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AN EVALUATION OF THE HYDRA-7 COUNTERMINE WEAPON SYSTEM

by

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June 2000

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The basic principle of Maneuver Warfare in the 21st century is the seamless integration of sea and land as maneuver space. Unfortunately, our inability to conduct counter-mine and counter-obstacle operations in the littorals severely curtails our ability to conduct Amphibious Warfare, a key ingredient to maneuver. Hydra-7, a possible solution to this problem, is one of the most promising counter-mine weapons under development, but its final performance level will depend on the effectiveness of sub-component technologies. These sub-component technologies have yet to reach maturity and may not perform as well as desired. This thesis provides analysis procedures and models to predict Hydra-7 effectiveness for a broad range of possible performance values of sub-component systems. The methodology will determine which of the sub-component technologies is most critical to the final performance of Hydra-7.

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AN EVALUATION OF THE HYDRA-7 COUNTERMINE WEAPON SYSTEM

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

The basic principle of Maneuver Warfare in the 21^{st} century is the seamless integration of sea and land as maneuver space. Unfortunately, our inability to conduct counter-mine and counter-obstacle operations in the littorals severely curtails our ability to conduct Amphibious Warfare, a key ingredient to maneuver. Hydra-7, a possible solution to this problem, is one of the most promising counter-mine weapons under development, but its final performance level will depend on the effectiveness of sub-component technologies. These sub-component technologies have yet to reach maturity and may not perform as well as desired. This thesis provides analysis procedures and models to predict Hydra-7 effectiveness for a broad range of possible performance values of sub-component systems. The methodology will determine which of the sub-component technologies is most critical to the final performance of Hydra-7.

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DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the planner.

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LIST OF DEFINITIONS AND VARIABLES

The Hydra-7, in its current form, consists of three parts, the impactor (I), the munition (M), and the system (S). The impactor kills the target, the munition, which is guided, delivers several hundred impactors to the objective, and the system is a group of munitions banded together and strapped to an aircraft. See Figures (1) and (2).

A	Area (ft^2)
R	Radius(ft)
Pk	Probability of kill
Ph	Probability of hit * Note-only used when $Pk < 1.0$.
N	Number of impactors
M	Number of munitions
D	Distribution
C	Coverage of area
Z	Number of sorties
X	Number of systems per sortie
S	Number of systems

Variables specific to an application or equation will be introduced as needed. Capital letters are used to better facilitate the use of subscripts for identification purposes. For example, $R_{munition}$ is the radius of a munition, while R_{mine} is the radius of a mine.

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EXECUTIVE SUMMARY

The basic principle of Maneuver Warfare in the 21st century is the seamless integration of sea and land as maneuver space. Unfortunately, our inability to conduct counter-mine and counter-obstacle operations in the littorals severely curtails our ability to conduct Amphibious Warfare, a key ingredient to maneuver.

Hydra-7, a possible solution to this problem, is one of the most promising counter-mine weapons under development. The mine clearance methodology of the Hydra-7 is to saturate the area to be cleared with high speed incendiary darts. However, its final performance level will depend on the effectiveness of sub-component technologies. These sub-component technologies have yet to reach maturity and may not perform as well as desired.

Analysis to predict Hydra-7 effectiveness for a broad range of possible performance values of sub-component systems was conducted. The results of the analysis helped to determine which of the sub-component technologies is most critical to the final performance of Hydra-7. Additionally, the results provided a range of system performance characteristics that will assist in the development of a Concept of Operations (CONOPS) governing future Amphibious Warfare.

Analysis was conducted on the output from a simulation in which Hydra-7's with a base case set of sub-component characteristics were employed against a simulated minefield. Performance characteristics of each sub-component of the Hydra-7 were modified between each simulation, and the results were compared to the base case. Changes in battlefield conditions were applied, and the simulations were repeated. Output was in the form of the total probability of killing a mine located anywhere in the minefield. The number of Hydra-7's required to achieve a specific probability of kill was calculated, and the associated number of aircraft sorties was determined.

Table (1) below shows the number of sorties required to clear a single 240 ft. by 240 ft. Initial Craft Landing Site (ICLS) of six inch mines. Other sub-component characteristics include the number of darts per Hydra-7 (6545), accuracy of the guidance system (7 meters), and the number of Hydra-7 systems carried on each sortie (10).

95% Clearance	96 % Clearance	97 % Clearance	98 % Clearance	99% Clearance
5.5 sorties per ICLS	6.0 sorties per ICLS	6.5 sorties per ICLS	7.0 sorties per ICLS	8.5 sorties per ICLS

Table 1.

Further analysis revealed the following:

(1) Halving the mine radius quadruples the required number of sorties.

(2) Halving the number of darts contained in a Hydra-7 munition doubles the required number of sorties.

(3) If the Hydra-7's are built so that the darts cover as large an area as possible, the effect of inaccuracies in guidance technology are minimized.

(4) Total probabilities of kill of greater than .98 are very expensive in terms of sorties, and are probably unreasonable.

The size of the targeted mine is the single most important battlefield condition in determining the number of sorties required. Killing mines that are less than four inches in diameter is very difficult with a dart, and is probably unreasonable. It is unlikely that the Hydra-7, whatever its final form, will be an effective solution to anti-personnel mines. However, its potential against anti-tank mines, which are generally larger than four inches in radius, appears to be good.

The most important sub-component characteristic is the performance of the dart dispensing system. The larger the pattern that the impacting darts make on the ground, the more flexible the weapon will be. Spreading the darts out over a larger area reduces the mine killing effectiveness within the pattern. However, this reduction can be overcome by overlapping impact areas. On the other hand, a small impact pattern with many darts has a good chance of killing mines within it. But, it is less resilient to guidance system inaccuracies. Engineers face the problem of evenly dispensing the darts over a large area while maintaining the dart velocity necessary to kill mines. This will not be an easy problem to solve, and is the single most critical design characteristic.

The Hydra-7 still has many engineering and design obstacles to overcome. However, over the range of variables studied in this analysis, its ability to perform is not tied to any single battlefield condition. Additionally, a functional Hydra-7 has employment possibilities as a breaching weapon in a conventional breach, it adds minimal logistic footprint, it can be carried organically within deployed units, and it can be employed with a high degree of pilot survivability directly in advance of assaulting troops.

In summary, there is no compelling reason not to pursue the Hydra-7. If the engineering and design problems are solved successfully, it should prove to be a powerful, flexible, and useful weapon.

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I. INTRODUCTION

> "*...the very shallow water region is a critical point for our offensive forces, and can easily, quickly, and cheaply be exploited by the enemy.*
>
> MajGen Edward J. Hanlon Jr., Directory of Expeditionary Warfare, *Sea Power*, May 1997.

A. BACKGROUND

The "Concept for Future Naval Mine Countermeasures in Littoral Power Projection" provides a framework for the execution of mine countermeasures (MCM) in support of Operational Maneuver From The Sea (OMFTS) in the time frame 2010 to 2015 [Ref. 1]. The basic principle of maneuver in the 21st century is the seamless integration of sea and land for use as maneuver space. Rather than phasing ashore, building up combat power, and then moving inland, the battlespace of the future envisions maneuver directly from the ship to the objective. The success of these operations will hinge on dominance of the battlespace and unencumbered movement in the air, on land, and at sea.

Currently, Joint and Naval forces are trained and equipped to gain and maintain freedom of maneuver on land, in the air, and in the open ocean. The ability to establish unencumbered maneuver when operating in the littorals, however, is severely limited. Already characterized by narrowness and limited maneuverability, the mining of littoral areas can literally paralyze unequipped operating forces. By the very nature of the mission, de-mining and breaching operations, even those involving forces both trained and equipped, are extremely hazardous.

Without question, the preferred method for dealing with the littoral threat is prevention. If intelligence gathering assets can determine a hostile nation's intent to mine, steps can be taken to prevent their doing so. Despite the attraction of this technique, the reality is that political restraints placed on the military leadership will often eliminate this option.

If prevention is unsuccessful, avoidance is preferred. If the minefield can be bypassed, it should be. This is true when conducting operations both on land and at sea. The Marine Corps' doctrine of Maneuver Warfare teaches leaders to avoid assaulting an enemy's strength whenever possible [Ref. 2]. A leader should always attempt to gain positional advantage through maneuver, and then attack the enemy where he is weakest. While this is certainly good advice, only the foolish leader fails to maintain the *ability* to conduct the frontal attack *when necessary*. In the way that a football team must establish a good ground game in order to have a truly effective passing attack, the attacker must have the ability to conduct an effective frontal attack in order to force the defender to establish a front. Once the defender establishes a front, the attacker has the ability to maneuver to a position of advantage. Without the threat of a frontal attack, defender can choose to defend with equal, if somewhat degraded strength, in all directions. This leaves the attacker with only one option, a frontal attack, which he is ill-equipped to conduct. This philosophy is equally true at sea, particularly in the restrictive maneuver space of the littorals, and should convince us that, although unattractive, the ability to conduct an opposed minefield breach is an absolute necessity.

Once begun, a breaching operation must be conducted rapidly, and without hesitation. Clearly then, a true littoral MCM capability includes an in-stride minefield breaching capability. This implies the integration of systems which are rapidly employable and deployable. They must be more accurate and effective than anything in the inventory thus far, and they must be available in a reasonable amount of time [Ref. 3].

Breaching of minefields in the littorals can realistically be broken down into two areas: chokepoint transits and ship-to-objective maneuver. While identification, classification, and neutralization of mines during chokepoint transit are currently substandard, the capability to conduct these operations does exist. Additionally, appropriate technologies and advancements in this type of operation appear to be on course, and the MCM community has confidence that this requirement will be met with satisfaction by the year 2015 [Ref. 3].

On the other hand, obstacle and mine identification, classification, and neutralization in support of ship-to-objective maneuver requires significant improvement. In general, the surface transited during Ship-To-Objective Maneuver (STOM) can be broken down into five areas: over the horizon (deep water), from just under the horizon to the forty foot depth (shallow water), from forty feet to ten feet (very shallow water, or VSW), from ten feet to the beach (surf zone or SZ), and on the beach itself (beach zone or BZ). Currently the U.S. does possess some breaching capability in both the deep and shallow water zones. However, recent wargames indicate that the VSW, SZ, and BZ regions lack a clearance capability, and future capabilities are contingent on *unproven* technologies. [Ref. 3 and 4]

B. FILLING THE VOID

In order to establish true battlespace dominance, the capability void that currently exists in breaching must be filled. New systems and technologies must be designed, and tactics must be developed which will provide operating forces with a true maneuver capability. This ability will complete the battlespace dominance triad, which will in turn facilitate the execution of maneuver in keeping with the concepts of OMFTS.

1. Current Systems

Currently, the two systems which are expected to perform the majority of BZ clearance are ground operated. These systems, the "Power Blade" and the "Grizzly" are based on current land mine clearance techniques, which requires that they be on the beach in order to use a mine clearance blade [Ref. 5]. This method of land mine clearance is common today, and will likely still be in use in the future. Unfortunately, the transition of these vehicles ashore requires a cleared landing site. Without this Initial Craft Landing Site (ICLS), these two systems can not be offloaded to clear the remainder of the BZ.

One current method of clearing the ICLS is through conventional bombing. However, this method is so destructive to the beach and the ICLS that it is counter-productive, and often renders the beach useless as a landing site. Future technologies may include the use of guidance technology with conventional warheads to increase accuracy and minimize damage to the ICLS. While this idea has potential for use in obstacle clearance, it is not envisioned as a counter mine capability. Other counter mine systems, both those that exist and those that are in the development stages, are geared towards mine clearance in water (SZ, VSW, SW), not for clearance on land. The MCM CONTECH '00

War Game Book, [Ref. 5] provides a listing of all unclassified current and future counter mine and counter obstacle systems. With the exception of the Hydra-7, none listed is designed to clear mines on land from a platform located at sea or in the air.

2. The Hydra-7 Mine/Countermine System

The Hydra-7 system, which is currently being designed for mine destruction on the beach, looks particularly capable. A "plug and play" weapon designed with interchangeable warheads capable of a variety of missions, it may have the flexibility to perform many tasks with a simple modification. An additional attractive feature of the Hydra-7 is the claim that it can be fired from any platform capable of dropping a Mk82 five hundred pound bomb. It may also eliminate many concerns of deployability due to the claim that it is reasonably small. Equivalent to the Mk82 in size and weight, addition of the Hydra-7 could result in no increase in total logistical footprint if Mk82's are replaced by Hydra-7's on a one to one basis. [Ref. 6]

Although not yet operational, and based on new principles of employment utilizing Global Positioning System (GPS) technology, it has repeatedly received very high acclaim from participants of wargames for its advertised employability, deployability, and survivability [Ref. 3]. Unfortunately, it is still in the very early stages of development, and its true effectiveness has yet to be demonstrated. If the Hydra-7 turns out to be as effective as advertised in breaching mines in the BZ, it will significantly enhance the overall breaching effort. Additionally, it may prove to be a valuable asset against other targets, to include obstacles and mines in the SZ. However, if for some reason it fails against mines in the BZ, then the combat capability void extends inland to the high water mark, and this future capability void must be exposed now.

For the purposes of this paper, one Hydra-7 system is composed of two major components, munitions and impactors. The other components which make up the Hydra-7 are not important to this discussion. Shown in Figure (1), each munition contains between 500 and 950 impactors, and each system is comprised of between five and seven munitions. Initially, the system is dropped from a ground attack jet without guidance. However, once released, the band holding the munitions together detaches, and each munition is independently guided to the target. As depicted in Figure (2), each munition approaches a pre-determined terminal velocity and altitude, and the impactors are dispensed, saturating the target area and killing mines through a combination of kinetic energy and chemical reaction. While the concept is fairly simple, the actual result on the ground is the result of some unknown distribution of munitions, each with an unknown distribution of impactors. [Ref. 6]

Also, the Hydra-7 must possess the characteristics which facilitate its employment as part of a combined arms team. Simultaneous employment of other weapons must not interfere with its performance.

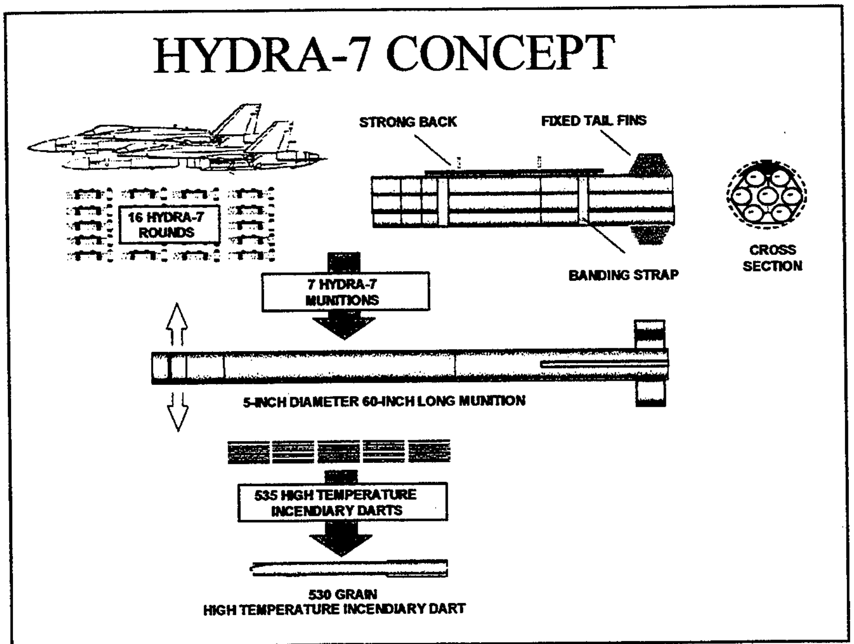


Figure 1: Hydra-7 Components

The Impactor (bottom): Kills the mine through kinetic and chemical-energy interaction.
The Munition (middle): Guides, accelerates, and dispenses impactors to objective.
The System (top): A group of munitions banded together for air delivery.

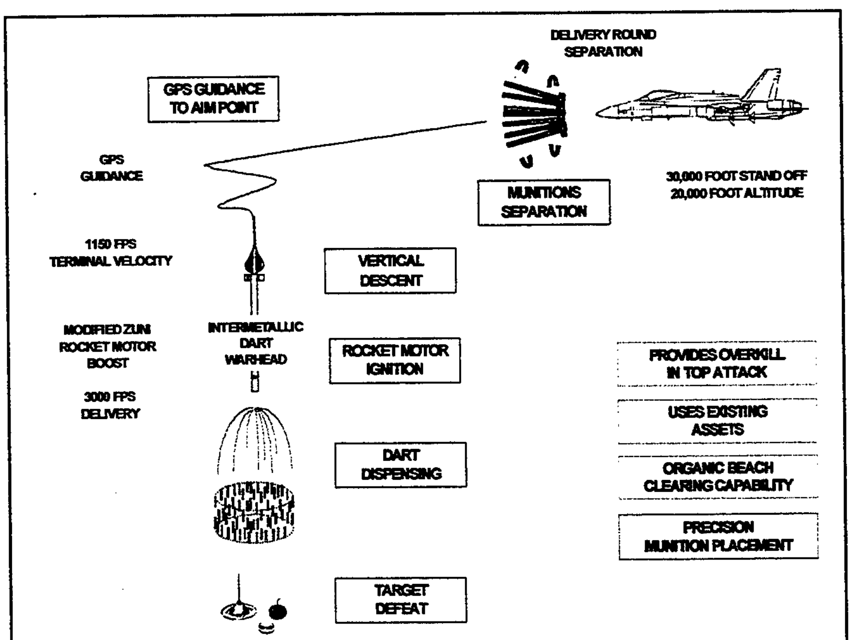


Figure 2: Hydra-7 Concept of Employment

Aircraft releases Hydra-7 systems, systems release munitions, munitions guide to target area, accelerate, and dispense impactors, impactors kill targets.
(Figures (1) and (2) provided by Lockheed Martin)

C. REQUIREMENTS

An analysis of the potential of any system would be incomplete if it did not begin with a clear understanding of the mission for which the system was being designed, and the requirements associated with the performance of that mission. The Hydra-7's initial design, should it work, is most suited for the clearance of the ICLS from a standoff distance. This mission is identified as the most significant required capability of the mid-term (FY09-FY14), and the requirements are summarized as follows [Ref. 7]:

1) The system must not use more than 10% of the Amphibious Task Force organic fixed wing Air Tasking Order (ATO) D-Day sortie rate, 20% threshold (upper bound).
2) The system must have the ability to clear up to 12 ICLS simultaneously within ten minutes of the launch of the first munition, 20 minutes threshold.
3) The system must be delivered from standoff distance, outside the range of enemy direct fire weapons.
4) Although not a specific requirement, the ability to conduct clearance of craft landing sites (CLS) in support of follow-on forces is desirable quality.

This analysis of the Hydra-7 weapon system will carefully evaluate its potential ability to meet the first requirement. The measure of effectiveness is the number of sorties required to achieve the desired probability of kill against a mine with radius R_{mine}.

II. SORTIE REQUIREMENT MODELS

A. METHODOLOGY

The first step in conducting this analysis is to design a perfectly efficient weapon, one for which every munition and every impactor lands exactly where intended. Next, the number of sorties required (Z) to clear an area the size of an ICLS will be determined. Subsequently this weapon will experience a series of compounding imperfections, starting with an inability to perfectly distribute the impactors within the munition, and eventually leading to cases where errors exist in the munition impact points. As weapon performance deteriorates, analytical solutions will no longer be possible, and simulation will be introduced.

B. THE PERFECT WEAPON

The most efficient, or perfect weapon, uses the fewest possible impactors to achieve Pk_{total} against a mine with radius R_{mine} in area A_{total}. The minimum number of impactors is achieved when the impactors are placed as far apart as possible, but still guarantee a kill. This occurs when the impact points are placed on the vertices of equilateral triangles, as shown in Figures (3) and (4). Despite the fact that it is extremely difficult to achieve the perfect weapon, these calculations are important because they give an indication of whether or not the weapon should be pursued. The perfect weapon provides an upper bound on system effectiveness, and if it requires an inordinate number of sorties, then further development of the Hydra-7 system may be unwise.

1. Circle Packing

The mine killing geometry is more easily visualized if the roles of the impactor and the mine are reversed. The number of circles of radius R_{mine} required to completely cover an area A is the same as the number of impactors required to cover A such that no mine of radius R_{mine} can survive. Determining the distance (d) between impactors which ensures contact with a mine of radius R_{mine}, but which maximizes the distance between impactors is accomplished through the application of circle packing problem techniques outlined by Washburn [Ref. 8: Ch. 1, p. 3].

Referring to Figures (3) and (4), the fraction of the entire area A covered by the circles is equal to the fraction of the repeating equilateral triangle which is covered ($C_{triangle}$). In this analysis, edge effects occurring at the boundary of A are ignored. Figure (3) shows the case where $C_{triangle} = 1.0$ (complete coverage), and Figure (4) the case where $C_{triangle} = .9069$. The distance between the centers of the circles (d) can be controlled through choice of the angle θ. [Ref. 8: Ch. 1, p. 4]

In particular, d is given by,

$$d = 2R_{mine}\cos(\theta). \tag{1}$$

When $\theta = 0$ (no circle overlap), $C_{total} = .9069$, and when $\theta = \pi/6$, then $C_{total} = 1.0$ (complete coverage). Equation 1.2-1 from Reference (8) provides Equation (2).

For $0 \leq \theta \leq \pi/6$,

$$C_{total} = \frac{\pi/2 + 3(\cos\theta \sin\theta - \theta)}{\sqrt{3}\cos^2\theta}. \tag{2}$$

Given C_{total}, θ can be determined from Equation (2). See Table (1).

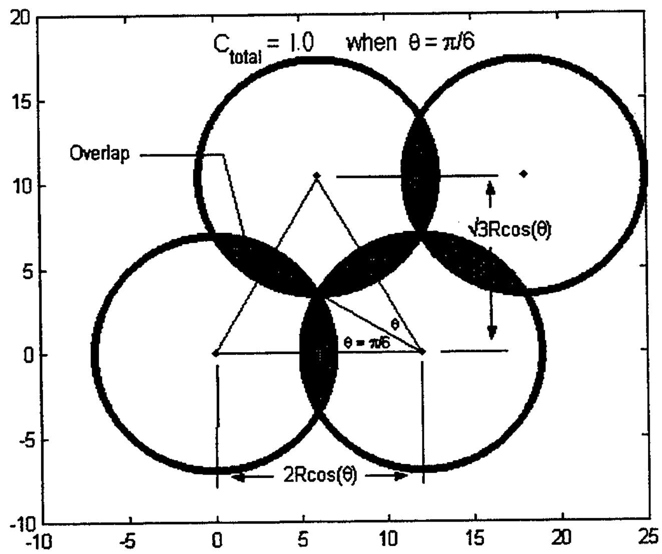


Figure 3: Coverage when $\theta = \pi/6$

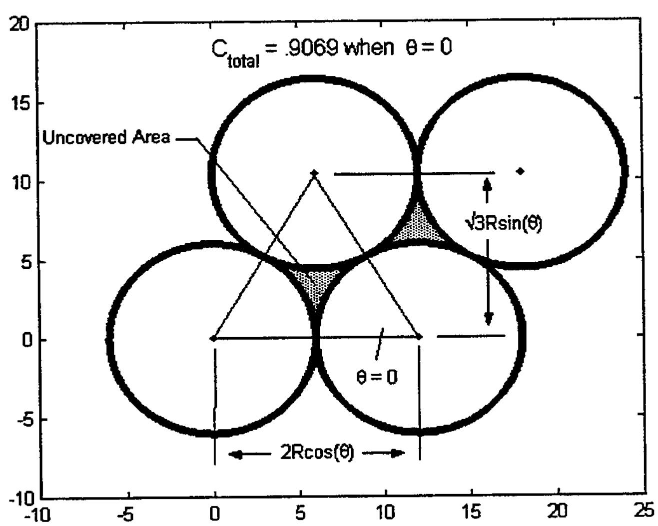


Figure 4: Coverage when $\theta = 0$

C_{total}	θ
1.00	π/6
.99	π/7.35
.98	π/8.25
.97	π/9.30
.96	π/10.5
.95	π/12
.90	0.00

Table 1. Values of θ for Given C_{total}'s

The area of each of the repeating triangles shown in Figures (3) and (4) is

$$A_{triangle} = \sqrt{3} R_{mine}^2 \cos^2 \theta. \tag{3}$$

Because each triangle covers 1/6 of three different circles, there are twice as many triangles as circles [Ref. 8: Ch. 1, p. 4]. Therefore, the number of impactors required to cover area A to a level of C_{total} is

$$N = \frac{A}{2\sqrt{3} R^2 \cos^2 \theta}. \tag{4}$$

When $N = N_{total}$, $A = A_{total}$, and $R = R_{mine}$;

$$N_{total} = \frac{A_{total}}{2\sqrt{3} R_{\mathrm{mine}}^2 \cos^2 \theta}. \tag{5}$$

In the case of the absolutely perfect weapon, where every mine with radius R_{mine} is killed, and every impactor is optimally and perfectly placed in A, $\theta = \pi/6$. Equation (5) then reduces to

$$N_{total} = \frac{.3849 A_{total}}{R_{mine}^2}. \qquad (6)$$

And finally, when $\theta = 0$ ($C = .9064$),

$$N_{total} = \frac{.2887 A_{total}}{R_{mine}^2}. \qquad (7)$$

Because the number of required impactors is inversely proportional to the square of R_{mine}, killing smaller and smaller mines becomes increasingly difficult with this weapon.

2. Solve for Sorties

Once N_{total} is known, and given the number of impactors per munition (N), the total number of munitions (m) is

$$m = \frac{N_{total}}{N}. \qquad (8)$$

Then given the number of munitions per Hydra-7 (M), the total number of Hydra-7's (S) can be found. The total number of sorties (Z) is then obtained by dividing S by the number of Hydra-7's that each aircraft can carry (X). That is,

$$S = \frac{m}{M}, \qquad (9)$$

$$Z = \frac{S}{X}. \qquad (10)$$

3. Solution 1: The Perfect Weapon

To solve for the number of sorties required under perfect conditions, some assumptions about the values of variables must be made.

N (Impactors per munition) = 935,

X (Hydra-7's per Sortie) = 10,

M (Munitions per Hydra-7) = 7,

R_{mine} (Mine radius) = .5ft,

Pk_{total} (the total probability of killing R_{mine}) = 1.0,

θ (from Table (1)) = $\pi/6$, and

A (The area to be cleared) = (240 ft) (240 ft) = 57600 ft^2.

From Equations (4) through (10),

$$
\begin{aligned}
N_{total} &= \frac{.3849\,(57600)}{.5^2}, \\
N_{total} &= 88681, \\
Z &= \frac{88681_{\text{impactors}}}{935_{\text{impactors/munition}}}\left(\frac{1}{7_{\text{munitions/Hydra}}}\right)\left(\frac{1}{10_{\text{Hydras/sortie}}}\right), \\
Z &= 1.35\,\text{sorties.}
\end{aligned}
$$

This is a relatively small number of sorties. From these computations, it appears that a perfect Hydra-7 weapon, where every impactor hits exactly where it is aimed, is a viable weapon. Of course, this level of accuracy is unrealistic, and degradation in system performance must be evaluated.

4. Solution 2: The Perfect Weapon Employed Imperfectly

If it were the case that a Pk_{total} of .95 was sufficient, then the required number of sorties would decrease. Because impactors kill mines with a probability of 1.0, C_{total} = Pk_{total}, and Table (1) shows that the appropriate value for θ is $\pi/12$. Equation (5) is then used to get N_{total}.

$$\begin{aligned}
N_{\text{total}} &= \frac{57600}{2\sqrt{3}.5^2\cos^2\left(\pi/12\right)}, \\
&= \frac{57600}{.8080}, \\
&= 71286, \\
Z &= \left(\frac{71286_{\text{impactors}}}{935_{\text{impactors/munition}}}\right)\left(\frac{1}{7_{\text{munitions/Hydra}}}\right)\left(\frac{1}{10_{\text{Hydras/sortie}}}\right), \\
&= 1.09\,\text{sorties.}
\end{aligned}$$

As expected, the reduction of Pk_{total} from 1.0 to .95 results in a lower sortie requirement.

C. A CONFETTI APPROXIMATION

Another scenario which can be modeled is the assumption that all of the impactors are uniformly and independently distributed over A_{total} [Ref. 9]. This situation is well modeled by the Confetti Approximation.

1. Description

The Confetti Approximation (CA) calculates the probability of covering a point target located in an area A_{total} by N disks which are "cut up" into n small pieces, each of which is uniformly and independently "thrown" onto A_{total}. The total area covered by the N disks (A_{N_disks}) is the area of each disk (A_{disk}) times N.

Using the CA,

$$P\{\text{Covering a point target with one piece of confetti}\} = \frac{A_{disk}N_{\text{disks}}}{nA_{total}},$$

$$P\{\text{Missing with one piece of confetti}\} = 1 - \frac{A_{disk}N_{\text{disks}}}{nA_{total}},$$

$$P\{\text{Missing with } n \text{ pieces}\} = \left(1 - \frac{A_{disks}N_{\text{disks}}}{nA_{total}}\right)^{n},$$

$$= e^{-\left(\frac{A_{disk}N_{disks}}{A_{total}}\right)} \text{ as } n \text{ becomes large.}$$

So, the probability of one or more confetti pieces covering the target is:

$$1 - e^{-\left(\frac{A_{disk}N_{total\_impactors}}{A_{total}}\right)}. \tag{11}$$

Equating the number of disks to the number of impactors,

$$Pk_{total} \cong 1 - e^{-\left(\frac{A_{disk}N_{\text{total\_impactors}}}{A_{total}}\right)}.$$

Solving for the total number of impactors yields

$$N_{total\_impactors} \cong \frac{-\ln(1 - Pk_{total})A_{total}}{A_{disk}} \tag{12}$$

Note that as was the case for the perfect weapon, the required number of impactors is inversely proportional to the square of R_{mine}.

2. Solution 3: Confetti Approximation

Using the same values for the variables as in Solution 2, the CA and perfect weapon approximations can be compared.

N (Impactors per munition) = 935,

X (Hydra-7's per Sortie) = 10,

M (Munitions per Hydra-7) = 7,

R_{mine} (Mine radius) = .5ft,

Pk_{total} (the total probability of killing R_{mine}) = .95,

A (The area to be cleared) = (240 ft) (240 ft) = 57600 ft^2.

$$N_{\text{total\_impactors}} = \frac{-\ln(1 - Pk_{total})A_{total}}{A_{mine}},$$

$$= \frac{-\ln(.05)57600}{\pi .5^2},$$

$$= 219703,$$

$$Z = \left(\frac{219703_{\text{impactors}}}{935_{\text{impactors/munition}}}\right)\left(\frac{1}{7_{\text{munitions/Hydra}}}\right)\left(\frac{1}{10_{\text{Hydras/sortie}}}\right),$$

$$= 3.35 \quad \text{sorties.}$$

As shown in Solution 2, a 95% clearance level costs approximately 1.09 sorties when a perfectly built munition is used. Solution 3 indicates that the 3.35 sorties are required when Confetti Approximation is used. This is not an excessive number of sorties for the proposed mission. So, it appears that the Hydra-7 remains a viable weapon for this mission even when the impactor guidance accuracy degrades to the point where all impactors are uniformly and independently scattered over the ICLS. But for smaller mines or large Pk_{total}'s, the required number of sorties can dramatically increase.

It would be convenient to assume that the CA provides an upper bound for Z for a realistic Hydra-7 weapon system. However, the assumption that the impactors can be delivered uniformly and independently within A possibly makes the CA optimistic. In reality, impactors will be distributed as part of a set of $N_{impactors/munition}$, and will therefore not be independent. As will be shown, this restriction on their placement will tend to increase the required number of sorties. The distribution of the impactors within the munition footprint, and the subsequent packing of the munitions within the ICLS can significantly degrade the performance of the Hydra-7 system. Simulation modeling will be required to quantify this change.

D. A UNIFORM APPROXIMATION

It is also noted that the Confetti Approximation is not the only analytical model which can be used to estimate Pk_{total} when the impactors are independently and uniformly distributed over A_{total}. In particular

$$Pk_{total} = 1 - \left(1 - \frac{A_{disk}}{A_{total}}\right)^{N_{total\_impactors}} \text{, so}$$

$$N_{total\_impactors} = \frac{\ln(1 - Pk_{total})}{\ln\left(1 - \frac{A_{disk}}{A_{total}}\right)}. \qquad (13)$$

This is approximately the CA result in Equation (12) when A_{disk}/A_{total} is small. Although slightly more difficult conceptually, the CA proved to be more useful in the results verification phase (Chapter III) of the study.

E. BUILDING PERFECT MUNITIONS

The above situation assumed that all of the impactors would be delivered independently, as if there was only one very large munition capable of carrying unlimited impactors. The truth is that impactors will be delivered to A in sets of size N, where each set is a munition. Also, it is unclear what final shape, or footprint, those N impactors will create when they impact on the ground. For the purposes of this study, the distribution of the impactors contained in a single muntion on the ground will be referred to as $D_{munition}$.

The actual value of N is also not known because the impactor dispensing system is still under development, and it is unclear how much space it will occupy within the encased munition. Without a dispensing system, the maximum number of impactors that will fit into the munition is 935. The final number will be smaller because the dispensing system is expected to be located within the munition itself, and will therefore reduce the space available for impactors. [Ref. 6]

Evaluation will assume that the footprint is circular, and that each impactor is perfectly placed within the circle (perfect $D_{munition}$). This result will be compared to the munition whose impactors are uniformly distributed within the circle (uniform $D_{munition}$). Once the circular munitions are built, they must be packed into A.

The perfect munition is built utilizing the circle packing techniques introduced in Chapter II, Section B. The impactors will be placed on the vertices of hexagons and packed in a honeycomb fashion. Although the munition is referred to as perfect, there is no requirement for $Pk_{munition}$ to be 1.0. In this case, perfect refers to the ability to individually place each impactor within the munition footprint. The choice of $Pk_{munition}$ will determine d, the distance between impactors, which in turn will determine $R_{munition}$.

1. Finding $R_{munition}$ for $Pk_{munition}$ less than .9069, Perfect $D_{munition}$

For $Pk_{munition}$'s of less than .9069, which occur when the edges of the circles do not touch, $R_{munition}$ is found using ratios of areas.

$$Pk_{munition} = \frac{\sum_{n=1}^{N} \text{Mine Area}}{\text{Total Munition Area}},$$

$$Pk_{munition} = \frac{N\left(\pi R_{mine}^2\right)}{\pi R_{munition}^2},$$

$$R_{munition} = \sqrt{\frac{N R_{mine}^2}{Pk_{munition}}}. \tag{14}$$

2. Finding $R_{munition}$ for $Pk_{munition}$ of .9069 or Greater, Perfect $D_{munition}$

a. Find R_{hex}

For $Pk_{munition}$'s of .9069 to 1.0, Equation (1) can be used to determine the distance between impactors (d) required to attain the desired Pk_{total}. The appropriate value for θ is taken from Table (1). R_{hex}, the radius of the hexagon, is found, and then converted to $R_{munition}$.

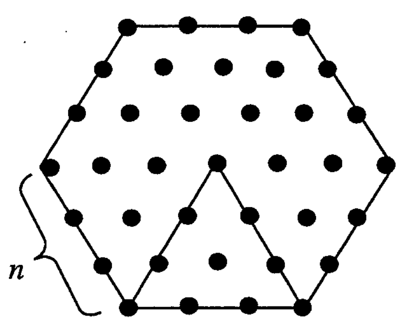
From Equation (1),

$$d = 2R_{mine}\cos\theta$$

R_{hex} is determined by multiplying the number of impactors on the radius of the hexagon (n) by d, the distance between them. It is not difficult to solve for the following relationship between N, the total number of impactors in the hexagon, and n, the number of impactors on the radius of the hexagon,

$$n = \left[\frac{N-1}{3} + \frac{1}{4}\right]^{1/2} + \frac{1}{2}. \tag{15}$$

The following simple example helps to clarify this relationship.



N = total number of impactors

$= 37$

n = number of impactors on radius

$= \left[\frac{N-1}{3} + \frac{1}{4}\right]^{1/2} + \frac{1}{2}$

$= 4$

Figure 5: The Perfect Hexagon

R_{hex} can now be found as,

$$R_{hex} = (n-1)\, d. \tag{16}$$

b. Find the Area of the Hexagon (A_{hex})

The area of a hexagon (A_{hex}) can be calculated by determining the area of one of the component equilateral triangles ($A_{triangle}$) (see Figure 5) and multiplying by six. Each triangle can be broken into two right triangles with area A_{RT}.

Solving for h, the height of the right triangle,

$$R_{hex}{}^2 = h^2 + \left(\frac{R_{hex}}{2}\right)^2,$$

$$h = \left(\frac{R_{hex}}{2}\right)\sqrt{3}. \tag{17}$$

Solving for A_{hex},

$$A_{RT} = \frac{1}{2} R_{hex} h,$$

$$A_{triangle} = 2(A_{RT}),$$

$$A_{hex} = 6\left[\frac{1}{2}\left(\frac{R_{hex}}{2}\right)\left(\frac{R_{hex}}{2}\right)\sqrt{3}\right],$$

$$= 2.5981 R_{hex}^2. \tag{18}$$

c. *Convert the Hexagon to a Circle*

The basis of the conversion is the assumption that a munition with a circular footprint will cover the same area as a munition with a hexagonal one. Given this assumption, R_{circle} can be found as follows.

Assumption: $A_{hex} = A_{circle}$.

From Equation (18):

$$2.5981 R^2{}_{hex} = A_{circle},$$

$$2.5981\ R^2{}_{hex} = \pi R^2{}_{circle},$$

$$R^2{}_{circle} = .827001\ R^2{}_{hex},$$

$$R_{circle} = .909395\ R_{hex}. \tag{19}$$

d. *Finding Total Munitions Required*

Once d is known, the number of munition circles required to cover area A is the product of the number of munitions on each axis. This is the likely manner in which the munitions will be employed. The decision is to place the munitions as shown in Figure (6), with the center of the bottom row of munitions on the line that represents the lower limit of A. This decision is based on the fact that the lower end of A is closest to the beach, and therefore each point on it has a high probability of being crossed by assaulting troops. The left edge of A, chosen arbitrarily, will intersect the centers of every other row of munitions.

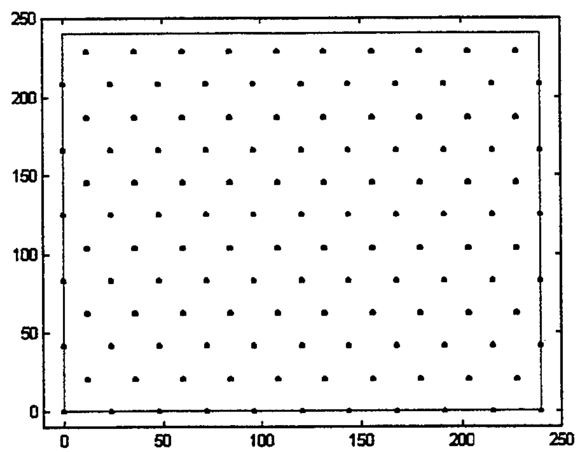


Figure 6: Munition Aiming Points

Each munition is expected to be individually and independently guided, and will require a set of coordinates to be input prior to launch. The distance between any two centers is d, so the change in X direction is just d, and the change in Y direction is $\cos(\pi/6)d$.

Let the length of the X axis of A be A_x. Likewise, let the length of the Y axis A be A_y. Also, let the number of munitions required on the X and Y axis be M_x and Y_x respectively.

$$M_x = \frac{A_x}{d} + 1 \tag{20}$$

$$M_y = \frac{A_y}{\cos\left(\frac{\pi}{6}\right)d} + 1 \tag{21}$$

$$M_{total} = M_x M_y \tag{22}$$

When the conditions are such that $Pk_{munition}$ is equal to the required Pk_{total}, Pk_{total} is achieved when A is completely covered with munitions, and Equation (1) can be used to find d. However, it is not very accurate, because it does not account for the increase in Pk_{total} which results from munition overlap. Calculating the Pk_{total} which results from the overlap of many $Pk_{munition}$'s is difficult. Finding the proper distance between the centers of imperfect circles which results in a desired Pk_{total} is equally difficult, but will be approached through simulation. However, Equation (1) does provide a d that guarantees Pk_{total} at a minimum, and will therefore suffice for the time being.

3. Solution 4: Munitions With Perfect Impactor Distribution

The same values for the variables are used as in Solution's 2 and 3. The new variables introduced for this problem are $Pk_{munition}$ and $D_{munition}$ (the distribution of the impactors within the munition footprint). Fot this study, $D_{munition}$ can only be one of two things, perfect or uniform. The actual distribution is still unknown, and could vary greatly from these two possibilities.

$D_{munition}$ = perfect

$Pk_{munition}$ = .95, so $\theta = \pi/12$

N (Impactors per munition) = 935,

X (Hydra-7's per Sortie) = 10,

M (Munitions per Hydra-7) = 7,

R_{mine} (Mine radius) = .5ft,

Pk_{total} (the total probability of killing R_{mine}) = .95.

Step (1): Building the munitions.

From Equation (1) and Table (1),

$d = 2R_{mine}\cos(\pi/12) = .9659.$

From Equation (15),

$$n = \left[\frac{935-1}{3} + \frac{1}{4}\right]^{1/2} + \frac{1}{2} = 18.15 \text{ impactors.}$$

From Equation (16),

$$R_{hex} = (n-1)d = 17.53\text{feet.}$$

From Equation (20),

$$R_{circle} = .90939(R_{hex}) = 15.94.$$

Step (2): Packing the munitions.

From Equation (1) and Table (1) for 100% Coverage,

$$d_{\text{between munitions}} = 2R_{munition}\cos\left(\pi/6\right) = 27.61 \text{ feet.}$$

From Equations (22) through (24),

$$M_x = \frac{240}{27.61} + 1 = 9.69,$$

$$M_y = \frac{240}{.866(d)} + 1 = 11.04,$$

$$M_{total} = M_x M_y = 106.98 = 107 \text{ munitions.}$$

From Equations (9) and (10),

$$Z_{sorties} = \frac{107_{\text{munitions}}}{7_{\text{munitions/Hydra}}} \frac{1}{10_{\text{Hydras/sortie}}} = 1.53 \text{ sorties.}$$

This result compares well to Solution (2), in which it cost 1.09 sorties to get 95% clearance when impactors were perfectly placed over A. Although delivering the impactors in perfectly built Hydra-7 munition circles did increase the sortie requirement, 1.53 sorties appears to be acceptable, and the Hydra-7 remains a viable weapon.

F. BUILDING IMPERFECT MUNITIONS

1. Uniform Impactor Distribution

Here the assumption is made that the distribution of the impactors within the footprint of the munition is uniform. Control over $Pk_{munition}$ is maintained by choice of $R_{munition}$. If the system design allows control of the radius of the munition, then the user may define the level of clearance that is appropriate, and therefore determine the required number of sorties. Once d, $Pk_{munition}$ and $R_{munition}$ are defined, the total number of munitions required can be found as described above. How the munitions are built, whether perfectly or uniformly, does not affect the technique used for packing the munitions within A.

Let Pk_{only1} be the probability that one impactor, uniformly distributed over a disk with area A, hits a mine with radius R. Let $Pk_{atleast1}$ be the probability that *at least* one out of N impactors, uniformly distributed over a disk with area A, hits a mine of radius R.

$$Pk_{atleast1}=1-[1-Pk_{only1}]^N, \text{ where}$$

$$Pk_{only1} = \frac{\pi R_{mine}^2}{\pi R_{munition}^2},$$

$$= \left(\frac{R_{mine}}{R_{munition}}\right)^2. \text{ So,}$$

$$Pk_{atleast1} = 1 - \left[1 - \left(\frac{R_{mine}}{R_{munition}}\right)^2\right]^N. \tag{23}$$

$R_{munition}$ can now be found.

$$\left(\frac{R_{mine}}{R_{munition}}\right)^2 = 1 - \sqrt[N]{(1 - Pk_{munition})},$$

$$R_{munition} = \frac{R_{mine}}{\left[1 - (1 - Pk_{munition})^{\frac{1}{N}}\right]^{\frac{1}{2}}}. \tag{24}$$

2. Solution 5: Munitions With Uniform Impactor Distribution

$D_{munition}$ = uniform

Pk_{total} = .95.

$Pk_{munition}$ = .95, so $\theta = \pi/6$* (Cover 100% of A)

N (Impactors per munition) = 935,

X (Hydra-7's per Sortie) = 10,

M (Munitions per Hydra-7) = 7,

R_{mine} (Mine radius) = .5ft,

A_{total} = (240ft) (240ft) = 57600 ft^2

Once again, the actual Pk_{total} would be greater than .95 because of the overlap caused when circles are packed to completely cover a plane. See Figure (3).

Step (1): Building the munition.

From Equation (24),

$$R_{munition} = \frac{.5}{\left[1-(1-.95)^{1/935}\right]^{1/2}} = 8.84 \text{ feet.}$$

Step (2): Packing the munitions.

From Table (1), when $C = 1.0$, $\theta = \pi/6$.

* Equation (1) is still accurate when finding the distance between impactors within the munition footprint because the impactor Pk is 1.0.

$$d_{between\ munitions} = 2R_{munition} \cos\pi/6 = 15.31 \text{ feet.}$$

Consolidating the steps shown in Solution (4) above,

$$Z = \frac{M_{total}}{7_{\text{munitions/Hydra}}} \frac{1}{10_{\text{Hydras/sortie}}} = \frac{318.52}{70} = 4.55 \text{ sorties.}$$

So, if the impactors are uniformly distributed over the munition footprint, rather than over A_{total}, the number of sorties increases from 3.35 to 4.53, which may still be an operationally reasonable number. Note, however, that the placement of the munitions within A_{total} has been perfect throughout the analysis.

G. INTRODUCTION OF FIRING ERRORS

Up to this point, each munition has been perfectly placed within A. This assumption has allowed the use of circle packing methods for determining the number of sorties required. However, it is an incorrect assumption. Due to the nature of air delivered ordnance, each independently guided munition will experience some error in accuracy.

1. Types of Error

Bias error results in a fixed error among all rounds fired at the target. Usually assumed to be caused by the "sights" or by the "shooter", error that is a result of a bias can result in a "tight group", the center of which is off of the intended target [Ref. 9]. In this analysis, the Hydra-7 saturates all of area A with impactors. Therefore, bias errors do not affect the clearance level *within* area A, they only affect the final location of it. Based on the assumption that assaulting forces will know the final location of A, and that they will use it as the Initial Craft Landing Site, bias errors are ignored in this study.

Dispersion error, or the error associated with the point of impact relative to the point of aim, is a "collection of independent random variables" [Ref. 9]. It will account for the majority of the degradation in weapon system efficiency. In the case of Hydra-7, the independent variables which together form the dispersion error can be reduced to global positioning system (GPS) error, guidance system error and flight error.

GPS error is likely to be the most significant, and therefore the most important, of the causes of dispersion error. While some of the causes of GPS inaccuracies may be eliminated or reduced in the future, there is no guarantee of this. To be complete, this study will assume that GPS error will be from 0 to 24 feet.

Guidance system error would be caused by failures in the inverse guidance law, which was developed and is currently undergoing testing by Lockheed Martin. Any errors found in this new guidance concept will manifest themselves in the form of dispersion errors, and will be a part of the final overall dispersion error tally.

Finally, even if the GPS transmitter and guidance technology perform perfectly, each munition is subject to error during flight. This will also increase the total dispersion error.

2. Effects of Dispersion Error

Dispersion error will be a combination of the above, and will be accounted for in the form of Circular Error Probable, or CEP, which is defined as the radius of the circle which contains half of the shots [Ref. 10]. The use of CEP vice standard deviation (σ) is common in firing theory. As shown below, assuming normally distributed dispersion errors, the two are closely related.

$$
\begin{aligned}
&1-e^{\frac{-CEP^2}{2\sigma^2}}=.5,\\
&-\ln .5=\frac{CEP^2}{2\sigma^2}\\
&\sigma=\frac{CEP}{\sqrt{2\ln 2}},\\
&\sigma=\frac{CEP}{1.1774}.
\end{aligned}
\tag{25}
$$

The introduction of CEP complicates the determination of the number of sorties required to achieve the required Pk_{total}. While the calculations for $R_{munition}$ remain the same, the method for determining the d, the distance between munition centers (Equation (1)) is no longer applicable.

The circle packing techniques introduced in Chapter II, Section B are valid for munition building because the Pk for each impactor is 1.0, but they have limited application in the employment of munitions whose $Pk_{munition}$ is less than 1.0.. This is because Equations (1) and (4) do not account for the increase in Pk_{total} that results from overlapping munitions.

Derivation of a method which yields an analytical solution is difficult, and will not be addressed here. Analytical results would be of limited use anyway, because the actual final position of each munition will be normally distributed about its point of aim. Point estimates make poor predictors of random CEP's. However, accurate approximations of the Pk_{total} achieved through combinations of $R_{munition}$, $Pk_{munition}$, and $d_{between\ munitions}$ can be found through simulation. The results of the simulation can then be used to dictate the appropriate positioning of munitions required to achieve Pk_{total}.

H. SOLVING FOR PK_{TOTAL} THROUGH SIMULATION

The simulation independent variables are the ratio of $R_{munition}$ to $d_{between\ munitions}$ (*RoverD*), $Pk_{munition}$, and ratio of CEP to $d_{between\ munitions}$ (*CEPoverD*). The use of unitless variables allows saving the simulation output in 2-dimensional matrices. The simulation is conducted using MATLAB [Ref. 10].

The simulation procedure is:

1. Specify *CEPoverD* .
2. Assuming d=1, determine munition aimpoints. Using independent, normal x- and y- errors determined from the specified CEP value, compute random munition impact points.
3. Specify *RoverD.*
4. Uniformly distribute 100 target mines over the ICLS.
5. Specify $Pk_{munition}$ and determine an average Pk .
6. Repeat steps 3. through 5. for all values of *RoverD* and $Pk_{munition}$. Call the matrix of averages "Pk_{batch}".
7. Repeat steps 2. through 6. 50 times. The final matrix, called "Pk_{total}", is the average of the 50 Pk_{batch} values.

This procedure will give a Pk_{total} value of each (*RoverD, CEPoverD,* $Pk_{munition}$) triplet. Table (2) shows simulation output for *CEPoverD* of 0.

1. Solution 6: Use of Simulation Results

The simulation is used to find $d_{between\ munitions}$. Otherwise, the problem solving technique is the same as was used in Solution (5).

$CEPoverD = 0$,

$D_{munition}$ = uniform,

$Pk_{total} = .9496$,

$Pk_{munition} = .8$,

N (Impactors per munition) = 935,

X (Hydra-7's per Sortie) = 10,

M (Munitions per Hydra-7) = 7,

R_{mine} (Mine radius) = .5ft,

A_{total} = (240ft) (240ft) = 57 600 ft^2

Step (1): Building the munitions.

Because $D_{munition}$ is uniform,

$$R_{munition} = \frac{R_{mine}}{\left[1-(1-Pk_{munition})^{\frac{1}{N}}\right]^{\frac{1}{2}}} = 12.05 \text{ feet.}$$

Step (2): Packing the munitions.

The row from Table (2) which intersects with the column labeled $Pk_{munition} = .8$ at the value .9496 is $RoverD = .8$. Given that $R_{munition}$ is 12.05 ft,

$$RoverD = \frac{R_{munition}}{d_{between\ munitions}},$$
$$d = \frac{R_{munition}}{RoverD} = \frac{12.05}{.8} = 15.06.$$

The sortie requirement can be found as follows:

$$M_x = \frac{A_x}{d} + 1 = \frac{240}{15.06} = 16.94,$$
$$M_y = \frac{A_y}{\cos\left(\frac{\pi}{6}\right)d} + 1 = \frac{240}{13.04} = 19.4,$$
$$M_{total} = M_x \times M_y = 328.64 \text{munitions}$$
$$Z_{sorties} = \left(\frac{328.64_{\text{munitions}}}{7_{\text{munitionsHydra}}}\right)\left(\frac{1}{10_{\text{Hydras/sortie}}}\right) = 4.69 \text{sorties}$$

This is a representative example, demonstrating the use of the simulation output to get $d_{between\ munitions}$ when CEP is zero. If the results are accurate, then the correct $d_{between\ munition}$ can be obtained for any combinations of input parameters.

RoverD	Pk(mun)=0.5	Pk(mun)=0.6	Pk(mun)=0.7	Pk(mun)=0.8	Pk(mun)=0.9	Pk(mun)=1.0
0.5000	0.4516	0.5419	0.6322	0.7226	0.8129	0.9032
0.5500	0.5217	0.6196	0.7155	0.8092	0.9009	0.9904
0.6000	0.5757	0.6726	0.7634	0.8482	0.9271	1.0000
0.6500	0.6242	0.7178	0.8018	0.8765	0.9425	1.0000
0.7000	0.6730	0.7624	0.8388	0.9033	0.9567	1.0000
0.7500	0.7187	0.8030	0.8716	0.9261	0.9683	1.0000
0.8000	0.7695	0.8473	0.9064	0.9496	0.9798	1.0000
0.8500	0.8168	0.8870	0.9362	0.9686	0.9885	1.0000
0.9000	0.8546	0.9175	0.9582	0.9819	0.9942	1.0000
0.9500	0.8861	0.9410	0.9736	0.9906	0.9977	1.0000
1.0000	0.9132	0.9594	0.9844	0.9958	0.9995	1.0000
1.0500	0.9312	0.9699	0.9894	0.9975	0.9997	1.0000
1.1000	0.9462	0.9781	0.9930	0.9985	0.9999	1.0000
1.1500	0.9558	0.9830	0.9949	0.9990	0.9999	1.0000
1.2000	0.9664	0.9879	0.9966	0.9994	1.0000	1.0000
1.2500	0.9750	0.9917	0.9979	0.9997	1.0000	1.0000
1.3000	0.9824	0.9948	0.9989	0.9999	1.0000	1.0000
1.3500	0.9882	0.9970	0.9995	0.9999	1.0000	1.0000
1.4000	0.9912	0.9979	0.9997	1.0000	1.0000	1.0000
1.4500	0.9934	0.9985	0.9998	1.0000	1.0000	1.0000
1.5000	0.9953	0.9991	0.9999	1.0000	1.0000	1.0000
1.5500	0.9966	0.9994	0.9999	1.0000	1.0000	1.0000
1.6000	0.9976	0.9996	1.0000	1.0000	1.0000	1.0000
1.6500	0.9983	0.9997	1.0000	1.0000	1.0000	1.0000
1.7000	0.9988	0.9998	1.0000	1.0000	1.0000	1.0000
1.7500	0.9993	0.9999	1.0000	1.0000	1.0000	1.0000
1.8000	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000
1.8500	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000
1.9000	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000
1.9500	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000
2.0000	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000

Table 2. CEP = 0

Pk_{total} as a function of *RoverD* and $Pk_{munition}$

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III. VERIFICATION OF RESULTS

Before the results of the simulation are put to use, it is necessary to establish some confidence in their accuracy. Although it is nearly impossible to establish absolute accuracy for most of the simulation results, it is possible to use known analytical solutions to paint a picture of what accurate simulations should look like.

A. ANALYTICAL VERIFICATION

A good first step in establishing simulation accuracy is to compare the results found using the simulation to those that can be found analytically. Analytical solutions are possible in a few limited cases. If the analytical solutions are not captured within a reasonable simulation-produced confidence interval, then the simulation results are questionable.

1. Explanation

The probability of kill (*Pk*) for a batch is the average of the probability of kill for each of the 100 independent and identically distributed (*iid*) mines in that batch. The probability of kill for a simulation run is the average of 50 batches. With this large number of trials in hand, the Central Limit Theorem says that the simulation-produced *Pk*'s are approximately normally distributed about the true *Pk*. The 95% confidence interval (CI) is then:

$$CI = \overline{Pk} \pm z_{\alpha/2}\sqrt{\frac{\text{var}(Pk)}{N}} \tag{26}$$

N = Number of trials

$z_{\alpha/2} = 1.96$ (for a 95% Confidence Interval)

The true Pk can be found analytically regardless of $Pk_{munition}$ as long as the edges of the munitions do not overlap. When the degradation caused by the edges of the circles leaving the area of interest is ignored, the "true" Pk is a ratio of areas. The effect of the decision to ignore the degradation caused by the edge effects will result in an analytical Pk that is always slightly larger than the simulation result.

$$Pk = \frac{A_{munitions}}{A_{total}} \times Pk_{munition} \tag{27}$$

If approximately 95% of the confidence intervals calculated using equation (26) contain the Pk calculated using equation (27), the simulation can not be rejected. As shown in the graph and examples below, the simulation does not fail this test.

When *RoverD* is .5, the edges of the circles are just touching, and coverage (*C*) is .9069. Therefore:

$$Pk_{total} = .9069 \times Pk_{munition} \tag{28}$$

2. Solution 6: Analytical vs. Simulated

Conditions: $Pk_{munition} = 1.0$, $RoverD = .5$.

Analytical Solution:

$Pk_{total} \geq .9069\ (Pk_{munition}) = .9069$

Simulated Result:

Row 1 ($RoverD = .5$), Column six ($Pk_{munition} = 1.0$), of Table (2) = .9032.

The effects of the edges on the simulation results in a Pk_{total} that is slightly lower than the analytical result. However, the 95% CI of the simulation result (.8937, .9127) does contain the analytical solution. Figures (6) and (7) show the relationship between many analytical results and independently generated simulation results.

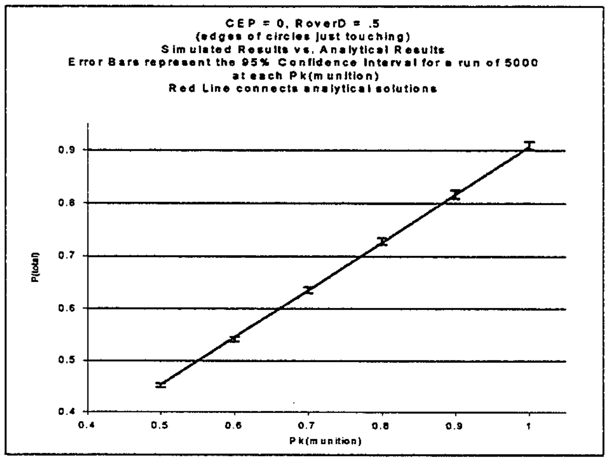

Figure 7: 95% Confidence Interval For All $Pk_{munition}$'s

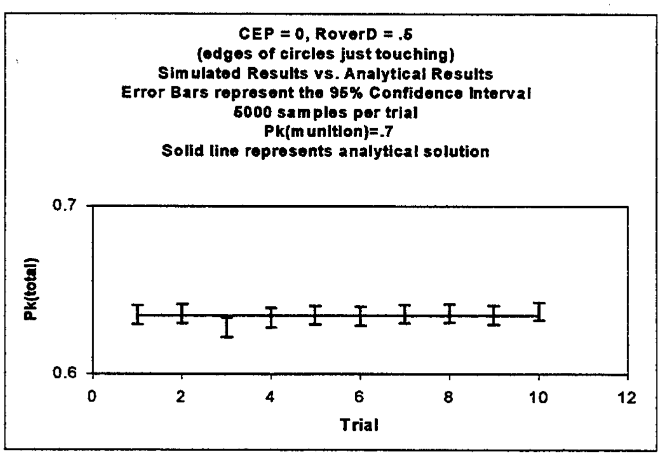

Figure 8: 95% Confidence Intervals for Ten Trials of $Pk_{munition}$ = .7

3. Analytical Verification Conclusion

Little disparity exits between Pk_{total}'s found using either method. Based on these results, there seems to be no reason to reject the simulation, particularly in the case where the CEP = 0.

B. CONFETI APPROXIMATION VERIFICATION

The Confetti Approximation is another useful indicator of simulation accuracy. As discussed in Chapter II, Section C, this method of approximating the probability of kill within area A treats each munition as if it has been cut into many small pieces of "confetti". Each of these pieces of confetti is then independently and uniformly "thrown" onto A. One important difference between this situation and the previous one is that now the disks which are to be cut up into confetti do not always have a Pk of 1.0. This problem is solved by multiplying the $A_{munition}$ by $Pk_{munition}$, thus converting the disk into a smaller one which does have a Pk of 1.0. For munitions,

$$Pk_{confetti} = 1 - e^{-\left(\frac{Pk_{munittion} \times A_{munition} \times M}{A_{total}}\right)} \tag{29}$$

Given any specific set of munition characteristics, the simulation generates M, the number of munitions required to achieve the desired Pk_{total}. For a given Pk_{total}, an increase in CEP results in an increase in M. Once M is known, and having already been given A_{total}, the Confetti Approximation formula will produce an estimated Pk_{total}.

When CEP is low, we expect the Confetti Approximation to underestimate the Pk_{total}. This underestimation is caused by the assumption that the pieces of confetti are randomly placed in A. This results in unwanted overlap, decreasing efficiency, and reducing Pk_{total}. As CEP increases, however, the placement of the munitions within A more closely resembles the uniform distribution assumption of the Confetti Approximation, and the two Pk_{total}'s start to converge. Finally, as CEP continues to grow, some of the simulated munitions leave the intended area A. Because of the Confetti Approximation assumption that all of the pieces of confetti remain within A, the Confetti Approximation Pk_{total} is greater than the simulated Pk_{total}.

Figures (8) and (9) show the results for scenarios involving $Pk_{munition}$'s of .5 and .9, obtained through uniform impactor placement. As a reminder, 935 impactors uniformly distributed to obtain Pk_{total}'s of .5 and .9 against a mine of radius .5 feet results in munitions with radii of 18.37 and 10.08 feet, respectively.

As expected, as CEP increases, the Confetti Approximation eventually overestimates the true Pk_{total}. The relationship between the simulated Pk_{total} and the $Pk_{\text{confetti}\_total}$ is as it should be. These results provide no compelling reason to question the accuracy of the simulation output. When combined with the accuracy displayed by the simulation in the case where an analytical solution to the true Pk_{total} was possible, it is reasonable to assume that the simulation estimates an accurate Pk_{total}.

CEP	TotalMunitions	SimPktot	Confetti Pktot
0.00	187.69	0.9000	0.8221
3.00	194.35	0.9000	0.8327
6.00	200.05	0.9000	0.8413
9.00	215.25	0.9000	0.8620
12.00	227.72	0.9000	0.8769
15.00	233.15	0.9000	0.8829
18.00	243.22	0.9000	0.8933
21.00	250.73	0.9000	0.9004
24.00	256.05	0.9000	0.9052

Table 3. Confetti Approx. vs. Simulation Results: $Pk_{munition} = .50$, $Pk_{total} = .90$

CEP	Total Munitions	SimPkTot	Confetti PkTot
0.00	375.7000	0.9900	0.9685
3.00	395.3800	0.9900	0.9737
6.00	409.7100	0.9900	0.9769
9.00	436.6100	0.9900	0.9820
12.00	474.1100	0.9900	0.9872
15.00	483.9200	0.9900	0.9883
18.00	495.1400	0.9900	0.9895
21.00	503.8300	0.9900	0.9903
24.00	519.5900	0.9900	0.9916

Table 4. Confetti Approx. vs. Simulation Results: $Pk_{munition} = .50$, $Pk_{total} = .99$

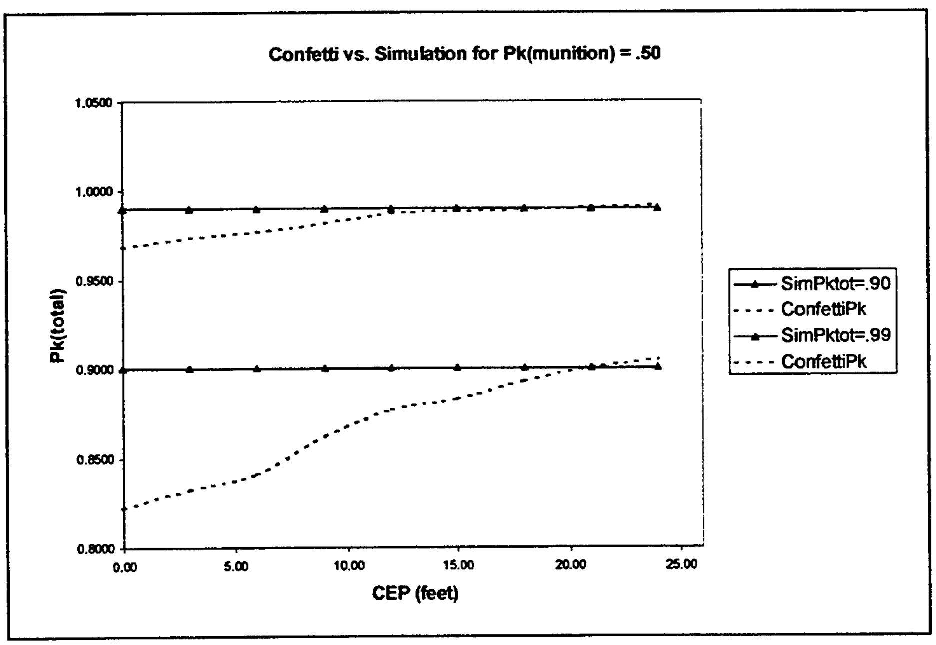

Figure 9: Confetti Approximation vs. Simulation Results at $Pk_{munition} = .5$

CEP	Total Muntions	SimPkTot	Confetti PkTot
0.00	197.58	0.9000	0.6269
3.00	254.43	0.9000	0.7190
6.00	389.24	0.9000	0.8566
9.00	400.30	0.9000	0.8643
12.00	421.22	0.9000	0.8777
15.00	468.21	0.9000	0.9033
18.00	481.59	0.9000	0.9095
21.00	496.29	0.9000	0.9159
24.00	500.75	0.9000	0.9178

Table 5. Confetti Approx. vs. Simulation Results: $Pk_{munition} = .90$, $Pk_{total} = .90$

CEP	Total Munitions	Sim PkTot	Confetti PkTot
0.00	487.9500	0.9900	0.9124
3.00	520.7600	0.9900	0.9256
6.00	784.7600	0.9900	0.9801
9.00	865.8900	0.9900	0.9867
12.00	902.6000	0.9900	0.9889
15.00	977.7300	0.9900	0.9924
18.00	978.0600	0.9900	0.9924
21.00	1028.8000	0.9900	0.9941
24.00	1215.8100	0.9900	0.9977

Table 6. Confetti Approx. vs. Simulation Results: $Pk_{munition} = .90$, $Pk_{total} = .99$

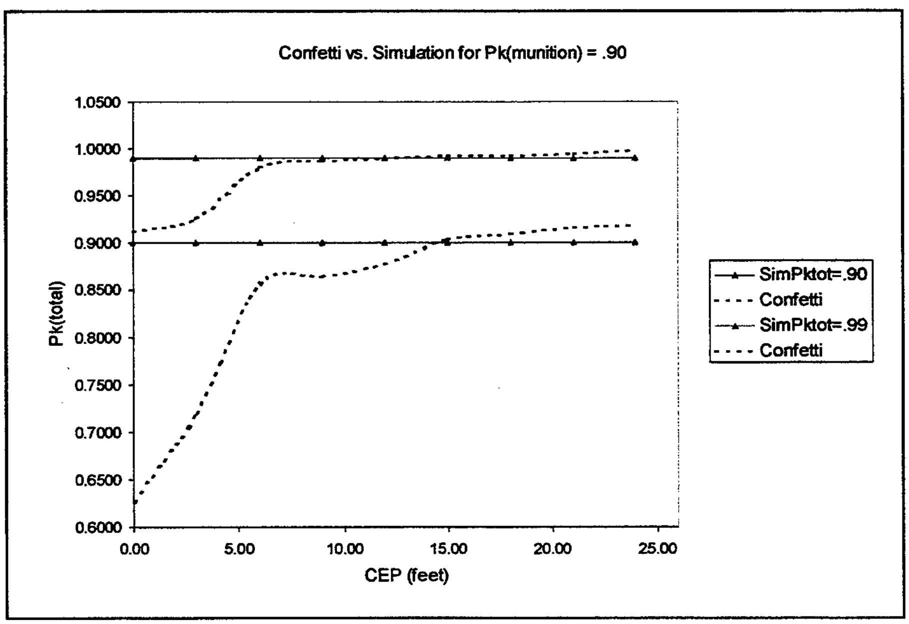

Figure 10: Confetti Approximation vs. Simulation Results at $Pk_{munition} = .90$

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IV. ANALYSIS

Given that the simulation results are reasonable, it is possible to determine the number of sorties required to clear area A under any number of conditions. Although it is possible to do so, the many combinations of independent variables makes it unreasonable to examine every possible case.

The variables are: (1) N - number of impactors per munition, (2) M - number of munitions per Hydra-7 , (3) S - number of Hydra-7's per sortie, (4) R_{mine} – the minimum mine radius which must be cleared to specifications, (5) A_{total} – the size of the area to be cleared, and (6) CEP – circular error probable, the distance from the point of aim within which 50% of the munitions will land on average.

Output graphs provide information for any combination of the following: (1) $D_{munition}$ – The distribution pattern of impactors within the munition footprint, (2) Pk_{total} – the level of clearance required, and (3) $Pk_{munition}$ – The probability that a mine of radius R_{mine} located within the area covered by a single munition is killed.

Analysis will be conducted on the impact on total sortie requirement of upper and lower values of each variable, with all other variables held constant. Although the effects of changes to combinations of variables will not be observed, the sensitivity of total sortie requirements to each independent variable will be. This sensitivity analysis will provide information on the variables which are most important to the future of the Hydra-7.

A. BASE CONDITIONS METHODOLOGY

An important first step in this sensitivity analysis is establishment of a starting point, or a set of baseline conditions. Results produced from variations to the base conditions are then compared to the base condition results to see the impact of the changes.

1. Base Conditions:

N (impactors per munition)= 935. This is the maximum number of impactors that, when perfectly packed, can be placed in one munition; M (munitions per Hydra-7) = 7; S (Hydra-7's per sortie) = 10; R_{mine} = .5 ft; A_{total} = 57600 ft^2 (the size of the ICLS); CEP = 3 feet. A graph of the results, shown below, provides solutions for any combination of Pk_{total}, $Pk_{munition}$, and impactor distribution.

2. Base Case Results

The munition with the perfectly placed impactors is much more efficient than the uniformly distributed one. In Figures (10) and (11), the same scale is maintained to allow better visual comprehension of the difference between the two. Also, on both graphs, a significant bend, or knee, occurs in the curve at Pk_{total} = .99. In the case of the uniformly distributed impactors, the number of sorties required to go from a 99% clearance level to a 100% clearance level nearly doubles regardless of the $Pk_{munition}$.

It is important to remember that these results are for a single simulation. Although each simulation consists of thousands of iterations, it is inaccurate to report the results as perfectly accurate. Statistically, it is impossible to achieve a 100% clearance level, which would require 100% clearance every time, in an infinite number of simulation runs. It is possible, however, to achieve 100% clearance in any one of the simulation runs.

On both graphs, all values are bounded by $Pk_{munition} = .5$ and $Pk_{munition} = 1.0$, however their positions are opposite on the two graphs. For the perfect distribution, $Pk_{munition} = 1.0$ is best, and $Pk_{munition} =.5$ is the worst. For the uniform distribution, $Pk_{munition} = .5$ is best, and $Pk_{munition} = 1.0$ is worst.

3. Base Case Conclusions

If the impactor distribution is uniform, larger munitions with lower $Pk_{munition}$ are more efficient. Also, it appears that Pk_{totals} of greater than .99 are going to be very hard to get when the target is a 6-inch radius mine.

B. CEP SENSITIVITY ANALYSIS

The sensitivity of the model to changes in CEP requires investigation. As stated previously, estimates of CEP as the result of GPS error are as large as 8 meters. For the purposes of this study, this will be used as the upper bound on CEP error.

1. Conditions

Changes to CEP alone will be compared to the base case conditions. CEP is increased to the upper limit of analysis.

Input Parameters: $N = 935$ (impactors per munition), $M = 7$ (munitions per Hydra-7), $S = 10$ (Hydra-7's per sortie), $R_{mine} = .5$ ft; $A_{total} = 57600$ ft^2, ***CEP = 24 feet.***

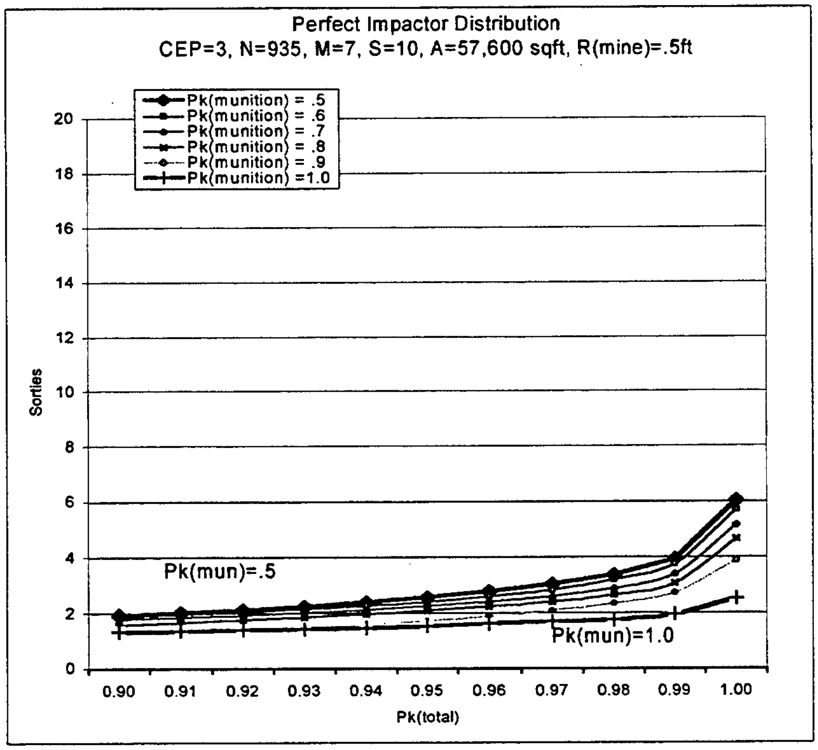

Figure 11: Base Conditions for Perfect $D_{munition}$

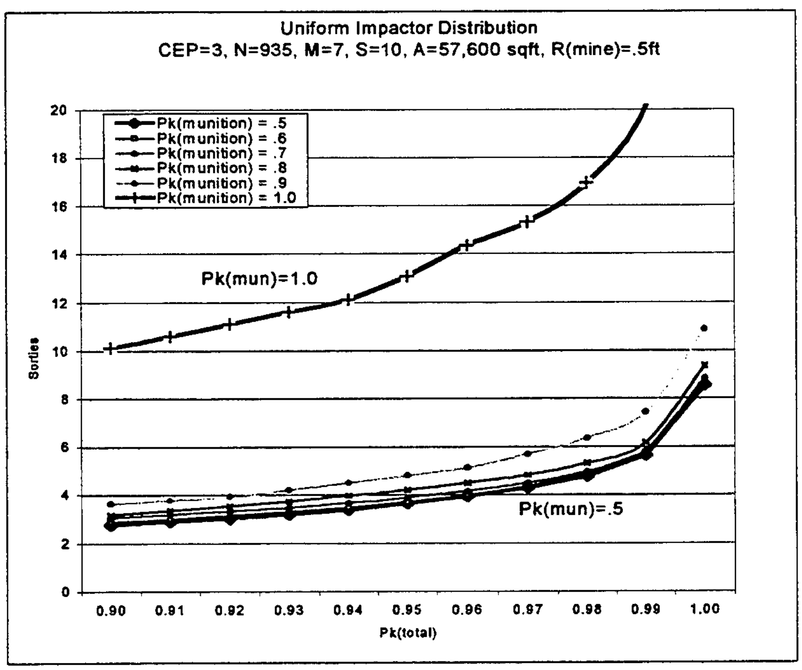

Figure 12: Base Conditions for Uniform $D_{munition}$

2. Analysis

Figures (13) and (14) show the results of a dramatic increase in CEP. In the case of the uniformly distributed impactors, it is clear that a larger, less efficient munition is the best design. This provides an interesting insight, and is explained by the dampening effect that larger munitions have on CEP. A closer look at Figure (13) reveals that with a larger munition CEP, the relative effectiveness of $Pk_{munition} = 1$ and $Pk_{munition} = .5$ have changed. In other words, .5 is now the best, and 1 is now the worst.

Final analysis of the overall impact of CEP is best shown in a graph of the effect of incremental increases in CEP on a single munition design over a range of Pk_{total}'s. For illustrative purposes only, a weapon with a $Pk_{munition}$ of .7 will be used. Figure (15) shows the results. Surprisingly, Figures (13) and (14) indicate that the number of required sorties has a high level of tolerance to significant increases in CEP. Figure (15) more clearly shows that the price of demanding Pk_{total} greater than .99 is very high.

3. Conclusion:

As expected, CEP is an important variable, the final value of which requires more research and testing. However, for the representative case of Pk_{total} of .95, a 700% increase in CEP over the base case caused less than a 50% increase in sortie requirement. For Pk_{total} values less than .98, CEP values as high as 24 feet failed to raise the number of sorties above 10. While the impact is somewhat greater for larger Pk_{total}'s, this model indicates a high level of robustness in the weapon. Thus, it appears that Hydra-7 will remain a viable weapon for CEP values over a broad range of dispersion errors.

It is likely that the final impactor distribution pattern will resemble the uniform distribution, rather than the perfect one. For that reason , the remainder of this paper will focus on the munition with the uniformly distributed impactors.

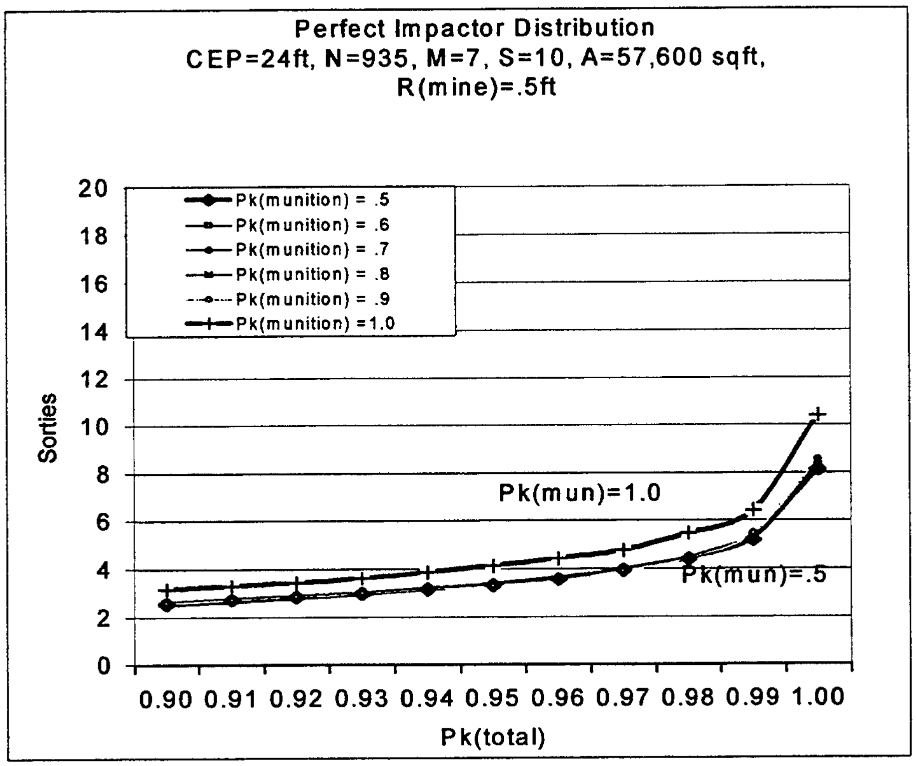

Figure 13: Effect of Increased CEP on Perfect Distribution

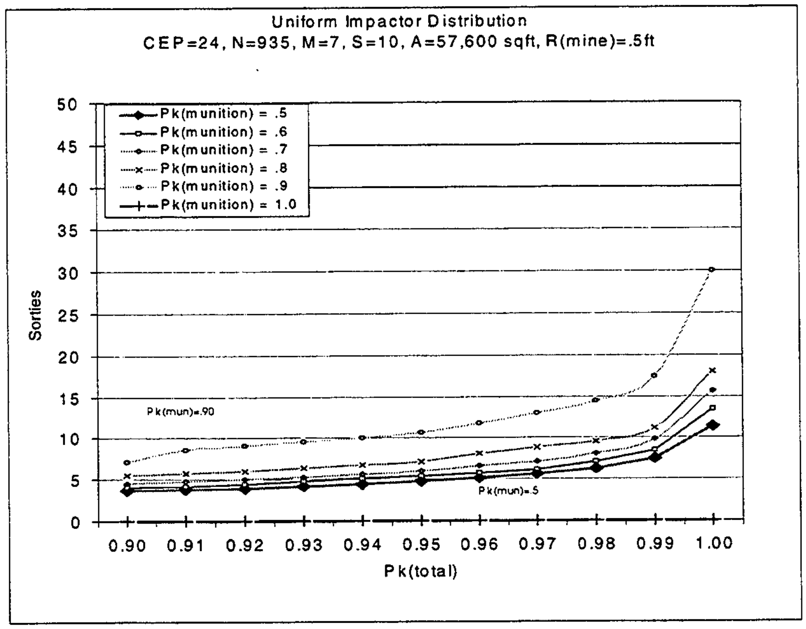

Figure 14: Effect of Increased CEP on Uniform Distribution

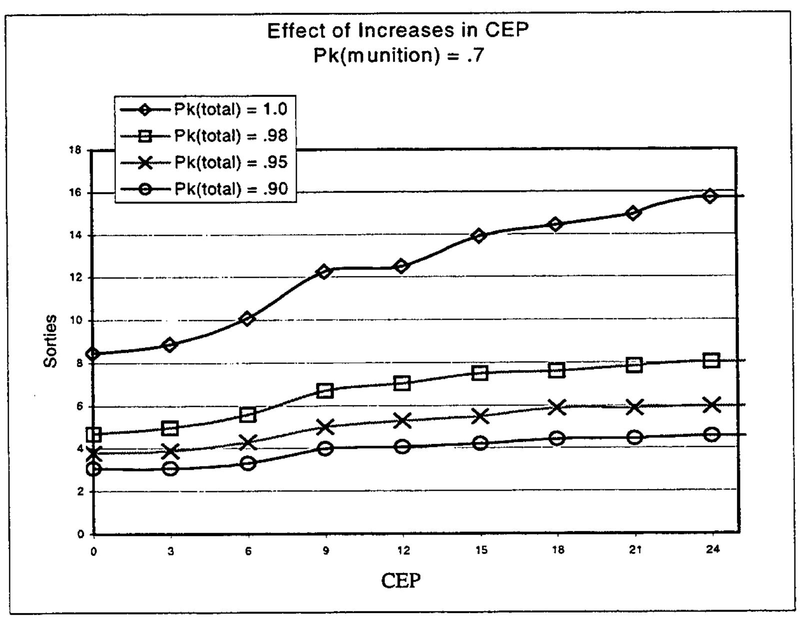

Figure 15: Effect of Increased CEP on $Pk_{munition}$ = .7

4. Why Pk_{total} Increases With $R_{munition}$

The conclusion reached above, that when CEP is large, a munition footprint with a larger radius and a lower probability of kill is more efficient than one with a smaller radius and a larger probability of kill, is not an intuitive one. However, the visual representation shown below will be helpful.

Intuitively, the reason that the Pk_{total} in Figure (16) is smallest is because the amount of uncovered area is largest. Also, as Pk_{total} approaches 1.0, the positive effect of munition overlap on Pk_{total} is negated. For the range of parameters examined in this study, and when $D_{munition}$ is uniform, the largest $R_{munition}$ produced the largest Pk_{total} with the fewest number of sorties. Figures (16) through (19) show circular munitions placed with the same CEP, or circular error probable, but in each figure $R_{munition}$ is increased. Even though the circles in Figure (19) have a lower $Pk_{munition}$ than the circles in Figure (16), the Pk_{total} for the area shown in Figure (19) is higher.

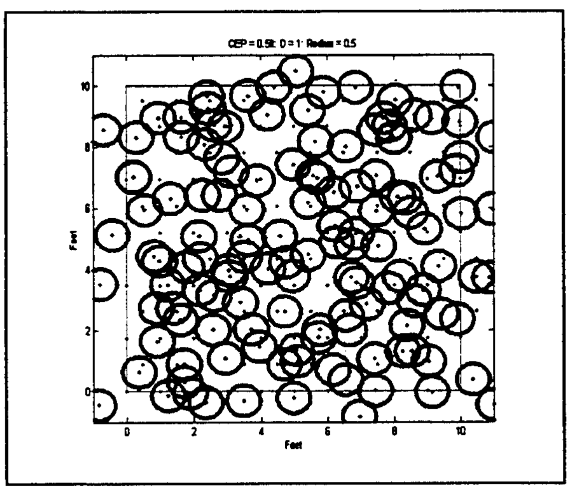

Figure 16: $Pk_{munition} = .998$, $Pk_{total} \approx .65$

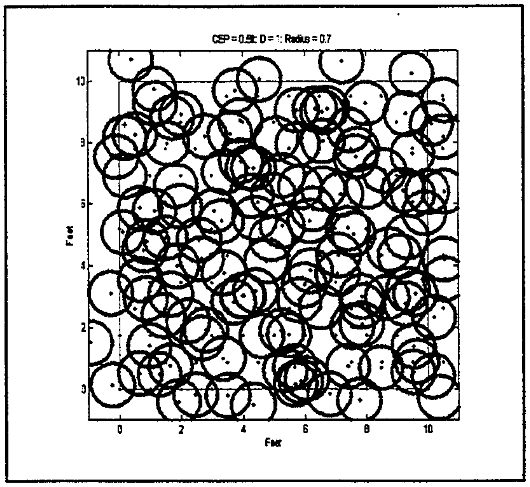

Figure 17: $Pk_{munition} = .96$, $Pk_{total} \approx .88$

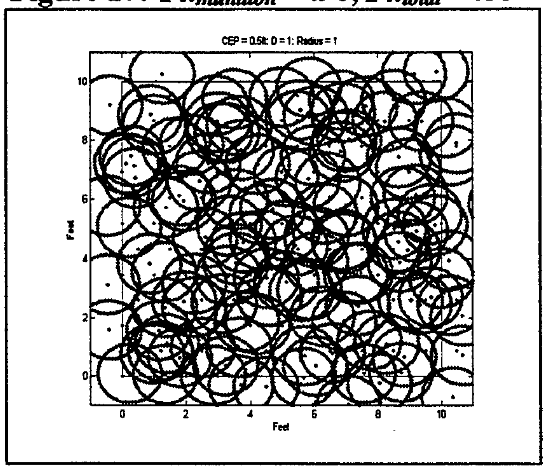

Figure 18: $Pk_{munition} = .79$, $Pk_{total} \approx .97$

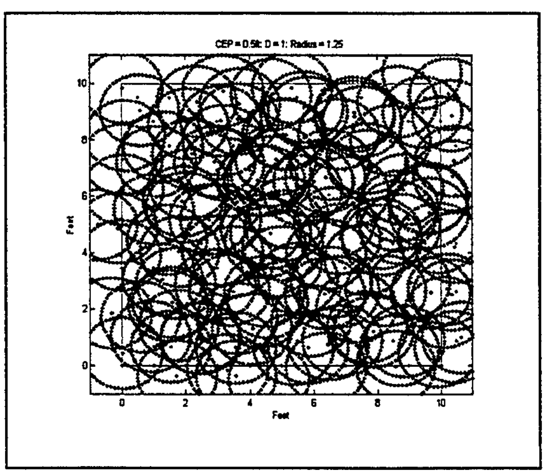

Figure 19: $Pk_{munition} = .63$, $Pk_{total} \approx .99$

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C. IMPACTORS PER MUNITION SENSITIVITY ANALYSIS

Current Lockheed Martin estimates are that the munition body has the ability to hold 935 impactors. However, it is likely that this is an optimistic estimate. The dispenser will require some space, therefore reducing the space available for impactors. Therefore, the system's sensitivity to decreases in the number of impactors per munition must be investigated.

1. Conditions

Input Parameters: ***N* = 500 (impactors per munition)**, $M = 7$ (munitions per Hydra-7), $S = 10$ (Hydra-7's per sortie), $R_{mine} = .5$ ft; $A_{total} = 57600$ ft^2, CEP = 3 ft.

2. Analysis

Fewer impactors means a smaller radius for a given $Pk_{munition}$, and consequently an increase in sorties. When compared to Figure (11), Figure (20) summarizes the impact that a reduction in N from 935 to 500 has on all of uniformly built munition designs. Figure (21) shows the effect of incremental reductions in N for $Pk_{munition} = .7$.

3. Conclusions

The number of sorties required to achieve a specified Pk_{total} is approximately inversely proportional to the number of impactors per munition. So, halving N nearly doubles the required number of sorties. As an example, Figure (20) shows that for a Pk_{total} of .95, when $N = 500$, 8 sorties are required, but when $N = 950$, only four sorties are required.

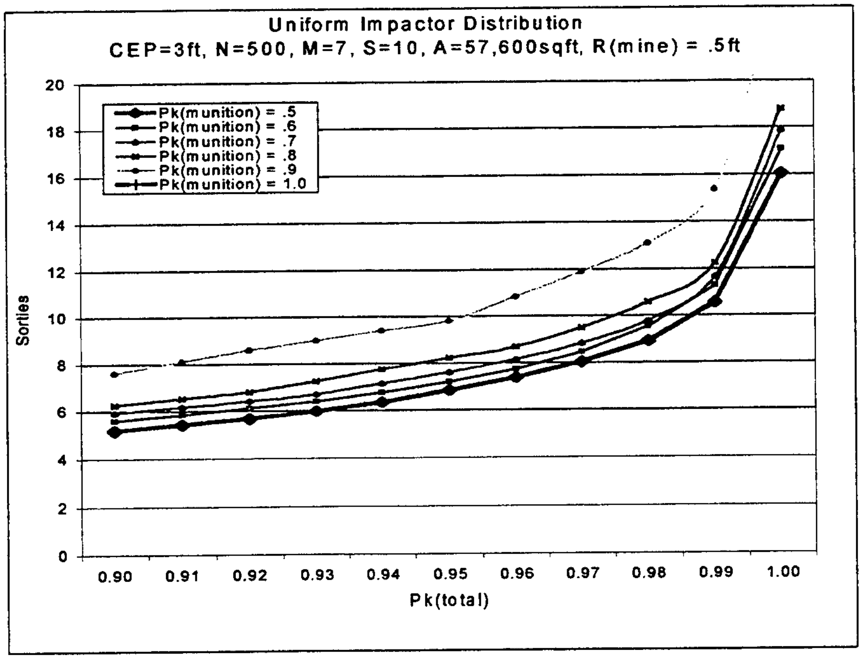

Figure 20: Effect of Impactors per Munition on Uniform Distribution
A reduction in the number of impactors per munition from the base case of 950 to 500 results in an increase in the number of sorties.

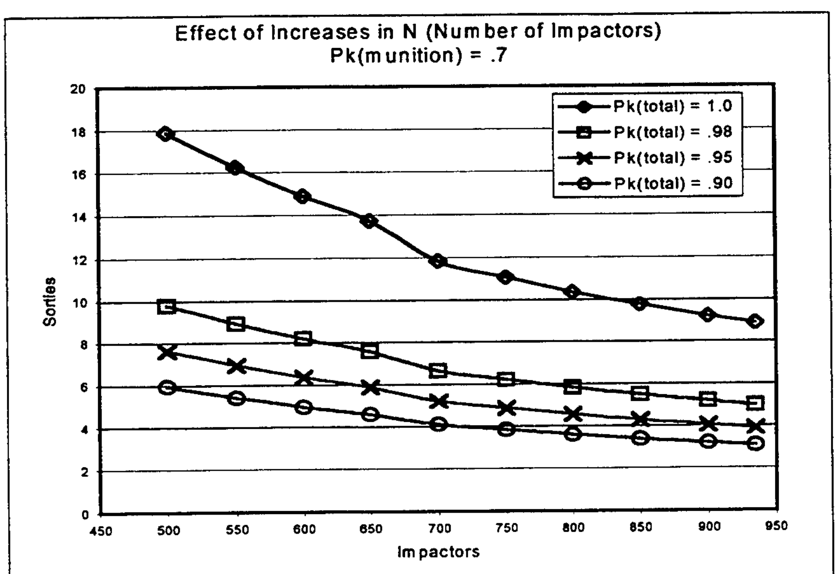

Figure 21: Effect of Impactors per Munition on $Pk_{munition}$ = .7
The effect of variations in the number of impactors per munition for a single munition design.

D. MINE SIZE (R_{MINE}) SENSITIVITY ANALYSIS

Of all of the parameters examined in this sensitivity analysis phase, probably none is as important as R_{mine}. Although N and CEP are important factors in determining sortie requirements, their final values will be determined by engineering constraints, and an optimal solution can be analytically determined. On the other hand, the size of the mine that needs to be cleared is open to debate. Some commanders will believe that A should be cleared to level Pk_{total} of all mines, regardless of size. Others argue that only anti-tank mines, which generally are larger than their anti-personnel counterparts, need to be cleared by Hydra-7.

A good solution is to make the Hydra-7 flexible enough to handle changing threats. If the engineers can build the dispenser to allow selection of $R_{munition}$, then the munition can be "dialed in" to achieve any $Pk_{munition}$ on any R_{mine}. However, this is unlikely. A more likely scenario is that the munition will be designed and built using today's best guess at what the future will look like. So, careful consideration of the appropriate R_{mine} is required.

1. Conditions

Input Parameters: N = 935 (impactors per munition), M = 7 (munitions per Hydra-7), S = 10 (Hydra-7's per sortie), $\boldsymbol{R_{mine} = .25\,ft}$; A_{total} = 57600 ft^2, CEP = 3 ft.

2. Analysis

When compared to Figure (12), Figure (22) shows the effect of changing R_{mine} from six inches to three inches. In this case, a 50% reduction in the variable R_{mine} results in a very large increase in the number of sorties. Figure (23) is a representative example of this relationship.

3. Conclusions

On Figure (23), the bend in the knees of the curves for $Pk_{munition} = .7$ just prior to R_{mine} = four inches indicates that any R_{mine} under four inches starts to become very expensive in terms of sorties. More generally, the relationship between R_{mine} and sorties appears to be:

$$\#sorties \propto \frac{1}{R_{Mine}^2}. \tag{27}$$

So, if R_{mine} is halved, the number of sorties quadruples. For example, the curve representing $Pk_{total} = .95$ shows that when $R_{mine} = 6$ inches, approximately six sorties are required. But, at R_{mine} =three inches, the number of sorties quadruples to 24. While this relationship holds across the range of values shown in Figure (23), it is also reasonably accurate between Figure (23) and Figure (12), the base case.

For $Pk_{munition} = .50$ and $Pk_{total} = .95$, Figure (12), with the base case of $R_{mine} = 6$ inches, shows a requirement for just under four sorties. In Figure (21), where $R_{mine} = 3$ inches, over 16 sorties are required. Again, halving R_{mine} quadrupled the sortie requirement. This generalization should also prove useful during the remaining design phase. It may also prove useful in procurement decisions.

Based of the results of this research, the Hydra-7 does not appear to be efficient in killing small mines. If there is a requirement to clear the ICLS of small mines, the Hydra-7 may not be the weapon for the job.

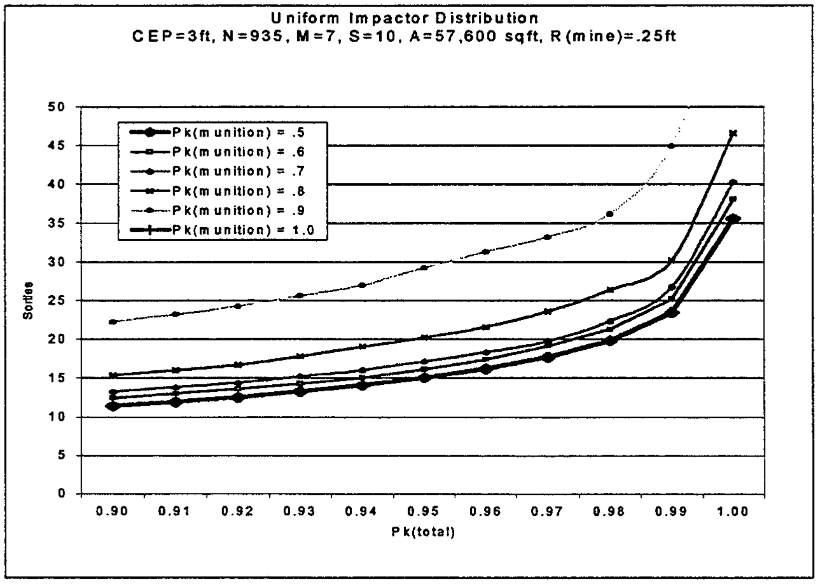

Figure 22: Effect of Decreased R_{mine} *on* Uniform Distribution
A change in R_{mine} to 3 inches. Note that the scale of the y axis has been changed from the base case graph (Fig(11)) to account for the large increase in sorties.

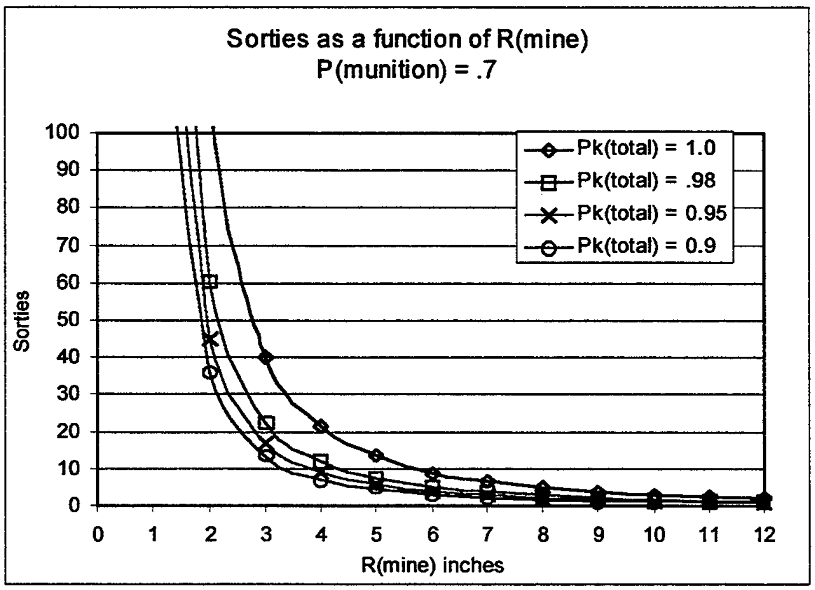

Figure 23: Effect of Decreased R_{mine} on $Pk_{munition}$ = .7
A graph of the relationship between R_{mine} and the number of sorties required to clear to a specified Pk_{total}.

E. OPTIMAL MUNITION DESIGN

It has been shown in the discussion of Figures (16) through (19) that, in the case of a munition with uniformly distributed impactors, a larger $R_{munition}$ is more efficient than a smaller one.

A munition designed to kill $R_{mine\_initial}$ at level $Pk_{munition}$ will kill R_{mine_new} to level $Pk_{munition\_new}$. If R_{mine_new} is less than $R_{mine\_initial}$, $Pk_{munition\_new}$ will be less than $Pk_{munition}$, resulting in an $R_{munition}$ that is larger than it would have been had the munition originally been designed to kill R_{mine_new} to level $Pk_{munition}$.

Therefore, although a decrease in R_{mine} will result in an increase in the number of sorties required, if the initial munition is designed as large as possible, the impact of reductions in $R_{munition}$ will be minimized.

This does not mean that the final number of sorties required to kill R_{mine_new} will not be too high. What it does mean, however, is that the munitions should be as large as possible, as long as $R_{munition}$ does not exceed the length of the shortest side of A. This is unlikely, however, as impactors must maintain very high velocity to work properly. An impactor that strays too far from the center of $R_{munition}$ is unlikely to maintain the required speed.

Also of interest, if the munitions are large, where $R_{munition}$ approaches the length of any side of A, it is very likely that some aiming point pattern other than the one used here would be most efficient. Under the current pattern, as the munitions grow, an increasing number of impactors will be lost out the sides of A. Intuitively, as $R_{munition}$ increases, the optimal aiming points will tend towards the center. At the point that the diameter of the munition is equal to the length of the sides of A, the aiming point of all munitions would be the center of A.

Given a final munition design with a characteristic $Pk_{munition}$ and $R_{munition}$ against a base case R_{mine}, the $Pk_{munition\_new}$ for that munition against R_{mine_new} can be found. It is not difficult to derive the following relationship:

$$Pk_{munition\_new} = 1 - \left[1 - \left(\frac{R_{mine\_new}}{R_{munition}} \right)^2 \right]^N . \tag{28}$$

Once $Pk_{munition\_new}$ is known, the distance between munition centers, d, can be found from the simulation output tables. Subsequently, munition aiming points can be found. See Figure (6).

F. OTHER VARIABLE SENSITIVITY ANALYSIS

Although the Excel spreadsheet with which sensitivity analysis is being conducted does have the option of changing the number of munitions per Hydra-7 *(M)*, the number of Hydra-7's per sorties (*S*), and the size of the target area *(A)* to see the effect on the sortie requirement, the results are unsurprising, and will therefore only be briefly addressed. Increasing the size of *A* will increase the requirement for munitions, independent of other variables, and will do so uniformly across the range of $Pk_{munition}$'s and Pk_{total}'s. Likewise, an change in the number of munitions per system, or in the number of systems that can fit on any aircraft, will result in a corresponding change in the number of sorties.

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V. CONCLUSIONS

In the base case, the number of sorties required to clear the ICLS (240ft by 240ft) of between 95% and 98% of six-inch mines is between four and five. Base Case conditions are 935 impactors per munition, 7 munitions per Hydra-7, 10 Hydra-7's per sortie, and a uniform impactor distribution in the munition footprint.

If the number of impactors per munition is reduced to 500, the sortie requirement to clear the ICLS of between 95% and 98% of six-inch mines is between seven and ten. The sortie requirement is inversely proportional to the number of impactors per munition.

If the CEP is increased to 24 feet from the base case value of three feet, the sortie requirement to clear the ICLS of between 95% and 98% of six-inch mines is between five and ten.

If the radius of the mine which needs to be cleared to level Pk_{total} is reduced to three inches from the base case value of six inches, the sortie requirement to clear the ICLS of between 95% and 98% of six inch mines is between 15 and 25 sorties. Halving of R_{mine} approximately quadruples the sortie requirement.

Bigger munitions are better. Munitions built to maximize $R_{munition}$ are much more efficient than munitions built to maximize $Pk_{munition}$, especially when impactors are assumed to be uniformly distributed.

R_{mine} is the most important "enemy controlled" variable when determining sortie requirements.

With the possible exception of an R_{mine} of less than 4 inches, a properly designed, the Hydra-7 Counter-Mine Weapon System appears to possess a high level of robustness to fluctuations in the variables addressed here.

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