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14. ABSTRACT

This project researched the development of a systematic feedback design methodology for shaping the steady-state response of complex systems. The design issues comprising this task include the stabilization of unstable systems and the design of feedback laws enabling a system to asymptotically track a signal and to asymptotically reject unwanted disturbances. The model used for complex systems are lumped nonlinear systems as well as linear and nonlinear distributed parameter systems

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EXECUTIVE SUMMARY

NONLINEAR CONTROL SYSTEMS

AFOSR #-F49620-01-10039

December 1, 2000-November 30, 2003

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Contents

1 Accomplishments/New Findings

The first three research tasks explicitly stated in our proposal focused on stabilizing a nonlinear system using only functions of the system output. Our research accomplishments on this set of tasks also considered practical stabilization of a system, whereby it is meant that a system can be stabilized to within an arbitrarily small neighborhood of the desired equilibrium. It is well known in the prior literature that semiglobal practical stability of a relative degree r, minimum-phase, nonlinear system can be achieved by feeding back (through sufficiently high gains) the output of the system and its derivatives up to order $r-1$. If, moreover, standard gain adaptation schemes are used, then global practical stability can be obtained.

If, however, only the output is available for measurement, then an (partial state) observer is needed, for the purpose of estimating the derivatives of the output. In the presence of modelling uncertainties, coarse estimates are provided by the so-called Khalil's observer. It is well-known, though, that if this observer is used, the subset of the state space (of the closed-loop system) in which the estimation error is zero is not an invariant subset, as opposed to the classical situation occurring when a true observer is used. Nevertheless, semiglobal practical stabilization can still be achieved and this implies, in particular, that the estimation error induced by the coarse observer converges to an arbitrarily small neighborhood of the origin.

In our investigation [21] we have analyzed and classified (in a number of simple cases), the structure of the limit set, contained in a fixed arbitrarily small neighborhood of the origin, to which the trajectories of the closed-loop system are steered by means of this type of dynamic feedback.

This investigation has also demonstrated that the use of a coarse estimator for the output and its derivatives up to order $r-1$ does not alter the structure of the limit set (contained in the small neighborhood of the origin) to which trajectories would be forced by means of high-gain partial state feedback. The "conservation" of the structure of the limit set is surprising to a certain extent, since the output feedback law is based on an approximate observer that does not guarantee the invariance of the subspace on which the observation error is zero. Nevertheless, we have determined that the use of the coarse estimator does not induce any additional "dimension" in the limit set. This provides a new insight into these high-gain output stabilization schemes and reveals an interesting property of Khalil's observer. In particular, if partial-state feedback is able to asymptotically stabilize (exponentially or even critically) the origin, then the output feedback which uses only a course estimation of the partial state yields the same result.

The next five research tasks in our proposal focused on output regulation of nonlinear systems, which is classically defined as the design of feedback laws that stabilize a system and enable it to asymptotically track and reject disturbances generated by an exogenous signal generator.

To this end, we have studied the design of an internal model-based semiglobal output feedback regulator for nonminimum phase nonlinear systems. By taking advantage of the design tool proposed in [23], we have shown how the problem of designing an internal model-based regulator can be approached by seeking an output feedback stabilizer of a suitably-defined extended "auxiliary" subsystem. The approach in question is not restrictive, when specialized to linear systems, because in this case stabilizability by output feedback of the extended auxiliary subsystem is a necessary condition for the output regulation problem to be solved. For nonlinear systems, the problem of designing an output feedback stabilizer for the extended auxiliary system has been approached by seeking a high-gain observer-based controller, as described above. The existence of this high-gain observer has been characterized in terms of necessary conditions which have been shown, when specialized for linear systems, to coincide with the standard non-resonance conditions between the modes of the exosystem and the zero dynamics of the controlled plant. Sufficient conditions for globally transforming the extended auxiliary subsystem into the observability normal form have also been given.

In the paper [19], we have laid the foundations for a non-equilibrium theory of nonlinear output regulation, giving a more general (non-equilibrium) definition of the problem, deriving necessary conditions, and, using these necessary conditions, we have presented a set of sufficient conditions and a design methodology for the solution of the problem in question. Our analysis leads to a non-equilibrium enhancement of the internal model principle, which can be expressed as a relationships between two uniformly stable attractors. The first is an attractor for the combined dynamics of the exogenous signal generator and the so-called zero-dynamics of the plant to be controlled, intrinsic to the formulation of the problem. The second is the uniformly stable attractor for the dynamics of the closed-loop system determined by the controller which solves the problem of output regulation, under hypotheses which are non-equilibrium enhancements of those familiar from the equilibrium case. This enhancement of the internal model principle asserts, roughly speaking, that any controller solving the problem of output regulation has to contain a copy of an attractor which may combine the dynamics of the exogenous system with certain nontrivial steady-state motions occurring in the plant to be controlled. In the simple case steady-state motions consist of just one equilibrium, and the analysis is only local, the theory we develop reduces to the one presented in [24]. On the other hand, the more general approach discussed here makes it possible to solve problems to which none of the existing design methods for output regulation are applicable.

As a first step in the development of systematic methods for regulation in the presence of uncertainties, we have extended the non-equilibrium approach of [19] to the case in which the exosystem is modelled by nonlinear differential equations. This is the case, in fact, when uncertain parameters affect the exosystem and the controlled plant. The results of this work are summarized in [20]. Generally speaking, the problem of output regulation is to have the regulated variables of a given controlled plant to asymptotically track (or reject) all desired trajectories (or disturbances) generated by some fixed autonomous system, known as the exosystem. The hypotheses assumed in [19] for the design of output regulators no longer include the assumption, common to all earlier literature, that the zero-dynamics of the controlled plant have a globally asymptotically stable equilibrium. Rather, this assumption is replaced with the (substantially weaker) hypothesis that the zero dynamics of the plant "augmented by the exsosystem," have a compact attractor. In [19], though, we have retained the (rather strong) assumption, typical of all earlier literature, that the set of all "feedforward inputs capable of securing perfect tracking" is a subset of the set of solutions of a suitable *linear* differential equation. In the work [20], we have shown how, within the new framework, the assumption of linearity can also be dropped.

A major theoretical issue in the design of feedback laws for robust nonlinear output regulation is the ability to robustly stabilize systems which can be interpreted as nonlinear systems whose zero dynamics possesses a nontrivial compact attractor. In this case, in fact, it is possible in most instances to reduce (asymptotically) the problem of output regulation in the presence of an exogenous input to the case where these signals are generated by a Poisson stable exosystem. In our work, we have developed the necessary theoretical background needed to prove that high-gain output feedback can be used to drive the trajectories of the closed loop to a compact attractor and, at the same time, the regulated variable to zero. This theory reposes on the enhancement of the small-gain theorem to the case in which one of the component systems does not possess a globally asymptotically stable equilibrium, but rather a Lyapunov stable compact attractor. Our result [22] reposes on the construction of suitable (not necessarily smooth) Lyapunov functions for compact attractors and shows that if the attractor of the zero dynamics is globally asymptotically and locally exponentially stable, then high-gain output feedback yields semiglobal asymptotic stability. Otherwise (i.e. if the attractor in question is just asymptotically stable) semiglobal practical stability can be obtained.

In the classical design of linear observers, asymptotic proxies for state variables are developed as outputs of a dynamical system operating in real time. In the stochastic case, this is also the principle feature of the Kalman filter. As part of our design of internal models, we also discovered that the development of real-time asymptotic proxies for state variables played a crucial role. In order to better understand this phenomenon, about a decade ago we analyzed the dynamical behavior of a fast form of Kalman filtering. This analysis led to a serendipitous discovery, viz. the solution of the classical rational covariance extension problem (see, e.g., the survey article [26]). This also solves a longstanding problem in speech processing that has led to the issuance of two patents, both cited in Section 7 of this report. As it turns out, the rational covariance extension problem is a special case of the Nevanlinna-Pick interpolation problem, which can be approached using similar methods. Since this has applications to circuits, signals and robust control, this important problem was the focus of the next six proposed research tasks.

In [29, 30] (also see [26]) we reformulated the Nevanlinna-Pick interpolation problem in terms of generalized moment problems, a setting that naturally accommodates the case with multiple interpolation points. In fact, interpolation conditions involving derivatives can be reformulated as generalized moment conditions where the corresponding basis function has been replaced with its derivative of appropriate order.

More precisely, in [29, 30] we derived a universal solution to the generalized moment problem, with a nonclassical complexity constraint, obtained by minimizing a strictly convex nonlinear functional. This optimization problem has been derived in two different ways. We have answered the question of why, intrinsically, there should always be an equivalent convex optimization problem. We have settled this question in a geometric way by path integration of a one-form which defines the generalized moment problem. We have shown that this one-form is closed and defined on a convex set, and thus is exact. Since its integral is therefore path-independent, it is intrinsic and turned out to be a strictly convex functional. We have also given a new derivation of this convex functional as the dual problem of a problem to maximize a cross entropy functional. In particular, these approaches give a constructive parameterization of all solutions to the Nevanlinna-Pick interpolation problem, with possible higher-order interpolation at certain points in the complex plane, with a degree constraint. In this regard, also see [5].

In [31] we study the generalized moment problem with complexity constraints in the case where the actual values of the moments are uncertain. In particular, we give an intrinsic geometric derivation of the Legendre transform and use it to describe convexity properties of the solution to the generalized moment problems as the moments vary over an arbitrary compact convex set of possible values.

In a well-known paper, Sarason generalized some classical interpolation problems for H^∞ functions on the unit disc to problems concerning operators on a coinvariant subspace $K = H^2 \ominus \phi H^2$ where ϕ is an inner function. These operators have norm not greater than one, and, among his results, he studied the structure of generalized interpolants for operators having norm one. In a variety of interesting cases, there is a unique such interpolant, which is given by the quotient of functions in K. In [32] we study the case where the operator is a strict contraction. There turns out to be an infinite number of interpolants that are such quotients, and we give a complete parameterization of these.

Our methodology is inspired by the engineering applications of classical interpolation problems in circuits, systems and signal processing, cases which all deal with the situation where ϕ is a finite Blaschke product and in which the quotient representation is physically natural. These are the problems we study in [25, 26]. We generalized this to the case of arbitrary inner functions by first constructing on a certain set a differential form which is exact (in an appropriate sense) and which gives rise intrincically to a convex optimization problem. Indeed, our method of proof reposes on a rigorous treatment of nonlinear optimization on certain (nonreflexive) Banach spaces. An example is given in [32] that suggests how this can be generalized to accommodate delay-differential systems.

An important problem in robust control is to develop systematic rules for selecting the parameters in the sensitivity function design so as to obtain low sensitivity in a designated part of the spectrum. A first step to address this problem was taken in [40]. In [37] a method for shaping the frequency response of a closed-loop system, based on the theory of Nevanlinna-Pick interpolation with a degree bound, is presented. It turns out that the spectral zeros of a certain function related to the closed-loop transfer function serve as design parameters. If necessary, some additional interpolation constraints can also be employed to increase the design flexibility. The main difference between this method and the existing H^∞ controller design methods is that we do not use the weighting functions to shape the frequency response of the sensitivity function. Instead, we tuned the spectral zeros of a positive real function related to the sensitivity function to obtain a desirable frequency response.

In [37] and [5] a robust algoritm is developed for solving the convex optimization problem in our theory of Nevanlinna-Pick interpolation with degree constraint. This algorithm, which is based on homotopy continuation with predictor-corrector steps, turns out to be quite efficient and numerically robust and avoids spectral factorization. The ill-conditoning intrinsic in the previous solvers is therefore avoided.

In particular, the problem of sensitivity optimization requiring rational Nevanlinna-Pick interpolation for multiple interpolation points which was studied in [5] and more generally in [4]. To solve the corresponding convex optimization problem, a homotopy continuation technique was used. These results were applied to benchmark problems in robust control. By constructing a controller satisfying all design specifications but having only half the McMillan degree of conventional H^∞ controllers, the advantage of the proposed method was also demonstrated. In [42] a certain shaping limitation of sensitivity functions was considered. The focus was on a frequency-wise infimum of gains of rational sensitivity functions with a degree constraint. An explicit infimum was derived for a special case. The result is useful for determining the inability of sensitivity functions of low degrees to achieve a specification in the frequency domain, and thus for motivating the use of higher degree sensitivity functions to fulfill the specification.

In [4] we took the first step in generalizing the theory of analytic interpolation theory with complexity constraint to the multivariable case. We parameterized a class of interpolants consisting of "most interpolants" of no higher degree than the central solution in terms of spectral zeros, and for each such interpolant we provided a convex optimization problem for determining it. We devised a numerically stable algorithm based on homotopy continuation to compute the interpolants. The potential advantage of the theory and the algorithm was illustrated by a benchmark multivariable control example: we constructed a controller satisfying all design specifications but having less than half the McMillan degree of conventional H^∞ controllers.

Finally, we considered a problem in spectral estimation and signal processing. Given the generalized covariance data Σ of a stationary stochastic process and an initial estimate Ψ for its spectral density, which may be inconsistent with the data, in [34] we formulated and solved the approximation problem of determining a closest approximant to Ψ in the sense of Kullback-Leibler which is also consistent with the data. In particular, we have shown that the minimizing function is unique.

This problem is relevant when statistics is specified in the form of a state covariance of a linear system driven by the unknown process. This is a rather general situation which, in particular, encompasses spectral analysis in linear arrays with ordinary partial autocorrelation function, as well as spectral analysis using filter banks. The basic techniques that we have developed in [34] should carry over to the case of a vector valued stochastic processes, where the distance measure is replaced by the matricial Kullback-Leibler-von Neumann generalization.

Our next seven tasks focused on the formulation of a rigorous theory of output regulation via state feedback for linear distributed parameter systems. The last three of these tasks deal with the special case of delay differential systems and our research progress on these have been published in [13], [15] and [11]. In the general case, we proposed a version of regulator theory similar to that previously developed by us in a more restricted case in [9] and to investigate solvability of the corresponding regulator equations.

There are numerous technical obstacles that had to be overcome en route to carrying out these tasks. For example, for unbounded B, even if A generates an analytic semigroup it may happen that $(A + B)$ is not such a generator. Further, for unbounded B and C (and even possibly K) expressions such as CB or BKC may make no sense. On the other hand there is considerable interest in the case of unbounded inputs and outputs that arise, for example, in the study of boundary control systems governed by partial differential equations. Typical applications include actuators and sensors supported at isolated points or on lower dimensional hypersurfaces in, or on the boundary of, a spatial domain.

Matters being so, after a considerable effort our work focused on extending our geometric approach to the class of regular linear systems ([45], [46], [47]) and our efforts have resulted in a work still in preprint form [16] which is under revision. Because this work has not yet appeared, we will summarize some of the key technical details.

The practical application of such a program relies on the existence of effective ways of knowing that a systems is indeed a regular linear system. The problem of deciding whether or not a system is a regular system, as it turns out, was not well understood in the literature. Our earliest successes in determining large classes of regular linear systems is reported in the works [18], [10].

A system is called regular provided the system is *Well Posed* and satisfies the *Regularity Condition.* In more detail, we consider regular linear systems

$$\begin{aligned} \dot{z} &= Az + Bu \\ y &= C_\Lambda z + Du \end{aligned}, \tag{1}$$

where C_Λ is the Λ-extension of the observation operator C (see [?])) defined for $z \in \mathcal{D}(C_\Lambda)$ by

$$C_\Lambda z \equiv \lim_{\lambda \to +\infty} C\lambda(\lambda I - A)^{-1} z \text{ exists.}$$

1. *Well Posedness*: A system (1) is well posed provided that B and C are *Admissible*, and there exists a *Transfer Function* $G(s) = C_\Lambda(sI - A)^{-1}B$ for some (hence, for every) $s \in \rho(A)$ (this means that

$$(sI - A)^{-1}BU \subset \mathcal{D}(C_\Lambda).$$

(For the definition and use of admissibility, we refer to the literature [45, 46].)

2. *Regularity*: A well posed system is called regular provided there exists a feed-through term $D \in \mathcal{L}(U, V)$, such that

$$\lim_{\substack{s \to +\infty \\ s \in \mathbb{R}}} G(s)\varphi = D\varphi, \quad \forall \ \varphi \in U$$

Let us define the space

$$Z_1 = \mathcal{D}(A) \subset Z \quad \text{with} \quad \|z\|_1 = \|(\beta I - A)z\| \ , \quad \beta \in \rho(A),$$

and the space

$$Z_{-1} \quad \text{the completion of} \quad Z \quad \text{with respect to} \quad \|z\|_{-1} = \|(\beta I - A)^{-1}z\|.$$

Then there are the dense embeddings

$$Z_1 \hookrightarrow Z \hookrightarrow Z_{-1}.$$

Assumption 1. *1. We assume that $B \in \mathcal{L}(U, Z_{-1})$, $C \in \mathcal{L}(Z_1, Y)$ are admissible*

2. $G(s) = C_\Lambda(sI - A)^{-1}B$ exists for $s \in \rho(A)$.

3. (A, B) stabilizable: There exists $K \in \mathcal{L}(Z_1, U)$ so that $(A + K_\Lambda B)$ is a stable generator.

Within this setting we have extended the solvability results for both the state and error feedback regulator problems. For example, we consider the case when full state measurements are available.

Problem 1. ***State Feedback Regulator Problem for Regular Systems:***

Find a feedback control law in the form

$$u(t) = K_\Lambda z(t) + Lw(t)$$

such that $K \in \mathcal{L}(Z,U)$, $L \in \mathcal{L}(W,U)$ *and*

(1.a) *the system* $\dot{z}(t) = (A+BK_\Lambda)z(t)$ *is stable, i.e.* $(A+BK_\Lambda)$ *is the infinitesimal generator of an exponentially stable* C_0 *semigroup, and*

(1.b) *for the closed loop system*

$$\dot{z}(t) = (A + BK_\Lambda)z(t) + (BL + P)w(t), \qquad (2)$$
$$\dot{w}(t) = Sw(t),$$

the error

$$e(t) = C_\Lambda z(t) - Qw(t) \in L^2_\alpha(0,\infty)$$

where for some $\alpha < 0$

$$L^2_\alpha(0,\infty) = \left\{ \phi \,\middle|\, \int_0^\infty |\phi(t)|^2 e^{-\alpha t}\, dt < \infty \right\}.$$

For this problem, in [16] we prove the following result.

Theorem 1. *Under the above assumptions, the state feedback regulator problem is solvable if and only if there exist mappings* $\Pi \in \mathcal{L}(W, \underset{\sim}{Z} \subset Z)$ *and* $\Gamma \in \mathcal{L}(W,U)$ *satisfying the "Regulator Equations"*

$$\Pi S = A\Pi + B\Gamma + P$$
$$C_\Lambda \Pi - Q = 0$$

Here the space $\underset{\sim}{Z}$ *is given by*

$$\underset{\sim}{Z} = \mathcal{D}(A) + (\lambda I - A)^{-1}PW + (\lambda I - A)^{-1}BU, \quad \text{for} \quad \lambda \in \rho(A).$$

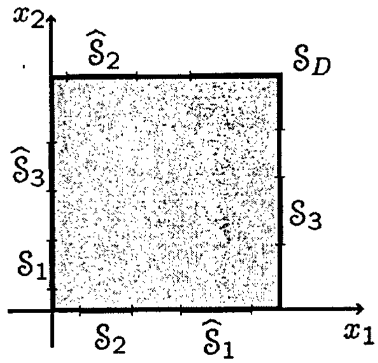
If Π *and* Γ *satisfy the regulator equations then a feedback law solving the problem of output regulation is given by* $u = K_\Lambda z + (\Gamma - K_L\Pi)w$.

A natural question that arises for boundary control systems in higher spatial dimensions, where the boundary control can also be infinite dimensional – Can the geometric theory of output regulation accommodate infinite dimensional input and output spaces. Indeed, what effect on the development results from an infinite dimensional exosystem generating signals to be tracked or disturbances to be rejected.

Examples are readily obtained. For example, the cancellation of acoustic signals would clearly require rejecting a disturbance produced by an infinite dimensional exosystem, such as a wave equation which would generate a signal with an infinite number of harmonics having known (natural) frequencies but unknown amplitudes or phases. As another example, repetitive control typically requires asymptotic tracking of an infinite saw-tooth wave. Similar remarks apply to general periodic signals. Finally, in several examples already presented here, the system to-be-controlled has as its output the restriction to the boundary of the solution to a distributed parameter system. For set-point control, in one dimension this allows for the specification of a function at one or two points, which can be accommodate by a one or two dimensional exosystem. For higher spatial dimensions, this allows for the specification of a desired function or wave-form on a continuum, which will most often require an infinite-dimensional exogenous signal generator.

Our preliminary research in this direction was carried out in a masters thesis [36] and, subsequently in a joint paper [14], in which was considered a problem of output regulation for a boundary controlled heat equation on a two dimensional domain for which the signal to be tracked was generated by the one dimensional wave equation.

In particular, we considered the temperature in a two-dimensional unit square, $\Omega = [0,1] \times [0,1]$, with coordinates $x = (x_1, x_2)$ and boundary of Ω denoted by $\partial\Omega$. The temperature distribution across the region is governed by the Heat Equation. In order to avoid technical difficulties which do not add any useful information concerning the main point, we considered case in which open loop heat plant is stable. This is accomplished by assuming that some intervals of $\partial\Omega$ will have homogeneous Dirichlet boundary conditions, i.e., the temperature will be held at 0 on those intervals. This part of the boundary will be denoted by $\mathcal{S}_D$, and it will be important that, by our assumption, S_D will consist of a finite union of intervals of positive length. It was further assumed that on the remainder of the boundary, we have Neumann boundary conditions, so that $\mathcal{S}_N = \partial\Omega \backslash \mathcal{S}_D$. We designate p non-overlapping input intervals $\mathcal{S}_j$, for $j = 1, \ldots, p$, and p non-overlapping output intervals, $\widehat{\mathcal{S}}_j$, for $j = 1, \ldots, p$, with each $\mathcal{S}_j$ and being $\widehat{\mathcal{S}}_j$ a subset of $\mathcal{S}_N$. We point out that the intersections $\mathcal{S}_i \cap \widetilde{\mathcal{S}}_j$ are not necessarily empty. Indeed, in the case of co-located actuators and sensors the $\mathcal{S}_i$ and $\widetilde{\mathcal{S}}_j$ will coincide. Finally, we define the set, $\mathcal{S}_0 = \mathcal{S}_N \backslash \cup \mathcal{S}_j$. A general depiction of the layout of these sections (in the case $\mathcal{S}_i \cap \widetilde{\mathcal{S}}_j = \emptyset$, $i, j = 1, \ldots, p$) is portrayed in the following figure.



Layout of the Various Intervals of the Boundary on Ω

In the specific examples treated in [36, ?] the controlled heat plant is given by the following initial-boundary value problem:

$$\frac{\partial}{\partial t} z(x,t) = \Delta z(x,t), \ x \in \Omega, \ t \geq 0, \ \Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}, \tag{3}$$

$$z(x,0) = z_0(x),$$

$$z(x,t)\Big|_{x \in \mathcal{S}_D} = 0, \tag{4}$$

$$\frac{\partial z}{\partial \nu}(x,t)\Big|_{x \in \mathcal{S}_j} = u_j(t), \ j = 1, \ldots, p, \ \ \frac{\partial z}{\partial \nu}(x,t)\Big|_{x \in \mathcal{S}_0} = 0. \tag{5}$$

Here $u_j(t)$ are the inputs and the state space of the plant is $\mathcal{Z} = L^2(\Omega)$.

It was assumed that the p outputs are given by the *average temperature* over the small regions $\widehat{\mathcal{S}}_j$ of the boundary, i.e.,

$$y_j(t) = \frac{1}{|\widehat{\mathcal{S}}_j|} \int_{\widehat{\mathcal{S}}_j} z(x,t) d\sigma_x, \quad y = [y_1(t), \cdots, y_p(t)]^T = Cz \tag{6}$$

where $|\widehat{\mathcal{S}}_j|$ denotes the length of the interval $\widehat{\mathcal{S}}_j$ and where $d\sigma_x$ is "surface measure" on the boundary of Ω. From this, we may note that the output C operator is defined in the following way

$$Cz(t) = \begin{bmatrix} \frac{1}{|\widehat{\mathbb{S}}_1|} \int_{\widehat{\mathbb{S}}_1} z(x,t) d\sigma_x \\ \vdots \\ \frac{1}{|\widehat{\mathbb{S}}_p|} \int_{\widehat{\mathbb{S}}_p} z(x,t) d\sigma_x \end{bmatrix}. \tag{7}$$

Introducing a standard formulation, we write the plant (68)-(5) in abstract form as

$$\frac{d}{dt} z = Az + Bu, \quad y(t) = Cz, \tag{8}$$

where $A : \mathcal{D}(A) \subset \mathcal{Z} \to \mathcal{Z}$, and $C : \mathcal{D}(C) \subset \mathcal{Z} \to \mathcal{Y} = \mathbb{R}^p$ are unbounded densely defined linear operators and $B : \mathcal{U} = \mathbb{R}^p \to \widetilde{H}^{-1}(\Omega)$ where $\widetilde{H}^{-\alpha}(\Omega)$ denotes the dual of $H^{\alpha}(\Omega)$, $\alpha > 0$ (see, e.g., [44]). $\widetilde{H}^{-\alpha}(\Omega)$ can be identified with a subspace of the space of distributions $H^{-\alpha}(\mathbb{R}^n) = [H^{\alpha}(\mathbb{R}^n)]^* \subset \mathcal{D}(\mathbb{R}^n)^*$:

$$\widetilde{H}^{-\alpha}(\Omega) = \left\{ f \in H^{-\alpha}(\mathbb{R}^n) : \operatorname{supp}(f) \subseteq \overline{\Omega} \right\}.$$

(See definition of B in (10)-(13) below.)

The operator $A = \Delta$ with domain

$$\mathcal{D}(A) = \left\{ \varphi \in \mathcal{Z} : \left. \frac{\partial \varphi}{\partial \nu} \right|_{x \in \mathbb{S}_N} = 0, \ \left. \varphi \right|_{x \in s S_D} = 0 \right\}$$

is an unbounded self-adjoint operator in the Hilbert space $\mathcal{Z} = L^2(\Omega)$ whose spectrum consists of real eigenvalues $\{\zeta_k\}_{k=1}^{\infty}$ satisfying

$$\zeta_{k+1} \le \zeta_k, \quad \zeta_k \xrightarrow{k \to \infty} -\infty, \tag{9}$$

and with associated orthonormal eigenfunctions $\varphi_k(x)$ satisfying

$$A\varphi_k = \zeta_k \varphi_k, \qquad \langle \varphi_n, \varphi_m \rangle = \delta_{nm}.$$

(Here and below we denote by $\langle \cdot, \cdot \rangle$ the inner product in $L^2(\Omega)$).

The input operator is then defined by

$$Bu(\eta) = \sum_{i=1}^{p} u_i(t) \frac{1}{|\mathbb{S}_i|} \int_{\mathbb{S}_i} \eta(x) d\sigma_x, \tag{10}$$

where $Bu \in \widetilde{H}^{-1}(\Omega)$ is a distribution and $\eta \in \mathcal{D}(\mathbb{R}^n)$ is a test function. Therefore,

$$Bu = \sum_{i=1}^{p} u_i b_i, \tag{11}$$

where b_i is the distribution which acts on a test function $\eta \in \mathcal{D}(\mathbb{R}^n)$ by the rule

$$b_i(\eta) = \frac{1}{|\mathbb{S}_i|} \int_{\mathbb{S}_i} \eta(x) d\sigma_x, \tag{12}$$

and

$$u = \begin{bmatrix} u_1 & \cdots & u_p \end{bmatrix}^T \in \mathcal{U} = \mathbb{R}^p. \tag{13}$$

We note that $b_i \in \widetilde{H}^{-1/2-\epsilon}(\Omega)$ for $\epsilon > 0$.

The exosystem is given by the one-dimensional wave equation on the interval $[0, 1]$ (with spatial coordinate ξ) and with homogeneous Dirichlet boundary conditions.

$$\frac{\partial^2}{\partial t^2} w(\xi, t) = \frac{\partial^2}{\partial \xi^2} w(\xi, t), \; \xi \in (0, 1), \; t \in \mathbb{R} \tag{14}$$

$$w(0, t) = w(1, t) = 0$$

$$w(\xi, 0) = \psi_0(\xi), \;\; \frac{\partial}{\partial t} w(\xi, 0) = \psi_1(\xi). \tag{15}$$

For this exosystem we are interested in reference outputs $y_j^{\text{ref}}(t)$ given as the displacements at a set of p points ξ_p in the interval $(0, 1)$

$$y_j^{\text{ref}}(t) = w(\xi_j, t), \; 0 < \xi_j < 1, \;\; y_{\text{ref}} = \widetilde{Q} w = \left[y_1^{\text{ref}}(t), \cdots, y_p^{\text{ref}}(t) \right]^T, \tag{16}$$

where $\widetilde{Q} w$ would be defined as

$$\widetilde{Q} w = \begin{bmatrix} w(\xi_1, t) \\ \vdots \\ w(\xi_p, t) \end{bmatrix}.$$

The exosystem can be formulated as an abstract dynamical system in an infinite dimensional state space in the usual way by first introducing new dependent variables

$$W = \begin{bmatrix} w \\ \frac{\partial}{\partial t} w \end{bmatrix} \equiv \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}, \;\; W(0) = \begin{bmatrix} \psi_0 \\ \psi_1 \end{bmatrix} \equiv W_0, \;\; S = \begin{bmatrix} 0 & 1 \\ \dfrac{\partial^2}{\partial \xi^2} & 0 \end{bmatrix} \tag{17}$$

and then writing the exosystem as:

$$\frac{d}{dt} W = SW, \;\; W(0) = W_0 \tag{18}$$

with reference outputs

$$y_{\text{ref}} = QW, \tag{19}$$

where

$$Q = \begin{bmatrix} \widetilde{Q} & 0 \end{bmatrix}.$$

The state space for (18) is

$$\mathcal{W} = H_0^1(0,1) \times L^2(0,1),$$

which is a Hilbert space with inner product defined by

$$\left\langle \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}, \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} \right\rangle_{\mathcal{W}} = \langle \varphi_1', \psi_1' \rangle_{L^2(0,1)} + \langle \varphi_2, \psi_2 \rangle_{L^2(0,1)}, \tag{20}$$

The spectrum of the operator S consists of eigenvalues

$$\lambda_n = n\pi i, \ n = \pm 1, \pm 2, ... \tag{21}$$

and associated normalized eigenfunctions (i.e., $\|\Phi_\ell\|_{\mathcal{W}} = 1$)

$$\Phi_\ell = \begin{bmatrix} \frac{1}{\lambda_\ell} \\ 1 \end{bmatrix} \sin(\ell\pi\xi), \ \ell = \pm 1, \pm 2, ...$$

Thus the exosystem is infinite dimensional with simple eigenvalues along the imaginary axis, and from (9) and (21) we note that the respective spectra of A and S are disjoint.

The error, defined as an output of the composite system, is given by the difference between the measured output and the reference output, i.e.,

$$e(t) = y(t) - y_{\text{ref}}(t),$$

where y and y_{ref} are defined in (6) and (19). Written another way,

$$y = Cz$$

and

$$y_{\text{ref}} = QW,$$

so that the error can be written in terms of (6) and (19) as

$$e(t) = y(t) - y_{ref}(t) = [C, -Q] \begin{bmatrix} z \\ W \end{bmatrix} (t). \tag{22}$$

With this development, the problem of output regulation considered in [36, ?] can be stated as

Problem 2 (The Main Problem). *Find a feedback control $u = \Gamma W$ so that for every initial condition of the plant and exosystem, the error satisfies*

$$e(t) \longrightarrow 0 \ \textit{as} \ t \longrightarrow \infty. \tag{23}$$

The plant for this example is a regular linear as follows from our work [10] and, for a finite dimensional exosystem, the main theorem of [16] would apply to provide a method for constructing a feedback law by solving the regulator equations. Unfortunately, this result cannot be applied directly to a problem of output regulation with infinite dimensional exosystem. However, mimicking the proof given in [16] in the present setting we can obtain formulas which allow us to solve the problem once we introduce an appropriate modification of the reference signals to be tracked.

The difficulties presented by an infinite dimensional exosystem present many new and interesting challenges but our research shows that the development of a fairly complete theory in this case is at least feasible.

The final six proposed research tasks involve extending the regulator theory for linear distributed parameter systems to nonlinear systems. As is widely appreciated, the study of Nonlinear Distributed Parameter systems exacerbates the difficulties encountered in the study of each of lumped nonlinear systems and linear distributed parameter systems, both analytically and computationally. Moreover, since nonlinear operations can produce either very large or very small quantities, the numerical difficulties stemming from the use of finite state, finite precision computers further exacerbates these difficulties.

Our preliminary calculations indicated that tracking problems can be solved for the special case of set-point control of nonlinear reaction diffusion systems with control inputs and outputs acting through the boundary of the spatial domain. Indeed, in [12] such results are rigorously established for a system whose zero dynamics consists of the now famous class of reaction-diffusion equations studied by Chaffee and Infante. In this case the structure of the attractor has been well-studied and is well-known, leading to a compact, Lyapunov stable attractor for the zero dynamics. We use the zero dynamics to design, following the non-equilibrium internal model principle, an infinite dimensional compensator and show how, using invariant manifold theory, the input-output behavior of the compensator can be realized by solving the distributed parameter version of the "regulator equations."

As an indication of the more general results we have obtained, yet unpublished, consider a nonlinear plant

$$z_t = Az - F(z) + Bu + d, \quad z(0) = z_0, \tag{24}$$

where $z(t)$ is the state in Hilbert space Z at time t. A is the generator of an exponentially stable C_0 semigroup in Z, $u \in U$ is a control input and $d(t) \in Z$ is a disturbance.

In order to simplify the discussion, we assume also that we have a an exosystem

$$\dot{w} = Sw, \quad w(0) = w_0 \tag{25}$$

acting in the space W with $\dim(W) = N$ and S is neutrally stable. We assume that S has the spectral representation

$$\begin{aligned} Sw &= \sum_{\ell=1}^{N} \lambda_\ell \langle w, \Phi_\ell^* \rangle \Phi_\ell, \quad w = \sum_{\ell=1}^{N} \langle w, \Phi_\ell^* \rangle \Phi_\ell, \\ S\Phi_\ell &= \lambda_\ell \Phi_\ell, \quad S^* \Phi_\ell^* = \overline{\lambda_\ell} \Phi_\ell^* \end{aligned} \tag{26}$$

where

$$\{\lambda_\ell\}_{\ell=1}^{N} \subset i\mathbb{R}, \quad \{\lambda_\ell\}_{\ell=1}^{N} = \overline{\{\lambda_\ell\}_{\ell=1}^{N}}.$$

Associated with the plant we have an output to be regulated $y(t)$ and a from the exosystem we have a reference output $y_r(t)$ to be tracked. We also assume that the disturbance is generated as an output of the exosystem:

$$y(t) = Cz(t), \quad y_r(t) = Qw(t), \quad d(t) = Pw(t). \tag{27}$$

The regulator equations for the systems (24), (51) for (a state feedback) control

$$u = \Gamma(w)$$

are given, via a pair of mapping, $\Pi : W \to Z$ and $\Gamma : W \to U$, by

$$\frac{\partial \Pi(w)}{\partial w} Sw = A\Pi(w) - F(\Pi(w)) + B\Gamma(w) + Pw \tag{28}$$

$$C\Pi(w) - Qw = 0. \tag{29}$$

Under our assumptions the composite system consisting of (24) and (51) can be written as

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} z \\ w \end{bmatrix} &= \mathcal{A} \begin{bmatrix} z \\ w \end{bmatrix} + \mathcal{F}(z, w), \\ \mathcal{A} &= \begin{bmatrix} A & (BG_L + P) \\ 0 & S \end{bmatrix}, \quad \mathcal{F}(z, w) = \begin{bmatrix} F(z) + G_N(w) \\ 0 \end{bmatrix}, \end{aligned} \tag{30}$$

where we have written

$$\Gamma(w) = \boldsymbol{G}_L w + \boldsymbol{G}_N(w), \quad \boldsymbol{G}_L \in \mathcal{L}(W, U), \quad \left. \frac{\partial \boldsymbol{G}_N}{\partial w} \right|_{w=0} = 0. \tag{31}$$

The system (30) has an N dimensional center manifold given in terms of a mapping Π as

$$\mathcal{S} = \left\{ \begin{bmatrix} \Pi(w) \\ w \end{bmatrix} : w \in W \right\}.$$

For this reason, we know that the regulator equations indeed have a solution.

Before turning to an example, we note, however, that there remains an open problem of paramount importance in any practical application, viz. the existence of a computationally accurate and effective method for approximating solutions to the regulator equations.

To this end, we introduce the notation

$$\Pi(w) = \mathfrak{P}_L w + \mathfrak{P}_N(w), \quad \mathfrak{P}_L \in \mathcal{L}(W, Z)$$

so that

$$\frac{\partial \Pi}{\partial w} = \mathfrak{P}_L + \frac{\partial \mathfrak{P}_N}{\partial w}.$$

With this we can write the regulator equations as

$$\begin{aligned} \left(\mathfrak{P}_L + \frac{\partial \mathfrak{P}_N}{\partial w}\right) Sw =& A\left(\mathfrak{P}_L w + \mathfrak{P}_N(w)\right) \\ &+ B\left(\boldsymbol{G}_L w + \boldsymbol{G}_N(w)\right) - F\left(\mathfrak{P}_L w + \mathfrak{P}_N\right)(w) + Pw. \end{aligned} \tag{32}$$

$$0 = C\Pi(w) - Qw = C\left(\mathfrak{P}_L w + \mathfrak{P}_N(w)\right) - Qw.$$

The first order approximation to the regulator equations in (32) can be written as

$$\mathfrak{P}_L Sw = A\mathfrak{P}_L w + B\boldsymbol{G}_L + Pw, \quad C\mathfrak{P}_L w - Qw = 0. \tag{33}$$

We can readily obtain a representation for $\mathfrak{P}_L$ and $\boldsymbol{G}_L$ using the spectral representation for S. Namely, from the first of these equations applied to Φ_ℓ we have

$$\lambda_\ell \mathfrak{P}_L \Phi_\ell = A\mathfrak{P}_L \Phi_\ell + B\boldsymbol{G}_L \Phi_\ell + P\Phi_\ell.$$

This gives

$$\mathfrak{P}_L \Phi_\ell = (\lambda_\ell I - A)^{-1}\left(B\boldsymbol{G}_L \Phi_\ell + P\Phi_\ell\right).$$

If we now apply the second result in (33) we have

$$Q\Phi_\ell = C(\lambda_\ell I - A)^{-1} B\boldsymbol{G}\Phi_\ell + C(\lambda_\ell I - A)^{-1} P\Phi_\ell.$$

We define the transfer function, for $\lambda \in \rho(A) = \mathbb{C}\backslash\sigma(A)$ ($\rho(A)$ the resolvent set and $\sigma(A)$ the spectrum of A) by

$$G(\lambda) = C(\lambda_\ell I - A)^{-1} B. \tag{34}$$

Then, under the standing assumption from linear theory, that no eigenvalue of S is a transmission zero of the linear plant, i.e., $G(\lambda_\ell) \neq 0$ for all $\ell = 1, 2, \cdots, N$ we have

$$\boldsymbol{G}_\ell \Phi_\ell = G(\lambda_\ell)^{-1} \Big[Q\Phi_\ell - C(\lambda_\ell I - A)^{-1} P\Phi_\ell \Big]. \tag{35}$$

Recalling that the eigenvectors Φ_ℓ form a basis for W we have, from (26) and (35) that for any $w \in W$ that

$$\boldsymbol{G}_L w = \sum_{\ell=1}^{N} \langle w, \Phi_\ell^* \rangle \; G(\lambda_\ell)^{-1} \Big[Q\Phi_\ell - C(\lambda_\ell I - A)^{-1} P\Phi_\ell \Big]. \tag{36}$$

From this we can also obtain simple formulas for the first order (linear) approximation to $\mathfrak{P}_L$. Namely, we have

$$\mathfrak{P}_L w = \sum_{\ell=1}^{N} \langle w, \Phi_\ell^* \rangle \left[(\lambda_\ell I - A)^{-1} (B\boldsymbol{G}_L + P)\Phi_\ell \right]. \tag{37}$$

The next step is to define an iterative scheme for obtaining successively better approximations to Π and Γ satisfying (53) and (54).

Returning to equation (32), we can remove the first order terms and solve for $\mathfrak{P}_N$ to obtain

$$\mathfrak{P}_N(w) = A^{-1} \left[F(\mathfrak{P}_L w + \mathfrak{P}_N(w)) + \frac{\partial \mathfrak{P}_N}{\partial w}(w) S w - B\boldsymbol{G}_N(w) \right]. \tag{38}$$

This expression can then be used this to define a sequence of iterations for $\mathfrak{P}_N$ and $\boldsymbol{G}_N$ as follows:

First define

$$\mathfrak{P}_N^{(1)}(w) = 0. \tag{39}$$

Then define

$$\mathfrak{P}_N^{(j+1)}(w) = A^{-1} \left[F(\mathfrak{P}_L w + \mathfrak{P}_N^{(j)}(w)) + \frac{\partial \mathfrak{P}_N^{(j)}}{\partial w}(w) S w - B\boldsymbol{G}_N^{(j)}(w) \right] \tag{40}$$

Due to our assumptions that Q and C are linear operators, our earlier calculations for linear terms gave,

$$C\mathfrak{P}_L w - Qw = 0$$

which implies that

$$C\mathfrak{P}_N(w) = 0$$

for all w. This, fact together with (38) gives

$$CA^{-1}\left[F(\mathfrak{P}_L w + \mathfrak{P}_N(w)) + \frac{\partial \mathfrak{P}_N}{\partial w}(w)Sw - \boldsymbol{B}\boldsymbol{G}_N(w)\right] = 0.$$

Solving this equation for $\boldsymbol{G}_N(w)$ we obtain

$$\boldsymbol{G}_N(w) = \left[CA^{-1}B\right]^{-1}\left[CA^{-1}\left(F(\mathfrak{P}_L w + \mathfrak{P}_N(w))\right)\right].$$

This formula gives the final ingredient, a means to obtain the iterations for $\boldsymbol{G}_N$. Namely, we first define

$$\boldsymbol{G}_N^{(1)}(w) = \left[CA^{-1}B\right]^{-1}\left[CA^{-1}\left(F(\mathfrak{P}_L w + \mathfrak{P}_N^{(1)}(w)\right)\right] \tag{41}$$

(here we note that $\mathfrak{P}_N^{(1)}(w) = 0$). Thus we define a sequence of approximations to $\Pi(w)$ and $\Gamma(w)$ given by

$$\widetilde{\Pi}_j(w) \equiv \mathfrak{P}_L w + \mathfrak{P}_N^j(w), \quad \text{and} \quad \Gamma_j(w) \equiv \boldsymbol{G}_L w + \boldsymbol{G}_N^j(w), \quad j = 1, 2, \cdots \tag{42}$$

where $\boldsymbol{G}_L$ is given in (36), $\mathfrak{P}_L$ is given in (37), $\mathfrak{P}_N^{(1)} = 0$, $\boldsymbol{G}_N^{(1)}$ is given in (41) and

$$\mathfrak{P}_N^{(j+1)}(w) = A^{-1}\left[F(\Pi_j(w)) + \frac{\partial \mathfrak{P}_N^{(j)}}{\partial w}(w)Sw - B\boldsymbol{G}_N^{(j)}(w)\right] \tag{43}$$

and

$$\boldsymbol{G}_N^{(j+1)}(w) = \left[CA^{-1}B\right]^{-1}\left[CA^{-1}\left(F(\Pi_j(w)) + \frac{\partial \mathfrak{P}_N^{(j)}}{\partial w}(w)Sw\right]\right. . \tag{44}$$

Thus we obtain a sequence of approximate controls given by

$$u_j = \Gamma_j(w)$$

which presumably converge to a solution of the regulator equations.

We now illustrate this approach in an explicit example from the Masters Thesis [39] at Texas Tech University.

Example 1 (Set-Point Burgers' Equation). Consider set-point control for a viscous Burgers' equation with homogeneous Dirichlet boundary conditions. In this case the regulator equations are nonlinear and we use a fixed point iterative scheme for obtaining approximate solutions.

Consider the nonlinear plant governed by Burgers' equation

$$\begin{aligned}
z_t(x,t) &= z_{xx}(x,t) - z_x(x,t)z(x,t) \\
z(0,t) &= u_0(t) \quad z(1,t) = u_1(t) \\
z(x,0) &= \varphi(x) \\
y(t) &= C(z)(t) = z(x_1,t) \quad 0 < x_1 < 1
\end{aligned} \tag{45}$$

where $z(t)$ is the state in Hilbert space $\mathcal{Z} = L^2(0,1)$ at time t.

Our control problem is to build a control u so that, for every initial condition and arbitrary constant reference signal $y_r(t) = M \in \mathbb{R}$ (for all t) we have

$$\lim_{t\to\infty} e(t) = y(t) - y_r(t) = 0. \tag{46}$$

The following theorem, which can be proved using the Hopf-Cole transformation [?], gives the exact feedback control for our problem of output regulation for Burgers' equation with Dirichlet boundary conditions.

Theorem 2. *For $M > 0$, find $\mu_0 > 0$ so that*

$$2\mu_0 \tanh\left(\mu_0(1 - x_1)\right) = M \tag{47}$$

Then controls u_0 and u_1 for (45) solving $y(t) \to M$ are given by

$$u_0 = 2\mu_0 \tanh\left(\mu_0\right), \quad u_1 = 0. \tag{48}$$

It can be shown that when $M > 0$ only one control is needed. To simplify the presentation we will consider this single input, single output (SISO) problem with a homogeneous boundary condition at $x = 1$. For this case, we define the state operator $A = d^2/dx^2$ in $\mathcal{Z} = L^2(0,1)$ with domain $\mathcal{D}(A) = H_0^2(0,1)$. The term $u(t)$ is the control input. The control operator can be found by considering the weak formulation of the problem.

In our SISO problem, we further restrict to the case in which the control is input at $x = 0$. So we set $u_0(t) = u(t)$ and $u_1(t) = 0$.

$$\begin{aligned} z_t(x,t) &= z_{xx}(x,t) - z_x(x,t)z(x,t) \\ z(0,t) &= u(t) \quad z(1,t) = 0, \\ z(x,0) &= \varphi(x). \end{aligned} \tag{49}$$

This problem can be reformulated with the control $u(t)$ in the differential equation with homogeneous boundary conditions as

$$\begin{aligned} z_t(x,t) &= z_{xx}(x,t) - z_x(x,t)z(x,t) + Bu(t) \\ z(0,t) &= 0 \quad z(1,t) = 0, \\ z(x,0) &= \varphi(x). \end{aligned} \tag{50}$$

where $B = -\delta_0'$.

Since we are solving a set point control problem, the exosystem is

$$\dot{w} = 0, \quad w(0) = M, \quad M > 0 \tag{51}$$

acting in the space $\mathcal{W} = \mathbb{R}$.

So we have

$$y(t) = Cz(t) = z(x_1, t), \quad y_r(t) = Qw(t) = M. \tag{52}$$

The regulator equations for the systems (50) and (51) for the control

$$u = \Gamma(w)$$

are given, via a pair of mappings, $\Pi : W \to Z$ and $\Gamma : W \to U$, by

$$\frac{\partial \Pi(w)}{\partial w} Sw = A\Pi(w) - F(\Pi(w)) + B\Gamma(w) = 0 \tag{53}$$

$$C\Pi(w) - Qw = 0. \tag{54}$$

The first regulator equations can be written in terms of $\Pi(x, w)$, $\Gamma(w)$ as

$$\Pi''(x, M) - \Pi'(x, M)\Pi(x, M) + B\Gamma(M) = 0, \quad \Pi(x_1, M) = M. \tag{55}$$

In order to solve this problem, we need to find the transfer function $G(s) = C(sI - A^{-1})B$. In the case of set-point control the only eigenvalue of the exosystem is, $s = 0$ so we will need to compute G(0). Thus we need to determine $A^{-1}\varphi$ for $\varphi \in L^2(0, 1)$ Recall that $A = d^2/dx^2$. Thus $A^{-1}\phi = \psi$ or equivalently $A\psi = \phi$. Applying the operator, we need to solve

$$\psi'' = \phi, \quad \psi(0) = 0, \quad \psi(1) = 0.$$

We find Ψ by constructing the Green's function. First, we need to find two functions $\{y_1, y_2\}$ that solve the homogeneous equation and satisfy the boundary conditions. So let $y_1 = x$ and $y_2 = 1 - x$. The Green's function is

$$G(x, \xi) = \begin{cases} x(1-\xi) & x < \xi \\ \xi(1-x) & \xi < x. \end{cases}$$

Thus our solution is

$$\psi = A^{-1}\phi = \frac{1}{W}\left\{(1-x)\int_0^x \xi\,\phi(\xi)\,d\xi + x\int_x^1 (1-\xi)\,\phi(\xi)\,d\xi\right\}, \tag{56}$$

where W is the Wronksian.

$$W = \det \begin{vmatrix} x & 1-x \\ 1 & -1 \end{vmatrix} = -1$$

So, for a test function η, we have

$$\begin{aligned}
(-A^{-1}B)(\eta) =& A^{-1}(\delta_0')(\eta) && (57)\\
=& \delta_0'(A^{-1}\eta) = \delta_0(-d/dxA^{-1}\eta) && (58)\\
=& \delta_0\left(-\int_0^x \xi\ \eta(\xi)\ d\xi + \int_x^1 (1-\xi)\ \eta(\xi)\ d\xi\right) && (59)\\
=& \int_0^1 (1-\xi)\ \eta(\xi)\ d\xi && (60)\\
=& T_{(1-x)}(\eta). && (61)
\end{aligned}$$

Therefore,

$$(-A^{-1}B) = 1 - x$$

and

$$C(-A^{-1}B) = 1 - x_1.$$

Next we briefly explain a numerical scheme for obtaining $u = \Gamma w$ for $M > 0$. Our approach here is based on an iterative algorithm using a fixed point method to solve nonlinear iterations. The main numerical method used cubic splines for a Galerkin finite element method to solve the second order ordinary differential equation at each step.

In the Galerkin approach we seek an approximate solution $\Pi^N(x, M)$ in the form

$$\Pi^N(x) = \sum_{j=1}^{N} c_j^N(t)\varphi_j(x). \tag{62}$$

where for each N we ask that the weak formulation holds for $\varphi_1, \varphi_2, \cdots, \varphi_N$. Substituting this expression into (50), taking inner products in $L^2(0,1)$ with the basis functions φ_m for $m = 1, \cdots, N$ we get

$$0 = \left\langle \Pi^{N''}, \varphi_m \right\rangle + \langle B\Gamma, \varphi_m\rangle - \left\langle \left(\frac{\Pi^{N2}}{2}\right)', \varphi_m \right\rangle.$$

Integrating by parts on the first and third inner product and using the boundary conditions, gave

$$0 = -\left\langle \Pi^{N'}, \varphi_m' \right\rangle + \varphi_m'(0)\Gamma + \left\langle \left(\frac{\Pi^{N2}}{2}\right), \varphi_m' \right\rangle.$$

This system of equations can then be expressed as

$$0 = \mathcal{S}\mathcal{C} + \Gamma(M)\Phi_0^p + \Psi(\mathcal{C}) \tag{63}$$

where

$$\mathcal{S} = -\left[\langle \varphi_i', \varphi_j' \rangle\right], \quad \Phi_0^p = \begin{bmatrix} \varphi_1'(0) \\ \varphi_2'(0) \\ \vdots \\ \varphi_N'(0) \end{bmatrix}, \quad \Psi(C) = \left\langle \left(\frac{\Pi^{N^2}}{2} \right), \varphi_m' \right\rangle.$$

Next we define $F(\mathcal{C})$ as

$$F(\mathcal{C}) = \mathcal{S}\mathcal{C} + \Psi(\mathcal{C}) + \Gamma(M)\Phi_0^p \tag{64}$$

and look for a zero of F with the property that the resulting Π^N also has $\Pi^N(x_1) = M$ (by the second regulator equation). Notice that by our construction of the basis functions each satisfy the Dirichlet boundary conditions so that, Π^N in (62), automatically satisfies these boundary conditions.

Initial values for the iterative scheme obtained from the linearized regulator equations for $\mathcal{C}$ and $\Gamma(M)$, i.e.,

$$0 = A\Pi(x, M) + B\Gamma(M), \quad \Pi(x_1, M) = M, \tag{65}$$

which implies

$$\Pi(x, M) = (-A^{-1}B)\Gamma(M).$$

Applying C gives

$$M = C\Pi = C(-A^{-1}B)\Gamma(M) = G(0)\Gamma(M).$$

From this we obtain

$$\Gamma_1(M) = G(0)^{-1}M = \frac{M}{1 - x_1}. \tag{66}$$

Then using this value for an approximate $\Gamma(M)$ and the spline approximation in the linearized version of equation (65) we arrive an the initial value for $\mathcal{C}$.

$$\begin{aligned} 0 &= \Pi^{N''} + B\Gamma \\ &= < \Pi^{N''}, \varphi_k > + < B, \varphi_k > \Gamma \\ &= \Pi^{N'}\varphi_k\big|_0^1 - < \Pi^{N'}, \varphi_k' > + \varphi_k'(0)\Gamma \\ &= \mathcal{S}\mathcal{C} + \Phi_0^p\Gamma. \end{aligned}$$

Thus

$$\mathcal{C}^1 = -\frac{M}{1-x_1}\mathcal{S}^{-1}\Phi_0^p. \tag{67}$$

Next we define fixed point iterations to update the coefficients.

For the fixed point method one approach would be to rewrite the equation $F(\mathcal{C}) = 0$, defined in (64), by first isolating $S\mathcal{C}$ on one side and then applying $\mathcal{S}^{-1}$ to obtain

$$\mathcal{C} = \widetilde{F}(\mathcal{C})$$

where

$$\widetilde{F}(\mathcal{C}) = S^{-1}\big(-\Psi(\mathcal{C}) - \Gamma(M)\Phi_0^p\big).$$

Then we want to solve this fixed point problem using the iterates

$$\mathcal{C}^{n+1} = \widetilde{F}(\mathcal{C}^n).$$

Updating Π^N produced

$$\Pi^{N+1}(x) = \sum_{j=1}^{N} C_j^{n+1}\varphi_j(x)$$

which was then used to update Γ.

At each step the value of $\Gamma(M)$, denoted Γ^{N+1}, was updated by using the regulator equations (53). In particular,

$$\Pi^{N+1} = A^{-1}\left(\left(\frac{{\Pi^{N+1}}^2}{2}\right)' - B\Gamma(M)\right)$$

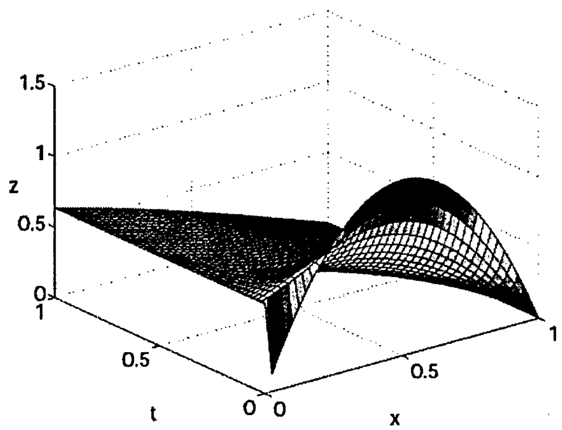
and then apply the second regulator equation and the definition of the transfer function implied

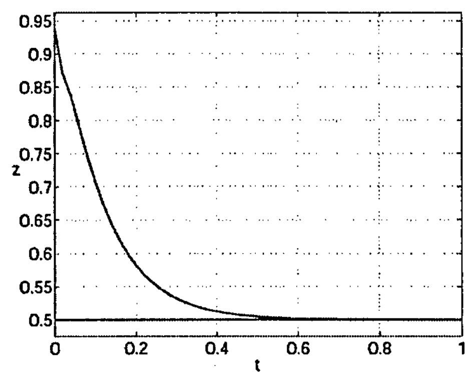
$$M = C\Pi^{N+1} = CA^{-1}\left(\frac{{\Pi^{N+1}}^2}{2}\right)' + G(0)\Gamma(M).$$

Solving for $\Gamma(M)$ will be the updated value

$$\Gamma(M)^{N+1} = G(0)^{-1}\left[M - CA^{-1}\left(\frac{{\Pi^{N+1}}^2}{2}\right)'\right].$$

The Matlab code starts with the initial Γ from (66) and $\mathcal{C}$ from (67). A while loop controls iterations until we are at the desired error which we set at 10^{-4}. Inside the loop we build $\widetilde{F}(\mathcal{C})$ which we use to calculate a new value for the coefficients. Once we have the updated coefficients, we can find a new Π and with that a new Γ. The error we track comes from the difference in successive iterates of $\mathcal{C}$ and from equation (63). Once we are within the desired tolerance, the approximate Γ is compared with the actual Γ found using a Hopf-Cole transformation. To illustrate the result, consider $n = 100, \varphi(x, 0) = 5x(1-x)$. The program is included in Appendix D. The following plots are of the solution surface and outputs y and y_r in the following Figures.





An important aspect of output regulation for nonlinear systems is the existence and characterization of the steady state response. For many reasons, the computational aspects of numerically obtaining the steady-state behavior of nonlinear distributed parameter systems are extremely challenging. We conclude by illustrating the computational challenges associated with identifying numerically the steady state behavior of Burgers' equation with Neumann boundary conditions.

For this problem it is clear that constants are stationary solutions but other than constant solutions the general long time behavior of trajectories was not clear. Indeed, numerical observations seemed to suggest a more complicated steady state behavior. We now briefly describe the phenomenon first observed in [2, 43] and investigated and explained, for Burgers' equation, in [1]. Consider the one-dimensional viscous Burgers' equation on the interval $[0, 1]$ subject to Neumann boundary conditions:

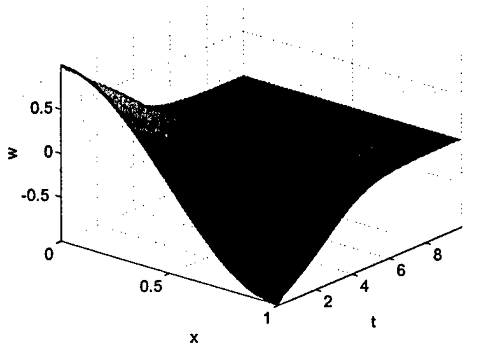
$$w_t - \epsilon w_{xx} + w w_x = 0, \; x \in (0,1), \quad t > 0, \; (\epsilon > 0 \text{ is the viscosity}), \tag{68}$$

$$w_x(0,t) = w_x(1,t) = 0, \quad \text{(Neumann Boundary Conditions)} \tag{1'}$$

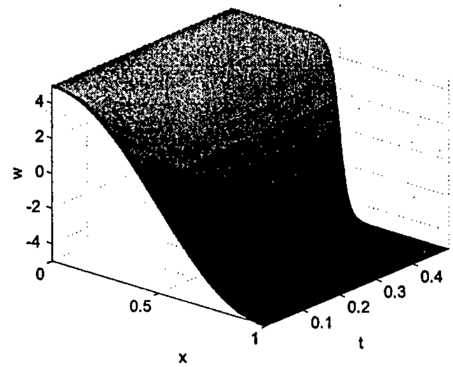
$$w(x,0) = \phi(x), \quad \text{(Initial Condition)}. \tag{1''}$$

(Here we use a subscript notation for partial derivatives.) It is easy to show that the only stationary solutions (i.e., $w_t \equiv 0$) of the problem (68), (1′) are constants: $w(x) = c$, $c \in \mathbb{R}$. It is also known [8, 38] that for any initial function $\phi \in L^2(0,1)$ the problem (68), (1′), (1″) has a unique strong solution defined for all $t \geq 0$ which is instantly classical (for $t > 0$). Moreover, applying results from [48] it can be shown that that for any $\phi \in L^\infty(0,1)$ the corresponding solution $w(x,t)$ tends to a stationary solution as $t \to \infty$, i.e., for every $\phi \in L^\infty(0,1)$ there exists a $c_\phi \in \mathbb{R}$ such that $w(x,t) \xrightarrow{t\to\infty} c_\phi$ in the $L^\infty(0,1)$-norm.

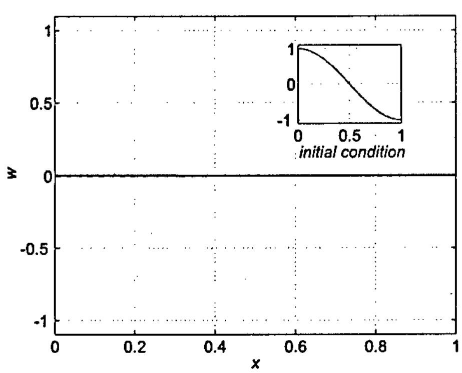
In [1, 2] it is shown that Burgers' equation with Neumann boundary conditions leaves invariant the set of so-called "antisymmetric" functions, (C^2-functions that are odd about $x = 1/2$ in the interval $(0,1)$) which we denote by $\mathcal{AS}(0,1)$. That is, a solution of (68), (1′),(1″) for initial data $\phi \in \mathcal{AS}(0,1)$ satisfies $w(x,t) = -w(1-x,t)$ for all $x \in [0,1]$ and for all $t > 0$. Thus the only stationary solution in $\mathcal{AS}(0,1)$ is $w \equiv 0$, i.e, for $\phi \in \mathcal{AS}(0,1)$, the corresponding solution $w(x,t) \xrightarrow{t\to\infty} 0$. However, if the value $\phi(0)$ (for $\phi \in \mathcal{AS}(0,1)$) is "large enough" and positive, then any sufficiently accurate numerical solution $w^N(x,t)$ will converge (for increasing t) to a nonconstant "steady state solution." This phenomenon, which has been rigorously verified, occurs independently of the particular numerical method used (e.g., finite difference, finite element, etc) or level of discretization (order of the approximate solution). But that it does depend of the particular floating point arithmetic on a given machine. This anomaly is explained in detail in [1]. It turns out that the equation (68) has a 1-parameter family of stationary solutions $h(\cdot,c) \in \mathcal{AS}(0,1)$ parameterized by $c \in \mathbb{R}$, which satisfy the nonhomogeneous Neumann boundary conditions: $h_x(0,c) = h_x(1,c) = \alpha_c$. Moreover, the $L^\infty(0,1)$-norm $\|h_x(\cdot,c)\|_\infty \xrightarrow{c\to\infty} \infty$ while $\alpha_c \xrightarrow{c\to\infty} 0$ (exponentially fast). We now demonstrate how this fact affects the numerical results. Consider the problem (68) with viscosity $\epsilon = 1/10$, first with initial condition $\phi_1(x) = \cos(\pi x)$ and then with $\phi_2(x) = 5\cos(\pi x)$. In the first two figures below we present the graphs of the corresponding numerical solutions $w_1^N(x,t)$ and $w_2^N(x,t)$ obtained using a Crank-Nicolson finite difference method (we emphasize that these results are independent of the particular numerical method or accuracy of approximation method). It is clear from the first figure below that $w_1^N(x,t)$, plotted for $(x,t) \in [0,1] \times [0,10]$, converges to the constant function $h_1(x) = 0$, while in the next figure we see that $w_2^N(x,t)$ converges very rapidly (on the time interval $0 \leq t \leq .5$) to a nonconstant stationary solution $h_2(x)$. In fact, $h_2(x)$ satisfies the nonhomogeneous boundary conditions $h_2'(0) = h_2'(1) = \alpha_c$.



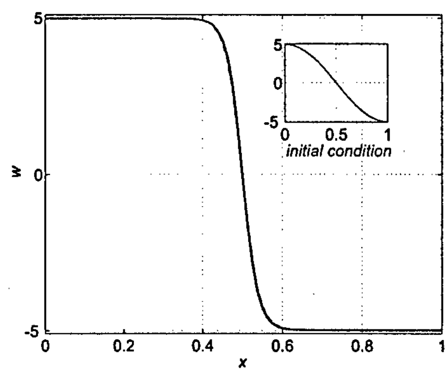
$\phi_1(x) = \cos(\pi x), \;\; 0 \le t \le 10$



$\phi_2(x) = 5\cos(\pi x), \; 0 \le t \le .5$



$\phi_1(x) = \cos(\pi x), \;\; t = 10$



$\phi_2(x) = 5\cos(\pi x), \; t = .5$

Since α_c is exponentially small (for c large enough) any computer, using finite precision arithmetic, could not distinguish between α_c and "zero" and, therefore, treated $h(x, c)$ (erroneously) as a stationary solution of (68)-(1″). The function $h(x, c)$ is given explicitly by

$$h(x,c) = \sqrt{2c}\tanh\left(\frac{\sqrt{2c}}{2\epsilon}(1/2 - x)\right), \quad h_x(0,c) = h_x(1,c) = -\frac{c}{\epsilon}\,\mathrm{sech}^2\left(\frac{\sqrt{2c}}{4\epsilon}\right) \equiv \alpha_c. \quad (69)$$

Attempts to circumvent this problem using a larger floating point number system (e.g. as high as 75 digits) available in a computer algebra system (e.g., Maple) very quickly depleted available memory even on an SGI Origin supercomputer with 16 gigabytes of memory. Even in this case for only slightly larger values of the amplitude the solutions still converge to a nonconstant stationary solution. The point is that no attempt to brute force the problem away with a bigger or more powerful computer will work. There are two important points that must be made. The problem arises because, on one hand, Burgers' equation with Neumann boundary conditions is extremely sensitive to perturbations in the boundary conditions, and, on the other hand, all computers today use a finite precision floating point arithmetic.

In this final report it is not possible to provide details of how finite precision arithmetic, the amplitude of the initial data $\phi(0)$, and the numerical approximation of the convective term in Burgers' equation combine to produce the convergence to the nonconstant stationary solution. We do, however, give a sample formula that is derived in [?] based on digits of accuracy and the value of $\phi(0)$ which can be used to determine when solutions will go to zero or converge to an incorrect answer. Let β denote the base for a computer system and d the number of digits. On the interval $[\beta^{m-1}, \beta^m]$, the floating point numbers are evenly spaced with separation β^{m-d}. Let $x_1, x_2 \in [\beta^{m-1}, \beta^m]$ be two floating point numbers. If $|x_1 - x_2| < \frac{1}{2}\beta^{m-d}$, then $x_1 = x_2$. Thus, if $|x_1 - x_2| < \frac{1}{2}\beta^{1-d}\beta^{m-1} \le \frac{1}{2}\beta^{1-d}|x_1|$, or, if $\frac{|x_1 - x_2|}{|x_1|} < \frac{1}{2}\beta^{1-d}$, then $x_1 = x_2$.

Floating Point Numbers $\alpha_2 = \alpha_1 + \beta^{m-t}$

Using a Crank-Nicolson finite difference scheme applied to Burgers' equation with spatial mesh size Δx and corresponding numerical solution denoted by $w^{\Delta x}(x,t)$, we arrive at the following conclusion:

$$\text{if} \quad \frac{8(\Delta x)\phi(0)}{\epsilon} \exp\left(\frac{-\phi(0)}{2\epsilon}\right) < \beta^{1-d} \quad \text{then} \quad w^{\Delta x}(x,t) \xrightarrow{t\to\infty} h(x,c),$$

$$\text{if} \quad \frac{8(\Delta x)\phi(0)}{\epsilon} \exp\left(\frac{-\phi(0)}{2\epsilon}\right) > \beta^{1-d} \quad \text{then} \quad w^{\Delta x}(x,t) \xrightarrow{t\to\infty} 0.$$

In our example depicted in the first two figures (above) we had $\Delta x = 1/80$, $\beta = 2$, $\epsilon = 1/10$, $d = 15$ so that $\beta^{1-d} \approx 6 \times 10^{-5}$ while for $\phi(0) = 1$ the left side above is approximately $7 \times 10^{-3} > \beta^{1-d}$ and for $\phi(0) = 5$ the left hand side is approximately $7 \times 10^{-11} < \beta^{1-d}$.

The explanation of the anomaly given in [1] and described above is immediately applicable only to Burgers' equation but the underlying reason that this anomaly occurs is a combination of the following three facts. It is the generalization of these important facts that will allow us to carry out the main objectives outlined below.

1) *The existence of a nearby problem with a nonconstant stationary solution:* The solution $w(x,t)$ of equation (68) with initial data (1″) ($\phi \in \mathcal{AS}(0,1)$) and with <u>nonhomogeneous</u> Neumann boundary conditions

$$w_x(0,t) = w_x(1,t) = \alpha \approx 0 \tag{70}$$

converges to the function $h(x)$ in (69) satisfying

$$-\epsilon h_{xx}(x) + h(x)h_x(x) = 0, \quad h_x(0) = h_x(1) = \alpha. \tag{71}$$

2) *Problem Sensitivity (ill-posedness):* The nonlinear ("elliptic") boundary value problem (71) for the stationary solution is ill-posed. Related phenomena in the numerical analysis of problems in fluid flow [3, 6] have recently been referred to as "problem sensitivity." In any case, the solution $h(x, \alpha)$ of (71) is not only discontinuous (in, e.g., $H^1(0,1)$-norm) as a function of the boundary parameter α but it even has a singularity as $\alpha \to 0$.

3) *Finite Precision Arithmetic:* In practice, floating point arithmetic is nearly always used in computational work and such arithmetic is always limited to a finite set of numbers and a finite precision arithmetic. Furthermore different machines have a different set of numbers and precision.

Our indications suggest that a similar anomaly takes place for a broad class of nonlinear n-dimensional ($n \geq 1$) parabolic equations containing convective type terms. Moreover, there is a strong numerical evidence that the same type of anomaly may occur for real hydrodynamic equations – Euler and Navier Stokes.

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5 Scientific Publications

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10. C. I. Byrnes, D. Gilliam, J. Hood and V. I. Shubov , *Examples of Output Regulation for Distributed Parameter Systems with Infinite Dimensional Exosystem*, Proceedings of 40th IEEE-CDC Proceedings, 547-548, (2001).
11. C. I. Byrnes, D. Gilliam, V. I. Shubov, "Examples of Regular Linear Systems Governed by Partial Differential Equations," Proceedings of *40th IEEE-CDC Proceedings*, 129-130, (2001).
12. F. Celani, C.I. Byrnes, and A. Isidori, *Compact attractors of nonlinear non-minimum-phase systems that are semiglobally practically stabilized*, CDC 2002, pp. 4306-4311.
13. F. Celani, C.I. Byrnes, A. Isidori, *Compact attractors of nonlinear minimum-phase systems that are semiglobally practically stabilized*, CDC 2001, pp. 3796-3801.
14. C. De Persis, A. Isidori, *The design of a fault detection filter with game-theoretical optimal sensitivity*, NOLCOS 2001, pp. 569-574.
15. C. De Persis, R. De Santis, A. Isidori, *Nonlinear actuator fault detection and isolation for a VTOL aircraft*, 2001 American Control Conference, pp. 4449-4454, Alington, VA, 2001.
16. T. T. Georgiou and A. Lindquist, *Kullback-Leibler approximation of spectral density functions*, Proceedings. 42nd IEEE Conference on Decision and Control, 2003,Volume: 4, pp 4237 - 4242.
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18. A. Isidori, L. Marconi, A. Serrani, *Observability Conditions for the Semiglobal Output Regulation of Non-Minimum Phase Nonlinear Systems*, pp. 55-60, CDC 2003
19. A. Isidori, L. Marconi, A. Serrani, *Robust nonlinear motion control of a helicopter*, CDC 2001, pp. 4586-4591.
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Books:

1. *Nonlinear control in the year 2000. Vol. 2.* Papers from the 2nd Workshop of the Nonlinear Control Network held in Paris, June 5–9, 2000. Edited by Alberto Isidori, Franoise Lamnabhi-Lagarrigue and Witold Respondek. Lecture Notes in Control and Information Sciences, 259. Springer-Verlag London, Ltd., London, 2001. xii+626 pp.
2. *Directions in mathematical systems theory and optimization*, Dedicated to Anders Lindquist on the occasion of his 60th birthday. Edited by Anders Rantzer and Christopher I. Byrnes. Lecture Notes in Control and Information Sciences, 286. Springer-Verlag, Berlin, 2003. xiv+389 pp.
3. A. Isidori, L. Marconi, A. Serrani, *Robust Motion Control: and Internal-Model Approach*, Springer Verlag, 2003.

Book Chapters:

1. C.I. Byrnes, *Toward a nonequilibrium theory for nonlinear control systems*, Nonlinear control in the year 2000, Vol. 1 (Paris), 253–275, Lecture Notes in Control and Inform. Sci., 258, Springer, London, 2001.
2. C.I. Byrnes, F. Celani, A. Isidori, *Omega limit sets of systems that are semiglobally practically stabilized*, Preprints of The Royal Swedish Academy of Sciences Institut Mittag-Leffler, REPORT No. 47, 2002/2003, spring ISSN 1103-467X ISRN IML-R- -47-02/03--SE+spring.
3. C. I. Byrnes, D. Gilliam A. Isidori and J. Ramsey, *On the steady-state behavior of forced nonlinear systems*, Preprints of The Royal Swedish Academy of Sciences Institut Mittag-Leffler, REPORT No. 06, 2002/2003, spring, ISSN 1103-467X, ISRN IML-R- -06-02/03--SE+spring.

4. C.I. Byrnes, D.S. Gilliam, V.I. Shubov, *Geometric theory of output regulation for linear distributed parameter systems.* Research directions in distributed parameter systems (Raleigh, NC, 2000), 139–167, Frontiers Appl. Math., 27, SIAM, Philadelphia, PA, 2003.
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6. C.I. Byrnes, D.S. Gilliam, A. Isidori, Y. Ikeda, L. Marconi, *Internal model based design for the suppression of harmonic disturbances*, Directions in mathematical systems theory and optimization, 51–70, Lecture Notes in Control and Inform. Sci., 286, Springer, Berlin, 2003.
7. C. I. Byrnes, T.T. Georgiou, A. Lindquist and A. Megretski, *Generalized interpolation in H^∞ with a complexity constraint*, Transactions of the American Mathematical Society, to be published (also available as Institut Mittag-Leffler Report No. 28 2002/2003 spring).
8. C.I. Byrnes, A. Isidori, L. Praly, *On the Asymptotic Properties of a System Arising in Non-equilibrium Theory of Output Regulation*, Preprints of The Royal Swedish Academy of Sciences Institut Mittag-Leffler, REPORT No. 18, 2002/2003, spring ISSN 1103-467X ISRN IML-R- -18-02/03- -SE+spring.
9. C. I. Byrnes and A. Lindquist, *A convex optimization approach to generalized moment problems*, Control and Modeling of Complex Systems: Cybernetics in the 21st Century: Festschrift in Honor of Hidenori Kimura on the Occasion of his 60th Birthday, K. Hashimoto, Y. Oishi and Y. Yamamoto, Editors, Birkhäuser, 2003, 3–21.
10. C.I. Byrnes, A. Lindquist, *The uncertain generalized moment problem with complexity constraint*, New trends in nonlinear dynamics and control, and their applications, 267–278, Lecture Notes in Control and Inform. Sci., 295, Springer, Berlin, 2003.
11. C.I. Byrnes, A. Lindquist, *The uncertain generalized moment problem with complexity constraint*, Preprints of The Royal Swedish Academy of Sciences Institut Mittag-Leffler, REPORT No. 04, 2002/2003, spring ISSN 1103-467X ISRN IML-R- -04-02/03- -SE+spring
12. C.I. Byrnes, A. Lindquist, *A convex optimization approach to generalized moment problems*, Control and modeling of complex systems (Tokyo, 2001), 3–21, Trends Math., Birkhäuser Boston, Boston, MA, 2003.
13. R. Nagamune and A. Lindquist, *Sensitivity shaping in feedback control and analytic*

interpolation theory, Optimal Control and Partial Differential Equations,J.L. Medaldi et al (editors), IOS Press, Amsterdam, 2001, pp. 404–413.

14. A. Serrani, A. Isidori, C.I. Byrnes, L. Marconi, *Recent advances in output regulation of nonlinear systems*, Nonlinear control in the year 2000, Vol. 2 (Paris), 409–419, Lecture Notes in Control and Inform. Sci., 259, Springer, London, 2001.

Theses:

1. P. Enqvist, *Spectral estimation by Geometric, Topological and Optimization Methods*, PhD thesis, Optimization and Systems Theory, Royal Institute of Technology, Stockholm, Sweden, 2001.
2. R. Nagamune, *Robust Control with Complexity Constraint: A Nevanlinna-Pick Interpolation Approach*, PhD thesis, Optimization and Systems Theory, Royal Institute of Technology, Stockholm, Sweden, 2002.
3. J.B. Hood, *Output Regulation for a Boundary Controlled Two-dimensional Heat Equation*, MS thesis, Texas Tech University, 2002.
4. R. Koskodon, *Approximation Methods for Output Regulation of Nonlinear Systems*, MS thesis, Texas Tech University, 2003.

6 Scientific Interactions/Transitions

In February 2002, Dr. Gilliam and Victor Shubov collaborated with Dr. John Burns of the AFOSR Center for Optimal Design and Control at VPI & State University. These discussions were concerned with the design of special sensors that damp high frequency oscillations. Applications include problems in regulation (such as active noise suppression) and various topics in hydrodynamics including applications to large eddy simulations (les). To date there have been four technical meetings in February 2002; December 2002; January 2003; August 2003; and October 2003.

Professors A. Lindquist and C.I. Byrnes were Scientific Leaders for The Semester on Mathematical Control and Systems Theory at the Mittag-Leffler Mathematical Institute, Royal Swedish Academy of Sciences, Djursholm, Sweden, January - June, 2003.

In addition to collaborative research with engineering research and development personnel at Boeing, St. Louis, MO, reported below as transitions, we have presented many invited lectures and colloquia nationally and internationally:

December 2000:

- Invited Speaker at the Plenary Panel *25 Seminal Papers in Control (1932-1981)*, 39th IEEE Conference on Decision and Control (Syndey, Australia), presented by Dr. A. Isidori.
- *Nonlinear robust output regulation for nonlinear systems*, Department of Physics, Washington University, St. Louis. Presented by James Ramsey.

February 2001 :

- *Synthetic speech and modern mathematics: What is the connection?* Seminar lecture presented by Professor Anders Lindquist at Stockholm University.
- *Nonlinear fault detection and isolation: a differential geometric approach*, Presented by Dr. A. Isidori at the annual GAMM Meeting, Zurich, (Invited plenary lecture).
- *Nevanlinna-Pick Interpolation with applications to systems and signals*, Modelling, Identification and Control Conference,Innsbruck Austria,presented by Dr. C.I. Byrnes.

April 2001 :

- *Nonlinear output regulation with adaptive internal model*, Presented by Dr. A. Isidori at KHT Stockholm, (Invited Lecture).
- *Nonlinear output regulation with adaptive internal model*, Presented by Dr. A. Isidori at Technical University of Delft, (Invited Lecture).
- *Nonlinear fault detection and isolation: a differential geometric approach*, Presented by Dr. A. Isidori at University of Lyon, (Invited Lecture).

May 2001 :

- *An analytic interpolation approach to robust control*, presented by Professor Anders Lindquist at Russian-Swedish Control Conference, Moscow (Plenary lecture).
- *Synthetic speech and modern mathematics: What is the connection?*, Seminar lecture presented by Professor Anders Lindquist at Uppsala University.

June 2001 :

- *Practical stabilizability and tracking for nonlinear systems: the nonequilibrium case*, London Mathematical Society Workshop on Mathematical Theory of Nonlinear Control, presented by Dr. C.I. Byrnes (Plenary Lecture).

July 2001 :

- *Nonlinear fault detection and isolation: a differential geometric approach*, Presented by Dr. A. Isidori at the IFAC NOLCOS Symposium (Invited plenary lecture).
- *Dynamics of airflows containing dust particles and fluid suspensions*, 5th SIAM Conference of Control and Applications, San Diego, CA., July 2001, Victor Shubov (Invited Lecture).
- *A global analysis approach to robust control*, presented by Professor Anders Lindquist at the 5th IFAC symposium on Nonlinear Control Systems (NOLCOS 2001 IFAC), Saint-Petersburg, Russia, (Plenary lecture).
- *Practical stabiliability and tracking for nonlinear systems: the nonequilibrium case*, at the 5th IFAC symposium on Nonlinear Control Systems (NOLCOS 2001 IFAC), Saint-Petersburg, Russia, presented by Dr. C.I. Byrnes (Plenary lecture).

August 2001 :

- *Partial Realization Theory: A Basic Paradigm in Signals, Systems and Control*, Presented by Dr. Anders Lindquist at the Fourth SIAM Conference on Linear Algebra in Signals, Systems and Control, Boston, USA, (Plenary lecture).

October 2001:

- *Output Regulation and Regulator Equations*, 'Invited lecture presented by Dr. David Gilliam at the Texas Tech Student SIAM Conference.

November 2001 :

- *Convex Optimization Algorithms for Classical Moment Problems, with Applications to Systems and Signals*, Presented by Dr. Christopher Byrnes at Cybernetics in the 21st Century: Information and Complexity in Control Theory, University of Tokyo, Japan, (Invited lecture).
- *A global analysis approach to robust control*, Presented by Dr. Anders Lindquist at Cybernetics in the 21st Century: Information and Complexity in Control Theory, University of Tokyo, Japan, (Invited lecture).
- *A short course on Nevanlinna-Pick interpolation theory*, Eight hours of lectures presented by Dr. Anders Lindquist at Abo Academi University, (Invited lectures).

December 2001:

- *Finesse et Geometrie: the Spirits of Nonlinear Control*, the Bode (plenary) Lecture delivered by A.Isidori at the annual IEEE Conference on Decision and Control, Orlando (Florida), Dec. 2001.
- *On the duality between filtering and Nevanlinna-Pick interpolation*, Presented by Professor Anders Lindquist at the 40th IEEE Conference on Decision and Control (CDC01), Orlando, Florida.
- *Examples of output regulation for distributed parameter systems with infinite dimensional exosystem*, Presented by Dr. David Gilliam at the 40th IEEE Conference on Decision and Control, Orlando Florida.
- *Examples of Regular Linear Systems Governed by Partial Differential Equations*, Presented by Dr. David Gilliam at the 40th IEEE Conference on Decision and Control, Orlando Florida.

February 2002 :

- *Convex Optimization Algorithms for Classical Moment Problems, with Applications to Systems and Signals*, Presented by Dr. Christopher Byrnes at the Mathematisches Forchungsinstitut Oberwohlfach, Oberwohlfach, Germany (Invited lecture).
- *Robust tracking of uncertain systems*, Presented by Dr. Alberto Isidori at the Mathematisches Forchungsinstitut Oberwohlfach, Oberwohlfach, Germany (Invited lecture).
- *Regular linear systems generated by parabolic equations*, Lecture presented by Dr. Victor I. Shubov at the Texas PDE Conference, San Antonio, TX.

March 2002 :

- *A New Approach to the Generalized Moment Problem, with Applications to Systems, Signals and Control*, Presented by Dr. Christopher Byrnes at the Department of Mathematics, Fudan University, Shanghai, China.

April 2002 :

- *A convex optimization approach to generalized moment problems*, Invited colloquium lecture by Professor Anders Lindquist at the Stieltjes Analysis Colloquium, Thomas Stieltjes Institute for Mathematics, Amsterdam, the Netherlands, April 8, 2002. May 2002
- *Shaping the Steady-State Response of Nonlinear Systems*, Invited Lecture presented

by Dr. Christopher I. Byrnes at AFOSR Conference on Future Directions in Control, Arlington, VA.

June 2002 :

- *A New Appraoch to the Generalized Moment Problem, with Applications to Systems, Signals and Control*, Presented by Dr. Christopher Byrnes at Universita' di Padova, Padova, Italy.
- *Shaping the Steady-State Response of Nonlinear Systems*, Invited Lecture presented by Dr. Christopher I. Byrnes at Universita' di Bologna, Bologna, Italy.

July 2002 :

- *Finesse et Geometrie: les esprits de l'automatique non-lineaire*, Invited lecture presented by A.Isidori at the Ecole Normale Superieure, Paris.
- *A convex optimization approach to generalized moment problems*, Invited lecture presented by Professor Anders Lindquist at MTNS 2002, Notre Dame.
- *Output Regulation for DPS with Infinite Dimensional Exosystem*, Invited lecture presented by Dr. David Gilliam at MTNS 2002, Notre Dame.

August 2002 :

- *Output Regulation for Nonlinear Systems*, one day workshop at the IFAC World Congress, Barcelona, presented by Drs. C.I. Byrnes, A. Isidori, L. Marconi, A. Serrani.

October 2002:

- Gordon McKay Lecture Series at University of California, Berkeley, Invited lecture presented by Dr. A. Lindquist.
- *Analytic interpolation with degree constraint with applications to systems and control and signal processing*, ' Distinguished Lecturer in EECS Joint Colloquium, UC Berkeley, Invited lecture presented by Dr. A. Lindquist.
- *Shaping the steady state response of nonlinear control systems*, Department of Mathematics and Statistics Texas Tech University Lubbock, Texas, Invited lecture presented by Dr. C.I. Byrnes.
- *Shaping the steady state response of nonlinear control systems*, Symposium on New Trends in Nonlinear Dynamics and Control, US Naval Postgraduate School, Monterey, California, Plenary Lecture presented by Dr. C.I. Byrnes.

- *Robust Tracking of Uncertain Trajectories, with Application to Helicopter Landing*, Symposium on New Trends in Nonlinear Dynamics and Control, US Naval Postgraduate School, Monterey, California, Plenary Lecture presented by Dr. A. Isidori.
- *A convex optimization approach to generalized moment problems*, Symposium on New Trends in Nonlinear Dynamics and Control, and their Applications, US Naval Postgraduate School, Monterey, California, Invited Lecture presented by Dr. A. Lindquist.

November 2002:

- *Internal model based design for the suppression of harmonic disturbances*, Directions in Mathematical System Theory and Optimization, Royal Institute of Technology, Stockholm, Sweden, Invited Lecture presented by Dr. C.I. Byrnes.
- *On the solution of the regulator equations*, SIAM Conf., Texas Tech University, Invited Lecture presented by Dr. D.S. Gilliam.
- *An optimization approach to generalized moment problems with complexity constraints*, seminar at Uppsala University, Sweden, Invited Lecture presented by Dr. A. Lindquist.

December 2002 :

- *Shaping the steady state response of nonlinear control systems*, 41st IEEE Conference on Decision and Control Las Vegas, Nevada, Plenary Lecture presented by Dr. C.I. Byrnes.
- *Identifiability of shaping filters from covariance lags, cepstral windows and Markov parameters*, 41st IEEE Conference on Decision and Control Las Vegas, Nevada, lecture presented by Dr. C.I. Byrnes.
- *The regulator equations for retarded delay differential equations*, 41st IEEE Conference on Decision and Control, Las Vegas, NV, lecture presented by Dr. D.S. Gilliam
- *Modeling modal based sensors for oscillatory systems*, 41st IEEE Conference on Decision and Control, Las Vegas, NV, lecture presented by Dr. D.S. Gilliam
- *An optimization approach to generalized moment problems with complexity constraints*, seminar at the Royal Institute of Technology, Stockholm, Sweden, Invited lecture presented by Dr. A. Lindquist.

January 2003:

- *Output regulation for distributed parameter systems with infinite dimensional exosystem*, Institut Mittag-Leffler, Djursholm, Sweden, Invited lecture presented by Dr. D.S.

Gilliam.

- *Shaping the Steady State Response of Nonlinear Control Systems*, Institut Mittag-Leffler, Djursholm, Sweden, Invited lecture presented by Dr. C.I. Byrnes.
- *Stability of gas and fluid flows containing particles*, Institut Mittag-Leffler, Djursholm, Sweden, Invited lecture presented by Dr. V. Shubov.
- *Regular linear systems governed by parabolic equations*, Institut Mittag-Leffler, Djursholm, Sweden, Invited lecture presented by Dr. V. Shubov.

February 2003:

- *Shaping the steady state response of nonlinear control systems*, Departments of Electrical Engineering and Systems Science and Mathematics Washington University St. Louis, Missouri, Invited lecture presented by Dr. C.I. Byrnes.
- *Output regulation with infinite dimensional exosystem and decay of generalized Fourier coefficients for Gevrey class functions*, Florida State University, Tallahassee, FL., Invited lecture presented by Dr. V. Shubov.

March 2003:

- *An optimization approach to generalized moment problems with complexity constraints*, seminar at Institut Mittag-Leffler, Djursholm, Sweden, Invited lecture presented by Dr. A. Lindquist.

April 2003:

- *Disturbance suppression via state feedback for Hamiltonian systems*, IFAC Workshop on Lagrangian and Hamiltonian Methods in Nonlinear Control, Sevilla, Invited lecture presented by Dr. A. Isidori.
- *A global analysis approach to robust control*, seminar at Institut Mittag-Leffler, Djursholm, Sweden, Invited lecture presented by Dr. A. Lindquist.

May 2003:

- *Toward a general non-equilibrium theory of output regulation*, Institut Mittag-Leffler, Invited lecture presented by Dr. A. Isidori .

June 2003:

- *A global analysis approach to robust control*, Workshop on Geometry in Nonlinear Control, Stephan Banach International Mathematical Center, Poland, Invited lecture presented by Dr. A. Lindquist.
- *Nonequilibrium Output Regulation for Nonlinear Distributed Parameter Systems*, Institut Mittag-Leffler, Djursholm, Sweden, Invited lecture presented by Dr. C.I. Byrnes

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July 2003:

- *Nonequilibrium output regulation for nonlinear control systems*, Computation, Control and Biological Systems, Department of Mathematics and Statistics, Montana State University, Bozeman, Montana, Invited lecture presented by Dr. C.I. Byrnes .
- *Nonequilibrium output regulation for distributed parameter systems*, Computation, Control and Biological Systems, Department of Mathematics and Statistics, Montana State University, Bozeman, Montana, Invited lecture presented by Dr. D.S. Gilliam.

September 2003:

- *Nonequilibrium Output Regulation*, AFOSR Contractors Meeting Dynamics and Control, Destin, Florida, Invited lecture presented by Dr. C.I. Byrnes.

November 2003:

- *Nonequilibrium Output Regulation*, New Directions in Control Theory and Applications, Texas Tech University, Lubbock, TX, November 14 - 15, Invited lecture presented by Dr. C.I. Byrnes.
- *Internal Model Adaptation in Non-equilibrium Theory of Output Regulation*, New Directions in Control Theory and Applications, Texas Tech University, Lubbock, TX, November 14 - 15, Invited lecture presented by Dr. A.Isidori.
- *Generalized Interpolation in H^∞ Solutions with Bounded Complexity*, New Directions in Control Theory and Applications, Texas Tech University, Lubbock, TX, November 14 - 15, Invited lecture presented by Dr. A.Lindquist.

Transitions:

In September of 2001 Dr. Byrnes, Gilliam and Isidori began a collaborative project on control of UAV, UCAV and tailless aircraft with Dr. Yutaka Ikeda Automatic Air Collision Avoidance Systems, Phantom Works, The Boeing Company (yutaka.ikeda@boeing.com).

The purpose of the project was to transition their prior work, and recent extensions thereof, on robust output regulation to flight control technologies for UCAVs.

Six technical meetings, were held on: September 15, 2001; October 15, 2001; November 8, 2001; March 7, 2002; April 19, 2002; and May 24, 2002. These working meetings focused on transitioning this work to flight control methodologies for take-off and landing of the Boeing TAFA model, with the transition of that work to UAVs and UCAVs being done by Boeing engineers. This project resulted in the publication of one paper on the suppression of harmonic disturbances in the measured roll and yaw rates in UCAV's:

Byrnes, Christopher I.; Gilliam, David S.; Isidori, Alberto; Ikeda, Yutaka; Marconi, Lorenzo Internal model based design for the suppression of harmonic disturbances. Directions in mathematical systems theory and optimization, 51–70, Lecture Notes in Control and Inform. Sci., 286, Springer, Berlin, 2003.

7 New Discoveries, Inventions or Patent Disclosures

We have made 3 patent disclosures and been granted 3 U.S. patents in the areas of speech processing and signal processing. Two of the patents relate to speech processing. The first is a new methodology for speech synthesis and processing, with contemplated applications in telephony. The second is a new methodology, based on the first, for speaker recognition with application in the use of speech as a biometric in security systems. We believe this methodology would be of major interest to the DoD and other agencies using security systems.

The third, and most recent, patent discloses a new approach to spectral estimation which is tunable to be higher resolution over prescribed frequency bands than other existing methods. We believe this methodology would also be of major interest to the DoD.

The status of the patents are:

The European patent was applied for the US Patent No 5,940,791 - Method and apparatus for speech analysis and synthesis using lattice ladder notch filters, with Drs. C.I. Byrnes and A.G. Lindquist as inventors.

On July 3, 2001 US Patent No. 6,256,609 - Method and apparatus for speech analysis and synthesis including speaker recognition, - was issued to Washington University , with Drs. C.I. Byrnes and A.G. Lindquist as inventors.

US Patent 6,400,310, Method and apparatus for a tunable high-resolution spectral estimator, granted to Drs. C.I. Byrnes, T. Georgiou and A.G. Lindquist through the University of Minnesota and Washington University on June 4, 2002.

Canadian and European applications pending for the international extension of the US Patent 6,400,310, Method and apparatus for a tunable high-resolution spectral estimator.

8 Additional Information, Awards and Honors

- 3 IEEE Fellows (Dr.s C.I. Byrnes, A. Isidori, A. Lindquist).

- At the 42nd IEEE CDC, Maui, Hawaii, in December 2003, C. I. Byrnes, T. Georgiou and A. Lindquist was awarded the 2003 IEEE George S. Axelby Award for the best paper in the IEEE Trans. on Aut. Control.
- The paper A convex optimization approach to the rational covariance extension problem, by C. I. Byrnes, S. V. Gusev and A. Lindquist was selected in 2000 to be published in an enhanced form in SIAM Review as a SIGEST paper.
- At the 40th IEEE CDC, Orlando, Florida, in December 2001, A.Isidori was awarded the 2001 Hendrik W. Bode Lecture prize from the Control Systems Society of IEEE.
- IFAC Best Paper Award, (C.I. Byrnes and A. Isidori), 1993 IFAC World Congress.
- IEEE George S. Axelby Award as the best paper in the IEEE Trans. on Aut. Control, 1991 (C.I. Byrnes and A. Isidori).

- Dr. C.I. Byrnes, elected Fellow of the Academy of Sciences of St. Louis in 1998.
- Dr. C.I. Byrnes was awarded an Honorary Doctorate of Technology from the Swedish Royal Institute of Technology, November 1998.
- Dr. C.I. Byrnes was elected in March 2001 as a Foreign Member of the Royal Swedish Academy of Engineering Sciences.
- The Graduate College Distinguished Research Award: C.I. Byrnes, 1988, ASU.
- Fellow, Japanese Society for the Promotion of Science: C.I. Byrnes, 1986.

- A. Isidori, The Differential Geometric Approach to the Detection of Faults in Nonlinear Systems, Plenary Lecture, NOLCOS 2001.
- A.Isidori, Finesse and Geometrie: the Spirit of Nonlinear Feedback, Bode Plenary Lecture, CDC 2001.

- Quazza Medal awarded to Dr. A. Isidori at 13th IFAC World Congress in San Francisco, 1996 for Pioneering and Fundamental Contributions to the Design of Nonlinear Feedback Systems.
- Alberto Isidori was listed in the Highly-Cited database among the top 10 most-cited authors in Engineering in the world for the period 1981-1999.

- Dr. A. Lindquist, Foreign Member of Russian Academy of Natural Sciences, 1997.
- Dr. A. Lindquist elected Member of the Royal Swedish Acad. of Engr. Sci., 1996.
- Dr. A. Lindquist, Honorary Member of Hungarian Operational Res. Soc., 1994.