missile and the target. For this purpose, it is convenient to first define two new variables ζ_1, ζ_2 such that

$$\zeta_1 = \eta \; , \; \zeta_2 = \dot{\eta}$$

The equations of motion for the engagement can then be written as:

$$\dot{\zeta}_1 = \zeta_2, \; \dot{\zeta}_2 = W - U$$

Next consider a differential game in which the missile tries to drive ζ_1 to zero in minimum time. The target attempts to prevent this, failing which, it attempts to maximize the time of

capture. This differential game can be cast as a two-sided optimal control problem. Towards this end, formulate the variational Hamiltonian [10] as:

$$H = 1 + \lambda_1 \zeta_2 + \lambda_2 (W - U)$$

The necessary conditions for optimality can be obtained by proceeding formally according to optimal control theory. The costate equations are:

$$\dot{\lambda}_1 = 0, \ \dot{\lambda}_2 = -\lambda_1$$

The optimal strategy for the target and the missile can be selected based on the the bang-bang control principle [10]. Since the target is attempting to maximize the final time, and because the control variable W appears linearly in the Hamiltonian, one has:

$$W = W_{max}, \text{ if } \lambda_2 > 0$$
$$W = W_{min}, \text{ if } \lambda_2 < 0$$

The missile is attempting to minimize the intercept time, which requires the Hamiltonian to be minimized. Moreover, since U appears linearly in the Hamiltonian with a negative sign, the missile optimal control is given by

$$U = U_{max}, \text{ if } \lambda_2 > 0$$
$$U = U_{min}, \text{ if } \lambda_2 < 0$$

Next, the costate equations can be integrated to yield:

$$\lambda_1 = c_1, \ \lambda_2 = -c_1 t + c_2$$

t is the current time and c_1 and c_2 are arbitrary constants. Since there are no restrictions on the relative velocity ζ_2 at the final time t_f, one has that: $\lambda_2(t_f) = 0$. Using this fact, the costate equations can be rewritten as:

$$\lambda_1 = c_1, \ \lambda_2 = c_1(t_f - t)$$

Since the difference between the final time and the current time is always greater than or equal to zero, the sign of the arbitrary constant c_1 essentially determines the sign of the costate λ_2. The costate equations suggest that the sign of the costate λ_2 will not change over the duration of the game. Consequently, the direction of the control U for the missile and the target control W remain unchanged throughout the engagement. Thus, if the sign of the arbitrary constant c_1 is known at the initial time, the direction of the controls can be determined.

However, determining the sign of the arbitrary constant c_1 is not simple, because the corresponding value of the state ζ_1 at the final time is specified to be zero. Actual determination will require integrating the state equations forward in time, and enforcing the specified terminal conditions.

An alternate approach is to use the observation that the target is attempting to increase the range between itself and the missile right from the very first instant. Thus, one has that $W = W_{max}$ at the initial time. Since optimal control theory requires the control to remain in the same direction throughout the game, this implies that $\lambda_2 < 0$. As a result, the optimal controls for the missile and the target are given by:

$$W_{Opt} = W_{max}, \quad U_{Opt} = U_{max}$$

i. e. both the missile and the target must use the maximum value of controls throughout the game. It may be observed that from any arbitrary initial conditions, capture will eventually occur if one can guarantee that $U_{Opt} > W_{Opt}$ throughout the duration of the game.

From the foregoing, one may conclude that the guidance calculations essentially consist of determining the actual control settings for the target and the missile such that the resulting values of U and W are the optimal values. Various approximations that facilitate the computation of the optimal values will be discussed in the next subsection.

Finding the Maximum Values of the Control Variables

The foregoing analysis reduced the guidance problem to that of finding the maximum values of U and W at each guidance interval. Approximations for both the target and the missile will be separately discussed in the following.

Target

The optimal value of target control variable to be used in the differential game can be found by examining the expression for W.

$$W = a_1 (\dot{x}_T - \dot{x}_M)^2 + a_2 (\dot{z}_T - \dot{z}_M)^2 + [a_1(x_T - x_M) \cos \gamma_T - a_2(z_T - z_M) \sin \gamma_T] \, a_{VT}$$
$$- [a_2(z_T - z_M) \cos \gamma_T + a_1(x_T - x_M) \sin \gamma_T] \, a_{nT} + a_2(z_T - z_M) \, g$$

Since the target is assumed to be flying with full throttle, the target's control variable is its acceleration component normal to the flight path a_{nT}. In most flight vehicles, the normal acceleration is constrained satisfy a specified limit: $|a_{nT}| \, \text{S} \, a_{max}$. Since the normal acceleration appears linearly in the expression for W, the control strategy for the target can be found to be:

$$\text{If } [a_2(z_T - z_M) \cos \gamma_T + a_1(x_T - x_M) \sin \gamma_T] < 0, \quad a_{nT} = a_{max}$$
$$\text{If } [a_2(z_T - z_M) \cos \gamma_T + a_1(x_T - x_M) \sin \gamma_T] > 0, \quad a_{nT} = -a_{max}$$

This control strategy will result in maximizing the miss distance between the two vehicles. Any deviation from this strategy will cause the target to be intercepted sooner than the optimal time.

Missile

The expression for U can be used to determine the missile angle of attack corresponding to the optimum value of the missile control variable. However, the development is not as simple as that of the target, because a more complex model of the missile is used in the analysis. The missile pseudo-control variable U is given by:

$$U = [a_1(x_T - x_M) \cos \theta - a_2(z_T - z_M) \sin \theta] \left(\frac{T - F_A}{m} \right)$$
$$- [a_1(x_T - x_M) \sin \theta + a_2(z_T - z_M) \cos \theta] \left(\frac{F_N + F_R}{m} \right) + a_2(z_T - z_M) \, g$$

It is important to note that the missile pitch attitude θ is a combination of the missile angle of attack and the flight path angle, see Figure 2. for details. The flight path angle depends on the missile velocity components. Since the aerodynamic forces on the airframe depend on the missile

angle of attack, it is convenient to eliminate the missile pitch attitude from the expression for U. In this case, U is given by:

$$U = [a_1(x_T - x_M) \cos(\alpha + \gamma) - a_2(z_T - z_M) \sin(\alpha + \gamma)] \left(\frac{T - F_A}{m}\right)$$
$$- [a_1(x_T - x_M) \sin(\alpha + \gamma) + a_2(z_T - z_M) \cos(\alpha + \gamma)] \left(\frac{F_N + F_R}{m}\right) + a_2(z_T - z_M) g$$

with $\gamma = \tan^{-1}(-\dot{z}_M/\dot{x}_M)$.

The aerodynamic forces F_A and F_N depend on the missile angle of attack, fin deflection and reaction jet thrust. Since the fin deflection and the reaction jets are used primarily for controlling the missile attitude, one may assume that these variables do not significantly influence U for the purposes of computing the guidance commands. A more refined approach may first compute the fin deflection and the reaction jet thrust required to maintain the net pitching moment at zero, and then use these values in the expression for U. In the interests of simplifying the computations, the former approach will be followed here. Thus, in the present work, the expression for the pseudo-control variable U will be optimized by assuming that the normal forces generated by the reaction jets and fin deflections are small.

Since the missile longitudinal acceleration is largely governed by the engine thrust, one may assume that it is relatively unaffected by the angle of attack. In this case, assuming that the missile longitudinal acceleration measurements are available, one may write:

$$U = [a_1(x_T - x_M) \cos(\alpha + \gamma) - a_2(z_T - z_M) \sin(\alpha + \gamma)] a_{xM}$$
$$- [a_1(x_T - x_M) \sin(\alpha + \gamma) + a_2(z_T - z_M) \cos(\alpha + \gamma)] \left(\frac{C_N(\alpha) \, \overline{q} \, s}{m}\right) + a_2(z_T - z_M) g$$

$C_N(\alpha)$ is the normal force coefficient, $\overline{q}$ is the dynamic pressure, and s is the reference area. The problem of finding U_{Opt} can be expressed in a succinct form as:

$$U_{Opt} = a_2(z_T - z_M) g + \max_{|\alpha| \le \alpha_{max}} \left\{ [a_1(x_T - x_M) \cos(\alpha + \gamma) - a_2(z_T - z_M) \sin(\alpha + \gamma)] a_{xM} \right.$$
$$\left. - [a_1(x_T - x_M) \sin(\alpha + \gamma) + a_2(z_T - z_M) \cos(\alpha + \gamma)] \left(\frac{C_N(\alpha) \, \overline{q} \, s}{m}\right) \right\}$$

The value of α that maximizes this expression will result in target intercept. Since α appears nonlinearly in this expression, a numerical one-dimensional search has to be employed to determine the optimum angle of attack. Numerically determined angle of attack can then be used to compute the normal acceleration command the autopilot. This completes the development of the guidance law.

It needs to be pointed out here that although game theoretic guidance laws have been previously discussed [11 - 13], they have all been based on either linearized missile models or on low-order model approximations. The approach advanced in this paper represents the first attempt at synthesizing a nonlinear game-theoretic intercept guidance law employing full-order missile model.

Guidance Law Evaluation

The intercept guidance law performance has been evaluated in a nonlinear rigid body simulation of a hypothetical missile, together with an autopilot for acceleration command tracking. Simulation results for two different engagements will be discussed in the following.

Interception of Non-Maneuvering Surface Targets

The first mission scenario considered is that of an aircraft launching the missile while climbing away from the target at about 45 degrees flight path angle. At the launch instant, the aircraft is at an altitude of 1500 meters. Such an engagement scenario can arise while approaching the target using a stealth aircraft and launching the missile as the aircraft is moving away from the threat area.

Immediately after the launch, the missile turns at the maximum permissible angle of attack, while accelerating towards the target. Note that no coasting phase was introduced immediately after release from the launch aircraft. Such an initial coast phase will be necessary in practical situations to ensure safe separation between the missile and the launch aircraft. Figure 4 shows a family of missile trajectories, with the launch aircraft flying away from the target at a flight path angle of 45 degrees. The target conditions represent intercept in front and behind the aircraft down range position at launch. It can be observed that in every case, the intercept trajectories are characterized by a tight turn, followed by a relatively straight flight towards the target.

The trajectory marked 'A' in Figure 4 is interesting, and somewhat unusual. The trajectory behavior can be explained as follows. As presently configured, the guidance law drives the missile towards the target while covering the least distance. The relative position of the target

with respect to the missile at launch determines the direction in which the trajectory will evolve. Thus, the trajectory 'A' represents the shortest path to the target. In reality, introduction of a coast phase right after launch from the aircraft may depress the flight angle at ignition sufficiently to prevent trajectories such as 'A' from occurring in real situations.

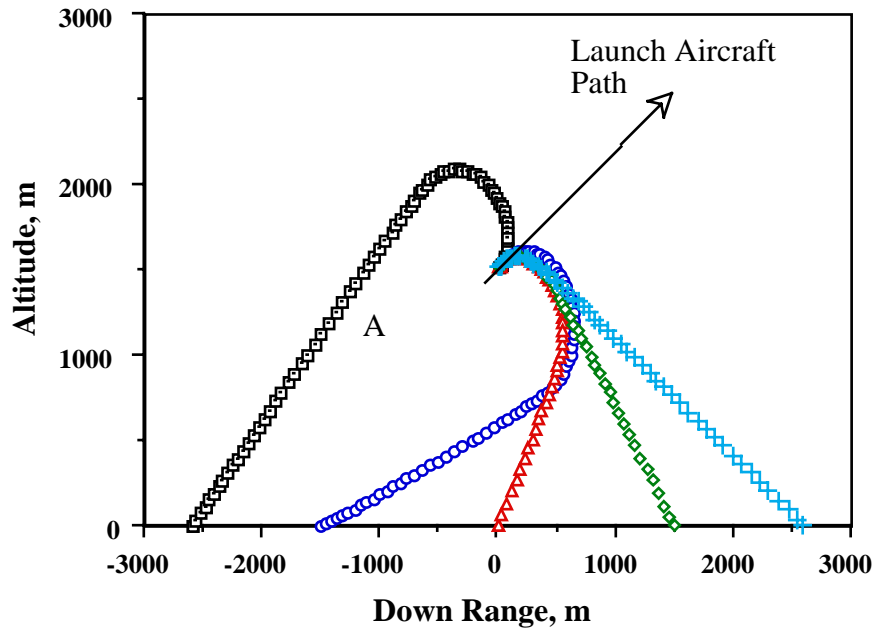The instantaneous missile range to the target for each air-to-surface missile trajectory is shown in Figure 6.

Fig. 4. A Family of Trajectories for the Air-to-Surface Mission
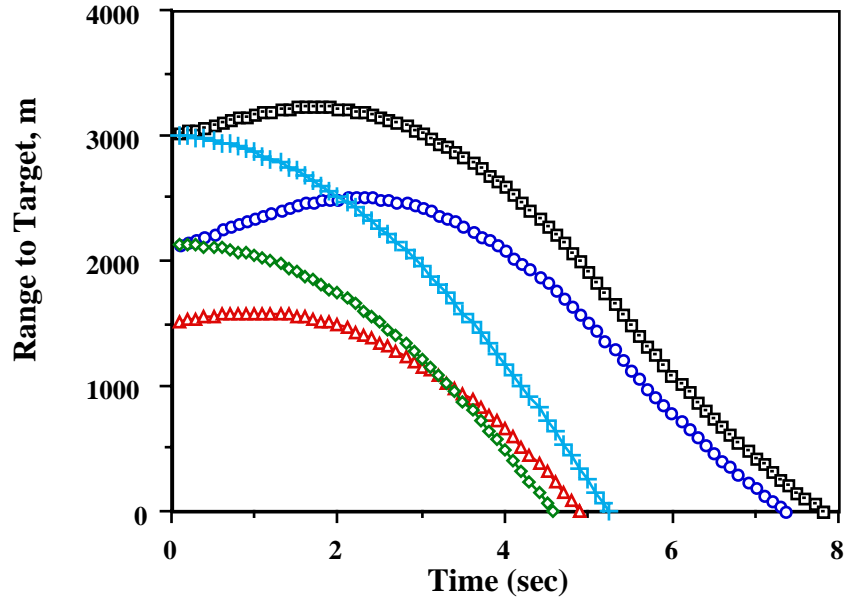
**Fig. 5. Range to Target for the Air-to-Surface
Missile Trajectories given in Fig. 4.**

Flight path angle histories along the air-to-surface trajectories given in Figure 7 illustrate the fact that the intercept trajectories consist of an initial turn at the highest possible rate, followed by near straight-line trajectories.
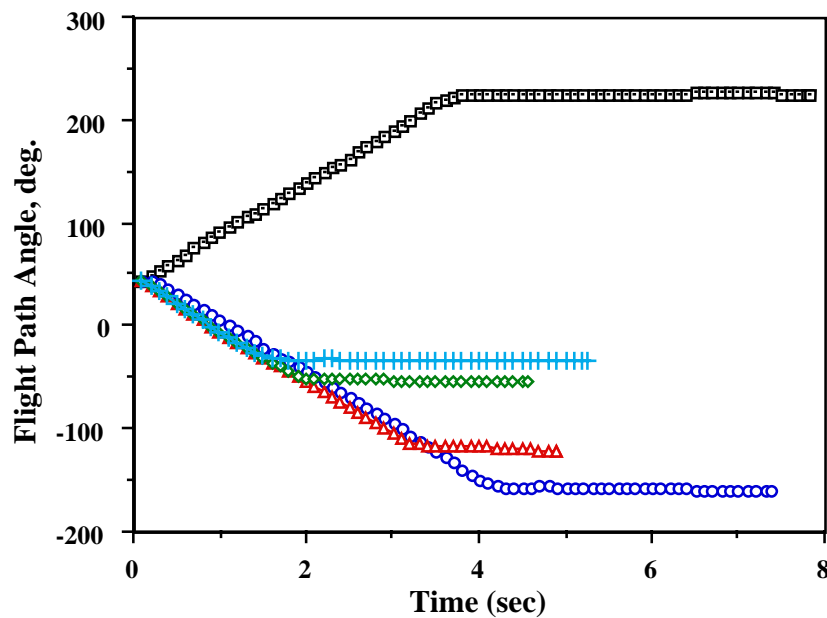

**Fig. 7. Flight Path Angle Histories for
Air-to-Surface Missile Trajectories given in Fig.4.**

Interception of Maneuvering Flying Targets

The interception of an actively maneuvering target next presented to illustrate the performance of the guidance law. The first engagement scenario consists of the launch aircraft in level flight at an altitude of 6000 meters, heading towards a target flying level at 5200 meters. The aircraft is assumed to be flying at 0.55 Mach. The missile is launched when the target is directly below the aircraft. The target is assumed to be capable of maneuvering with 9 g normal acceleration. The target airspeed at missile launch is 132 m/s. As soon as the target detects the missile launch, it maneuvers in an attempt to increase the distance between itself and the missile. The superior acceleration capabilities of the missile defeat this evasive tactic, and the target is intercepted at about 4.25 seconds after the launch. Note that the evasive maneuver employed by the target in the present research is completely dependent on the chosen performance index for the differential game. For instance, the target may employ a different evasive maneuver if the performance index is changed from flight time to terminal energy.

Trajectories of the launch aircraft, target, and the missile are shown in Figure 7. As in the air-to-surface engagements, the missile initially flies at a high angle of attack, and subsequently maintains moderate angles of attack until intercept.
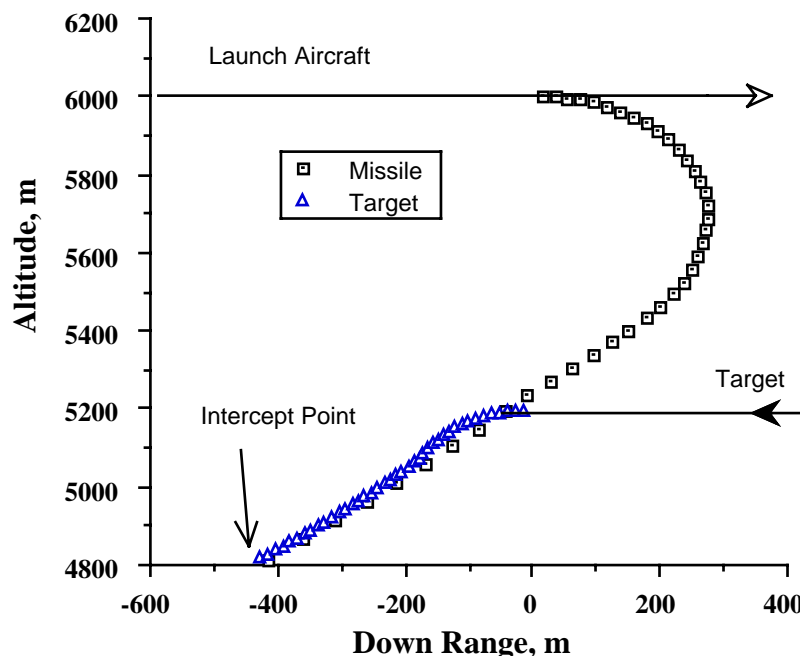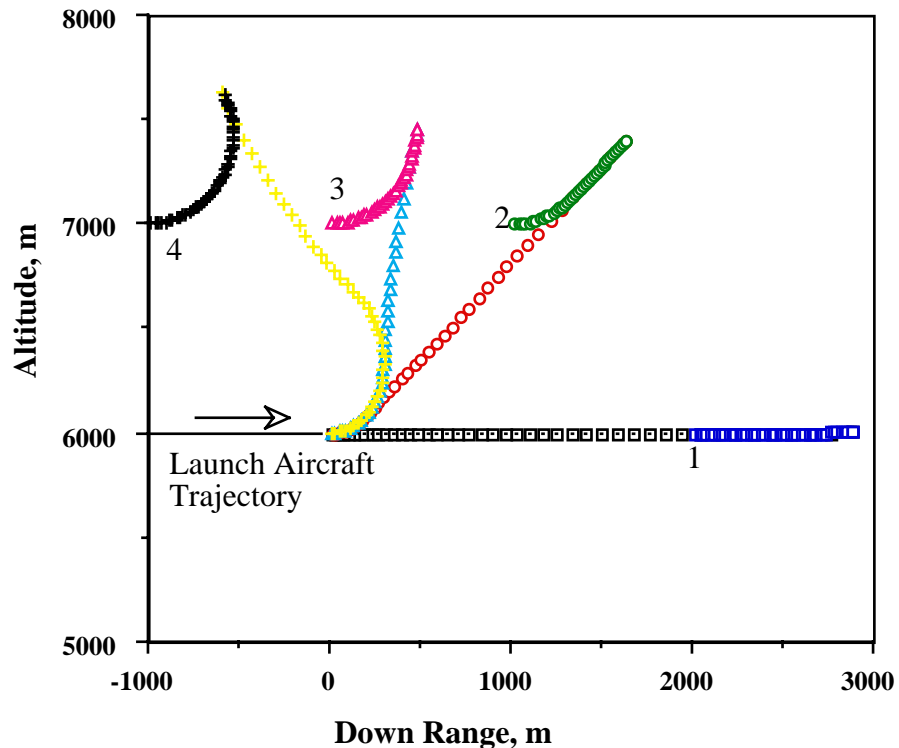

Fig. 7. Interception of an Actively Maneuvering Target

Several more intercept trajectories against maneuvering targets are given in Figure 8. In each case shown in Figure 8, the target is initially flying level with a speed of 200 m/s, and has a 9 g normal acceleration capability. Target position at missile launch for each case is identified by a numeral in Figure 8.

Fig. 8. Air-to-Air Missile Trajectories Against

Maneuvering Targets

(1, 2, 3, 4: Target at Missile Launch)

The behavior of the targets in each scenario given in Figure 8 can be explained by recalling the optimality criterion employed in the derivation of the target evasion guidance law. Since the optimality criterion for the target is to maximize the time of capture, at each instant, the target chooses the sign of its normal acceleration to drive the intercept time as high as possible.

Flight path angle histories for the air-to-air missile engagements are given in Figure 5.22. Once again, high angle of attack initial maneuvering, followed by relatively gentle trajectories are clearly observable. Finally, missile pitch attitude histories along the engagement are given in Figure 5.23.
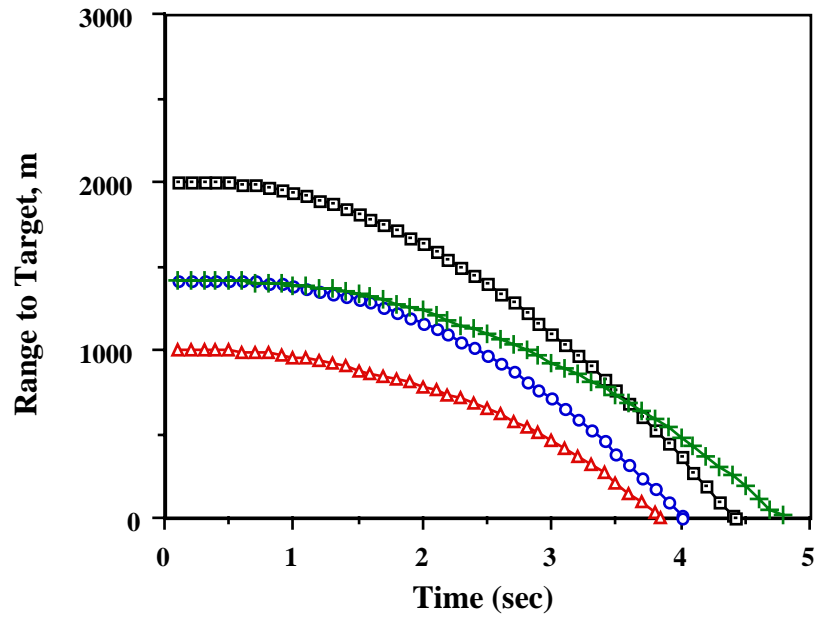
Fig. 5.21. Range-to-Target as a Function of Time

for the Air-to-Air Missile Trajectories given in Figure 5.18.
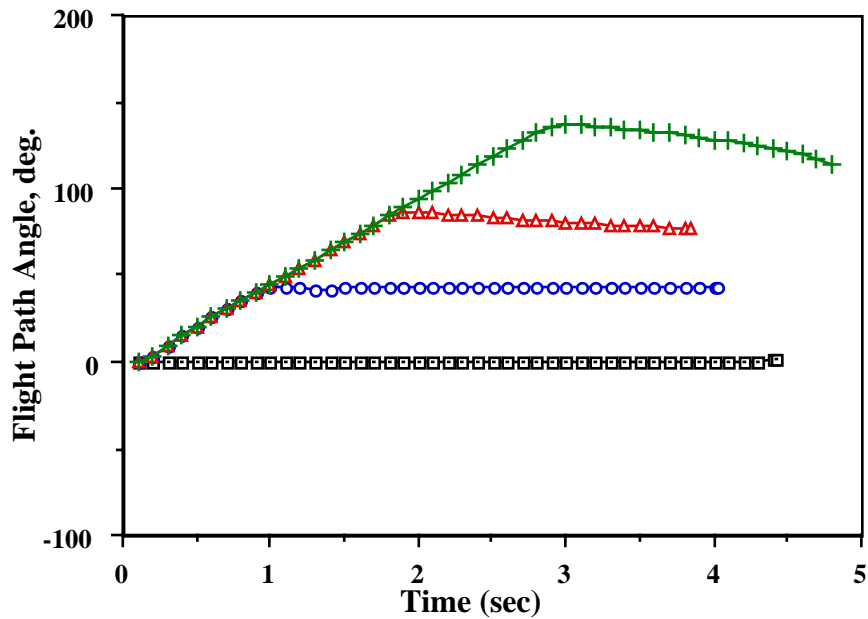

Fig. 5.22. Evolution of Flight Path Angle Along the Air-to-Air Missile Trajectories given in

Figure 5.18.

The numerical results presented in this section amply demonstrate the capabilities of the guidance law in various engagement scenarios. The methods developed in this paper can be extended to deal with more complex missile models.

Conclusions

This paper presented the development of a novel guidance law using feedback linearization and differential game theory. The missile model was first transformed into a linear time invariant form. In the case of a maneuvering target, the target model was also similarly transformed. Using a performance index involving the mini-maximization of the intercept time, differential game theory was applied to synthesize the guidance law. Adjustable weights in the guidance law can be used to fix the missile flight path angle at target intercept

The guidance law was integrated with an autopilot in a nonlinear missile simulation and evaluated against various targets. Several air-to-surface engagements involving launch against targets in front and behind the aircraft were studied. In every case, the missile accomplishes target intercept, with over 210 degrees flight path angle changes in certain scenarios. Although the emphasis of the study was on air-to-surface missions, a few air-to-air engagement scenarios were also examined. For air-to-air engagements, the target was assumed to be capable of 9 g evasive maneuvers. Successful interceptions under various engagement geometries were demonstrated.

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