# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

**TECHNICAL NOTE 2800** 

SOLUTIONS OF LAMINAR-BOUNDARY-LAYER EQUATIONS WHICH

**RESULT IN SPECIFIC-WEIGHT-FLOW PROFILES LOCALLY** 

**EXCEEDING FREE-STREAM VALUES** 

By W. Byron Brown and John N. B. Livingood

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#### TECHNICAL NOTE 2800

# SOLUTIONS OF LAMINAR-BOUNDARY-LAYER EQUATIONS WHICH RESULT IN SPECIFIC-

# WEIGHT-FLOW PROFILES LOCALLY EXCEEDING FREE-STREAM VALUES

By W. Byron Brown and John N. B. Livingood

#### SUMMARY

Solutions of the laminar-boundary-layer equations when large temperature changes in the boundary layer and large pressure changes in the main stream occur simultaneously were found to be very sensitive to the behavior of the third-order derivative of the boundary-layer stream function as the specific weight flow approached its free-stream value. (The specific weight flow is proportional to the first derivative of the stream function.) Theoretically, all derivatives of the specific weight flow should vanish at the outer edge of the boundary layer; however, in numerical solutions, only a restricted number of these conditions can be applied. Under assumed constant wall temperature and small Mach numbers, solutions of the laminar-boundary-layer equations for stream-to-wall temperature ratios of 2 and 4, Euler numbers of 0.5 and 1, and rates of cooling-air flow through the porous wall signified by values of the coolant flow parameter of 0, -0.5, and -1 previously reported did not fulfill the condition that the third-order derivative of the stream function vanish at the outer edge of the boundary layer. New solutions which not only fulfilled this condition but also which resulted in very small values of higher-order derivatives were therefore obtained. The resulting specific-weight-flow, velocity, and temperature distributions and the local heat-transfer coefficients are tabulated and are compared with those determined previously. Friction coefficients and dimensionless displacement, momentum, convection, and thermal boundary-layer thicknesses are also tabulated.

The new solutions resulted in specific weight flows which exceeded the free-stream values. These excesses ranged from 2 percent for the impermeable wall, a stream-to-wall temperature ratio of 2, and an Euler number of 0.5 to 15 percent for the permeable wall with a coolant flow parameter of -1, a stream-to-wall temperature ratio of 4, and an Euler number of 1, the most severe case considered.

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#### INTRODUCTION

General solutions of the laminar-boundary-layer equations are obtainable for free-stream velocity distributions which are proportional to a power of the distance from the stagnation point and which are found in incompressible flow around infinite wedges. These wedge solutions have proved useful as good approximations for other types of flow (reference 1) and have served as a basis for methods of determing local heat-transfer coefficients for bodies of arbitrary shape (references 2 and 3).

Wedge solutions were first presented in reference 4 for conditions of constant property values and small Mach numbers. Extension to highspeed flow is accomplished in reference 5, in which the wedge solutions are used to approximate heat transfer to bodies of arbitrary shape. Reference 6 includes the effects of aerodynamic heating. Extensions to include variable wall temperatures are made in references 7 and 8.

In reference 9, the effects of transpiration cooling are included for constant property values, constant wall temperature, and small Mach numbers. The extension to include variable fluid properties is presented in references 1 and 10.

Tables of laminar-boundary-layer solutions with variable fluid properties and a pressure gradient in the main stream are given in reference 10 for conditions of constant wall temperature and small Mach numbers. When large temperature changes occurred in the boundary layer and were accompanied by large pressure changes in the main stream, a new type of velocity profile not found in any of the earlier referenced work was sometimes obtained. In particular, when the wall was four times as hot as the stream and a stagnation-point pressure gradient was present, the velocity increased very rapidly from zero at the wall to a value 20 percent above the free-stream value one-third of the way through the boundary layer and then decreased gradually to the free-stream value as the boundary-layer density approached the stream value. This behavior is explained by E. R. G. Eckert to be caused by the fact that the warmer layers in the boundary layer are more accelerated than the outside flow because of their lower density when both are subject to the same pressure drop.

In view of this result, an examination of the tabular distributions of reference 10 was made to determine whether some temperature- and pressure-drop effect could be found in the opposite case, that is, when the boundary layer was much cooler than the free stream and consequently denser. This, it was thought, might cause the specific weight flow (product of density and velocity) to exceed the free-stream value near the wall because of the high density value there. When this alternative

was inserted in the analysis by permitting the specific weight flow to exceed its free-stream value and to decrease toward it until the profile became horizontal for the second time, it was found to fit well and to yield a smoother blending of the boundary-layer and the free-stream values of specific weight flow, temperature, and velocity than that of reference 10 in which the free-stream value of the specific weight flow was attained but not exceeded when the profile became horizontal the first time. At this point the curvature of the specific-weight-flow profile did not approach zero as the slope did when large pressure gradients in the main stream and strong cooling occurred together. In all other cases considered, the specific-weight-flow profile curvature did approach zero.

Accordingly, 12 cases of reference 10 that involved cooling at the wall combined with large pressure gradients in the main stream were recalculated at the NACA Lewis laboratory; these new solutions, for which the second- and higher-order derivatives of the boundary-layer stream function vanish at the outer edge of the boundary layer are presented herein. Stream-to-wall temperature ratios of 2 and 4, Euler numbers (nondimensional pressure gradient parameter) of 0.5 and 1, and nondimensional flow rates through the porous walls of 0, -0.5, and -1 were used in these recalculations. It should be noted that distributions presented in reference 10 for an Euler number of 0.4 would likewise be revised by the present method. These revised values were not calculated, however, because it was believed that the Euler numbers 0.4 and 0.5 were not sufficiently far apart to justify the recalculation. Velocity, specific-weight-flow, and temperature distributions are tabulated as well as the dimensionless stream function of Falkner and Skan and its derivatives and the dimensionless temperature function of Pohlhausen and its derivatives.

For each case, dimensionless forms of displacement, momentum, convection, and thermal boundary-layer thicknesses, Nusselt numbers, and wall friction coefficients are given.

#### SYMBOLS

The following symbols are used in this report:

С

 $c_{D}$ 

constant of proportionality

C<sub>f,w</sub>

.

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specific heat at constant pressure

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Euler number,  $\frac{-x \frac{dp}{dx}}{\rho_{\infty} U_{\infty}^{2}}$ ;  $U_{\infty} = Cx^{Eu}$ Eu dimensionless stream function f first, second, and third derivatives of f with respect '' f'', f'' to ŋ heat-transfer coefficient Η thermal conductivity k Nusselt number,  $\frac{Hx}{k_{w}}$ Nu  $\frac{\mathrm{T}}{\mathrm{T}_{\mathrm{W}}} = 1 + \theta \left( \frac{\mathrm{T}_{\mathrm{w}}}{\mathrm{T}_{\mathrm{W}}} - 1 \right) = \mathrm{P}$ P Prandtl number,  $\frac{c_{p,w} \mu_w}{k}$ Pr pressure р Reynolds number,  $\frac{U_{\infty}\rho_W x}{u_{\infty}}$ Re fluid temperature Т refers to wall temperature and coolant upon emergence from тw porous wall fluid velocity at edge of boundary layer U fluid velocity parallel to wall in boundary layer u fluid velocity normal to wall in boundary layer v distance along surface х distance normal to surface У exponent of temperature for specific heat,  $\,c_{\rm p} \, \varpi \, T^{\alpha}$ α displacement boundary-layer thickness δ\*

δ<sub>c</sub>

η

θ

μ

ρ

convection boundary-layer thickness

 $\delta_i$  momentum boundary-layer thickness

 $\delta_t$  thermal boundary-layer thickness

 $\epsilon$  exponent of temperature for thermal conductivity, k  $\infty T^{\epsilon}$ 

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(1)

dimensionless boundary-layer coordinate,  $y \sqrt{\frac{\rho_W U_{\infty}}{\mu_W x}}$ 

temperature-difference ratio,  $\frac{T - T_w}{T_{\infty} - T_w}$ 

 $\theta', \theta''$  first and second derivatives of  $\theta$  with respect to  $\eta$ 

absolute viscosity of fluid

density of fluid

 $\tau_w$  shear stress at wall

 $\psi$  stream function

 $\omega$  exponent of temperature for viscosity,  $\mu \mathbf{o} T^{\omega}$ 

Subscripts:

w wall

∞ main stream

#### ANALYSIS

#### Laminar-Boundary-Layer Equations

The equations of the laminar boundary layer for steady-state flow of a viscous fluid with heat transfer, presented in references 1 and 10 and repeated for convenience herein, are

Momentum equation:

$$\operatorname{bn} \frac{\operatorname{gn}}{\operatorname{gn}} + \operatorname{bn} \frac{\operatorname{gn}}{\operatorname{gn}} = \frac{\operatorname{gn}}{\operatorname{gn}} \left( \operatorname{h} \frac{\operatorname{gn}}{\operatorname{gn}} \right) - \frac{\operatorname{gn}}{\operatorname{gn}}$$

(3)

(4)

(5)

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Continuity equation:

 $\frac{\partial x}{\partial t} (\rho u) + \frac{\partial y}{\partial t} (\rho v) = 0$  (2)

Energy equation:

$$c^{b}\left(bn \frac{\partial x}{\partial L} + bn \frac{\partial x}{\partial L}\right) = \frac{\partial x}{\partial }\left(k \frac{\partial x}{\partial L}\right) + h\left(\frac{\partial x}{\partial n}\right)_{S} + n \frac{\partial x}{\partial b}$$

The boundary conditions are

u = 0,  $v = v_W$ , and  $T = T_W$  when y = 0

 $\mathbf{a}$ nd

$$u \rightarrow U_{\infty}; \frac{\partial u}{\partial y} \rightarrow 0, \frac{\partial^{2} u}{\partial y^{2}} \rightarrow 0; \dots \frac{\partial^{n} u}{\partial y^{n}} \rightarrow 0$$
$$T \rightarrow T_{\infty}; \frac{\partial T}{\partial y} \rightarrow 0, \frac{\partial^{2} T}{\partial y^{2}} \rightarrow 0; \dots \frac{\partial^{n} T}{\partial y^{n}} \rightarrow 0$$

#### Assumptions

For simplification of the analysis, the following assumptions are made:

(1) The Mach number is small.

μ

(2) The Euler number is constant.

(3) The wall temperature is constant.

(4) The fluid property variations are expressible in terms of the absolute temperature as follows:

$$c_p \mathbf{\alpha} T^{\alpha} \rho \mathbf{\alpha} T^{-1}$$

The second and third terms in the right member of equation (3) are proportional to  $M^2$  and hence approach zero as  $M^2$  does. By virtue of assumption (1), these two terms may be neglected. Furthermore, for the same reason,  $\rho_w$  and  $T_\infty$  are considered constants.

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#### Transformation to Ordinary Differential Equations

The transformation from partial to total differential equations is accomplished by the change in variables

$$\eta = y \sqrt{\frac{\rho_{w} U}{\mu_{w} x}}$$

$$\theta = \frac{T - T_{w}}{T_{\infty} - T_{w}}$$

$$f = \frac{\rho_{w} \psi}{\sqrt{\mu_{w} x U_{\infty} \rho_{w}}}$$

(6)

where  $\eta$  is the dimensionless independent variable introduced by Blasius and f and  $\theta$  are the dimensionless dependent variables representing the stream function and the temperature, respectively.

Substitution of  $\eta$ , f, and  $\theta$  in the partial differential equations, use of the abbreviation  $P = \frac{T}{T_W} = 1 + \theta \left(\frac{T_\infty}{T_W} - 1\right)$ , and the simplifying assumptions yield (references 1 and 10) the energy equation

$$-\theta'' = \frac{\mathrm{Eu} + 1}{2} \operatorname{Pr}_{W} \operatorname{P}^{\alpha - \epsilon} \theta' \mathbf{f} + \epsilon \left(\frac{\mathrm{T}_{\infty}}{\mathrm{T}_{W}} - 1\right) \operatorname{P}^{-1} \theta'^{2}$$
(7)

and the momentum equation

$$f''' = Eu P^{-\omega} f'^{2} - \frac{Eu + 1}{2} P^{-\omega} ff'' - Eu \frac{T_{w}}{T_{w}} P^{-\omega-1} - \frac{Eu + 1}{2} \left(\frac{T_{w}}{T_{w}} - 1\right) P^{-\omega-1} ff'\theta' - \left(\frac{T_{w}}{T_{w}} - 1\right) P^{-1}f'\theta'' - \left(\frac{T_{w}}{T_{w}} - 1\right) P^{-1}f'\theta'' - \left(\frac{T_{w}}{T_{w}} - 1\right)^{2} P^{-2}f'\theta'^{2}$$

$$(\omega + 2) \left(\frac{T_{w}}{T_{w}} - 1\right) P^{-1}f''\theta' - \omega \left(\frac{T_{w}}{T_{w}} - 1\right)^{2} P^{-2}f'\theta'^{2}$$

$$(8)$$

The boundary conditions are

$$f' = 0$$
,  $f = f_W$ , and  $\theta = 0$  when  $\eta = 0$ 

and

(9)

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$$\theta \rightarrow 1; \theta' \rightarrow 0, \theta'' \rightarrow 0, \ldots, \theta^n \rightarrow 0$$

$$f' \rightarrow \frac{T_w}{T_w}; f'' \rightarrow 0, f''' \rightarrow 0, \ldots f^n \rightarrow 0$$

# Discussion of Boundary Conditions

Satisfying so many boundary conditions simultaneously is difficult. Initial values must be assumed for f" and  $\theta$ ' to begin the calculation; then the step-by-step numerical solution of equations (8) and (9) can be carried out until the boundary layer merges with the free stream. For constant property values,  $T_{\infty}/T_{W} = P = 1$  and equation (7) reduces to

$$-\theta'' = \frac{\mathrm{Eu} + 1}{2} \operatorname{Pr}_{W} \theta' \mathbf{f}$$

Consequently,  $\theta'' \rightarrow 0$  when  $\theta' \rightarrow 0$ . For constant property values, equation (8) reduces to

$$f''' = Eu f'^2 - \frac{Eu + 1}{2} ff'' - Eu$$

and  $f'' \rightarrow 0$  when  $f' \rightarrow 0$  and  $f' \rightarrow 1$ . The solutions for constant property values can therefore be obtained by adjusting the initial values of f'' and  $\theta'$  at the wall so that, at the free-stream boundary,  $\theta \rightarrow 1$  when  $\theta' \rightarrow 0$  and  $f' \rightarrow 1$  when  $f'' \rightarrow 0$ .

For variable property values, equation (7) shows that  $\theta" \rightarrow 0$  when  $\theta' \rightarrow 0$ , as before. However, equation (8) reduces to

$$f''' = Eu P^{-\omega} f'^2 - Eu \frac{T_w}{T_\infty} P^{-\omega-1} - \frac{Eu + 1}{2} \left( \frac{T_\infty}{T_w} - 1 \right) P^{-\omega-1} ff' \theta' -$$

$$\left(\frac{T_{\infty}}{T_{W}} - 1\right) P^{-1}f'\theta'' - \omega \left(\frac{T_{\infty}}{T_{W}} - 1\right)^{2} P^{-2}f'\theta'^{2}$$

when  $f'' \rightarrow 0$ , and for  $f''' \rightarrow 0$  it is necessary for  $\theta' \rightarrow 0$ ,  $\theta \rightarrow 1$ , and  $f' \rightarrow \frac{T_w}{T_w}$  simultaneously.

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According to reference 10,

$$\frac{\rho u}{\rho U} = f' \frac{T_{\infty}}{T_{W}}$$

so that the specific weight flow is proportional to f'; that is, the variation of f' can be used to measure the variations of the specific weight flow.

Former method. - The solutions reported in reference 10 were obtained by assuming that the free-stream boundary conditions were satisfied when  $f'' \rightarrow 0$  the first time and  $f' \rightarrow T_w/T_\infty$ . These calculations show that for the flat plate, where Eu = 0, the boundary-layer specific weight flow and temperature merge with the free stream at points sufficiently near each other that f''' is quite close to zero when f'' = 0. This is likewise true when Eu has small negative values (adverse pressure gradient).

When the Euler number has large positive values, 0.5 and 1, f' builds up to the free-stream value  $T_w/T_\infty$  very rapidly. For example, when f" = 0, f''' = -0.014 when Eu = 1,  $T_\infty/T_w = 4$ , and  $f_w = 0$ . If, in addition, there is flow through the porous wall ( $f_w = -1$ ) the situation becomes worse, with f''' = -0.042 when f" = 0.

At the free stream, the curvature of f' should be zero, that is,  $f''' \rightarrow 0$ . The foregoing discussion shows that the method reported in reference 10 does not satisfy this condition sufficiently well for large positive Euler numbers combined with large temperature ratios. Consequently, a modification in the solution for these cases was necessary.

<u>Present method</u>. - In an effort to satisfy the free-stream boundary conditions more accurately for the cases of large positive Euler numbers and temperature ratios, f' was allowed to exceed the main-stream value and initial values of f" and  $\theta$ ' were adjusted so that  $f' \rightarrow T_w/T_{\infty}$ when  $f'' \rightarrow 0$  the second time. The result of this modification, as will be shown later, was that f''' approached zero quite closely even for large positive Euler numbers and temperature ratios. Moreover, it will also be shown that  $f''' \rightarrow 0$  at values of  $\eta$  (or y) only slightly less than those for which  $\theta'$  approached zero.

# NUMERICAL CALCULATION AND RESULTS

Equations (7) and (8) were solved with physical properties appropriate for air in the range  $600^{\circ}$  to  $2400^{\circ}$  F obtained from logarithmic plots of data taken from reference 11. These values were

(10)

 $Pr_{W} = 0.7$   $\omega = 0.7$   $\epsilon = 0.85$   $\alpha = 0.19$ 

The cases recomputed here are 12 in all as follows:

$$f_w = 0, -0.5, -1$$
  
 $\frac{T_{\infty}}{T_w} = 2, 4$   
Eu = 0.5, 1

#### Method of Calculation

The system of differential equations (7) and (8) was solved by Picard's method (reference 12). Values of f" and  $\theta$ ' were assumed for  $\eta = 0$  and calculations were carried through the boundary layer until f"  $\rightarrow 0$  (second time) and  $\theta' \rightarrow 0$ . Here the value of f' should be  $T_w/T_\infty$  and that of  $\theta$  should be 1. In order to ensure that the higher derivatives also approach zero, the differences in the f''' and  $\theta$ " columns should be approaching zero. If not, the initial values were adjusted until the desired values at the boundary-layer border were attained.

The velocity and specific-weight-flow distributions were found as in reference 10 by use of the relations

$$\frac{u}{U_{\infty}} = f' P$$

$$\frac{\rho u}{\rho_{\infty} U_{\infty}} = f' \frac{T_{\infty}}{T_{W}}$$

(11)

(12)

The displacement, momentum, and convection thicknesses were found as in references 1 and 10 by the equations

$$\frac{\delta^{*}}{x} \sqrt{Re} = \int_{0}^{\infty} \left(1 - f' \frac{T_{\infty}}{T_{w}}\right) d\eta$$

$$\frac{\delta_{i}}{x} \sqrt{Re} = \frac{T_{\infty}}{T_{w}} \int_{0}^{\infty} f' (1 - P f') d\eta$$

$$\frac{\delta_{c}}{x} \sqrt{Re} = \frac{T_{\infty}}{T_{w}} \int_{0}^{\infty} f' (1 - \theta) d\eta$$

Another thickness called the thermal boundary-layer thickness is defined in reference 3 as

 $\delta_{t} = \int_{0}^{\infty} \frac{T - T_{\infty}}{T_{w} - T_{\infty}} d\eta$ 

By means of relations (6), this equation can be reduced to the nondimensional form

$$\frac{\delta_{t}}{x}\sqrt{Re} = \int_{0}^{\infty} (1 - \theta) d\eta \qquad (12a)$$

The specific-weight-flow, velocity, and temperature distributions for each of the 12 cases considered are presented in table I. Also included in table I are the local heat-transfer coefficients and friction coefficients, obtained as in references 1 and 10 from the relations

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = \theta_{\mathrm{W}}$$

and

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and values of the dimensionless displacement, momentum, convection, and thermal boundary-layer thicknesses, obtained by integrations made from the distributions in the tables according to equations (12) and (12a).

 $\frac{C_{f,W}}{2}\sqrt{Re} = f_W''$ 

Table II is a summary table including local heat-transfer coefficients and friction coefficients and values of the dimensionless displacement, momentum, convection, and thermal boundary-layer thicknesses. This table contains the results presented in reference 10, with corrections made for the 12 cases that were recalculated and reported herein and with the additional values of the dimensionless thermal boundarylayer thickness; all thickness calculations were reviewed, and several minor corrections in addition to the 12 cases referred to previously were included.

In the calculations reported herein, four decimal places were carried in all cases except a few sensitive ones in which five decimals were necessary. These were all rounded off to three decimals in the distributions presented in table I because it was believed that the fourth decimal might be affected by cumulative errors in such long step-by-step calculations. In table II, where initial values of f" and  $\theta$ ' are listed, all four decimals are given.

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An inspection of table I shows that the exact stream values of f' and  $\theta$  are not always attained. To attain these exact stream values would have entailed excessive labor without corresponding increases in the precision of  $\frac{Nu}{\sqrt{Re}}$  and  $\frac{C_{f,W}}{2} \sqrt{Re}$ . A comparison of the results between the last two trial solutions for the case of greatest deviation from exact stream values  $(T_{o}/T_{w} = 4, Eu = 1, and f_{w} = -0.5)$  was made. It was found that if f' were to approach 0.250 instead of 0.252 and  $\theta$ were to approach 1.000 instead of 0.999 at the free stream, the following approximate changes would be expected:

Nu √Re	would change from 0.22522 to 0.22536
f <mark>"</mark>	would change from 0.44647 to 0.44650
$\frac{\delta^* \sqrt{Re}}{x}$	would change from 0.000 to -0.004
$\frac{\delta_{i}\sqrt{\text{Re}}}{x}$	would change from 1.867 to 1.856
$\frac{\delta_c \sqrt{Re}}{x}$	would change from 2.932 to 2.929
$\frac{\delta_t \sqrt{Re}}{x}$	would change from 3.233 to 3.226

In only one case is the third significant figure affected except for the value of  $\frac{\delta^* \sqrt{\text{Re}}}{x}$ , which results in an exceptional value for the particular case considered.

#### COMPARISON WITH PREVIOUS RESULTS

Specific-weight-flow, velocity, and temperature profiles obtained by both the method of reference 10 and the present method are shown in figure 1(a) for the case of maximum deviation and in figure 1(b) for the case of minimum deviation of specific-weight-flow values from stream values for the impermeable wall; the maximum deviation (fig. 1(a)) was obtained for a stream-to-wall temperature ratio  $T_{\infty}/T_W$  of 4 and an Euler number Eu of 1, whereas the minimum deviation (fig. 1(b)) resulted from a stream-to-wall temperature ratio of 2 and an Euler number

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of 0.5. Figure 1(a) shows an excess of the specific-weight-flow profile of 7 percent above the free-stream value, a considerable increase in the velocity profile, and relatively little change in the temperature profile. Moreover, the new specific-weight-flow profile approaches the freestream value with zero curvature and at a value of the nondimensional boundary-layer coordinate  $\eta$  which is now only slightly less than the corresponding values for the velocity and temperature profiles. In addition, the abrupt change in curvature of the velocity profile obtained by the method of reference 10 is eliminated and the new velocity profile is a smooth curve through the entire boundary layer.

Figure 1(b), the minimum deviation case for an impermeable wall, shows a specific-weight-flow profile excess of only about 2 percent, only a slight increase in the velocity profile, and temperature differences so slight that no change in the temperature profile can be plotted.

Much larger differences in the various profiles result for permeable walls. Figure 1(c) shows the comparison for a stream-to-wall temperature ratio of 4, an Euler number of 1, and a flow rate through the porous wall, designated by  $f_W$ , of -1, the most severe case considered. For this permeable case, the specific-weight-flow values exceeded the free-stream value by over 15 percent, and a much more pronounced increase in the velocity profile and even a considerable increase in the temperature profile resulted from the present method. In this case, as before, the abrupt change in curvature of the velocity profile obtained by the method of reference 10 no longer appears in the new solution. A comparison of figures 1(a) and 1(c) confirms the fact that thicker boundary layers result when porous cooling is considered.

Figure 2 shows a plot of the heat-transfer coefficient against the Euler number for values calculated by the present method for various stream-to-wall temperature ratios and flow rates through the porous wall designated by  $f_W = 0$ , -0.5, and -1 and for values obtained by the method of reference 10 for Euler numbers of 0.5 and 1 designated by circles for  $T_{\infty}/T_{w} = 2$  and by triangles for  $T_{\infty}/T_{w} = 4$ . The upper two curves for the impermeable wall ( $f_w = 0$ ), that is, those for  $T_{\infty}/T_w$  values less than unity, are presented in reference 13. The velocity values for these temperature ratios exceeded the free-stream values. No changes resulted when the two methods were used for a value of  $T_{w}/T_{w} \leq 1$ . It can be seen from figure 2 that the curves group themselves according to the coolant flow rates considered. Moreover, in each instance, the magnitude of the changes encountered by applying the two methods are nearly the same. Percentagewise, however, the variations increase as the coolingair flow rate through the porous wall increases (or as the porous-flow parameter decreases).

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Comparisons of the dimensionless displacement thickness of the boundary layer obtained by the two methods are shown in figure 3 for a range of Euler number, coolant flow rates through the walls of 0 and -1, and stream-to-wall temperature ratios of 2 and 4. In each case, the dimensionless displacement thickness decreased when the present method of solution was applied. These decreases result directly from the fact that the new specific-weight-flow profiles exceed the free-stream values; the integral in the definition of this boundary-layer thickness is measured by the area between the unit ordinate and the specific-weight-flow profiles. From figure 3(a), for a stream-to-wall temperature ratio of 2, it can be seen that for an Euler number of 1, the nondimensional displacement thickness decreases by about 16 percent for an impermeable wall and about 23 percent for the permeable wall. From figure 3(b), the corresponding decreases for a stream-to-wall temperature ratio of 4 are about 80 and 116 percent, respectively. For a permeable wall, figure 3(b) also shows a negative displacement thickness for large Euler numbers. From these figures it can be seen that, as the Euler number decreases, the curves representing the solutions obtained by the two methods converge.

In the calculation of the dimensionless momentum and convection thicknesses, the influence of changes in the velocity profile are present along with those of the specific-weight-flow profiles. These influences tend to counteract each other, with the net result that there are only slight differences in the momentum and convection thicknesses. For this reason, these thicknesses are not plotted. The new values are tabulated in table II; the values obtained by the method of reference 10 are available in table II of reference 10.

#### SUMMARY OF RESULTS

Under assumed conditions of constant wall temperature and small Mach numbers, solutions of the laminar-boundary-layer equations which fulfill the vanishing of the second- and higher-order derivatives of the boundary-layer stream function at the free-stream boundary were obtained for cases where large temperature changes in the boundary layer and large pressure changes in the main stream occur simultaneously. Stream-to-wall temperature ratios of 2 and 4, Euler numbers of 0.5 and 1, and porous flow rates characterized by values of the flow parameter of 0, -0.5, and -1 were considered in these calculations. Previously determined results for stream-to-wall temperature ratios less than unity resulted in velocities which exceeded the free-stream values.

The results of this analytical investigation are summarized as follows:

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1. The new specific-weight-flow profiles were found to exceed the free-stream values. For an impermeable wall, a stream-to-wall temperature ratio of 2, and an Euler number of 0.5, the excess of the specific weight flow over the free-stream value was 2 percent; for a permeable wall with a coolant flow parameter of -1, a stream-to-wall temperature ratio of 4, and an Euler number of 1, the excess was 15 percent.

2. Velocity and temperature values throughout the boundary layer were larger than those previously obtained; maximum differences were found for a stream-to-wall temperature ratio of 4, an Euler number of 1, and a flow rate through the porous wall of -1.

3. Specific-weight-flow profiles approached the free-stream values with zero curvatures at values of the dimensionless boundary-layer coordinate only slightly less than the corresponding values for the velocity and temperature profiles.

4. Heat-transfer coefficients increased in value over those obtained previously; the percentage change in heat-transfer coefficients increased as the cooling-air flow rate increased.

5. Considerable reductions in the nondimensional displacement boundary-layer thicknesses resulted from application of the new method. Largest decreases were obtained for an Euler number of 1. Negative values resulted for permeable walls, large temperature ratios, and large Euler numbers. As the Euler number decreased, the solutions obtained by the two methods converged.

6. Only slight changes in the dimensionless momentum and convection thicknesses resulted when the present method of solution was used.

7. Values of the dimensionless thermal boundary-layer thickness were calculated and are included.

#### CONCLUSION

The specific-weight-flow profiles determined from analytical solutions of the laminar-boundary-layer equations, under assumed conditions of constant wall temperature and small Mach numbers, exceed the freestream values when large pressure changes in the main stream and streamto-wall temperature ratios greater than unity occur simultaneously.

Lewis Flight Propulsion Laboratory National Advisory Committee for Aeronautics Cleveland, Ohio, July 31, 1952

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# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL

(1) 
$$T_{\infty}/T_{w} = 2$$
; Eu = 0.5;  $f_{W} = 0$ 

 $\frac{\delta^* \sqrt{Re}}{x} = 0.622; \ \frac{\delta_1 \sqrt{Re}}{x} = 0.838; \ \frac{\delta_c \sqrt{Re}}{x} = 1.372; \ \frac{\delta_t \sqrt{Re}}{x} = 1.904$ 

η	f	f'	f"	f"'	θ	θ'	θ"	u Uco	<u>ρυ</u> Ρ <b>ω <sup>Մ</sup>ω</b>
								Pf'	2f'
0 .2 .4 .6 .8 1.0 1.2 1.4 1.6 1.8 2.0 2.2	0 .012 .046 .095 .156 .228 .306 .391 .480 .572 .667 .764	0 .119 .210 .280 .334 .376 .409 .434 .454 .454 .469 .481 .490	0.684 .516 .396 .238 .185 .144 .112 .087 .067 .051 .038	-0.992 704 517 389 298 231 181 142 113 089 071 056 044	0 .078 .151 .220 .286 .347 .406 .461 .512 .561 .606 .648 .686	0.402 .377 .356 .336 .318 .300 .283 .266 .250 .233 .217 .201 .86	-0.137 114 101 093 089 086 085 084 083 082 081 079 077	Pr 0 .128 .241 .341 .429 .506 .575 .634 .687 .733 .773 .807 .837	21 0 .238 .419 .559 .668 .752 .817 .869 .908 .939 .962 .980 .993
2.4 2.6 2.8 3.2 3.6 4.0 4.4 4.8 5.2 5.6 6.0 6.4 6.8 7.2 7.6 8.0 8.4 8.8 9.6 10.0	.963 .963 1.063 1.266 1.470 1.673 1.877 2.080 2.282 2.484 2.685 2.886 3.087 3.288 3.488 3.689 3.889 4.090 4.290 4.490 4.691	.501 .505 .508 .510 .509 .509 .508 .507 .505 .504 .503 .502 .502 .501 .501 .501	.020 .020 .014 .005 0 002 003 003 003 003 003 002 002 001 0 0	035 027 016 009 004 001 0 .001 .001 .001 .001 .001 .001 0 0	.722 .754 .811 .857 .894 .923 .946 .962 .974 .983 .989 .993 .999 .999 .999 .999 1.000 1.000 1.000	.100 .170 .156 .128 .103 .082 .063 .048 .035 .026 .018 .012 .008 .005 .003 .002 .001 .001 0 0	075 072 065 058 050 042 035 028 022 016 012 009 006 004 003 002 001 001 0 0	.863 .885 .921 .946 .964 .977 .986 .991 .995 .997 .999 1.000 1.001 1.002 1.002 1.002 1.002 1.002 1.002	1.003 1.009 1.017 1.019 1.018 1.016 1.013 1.010 1.008 1.006 1.005 1.004 1.003 1.002 1.002

 $\frac{\delta^* \sqrt{\text{Re}}}{x}$ 

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# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

	(2) $T_{co}/T_{W} = 2;$	$Eu = 1; f_w = 0$	
	δ <sub>i</sub> <sub>\</sub> Re	$\delta_{a}/Re$	$\delta_{\pm}\sqrt{\text{Re}}$
= 0.433;	$\frac{1}{x} = 0.693;$	$\frac{c}{x} = 1.215;$	<u></u> x

Г										
	η	f	f'	f"	f"'	θ	θ'	θ"	u U <sub>co</sub> Pf'	_ρu ρœŪœ 2f'
ľ	0	0	0	0.000	1 000	0	0.470	0.107		
l	<b>1</b>	004	007	750	-1.000	0	0.476	-0.193	0	0
	.1	016	157	.750	-1.352	.047	.458	172	.087	.167
	• <u>2</u> 7	034	.100	.030	-1.108	.092	.441	156	.167	.306
L	•О И	.054	.611	.000	918	.135	.426	146	.240	.422
	• <del>*</del>	.000	.200	.401	/65	•1/1	.412	138	.306	.521
	.5	.000	.302	.301	642	.218	.399	132	.367	.604
	.0	15/	367	.366	541	.257	.386	128	.423	.674
ŀ	• '	102	.307	.6/6	456	.295	.3/3	125	.475	.733
	••	.136	.396	.230	38/	.352	.361	123	.522	.783
1	1 0	.636	.413	.195	529	.367	.349	122	.564	.826
	1 1	•674 310	.431	.104	280	.401	.337	120	.604	.862
	1 2	.510	.440	.130	239	.434	.325	119	.640	.892
1	1 1	.303	.433	.110	204	.466	.313	118		.917
	1.4	-407 EEA	•4/0	1.081	149	.526	.289	117	.730	.956
	1.0	.004 657	.492	.056	109	.582	.266	114	.778	.984
	2.0	.000 754	•501 507	.057	079	.633	.243	111	.818	1.002
ĺ	2.0	.104	.507	.025	058	.679	.221	108	.851	1.014
	2.4	.000	.511	.014	041	.721	.200	104	.879	1.021
	6.4 0 C	.908	.513	.006	029	.759	.180	099	.902	1.025
	2.0	1.061	.514	.002	020	.793	.161	093	.921	1.027
	2.0	1.104	.514	002	013	.824	.143	088	<b>.9</b> 37	1.027
	2.0	1.200	.513	004	008	.851	.126	082	.949	1.026
	3.6	1 577	-212	005	005	.874	.110	075	.960	1.024
	1 0	1 777	.510	006	0	.913	.083	062	.975	1.019
	4.0	1 070	.507	006	.002	.941	.060	050	.985	1.015
	4.4	2 101	.505	005	.003	.961	.043	039	.991	1.010
	4.0 5 0	2 702	.504	004	.005	.976	.029	029	.995	1.007
	5.4	2.382	.502	003	.002	.985	.019	021	.997	1.005
		2.505	.501	002	.002	.992	.012	014	.999	1,003
	6.0	2.103	.501	001	.001	.996	.008	010	.999	1.002
	0.4	2.904	.500	001	.001	.998	.005	006	1.000	1.001
	7 0	J.184	.500		.001	1.000	.003	004	1.000	1.000
	7.6	3.384	.500	0	0	1.000	.001	002	1.000	1.000
		3.584	.500	0	0	1.001	.001	001	1.000	1.000
	0.0	J. 184	.500	0	0	1.001	0	001	1.000	.999
,	0.4	5.983	.500	0	0	1.001	0	0	1.000	.999
i	8.8	4.183		· ·		1.001	0	0	1.000	

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# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

(3)  $T_{\infty}/T_{w} = 2$ ; Eu = 0.5;  $f_{w} = -0.5$ 

 $\frac{\delta^{*}\sqrt{Re}}{x} = 0.772; \ \frac{\delta_{1}\sqrt{Re}}{x} = 1.044; \ \frac{\delta_{c}\sqrt{Re}}{x} = 1.771; \ \frac{\delta_{t}\sqrt{Re}}{x} = 2.476$  $\theta$ " θ θ' f" f"' u ρu f f' η Ūœ ρœŪœ Pf' 2f' 0 0.015 0 0.230 0.477 -0.368 0 0 -0.500 0 .093 .177 .014 .046 .233 -.491 .088 .408 -.325 .2 .327 .093 .236 .011 .179 -.466 .347 -.286 .164 .4 .260 .455 .238 .007 -.250 .141 -.426 .228 .293 .6 .335 .563 -.218 .188 .239 .001 .282 .247 -.375 .8 .236 .238 -.005 .404 .653 -.188 .206 -.314 .327 1.0 -.011 .468 .729 .237 -.162 .284 .364 .171 1.2 -.245 .791 .234 -.018 .526 -.138 .331 -.169 .396 1.4 .141 -.024 .580 .842 .230 .421 .116 -.117 .377 -.087 1.6 -.030 .629 .884 -.099 .423 .224 -.001 .442 .094 1.8 .218 -.035 .673 .918 .467 .089 .459 .076 -.084 2.0 .750 .967 .202 -.044 -.058 .551 .483 .048 .278 2.4 .997 .811 .628 .183 -.050 -.040 .498 .028 .475 2.8 -.054 .860 1.014 -.026 .697 .162 .507 .015 3.2 .676 .898 1.022 -.054 .757 .140 .511 .007 -.017 3.6 .880 1.025 -.052 .927 -.010 .809 .119 .513 .001 4.0 1.084 1.025 .098 -.049 .949 -.002 -.006 .852 1.289 .512 4.4 1.023 -.044 .965 -.003 -.002 .888 .080 1.494.511 4.8 -.039 1.020 .917 .063 .977 -.004 0 1.698 .510 5.2 -.033 .985 1.016 ,939 .049 -.004 .001 5.6 1.902 .508 -.027 .991 1.013 .507 .956 .037 .001 6.0 2.105 -.004 -.022 1.011 .995 .505 -.003 .002 .969 .027 2.307 6.4 -.017 .998 1.009 -.002 .002 .978 .020 2.509 .504 6.8 1.007 -.012 1.000 .985 .014 2.711 .504 -.002 .002 7.2 1.006 -.009 1.001 .001 .990 .010 .503 -.001 7.6 2.912 -.007 1.002 1.005 .993 .006 .503 -.001 .001 3.113 8.0 -.005 1.002 1.005 .502 .001 .995 .004 3.314 0 8.4 1.005 -.003 1.003 0 .997 .003 0 .502 8.8 3.515 -.002 1.003 1.005 .997 .002 0 .502 0 9.2 3.714 -.001 1.004 .998 .001 3.915 9.6 -.001 1.004 .998 .001 4.116 10.0 **.9**99 -.001 1.004 0 10.4 4.317 1.004 .999 0 0 4.51810.8 .999 0 0 1.004 4.719 11.2

# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

(4) 
$$T_{oo}/T_w = 2$$
; Eu = 1.0;  $f_w = -0.5$ 

$$\frac{\sqrt{\text{Re}}}{x} = 0.507; \frac{\delta_1 \sqrt{\text{Re}}}{x} = 0.880; \frac{\delta_c \sqrt{\text{Re}}}{x} = 1.638; \frac{\delta_t \sqrt{\text{Re}}}{x} = 2.151$$

η	f	f'	f"	f'''	θ	θ'	θ"	u	ρu
			•					U <sub>co</sub>	$\rho_{\infty} U_{\infty}$
								Pf'	2f'
0	-0.500	0	0.642	-0.623	0	0.256	0.034	0	0
.2	488	.116	.524	547	.052	.263	.031	.122	.233
.4	455	.211	.423	470	.105	.268	.025	.233	.422
.6	405	.286	.336	396	.159	.272	.016	.332	.573
.8	342	.346	.264	328	.214	.274	.005	.420	.693
1.0	267	.393	.205	267	.268	.274	006	.498	.786
1.2	185	.429	.156	216	.323	.271	018	.567	<b>.8</b> 58
1.4	096	.456	.118	172	.377	.266	029	.628	.912
1.6	003	.477	.087	136	.429	.259	040	.681	.953
1.8	.094	.492	.063	106	.480	.251	049	.728	. 983
2.0	.193	.502	.044	082	.530	.240	057	.768	1.005
2.2	.295	.510	.030	063	.576	.228	063	.803	1.019
2.4	.397	.514	.019	048	.621	.215	068	.834	1.029
2.8	.604	.519	.005	026	.701	.187	073	.883	1.038
3.2	.812	.519	003	013	.770	.157	073	.919	1.039
3.6	1.019	.517	006	005	.827	.129	069	.945	1.035
4.0	1.226	.514	007	001	.873	.102	063	.964	1.029
4.4	1.431	.512	007	.002	.909	·.079	054	.977	1.023
4.8	1.635	.509	006	.003	.937	.059	045	.986	1.018
5.2	1.838	.507	005	.003	.957	.043	036	.992	1.013
5.6	2.040	.505	004	.003	.971	.030	028	.995	1.010
6.0	2.242	.504	003	.003	.981	.020	021	. 998	1.007
6.4	2.443	.503	002	.002	.988	.013	015	1.000	1.006
6.8	2.644	.502	001	.002	.992	.009	010	1.001	1.004
7.2	2.845	.502	0	.001	.995	.005	006	1.001	1.004
7.6	3.046	.502	0	.001	.997	.003	005	1.002	1.003
8.0	3.247	.502	0	0	.998	.002	002	1.002	1.003
8.4	3.447				.998	.001	002	1.002	
8.8	3.648			1	.999	.001	001	1.003	
9.2	3.849				.999	0	0	1.003	
9.6	4.049				.999	0	0	1.003	

# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

(5) 
$$T_{co}/T_{W} = 2$$
; Eu = 0.5;  $f_{v} = -1.0$ 

$$\frac{\delta^* \sqrt{Re}}{x} = 0.965; \frac{\delta_1 \sqrt{Re}}{x} = 1.310; \frac{\delta_c \sqrt{Re}}{x} = 2.298; \frac{\delta_t \sqrt{Re}}{x} = 3.236$$

	η	f	f'	f" -	f"'	θ	θ'	θ"	u Uso Pf'	ρu ρω Uω 2f'
1	0	-1.000	0	0.323	-0.100	0	0.106	0.046	0	0
	.2	994	.062	.302	108	.022	.116	.048	.064	.125
	.4	975	.121	.280	113	.046	.125	.050	.126	.241
	.6	946	.174	.257	117	.072	.135	.050	.187	.349
	.8	906	.223	.233	118	.100	.145	.049	.246	.447
	1.0	857	.268	.210	117	.130	.155	.046	.303	.535
	1.2	799	.307	.187	114	.162	.164	.043	.357	.615
	1.4	734	.342	.164	109	.196	.172	.038	.409	.685
	1.6	662	.373	.143	102	.231	.179	.032	.459	.747
	1.8	585	.400	.124	094	.267	.185	.026	.507	.800
	2.0	503	.423	.106	086	.304	.189	.019	.552	.846
	2.2	416	.442	.089	078	.343	.192	.011	.594	.885
	2.4	326	.459	.074	069	.381	.194	.004	.633	.917
	2.8	137	.483	.050	053	.459	.192	011	.705	.966
ļ	3.2	.060	.499	.032	039	.534	.185	023	.766	.999
I	3.6	.261	.509	.018	028	.606	.174	033	.818	1.018
	4.0	.466	.514	.009	019	.673	.159	041	.861	1.029
l	4.4	.672	.517	.002	012	.733	.142	045	.895	1.033
ļ	4.8	.879	.517	001	008	.787	.124	046	.923	1.034
ĺ	5.2	1.086	.516	004	004	.832	.105	045	.945	1.032
l	5.6	. 1.292	.514	005	002	.871	.088	043	.962	1.028
	6.0	1.497	.512	005	0	.903	.071	039	.974	1.024
l	6.4	1.701	.510	005	.001	.928	.057	034	.984	1.020
ĺ	7.2	2.108	.506	004	.002	.964	.033	024	.994	1.013
	8.0	2.512	.504	002	.002	.984	.018	015	1.000	1.008
ĺ	8.8	2.914	.502	001	.001	.994	.009	008	1.002	1.005
	9.6	3.316	.502	0	.001	.999	.004	004	1.003	1.004
	10.4	3.717	.502	0	.001	1.001	.001	002	1.004	T.003
	11.2	4.119	÷			1.002	0	001	1.004	
	TS'0	4.520				1.002	υ	0	1.004	
L			· ·		1 A A A A A A A A A A A A A A A A A A A	1			•	

# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

(6)  $T_{\infty}/T_{w} = 2$ ; Eu = 1;  $f_{w} = -1.0$ 

<b>r</b>	•	$\frac{\delta^* \sqrt{\text{Re}}}{x}$	= 0.60	8; $\frac{\delta_i \sqrt{1}}{x}$	Re = 1.1	48;	$\frac{\sqrt{\text{Re}}}{\text{x}} =$	2.228; -	$\frac{\delta_t \sqrt{Re}}{x} =$	2.891
	η	f	f'	f"	f'''	θ	θ'	θ"	u	_ρu
ĺ			1						Ŭœ	ρ <sub>α</sub> υ <sub>α</sub>
$\vdash$									Pf'	2f'
	0	-1.000	0	0.445	-0.182	0	0.105	0.064	0	0
	•2	991	.085	.406	202	.022	.119	.069	.087	.170
	.4	966	.162	.364	214	.048	.133	.073	.170	.324
	•6	927	.231	.321	218	.076	.148	.074	.248	.462
	.8	875	.291	.278	215	.107	.162	.073	.322	.581
	1.0	811	.342	.236	204	.140	.176	.069	.390	.684
	1.2	739	.385	.196	189	.177	.190	.062	.453	.770
1	1.4	658	.421	.160	170	.216	.201	.053	.512	.841
	1.6	571	•450	.128	150	.257	.211	.042	.565	.899
	1.8	478	.472	.100	129	.300	.218	.030	.614	.945
	2.0	382	.490	.077	108	.345	.223	.018	.659	.980
	2.4	181	.513	.041	073	.434	.225	007	.736	1.026
ļ	2.8	.026	.524	.017	046	.523	.217	029	.798	1.048
	3.2	.237	.528	.003	027	.607	.202	046	.849	1.056
	3.6	.448	.528	005	014	.684	.181	057	.888	1.055
	4.0	.659	.525	009	006	.752	.157	062	.919	1.050
	4.4	.868	.521	010	001	.810	.132	062	.943	1.042
	4.8	1.076	.517	010	.002	.858	.108	059	. 960	1.034
	5.2	1.282	.513	009	.003	.896	.085	053	.973	1.026
	5.6	1.486	.510	008	.004	.926	.065	046	.982	1.020
	6.0	1.690	.507	006	.004	.949	.048	038	.988	1.014
	6.4	1.892	.505	005	.003	.965	.035	030	.992	1.010
	0.8	2.094	.503	004	.003	.977	.024	023	.995	1.007
	1.2	2.295	.502	003	.002	.985	.016	017	.997	1.004
	1.6	2.496	.501	002	.002	.990	.011	012	.998	1.002
e e		2.696	.501	001	.001	.994	.007	008	.998	1.001
		2.896	.500	001	.001	.996	.004	005	.998	1.000
ì		3.096	.500	001	.001	.997	.002	003	.998	1.000
		3.296	.500	U	0	.998	.001	002	.998	.999
: יר	0.6	3.496	ĺ			.998	.001	001	.999	
Т( Т(		2.696			· .	.999	0	001	.999	
т( л 4		2.896	Í			.999	0	0	.999	
τı	0.0	4.096				.999	0	0	.999	
		1	J J	I		I	,	1		

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### TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

(7) 
$$T_{w}/T_{w} = 4$$
; Eu = 0.5;  $f_{w} = 0$   
 $\frac{\delta^{*}\sqrt{Re}}{x} = 0.272$ ;  $\frac{\delta_{1}\sqrt{Re}}{x} = 1.676$ ;  $\frac{\delta_{c}\sqrt{Re}}{x} = 2.399$ ;  $\frac{\delta_{t}\sqrt{Re}}{x} = 2.790$ 

-	f	÷1	-P <sup>11</sup>	e 11 1	A	A1	A"	11	011
11	Ţ	T	÷ .		Ū	Ŭ	Ů	Tra	<u>Om Um</u>
								Pf'	4f'
							·		
0	0	0	0.554	-1.893	· 0	0.394	-0.396	0	0
.1	.002	.047	.406	-1.160	.038	.360	297	.053	.190
.2	.009	.083	.312	773	.072	.333	234	.101	.332
.3	.019	.111	.248	545	.104	.312	192	.146	.444
.4	.031	.133	.201	400	.135	.295	161	.187	.533
.5	.045	.151	.166	304	.164	.280	139	.226	.606
.6	.061	.167	.139	237	.191	.267	122	.262	.666
.7	.078	.179	.118	189	.217	.255	108	.296	.718
.8	.097	.190	.100	153	.242	.245	097	.328	.761
.9	.116	.200	.086	126	.266	.236	089	.359	.798
1.0	.137	.208	.075	105	.289	.227	081	.388	.851
1.1	.158	.215	.065	089	.312	.220	075	.415	.859
1.2	.180	.221	.057	076	.333	.212	070	.442	.885
1.3	.202	.226	.050	065	.354	.206	066	.466	.904
1.4	.225	.231	.044	056	.374	.199	062	.490	.923
1.6	.272	.239	.034	042	.413	.187	056	.534	.954
1.8	.320	.245	.027	033	.449	.177	051	.575	.9/9
2.0	.370	.249	.021	026	.484	.167	047	.611	.998
2.2	.420	.253	.016	020	.516	.158	044	.645	1.012
2.4	.471	.256	.013	016	.547	.150	041	.676	1.024
2.6	.522	.258	.010	013	.576	.142	039	.705	1.055
2.8	.574	.260	.007	011	.604	.134	037	.731	1.040
3.0	.626	.261	.005	009	.630	.127	035	.755	1.044
3.2	.679	.262	.004	007	.655	.120	033	.777	1.048
3.4	.731	.263	.002	- 006	.678	.113	032	./9/	1.050
3.6	.784	.263	.001	005	.700	.107	030	.815	1.052
3.8	.836	.263	0	004	.721	.101	029	.852	1.055
4.0	.889	.263	0	003	.740	.096	028	.848	1.055
4.4	.994	.263	001	002	.777	.085	026	.875	1.051
4.8	1.099	.262	002	001	.809	.075	025	.898	1.040
5.2	1.204	.261	002	001	.837	.066	021	.91/	1.045
5.6	1.308	.260	003	0	.862	.058	020	.955	1 036
6.0	1.412	.259	003	0	.885	.051	010	.540	1.032
6.4	1.515	.258	005	0.	.902	039	- 014	965	1.028
6.8	1.618	.257	002	0	033	032	- 013	972	1.024
7.2	1.721	.256	002		945	028	011	.978	1.020
7.6	1.823	.200	002	0	955	.023	010	.983	1.017
8.0	1.940	.404	- 002	0	971	.016	008	.989	1.012
0.0	2.120	200	- 001		.981	.011	006	.994	1.008
3.0	2.330	252			989	.007	004	.996	1.004
10.4	0 730	250		l n	993	.005	003	.997	1.002
12.0	0 030	250			.997	.003	002	.998	1.000
12.0	0.706	200		Ŭ	999	.002	001	.998	.999
17.0	2 220	000		1	1.000	001	001	.998	.998
12.0	2 521	250	· ·		1.000	.001	0	998	.998
14.4	2 727	240			1,001	0	l o	.998	.998
10.0	2 070	. 4 3			1 001	lõ	0	.998	
I TO*O	1 0.300	1	1	1	1	· <b>· · ·</b>	1 -	1	i

**4**Y

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# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

(8)  $T_{\infty}/T_{w} = 4$ ; Eu = 1;  $f_{w} = 0$ 

NACA

$\mathbf{A}$ $\mathbf{X}$ $\mathbf{X}$	$\frac{\delta^* \sqrt{Re}}{x} = 0.08$	1; $\frac{\delta_1 \sqrt{\text{Re}}}{x} = 1$	.403; $\frac{\delta_c \sqrt{Re}}{x} =$	2.113; $\frac{\delta_t \sqrt{Re}}{x} = 2.370$
--	---------------------------------------	--	--	---

η	f	f'	f"	f'''	θ	<i>θ</i> '	θ"	u	ρυ
								U.	ρ <sub>∞</sub> U <sub>∞</sub>
								Pf'	4f'
. 0	0	0	0.722	-2.976	0	0.466	-0.554	0	0
.05	.001	.033	.594	-2.202	.023	.441	464	.035	.131
.10	.003	.060	.498	-1.683	.044	.419	397	.068	.240
.15	.007	.083	.423	-1.320	.065	.401	345	.099	.332
.20	.011	.102	.364	-1.058	.084	.385	304	.128	.410
.25	.017	.120	.316	863	.103	.370	271	.156	.478
.30	.023	.134	.277	715	.121	.357	244	.183	.537
.4	.038	.159	.216	508	.156	.335	202	.233	.635
.5	.055	.178	.173	375	.188	.317	172	.279	.713
.6	.074	.194	.140	285	.219	.301	150	.321	.775
.7	.094	.206	.115	222	.249	.286	133	.361	.826
.8	.115	.217	.095	176	.277	.274	119	.397	.868
.9	.137	.226	.079	142	.303	.262	108	.431	.903
1.0	.160	.233	.066	116	.329	.252	099	.463	.932
1.1	.184	.239	.056	096	.354	.242	092	.493	<b>.9</b> 56
1.2	.208	.244	.047	080	.378	.234	086	.521	.976
1.4	.257	.252	.033	057	.423	.217	076	.572	1.008
1.6	.308	.258	.024	041	.465	.203	068	.617	1.030
1.8	.360	.262	.016	031	.504	.190	063	.657	1.046
2.0	.413	.264	.011	023	.541	.178	058	.693	1.057
2.2	.466	.266	.007	017	.575	.167	054	.725	1.064
2.4	.519	.267	.004	013	.608	.156	051	.754	1.069
2.8	.626	.268	0	007	.666	.137	045	.804	1.072
3.2	.734	.268	- 002	004	.718	.120	041	.844	1.070
3.6	.840	.266	004	002	.763	.105	037	.876	1.065
4.0	.946	.265	004	001	.802	.091	033	.901	1.059
4.4	1.052	.263	004	0	.836	.078	030	.922	1.052
4.8	1.157	.261	004	0	.865	.067	027	.939	1.045
5.2	1.261	.260	004	.001	.889	.057	024	<b>.9</b> 53	1.039
5.6	1.365	.258	004	.001	.910	.048	021	.963	1.032
6.4	1.570	.256	003	.001	.943	.033	016	.978	1.022
7.2	1.774	.254	002	.001	<b>.9</b> 65	.022	012	<b>.9</b> 88	1.015
8.0	1.976	.252	001	.001	.979	.014	008	.994	1.010
8.8	2.178	.252	001	0	.988	.009	005	.997	1.006
9.6	2.379	.251	001	0	.994	.005	003	.999	1.004
10.4	2.579	.250	0	0	.997	.003	002	.999	1.002
11.2	2.780	.250	0	0	.999	.002	001	1.000	1.001
12.0	2.980	.250	0	0	1.000	.001	001	1.000	1.000
12.8	3.180	.250	0	0	1.000	0.	0	1.000	1.000
13.6	3.380				1.001	0	0	1.001	
14.4	3.580				1.001	0	0	1.001	

# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

(9) $T_w = 4$ ; Eu = 0.5; $f_w = -0.5$										,
8	* 100		$\delta_{\dagger}\sqrt{\text{Re}}$		$\delta_{c} \sqrt{Re}$	-	δ <sub>t N</sub> R	ē		
<u> </u>	$\frac{V}{X} = 0$	0.318;	<u></u>	= 2.155;		= 3.14	2; <u>x</u>	- = 3.6	70	
	л								· · · · · · · · · · · · · · · · · · ·	-
η	f	f'	f"	f	θ	<i>θ</i> '	θ"	u	_pu	
•			· ·					Ūω	$\rho_{\omega} U_{\infty}$	
								Pf'	4f'	
0	-0.500	0	0.345	-0.579	0	0.209	-0.056	0 · 0	0	
.2	494	.059	.252	371	.041	.199	042	.066	.236	
.4	477	.103	.191	255	.080	.191	034	.128	.412	
.6	- 453	.137	.148	183	.117	.185	028	.185	.546	
.8	- 423	.163	.116	135	.154	.180	025	.238	.652	
1.0	389	.184	.092	103	.189	.175	023	.288	.735	
12	- 350	.200	.074	080	.224	.170	022	.335	.801	
1 4	- 309	.214	.060	063	.258	.166	021	.379	.855	
16	- 265	225	.049	050	.290	.162	021	.420	.898	
1.0	- 219	233	.040	041	.322	.158	021	.459	.934	
20	- 172	241	.010	033	.354	.153	021	.496	.962	
2.0	- 123	246	026	027	.384	.149	021	.530	.986	
2.2	125	251	.020	- 022	.413	.145	021	.563	1.005	
2.4	073	-201	.011	- 016	470	.137	021	.622	1.032	
2.0	.025	.200	.009	- 011	523	128	021	.674	1.050	
3.4	.133	- 404 265		- 008	572	120	021	.720	1.060	
3.6	.239	.205	.003	005	618	111	- 021	.761	1.066	ļ
4.0	.345	.200	.002	003	.010	103	- 021	797	1.068	
4.4	.452	.407	0 001	004	.001	.105	- 021	828	1 067	
4.8	.559	.267	001	003	737	.035	- 020	.020 855	1.065	
5.2	.665	.266	002	002	.131	.000	- 019	.000	1 062	
5.6	.772	.265	002	001	.110	073	- 019	800	1 058	
6.0	.878	.264	005	001	.000	.071	- 017	916	1 053	
6.4	.983	.263	005		.041	.004	- 015	9/3	1 043	
7.2	1.193	.261	005		.010	.051	- 013	063	1 034	
8.0	1.400	.258	003	0	.909	.039	- 013	.303	1 026	
8.8	1.606	.256	002	1001	.936	.050	011		1 019	l
9.6	1.811	.255	002	.001	.956	.022	009	. 300	1.013	
10.4	2.014	.253	002	.001	.9/1	.015	007	.992	1.014	
11.2	2.216	.252	001	0	.981	.011	005	.996	1.010	
12.0	2.418	.252	001	0	.988	.007	004	.998	1.000	
12.8	2.619	.251	001	0	.993	.005	003	.999	1.004	
13.6	2.820	.251	0.	0	.996	.003	002	1.000	T.002	
14.4	3.020	.250	0	0	.998	.002	001	T.000	T.002	
15.2	3.220	:250	0	0	1.000	.001	001	1.001	1.00L	
16.0	3.420	.250	0	0	1.000	.001	0	1.001	T.OOT	
16.8	3.621				1.001	0	0	1.001		l
17.6	3.821	14. 			1.001	0	0	1.001		1

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# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

(10)  $T_{\infty}/T_{w} = 4$ ; Eu = 1;  $f_{w} = -0.5$ 

			-					NA	CA
<u>8</u> *	$\frac{\sqrt{Re}}{x} = 0$	ع _ ;000	$\frac{1}{x}$ =	<b>1.867;</b>	$\frac{\delta_c \sqrt{Re}}{x}$	= 2.932	$\frac{\delta_t \sqrt{Re}}{x}$	- = 3.23	3
ŋ	f	f'	f"	f'''	θ	θ'	θ"	u U∞ Pf'	ρυ ρ <sub>∞</sub> U <sub>∞</sub> 4f'
0	-0.500	0	0.446	-0.841	0	0.225	-0.051	0	0
.1	498	.041	.372	662	.022	.221	043	.044	.163
.2	492	.075	.313	531	.044	.217	037	.085	.300
.3	483	.104	.265	432	.066	.213	033	.124	.415
.4	472	.128	.225	356	.087	.210	030	.161	.512
• .5	458	.149	.193	297	.108	.207	028	.197	.596
.6	442	.167	.166	249	.128	.204	026	.231	.668
.7	424	.182	.143	210	.149	.202	025	.264	.729
.8	405	.196	.123	179	.169	.199	<u>°</u> .024	.295	.782
1.0	364	.217	.092	132	.208	.195	023	.352	.868
1.2	319	.233	.070	099	.246	.190	023	.405	.932
1.4	271	.245	.052	075	.284	.185	024	.454	.981
1.6	221	.254	.039	057	.321	.180	024	.499	1.017
1.8	169	.261	.029	044	.356	.175	025	.540	1.044
2.0	117	.266	.021	034	.391	.170	026	.578	1.064
2.2	063	.270	.015	027	.424	.165	026	.613	1.079
2.4	009	.272	.010	021	.457	.160	027	.645	1.089
2.8	.101	.275	.004	013	.519	.149	028	.703	1.100
3.2	.211	.276	0	008	.576	.138	028	.752	1.102
3.6	.321	.275	003	005	.629	.126	028	.793	1.099
4.0	.430	.273	004	003	.677	.115	028	.829	1.094
4.4	.540	.272	005	001	.721	.104	027	.859	1.086
4.8	.648	.270	005	0	.761	.094	026	.884	1.078
5.6	.862	.265	005	.001	.827	.074	024	.924	1.062
6.4	1.072	.262	004	.001	.879	.056	020	.952	1.047
7.2	1.280	.259	004	.001	.918	.041	017	.971	1.034
8.0	1.486	.256	003	.001	.946	.029	013	.983	1.025
8.8	,1.690	.254	- :002	.001	.965	.020	010	.991	1.018
9.6	1.893	.253	001	.001	.979	.013	007	.996	1.012
10.4	2.096	.252	001	0	.987	.009	005	1.000	1.009
11.2	2.297	.252	0	0	.993	.005	004	1.002	1.007
12.0	2.499	.252	0	0	.996	.003	002	1.003	1.006
12.8	2.700	.252	0	0	.998	.002	001	1.005	1.006
13.6	2.901				.999	.001	001	1.005	
14.4	3.102				.999	0	001	1.006	•
15.2	3.304				.999	0	0	1.006	
16.0	3.505				.999	0	0	1.006	

1.006

# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Continued

(11) 
$$T_{\infty}/T_{w} = 4$$
; Eu = 0.5;  $f_{w} = -1.0$ 

$$\frac{\delta^* \sqrt{Re}}{x} = 0.402; \frac{\delta_1 \sqrt{Re}}{x} = 2.882; \frac{\delta_c \sqrt{Re}}{x} = 4.255; \frac{\delta_t \sqrt{Re}}{x} = 4.954$$

η	f	f'	f"	f'''	θ	θ'	θ"	u	ρυ
								Ūα	$\rho_{\infty} U_{\infty}$
								Pf'	4f'
0	-1.000	0	0.196	-0.102	0	0.078	0.025	0	0
.4	985	.070	.157	093	.033	.088	.025	.077	.282
.8	946	.126	.122	081	.070	.098	.023	.153	.504
1.2	886	.169	.092	067	.111	.106	.019	.225	.675
1.6	812	.201	.069	053	.155	.113	.015	.294	.803
2.0	727	.224	.050	041	.202	.118	.011	.360	.897
2.4	634	.241	.036	031	.250	.122	.006	.422	.965
2.8	535	.253	.025	023	.299	.124	.002	.480	1.013
3.2	432	.262	.017	017	.348	.124	002	.535	1.046
3.6	326	.267	.011	013	.398	.122	005	.586	1.068
4.0	218	.270	.006	009	.446	.120	008	.633	1.082
4.4	<b></b> 110	.272	.003	007	.494	.116	010	.676	1.089
4.8	001	.273	.001	005	.539	.112	012	.715	1.092
5.2	.109	.273	001	004	.583	.107	014	.751	1.092
5.6	.218	.272	002	002	.624	.101	015	.783	1.090
6.0	.327	.272	003	002	.664	.095	016	.812	1.086
6.4	.435	.270	003	001	.700	.088	016	.838	1.081
6.8	.543	.269	004	001	.734	.082	016	.861	1.076
7.2	.650	.267	004	0	.766	.076	016	.882	1.070
8.0	.863	.264	004	0	.821	.063	015	.916	1.058
8.8	1.073	.261	004	0	.867	.051	014	.941	1.046
9.6	1.281	.259	003	.001	.903	.040	012	. 960	1.035
10.4	1.488	.256	003	.001	.932	.031	011	.973	1.026
11.2	1.692	.255	002	.001	.953	.023	009	.983	1.018
12.0	1.895	.253	002	.001	.969	.017	007	.989	1.012
12.8	2.097	.252	001	0 /	.980	.012	005	.993	1.008
13.6	2.298	.251	001	0	. 988	.008	004	.996	1.005
14.4	2.499	.251	001	0	.993	.005	003	.997	1.002
15.2	2.700	.250	0	0	.997	.003	002	.999	1.001
16.0	2.900	.250	0	0	.999	.002	001	.999	1.000
16.8	3.099	.250	0	0	1.001	.001	001	.999	.999
17.6	3.299	.250	0	0	1.002	.001	001	1.000	.998
18.4	3.499		·	}	1.002	0		1.000	
19.2	3.698				1.002	0	0	T.000	
20.0	3.898	[			1.002	0	0	T.000	

# TABLE I - VELOCITY, WEIGHT-FLOW, AND TEMPERATURE DISTRIBUTIONS IN LAMINAR BOUNDARY LAYER WITH VARIABLE FLUID PROPERTIES, PRESSURE GRADIENT IN MAIN STREAM, AND FLOW THROUGH POROUS WALL - Concluded

δ <sup>*</sup> <sub>A</sub> /Re	$\delta_{1}\sqrt{\text{Re}}$	$\delta_{c}\sqrt{Re}$	$\delta_{\pm \lambda}/\text{Re}$
$\frac{1}{x}$ =	$-0.117; \frac{1}{x} =$	2.602; $\frac{1}{2}$ = 4.147;	$\frac{1}{3}$ = 4.448

(12)  $T_{co}/T_{w} = 4$ ; Eu = 1;  $f_{w}$ 

ŋ	f	f'	f"	f '"	θ	θ'	θ"	u U∞ Pf'	$\frac{\rho u}{\rho_{\infty} U_{\infty}}$ 4f'
$\begin{array}{c} 0\\ .2\\ .4\\ .6\\ .8\\ 1.0\\ 1.2\\ 1.4\\ 1.6\\ 2.8\\ 3.2\\ 3.6\\ 4.0\\ 4.4\\ 5.2\\ 5.6\\ 6.0\\ 4.4\\ 5.2\\ 5.6\\ 6.4\\ 6.8\\ 7.2\\ 8.0\\ 8.8\\ 9.6\\ 10.4\\ 11.2\\ 12.0\\ 12.8\\ 13.6\\ 14.4\\ 15.2\\ 16.0\\ \end{array}$	-1.000 995 981 958 929 894 853 809 761 711 658 549 435 320 205 089 .026 .140 .252 .364 .474 .583 .691 .798 1.009 1.216 1.422 1.625 1.828 2.029 2.229 2.429 2.629 2.828 3.027	0 .049 .093 .131 .163 .190 .213 .231 .246 .258 .267 .280 .289 .289 .289 .289 .289 .289 .289 .289	0.261 .232 .203 .175 .148 .124 .102 .083 .066 .052 .041 .023 .011 .003 002 005 006 007 008 007 008 007 006 007 006 007 006 007 007 006 005 004 003 002 001 001 001 0 0 0	-0.142 146 144 138 128 128 102 089 076 064 054 054 054 024 015 010 006 003 001 .001 .001 .001 .001 .001 .001 .00	0 .015 .032 .050 .070 .092 .114 .138 .164 .190 .217 .272 .329 .386 .443 .550 .600 .647 .691 .731 .767 .800 .830 .830 .879 .917 .945 .965 .978 .987 .993 .995 .998 .999 1.000	0.073 .080 .088 .096 .103 .110 .117 .123 .128 .133 .128 .133 .136 .141 .143 .143 .143 .143 .143 .143 .140 .135 .129 .121 .113 .105 .096 .087 .078 .070 .054 .041 .029 .020 .014 .009 .005 .003 .002 .001 .001	0.037 .039 .039 .038 .037 .034 .028 .024 .020 .016 .009 .011 005 010 014 017 019 021 022 022 022 022 021 022 022 022 022 022 022 022 022 021 021 003 005 003 002 001 001 002	Pf' 0 .052 .102 .150 .197 .242 .286 .327 .367 .404 .441 .508 .569 .624 .673 .718 .757 .793 .824 .851 .875 .896 .913 .928 .952 .969 .980 .987 .995 .995 .995 .995 .995	4f' 0 .197 .371 .522 .652 .760 .850 .924 .984 1.031 1.068 1.118 1.145 1.156 1.157 1.152 1.143 1.152 1.120 1.085 1.074 1.064 1.047 1.033 1.022 1.014 1.000 .998 .997 .996 .995
17.6	3.425 3.624				1.000 1.000	0	000	.995 .995	-

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fw	$\frac{T_{\infty}}{T_{W}}$	Eu	Nu √Re	C <sub>f,W</sub> 2√Re	$\delta^* \frac{\sqrt{Re}}{x}$	$\delta_1 \frac{\sqrt{Re}}{x}$	$\delta_c \frac{\sqrt{Re}}{x}$	$\delta_t \frac{\sqrt{Re}}{x}$
			θ'w	f <sub>w</sub> "				
0	1	-0.0904 0868 0826 0741 0654 0476 0 .50 1.00	0.1993 .2215 .2310 .2435 .2528 .2673 .2927 .4162 .4958	0 .0581 .0870 .1296 .1637 .2202 .3320 .8997 1.2326	3.498 2.971 2.763 2.510 2.336 2.092 1.721 .855 .648	0.868 .852 .838 .812 .788 .747 .662 .377 .290	0.626 .692 .720 .752 .773 .801 .835 .792 .708	2.737 2.505 2.415 2.307 2.233 2.128 1.959 1.407 1.187
	2	-0.1178 09 05 0 .50 1.00	0.1890 .2522 .2756 .2944 .4020 .4760	0 .1634 .2434 .3125 .6836 .9090	4.582 2.430 1.882 1.537 .622 .433	1.663 1.501 1.378 1.271 .838 .693	1.076 1.408 1.478 1.495 1.372 1.215	3.721 2.928 2.713 2.547 1.904 1.611
	4	-0.1351 09 05 0 .50 1.00	0.1794 .2642 .2790 .2952 .3940 .4662	0 .1934 .2397 .2874 .5541 .7220	6.950 2.297 1.810 1.438 .272 .081	3.109 2.719 2.582 2.455 1.676 1.403	1.834 2.595 2.651 2.663 2.399 2.113	5.596 4.053 3.863 3.678 2.790 2.370
$-\frac{1}{2}$	1	-0.0418	0.1029	0	4.272	0.954	0.807	3.677
		0 .50 1.00	.1661 .2594 .2934	.1648 .6974 .9692	2.459 1.033 .783	.827 .443 .345	.974 .994 .918	2.650 1.776 1.530
	2	0 .50 1.00	0.1602 .2304 .2560	0.1476 .4770 .6415	2.381 .772 .507	1.605 1.044 .880	1.778 1.771 1.638	3.456 2.476 2.151
	4	-0.0644 0 .50 1.000	0.0796 .1506 .2088 .2252	0 .1263 .3448 .4465	7.219 2.460 .318 0	3.485 3.100 2.155 1.867	2.620 3.237 3.142 2.932	7.329 5.012 3.670 3.233
-1	1	-0.0072 0 .05 .15 .50 1.00	0.0251 .0516 .0881 .1129 .1392 .1457	0 .0355 .1410 .2703 .5344 .7565	6.398 4.396 2.796 2.008 1.252 .945	1.116 1.073 .911 .750 .525 .405	1.072 1.150 1.241 1.280 1.270 1.208	5.997 4.414 3.317 2.806 2.274 1.995
· .	2	0 .05 .15 .50 1.00	0.0406 .0692 .0886 .1062 .1052	0.0242 .0892 .1678 .3228 .4445	4.931 2.985 1.989 .965 .608	2.114 1.911 1.704 1.310 1.148	2.167 2.299 2.343 2.298 2.228	5.941 4.625 3.959 3.236 2.891
	4	0 .05 .15 .50 1.00	0.0262 .0510 .0656 .0780 .0726	0.0125 .0542 .1030 .1959 .2611	6.409 3.405 2.040 .402 117	4.160 3.881 3.620 2.882 2.602	4.002 4.247 4.309 4.256 4.147	9.007 6.995 6.054 4.954 4.448
0	1	-0.06 -0.04 0 .50 1.00	0.2062 .2554 .2900 .4412 .5298	0 .1735 .3462 1.2754 1.8000	3.043 2.309 1.898 .980 .768	0.442 .406 .347 .116 .065	0.348 .420 .457 .460 .415	2.215 1.846 1.645 1.109 .929
	$\frac{1}{4}$	0	0.2884	0.3556	2.031	0.182	0.246	1.482
		1 2 1	.4801 .5812	1.9299 2.7842	1.033 .820	031 064	.270 .246	.919 .764

TABLE II - SUMMARY OF HEAT-TRANSFER AND FRICTION PARAMETERS AND BOUNDARY-LAYER THICKNESSES<sup>1</sup>

 $\ensuremath{^1}\xspace{\textsc{The}}$  last figure in the boundary-layer thicknesses is doubtful.











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Figure 3. - Nondimensional displacement thickness with and without porous flow.



(b) Stream-to-wall temperature ratio, 4.

Figure 3. - Concluded. Nondimensional displacement thickness with and without porous flow.

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<ol> <li>Flow, Laminar         <ol> <li>Flow, Laminar</li></ol></li></ol>	NACA	<ol> <li>Flow, Laminar         <ol> <li>Flow, Laminar</li></ol></li></ol>	NACA
NACA TN 2800 National Advisory Committee for Aeronautics. SOLUTTONS OF LAMINAR-BOUNDARY-LAYER EQUATIONS WHICH RESULT IN SPECIFIC-WEIGHT- FLOW PROFILES LOCALLY EXCEDING FREE- STREAM VALUES. W. Byron Brown and John N. B. Livingood. September 1952. 36p. diagrs., 2 tabs. (NACA TN 2800) Revised solutions of the laminar-boundary-layer equations for cases which involved cooling at the wall combined with large pressure gradients in the main stream produced specific-weight-flow profiles which locally exceeded free-stream values. Heat-transfer and friction coefficients, boundary-layer thicknesses, and velocity, temperature, and specific-weight-flow distributions resulting from the revised solutions are presented for Euler numbers of 0.5 and 1, stream- to-wall temperature ratios of 2 and 4, and cooling-	Copies obtainable from NACA, Washington (over)	NACA TN 2800 National Advisory Committee for Aeronautics. SOLUTIONS OF LAMINAR-BOUNDARY-LAYER EQUATIONS WHICH RESULT IN SPECIFIC-WEIGHT- FLOW PROFILES LOCALLY EXCEEDING FREE- STREAM VALUES. W. Byron Brown and John N. B. Livingood. September 1952. 36p. diagrs., 2 tabs. (NACA TN 2800) Revised solutions of the laminar-boundary-layer equations for cases which involved cooling at the wall combined with large pressure gradients in the main stream produced specific-weight-flow profiles which locally exceeded free-stream values. Heat-transfer and friction coefficients, boundary-layer thicknesses, and velocity, temperature, and specific-weight-flow distributions resulting from the revised solutions are presented for Euler numbers of 0.5 and 1, stream- to-wall temperature ratios of 2 and 4, and cooling-	Copies obtainable from NACA, Washington (over)
<ol> <li>Flow, Laminar         <ol> <li>Flow, Laminar</li></ol></li></ol>	NACA	<ol> <li>Flow, Laminar         <ul> <li>(1.1.3.1)</li> <li>Heat Transfer, Aero-dynamic</li> <li>(1.1.4.2)</li> <li>Turbine Cooling (3.7:2)</li> <li>Turbine Cooling (3.7:2)</li> <li>Heat Transfer Theory and Experiment (3.9.1)</li> <li>Brown, W. Byron</li> <li>Livingood, John N. B.</li> <li>II. NACA TN 2800</li> </ul> </li> </ol>	NACA
NACA TN 2800 National Advisory Committee for Aeronautics. SOLUTIONS OF LAMINAR-BOUNDARY-LAYER EQUATIONS OF LAMINAR-BOUNDARY-LAYER EQUATIONS OF LAMINAR-BOUNDARY-LAYER EQUATIONS OF LAMINAR-BOUNDARY-LAYER FLOW PROFILES LOCALLY EXCEEDING FREE- STREAM VALUES. W. Byron Brown and John N. B. Livingood. September 1952. 36p. diagrs., 2 tabs. (NACA TN 2800) Revised solutions of the laminar-boundary-layer equations for cases which involved cooling at the wall combined with large pressure gradients in the main stream produced specific-weight-flow profiles which locally temperature, and specific-weight-flow distributions resulting from the revised solutions are presented for Euler numbers of 0.5 and 1, stream- to-wall temperature ratios of 2 and 4, and cooling-	Copies obtainable from NACA, Washington (over)	NACA TN 2800 National Advisory Committee for Aeronautics. SOLUTIONS OF LAMINAR-BOUNDARY-LAYER EQUATIONS WHICH RESULT IN SPECIFIC-WEIGHT- FLOW PROFILES LOCALLY EXCEEDING FREE- STREAM VALUES. W. Byron Brown and John N. B. Livingood. September 1952. 36p. diagrs., 2 tabs. (NACA TN 2800) Revised solutions of the laminar-boundary-layer equations for cases which involved cooling at the wall combined with large pressure gradients in the main stream produced specific-weight-flow profiles which locally exceeded free-stream values. Heat-transfer and friction coefficients, boundary-layer thicknesses, and velocity, temperature, and specific-weight-flow distributions resulting from the revised solutions are presented for Euler numbers of 0.5 and 1, stream- to-wall temperature ratios of 2 and 4, and cooling-	Copies obtainable from NACA, Washington

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air flow rates through porous walls designated by flow parameters of 0, -0.5, and -1.

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