NAVAL POSTGRADUATE SCHOOL
Monterey, California

Joint Interoperability of Theater Missile Defense
Systems: Extending the Scope of Testing

by

Robert A. Koyak

October 2000

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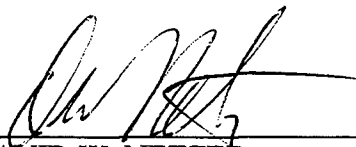
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13. ABSTRACT (Maximum 200 words)

Assuring the interoperability of a theater missile defense (TMD) family of systems (FoS) is a challenging problem with many different facets. Under a concept of interoperability that is based on competition among interfacing systems for reporting responsibility on tracks, the least performing system can degrade the performance of the entire family. A program of interoperability testing must therefore emphasize operability in addition to conformance to message standards. This report examines interoperability testing of a TMD FoS from perspectives ranging from the statistical validity of tracking algorithms to the integrity of track messaging. Recommendations for extending interoperability testing in these areas are made. Testing should be conducted in a manner that focuses attention on deficiencies in both the concept and operations of a TMD FoS. In particular, interoperability testing should be structured to demonstrate the effects of system-level registration errors (bias) on FoS operations. A three-tiered "test, registration, re-test" procedure can satisfy this objective with minimal disruption to testing under current (TADIL J) message standards.

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Joint Interoperability of Theater Missile Defense Systems: Extending the Scope of Testing

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Executive Summary

Assuring the interoperability of a theater missile defense (TMD) family of systems (FoS) is a challenging problem with many different facets. Procedures for the management of communication between constituent systems must be carefully articulated to ensure that information is processed in both a timely and accurate manner. But, necessary as they are, such procedures alone cannot assure interoperability. A family of systems that complies with a set of communication standards may fail to interoperate for a variety of reasons:

- Measurements obtained from a constituent system may be subject to systematic error (bias)

- The communication standard either may not require or may not facilitate the transmission of critical information between systems

- Data processing may be based on incorrect assumptions

- Coordination of data processing across systems may be inadequate.

The Joint Interoperability Test Command (JITC) has recognized that interoperability testing of a TMD FoS requires more than testing for standards conformance. The purpose of this report is to recommend testing concepts and metrics that the JITC can adopt to augment its interoperability testing program for TMD FoS, particularly in the area of functional interoperability. The recommendations, which are summarized below, pertain specifically to a TMD FoS that communicates over a Joint Data Network (JDN) with TADIL J messaging.

Recommendation 1. In TMD FoS interoperability testing based on a TADIL J network messaging platform (e.g. JDN), interfacing systems should be instructed that all J3.6 messages must contain full covariance matrices.

The JITC has observed that covariance matrices are not routinely transmitted during TMD FoS testing. This omission has serious, negative consequences for interoperability. The TADIL J message standard (MIL-STD-6016A) does not present a clear requirement for transmitting complete covariance matrices with a track report. It is recommended that the message standard make doing so a requirement. In the meantime, the JITC should require that all systems routinely include the full covariance matrix in their track reports during interoperability testing.

> **Recommendation 2.** Transmission of a numerically accurate covariance matrix is an interoperability issue. The JITC should conduct specialized testing to examine whether each of the interfacing systems can calculate, encode, and decode covariance matrices accurately.

Covariance matrices are required to estimate trajectories, impact points, launch points, and uncertainties associated with each; to correlate local tracks to network tracks; and, to calculate track quality (TQ) numbers for assigning reporting responsibility (R^2) on a particular track. The JITC should conduct non-event based testing, possibly on an individual system basis, to evaluate each system's ability to transmit and process covariance information accurately. Testing can be conducted using a pre-determined set of covariance matrices that the JITC transmits to each system for reconstruction, and another set that each system transmits to the JITC.

> **Recommendation 3**. Interoperability problems that arise during TMD FoS testing may result from problems within a particular system. A TADIL J message stream obtained during FoS testing may not reveal enough information to pinpoint the source of a problem. Interoperability testing should include a component in which interfacing systems operate in non-FoS (autonomous) mode. In autonomous mode, each interfacing system tracks what it can, and provides full messaging based on its local tracks, without regard to reporting responsibility (R^2) rules.

A TMD FoS is fragile in the sense that one interfacing system can degrade the performance of the entire family. A system with sensors that exhibit severe, but undetected, bias can thwart interoperability by overstating its track quality, thereby acquiring R^2 and preventing more qualified systems from reporting on the track. It is therefore difficult, if not impossible, to separate FoS interoperability from system operability under the JDN concept. The JITC should not be restricted to assessing interoperability solely on the basis of joint (TADIL J) messaging.

Test scenarios should be conducted in both FoS (joint) mode and in non-FoS (autonomous) mode. In addition to allowing the JITC to isolate problems that are specific to an individual system, a comparison of both modes of testing will allow the JITC to measure the benefits of joint tracking relative to the same assets operating autonomously.

Recommendation 4. Estimation of track position-velocity missile state vectors cannot be reduced to a single, valid formulation. In the absence of a single, standard tracking algorithm that each interfacing system of a FoS must adopt, tracking algorithms can be expected to vary across systems. As part of interoperability testing, the JITC should have each system declare, in the form of a written report, important aspects of its tracking algorithms as they pertain to tracking ballistic missiles.

The astrodynamics of missile trajectories is a well-studied area of physics, and it is expected that each system in a FoS will reflect the current state of knowledge in its tracking algorithms. However, there are modeling issues that may be handled differently across systems, perhaps unsatisfactorily in some cases. Inspection of computer code that is used to implement motion models may not be possible, due to the desire of developers to protect their intellectual property; and it may not be productive for the JITC to undertake such inspection in any case. As an alternative, the JITC should develop a questionnaire to be submitted to each system in a FoS test. The questionnaire should elicit detailed information about the modeling decisions that were incorporated into each system's tracking software.

Recommendation 5. The JITC should assess the models that are used for generating missile trajectories in event simulators. A criterion for suitability is whether live-missile trajectories (e.g. Coral Talon) are within a "high confidence set" of trajectories that can be obtained if the event simulator is primed with the same initial conditions as the truth trajectory.

Modeling and simulation (M&S) is an important tool for testing TMD FoS interoperability. Many of the interoperability shortcomings that have been observed to date are manifested in simulated event scenarios. But there is concern that simulated missile trajectories may not be fully realistic. If that is the case, a TMD FoS that is trained primarily on simulated events may perform disappointingly in real-world situations. Unfortunately, there are relatively few data on real missile trajectories that can be used to develop a truly complete empirical model. But existing data may allow comparisons to be made between simulated and real missile tracks. The JITC should use real data to "prime" event simulators to measure the discrepancy between true and simulated trajectories. By doing so the JITC can achieve a better understanding of the suitability of its testing tools.

> **Recommendation 6.** Data registration is an important activity to ensuring the interoperability of a TMD FoS. Unfortunately, TADIL J does not provide the messaging resources that are needed to make on-going and interactive data registration an integral part of the operation of a TMD FoS. It is therefore unlikely that the JITC can address this situation directly in testing. But the JITC can, in a realistic manner, incorporate a registration interval into its interoperability assessment procedures.

The data registration provisions of the JDN interoperability concept are not well articulated. TADIL J provides few messaging resources for systems to share information that is needed to make data registration an integral part of joint tracking activity. As a result, systems can introduce biased measurements into the network, encountering virtually no firewalls. It is doubtful that a TMD FoS can ever become truly interoperable without overcoming this deficiency. To become so would require modification of MIL-STD-6016A or the adoption of a different messaging concept. It may be beyond the scope of the JITC to address this concern directly. Instead, the JITC should attempt to measure the improvement that would be achieved by integrating data registration procedures into TMD FoS operations. This can be done by conducting testing in three stages: (1) pre-test, (2) registration interval, and (3) re-test. The pre-test entails joint tracking as currently done in interoperability testing. It is followed by a registration interval, which gives each system the opportunity to track objects with known trajectories. Systems use this information to refine its data registration before the re-test is conducted. Comparison of test metrics between the pre-test and re-test will allow the JITC to measure improvement due to the use of data registration procedures.

> **Recommendation 7.** The JITC should adopt metrics related to *track latency*, which is the difference in time reported in a space track message and time as appropriate to the position and velocity of the object reported in the same track message. Track latency reflects the time required to process and transmit a space track message that is not fully reflected in the reported time.

In tracking a high-velocity target such as a ballistic missile, a small timing error can translate into a large physical distance error. Track latency errors may occur even if a system is capable of measuring track position and velocity without error and if its clock is perfectly synchronized. By measuring track latency the JITC will be able to isolate tracking errors due to latency and tracking errors due to other causes.

1. Introduction

This reports describes research that was conducted on statistical aspects of interoperability testing of theater missile defense (TMD) families of systems (FoS), during the summer of 2000. The interoperability problem in this context is a deep one, as anyone who is familiar with it knows. Setting aside the challenge of engaging a hostile ballistic missile with defensive countermeasures, it is a challenge simply to come to a unified understanding of what exists in the air space. Different systems must be able to "talk" to each other in a timely and accurate manner before a FoS defensive concept can hope to supersede the benefits of a "single system" defensive concept.

The communications backbone of the TMD FoS concept that the Joint Interoperability Test Command (JITC) has tested is TADIL J and its associated standard (MIL-STD-6016A). TADIL J is the networking platform upon which the Joint Data Network (JDN) is based. TADIL J is a well-endowed resource for sharing technical information on air or space threats between the interfacing systems of a FoS. However, the message standard does not attempt to enforce rigid uniformity on the content and transmission of technical information. As a result, flexibility is maintained somewhat at the expense of coherence and completeness in communicating this information.

The nearly uniform assessment of the TMD FoS concept is that it falls short of expectations when it is loosely confederated. Data registration is an area where this is particularly evident. A TMD FoS is not robust to measurement biases that are exhibited by even a few of its sensors. Under ideal conditions where there is no bias, interoperability issues are substantial. Where such biases are present, interoperability problems may be insurmountable. This is because each interfacing system depends on the "good faith" effort of every other system to report its information free of removable systematic errors.

Data registration is a collection of procedures, including calibration, that are designed to remove measurement bias. MIL-STD-6016A recognizes the importance of this activity, and recommends that interfacing systems adopt data registration procedures. But there is little standardization or enforcement. As a result, the JITC has observed classical manifestations of systematic errors in TMD FoS testing, which include the creation of multiple tracks for the same object, the premature dropping of valid tracks, the creation of tracks that do not represent real objects, and tracks that are not physically plausible.

Another problem that loose confederation poses is that system software is not standardized. Different tracking and correlator algorithms may be in use simultaneously. Statistical criteria for associating objects to tracks may be different. In TMD FoS testing under JDN, interfacing systems hold their own local tracking data, and systems' software tools are essentially "black boxes." This makes it difficult to trace interoperability problems to their cause.

An alternative concept of joint tracking is the Joint Composite Tracking Network (JCTN), which is still under development. Unlike JDN, JCTN requires that each system run the same software for basic tracking tasks. In the developmental stage, benchmarking software is integrated into the software, which facilitates the automatic generation of testing metrics whenever FoS testing is conducted. However, the messaging platform to support this concept has yet to be developed, and to do so will face significant challenges.

This report is organized into five sections. Section 2 discusses missile tracking from statistical and physical viewpoints. The models that are used in tracking are described in this section. Section 3 discusses the TADIL message standards and how their technical content affects interoperability. In particular, the TQ Number concept of assigning Reporting Responsibility is discussed. Section 4 is a review of metrics that are used for measuring interoperability by the JITC, and in benchmarking by JCTN. Many of the JCTN benchmarks are related to interoperability, and some can be adapted for use by the JITC. Section 5 offers seven recommendations for improving interoperability testing of a TMD FoS, with respect to the technical content of space track messages. New performance metrics are also proposed in Section 5.

2. Technical Background

A TMD FoS that is designed to track and engage missile threats must contend with a myriad of technical problems that have eluded solution for decades. The design of increasingly accurate sensors and faster computers offers new possibilities for bring workable multisensor tracking systems into fruition. Counterbalancing this is that the ability of adversaries to implement countermeasures is also increasing both in scope and effectiveness. What this means is that not only must the FoS be able to track a single, high-velocity ballistic object accurately, it must also be able to do so for many such tracks, while distinguishing those that represent ballistic missiles from those that do not.

Although one system in the family may use analytical techniques that differ from those of another, there are a number of principles that should be reflected in any analytical approach. These principles are discussed briefly in this chapter.

2.1 The Astrodynamics of Ballistic Missile Trajectories

The flight of a ballistic missile from the time of launch to the time of impact with the surface of the earth can be divided into two phases, depending on the forces that act upon the vehicle:

- The *boost phase* occurs from the time of launch until the time that the motors no longer provide thrust to the missile. The boost phase may last from less than a minute to several minutes, depending on the design of the missile. In addition to gravity, atmospheric lift and drag, and possibly other (minor) perturbing forces, the missile is subject to a force provided by the use of its fuel. The latter is a combined effect due to the kinetic energy released when fuel is burned, and the gradual loss of mass that the missile experiences at the same time. The boost phase may consist of several (two or three) distinct *subphases* of boosting, and some missiles may be capable of producing "corrective" thrust during midcourse.

- The *ballistic phase* occurs from the end of the boost phase until the time of impact with the surface of the earth. The ballistic phase of a theater-area missile may last from several minutes to over one-half hour, depending on the range of the missile. During the ballistic phase the only forces that act upon the missile are those due to gravity, atmospherics, and perturbations of external origin.

In some treatment of this subject, the ballistic phase is further divided into *exo-atmospheric* and *re-entry* phases. During the exo-atmospheric phase, the missile is at sufficient altitude that the density of the atmosphere does not significantly affect motion of the missile. During the re-entry phase the missile enters the lower atmosphere before its imminent impact with the earth. Where the transition from the exo-atmospheric to the re-entry phase occurs is somewhat arbitrary, although the distinction may have practical value in certain contexts. Atmospheric density is an approximately exponentially decreasing function of altitude. At altitudes above 30 km atmospheric density is widely

regarded negligible. By accurately modeling atmospheric density, both the exo-atmospheric and re-entry phases can be described in a unified manner.

2.1.1   Coordinate Systems for Missile Tracking

Physical laws dictate that that the motion of a negligibly small body (i.e., a missile) with respect to Earth, not accounting for other-body gravitational forces, is described relative to the center of mass of the earth in inertial coordinates. An Earth-centered inertial (ECI) coordinate system is therefore the preferred one for describing the trajectory of a ballistic missile. The commonly used Earth-centered Earth-fixed (ECEF) coordinate system has its origin at the center of the earth, with axes passing through the points 0° Latitude 0° Longitude (equator at Greenwich Meridian), 0° Latitude 90° E Longitude, and the North Pole. Because the ECEF coordinate system rotates with the earth, it is not inertial. It is, however, possible to "fix" the ECEF coordinates at the time that a missile is first detected and correct for rotation of the earth as time progresses.

From the point of view of a sensor located on the surface of the earth, a polar coordinate system centered at the position of the sensor more naturally corresponds to its measurement process. Three characteristics typically are measured:

- the *range*, or distance of the missile from the sensor

- the *azimuth angle* or angle of rotation (at the sensor) from "true north" to the missile.

- the *elevation angle* that a line from the missile to the sensor makes with the plane tangent to the earth at the sensor location

These attributes comprise what is often referred to as the sensor-RAE (range, azimuth, elevation) coordinate system. The sensor-RAE coordinate system is non-inertial because the sensor moves with the rotation of the earth.

For radars in particular, estimates of the RAE quantities are obtained directly through processing of the radar cross-section (RCS). It has been suggested that measurement errors are nearly independent in sensor-RAE coordinates, which is an advantage in the design of efficient estimation strategies. By contrast, infrared (IR) sensors provide only "line of sight" (LOS) measurements from the sensor to the target. From one LOS measurement two angles can be determined but not the range. Using two LOS measurements it is possible to determine the range using triangulation.

It is possible to convert between a sensor-RAE coordinate system and the ECEF coordinate system using simple mathematical relationships. Such conversion is necessary to express data from multiple sensors in a common reference frame. But, conversion from sensor-centered to earth-centered coordinates can impart systematic tracking errors if the position of the sensor in ECEF coordinates is not known accurately.

This underscores the need to have well-developed and followed procedures for sensor alignment in any multisensor tracking system.

When tracking is conducted in a Cartesian coordinate system centered at the earth, for display purposes it is sometimes convenient to convert positional information to geodetic latitude, longitude, and altitude. This applies in particular to the estimation of a missile's launch and impact points. Conversion between Cartesian (ECEF) and geodetic coordinates is mathematically straightforward. The WGS-84 Earth Model provides Earth data (based on non-sphericity of the earth) that are currently considered to be the most accurate for performing these conversions.

2.1.2 Dynamic modeling

The motion of a missile relative to the center of the earth can be described using a nonlinear dynamic model. Assuming that Earth gravitation is the only force that acts upon the missile, and that the earth is perfectly spherical, the following relationship due to Netwon's Second Law holds:

$$\ddot{r}(t) = -\frac{\mu r(t)}{|r(t)|^3}$$

The symbol $r(t)$ refers to the three-dimensional position of the missile in an interial coordinate system such as ECI, or ECEF at a fixed point in time. Position is described as a function of time, denoted t. The symbol $|r(t)|$ is the norm of the position vector, given by $|r(t)| = \sqrt{r_x^2(t) + r_y^2(t) + r_z^2(t)}$. The symbol $\ddot{r}(t)$ refers to the three-dimensional acceleration vector, which is obtained by taking the second derivatives of each of the $r(t)$ coordinates with respect to time. Similarly, $\dot{r}(t)$ refers to the first derivatives, or the velocity of the missile in three-dimensional space. The symbol μ refers to the earth's gravitational constant, given by $\mu = 3.986012 \times 10^5$ km^3/s^2. It is assumed here that metric units are used, so that $r(t)$ is measured in kilometers (km), $\dot{r}(t)$ in kilometers per second (km/s), and $\ddot{r}(t)$ in kilometers per squared second (km/s^2).

The position and velocity of the missile can be combined into a single 6-dimensional state vector $x(t) = [r(t), \dot{r}(t)]$. The first derivative of $x(t)$ is then completely described in terms of $x(t)$:

$$\dot{x}(t) = [\dot{r}(t), \ddot{r}(t)] = \left[\dot{r}(t), \quad -\frac{\mu r(t)}{|r(t)|^3} \right] = F(x(t)).$$

From this relationship, is is seen that $x(t)$ is the solution to a first order, nonlinear differential equation. Solutions to this equation have several well-known properties:

- For an object that does not escape Earth's gravitational field (e.g., a ballistic missile), $r(t)$ can be described as an ellipse with one of its two foci located at the center of the earth.

- Knowing both $r(t)$ and $\dot{r}(t)$ at one point in time, or $r(t)$ at three distinct points in time, is sufficient to characterize the elliptical trajectory.

Of course, the earth is not perfectly spherical, and Earth's gravity is not the only force that acts upon a missile during its flight. The slight bulge at the equator of the earth exerts a torque on the elliptical plane of the missile's trajectory. Stated another way, Earth's gravitational force is greatest at the equator, and decreases slightly as geodetic latitude approaches that of the North or South Pole. This effect is usually accounted for by correcting the gravitational constant μ, depending on latitude, using formulas that are widely available. Although the resulting correction is usually small, it may be important if it is desired to take countermeasures against a high-velocity target.

Other forces that act upon the missile are orders of magnitude greater than the variable effect of gravity. During the boost phase, the missile is subject to thrust from its motors, continual diminution of its mass, and atmospheric drag and lift as the missile departs the lower atmosphere. To account for these factors requires detailed information about the design of the missile and the manner in which it was launched, which may or may not be available. During the ballistic phase, the missile is continually subject to atmospheric lift and drag, which become increasingly significant forces as the missile passes through the lower atmosphere towards the surface of the earth. During both phases the *ballistic coefficient* captures features of the missile that determine how atmospheric lift and drag affect its motion.

It is possible to extend Newton's gravitational model to account for the factors described above, resulting in a nonlinear dynamic model of the same generic form $\dot{x}(t) = F(x(t))$. This subject is discussed in Stevens and Lewis (1992) and in other published sources. Specification of the dynamic model may vary depending on the context and information about the missile that is available. The transition from the boost to the ballistic phase represents a change in the astrodynamic properties of the missile, and two different dynamic models may be used.

The ballistic coefficient presents a modeling issue that requires careful attention. If the missile is of known type, intelligence may supply information that, if correct, would allow the ballistic coefficient to be determined precisely. However, operating under the assumption of correctness may be risky, and in any case it is necessary to consider the case where such information is absent. An effective approach is to incorporate the ballistic coefficient (or drag force, determined from the ballistic coefficient) into the state vector. This approach is discussed in Cardillo, Mrstik and Plambeck (1999), where the authors report considerable improvement in tracking ballistic missiles at Kwajalein Missile Range as a result of this modification.

A dynamic motion model cannot perfectly predict the trajectory of a ballistic missile. One reason is that a model accounts for known physical forces only to within a certain degree of accuracy. Refinements to the model (e.g. to account for variations in the Earth's gravitational field) can be made to reduce, but not eliminate, these approximation errors. Another reason is that a missile is subject to perturbations in its trajectory due to forces that are not fixed and predictable. These perturbations are often accounted for by including a random component, referred to as *process noise,* in the model. The dynamic model then takes the form of a first-order, nonlinear *stochastic* differential equation: $\dot{x}(t) = G(x(t), \varepsilon(t))$, where $\varepsilon(t)$ is a random vector representing the process noise. If the process noise is additive, then $G(x(t), \varepsilon(t)) = F(x(t)) + Q\varepsilon(t)$, where Q is a matrix of known form. The process noise $\varepsilon(t)$ is usually of smaller dimension than the state vector $x(t)$. For example, if the state vector is 6-dimensional, the process noise may be 3-dimensional and apply only to the derivatives of the velocity. In this case Q would be a 6×3 matrix.

The probability distribution that is used to describe the process noise is an important aspect of the model that has received little attention in the published literature. There is a tendency to use Gaussian (normal) distributions with simple covariance structures, which is computationally convenient but not necessarily grounded in astrodynamical principles.

An alternative to modeling motion in Cartesian (ECI or ECEF) coordinates is to use polar (sensor-RAE) coordinates. As noted previously, this may be desirable from the point of view that sensor-RAE coordinates more naturally express how sensor measurements are obtained. Cardillo et al. (1999) provide dynamic modeling equations for tracking ballistic missiles in sensor-RAE coordinates.

The models described above apply to "continuous" time, but in measurement situations the position and velocity of a target are usually observed at discrete, equally-spaced time points. Continuous-time nonlinear models are often linearized using the definition of a derivative. For example, $x(t + \Delta) \approx x(t) + \Delta G(x(t), \varepsilon(t))$ is the discreteized, linear approximation to the model $\dot{x}(t) = G(x(t), \varepsilon(t))$.

In summary, there is no one "correct" way to formulate a dynamic model that describes the motion of a ballistic missile. How it is formulated depends on the information available (e.g. the ballistic coefficient) and the coordinate system that is used. Regardless of how the dynamic model is formulated, the following elements should be present:

- Gravitational force represented by Newton's formula. A "flat Earth" version of this formula may be acceptable if tracking is done over short distances (e.g, during the boost phase).

- Earth's gravitational field represented as variable due to non-sphericity of the earth. Approximations are available using first- and second-order zonal polynomials.

- Atmospheric drag represented using the ballistic coefficient (known or estimated) and density of the atmosphere. Atmospheric density can be approximated as a function of altitude using available formulas (Zarchan, 1999).

- Thrust and pitch-over angle included during the boost phase.

- Earth's rotation included as a continual angular displacement if tracking is performed in non-inertial coordinates.

- Process noise incorporated into the model in a scientifically defensible manner.

2.2 The Measurement Process

A sensor such as a radar does not perfectly measure the position and velocity of a target that it tracks. Data processing is subject to random error and bias (systematic error) that have different characteristics across sensors. Usually, these errors are expressed in sensor-RAE coordinates, which is the "natural" coordinate system for sensor measurement. Radars with Doppler can use this information to improve range measurement, to produce velocity measurements, or both. The following assumptions are typically made:

- Sensor measurements are unbiased, in range, azimuth, and elevation;

- Random errors follow normal (Gaussian) probability distributions, with covariance matrices that may depend upon time;

- Random errors for range, azimuth, and elevation are statistically independent;

- Random errors are independent across time.

Operating under these assumptions greatly simplifies the estimation of missile trajectories. However, a violation of any one of them could have undesirable consequences. If a sensor is miscalibrated, for instance, systematic errors will be present in all of its measurements. Data registration, which is discussed in section 2.7 below, is a process by which an operator or system user attempts to remove systematic errors from sensor measurements. Modeling of the measurement process assumes that effective data registration procedures are in place, so that only random errors need to be considered.

Table 2-1. Sources of Angle, Range, and Doppler Random Errors
In Radar Measurements (from Barton, 1988)

Class of Error	Noise Component	Affects Angles	Affects Range	Affects Doppler
Radar-dependent tracking errors	Thermal noise	X	X	X
	Multipath	X	X	X
	Clutter and clutter jamming	X	X	X
	Torque caused by wind gusts	X		
	Servo noise	X		
	Antenna deflection due to acceleration	X		
	Jamming variation in receiver delay		X	X
Radar-dependent translation errors	Bearing wobble	X		
	Data gear nonlinearity and backlash	X		
	Data takeoff nonlinearity and granularity	X		
	Pedestal deflection caused by acceleration	X		
	Phase shifter error	X		
	Range-doppler coupling		X	
	Internal jitter		X	
	Data encoding		X	
	Range oscillator stability	X		
	VCO[a] frequency measurement			X
	Radar frequency stability			X
Target-dependent tracking errors	Glint	X	X	
	Dynamic lag variation	X	X	X
	Scintillation or beacon modulation	X	X	
	Beacon jitter	X		
	Target rotation (glint)			X
	Target modulation			X
Propagation errors	Variations in ionospheric refraction	X	X	X
	Variations in tropospheric refraction	X	X	X
Apparent or instrumentation errors	Vibration or jitter in reference instrument	X		
	Film transport jitter	X		
	Reading error	X		
	Granularity error	X		
	Variation in parallax	X		

[a]Voltage-controlled oscillator

Chapter 11 of Barton (1988) gives a thorough discussion of errors that occur in radar measurements. Table 2-1 (which combines information from Tables 11.1, 11.6, and 11.7 of Barton) illustrates that random errors in angle (azimuth and elevation), range, and Doppler measurements are subject to a number of common factors. This overlap gives ample reason to question the treatment of measurement errors in sensor-RAE coordinates as statistically independent, although a common error source may, in principle, affect the coordinates in different ways.

Some of the noise components listed in Table 2-1 appear capable of persisting for periods of time, possibly across consecutive radar measurements. If that is the case, then the assumption of independent random errors across time is likely to be violated. Barton's (1988) spectral analysis of angle-tracking errors, which found a pronounced cyclic component and other evidence of departure from a white-noise spectrum, suggests that there may be good reason to closely examine the validity of this assumption.

The assumption of normality, often made for convenience reasons, deserves scrutiny. One of the defining traits of a normal distribution is that large deviations (i.e., values more than three standard deviations from the mean) rarely occur. This assumption justifies the use of estimation procedures that are optimized for normality, but perform badly in the presence of even a small number of large deviations. These procedures include the Kalman filter, which is widely used for tracking ballistic missiles.

Finally, it should be mentioned that any statement of the precision of a sensor (e.g. in the form of a covariance matrix) should reflect its *practical* operation when, where, and how the sensor is to be employed. Manufacturer's specifications, which often assume that equipment is new, properly calibrated, and operated under ideal conditions, can be overly optimistic in assessing accuracy, particularly in the presence of errors that originate from external sources (see Table 2-1). Determining the operational accuracy of a sensor is a laborious process that requires ongoing testing and experimentation.

2.3 Estimation Techniques for Ballistic Missile Trajectories

An algorithm that is used to estimate the trajectories of ballistic missiles must be both good and fast. To illustrate, consider a ballistic missile that impacts the earth 600 km (ground distance) from the launch site, and achieves a maximum height (at apogee) of 130 km. Ignoring boost-phase issues and atmospheric drag, this missile will impact the earth approximately 6 minutes after launch and achieve a forward velocity of about 8,500 km/hr at the time of impact. An algorithm that uses crude approximations may be time-efficient but unable to localize a missile during its flight in a usable way. Similarly, a highly accurate algorithm that is computationally intensive is useless if it cannot provide its answers quickly.

Electrical engineers and other scientists have developed a variety of tracking algorithms over the last four decades, and new algorithms continue to appear. These algorithms can be classified in a number of different ways. Most of the algorithms that

have been described in the published literature for estimation of missile trajectories can be broadly classified as either *fixed-coefficient filters* or *Kalman filters*.

Filtering techniques are based on linking a state equation to another equation that describes the measurement process. Ideally, all of the relationships in the two equations are linear, and all of the noise is Gaussian. In this case the discrete-time filtering model has the following form:

State equation: $\qquad x(k+1) = F_k x(k) + Q_k \varepsilon(k)$

Measurement equation: $\quad y(k) = H_k x(k) + \eta(k)$

The state equation describes how the state (missile position, velocity, etc.) changes from time k to time $k+1$. The current state is multiplied by a known matrix F_k, and process noise represented by the remaining term is then added. The process noise $\varepsilon(k)$ is assumed to be Gaussian, uncorrelated across time (white noise), with each coordinate having mean 0 and variance 1. Covariance and other linkage issues are addressed by the matrix Q_k, which is not known and must be estimated.

The true state vector $x(k)$ is not directly observable, and it is affected by measurement error. The measurement equation describes how the sensor-data vector $y(k)$ is related to the state vector. The state vector is premultipled by a known matrix H_k and Gaussian measurement error $\eta(k)$ is then added. By assumption, this measurement error is uncorrelated across time and with the process noise. The covariance matrix of the measurement error is assumed to be known.

The goals of estimation are the following:

- Estimate the current state $x(k)$ based on all data $y(1)$, ..., $y(k)$ available at that time;

- Predict a future state $x(k+r)$ based on all data $y(1)$, ..., $y(k)$ available at time k;

- Provide a smoothed estimate of a past state $x(k-r)$ based on all data $y(1)$, ..., $y(k)$ available at time k;

- Provide estimated covariance matrices for all estimated, predicted, and smoothed quantities.

Under the model described above, the Kalman filter provides a statistically optimal means for satisfying all of these goals. In addition, the calculations are algebraically straightforward and well-suited to real-time implementation. When a new observation $y(k+1)$ enters the system, estimates and predictions are easily updated. The Kalman filter is the subject of numerous books and articles in the scientific literature. Grewal and Andrews (1993) is a good introductory treatment.

The fixed-coefficient filter, usually of α-β or α-β-γ type, is a simpler alternative to the Kalman filter that has also been used in motion-tracking problems. It is computationally faster than the Kalman filter, but less efficient in extracting information from data. Both Kalman and fixed-coefficient filter missile trackers have been used with radars at Kwajalein Missile Range (Cardillo et al. 1999). A discussion of fixed-coefficient filters can be found in Blackman (1986).

In many applications, including the estimation of ballistic missile trajectories, the state and measurement equations are nonlinear. For ballistic missile trajectories:

- The state equation is based on the nonlinear dynamic motion model introduced in section 2.1.2;

- A different nonlinear state equation is needed for the boost and ballistic phases, due to a discontinuity in the dynamics of the missile at the transition;

- The measurement equation is also nonlinear if tracking is done in Earth-centered Cartesian coordinates, but sensor measurements are obtained in sensor-centered polar coordinates (which is usually the case).

Nonlinear estimation raises computational difficulties that are especially troublesome when tracking a high-velocity target such as a missile. A number of approaches have been developed to deal with this problem, and it remains an active area for research. As computer hardware continues to improve, what is regarded as the "best" solution today may not be the best solution in the future.

One way to deal with nonlinearity is to use the *extended Kalman filter* (EKF) based on Taylor-series approximations of the state and measurement equations. In some applications the EKF is used only to update covariance matrices, and the nonlinear dynamic equation is used to update the state vector. Linearization and other simplifications of the Kalman filter calculations, often based on heuristics, are driven by the need to balance computational speed and accuracy. There are many potential variations of these concepts, and apparently a variety of them have been used to design trackers for ballistic missiles.

For ballistic-phase tracking, Cardillo et al. (1999) studied several extended Kalman filters and fixed-coefficient filters either in use or designed for use at the millimeter wave (MMW) radar located at the Kwajalein Missile Range, including the tracking filter developed by the authors. Each of the filters tracked a large number of real and simulated ICBM and TBM trajectories, and their accuracies were compared. The authors' KB(7,3) filter[1], which tracks in sensor-RAE coordinates and has a "seventh state" for the ballistic coefficient, was found to be "clearly superior" to the others in

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[1] KB(7,3) stands for Kalman ballistic filter, with 7 states (range, azimuth, elevation, range rate, azimuth rate, elevation rate, and ballistic coefficient) and 3 sensor measurements (range, azimuth, elevation).

terms of both accuracy and adaptability to a variety of missile and atmospheric conditions.

Precise tracking in the boost phase is more difficult, as it depends on the launch characteristics and design of the missile. The primary objective of boost-phase tracking is to back-propagate the trajectory so that the launch point can be estimated. A nonlinear dynamic model for boost-phase tracking is provided in Li, Kirubarajan, Bar-Shalom, and Yeddanapudi (1999). Li et al. use fully nonlinear maximum likelihood estimation in "batch mode" (non-real time) to estimate the boost-phase trajectory. A Kalman filter approach to tracking in the boost phase is provided in Blackman and Popoli (1999).

Integrating a boost-phase and ballistic-phase dynamic model is advantageous particularly when the target is acquired at the early stages. A method for performing the integration is to use an Interacting Multiple Model (IMM) filter. The IMM filter uses estimated transition probabilities to discriminate between the models. San Jose (1998) found that an IMM model performed well in tracking short-range ballistic missiles where a switch-over from the boost phase to the ballistic phase occurred during the period of observation. When the transition probabilities suggest that the initial boost stage is likely to have ended, the parameters of the boost-phase model are reset in case a second boost stage is detected. Blackman and Popoli (1999) recommend a three-model IMM filter, with the additional model used to allow for the possibility of a maneuvering target.

Although the missile trajectory estimation problem has been studied for many years, and the dynamics of motion have also been understood for quite some time, new algorithms continue to be developed, and it is hard to say that a clear "winner" has emerged. Different strategies reflect tradeoffs between accuracy, speed, and versatility. In particular, elaborated IMM models reflect the desire for the integrated tracking of a missile during its boost stage(s), during its ballistic phase, and possibly allowing for post-boost phase maneuvers. The effectiveness of any estimation strategy should be judged on a combination of scientific validity and performance.

2.4 Impact Point Prediction

With state and measurement equations specified, estimation of the point on the surface of the earth where the missile will make impact is conceptually straightforward. At time k, the estimated state vector $\hat{x}(k|k)$ contains the most up-to-date information available about the properties of the missile trajectory. Starting from this estimate, the dynamic state equation can be propagated forward, in continuous time and without process noise, until the norm of the state vector is equal to the radius of the earth at the indicated latitude (in Earth-centered Cartesian coordinates), or until the elevation is zero. Both the time of impact and the impact point can be estimated in this manner.

Propagation entails numerical integration of the first-order nonlinear differential equation associated with the state equation. It is important that this be done using a good-quality numerical integrator, because integration is prone to the accumulation of

small errors that, in the whole, can be substantial. Runge-Kutta (order 4) and its variants are considered to be well-suited for integration of the dynamic equation associated with missile trajectories.
Runge-Kutta can also be used to update the state equation during the operation of a Kalman filter.

Due to process noise in the state equation, the impact point is formally described as the boundary crossing of a diffusion process with nonlinear drift. Probabilistic treatment of boundary crossings is not a simple subject, and tracking a high-velocity missile in real time requires rapid turnaround of any solution. Simpler heuristic methods for finding a 95 percent prediction region (ellipsoid) for the impact point can be devised by propagating the prediction covariance along with the state vector using the Kalman filter. The accuracy of any heuristic method should be carefully evaluated before accepting it as valid.

2.5 Launch Point Estimation

An estimate of the launch point can be obtained, in principle, by back-propagation of the boost-phase trajectory. Using the ballistic-phase trajectory for this purpose is inherently problematical, unless reliable information is available on both the missile and its launch profile. Early detection, possibly using satellite-based infrared (IR) sensors, is therefore critical to timely estimation of the launch point.

Simple back-propagation schemes, based on polynomial interpolation of a small number of boost-phase position estimates, are reputed to be highly inaccurate. The methodology suggested by Li et al. (1999) for boost-phase line-of-sight (LOS) measurements (e.g., those obtained with an IR sensor) is a more rigorous approach, although it is prone to numerical ill-conditioning due to the small change in angles from the sensor to the missile over short periods of time. The launch-point estimation problem is best described as a continuing area of research, for which simple techniques may provide disappointing levels of accuracy.

2.6 Multiple-Target Tracking and Correlation

The computational burden of tracking a single ballistic missile in real time is compounded when there are potentially many missiles. At every time period, a new radar scan may detect a variety of objects, some representing previously-detected missiles that have moved, others representing non-missile objects such as aircraft, and others that are "false positives" representing clutter. The sensor must associate each of these objects with tracks that were recognized in the immediately preceding time period, disregarding those that do not represent ballistic missiles. It is possible that a given object can be matched to more than one track, and that a given track can have nothing in the current radar scan that is a plausible match to it.

There is a substantial literature on the multiple target tracking (MTT) problem, including Blackman (1986), Bar-Shalom (1989), Blackman and Popoli (1999), and Stone, Barlow, and Corwin (1999). A very cursory summary of the main ideas will be discussed here. The term *correlation* refers to the process by which objects recognized by the sensor are associated with the currently-understood state of the air space.

There are several broadly different concepts of how this process can be executed. The simplest is to assign an object to at most one track, and no more than one object to a track. Before these associations are made, it is necessary to propagate all of the state vectors for existing tracks to the current time, along with their estimated covariance matrices. A rejection region or *gate* is then formed around each of the estimated states, based on the covariance matrix of the estimated state and that of a presently-detected object. If the object falls within the gate it is a candidate for correlation to the associated track. Objects that do no fall within the gate of any track are candidates for initiating new tracks. Rules are developed to resolve basic ambiguities:

- For selecting one object, among several within the same gate, to update the corresponding track;

- For selecting one track, among several whose gates contain an object, to be updated by the object in question.

This association logic is, of course, subject to errors. One of the problems in multi-target tracking is to incorporate additional uncertainty due to correlation errors into the Kalman filters that are used to for track estimation. Failure to do so will overstate the precision of the predictions, which in turn makes subsequent correlation even more error-prone, in addition to invalidating the uncertainty assessments themselves. Blackman (1986) describes heuristic methods for making covariance adjustments that account for correlation error.

More sophisticated association procedures, such as multiple hypothesis tracking (MHT) and probabilistic data association (PDA), have been developed as alternatives to the somewhat simplistic association paradigm described above. A severe limitation of their use in multiple-target tracking of ballistic missiles is their computational burden. The objective is to balance information management with the need for rapid turnaround. Finding the right balance is a continuing area of development and research.

2.7 Sensor Bias, Registration and Multiple-Sensor Fusion

A sensor that detects a ballistic-missile target is subject to measurement error in stating the position and velocity of the target. Ideally, this error will consist only of random errors of known magnitude. Techniques such as the Kalman filter can account for these errors, producing updated state estimates and covariances that reflect increasing precision as more information becomes available. But there are some errors that will not gradually dampen out. These are the systematic errors, or biases, that affect all

measurements the same way. The main sources of bias to which sensors such as radar are vulnerable (Helmick and Rice, 1993) are listed below:

- Calibration (or offset) errors;

- Attitude (or orientation) errors;

- Sensor location errors;

- Timing errors.

These sources of bias can affect measurements in different ways:

Calibration. Sensors such as radar must be calibrated periodically to prevent drift in their measurements. Failure to perform effective calibration can lead to systematic angle errors, range errors, or both. The estimated track will then differ from the truth by a distance that can depend on the distance of the target from the sensor. Barton (1988) describes in detail the calibration issues that affect radar sensors.

Attitude. The rotational orientation of the sensor may be misaligned due to errors in the gyros of its inertial measurement unit (IMU). The estimated track will impart the same rotational error relative to the sensor in all measurements.

Location. Errors in the navigation system associated with a sensor, or an error in applying the Global Positioning System (GPS) to find the position of a stationary sensor, will typically result in a constant difference of position between the estimated and true tracks.

Timing. Errors in the clock of a sensor will cause the estimated track to either lag or fall ahead of where the track should be placed at a given time. For a ballistic missile, a timing error on the order of a fraction of a second can result in a substantial positional error.

Bias errors are a serious problem when a single sensor is engaged in tracking a single high-velocity target such as a missile. In the presence of many potential targets, or data from many sensors, the difficulties are compounded. Sensors that detect a common target can fail to recognize it as a single entity, which can lead to the initiation of false tracks or to erratic tracking. Similarly, a valid track can be dropped due to a failure of misaligned sensors to continue its detection. Dana (in Bar-Shalom, 1989) developed simple guidelines for effective data registration.

Data registration refers to a process by which data from multiple sensors are expressed in a common reference frame that reflects space-and-time reality. This process goes beyond the autonomous actions of sensor managers as they attempt to correct the identified biases in their individual systems. Relative alignment of sensors is needed to make data association paradigms workable. As a matter of policy, a range of procedures

can be used to align sensors in a multi-sensor tracking system to set system clocks, determine sensor locations, etc.

Data registration can also be built into the tracking software to identify biases that had not been removed previously. Methods have been developed using the Kalman filter (Helmick and Rice, 1993), and more recently, using neural networks (Karniely and Siegelmann, 2000). The latter, in particular, is computationally intensive and requires a substantial quantity of data. On the other hand, data registration should be viewed as an *ongoing* process, rather than a technique to be employed once threatening targets have been detected. Karniely and Siegelmann found that their neural network registration method successfully detected complex bias patterns that a Kalman filter was unable to detect.

3. Interoperability and Department of Defense Message Standards

A theater-area missile defense (TMD) family of systems (FoS) with TADIL A/B or TADIL J messaging is expected to conform to standards:

- MIL-STD-6011B (henceforth abbreviated 6011B) for TADIL A/B,

- MIL-STD-6016A (henceforth abbreviated 6016A) for TADIL J.

6011B addresses missile tracking only in the context of reporting air tracks. TADIL A/B is, at best, a minimal platform for meeting the composite tracking requirements of a TMD FoS. In contrast, TADIL J provides specific messaging resources to meet these requirements, and has been designated for use over the Joint Data Network (JDN) to communicate missile tracks between participants in a TMD FoS. This section will focus mainly on 6016A, because TADIL J is the messaging platform that was designed for achieving interoperability within a TMD FoS.

3.1 TADIL Message Standards

The message standards for TADIL A/B and TADIL J address similar issues pertaining to interoperability of a TMD FoS, but the former is less specific in many respects. The important issue of track quality (TQ) number reporting is illustrative. The TQ number is an assessment, by a participating system, of its reliability in estimating the position and velocity of a possible ballistic missile track. 6016A gives detailed instructions on how this quantity is to be calculated (on a 0–15 scale), but 6011B offers no instructions other than that the TQ number is to represent reliability on a scale where 0 = lowest and 7 = highest. Moreover, 6016A provides for the reporting of covariance information whereas 6011B does not.

Compliance with either message standard should be viewed as necessary, but not sufficient conditions for achieving interoperability. Coherent communication is necessary for systems to work together, but unless what is communicated is both sufficient and correct interoperability will not be achieved. This especially applies to joint surveillance of an air space that may contain high-velocity ballistic missiles.

3.2 Track Quality and Reporting Responsibility

In a multi-sensor environment there is a need to organize data processing so that accuracy, timeliness, and proper management of communication resources is achieved. But these are conflicting goals. An alternative concept of multi-sensor tracking is to have each sensor or system report its information to a central processor at every reporting interval, which then updates existing tracks, initiates new tracks, and drops obsolete or

erroneous tracks based on all available information. In theory, a "composite" tracker of this kind should be able to achieve greater accuracy than to have one system report on a track at any given time, and to have all systems process data autonomously. But the communication resources that the former would require exceeds what is contemplated in the TADIL message standards, and its workability in concept is not assured.

6016A conceives of joint tracking where only one joint user (JU) officially reports on a track at any given time. The reporting JU is assigned *reporting responsibility* (R^2) for the track. It is the responsibility of the JU with R^2 to perform the calculations needed to update the track with new information from the previous to the current time period, and to report the outcome to all other JUs in the network participation group (NPG). R^2 is awarded on the basis of "competition" between different JUs that are capable of reporting on a track. The JU that can report positional and velocity information with the greatest accuracy, according to its own claim, is assigned R^2.

The assignment of R^2 is based on a *track quality* (TQ) number that a JU calculates and transmits to express the reliability of its information about the track. 6016A offers specific instructions on how a TQ number is calculated. It is derived from a quantity B that based on the 6×6 covariance matrix of three-dimensional position and velocity measurements (in ECEF coordinates):

$$B = \sum_{j=1}^{3} [\mathrm{Var}(r_j) + \Delta_t^2 \mathrm{Var}(\dot{r}_j)]$$

where r_j is one of the three-dimensional position coordinates estimated by the sensor, $\dot{r}_j$ is the corresponding velocity coordinate, and Δ_t is a time increment, taken to be 6 seconds by default (6016A, p. 4-200). The quantity B is referred to a "look-up" table (6016A, p. 5.1-752) to determine the TQ number.

In some of its interoperability testing the JITC has observed message streams that contain both TADIL A/B and J messages. This mixture presents the FoS with the need to compare TQ numbers that were derived using different scales and different criteria.

3.2.1 Factors That Affect The Accuracy of TQ Numbers

Despite its more careful formulation, there are a number of reasons why TQ numbers reported on TADIL-J may fail to reflect true reliability:

1. Data registration was either neglected or not effectively executed, so that substantial **sensor bias** remains;

2. Covariance matrices do not accurately reflect **random measurement errors** of the sensor;

3. Covariance matrices do not accurately reflect **process noise** in the dynamic state model used to describe motion of the missile;

4. The covariance matrices do not accurately reflect **correlator errors**;

5. The JU **did not follow the guidelines** for calculating the TQ number.

Experience with TMD FoS testing at the JITC suggests that any or all of these problems may be present.

3.2.2 Effect of TQ Number Inaccuracy on Interoperability

Under the TQ-number concept for assigning R^2 even one JU that overstates its accuracy can degrade the system. Data registration, which is a basic requirement of any multi-sensor tracking system, must be rigorously observed if TQ numbers are to be used in the manner suggested by the TADIL message standards. There is no agent under this concept that can down-weight or disregard data from a sensor that appears to be misaligned.

Data registration will not be perfect. The bias adjustments themselves will be prone to error, mobile sensors will move, and sensors may drift gradually into miscalibration over time. At times when R^2 shifts from one sensor to another, spatial offsets in the track of some order of magnitude will occur. Depending on the magnitude of these offsets, the network interface will display tracks that "criss-cross" in physically unexplainable ways, display multiple tracks for the same object, and drop valid tracks prematurely. Neglected or poorly followed data registration procedures will increase the magnitude of the offsets and exacerbate these problems.

Even in the absence of registration errors, the association of sensor information to tracks will be subject to errors due to measurement uncertainty. During correlation, statistical "gating" criteria similar to hypothesis testing are used to determine which existing tracks (propagated to the current time) match a sensor's measurements. If none of the tracks match, a new track may be initiated. Similarly, if a track does not match to at least one sensor's measurements, it may be dropped. Random errors in sensor measurements, and process noise in the missile dynamics, make gating an imperfect process. Thus, false tracks and dropped tracks can be expected to occur under the TQ-number concept even under the best of circumstances. The questions that should be asked are, what are the lowest erroneous tracking rates that can be achieved, and can the FoS under consideration achieve these rates?

3.3 Quantitative Requirements for Space-Track Messaging

6011B requires that a JU report the TQ number, position, and velocity of a track using TADIL A/B messaging. For space-track TADIL J messaging, 6016A provides for the reporting of a larger set of track-related quantities, some of which are required, but others are subject to ambiguous reporting requirements.

3.3.1 Units of Measurement

Both TADIL A/B and TADIL J use English units for measuring distance, but their coordinate systems are different. TADIL A/B uses "flat Earth" topocentric coordinates centered at a System Coordinate Center (SCC). The positional measurements are East, North, and Height oriented. East and North are measured in data miles, in increments of ¼ data mile (1,500 feet). A maximum displacement of 511.75 data miles (581.5 standard miles) can be reported in either direction from the SCC in the East and North coordinates. Velocity is measured in data miles per hour, in increments of 28 1/8 dm/hr, in the East and North directions. A maximum absolute velocity of 3,571 7/8 dm/hr (about 1.13 standard miles per second) can be reported. Height is measured in feet, in increments of 500 feet. A maxium height of 127,000 feet (appx. 24 standard miles) can be reported. There is, however, no provision for reporting velocity in height. This somewhat limits the scope of missile reporting that is possible on TADIL A/B.

TADIL J uses ECEF coordinates based on the WGS-84 Earth Model for an oblate Earth. All coordinates are measured in feet, in increments of 10 feet. A maximum displacement of 41,943,030 feet (7,944 miles) from the center of the earth, in any coordinate direction, can be reported. Velocities are measured in feet per second, in increments of 3.33 feet per second. A maximum absolute velocity of 27,276 ft/sec can be reported in any coordinate direction. Since the radius of the earth is approximately 3,963 miles virtually any theater-wide ballistic missile can be tracked on TADIL J.

A mixture of coordinate systems and measurement units results if a TMD FoS has both TADIL A/B and J messaging. Although the conversions are not difficult to make, they present an opportunity for error. Additionally, a FoS concept that includes allied nations' assets should be aware of metric-English unit conversion issues. A confusion of feet with meters can sometimes produce seemingly plausible values.

3.3.2 Covariance Reporting

TADIL A/B messaging requires the reporting of the track number (TN), time, TQ number, and positional information in the units described above[2]. It also requires the reporting of velocity, but only for the surface (East, North) coordinates. There is no provision for reporting variances, covariance matrices or other measures of statistical

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[2] Other information is reported as well, on both TADIL A/B and TADIL J, but not covered here if it does not pertain to obtaining an accurate description of the motion of the tracked object.

accuracy, or other quantitative attributes of a missile trajectory (e.g., the impact point). The only information about the missile vehicle is a data field that indicates whether or not the track is a missile.

TADIL J, by contrast, provides for all of the above, plus complete velocity information, covariances, the impact point prediction (and its covariance matrix), the launch-point estimate (and its covariance matrix), the missile type (from an extensive list of known types), and the ballistic coefficient. Some of these quantities must be reported every time, others only under specified conditions, while others are subject to ambiguous reporting requirements. 6016A is particularly unclear about when the position-velocity covariance matrix must be reported. Perhaps as a result, the JITC has observed that covariance matrices are not consistently reported in TMD FoS testing. Without the covariance matrix it is difficult to compare stated accuracy to actual performance, to verify that the TQ number was calculated properly, and difficult for another JU to correlate its sensor data to the registry of existing tracks.

If a JU with R^2 on a track does not report covariance information in the J3.6 message, another JU can request this information by issuing a J7.1 message with the covariance indicator set. A "provide only upon request" policy (if one exists) may reflect an information management strategy to minimize laborious data processing and messaging. If this is the case, the benefits should be reviewed in light of the difficulties that not having this information creates. To wait for a request before transmitting covariance information adds to the message stream and delays the resolution of tracking issues through data processing.

6016A is also unclear on when the full 6 × 6 matrix should be reported, or when a JU may report two 3 × 3 covariance matrices representing the positon and velocity separately. The latter is appropriate *only* if it is known that position and velocity estimates are uncorrelated. The validity of this assertion is doubtful. In Kalman filter state equations, position and velocity are coupled. Additionally, sensors that use Doppler to measure velocity are subject to error sources that affect Doppler, range, and angle measurements simultaneously (see section 2.2). The small savings in processing time and messaging that may be achieved by a partial transmission of covariance information should be weighed against the consequences of making what is a potentially false assumption.

Under 6016A a covariance matrix is supposed to reflect all known sources of error, including random error and systematic error (bias). Systematic error arises from sensor measurement bias and from registration errors. Random error is a part of the uncertainty of sensor measurement, and it also arises from process noise in the dynamics of missile motion. An additional source of error is due to the use of a correlator algorithm. Correlator error refers to uncertainty in assigning sensor data to tracks.

Standard usage of the term "covariance matrix" refers to the random error component only. If the Kalman filter is used, this matrix is updated in real time along with the state vector using a recursion relationship. The resulting covariance matrix

reflects both random error and process noise at any given time. Bias is sometimes incorporated into a covariance matrix by treating it as a "random effect." However, if the bias is known or carefully estimated, it is usually preferable to calibrate rather than to adjust the covariance matrix. Random effects are not "independent" across a series of measurements, which makes it difficult to interpret confidence ellipses and prediction regions that reflect their influence. Similarly, correlator errors are difficult to estimate, and are not included in the usual concept of a covariance matrix. Nonetheless, correlator errors do increase the uncertainty, and their existence should be reflected.

Because 6016A does not offer guidance on how all sources of error should be reflected in a covariance matrix, it is likely that not all JUs will estimate it the same way. Covariance matrices, and the TQ numbers derived from them, can be expected to vary among JUs for this reason alone.

In TADIL J messaging a covariance matrix is not transmitted directly. 6016A requires that the 6×6 covariance matrix of position and velocity, denoted P, be mathematically encoded before it is communicated to other JUs. The encoding procedure is described in section 4.4.6.9 of the standard, and briefly summarized below:

1. Extract the diagonal of P (the variances), and take their square roots (the root variances).

2. Use the root variances to scale P to a 6×6 correlation matrix C.

3. Precompensate C by multiplying its off-diagonal elements by a number slightly smaller than one $(1 - 2^{-22})$ to preserve its positive definiteness under roundoff error. Call this new matrix C_p.

4. Find the Cholesky (root) matrix, U, of C_p. The Cholesky matrix is a 6×6 matrix with zeros below the main diagonal (i.e., at most 21 of the 36 entries of U are not zero).

5. Take the (base 10) logarithms of the root variances.

6. Transmit the log-root variances and sufficient information from U to allow reconstruction of P by the receiver. This requires the transmission of 165 bits in the J3.6 message.

These steps are reversible: given the Cholesky matrix and the logarithms of the root variances it is possible to reconstruct P.

If only partial covariance information is transmitted, encoding is applied separately to the upper 3×3 submatrix of P that relates to positional coordinates, and to the 3×3 lower submatrix of P that relates to velocity coordinates. As discussed earlier, the use of partial covariance information assumes that positional and velocity estimates do not cross-correlate.

The following points can be made about the encoding procedure:

1. <u>Transmitting the Cholesky matrix does not conserve bandwidth</u>. If the elements of C_p above the main diagonal are transmitted instead of what is transmitted on U, the same number of bits (165) is required[3].

2. <u>The benefit of transmitting Cholesky matrices should be clearly defined</u>. Kalman filter propagation at the JU level may be able to use Cholesky matrices to advantage, but only if software is designed accordingly. If JUs reconstruct the original covariance matrix, any advantage is lost. 6016A does not address this point, and even appears to imply that the reconstruction will be performed.

3. <u>The encoding procedure should be evaluated as a whole</u>. This should be done with respect to both numerical accuracy, and computational burden, at both the transmitter and receiver ends. This is further addressed in section 3.3.3 below.

The transmission of complete covariance information in a numerically efficient and sound manner is an interoperability issue. Adding to the computational burden of JUs without tangible benefit reduces response time, and imparts numerical error, both of which make tracking high-velocity objects a greater challenge.

3.3.3 About Numerical Accuracy

Each covariance element that is transmitted in the J3.6 TADIL J message is assigned 10 bits for its absolute value, plus one bit for a sign (+1 or −1) if needed. Numerically, 10-bit coding is somewhat "coarse" in that it allows only 1,024 distinct numbers to be represented. For an individual number, this may be good enough as an approximation, but in the context of joint tracking it can pose several problems. One problem is that covariance matrices may become ill-conditioned. Another problem is that numerical error may build up over repeated application of the Kalman filter. Numerical error propagation is recognized as a significant problem in usage of the Kalman filter (Grewal and Andrews, 1993).

It is doubtful that the precompensation described in section 3.3.2 will have much effect on numerical stability. Applying a multiplicative factor of $1 - 2^{-22}$ will make little or no difference in how a covariance matrix is encoded with 10-bit digitization.

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[3] The precompensated correlation matrix C_p is symmetric and has ones on its main diagonal, so only the 15 elements above the main diagonal are informative.

3.3.4 Launch-Point Estimate Reporting

6016A assigns R^2 for the launch point in a different manner than for space tracks in general. Only sensors that detect the object shortly after launch would normally be eligible for R^2. The standard further states that "back azimuth" should be used to estimate the launch point. But as noted in section 2, simple back-extrapolation schemes are not regarded as very reliable for this task. Missile motion dynamics over one or several boost phases, followed by transition into a ballistic phase, makes reversing the trajectory a difficult problem. Recent research by Li et al. (1999) may offer a better approach. Their maximum likelihood (ML) estimation technique also provides an estimated covariance matrix for the true launch point.

3.3.5 Impact-Point Prediction Reporting

6016A assigns R^2 for the impact point to the same JU that has R^2 for the space track. The impact point can be estimated by integrating the dynamic state model (without process error) forward in time until impact with the earth is achieved. However, the prediction covariance matrix for the impact point should reflect two sources of variability:

- Error in estimating the position and velocity of the track, given by the covariance matrix reported under TADIL J;

- Process noise, or variability in the dynamics of motion.

There is an implicit assumption that the sensors are unbiased: if they are not, then the prediction covariance matrix may fail to "localize" the impact point in a high-probability elliptical region.

The JITC has often observed that, when R^2 shifts from one JU to another, not only do the impact-point predictions change, the 95 percent probability ellipses also change, both in shape and size. This may be the result of different sensor biases, or the use of different techniques to predict the impact point and estimate its covariance matrix. Sensor biases are better addressed through calibration and data registration procedures than by attempting to account for them in covariance matrices. If a JU knows its sensor biases, it should try to remove them; if a JU does not know its sensor biases, it is not in a good position to account for their effects.

Accounting for process noise in the prediction covariance matrix poses a problem for joint tracking under TADIL J. Consider a JU that has had R^2 on a track for a period of time. From its repeated measurements and use of the Kalman filter, the JU obtained the covariance matrix of the current state estimate *and* the covariance matrix of the process noise. This enables the JU to estimate both the impact point and its associated prediction covariance matrix. Now suppose that a different JU assumes R^2. It continues tracking using its own sensor measurements, and the Kalman filter outputs of its

predecessor. *But, the process noise covariance matrix was not transmitted.* Because TADIL J makes no provision for the transmission of this "other" covariance matrix, it must be estimated anew each time R^2 changes.

This represents a loss of information to the network, and it is disadvantageous for joint tracking. Every time R^2 changes and the process noise covariance matrix is re-estimated, it is done so using only a small number of state-equation residuals, which makes the estimate unreliable. If the process noise is substantial, this can lead to visual disruption in the 95% prediction regions for the impact point at the times when R^2 changes.

3.4 Correlation Under TADIL Message Standards

Both 6011B and 6016A discuss correlation, although 6016A covers this subject in greater detail. Under both standards it is the obligation of the JU to correlate its sensor data to the existing registry of tracks. If no association can be made to an existing track, the JU may initiate a new track for the object. 6016A does not require that one, specific correlator be used by all JUs, or even that the correlator methodology be the same.

The correlation process can be briefly described as follows:

1. Each existing track has a time tag, a position-velocity state vector (six-dimensional), and a position-velocity covariance matrix (6 × 6, see section 3.2.2 above).

2. If a JU has sensor data at the "current" time to be correlated to the existing registry, then that JU must propagate each of the state vectors and covariance matrices forward to the current time, using standard Kalman filter techniques[4].

3. The state and sensor covariance matrices are combined[5] to form "gating" regions. All tracks in the registry whose gates contain the sensor data are eligible for association. If there is more than one such track, association logic (e.g. nearest neighbors) is used to choose the "best" match.

4. If no track in the registry contains the sensor data in its gate, then a new track may be initiated in the registry.

Step 2 of this process requires full, and reliable, covariance information from all JUs that have R^2 on a track. Without it, another JU cannot properly update the state vectors or form the necessary gating regions. This again points to transmission of the covariance matrix as an interoperability issue.

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[4] In practice, some tracks may be ruled out quickly and not require propagation, thus reducing the computational burden of the JU.

[5] The combination must occur in either the coordinate system of the sensor or that of the state vector. Some sensors, such as IR, may not provide the measurement equivalent of a full six-dimensional state vector.

6016A does not offer specific, quantitative guidelines on correlation. The JUs may use different correlators that operate under different criteria for gating and matching sensor data to tracks. It is possible, for example, that different tradeoffs between the generation of false tracks and the dropping of valid tracks may be in place simultaneously throughout the FoS.

3.5 Data Registration Under TADIL Message Standards

Both 6011B and 6016A recognize the importance of having sensors and their measurements properly aligned with each other. The term *data registration* is used here in the same sense as section 2.7: it refers to a process in which a sensor's location and orientation are correctly determined, its measurement biases are removed through calibration, its clock is set, and any remaining relative biases between sensors are detected (and removed) by means of a data analysis procedure[6]. In 6016A four types of error that data registration is designed to address are identified:

1. Geodetic position errors

2. Sensor errors

3. Data processing errors

4. Remote interface unit (IU) registration errors.

Table 4.1-5 (6016A, p. 4-18) summarizes these errors, and Figure 4.1-3 (6016A, p. 4-23) gives a diagram of the data registration process. 6016A provides acceptable error tolerances for JUs to use as data registration standards.

The data registration activities described in 6016A can be best described as autonomous: each JU is responsible for its own compliance, but compliance is not monitored. There is no messaging from a JU to the network concerning its data registration activity. This activity is also to be carried out "with minimal interference to, or curtailment of, normal data system operation." (6016A, p. 4-21). Some interactive data registration issues are covered under precise participant location and identification (PPLI) activity (section 4.3 of 6016A). PPLI addresses synchronization of the JUs relative to each other, with respect to system timing and relative navigation. TADIL J provides message space to conduct these PPLI activities.

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[6] The TADIL message standards define *sensor* registration as one activity under the scope of *data* registration, as is done here. See section 4.2.2 of 6011B and section 4.1.4 of 6016A. In the scientific literature, sensor registration usually refers to the entire scope of data registration activity.

3.5.1 The Data Registration Imperative

Although the TADIL message standards recognize the importance of data registration, few of its facets are included in network messaging. If a JU transmits data with registration errors, which are unlikely to be reflected in TQ numbers, the network will not be aware of this and interoperability will be degraded. In addition, the treatment of data registration as an item of secondary importance may promote its neglect, to the detriment of interoperability.

Effective data registration requires more careful articulation than what is provided in the TADIL message standards. Consider a TMD FoS that operates over a certain geographic area. Each of its sensors must be calibrated on an ongoing basis. Mobile sensors must recalculate their locations. But, corrective procedures will be successful to only a certain extent. Relative biases will remain between sensors due to errors in performing calibration, or neglect of calibration, by even one sensor. Because of the decentralized nature of data processing in the JDN, data registration should be made an integral part of TMD FoS operations.

3.5.2 Integrating Data Registration Into Operations

There is a good argument to be made that a TMD FoS cannot be made interoperable unless the network exchanges information on data registration. To bring this about would require modification of the operational concept of a TMD FoS that uses TADIL J messaging over a JDN. Integrating data registration into operations would have several tangible benefits:

- Interoperability problems that arise from registration errors would be traceable to their cause;

- Real-time data registration procedures would be made feasible;

- Systems would be aware that data registration procedures must be integrated into their operations.

In the current concept of interoperability testing, the biases of each system can be assessed *post facto* by comparing the content of track messages to "truth" trajectories. This type of analysis is important, but it is retrospective. Biased tracks can enter the system without detection, which degrades interoperability.

On an operational level, integrating data registration into network messaging would require that a "dummy object", with a trajectory known to all JUs, be provided for the systems' sensors to track. Ideally, this would be a physical object (e.g., an aircraft, UAV, or satellite), but a simulated object may be at least partially adequate. The message stream would contain each JU's track of the dummy object in addition to its regular surveillance tracks.

Making data registration an ongoing and interactive part of TMD FoS operations would facilitate the use of bias-estimation (and removal) techniques that are designed for this purpose. Two of these techniques (Helmick and Rice, 1993; Karniely and Siegelmann, 2000) are discussed in section 2.7. Karniely and Siegelmann use data from an episode of multi-sensor tracking to "train" a neural network that characterizes the biases between sensors. This characterization defines the adjustments that are needed to calibrate subsequent sensor measurements. Because biases change with time, it is necessary to retrain the neural network periodically.

Integrating data registration into the operation of a TMD FoS would present a challenge for interoperability testing. Under current testing, the FoS is configured to operate for the duration of a short, threat-laden episode in the context of a live missile fire or a simulated event. The FoS does not formally experience a maturation period when data registration procedures can be established.

Understandably, it may be difficult for the JITC to alter its testing in line with the above suggestions, but a modest augmentation of its current testing regime may be a useful first step:

ADDING DATA REGISTRATION TO INTEROPERABILITY TESTING

1. Conduct testing in three stages: (1) **FoS pre-test**, (2) **registration interval**, and (3) **FoS re-test**. The pre-test refers to a joint tracking event (real or simulated) as used in current interoperability testing.

2. In the registration interval, the FoS conducts joint tracking of objects with "truth" positions messaged in real time over the network on TADIL J. Each JU that detects the track issues a J3.6 message that includes TQ numbers and covariance matrices (but not launch-point estimates or impact points), at every reporting opportunity.

3. The registration interval may use live or simulated missile tracks, aircraft tracks, etc.

4. Each JU can use registration interval messages to conduct data registration.

5. Start the re-test upon completion of the registration interval.

6. A system is not evaluated on how well it performed in the registration interval, but it can be evaluated on how well it used the registration interval to improve its performance in the re-test.

This augmentation should be viewed as a demonstration project, rather than as a comprehensive data registration procedure of the type described earlier. Nonetheless, each JU would acquire information that it could use to reduce its registration errors. There are many potential variations of this three-tiered testing concept. The JITC should determine which variation is the most appropriate for the FoS that is under test.

4. Joint Interoperability Testing: Current Approaches

Over the last several years the JITC has conducted interoperability testing of TMD FoS based on the JDN concept, in which TADIL J messaging is the central communications platform. The JITC has also conducted interoperability certification testing of the PATRIOT Advanced Capability Level 3 (PAC-3) system relative to the same concept. Testing to date has emphasized the following three of five identified layers: (1) physical layer, (2) information layer, and (3) functional layer. These layers are hierarchical, in the sense that functionality cannot be achieved without a workable information layer, nor can information integrity be assured without the physical layer to support it.

The material addressed in this report assumes that testing has proceeded beyond the physical layer to the information and functional layers. Discussion of covariance matrix transmission in section 3.4 identified interoperability issues in the information layer. In the functional layer interoperability is addressed in topics such as the validity of tracking algorithms, the TQ-number concept, correlator performance, and data registration.

The JITC has recognized that interoperability testing and certification under the Joint Composite Tracking Network (JCTN) concept may be conducted at a future time. JCTN, which was still under development at the time this report was written, refers to "a joint telecommunications network and processing capability to enable composite tracking among joint, heterogeneous mixes of sensors, and to support appropriate levels of cooperative engagement of targets by weapons systems."[7] JCTN stands in contrast to JDN in several important respect, the most important of which is that, under JCTN, each system must process tracking information using "the same algorithms running on processors with virtually identical architectures."[8] It is likely that JCTN will require message standards that are neither a subset nor a superset of 6016A in order to support it.

JCTN is being designed to overcome some of the interoperability problems at the functional level that have been exhibited in TMD FoS testing under the JDN concept. Its success remains to be proven, although it is modeled on the U.S. Navy's well-received Cooperative Engagement Capability (CEC) concept. In conjunction with the development of JCTN, performance metrics are also being developed and distributed to appropriate parties in the form of benchmarking software[9].

Most of the JCTN performance metrics are related to interoperability. Some of the metrics are similar in scope to what the JITC has been using in testing under the JDN concept. It would be desirable to have a set of consistent metrics so that incremental improvements can be measured as TMD FoS concepts evolve. Comparisons can then be made for a given FoS configured under JDN and JCTN.

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[7] Joint Composite Tracking Network (JCTN) Summary Tutorial, p. 3B.
[8] id., p. 4A.
[9] Version 1.08 of the JCTN Benchmarking Environment was released on June 5, 2000.

It is useful to review the benchmarking metrics that have been developed for JCTN (Rothrock and Drummond, 1999), which stress interoperability respects that are in some ways similar to those that the JITC has developed. The JCTN metrics are calculated in benchmarking software that is integrated by the interfacing systems into their test articles. This integration allows the test articles to be treated as "black boxes," to be judged by performance without the need to view source code. The JCTN concept of uniformity between systems' software makes this integrated testing possible.

JCTN is not designed to operate under a R^2 concept based on TQ numbers. For this reason much of the testing that is done under the JDN concept (involving, e.g. R^2 violations, TQ-number issues) has no analog under JCTN. None of the JCTN metrics address estimation of launch points or prediction of impact points, which are recognized as important data in 6016A and by the JITC. Similarly, the integrated structure of JCTN suggests the use of metrics that would not be applicable to JDN.

Table 4-1 below summarizes the JCTN benchmarking metrics, with comments on their relation to metrics that are used by the JITC in its TMD FoS testing. The metrics are calculated with respect to a set of *scoring times* that are randomly chosen from the beginning to the end of the test event. The random selection of scoring times is designed to prevent "gaming" of the metrics. Some of the metrics require local tracking information that is not readily available under JDN, and others are closely tailored to the JCTN concept. Formulas for the JCTN benchmarking metrics can be found in Rothrock and Drummond (1999).

Table 4-1. JCTN Benchmarking Metrics (see note at end)

FUNCTIONAL AREA	JCTN METRIC	CALCULATION	RELATION TO JITC METRICS
Completeness history	1.1 Composite completeness	Ratio of the number of valid composite tracks to the number of valid objects that should be tracked. Calculated at each scoring time.	MOPs 1-1-5, 1-1-6, 2-1-3, 2-2-3, 2-2-6 (Dendritic)
	1.2 Relative completeness	Ratio of the number of valid composite tracks to the maximum, over all sensors, of the number of local tracks that can be associated with a valid object. Calculated at each scoring time.	Cannot calculate unless systems reveal their local tracking.
Timeliness	2.1 Composite track initiation time	Time at which a valid object is first assigned a composite track. If no composite track is assigned then the time is set to a maximum value. Calculated for each valid object.	JMOP 1.5 (PAC-3 LUT) MOPs 1-1-4, 2-1-1, 2-2-1, 2-2-4 (Dentritic)
	2.2 Composite track relative initiation time	Composite track initiation time (2.1) minus the time that a local track is first assigned to the object (= 0 if difference is negative). Calculated for each valid object.	Cannot calculate unless systems reveal their local tracking.
	2.3 Composite track convergence time	Time from when a valid object is first assigned a composite track to when the position error is reduced below certain thresholds (e.g. 100 m). Calculated for each valid object.	Not calculated currently, but calculation is possible.
Track continuity	3.1 Cumulative swaps of composite tracks	For a fixed valid object and a fixed scoring time, count the number of "object swaps" that have occurred among composite tracks for that object up to that time. Average these counts across valid objects and Monte Carlo-event simulations.	JMOP 1.10 (PAC-3 LUT)
	3.2 Cumulative broken composite tracks	Similar to (3.1) but with counting of the number of scoring times that no composite track is associated with the valid object.	Can be calculated with information analogous to that used for the above.

Table 4-1. JCTN Benchmarking Metrics (see note at end)

FUNCTIONAL AREA	JCTN METRIC	CALCULATION	RELATION TO JITC METRICS
Ambiguity	4.1 Composite track redundant track	Ratio of the number of composite tracks that are assignable to valid objects to the number of valid composite tracks.	Requires that the JITC be able to correlate tracks, *non-uniquely*, to objects. Local track information needed. Similar in concept to JMOP 1.6 (PAC-3 LUT)
	4.2 Composite track spurious track	Ratio of the number of composite tracks that are not assignable to valid objects to the number of valid composite tracks.	Ditto.
Accuracy	5.1 Composite track accuracy	For a fixed valid object, the root mean squared errors (RMSE), integrated to each scoring time, comparing the composite track six-state position-velocity vectors to the truth.	MOPs 2-1-2, 2-2-2 (Dendtritic)
	5.2 Composite track covariance consistency	For a fixed valid object and a fixed scoring time, the chi-square statistic (Mahalanobis distance), based on the position-velocity covariance matrix, for composite minus truth state vectors. Averaged across Monte Carlo simulations.	Not calculated by the JITC
Cross-platform commonality history	6.1 Ratio of non-common composite track numbers	For a given pair of tracking processors and a fixed scoring time, the ratio of the number of composite track numbers that are different between them to the number of composite track numbers in the union of the two sets.	Not calculated by the JITC
	6.2 Composite track state estimate differences	For a given pair of tracking processors, the combined distances across composite tracks and time of the Euclidean distances between state vectors.	Ditto.

Table 4-1. JCTN Benchmarking Metrics (see note at end)

FUNCTIONAL AREA	JCTN METRIC	CALCULATION	RELATION TO JITC METRICS
JCTN loading	7.1 Communication data loading	Sum of sensor data rates for invoked pruning and compression	Analogous to information layer interoperability testing for JDN
	7.2 Processor loading	Peak number of floating point operations (FLOPS) per processor per scenario second	Ditto.

Note: JITC metrics refer to the following documents:

PAC-3 LUT: Detailed Event Plan (DEP) for the PATRIOT Advanced Capability User Test (LUT) (Draft, February 2000)
Dendritic: Operational Issues for Joint TMD FoS Interoperability "Plug and Fight" (JITC, 27 September 1999)

5. Extending the Scope of Interoperability Testing

In this section some recommendations are made for extending the scope of interoperability testing of a TMD FoS. The aim of these recommendations is to address real or potential interoperability problems that current testing or test metrics are not designed to address. An attempt has been made, where possible, to offer recommendations that are compatible with the TADIL J message standard (6016A). In the second subsection new metrics are suggested for assessing the accuracy of TQ numbers and trajectories.

5.1 Recommendations

Recommendation 1. In TMD FoS interoperability testing based on a TADIL J network messaging platform (e.g. JDN), interfacing systems should be instructed that all J3.6 messages must contain full covariance matrices.

Notes:

1. Full covariance matrices are required by systems in order to correlate their local tracks to the registry of network tracks.

2. If a system cannot produce a full a covariance matrix in accordance with message standards, its other computational activities are questionable as well.

3. Transmission of partial covariance matrices is inadequate. Partial covariance matrices fall short of what is needed by some correlators, and they give an incomplete picture of uncertainty in the track position-velocity vector.

Metrics:

MET-R1-1 Covariance Completeness	Proportion of time that full covariance matrix is transmitted by a system.
MET-R1-2 Covariance Latency	Amount of time that elapses from when a system (a) receives a J3.6 space track, (b) responds with a J7.1 message with request for covariance matrix, and (c) receives a new J3.6 space track with the requested information.

Recommendation 2. **Transmission of a numerically accurate covariance matrix is an interoperability issue. The JITC should conduct specialized testing to examine whether each of the interfacing systems can calculate, encode, and decode covariance matrices accurately.**

Notes:

1. A collection of J3.6 space track messages, each containing a full, 6-by-6 covariance matrix, should be developed by the JITC for this testing. The collection should be designed to reflect realistic conditions. Impact and launch point information in the J3.6 message may be omitted.

2. The JITC or a designated agent transmits a series of J3.6 messages from the collection. Each system receives the messages, and reconstructs the covariance matrices. The reconstructed covariance matrices are provided to JITC for accuracy evaluation.

3. Similarly, each system is provided with a set of covariance matrices to encode and transmit. JITC evaluates the TADIL J messages for covariance accuracy.

4. Adaptation of the above procedures may be necessary to account for messaging capabilities, and capabilities of the systems' software.

Metrics:

MET-R2-1 Covariance Reconstruction	Compare reconstructed covariance matrix to the original. Error is measured by a weighted sum of squares: $$E^2 = \sum_{i=1}^{6} \sum_{j=1}^{6} \frac{(\hat{v}_{ij} - v_{ij})^2}{v_{ii}\, v_{jj}}$$ where v_{ij} is an element from the true covariance matrix, and $\hat{v}_{ij}$ is the same element from the matrix after reconstruction. Partial error measures, using only the upper 3×3 (position) or lower 3×3 (velocity) covariance matrices can be used as supplementary metrics.
MET-R2-2 Covariance Transmission	Use the same error measure as in MET-R2-1

Recommendation 3. **Interoperability problems that arise during TMD FoS testing may result from problems within a particular system. A TADIL J message stream obtained during FoS testing may not reveal enough information to pinpoint the source of a problem. Interoperability testing should include a component in which interfacing systems operate in non-FoS (autonomous) mode. In autonomous mode, each interfacing system tracks what it can, and provides full messaging based on its local tracks, without regard to reporting responsibility (R^2) rules.**

Notes:

1. Event-based testing should be conducted in two tiers:

   Tier 1. Full local track messaging, non-interactive mode
   Tier 2. TADIL J compliant messaging (with R^2 rules), interactive mode.

   Non-interactive mode testing may be performed without FoS assemblage under some circumstances. Each system interacts with an event simulator and tracks what it can.

2. Both tiers should employ the same metrics (where possible) related to functional interoperability. This will serve two purposes: (1) it will give reliable performance metrics for each individual system, and (2) it will allow "change in interoperability" to be measured as the TMD FoS progresses from Tier 1 to Tier 2.

3. In Tier 1 testing the JITC can construct a "composite tracking" picture by examining the reported TQ numbers on each of the tracks, and assigning R^2 in accordance with the rules specified in 6016A. The performance of this "autonomous FoS" can be compared to Tier 2, to measure the benefit of interactive, composite tracking.

Metrics:

   Use the same metrics currently used by the JITC, by JCTN (where applicable), and those suggested in section 5.2.

Recommendation 4. **Estimation of track position-velocity missile state vectors cannot be reduced to a single, valid formulation. In the absence of a single, standard tracking algorithm that each interfacing system of a FoS must adopt, tracking algorithms can be expected to vary across systems. As part of interoperability testing, the JITC should have each system declare, in the form of a written report, important aspects of its tracking algorithms as they pertain to tracking ballistic missiles.**

The declaration should include the following items:

1. Coordinate systems used for both the state and measurement equations.

2. Type of tracker algorithm used (Kalman filter, EKF, IMM, etc.).

3. Description of the state equation, and the elements of the state vector.

   - This includes the method used to incorporate process noise into the state equation, and the assumptions made about process noise.

   - Oblate earth gravity model used.

4. Method for incorporating the ballistic coefficient (atmospheric drag) into the model

   - If ballistic coefficient is provided in the J3.6 message but is incorrect, how does the model adjust?

   - If ballistic coefficient is not provided in the J3.6 message, how does the model adjust?

   - Model for atmospheric density used.

5. Method for estimating the launch point and its covariance matrix.

6. Method for predicting the impact point.

   - Numerical integration method used to propagate the state equation.

   - Method for estimating the covariance matrix of the impact point.

7. Is the WGS-84 model used for conversion between ECEF and geodetic coordinates?

Notes:

1. This information may be useful in understanding why a system performs the way that it does when the message stream is too scant to draw reliable conclusions based on performance.

2. At the present time, use of the WGS-84 model for coordinate conversion is strongly suggested if not required by 6016A.

3. Allowance for an either unspecified or incorrectly specified ballistic coefficient should be made in all algorithms.

4. At a future time, it may be necessary to standardize some of the aspects of tracking algorithms listed above, or others, in order to promote interoperability.

Recommendation 5. The JITC should assess the models that are used for generating missile trajectories in event simulators. A criterion for suitability is whether live-missile trajectories (e.g. Coral Talon) are within a "high confidence set" of trajectories that can be obtained if the event simulator is primed with the same initial conditions as the truth trajectory.

Notes:

1. This is a difficult problem, because there is not a large collection of live ballistic missile trajectories, with known (or nearly known) positions and velocities at closely spaced times, that can be used as reference set.

2. Dynamic modeling of even a small number of truth trajectories may give insight on how process noise should be included in models for tracking and simulation.

3. Developers of event simulators should reveal the models that they use to simulate missile trajectories.

Recommendation 6. Data registration is an important activity to ensuring the interoperability of a TMD FoS. Unfortunately, TADIL J does not provide the messaging resources that are needed to make on-going and interactive data registration an integral part of the operation of a TMD FoS. It is therefore unlikely that the JITC can address this situation directly in testing. But the JITC can, in a realistic manner, incorporate a registration interval into its interoperability assessment procedures.

Notes:

1. Details of the proposed three-tiered test sequence are given in section 3.5.2. The idea is that after a test event is conducted (pre-test), the FoS participates in a registration interval where missile tracks *with known trajectories* are presented to them. The registration interval allows the systems to detect (and attempt to correct) its registration errors before the re-test begins.

2. If an event simulator is used, the following three-step test sequence can be followed:

   - Pre-test using an event simulator (simulations $1, \ldots, m$)

   - Registration interval

- Re-test using the same event simulator (simulations $m+1, \ldots, 2m$)

where m is the total number of Monte Carlo runs before and after the registration interval. MOPs and MOEs can be obtained for the first m simulations and the last m simulations.

Metrics:

The "after minus before" differences in MOPs and MOEs used by the JITC currently, by JCTN, and those suggested in section 5.2 below can be used to measure the benefits of performing data registration. Systems can be individually evaluated in a similar manner, to assess their ability in using the registration interval effectively.

Recommendation 7. **The JITC should adopt metrics related to *track latency*, which is the difference in time reported in a space track message and time as appropriate to the position and velocity of the object reported in the same track message. Track latency reflects the time required to process and transmit a space track message that is not fully reflected in the reported time.**

Notes:

1. Measuring track latency is a potentially difficult problem, due to the need to time-align the reported track with the "truth track." If a system has tracking bias it will be difficult to construct this measure reliably. Use of a data registration pre-test may alleviate this problem to some extent.

2. Tracking error can be decomposed into track latency and spatial error if the former is provided. Spatial error can be found by making a latency adjustment to the reported tracks, and finding the Euclidean distances between the adjusted tracks and the truth tracks.

Metrics:

MET-R7-1 Track Latency	Time adjustment (+/−) to the reported track times such that the total squared (or absolute) error between the position and/or velocity vectors in the reported tracks and the same vectors in the truth tracks is minimized.

5.2 Additional Metrics for Interoperability Testing

The metrics suggested below are different from those currently used by the JITC, or by the JCTN benchmarking metrics software. As suggested in the recommendations made in section 5.1, these metrics can be generated for a given test under the testing variations that were described:

How one metric can be used to generate four measures
(A, B, C, D) under different testing variations

	Before data registration pre-test	After data registration pre-test
Tier 1 (non-interactive)	A	C
Tier 2 (full FoS operation)	B	D

FoS synergy effects: B – A, D – C
Data registration effects: C – A, D – B

METRIC	DESCRIPTION
MET-A-1a **Track accuracy** **(ballistic coefficient correctly reported)** **Submeasures:** **Component errors.** **Altitude, range, azimuth errors.** **Geodetic error.**	Distance measures between reported and a truth trajectory indexed to a range of test times. A <u>correct ballistic coefficient</u> is reported initially. Systems follow their own procedures for reporting the ballistic coefficient in the J3.6 message. Track distance measures (each is a function of time): (a) Full relative. $d^2 = \sum_{j=1}^{3} \dfrac{(\hat{r}_j - r_j)^2}{|r|^2} + \sum_{j=1}^{3} \dfrac{(\hat{v}_j - v_j)^2}{|v|^2}$ (b) Position relative. Same as (a) but only first sum is used. (c) Velocity relative. Same as (a) but only second sum is used. (d) Full chi-square. $d^2 = (\hat{x} - x)' V^{-1} (\hat{x} - x)$ (uses six-dimensional state vector and 6×6 covariance matrix) (e) Position chi-square. $d^2 = (\hat{r} - r)' V_{rr}^{-1} (\hat{r} - r)$ (uses three-dimensional position vector and 3×3 covariance matrix) (f) Velocity chi-square. Similar to (e), but using velocity components.

METRIC	DESCRIPTION
MET-A-1b	Same as MET-A-1a, except that the ballistic coefficient is omitted from the J3.6 message.
MET-A-1c **Track accuracy** **(ballistic coefficient incorrectly reported)**	Same as MET-A-1a, except that an incorrect ballistic coefficient is made available initially.
MET-A-2a,b,c **Accuracy of the ballistic coefficient as reported in the last track on an object.**	Compare the ballistic coefficient reported in the J3.6 message obtained from the last JU that reports a track on a real object, to the true ballistic coefficient for that object. Do this for the three scenarios (a,b,c) listed above.
MET-A-3 **Latency-adjusted error (position and/or velocity)**	Latency-adjusted distance between reported tracks and truth tracks. See MET-R7-1 for a definition of track latency. The metrics defined in MET-A-1a can be used after track latency adjustment is made.
MET-A-4 **Launch point estimation error** **Submeasures:** **Geodetic error.** **Error in direction of trajectory (plane of orbit), and in the perpendicular direction.**	Distance, in earth-surface miles, between the estimated and actual launch point. This distance is best obtained in a simulation setting, where a series of estimated launch points are available for comparison to "truth". An alternative error measure is the following: $$d^2 = (\hat{x}_e - \mu_e)'V_e^{-1}(\hat{x}_e - \mu_e)$$ where the difference in the two-dimensional vectors for estimated and actual launch point is "scaled" by the covariance matrix of the estimated launch point. The covariance matrix can be inferred from the information reported on positional accuracy (J3.0 message).

METRIC	DESCRIPTION
MET-A-5	Distance, in earth-surface miles, between the predicted and true impact point. If an event simulator is used, the algorithm used to generate the truth track can yield a mean impact point, μ_p and a covariance matrix Σ_p for the impact point. An alternative measure of the impact point prediction error is then given by $$d^2 = (\hat{x}_p - \mu_p)'(\Sigma_p + V_p)^{-1}(\hat{x}_p - \mu_p)$$ where V_p refers to the covariance matrix of the impact point estimate (obtainable from the J3.0 message). If an event simulator is not used (i.e. a live missile trajectory), a dynamic model can be fit to the truth track and used to obtain the alternative measure. Both the distance and the alternative measure are best obtained in a simulation setting, where a series of predicted impact points are available for comparison to "truth"
MET-A-6 **TQ Number** **accuracy**	TBD. This is a subject of ongoing research. One possible approach is to view the TQ Number as a "null hypothesis" that the covariance parameter defined by B (the look-up table number) falls within a specified range. Data for conducting the hypothesis test consists of the difference between the estimated and true track. In order to have adequate power for rejecting the null hypothesis when it is false, sufficient data will be necessary.

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LIST OF ACRONYMS

6011B	MIL-STD-6011B (TADIL A/B message standard)
6016A	MIL-STD-6016A (TADIL J message standard)
CEC	Cooperative Engagement Capability (USN)
ECEF	Earth-centered, Earth-fixed (coordinate system)
ECI	Earth-centered inertial (coordinate system)
EKF	Extended Kalman filter
FoS	Family of systems
GPS	Global Positioning System
ICBM	Intercontinental ballistic missile
IMM	Interacting Multiple Model
IMU	Inertial measurement unit
IR	Infrared
JCTN	Joint Composite Tracking Network
JDN	Joint Data Network
JITC	Joint Interoperability Test Command
JMOP	Joint measure of performance
JU	Joint user
LOS	Line of sight (sensor measurement type)
LUT	Limited user test
MHT	Multiple hypothesis tracking
ML	Maximum likelihood
MMW	Millimeter wave (radar)
MOP	Measure of performance
MTT	Multiple target tracking
NPG	Network participation group
PAC-3	PATRIOT Advanced Capability Level 3
PATRIOT	Phased Array Tracking Radar to Intercept on Target
PDA	Probabilistic data association
R^2	Reporting responsibility
RAE	Range, azimuth, and elevation (coordinate system)
RCS	Radar cross-section
SCC	System coordinate center
TMD	Theater missile defense
TN	Track number
TQ	Track quality
WGS-84	World Geodetic System (1984)

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