AFIT/EN-TR-01-07
TECHNICAL REPORT
February 2001

Reflectivity and Transmissivity through Layered, Lossy Media: A User-Friendly Approach

Kenneth J. Pascoe

GRADUATE SCHOOL OF ENGINEERING AND MANAGEMENT
AIR FORCE INSTITUTE OF TECHNOLOGY
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 074-0188

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From – To)*
02-15-2001	Technical Report	Jan 2001 - Feb 2001

4. TITLE AND SUBTITLE

REFLECTIVITY AND TRANSMISSIVITY THROUGH LAYERED, LOSSY MEDIA: A USER-FRIENDLY APPROACH

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Pascoe, Kenneth J., Captain, USAF

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(S)

Air Force Institute of Technology
Graduate School of Engineering and Management (AFIT/EN)
2950 P Street, Building 640
WPAFB OH 45433-7765

8. PERFORMING ORGANIZATION REPORT NUMBER

AFIT/EN-TR-01-07

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
The theory behind the use of layers of radar absorbing materials or other dielectric materials is identical to the theory of optical reflection and transmission through layered media. This report is intended to be of use to students studying the application of layered media to a radar cross-section reduction problem. In this report, we survey several established optics and electromagnetics texts. We critique them and attempt to reconcile differences. We arrive at a single consistent theory which fully considers lossy materials. Layers are depicted as matrices which can be multiplied to combine the effects of several adjacent layers. We can then find the transmissivity and reflectivity of the entire multiple-layer structure. This theory is implemented in the MATLAB language in a user-friendly format.

15. SUBJECT TERMS
Electromagnetic Wave Reflections, Electromagnetic Properties, Reflectivity, Transmissivity, Optical Properties, Dielectric Properties

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			Prof V. P. Pyati, ENG
U	U	U	UU	22	**19b. TELEPHONE NUMBER** *(Include area code)* (937) 255-3636, ext 4576

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18

Form Approved
OMB No. 074-0188

Abstract

The theory behind the use of layers of radar absorbing materials or other dielectric materials is identical to the theory of optical reflection and transmission through layered media. This report is intended to be of use to students studying the application of layered media to a radar cross-section reduction problem. In this report, we survey several established optics and electromagnetics texts. We critique them and attempt to reconcile differences. We arrive at a single consistent theory which fully considers lossy materials. Layers are depicted as matrices which can be multiplied to combine the effects of several adjacent layers. We can then find the transmissivity and reflectivity of the entire multiple-layer structure. This theory is implemented in the MATLAB language in a user-friendly format.

Table of Contents

Table of Figures

1. Introduction

This report details an attempt to apply electromagnetic and optical theory to the design of layered dielectric absorbers. We survey several established optics and electromagnetics texts. We critique them and attempt to reconcile differences. We arrive at a single consistent theory which fully considers lossy materials. This theory is implemented in the MATLAB language in a user-friendly format.

AFIT teaches a sequence of courses in electromagnetics leading to a capstone course in radar cross-section (RCS) measurement, analysis and reduction. Some RCS reduction techniques involve the use of layers of dielectric media. This report is intended to be of use to students studying the application of layered media to an RCS reduction problem.

2. Theory

The theory behind the use of layers of radar absorbing materials or other dielectric materials is identical to the theory of optical reflection and transmission through layered media.

Several books discuss theory of reflection and transmission through a single interface (two layers), but treatment of three layers is a bit less common, and the available coverage is less comprehensive. Discussion of arbitrarily layered material is more rare and limited. Our goal is treatment of arbitrary numbers of layers, each of which may be lossy. We restrict our attention to uniform plane waves arriving through air, but allow for any angle from normal (to the surface) to 85 degrees from normal. We consider two polarization components, called Transverse Electric (TE) and Transverse Magnetic (TM). All plane waves can be treated as the sum of TE and TM components. TE waves are those in which the electric field is directed perpendicular to the plane of incidence, the plane formed by the incident ray and the surface normal. TE waves are also called perpendicularly-polarized (Ruck, et al., 1970:473), or s-polarized (Macleod, 1986:22). Similarly, TM waves have the magnetic field directed normal to the plane of incidence. TM waves are also called parallel-polarized (Ruck, et al., 1970:473) or p-polarized (Macleod, 1986:22). In this report, the terms TE and TM are used hereafter.

A complicating factor in the study of layered media is the confusion and disagreement among various writers on the subject. Ruck writes a cautionary footnote, in which he criticizes the frequent statement that reflectivity R plus transmissivity T always add to 1 due to conservation of energy. He points out the correct relationship between reflection and transmission coefficients (ρ and τ), which are field quantities related to the square roots of reflectivity and transmissivity, respectively: $1 + \rho = \tau$. The $R + T = 1$ relationship only applies to a lossless dielectric layer with the same medium on both sides (Ruck, et al., 1970:474). Unfortunately, Ruck contradicts his own warning in the development of Figure 7-6 in the same chapter, shortly after the footnote appears (Ruck, et al., 1970:484).

2.1 Comparison of existing theory

The initial goal was to apply the matrix method used by Born and Wolf. This method, developed for optical theory, can determine reflectivity and transmissivity through an arbitrary number of lossless dielectric layers. The characteristic matrix method assumes a lossless dielectric substrate, but can be modified to account for a perfect electrical conductor (PEC) substrate (Born and Wolf, 1975:51-70). The limitations of Born and Wolf's approach (lossless materials) drove a search for more comprehensive theory. Stratton and Balanis both cover lossy materials in detail, but neither offers a technique for an arbitrary number of layers. Either author's technique can be extended to multiple layers, but only at the cost of greatly increased complexity (Stratton, 1941:500-516, Balanis, 1989:206-236). Balanis adopts a practical approximation for multiple layers, but his but it is limited to cases where the reflection coefficient at each interface is small (Balanis, 1989:229-230).

Tuley develops analysis techniques for layered media above a PEC or air surface, but assumes a conductive sheet at the interface of each layer (Salisbury screens and Jaumann absorbers). His discussion of material with no conductive sheets is limited to single layers on a PEC surface (Dallenbach layers). In both cases, Tuley considers lossy material (Knott, et al., 1993:298-327).

Ruck uses a matrix method which is similar to that of Born and Wolf, but not identical. While Ruck's matrix is different, he arrives at the same reflection and transmission coefficients as do Born and Wolf. Born and Wolf don't cover lossy materials within their characteristic matrix technique. Ruck mentions it, but does not develop enough detail to allow the reader to properly implement his theory. In fact, Ruck's lossy material example in Figure 7-6 has multiple errors. One is the contradiction of the footnote cited above. Others involve conflicting definitions of relative permittivity (Ruck, et al., 1970: 483, 484) and loss tangent (Ruck, et al., 1970: 477, 483).

Macleod uses the same matrix as Ruck, but deals more explicitly with lossy materials. He develops an expression for 'Absorptance,' the energy absorbed in lossy layers, and notes that

$$1 = R + T + A$$

where R, T and A stand for Reflectance, Transmittance, and Absorptance (Macleod, 1986:34-39). Other authors call the first two terms Reflectivity and Transmissivity, so the third will hereafter be called Absorptivity for consistence. There is no 'absorption coefficient' comparable to the reflection and transmission coefficients discussed by many authors. The equation is consistent with conservation of energy and is an interesting addition to Ruck's footnote discussed above. Macleod's book, although narrowly focused on thin-film optical filters, has the most generally applicable theory.

2.2 Characteristic Matrix Method for Lossy Materials

One benefit of the characteristic matrix method (Born and Wolf, 1975, Ruck, et al., 1970, Macleod, 1986), is that any number of layers can be analyzed. The computations do not grow in complexity as layers are added. Although more matrices must be computed and multiplied together, these steps are easily handled with a simple program, such as the one listed in the Appendix. As the number of layers changes, the equations for transmission and reflection coefficients are unchanged, as are the equations for reflectivity, transmissivity, and absorptivity.

The method assumes plane waves incident through a lossless medium (usually taken to be air) on infinite planar surfaces. If the first layer is lossy, the theory described in this paper does not apply (Macleod, 1986). Although this is a severe limitation, as a practical matter we are nearly always interested in transmission through air to a target. The plane wave assumption is a good one in the far zone of the radiation source. The assumption of the infinite planar surface is useful as an approximation when the surface area is smooth and large relative to a wavelength.

Figure 1 shows the material layers through which the incident energy passes. The solid arrows indicate the basic structure of the problem, while the dotted arrows indicate some of the many rays produced by multiple reflections. In fact, there are many more rays than shown, as reflected and transmitted rays are produced every time a ray strikes an interface. The matrix method combines the effects of each intermediate layer (Layers 2, 3 and 4 in Figure 1) into a single layer. The reflectivity is referenced to the interface of Layers 1 and 2, while the transmissivity is referenced to the interface of the last layer and the next-to-last. The problem is thus simplified into that shown in Figure 2. The reflectivity and transmissivity calculated here consider the combined effects of all rays transmitted into the first and last layers, respectively.

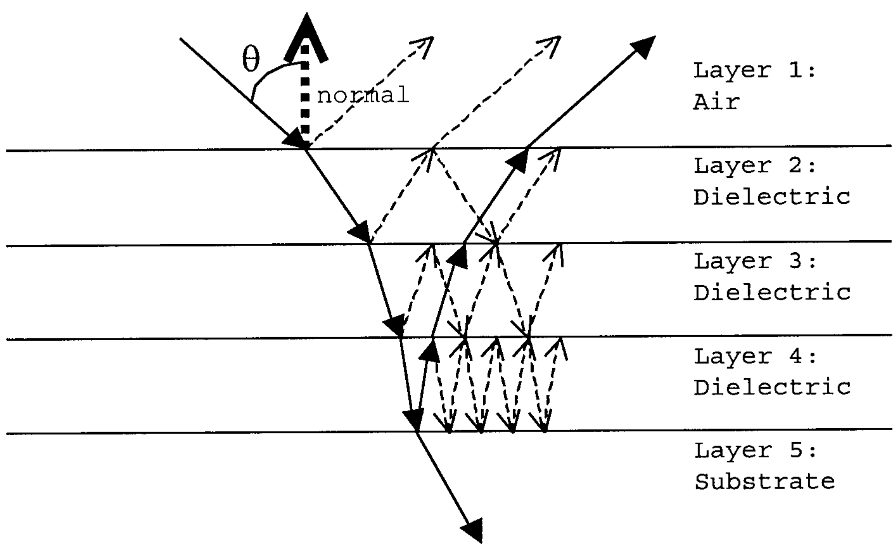

Figure 1. Geometry of the problem (most rays omitted)

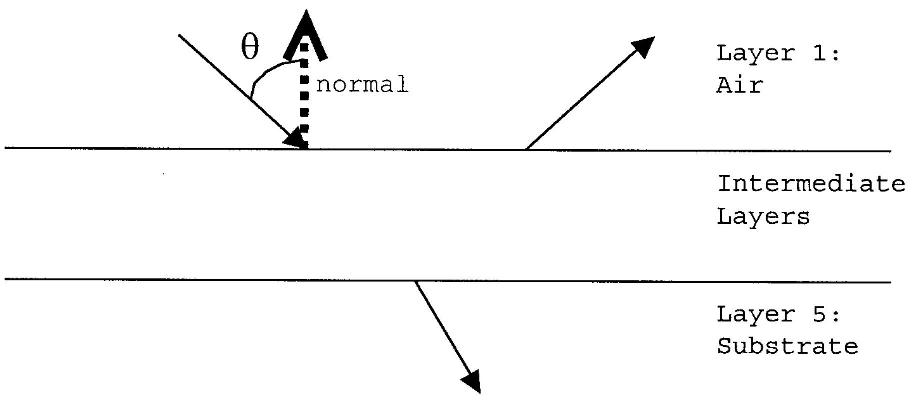

Figure 2. Simplified problem geometry

The characteristic matrix method takes into account the thickness and material properties of each layer, as well as the incident angle through air, where 0 degrees is normal to the surface.

Y is the admittance (inverse of the impedance) of the material layer. It is determined from the electric permittivity ε, the conductivity σ, the magnetic permeability μ, and the frequency ω (radians per second).

$$Y = \sqrt{\frac{\sigma + j\omega\varepsilon}{j\omega\mu}}.$$

Conductivity can be treated as the imaginary component of permittivity, or as a separate term. Different authors use different conventions, but all are accommodated by the equation above if we note that ε and μ can be complex.

The propagation constant γ is found from the same material properties which define the admittance. The real and imaginary parts of γ are known as the attenuation and phase constants, α and β (Balanis, 1989:107). Older authors instead refer to the wave number k, which is complex in the case of lossy material (Stratton, 1941:501). If k is real, it is equivalent to the phase constant β, while if it is complex, it is related to γ and the material properties by

$$\gamma = \sqrt{(\sigma + j\omega\varepsilon)(j\omega\mu)} = jk\,.$$

Given the incident angle, the angle through all remaining layers can be found using Snell's Law of Refraction. The subscripts m and n refer to two adjoining layers. To properly allow for the effects of attenuation, the angle θ must be considered complex (Balanis, 1989:210).

$$\gamma_m \sin\theta_m = \gamma_n \sin\theta_n$$

The 'tilted admittance' (Macleod, 1986:25) is a term comprising both the material admittance and a factor due to incident angle θ and polarization. The tilted admittance accounts for the cosine factor present in the phase velocity of the transmitted wave. It is found as follows, based on polarization,

$$\eta = \mathrm{Y}\cos\theta \quad \text{for TE polarization and}$$

$$\eta = \mathrm{Y}/\cos\theta \quad \text{for TM polarization.}$$

We use γ, θ and the layer thickness d to develop an intermediate quantity δ which is input directly into the matrix (Macleod, 1986:33).

$$\delta = \mathrm{j}\gamma\,\mathrm{d}\cos\theta\,.$$

So, given the material properties, incident wave frequency polarization and angle, we can now develop the matrix $\underline{\underline{M}}$ which is the key to our effort

$$\underline{\underline{M}} = \begin{bmatrix} \cos\delta & \left(j/\eta\right)\sin\delta \\ j\eta\sin\delta & \cos\delta \end{bmatrix}.$$

A separate matrix $\underline{\underline{M}}_i$ is developed for each layer, exclusive of the first (air) and last (substrate). The matrices for each intermediate layer are then multiplied, in order, starting with the layer next to air. The resulting product describes the total reflection, transmission, and absorption effects of the intermediate layers. For example, let layer 1 be air, and layer 8 be a PEC surface. The matrix describing the six dielectric layers between air and PEC is (Born and Wolf, 1975:55-59, Ruck, et al., 1970:480-481, Macleod, 1986:35-36)

$$\underline{\underline{M}}_{total} = \underline{\underline{M}}_2 \underline{\underline{M}}_3 \underline{\underline{M}}_4 \underline{\underline{M}}_5 \underline{\underline{M}}_6 \underline{\underline{M}}_7 .$$

Once the total characteristic matrix is found, the reflection coefficient ρ, reflectivity R, transmission coefficient τ, transmittivity T, and absorptivity A can be found. Reflectivity is the ratio of energy reflected to energy incident on the medium. Transmissivity is the ratio of energy transmitted through the medium to energy incident on it. Absorptivity is the ratio of energy absorbed by the medium to energy incident on it (Macleod, 1986:38). We will use the elements of the $\underline{\underline{M}}_{total}$ matrix to determine R, T and A.

$$\underline{\underline{M}}_{total} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}.$$

The reflection coefficient is (subscript L denotes the last layer, the substrate)

$$\rho = \frac{\eta_1 \left(m_{11} + m_{12}\eta_L\right) - \left(m_{21} + m_{22}\eta_L\right)}{\eta_1 \left(m_{11} + m_{12}\eta_L\right) + \left(m_{21} + m_{22}\eta_L\right)}.$$

The reflectivity is

$$R = \rho \rho *$$

where the asterisk denotes the complex conjugate. The transmission coefficient is

$$\tau = \frac{2\eta_1}{\eta_1 \left(m_{11} + m_{12}\eta_L\right) + \left(m_{21} + m_{22}\eta_L\right)}.$$

The transmissivity is

$$T = \mathrm{Re}\left(\frac{\eta_L}{\eta_1}\right)\tau\tau^* .$$

Absorptivity, unlike reflectivity and transmissivity, does not naturally break down into coefficients related to field strength (Macleod, 1986). The absorptivity is

$$A = \frac{4\eta_1 \mathrm{Re}\left[\left(\mathrm{m}_{11} + \mathrm{m}_{12}\eta_L\right)\left(\mathrm{m}_{21} + \mathrm{m}_{22}\eta_L\right)^* - \eta_L\right]}{\left[\eta_1\left(\mathrm{m}_{11} + \mathrm{m}_{12}\eta_L\right) + \left(\mathrm{m}_{21} + \mathrm{m}_{22}\eta_L\right)\right]\left[\eta_1\left(\mathrm{m}_{11} + \mathrm{m}_{12}\eta_L\right) + \left(\mathrm{m}_{21} + \mathrm{m}_{22}\eta_L\right)\right]^*} .$$

3. MATLAB Implementation

The complete 'basic version' of the MATLAB code `reftran.m` is listed in the Appendix. The 'test bed' versions discussed below are not included but can be developed easily by readers familiar with MATLAB.

The medium consists of several layers. The first is air through which the incident wave travels. The last is either a perfect electrical conductor (PEC) surface or a semi-infinite dielectric layer which may be specified to be air. The intermediate layers are dielectric materials which may be lossy. The thickness and material properties of each layer are assumed constant throughout the layer.

The user inputs a range of incident angles and the material properties of each layer at the command line. The output is a numerical and graphic depiction of transmissivity, reflectivity, and absorptivity. Additionally, reflection and transmission coefficient magnitudes are plotted.

The code is documented with comment statements including a list of variables. Although it currently inputs one material layer at a time via the command line, it can be modified to accept vectors of material data.

4. Results and Conclusions

The basic version of the `reftran.m` code was compared to data from Table III in Born and Wolf. We used variants of the code to match figures provided in various texts. These 'test bed' versions of the code were modified to vary the thickness or electric permittivity of a single intermediate layer at normal incidence.

The basic code shown in the appendix, or the test bed versions of it, were able to match Ruck, et al., figures 7-2 and 7-5, Born and Wolf figures 1.18, 1.19 and 13.5, Stratton figures 95 and 97, and Knott, Shaeffer and Tuley, figures 8.12 and 8.13. Some of these are shown below.

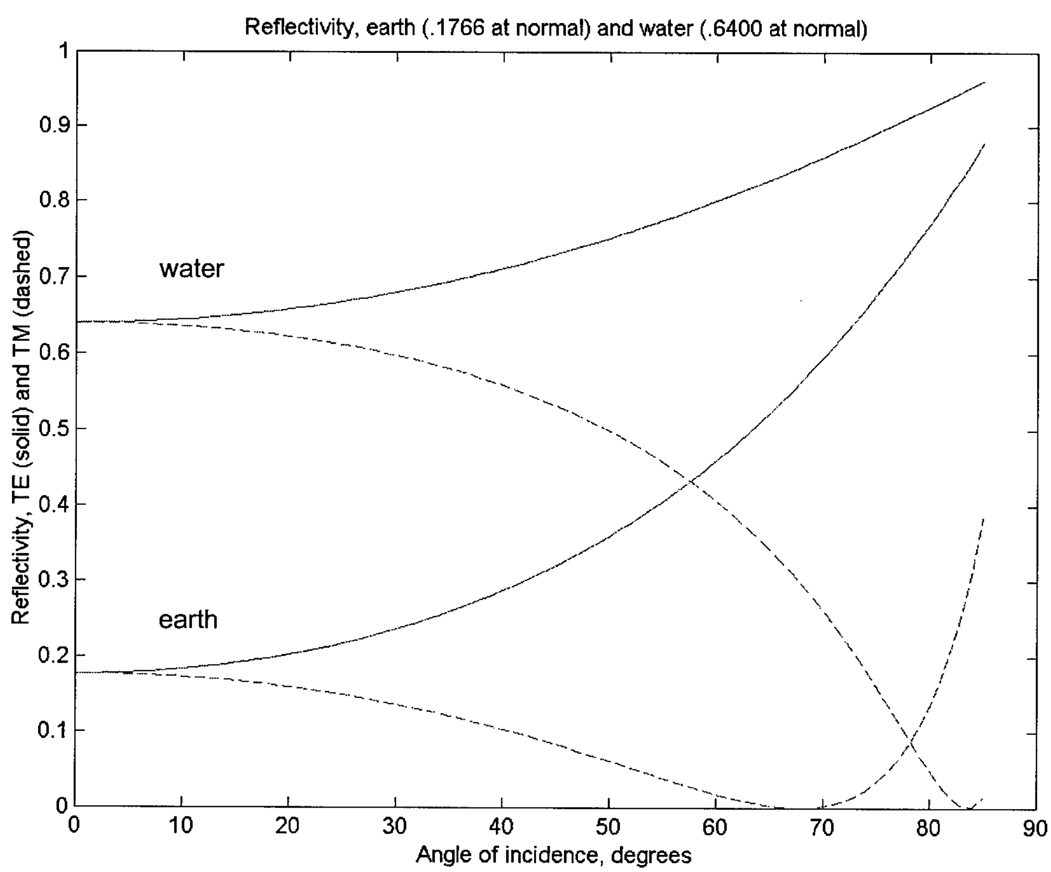

Figure 3. Reflectivity of fresh water and earth, TE and TM polarization

Figure 3 is the information shown in Figure 95 in Stratton, produced by `reftran.m`. Stratton deals with two layers in that case, so the thickness of layer two is set to zero; the resulting matrix M is the identity matrix. The frequency is 10.714 GHz. The relative electric permittivity of water is 81 + j (4.153 x 10^{-6}). For earth, relative permittivity is 6 + j (2.80 x 10^{-6}). Conductivity is set to zero in each case, as the imaginary part of the permittivity is used to represent conductivity instead. Relative magnetic permeability is set to 1 for each.

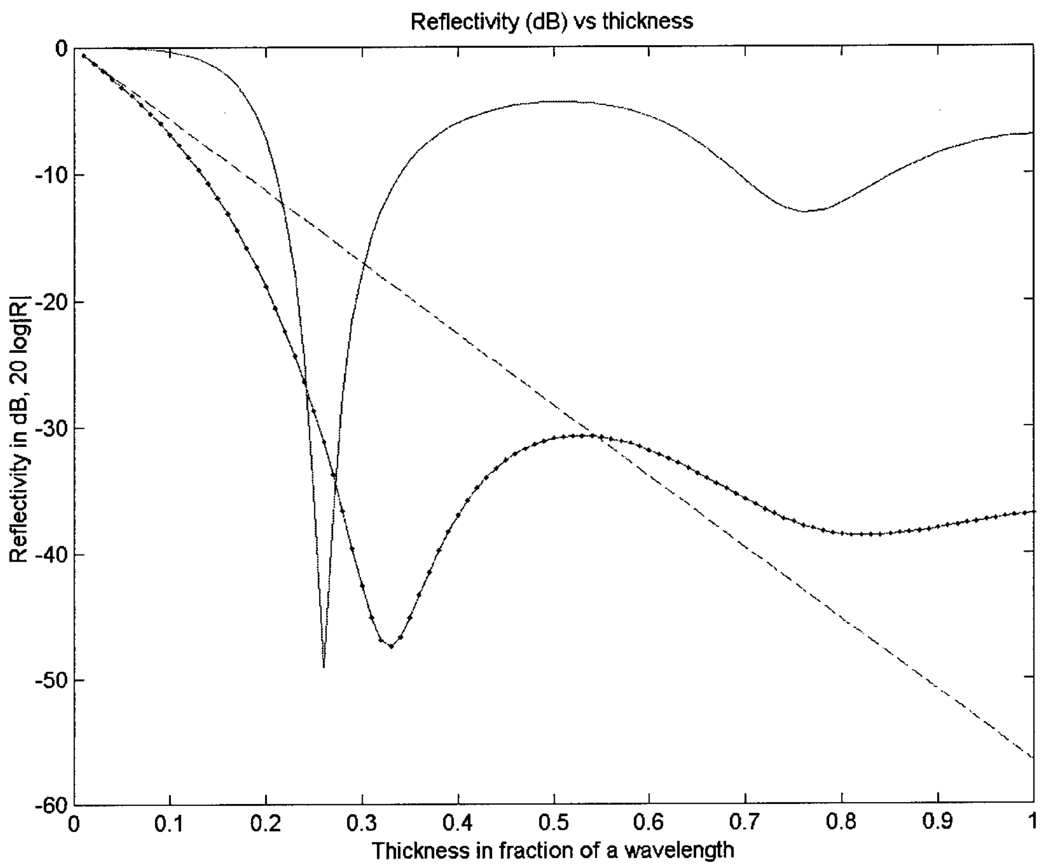

Figure 4. Reflectivity (dB) of arbitrary lossy media

Figure 4 shows the same information as in Knott, Shaeffer and Tuley Figure 8.12, calculated by the 'test bed' version of `reftran.m`. It shows the reflectivity of a single layer of a conjectural lossy material at normal incidence, with varying material thickness, above a PEC surface. The solid line represents a layer in which relative permittivity and permeability are 15.035 + j 5.472 and 1, respectively. The solid line with dots is due to permittivity 21.651 + j 12.500 and permeability 15.035 + j 5.472. The diagonal dashed line is due to permittivity and permeability both equal to 3.864 + j 1.035. Conductivity is again set to zero.

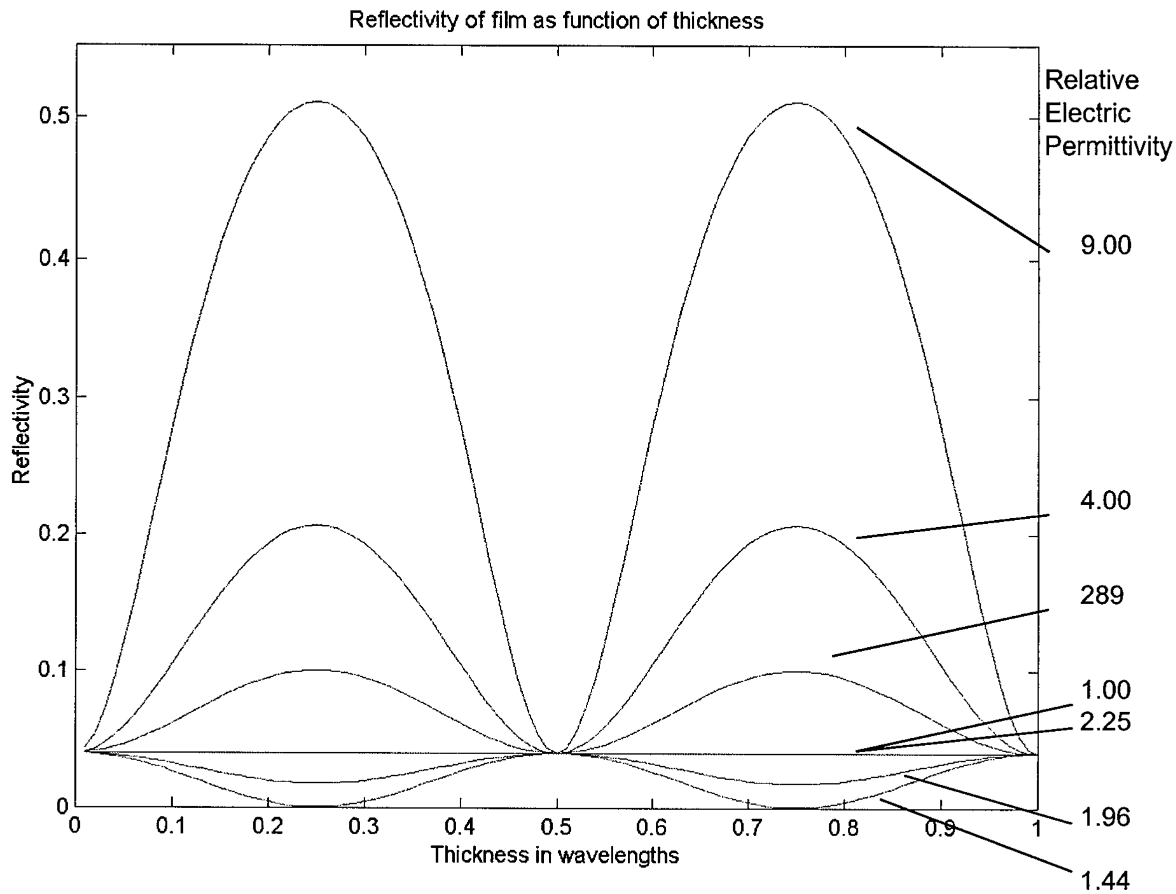

Figure 5. Reflectivity of lossless dielectric, varying thickness and permittivity

Figure 5 is the `reftran.m` 'test bed' reproduction of Born and Wolf figure 1.18. There are three layers, all with relative magnetic permeability of 1 and conductivity of zero. The first is air, the last has relative permittivity 2.25, and the middle varies in both thickness and permittivity. The permittivity for each curve is shown at right. Permittivities 1.00 and 2.25 result in the same curve, with a constant reflectivity of 0.04.

An area for future work is extension of this theory to include the conductive layers used in Salisbury screens and Jaumann absorbers (Knott, et al., 1993).

References

1. Constantine A. Balanis. Advanced Engineering Electromagnetics. John Wiley & Sons, 1989.

2. Max Born and Emil Wolf. Principles of Optics: Electromagnetic Theory of Propagation, Interference, and Diffraction of Light, Fifth Edition. Oxford, New York, Pergamon Press, 1975.

3. Eugene F. Knott, John F. Shaeffer, Michael T. Tuley. Radar Cross Section, 2nd Ed. Artech House, 1993. Tuley is the sole author of Chapter 8, Radar Absorbing Materials, which is cited here.

4. Hugh Angus Macleod. Thin-Film Optical Filters. Macmillan Pub. Co., 1986.

5. George T. Ruck, Donald E. Barrick, William D. Stewart and Clarence K. Kirchbaum. Radar Cross Section Handbook, Volumes 1 and 2. Plenum Press, 1970. Ruck is the sole author of Chapter 7, Planar Surfaces, which is cited here.

6. Julius Adams Stratton. Electromagnetic Theory. McGraw-Hill Book Company, Inc., 1941.

Appendix: MATLAB Code `reftran.m`

```
% reftran.m
% Reflectivity/Transmissivity of multilayer dielectric media
% Kenneth J. Pascoe, Capt, USAF
% AFIT/ENG
% Department of Electrical and Computer Engineering

% Introduction~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
% This program calculates the reflectivity and transmissivity of an
% absorber composed of several dielectric layers, which may be lossy.
% The user inputs the thickness and characteristics of several layers.
% The first layer is air, layers 2+ are dielectric materials, and the
% last layer is either dielectric (may be air) or a perfect electric
% conductor (PEC). The program is designed to support absorber design
% for low observable vehicles and weapons.
fprintf('This program determines the reflectivity and transmissivity\n')
fprintf('of layered media which may be backed by PEC.\n')

% List of Variables~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
% convert program to match these!!!
% a      scalar integer angle index
% Absorp array  real    absorptivity for each angle
% AbsorN array  real    intermediate quantity (numerator)
% b      scalar integer layer index
% Ahi    scalar real    user input highest incident angle
% Ainc   scalar real    user input incident angle increment
% AL     scalar integer number of incident angles
% Alow   scalar real    user input lowest incident angle
% Ang    matrix real    angles of incident rays through all layers
% Angles array  real    angles of incident rays in air layer
% D      scalar complex intermediate quantity (delta)
% d      array  real    thickness of each layer, meters
% dm     array  real    thickness of each layer, millimeters
% e      array  complex permittivity of each layer
% er     array  complex relative permittivity of each layer
% f      scalar real    frequency
% G      array  complex propagation constant (gamma) in each layer
% M      matrix complex characteristic matrix of one layer, one angle
% Mtot   matrix complex characteristic matrix of layers 2:L-1, one angle
% L      scalar integer number of layers
% m      array  complex permeability of each layer
% m11    array  complex element of Mtot for each angle
% m12    array  complex element of Mtot for each angle
% m21    array  complex element of Mtot for each angle
% m22    array  complex element of Mtot for each angle
% mr     array  complex relative permeability of each layer
% PEC    scalar integer flag =1 if Layer L is PEC
% Pol    scalar integer flag for TE or TM polarization
% ref    array  complex reflection coefficient
% refD   scalar complex intermediate quantity (denominator)
% Reflec array  real    reflectivity for each angle
% refN   scalar complex intermediate quantity (numerator)
% S      scalar real    conductivity (sigma) of last layer
% tran   array  complex transmission coefficient
% Transm array  real    transmissivity for each angle
% Y      array  complex admittance in each layer
% Yt     matrix complex 'tilted admittance' in each layer
```

```
% Wave Properties~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
f=input('\nIncident wave frequency?\n');
w=2*pi*f;
fprintf('Incident wave polarization?\n');
fprintf('1 = Transverse Electric (TE or perpendicular)\n');
Pol=input('2 = Transverse Magnetic (TM or parallel)\n');
if Pol~=1 & Pol~=2
   fprintf('Error, must be 1 or 2, please start over\n')
   break
end
fprintf('Range of incident angles (0 to 85 degrees, 0 is normal \n')
Alow=input('to surface).  Lowest angle (degrees)\n');
if Alow<0 | Alow>85
   fprintf('Error, must be 0 to 85, please start over\n')
   break
end
Ahi=input('Highest angle (degrees)\n');
%if Ahi<0 | Ahi>85
if Ahi<0 | Ahi>85
   fprintf('Error, must be 0 to 85, please start over\n')
   break
end
Ainc=input('Angle increment (degrees)\n');
if Ainc<0 | Ainc>(Ahi-Alow)
   fprintf('Error, will use 1-degree increment\n')
   Ainc=1;
end
if Ahi==Alow
   Angles=Alow
else
   Angles=Alow:Ainc:Ahi
end
AL=length(Angles);

% Target Properties~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
fprintf('Number of layers? (At least 3) Layer 1 is air, last layer');
L=input('\nis either dielectric (may be air) or PEC\n');
if L~=round(L)
   fprintf('Error, must have integer number of layers\n')
   break
end
if L<3
   fprintf('Error, must have 3 or more layers\n')
   break
end
PEC=0;
fprintf('\nLayer 1 is air\n')
fprintf('Relative electric permittivity = 1 \n')
er(1)=1;
fprintf('Relative magnetic permeability = 1 \n')
mr(1)=1;
fprintf('Static Conductivity = 0 \n')
S(1)=0;
for b=2:L-1
   fprintf('\nLayer %g is dielectric\n',b)
   d1(b)=input('Thickness (mm) \n');
   fprintf('Relative electric permittivity\n')
   er(b)=input('Input complex values as x-i*y\n');
   fprintf('Relative magnetic permeability\n')
   mr(b)=input('Input complex values as x-i*y\n');
```

```
   S(b)=input('Static conductivity (real)\n');
   if imag(S(b))~=0
      S(b)=real(S(b));
      fprintf('Imaginary part of sigma ignored\n')
   end
   if imag(er(b))~=0 | imag(mr(b))~=0
      fprintf('Layer %g is lossy\n',b)
   end
end
d=.001*d1;  % thickness (meters)
fprintf('\nIf Layer %g is PEC, enter 1 now, otherwise enter 0',L);
PEC=input('\n');
if PEC==1
   fprintf('Layer %g is PEC\n',L)
   S(L)=Inf;  % can use a real conductivity here
   % treat (sigma+j*omega*epsilon)/(j*omega*epsilon0) as er
   er(L)=1-i*S(L)/(w*8.854185e-12);
   mr(L)=1;  % dummy value
else
   fprintf('Layer %g is dielectric\n',L)
   fprintf('Relative electric permittivity\n')
   er(L)=input('Input complex values as x+i*y\n');
   fprintf('Relative magnetic permeability\n')
   mr(L)=input('Input complex values as x+i*y\n');
   S(L)=input('Static conductivity (real)\n');
   if imag(S(L))~=0
      S(L)=real(S(L));
      fprintf('Imaginary part of sigma ignored\n')
   end
   if imag(er(L))~=0 | imag(mr(L))~=0 | S(L)~=0
      fprintf('Layer %g is lossy, loss will not affect answer',L)
   end
end
e=er*8.854185e-12;
m=mr*4*pi*1e-7;

% Angles, Propagation Constants, Admittances~~~~~~~~~~~~~~~~~~~~~~~~~~
% calculate angles thru each layer via Snell's law
Ang=[Angles*pi/180; zeros(L-1,AL)];
G(1)=sqrt((S(1)+j*w*e(1))*(j*w*m(1)));     % propagation const in air
Y(1)=sqrt((S(1)+j*w*e(1))/(j*w*m(1)));  % admittance of air
for b=2:L-1
   G(b)=sqrt(j*w*m(b)*(S(b)+j*w*e(b)));
   Y(b)=sqrt((S(b)+j*w*e(b))/(j*w*m(b)));
   Ang(b,:)=asin(sin(Ang(b-1,:))*G(b-1)/G(b));
end
% find attenuation through all layers (except 1, L)
if PEC==1
   G(L)=Inf*(1+j);      % sqrt(Inf*j*w*m)
   Y(L)=Inf*(-1+j);      % sqrt(Inf/(j*w*m))
   Ang(L,:)=0;
else
   G(L)=sqrt((S(L)+j*w*e(L))*(j*w*m(L)));
   Y(L)=sqrt((S(L)+j*w*e(L))/(j*w*m(L)));
   Ang(L,:)=asin(sin(Ang(L-1,:))*G(L-1)/G(L));
end
% reflected ray angles are negatives of incident ray angles

% Reflectivity and Transmissivity~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
m11=zeros(1,AL);
```

```
m12=zeros(1,AL);
m21=zeros(1,AL);
m22=zeros(1,AL);
Yt=zeros(L,AL);
refN=zeros(1,AL);
refD=zeros(1,AL);
ref=zeros(1,AL);
tran=zeros(1,AL);
% find tilted admittance
if Pol==1   % TE case
   for b=1:L
      Yt(b,:)=Y(b).*cos(Ang(b,:));
   end
else        % TM case
   for b=1:L
      Yt(b,:)=Y(b)./cos(Ang(b,:));
   end
end
% find ref & tran coeff
for a=1:AL
   Mtot=eye(2);
   % find characteristic matrix of each layer and overall
   for b=2:L-1
      D=j*G(b)*d(b)*cos(Ang(b,a));
      M=[cos(D) -i/Yt(b,a)*sin(D); -i*Yt(b,a)*sin(D) cos(D)];
      Mtot=Mtot*M;
   end
   m11(a)=M(1,1);
   m12(a)=M(1,2);
   m21(a)=M(2,1);
   m22(a)=M(2,2);
end
% find transmissivity, reflectivity, absorptivity
if PEC==1
   Yt(L,:)=1;   % dummy value
   ref=(m12.*Yt(1,:)-m22)/(m12.*Yt(1,:)+m22);
   tran=0;
   AbsorN=real(m12.*conj(m22));
   Absorp=4*Yt(1,:).*AbsorN./(abs(m12.*Yt(1,:)+m22).^2)
   Transm=0
else
   Yt(L,:)=Y(L)./cos(Ang(L,:));
   refN=Yt(1,:).*(m11+Yt(L,:).*m12)-m21-Yt(L,:).*m22;
   refD=Yt(1,:).*(m11+Yt(L,:).*m12)+m21+Yt(L,:).*m22;
   ref=refN./refD;
   tran=2*real(Yt(1,:))./refD;
   AbsorN=real((m11+Yt(L,:).*m12).*conj(m21+Yt(L,:).*m22)-Yt(L,:));
   Absorp=4*Yt(1,:).*AbsorN./(refD.*conj(refD))
   Transm=real(Yt(L,:))./real(Yt(1,:)).*tran.*conj(tran)
end
Reflec=ref.*conj(ref)

% Plots~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
figure(1)
if PEC==1
   plot(Angles,Reflec,'r-')
   ylabel('Reflectivity')
else
   plot(Angles,Reflec,'r-',Angles,Transm,'b:')
   ylabel('Reflectivity (red) and Transmissivity (blue)')
```

```
end
xlabel('Angles (degrees)')
title('Reflectivity and Transmissivity vs angle')

figure(2)
if PEC==1
   plot(Angles,abs(ref),'r-')
   ylabel('Reflec Coeff')
else
   plot(Angles,abs(ref),'r-',Angles,abs(tran),'b:')
   ylabel('Reflec Coeff (red) and Trans Coeff (blue)')
end
xlabel('Angles (degrees)')
title('Abs Value of Ref & Tran Coeff vs angle')
```