A TRIDENT SCHOLAR PROJECT REPORT

NO. 236

"DYNAMIC ANALYSIS OF BEACH RENOURISHMENT"

UNITED STATES NAVAL ACADEMY
ANNAPOLIS, MARYLAND

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (*Leave Blank*)

2. REPORT DATE

1996

3. REPORT TYPE AND DATES COVERED

4. TITLE AND SUBTITLE

Dynamic analysis of beach renourishment

5. FUNDING NUMBERS

6. AUTHOR(S)

Beckette, Michael C.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

United States Naval Academy
Annapolis, MD 21402

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

USNA Trident Scholar report; no. 236 (1996)

11. SUPPLEMENTARY NOTES

Accepted by the U.S.N.A. Trident Scholar Committee

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

UL

13. ABSTRACT (*Maximum 200 words*)

This project attempts to develop a spreadsheet model that analyzes the dynamic evolution of a beach renourishment project to various natural forces. Beach nourishment is a non-structural alternative used to control coastal erosion. It provides additional sand to an area so that the beach may continue to act as a storm energy absorption area that protects the property behind it from storm-induced wave attack. Because beach nourishment is not a solid structure, it reacts to the environmental forces placed on it. This project develops a probabilistic model that takes into account the following forces; background erosion that removes sand from the system, longshore diffusion or spreading of the project, and storm-induced erosion. Because the beach system is constantly eroding and changing, it is necessary to periodically renourish a project. This in effect returns the project to its original dimensions so that strom protection will not be lost. The dynamic model has been developed so that it can determine the renourishment interval necessary for a project. The results from this dynamic model can then be compared to the traditional static method used by the Army Corps of Engineers in order to analyze the accuracy of present designs.

14. SUBJECT TERMS

Erosion Beach Corps of Engineers Coastal

15. NUMBER OF PAGES

120

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

U.S.N.A. --- Trident Scholar project report; no. 236 (1996)

"DYNAMIC ANALYSIS OF BEACH RENOURISHMENT"

by

Midshipman Michael C. Beckette, Class of 1996
United States Naval Academy
Annapolis, Maryland

(signature)

Certification of Advisors Approval

Professor Bruce Johnson
Department of Naval Architecture, Ocean, and Marine Engineering

Bruce Johnson
(signature)

6 May 1996
(date)

Associate Professor David L. Kriebel
Department of Naval Architecture, Ocean, and Marine Engineering

(signature)

6 May 1996
(date)

Acceptance of Trident Scholar Committee

Professore Joyce E. Shade
Chair, Trident Scholar Committee

J. E. Shade
(signature)

7 May 1996
(date)

USNA-1531-2

ABSTRACT

This project attempts to develop a spreadsheet model that analyzes the dynamic evolution of a beach renourishment project to various natural forces.

Beach nourishment is a non-structural alternative used to control coastal erosion. It provides additional sand to an area so that the beach may continue to act as a storm energy absorption area that protects the property behind it from storm-induced wave attack. Because beach nourishment is not a solid structure, it reacts to the environmental forces placed on it. This project develops a probabilistic model that takes into account the following forces; background erosion that removes sand from the system, longshore diffusion or spreading of the project, and storm-induced erosion.

Because the beach system is constantly eroding and changing, it is necessary to periodically renourish a project. This in effect returns the project to its original dimensions so that storm protection will not be lost. The dynamic model has been developed so that it can determine the renourishment interval necessary for a project.

The results from this dynamic model can then be compared to the traditional static method used by the Army Corps of Engineers in order to analyze the accuracy of present designs.

KEYWORDS

Erosion
Beach
Corps of Engineers
Coastal

TABLE OF CONTENTS

LIST OF FIGURES AND TABLES

INTRODUCTION

Powerful storms as well as everyday wave action continually erode many beaches and coastlines throughout the United States. These beaches are important to oceanfront communities because they perform the important function of absorbing storm wave energy. In this way the beach absorbs the intense force of the storm and protects the community behind it from much of the storm's fury. However, as the beach continues to erode due to these storms, it provides less and less protection. As the beach width narrows, the storm waters may reach the landward property along the coast and produce extensive damage. In response to this beach erosion a number of different options of shoreline protection have been attempted in the United States.

Most of the early alternatives used in shore protection prior to the 1950's were permanent structures that altered the physical appearance and natural order of the coastline (Sudar et al, 1995). These include groins and jetties which extend perpendicular to shore and trap sand as it moves in the longshore direction. Offshore or detached breakwaters are another alternative that has been used. Breakwaters are designed to limit the degree of wave energy that hits the beach. Thus, the breakwater absorbs part of the storm's energy and creates a more sheltered area for the shoreline behind. Finally, many communities have installed seawalls or revetments that provide a permanent hardened shoreline that prevents further erosion behind the structure. These permanent structures have their disadvantages however. Even though they provide different degrees of storm protection, they alter the physical appearance of the shoreline, add possible recreation and navigation hazards to the littoral waters, and may cause additional erosion in certain areas near the structures. To counter this erosion many communities have opted for dynamic, natural storm protection in the form of beach nourishment or beachfill instead.

Beach nourishment is the process of artificially placing sand onto a beach in order to counter the effects of erosion. The additional sand works to provide storm protection as well as to increase the length and width of the recreational beach area. One advantage of beach nourishment is that it provides shore protection without compromising the natural beauty of an area through the construction of large manmade structures. The major disadvantage of beach nourishment is that the area continues to constantly erode. Therefore, simply placing sand onto a beach does not fully solve an erosion problem. An area must be periodically renourished in order to maintain a desired level of storm protection. The frequency of this renourishment or replenishment is what drives the life cycle cost of a beach nourishment project. This need to continually maintain the project, including frequency and cost, is the one of the primary subjects of this study.

There has been considerable debate among experts in this field as to whether or not beach nourishment, as it has been undertaken to date, is an economically viable option for shore control. Some scholars, such as Professor Orrin Pilkey of Duke University, argue that the methods used for the estimation of the volume requirements for replenished beaches are not adequate. He states that at the present time the Army Corps of Engineers under-designs beach restoration projects and at the same time overestimates the lifespan of these projects (Pilkey, 1988). His major contention is that nourished beaches must be renourished more frequently than originally predicted. The standard practice is to calculate the annual volume required for periodic renourishment as being equal to the "historic annual erosion losses from the natural beach" (Pilkey, 1988). He believes that beaches which have been artificially nourished erode at a much faster rate than the historical average for the region. Pilkey bases his contentions on his own comprehensive survey of previous beach nourishment projects. He has found that the majority of large projects studied lose sand at a faster rate than originally predicted.

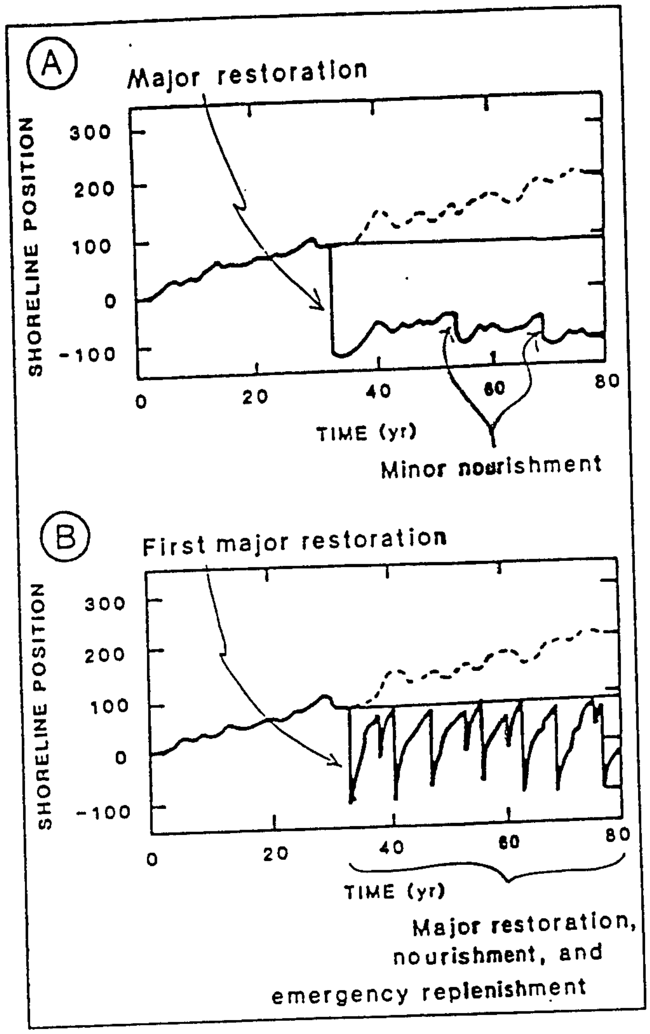
In order to solve this problem and accurately predict the volume of sand necessary for a project, Pilkey has devised a model which is based on the geographic area in which a project is located. This model does not base long-term nourishment requirements on the historical average erosion rate for a region. Instead, these maintenance requirements are based on an assumed lifespan for a region that is determined from "informal information" taken from his study of previous projects in a region. He thus developed the following equation for the volume of sand required for renourishment during the lifespan of a project:

$$V_m = \left(\frac{M}{n}\right) V_i$$

(Pilkey, 1988) (1)

V_m = the total volume of sand required to maintain a given length
n = assumed interval of required major restoration
    n = 9 years for Florida
    n = 3 years for New Jersey
    n = 5 years for East Coast barrier islands
M = design life of project
V_i = volume of initial fill placed on beach

Figure 1 shows an example of the disparity between the current predicted response of a project and Pilkey's model. Figure 1a shows how the current beach is eroding at a



(Pilkey, 1988)

Figure 1. Comparison of Pilkey's ideas on beach nourishment to the Army Corps'

historical rate as shown by the bold line at the left and continued by the dashed line. The top figure shows how the Corps of Engineers believes that the beach will continue to erode at the historical rate. Therefore, it is projected that only minor periodic renourishments will be necessary. In Figure 1b, Pilkey illustrates how he believes the beach to react. He shows a much more rapid loss of sand that requires major restoration at frequent intervals that correspond to the *n* parameter developed in Equation 1. This second representation would result in a considerably more expensive project than the original prediction.

Therefore, it is Pilkey's contention that beaches need to be nourished more often than earlier predictions in order to provide the protection that is desired. The major drawback of Pilkey's argument, however, is that he lacks sufficient documentation of project performance that can be verified by others studying the problem of beach renourishment. Despite this drawback, one of the goals of this project is to investigate Pilkey's belief that renourished beaches erode more quickly than natural beaches and attempt to test his ideas on the frequency of renourishment required for a project.

Not all parties in this debate side with Pilkey's ideas. The largest group in opposition to Pilkey is the professional coastal engineers such as Dr. James Houston of the Army Corps of Engineer's Coastal Engineering Research Center. He contends that Pilkey's statements do not accurately reflect the true response of beaches to environmental forces. The major point that Houston disputes is the faster erosion of replenished beaches as compared to natural beaches. Houston questions the validity of Pilkey's findings in this area mostly due to the lack of "fundamental documentation," and "lack of quality control" in how Pilkey defines beach loss and other parameters associated with monitoring the process of beach nourishment projects (Houston, 1991).

Houston refers to Pilkey's "comprehensive study" of beach fill projects as lacking "scientific rigor." Houston has found numerous errors in Pilkey's study that resulted in incorrect measurements of the erosion rates of projects. For example, errors such as confusing fill locations, use of incorrect conversion factors, and confusion between sand transport rates and erosion rates were found by Houston (Houston, 1991). He believes that these errors have led to Pilkey overestimating the erosion of beachfill projects as compared to unnourished beaches.

The other major discrepancy that Houston found was Pilkey's lack of an adequate definition of beach loss. This problem resulted in Pilkey defining projects as "lost" when in fact they were simply adjusting into their designed positions in a manner predicted by design engineers. Pilkey defined a project as lost when 50% of the visible sand (above mean sea level) was eroded. However, as Houston points out, and as will be explained in detail later, "sand ... on the subaerial portion of the profile during construction ... is allowed to rework over the first year and distribute sand along the complete profile" (Houston, 1991). For ease of construction, the initial profile as built by a contractor is not the actual design that has been planned by the designers. The original loss of beach width after construction is not in reality a true loss of sand from the beach system, but rather a planned out occurrence that results in the beach molding from an artificially steep construction profile to its natural profile. Houston believes that Pilkey fails to note this major design condition, and therefore greatly underestimates the performance of beach nourishment projects.

It is this disparity of ideas that forms the basis of this project. Specifically, this study focuses on developing a computer spreadsheet model for predicting the erosion and life-span of a beachfill project starting from the initial post-construction beach profile and extending until the first major renourishment of the project. In general, this model contains three erosion forcing mechanisms, including: (1) the long-term chronic erosion trend that occurs on beaches for which nourishment would be used, (2) the longshore spreading of a nourishment project that occurs as it molds to the incoming wave energy, (3) and the probabilistic effects of major storms suddenly eroding the beach. In addition to modeling erosion, this spreadsheet model also treats the economic aspects of the beach replenishment problem. The economic study will attempt to determine the accuracy of present cost modeling efforts when compared to a more probabilistic based method that accounts for storm occurrences.

This spreadsheet model will then be applied to selected hypothetical beachfill projects in order to determine the expected renourishment interval or project life-span. This model can be utilized to test previously constructed projects as well as to determine the effectiveness of new or proposed designs. From these results it will be possible to look at the ideas of both Pilkey and Houston and make a determination as to the validity of their claims. This model represents an unbiased look at their debate which utilizes documented scientific equations and theories so that its results can be verified and studied further.

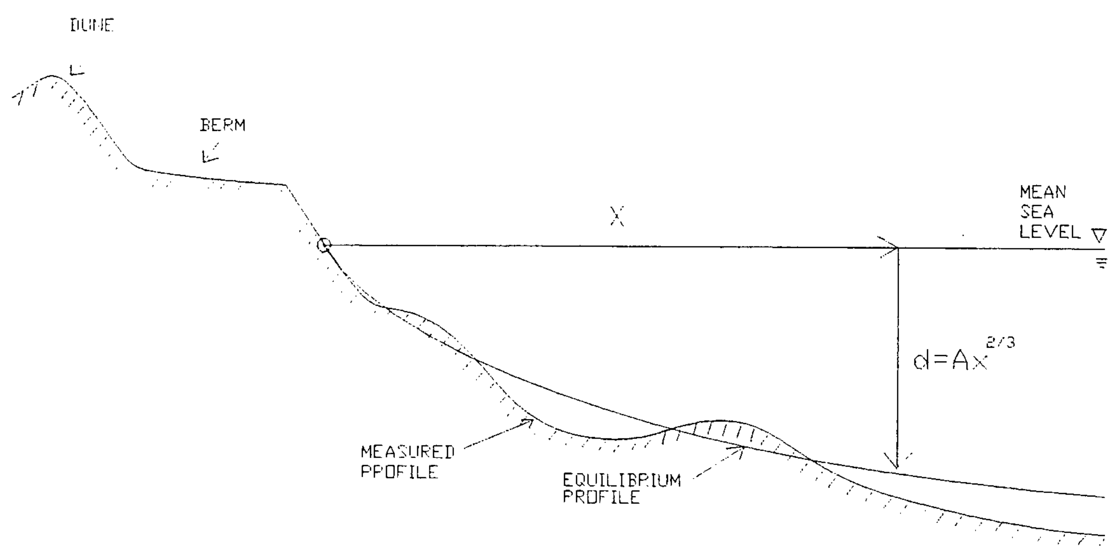
EROSION PROCESSES FOR BEACH NOURISHMENT

Beach Equilibrium Profiles

Visiting various beaches across the country makes it clear that they do not all have the same shape. The slope and general appearance of the visible beach varies from location to location. Offshore these differences continue as sandbars and depressions are found in different areas. However, beach profiles do have some general similarities. They all possess a general concave-upward shape when the sandbars and troughs are smoothed out. They also have bottom slopes that are steepest near the shoreline and which are milder offshore. In order to analyze the response of a general beach to different storms or sea-level changes it is first necessary to develop a model of the general shape of a beach that would be useful in any locale.

Professor Robert Dean of the University of Florida developed such a model in 1977 with the "Equilibrium Beach Profile" (Kriebel and Dean, 1993). These equilibrium profiles represent the smoothed shape that a beach will take after it has been allowed to mold to the wave environment which acts on it. Figure 2 shows an example of this equilibrium profile compared to the actual shape of the beach. The beach system begins inland with the beach's dune. The dune is roughly trapezoidal in shape and is formed naturally as vegetation traps wind blown sand. The dune provides the final protection from storms by protecting inland areas from raised water levels and by providing a reservoir of sand that is only eroded in extreme storms.

Seaward from the dune is the backshore and the berm. The berm is characterized as



(Kriebel, 1995)

Figure 2. Comparison of equilibrium profile to actual profile

the generally flat, wide portion of the beach where most recreational beach activities take place. This section is the part of a beach profile is raised from sea level to the upper limit of normal wave uprush but not to the extent of the dune. It is also the area that is most susceptible to storm erosion.

The basis of Dean's theory for an equilibrium shape of the beach begins seaward of the berm and divides the beach profile into two main sections. The first is a linearly sloping beach face that extends from the berm crest to the point just below the shoreline elevation. The second offshore portion is an equilibrium form given by the power law curve

$$d = A x^{\frac{2}{3}}$$

(Kriebel , 1995) (2)

d = water depth
x = distance offshore
A = sediment size parameter

The steepness parameter A correlates primarily with sediment grain size or sediment fall velocity (ω) in the equation:

$$A = 2.25 \left(\frac{\omega^2}{g}\right)$$

(Kriebel , 1995) (3)

such that larger sediment sizes have larger A values. Therefore, the larger sediment (large A) produces a beach with a steeper overall slope than a profile that is composed of smaller sized sediment as shown in Figure 3.

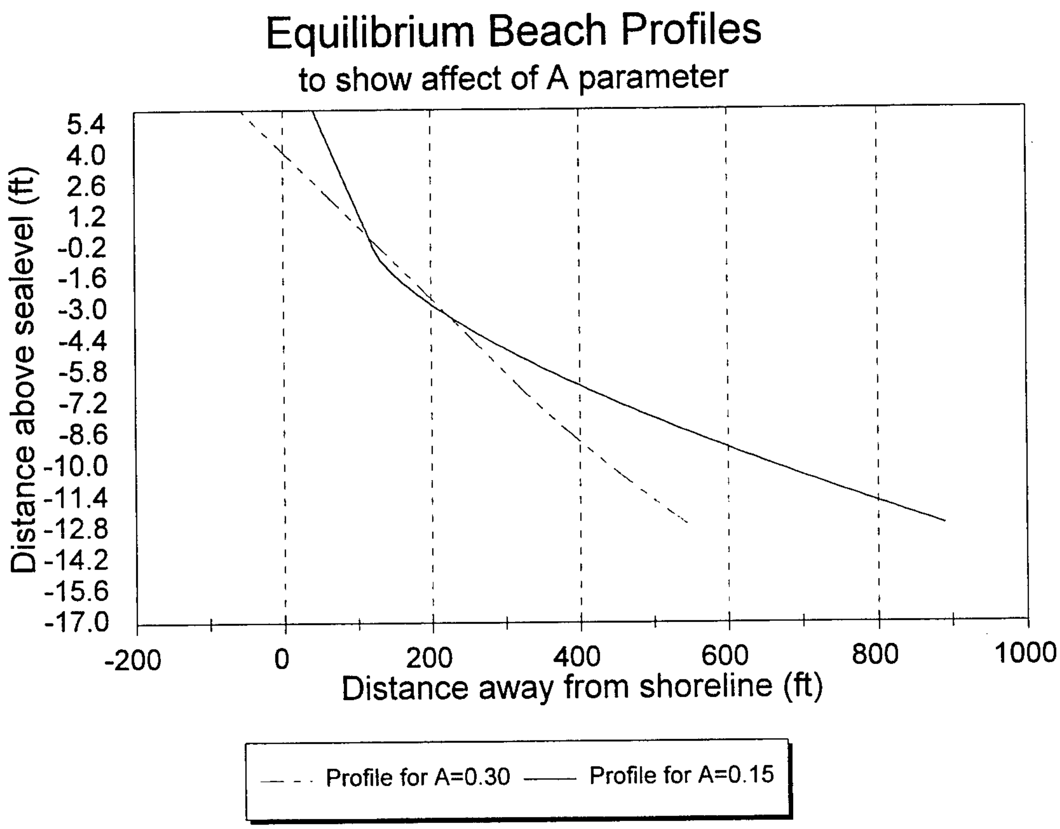


Figure 3. Comparison of sediment size on profile shape

The transition from a linear beach face to a concave equilibrium profile is at the point where the slopes of the two sections are tangent to one another at a depth denoted as d_T. Dean's equilibrium profile may then be summarized as:

$x = d/m \qquad \text{for } d<d_T$

$x = x_0 + (d/A)^{3/2} \qquad \text{for } d>d_T$

(4)

m= the linear beach slope
x_0= the distance from the still-water shoreline to the virtual origin of the power law portion of the profile
$x_0=d_T/3m$ (5)
$d_T=4A^3/9m^2$ (6)

This concave portion of the profile continues seaward until it reaches a depth known as the depth of closure, d_c. The depth of closure is the seaward limit of significant beach profile change over the long term. Thus, in normal conditions, this is the depth at which the beach profile will finally reach equilibrium. The depth of closure can be determined using methods given in the *Shore Protection Manual* of the Army Corps of Engineers (1984). These methods may be expressed approximately as:

$$d_c \cong 9 * (H_s)$$

(Army Corps, 1984) (7)

d_c = depth of closure
H_s = mean annual significant wave height

This equilibrium beach profile method makes it possible to plot the shape, and eventually analyze the response of beaches in any location by considering the sediment size in the area and upland parameters such as initial slope, berm, and dune measurements that can be taken easily.

Profile Modification by Beach Nourishment

Beach nourishment attempts to advance the entire beach profile seaward by some predetermined amount. Figure 4 provides an example of how the beachfill moves the width of the beach seaward in the same shape as the original profile. However, when a beach nourishment project is undertaken, it is impossible to place sand onto a beach so that it matches the equilibrium shape to a significant degree. In a typical project, offshore sand is deposited with hydraulic pumping. This sand is either brought in from another area or dredged from sand deposits farther offshore. This sand-water slurry is pumped through a pipeline and then discharged into the prescribed area of the project. This crude placement method does not allow for accurate placement of sand. The regions such as the dune and berm can be formed by bulldozing the sand that has been placed into the general shaped desired. However, the beach profile below mean sea level will still only be a rough estimate of its final shape because this area is subject to wave action that will shape its final position.

In most beach nourishment projects, sand is placed offshore in a trapezoidal shape with a linear slope on a construction template as shown in Figure 4. Within the first three to six months of construction, the wave climate in an area generally molds the profile from its initial linear construction shape to its equilibrium shape. Once this "equilibration" phase has taken place, the beach profile fits into Dean's model. In order for the beach profile to conform to its natural shape, the construction berm must erode and the eroded sand must be deposited into a concave shape that extends until the depth of closure. It is possible to determine the new berm width and shape of a project through a conservation of mass of the sand in the beach profile. The volume of sand that has been eroded from the construction

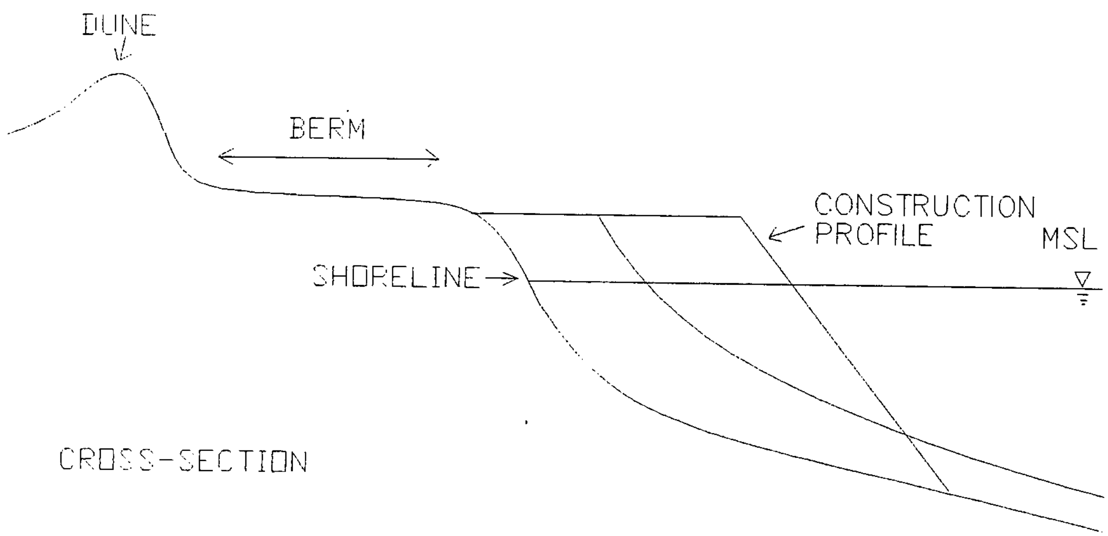
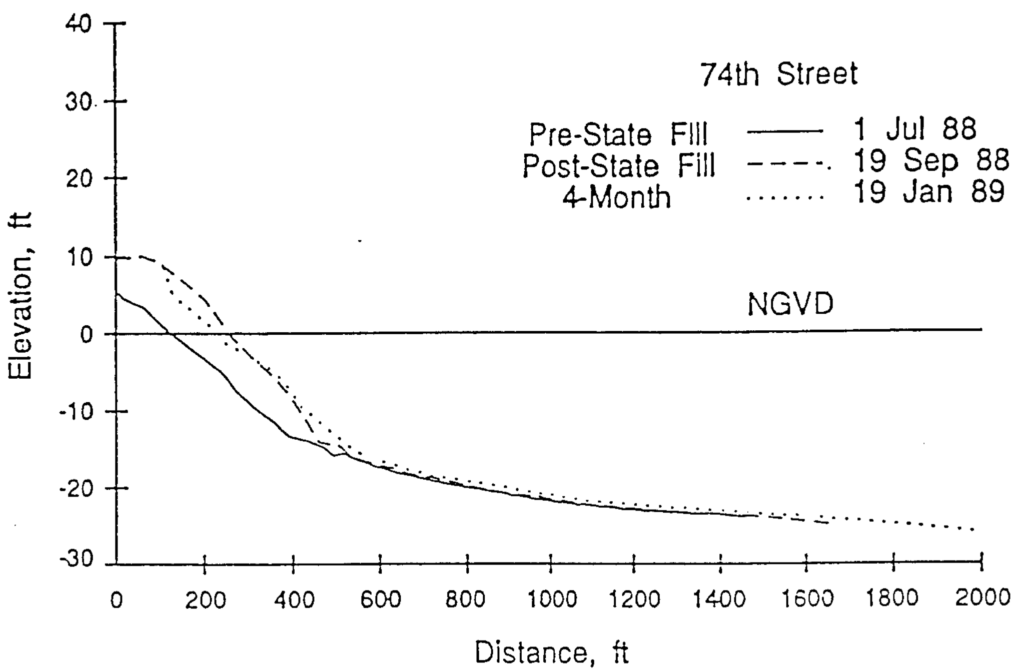


Figure 4. Construction template to desired project shape (MSL = mean sea level)

profile must equal the volume of sand that is deposited further offshore with its seaward limit being at the depth of closure.

Figure 5 shows an example from the 1988 renourishment project at Ocean City, Maryland. The figure shows how the initial beachfill (construction profile) follows a generally trapezoidal shape. However, after four months time, the berm has eroded a small distance, and the sand conformed into a concave shape that very nearly matches the beach profile before the project was undertaken. This remolding or "equilibration" of beachfill projects leads to a public perception, also held by Professor Pilkey, that the project has eroded away and is "lost." In reality, however, the project has simply molded itself to the natural considerations of the project site. This initial "loss" of the project is a factor that has been taken into account by design engineers and should not be counted in an objective analysis of project performance.



(Kriebel, 1995)

Figure 5. Equilibration examples from Ocean City, Maryland

(NGVD = fixed datum equal to mean sea level in 1929)

Background Erosion

Because beachfill projects are placed in areas already subjected to progressive erosion, a beachfill begins to erode immediately after it has achieved its equilibrium shape. Erosion is a constant process that is caused mainly by: 1) spatial gradients in the net longshore transport of sand in the littoral region or 2) sea level rise over long time scales. Sea level rise is generally not significant over very short time scales (a few years), therefore, most net erosion is due to an imbalance of longshore transport into or out of an area.

Longshore transport refers to the movement of sand in a direction parallel to the shoreline. This transport occurs as the sand along the seabed is stirred by incoming wave energy, picked up into the water column, and transported along with the longshore current. The longshore current itself is caused by the breaking of waves at an angle to the shoreline. The magnitude of longshore transport is determined by the wave height and the angle at which the wave is hitting a beach. The Army Corps of Engineers *Shore Protection Manual* (1984) defines the longshore transport rate Q_{ls} in yd^3/yr as:

$$Q_{ls} = 7500 * P_{ls} \tag{8}$$

where P_{ls}, the longshore component of wave power per foot of shoreline as defined by:

$$P_{ls} = \frac{1}{16}\rho g^{\frac{3}{2}} H_b^{\frac{5}{2}} \sin 2(\alpha_b - \beta) \tag{9}$$

where H_b = breaking wave height as defined as the average of the 1/3 highest breaking waves
α_b = breaking wave angle
β = shoreline orientation angle
ρ = density of water
g = acceleration of gravity

The factor of 7500 in Equation 8 converts units of longshore wave power from ft-lb/sec-ft to yd^3/yr. Thus, the driving forces behind longshore transport are wave height, because transport is proportional to $H^{5/2}$, as well as wave angle since transport is proportional to $\sin 2(\alpha_b - \beta)$. The longshore transport therefore can change direction and magnitude daily as the incoming waves change both height and angle.

In order to determine the background erosion rate in an area due to longshore transport, a control volume must be established for the region under study. Erosion is then the result of a loss of sand from within this control volume. It is due to the imbalance in longshore transport Q_{ls} across control boundaries which occurs when the transport out of the control volume is greater than the transport into it. Figure 6 is an example of a control volume that has been established for an area under concern. The longshore distance is identified as Y while the offshore direction is defined as positive (+) X (this notation will be followed for the remainder of the report). The volume loss in an area for time Δt in an area defined as having shoreline length Δy and erosion Δx is determined as:

$$(Q_{out} - Q_{into})\ \Delta t = \Delta x * \Delta y (B + d_c)$$

(Army Corps, 1984) (10)

Q_{out} = longshore transport out of control volume
Q_{into} = longshore transport into control volume

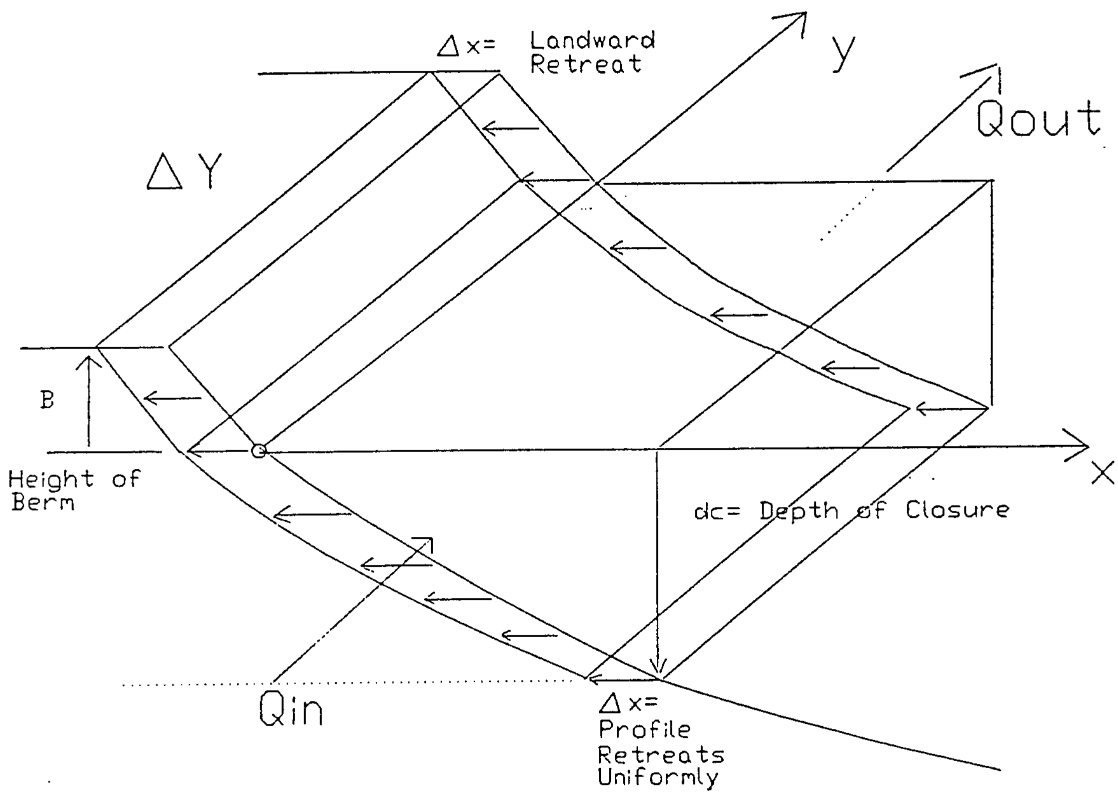


Figure 6. Longshore transport differential in a control volume

B = Berm height
d_c = depth of closure

The erosion rate for the control volume can then be found by rewriting Equation 10 into a new form as shown by

$$\frac{\Delta x}{\Delta t} = \frac{1}{B + d_c} \frac{\Delta Q}{\Delta y}$$

(Army Corps, 1984) (11)

$\Delta Q/\Delta y$ = spatial gradient of longshore transport

Thus, the erosion rate is mainly determined by the spatial gradient or imbalance of longshore transport.

This yearly erosion rate is the "background erosion" for an area and controls the distance that the shoreline is eroded each year. The distance eroded is the factor that influences a beach's storm protection capacity. It is also important to note that the sand lost to longshore transport or background erosion is taken out of the beach system being studied by longshore transport. Unlike the equilibration of the beach profile discussed previously, background erosion represents a net sand loss to the beach system (control volume).

Longshore Diffusion

The historical or background beach recession does not act alone in eroding a beachfill. Another effect that acts on the beach planform is longshore diffusion or spreading of the project. This diffusion occurs when a beach nourishment project is undertaken by placing a large quantity of sand out into the nearshore region over a finite length of shoreline. This placement of sand onto a long, uninterrupted shoreline acts as a perturbation that is smoothed out over time by longshore diffusion (Dean and Yoo, 1992). Pelnard-Considere (1956) found that based on the concepts of sand conservation and longshore transport, the effect on the beach can be modeled by the diffusion equation which has the same form as the heat-conduction equation

$$\frac{\partial x}{\partial t} = K \frac{\partial^2 x}{\partial y^2}$$

(Dean and Yoo, 1992) (12)

The parameter K is the longshore diffusivity constant which results from taking the differential form of Equation 11 and substituting Equation 8 and Equation 9 in for longshore transport. This produces K as defined by the following equation:

$$K = \frac{C H_b^{\frac{5}{2}} \left(\frac{g}{\kappa}\right)^{\frac{1}{2}}}{8(s-1)(1-p)(d_c+B)}$$

(Dean and Yoo, 1992) (13)

C = nondimensional sediment-transport constant = 0.77
g = gravity acceleration= 32.17 ft/sec^2
κ = proportionality constant = 0.78

s = ratio of mass densities of sediment to water = 2.60
p = in place sediment porosity = 0.35
d_c = depth of closure
B = berm height
H_b = mean breaking wave height

From the diffusion equation it is possible to interpret a number of important observations. The equation reveals that the planform erosion rate, $\partial x/\partial t$, is highest when the shoreline curvature, $\partial^2 x/\partial y^2$, is the highest. From this interpretation, it follows that there is no longshore diffusion when the shoreline is straight. Such a condition often exists on a natural, unnourished shoreline. However, when a beachfill project is present, longshore diffusion occurs due to the perturbation of the shoreline. Erosion is highest at the ends of the project where the beach width rapidly changes from a widened beach within a project to an original beach width outside the project area as shown in Figure 7. The overall effect of this longshore diffusion is to spread the sand out in an attempt to return the beach to its original straight state. It is important to note that the sand moved by this force is not totally lost from the beach system but simply moved into different areas in the longshore direction. While the sand is lost from within project boundaries, beaches adjacent to the project gain sand at the expense of the original beachfill.

This differential equation has been solved for beach fill placed on a straight reach of beach as shown in Figure 7. The following equation represents the solution that is obtained for longshore diffusion model to account for the movement of sand that is placed on a beach that extends from -a to +a:

$$y = \frac{X}{2}\left[erf\left(\left(\frac{a}{2\sqrt{Kt}}\right)\left(1-\frac{y}{a}\right)\right) + erf\left(\left(\frac{a}{2\sqrt{Kt}}\right)\left(1+\frac{y}{a}\right)\right)\right]$$

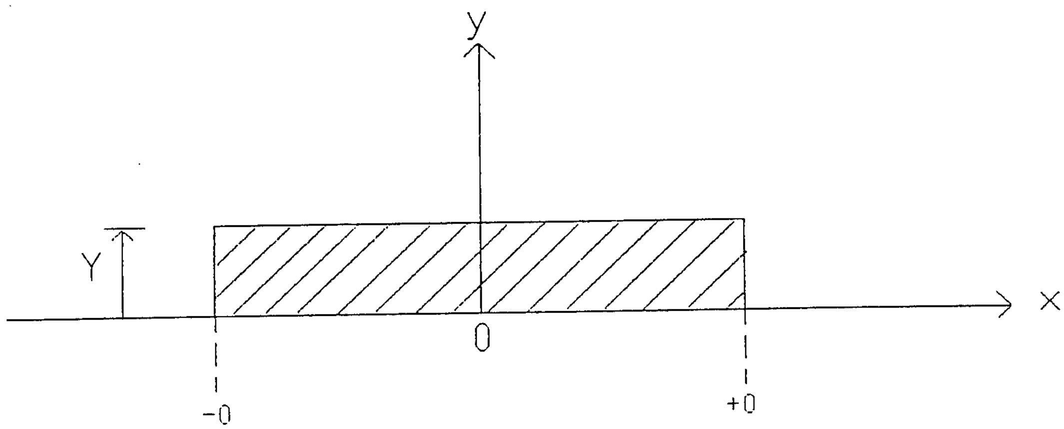


Figure 7. Plan view of longshore diffusion setup after construction

(Walton and Chiu, 1979) (14)

x = position of the shoreline at any instant in time
X = initial position of shoreline after construction profile reaches equilibrium
t = time in years
L = overall length of the project in miles
a = L/2
y = position along the coastline as measured from center of project being 0
erf = error function

The error function term in the equation is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

(Greenberg, 1978) (15)

The general solution as derived by Walton and Chiu was based on a constant wave height throughout the year, thus giving a constant diffusivity, K. The result of this longshore diffusion as shown in Figure 8 is to produce a curved shape to the initially rectangular nourishment planform. In the first few months, the end points of the project erode back close to fifty percent of their original length. These endpoints then display only slight erosion for the remaining time analyzed. This can be contributed to the large differential in position x between the project and natural shoreline bordering it. As discussed before, the general diffusion equation predicts that erosion will be largest in this area due to the gradient in shoreline positions. Near the center of the project, where there is no curvature, initially there is little beach erosion. After some time, however, the project begins to flatten out as the diffusion eliminates the perturbation in the shoreline that the project caused and the center of the project eventually erodes as well.

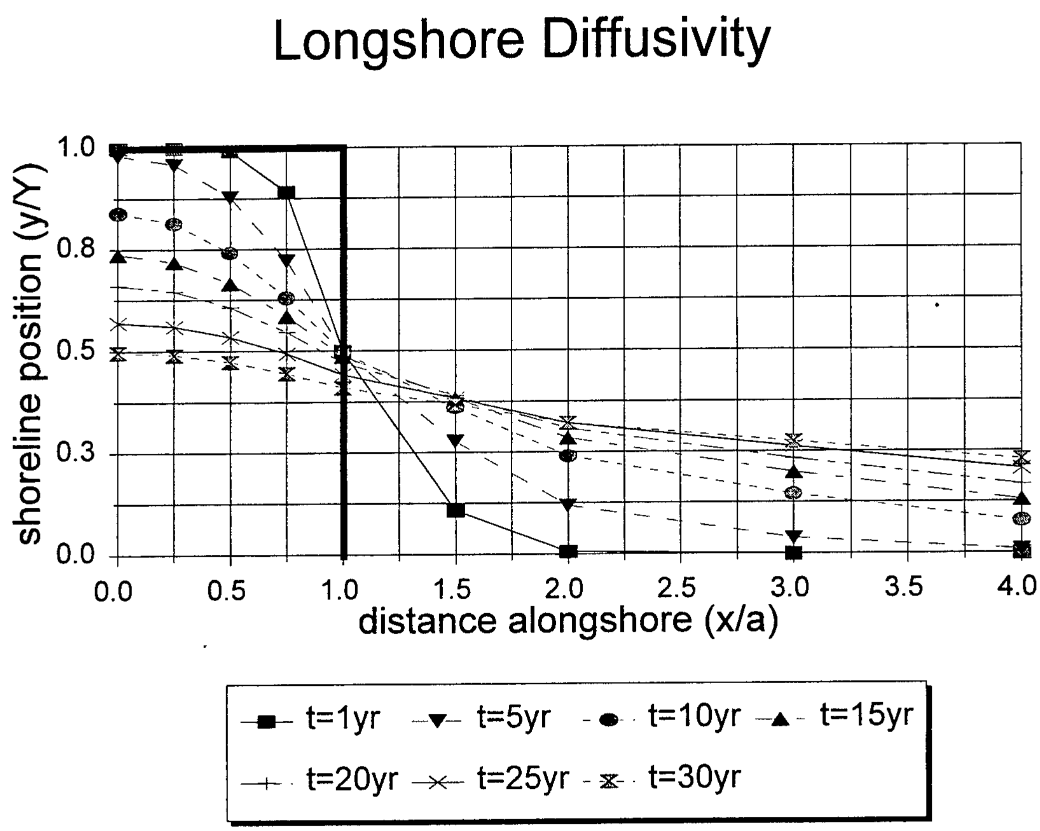


Figure 8. Example of symmetrical longshore spreading of 4 mile long project.

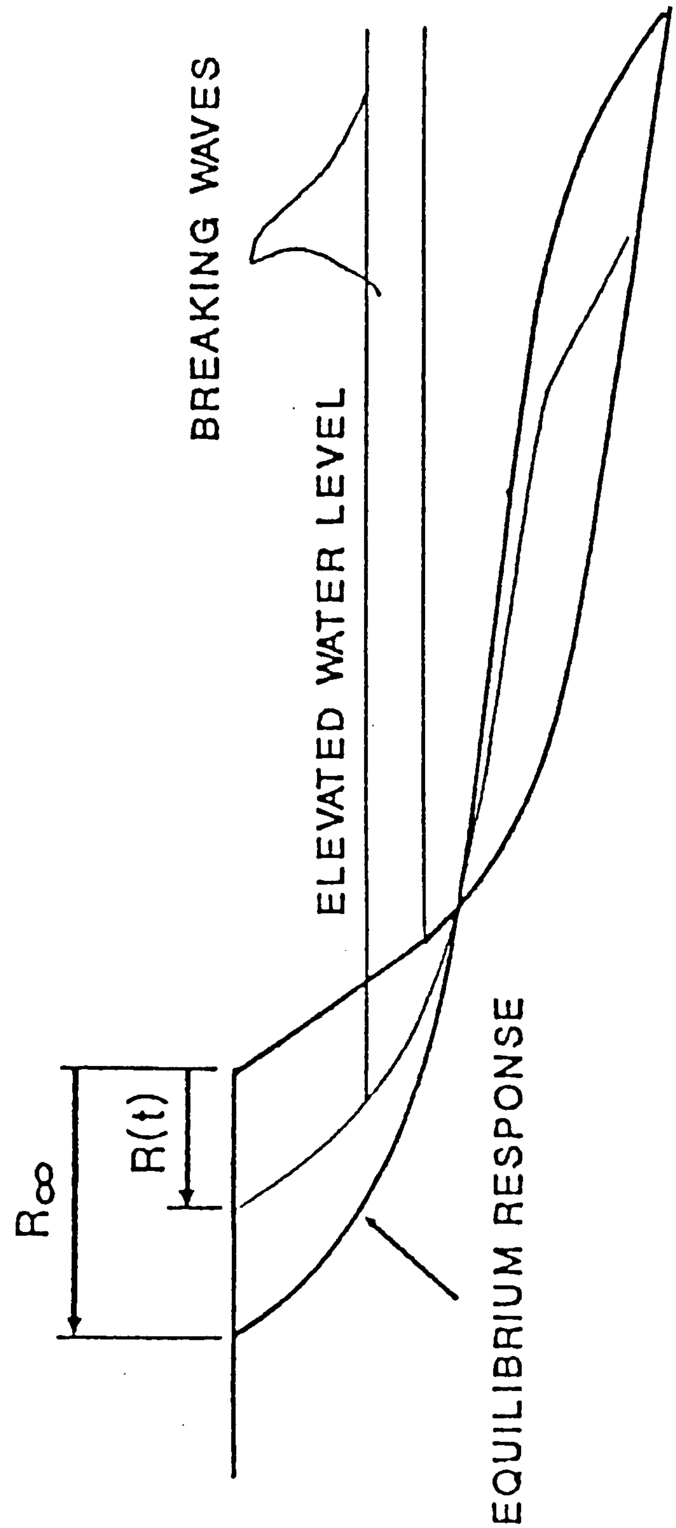
Due to the concept of sand conservation, the sand that is eroded does not simply disappear, but rather it accretes just outside of the project area. Thus, a beach nourishment project will, over time, provide sand to areas outside of the immediate project area if no fixed structure such as a groin is used to hold sand within the inital fill region. If a project is left unaltered the final effect of longshore diffusion will be to continue to spread the beachfill until little or no pertubation into the ocean is present. The longshore spreading will cease when there is no position gradient due to the fill, thus when the beach is straight again.

Storm Erosion

After longshore or progressive erosion mechanisms have been accounted for, it is necessary to analyze the effects of storms on the beach. Storms act as the major stressors on the beach system. They cause the most damage to the beach as well as the structures that the beach is designed to protect. However if a beachfill project is designed to withstand a certain level storm, erosion will occur, but sufficient sand will remain in the beach berm and dune to prevent storm waves from reaching structures behind the beach. In order to analyze the erosion that a storm causes, the Kriebel and Dean (1993) convolution model is used to estimate the beach response due to storm conditions. This model is general enough that with the input of a few location specific variables it can be used for a variety of locations.

In this model the basic premise is that beaches subjected to a steady-state erosion condition will respond toward equilibrium in an approximately exponential manner. The forcing mechanism causing this change is the water level rise or storm surge S that come about with the onset of storms. The two main parameters that determine the magnitude of the erosion response are R_∞ and T_s. The first term, R_∞, represents the maximum potential response that a beach will have after it reaches equilibrium under the storm surge. T_s represents the erosion time scale that governs the exponential rate at which the profile responds toward a new equilibrium. Figure 9 shows how these terms are visually depicted.

In order to determine the exact level of erosion, the term R_∞ must be determined. This term represents the maximum possible erosion expected if the peak storm surge held steady indefinitely. To obtain this, it is assumed that the entire profile of the beach rises by S and maintains its equilibrium shape relative to the water level. In order for the profile to



(Kriebel and Dean, 1993)

Figure 9. Definition sketch of R∞, R(t)

rise with the storm surge, there must be a deposition of sand offshore. This offshore sand is supplied by the erosion of the beachface as is shown in Figure 10. The conservation of sand in the beach system dictates that the volume of sand which is deposited offshore must equal the volume of sand that is eroded from the beachface.

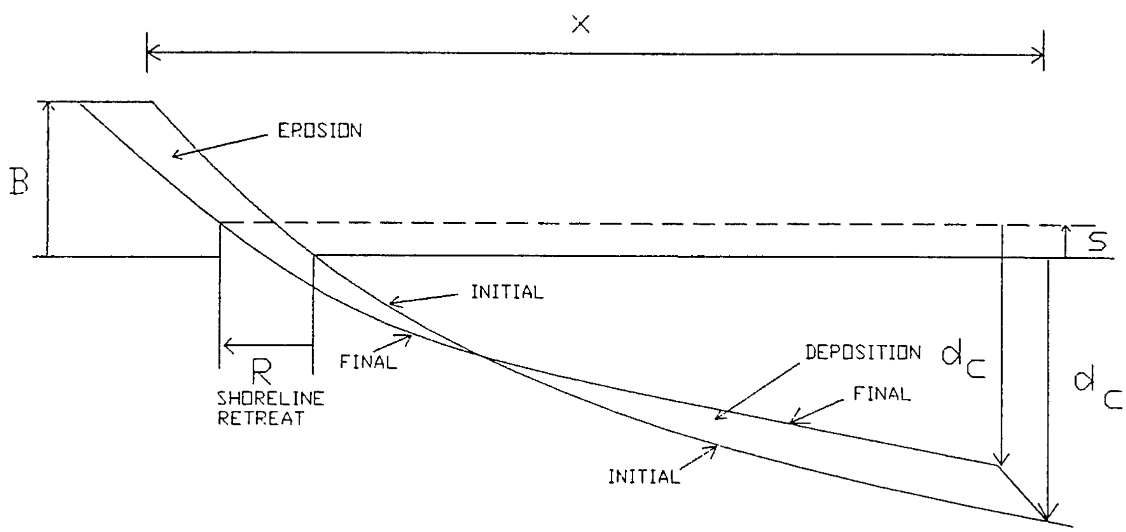
Through the conservation of eroded and deposited sand volumes, an equation for the maximum erosion potential can be determined. The final form as given below can account for the presence of dunes and backshore distance.

$$R\infty = \frac{S(x_b - \frac{d_b}{m})}{B + D + d_b - \frac{S}{2}} - \frac{W(B + d_b - \frac{S}{2})}{B + D + d_b - \frac{S}{2}}$$

(Kriebel and Dean, 1993) (16)

S= peak storm surge
B= berm elevation
D= dune elevation
W= backshore width or berm width
m = beach slope
x_b= surf zone width given as $x_b = x_0 + (d_b/A)^{3/2}$ (17)
d_b= wave breaking depth = H_b / 0.9 where the breaking height of a specific storm is used.

However, if no dune is present in an area then second term in the equation can be eliminated from the equation so that the equation becomes:



(Kriebel, 1995)

Figure 10. Beach profile response to sea level rise

$$R\infty = \frac{S(x_b - \frac{d_b}{m})}{B + d_b - \frac{S}{2}}$$

(Kriebel and Dean, 1993) (18)

The erosion time scale, T_S, could not be found analytically or with geometric profile forms. Kriebel and Dean determined this response by running multiple numerical tests and then developing a best fit solution for the time scale. It was found that the time scale was most dependent on breaking wave height and sediment size such that the larger wave heights and smaller sediment size result in much larger time scales. This is because the larger storms define a wider surf zone that requires moving sand farther offshore which requires more time for the beach to obtain equilibrium. After studying the relationships uncovered by the numerical tests, an empirical expression for the erosion time scale in hours was determined as:

$$T_S = \frac{C_1}{3600} \frac{H_b^{3/2}}{g^{1/2} A^3} (1 + \frac{d_b}{B} + \frac{m x_b}{d_b})^{-1}$$

(Kriebel and Dean, 1993) (19)

where the coefficient C_1 is determined from the slope of the data to be 320. In cases in which a dune is present, the berm elevation B should be replaced by the total berm plus dune elevation of B+D.

The equations as developed above are for the maximum potential erosion of a storm, R_∞. However, storms do not always have the peak storm surge possible nor do they maintain this storm level indefinitely. Therefore, a method of determining the actual degree of erosion, R(t) based on the duration of the storm, T_D is necessary. The governing differential equation for this time-dependent storm erosion is

$$\frac{dR(t)}{dt}=\frac{1}{T_s}[R_\infty f(t)-R(t)]$$

(Kriebel and Dean, 1993) (20)

f(t) = time dependent erosion forcing function

The general solution of this differential equation can be expressed in the form of a convolution integral as:

$$R(t)=\frac{R\infty}{T_s}\int_0^t f(\tau)e^{-(t-\tau)/T_s}d\tau$$

(Kriebel and Dean, 1993) (21)

where τ = a time lag. This type of response is not restricted only to beaches but has applications in civil engineering and other disciplines. The important ideas to note from this convolution equation are that the erosion will lag behind the water level changes and the erosion will be damped relative to the maximum erosion potential of the system.

For storm erosion, the storm surge provides the erosion forcing and can be approximated by $f(t) = \sin^2(\sigma t)$ where $\sigma = \pi/T_D$ and where T_D= the total storm surge duration defined as the time from the beginning to the end of the water level rise. The actual water level during a storm would be given by S $\sin^2(\sigma t)$ where S is the peak storm surge.

With this sine-squared storm surge, Equation 21 may be integrated directly to obtain a solution for time-dependent erosion response. This equation, with the input of some of its variables, can be the basis for spreadsheet analysis of erosion due to different level storms. The basic equation is

$$\frac{R(t)}{R_\infty} = \frac{1}{2}[1 - \frac{\beta^2}{1+\beta^2}\exp(\frac{-2\sigma t}{\beta}) - \frac{1}{1+\beta^2}(\cos(2\sigma t) + \beta\sin(2\sigma t))]$$

(Kriebel and Dean, 1993) (22)

where $\beta = 2\pi\, T_s / T_D$.

As this response is analyzed it can be shown that the erosion lags the storm surge as it should. In addition, after a certain time a recovery period begins at a slower exponential rate than the erosion. This recovery is due to the fact that the beach profile is also out of equilibrium at the end of a storm when the water level falls. However, due to a number of unaccounted effects, the convolution method is not good for estimating the extent of recovery so that will not be analyzed here.

The storm induced erosion will increase to some maximum value and then begin to lessen toward the end of the storm. By varying σt from 0 to π, it is possible to determine at what time during the storm the maximum erosion occurs and the extent of this maximum erosion. The other important quantity that can determine the extent of erosion is the term β which is defined as $2\pi T_s/T_d$. When β is large, such as for a short duration hurricane, the erosion may only reach 30% of R_∞ while a smaller value of β that occurs during a longer storm like a northeaster can result in erosion up to 90% of R_∞. An example of both of these cases is shown in Figure 11.

Once the extent of actual erosion, R(t), has been determined, it is possible to find the volume of sand that has been eroded from a project. The volume of sand that is eroded from the beach project during a storm is found by first determining the cross-sectional area of sand eroded from a one foot unit width of beach. This area of erosion is found as

$$A = R_\infty D + (R_\infty + W)B + \frac{S^2}{2m} - \frac{2S^{\frac{5}{2}}}{A^{\frac{3}{2}}}$$

(Kriebel an Dean, 1993) (23)

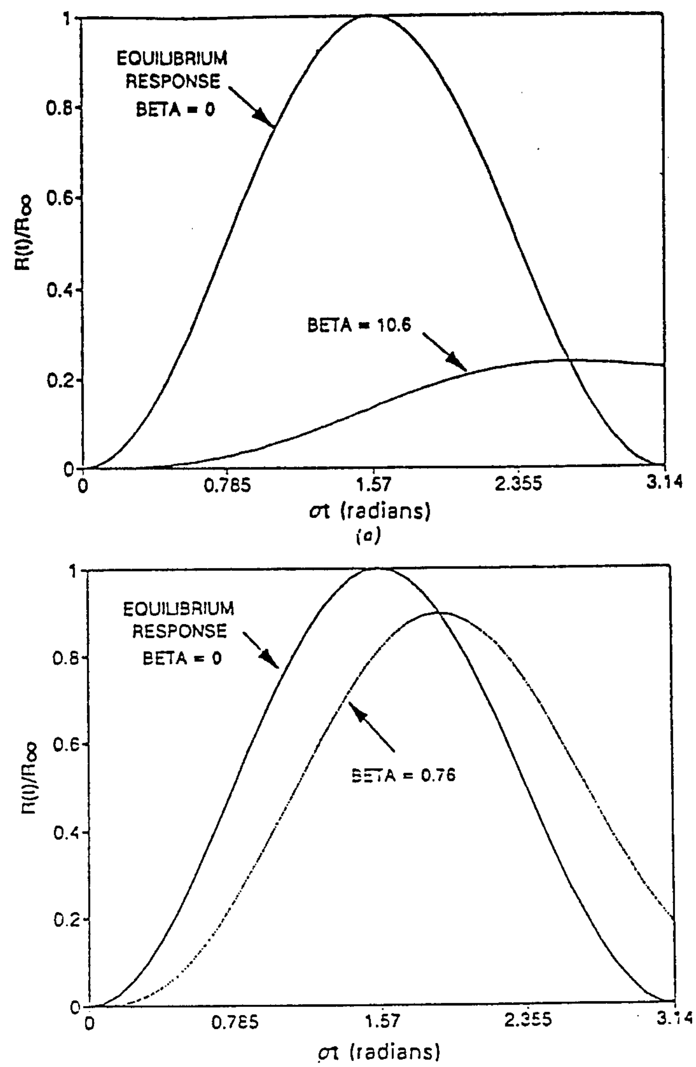
when a dune is present at a project and the following equation when an area has no dune behind it or if the erosion does not reach the dune:

$$A = R_\infty B + \frac{S^2}{2m} - \frac{2S^{\frac{5}{2}}}{5A^{\frac{3}{2}}}$$

(Kriebel and Dean, 1993) (24)

The volume of sand eroded from the project can then be found by multiplying the cross-sectional area that is eroded by the length of the project in question.

Therefore, the model developed by Kriebel and Dean determines the time-varying erosion of a beach due to the presence of a storm. This model evaluates the erosion based on the intensity and duration of the storm, thus producing a simplified but realistic erosion



(Kriebel and Dean, 1993)

Figure 11. Variation of R(t) with change in storm duration

of a beach due to the stress of a storm impacting a region.

SPREADSHEET MODEL FOR BEACH FILL RESPONSE

Model Overview

Utilizing the theories developed in the earlier sections, the major focus of this project was to develop a realistic spreadsheet model that accurately models the life-cycle of a beach nourishment project. Before describing how each individual erosion process has been replicated on the spreadsheet, it is necessary to provide an overview of how this modeling will occur.

The first step in this model is to establish the initial geometry of the project. This includes the major beach parameters such as the berm height, berm width, dune height, project length, etc. The selection of these parameters will determine the response of the project to all possible methods of erosion. Once this design profile has been determined, a construction profile for the project must be established. The model will then account for the equilibration of the constructed beachfill back to the design equilibrium profile. This step will provide the actual volume of sand placed onto the project as well as determining the width of berm that is "lost" to equilibration.

This model will then use this design profile (following initial equilibrium) to model the project against the various erosion processes in order to determine the response of the beachfill. The model will be run every season, or quarter of the year, in order to provide an evaluation of how the project performs through time. In each quarter of the year, the project will be modeled for background erosion, longshore diffusion, and storm erosion. All of these erosion mechanisms will have statistical variability built into them in order to increase the realism of the model.

This process will continue until the beach width diminishes to a point where the project no longer satisfies its major function of storm protection. The end result of this modeling is to define the time when the project reaches "failure." In this study, this failure point is defined as the time when the area of the project three-quarters of the way from the center has produced dune erosion of 20 feet. It is assumed that any erosion after this point would begin to cause damage to the structures behind the beachfill. The 20 foot criterion was selected because most projects are designed with a dune width of 25 feet. Any erosion past the 20 foot mark would leave any structures behind the project highly vulnerable to storm attack. Thus, the project must be rebuilt at this time. Additionally, seasons in which the storm erosion stopped short of eroding the dune would be modeled as returning to their pre-storm condition. This is because the natural wave action in the area will return much of the sand back onto the beach. Other criteria are possible, but this provides a reasonable minimum storm protection level for the project.

When this renourishment time is reached, the volume and cost to rebuild the project will also be determined. This simulation is repeated N times in order to produce a statistical description of project renourishment interval. The renourishment interval as defined by this spreadsheet model will be compared economically to a model currently used by the Army Corps of Engineers. The comparison of these two different approaches will attempt to determine if present methods are an accurate measure of the time and cost involved in a beach nourishment project.

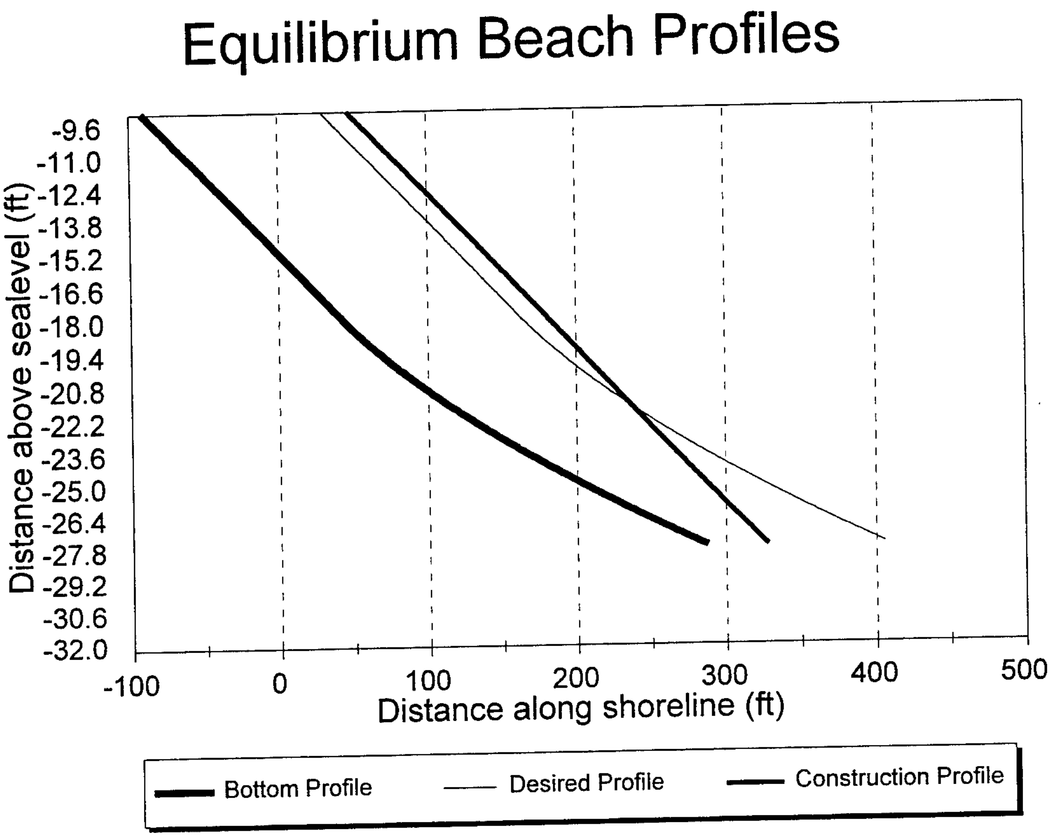
The next section will fully detail the individual models that are combined in this project. The statistical methods used to increase the degree of realism in the projects will also be explained.

Beach Equilibrium Profiles

In order to accurately evaluate the effectiveness of a beach nourishment project, any model must take into account the initial equilibration period in which the construction template is remolded into an equilibrium profile. The spreadsheet model developed in this project allows for this initial equilibrium period. It determines the necessary construction berm width for a desired beachfill width as well as, most importantly, the actual volume of sand to be placed onto a project. The model uses the conservation of mass argument that has been explained above.

The first step that is necessary is to input the beach parameters such as sediment size parameter A, natural beachface slope m, construction beach slope m_c, initial berm width W_i, berm height B, project length L, and most importantly the desired berm width W_d as shown in Figure 12. This is the most important value since it determines the amount of storm protection that an area receives as well as the width of its recreational beach. The width added to the berm, W_d - W_i, will determine the volume of sand that is added per unit length to the beach.

It is then necessary to vary the construction berm width in an iterative fashion until a conservation of eroded and deposited sand volumes is reached. The model then determines the volume of sand that was actually placed onto the project by adding the amount determined by the construction template with the amount placed on a trapezoidal dune which does not need an equilibration period. Figure 12 shows an example of the spreadsheet inputs as well as the graphical output of the beachfill project. The graph from the model matches Figure 4 as it shows how the original construction template compares to the desired beach



Berm width (W)	145	ft
Construction Width (Wc)	163	ft
Volume Difference:	(2,518)	drive toward zero
Total Volume of sand added	1,161,435	yd^3
Width Difference:	18	ft

Figure 12. Equilibrium beach profile model inputs

profile.

From this equilibration period, it is evident that beachfill projects "lose" some of their initial beach width as the construction template molds into the desired design profile. The loss of visible beach during this period is one of the subjects which divide Pilkey and Houston in their debate. Houston holds the view that the project should be judged after equilibration and that the initial beach loss is simply a natural result of letting the sand conform to the existing beach profile. Pilkey, on the other hand, views beachfill performance from the construction template. Therefore, he considers the initial beach width loss during equilibration when he states how beachfill projects erode at a faster rate than natural beaches. In claiming that the nourishment project erodes faster, Pilkey fails to take note that the sand is simply moving into a pre-designed position that a natural beach already has. Therefore, this sand is not lost, rather it is redistributed into a more natural form.

Statistical Variability of Background Erosion

When modeling the background erosion trend, it is necessary to begin with the general yearly erosion rate in an area. However, since waves are not continually of the same nature every year, this model takes into account some of the variability of the erosion rate from year to year. Because this spreadsheet model uses quarters of the year as its time step, any yearly erosion rate for an area is broken into a quarterly rate. In order to account for the variability of the erosion rate from quarter to quarter, the mean erosion rate is then specified along with a standard deviation. The necessary data for this input is found from historical erosion records. The model assumes that the erosion rate has a Gaussian or normal probability density function. Using the spreadsheet's random number generation tool, the model selects a quarterly erosion rate from the Gaussian random number distribution as shown in Figure 13.

Figure 14 shows an example of this random background erosion through a thirty year time period. From this it is possible to see that the erosion follows a general trend as defined by the mean erosion rate. However, the variability inherent in the model causes the differing degrees of erosion and in some cases accretion from quarter to quarter. This varying value provides the more realistic view of the uncertain nature of the wave climate in a given region than simply using the mean erosion rate every quarter. It is anticipated that the random chance of several quarters of severe erosion, coupled with other causes, could require premature or unplanned renourishment of the project.

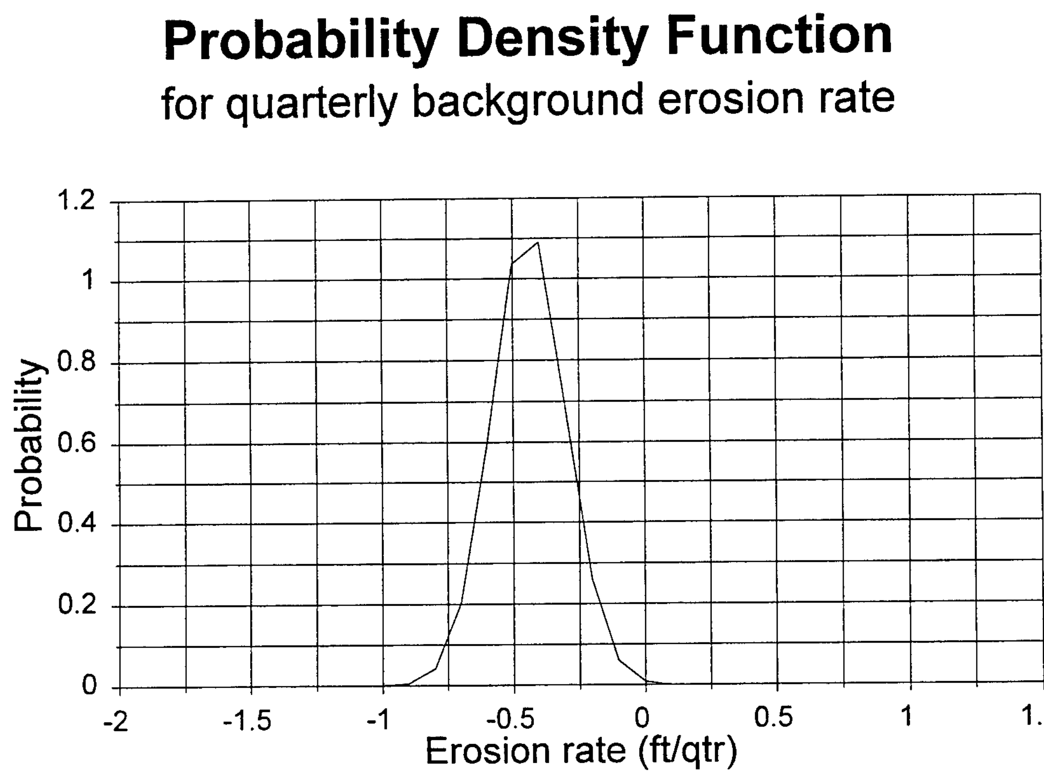


Figure 13. Normal distribution of erosion rate

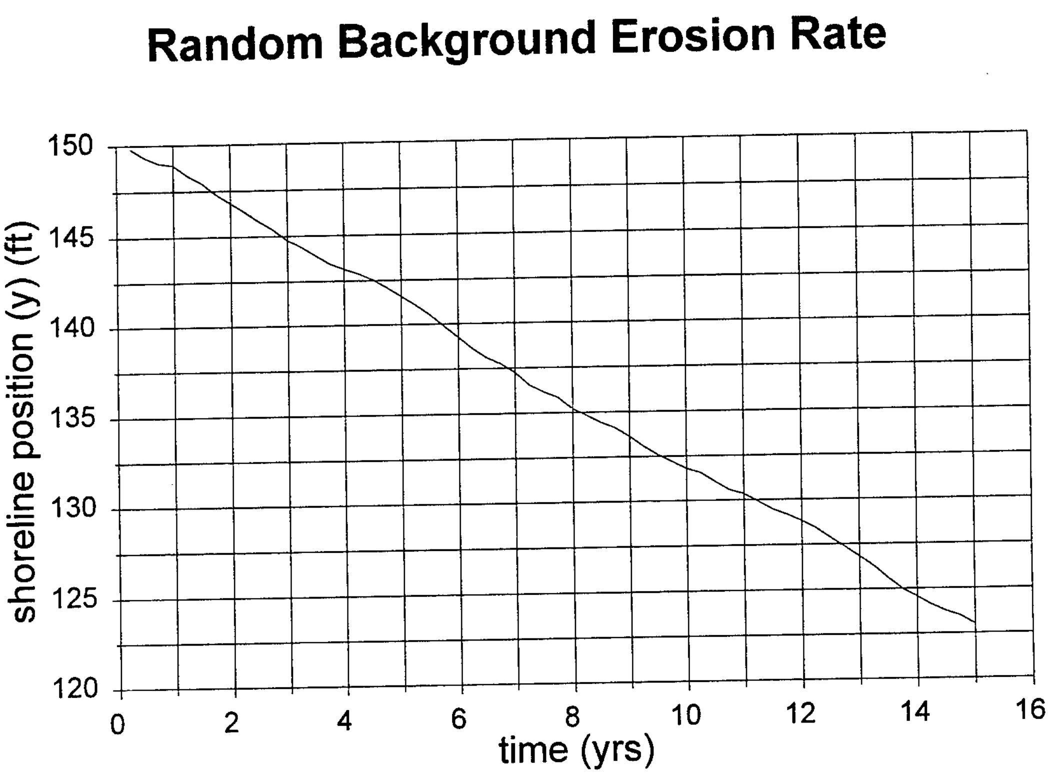


Figure 14. Example of random erosion rate over time

Statistical Variability of Longshore Diffusion

In this study, the variability of longshore diffusion due to randomly varying wave conditions is also investigated. As is known, wave direction and height change throughout the year. According to previous studies by Dean and Yoo (1992), the changes in incoming wave direction do not have a significant effect on longshore diffusion. Following their recommendations, the present diffusion model neglects the effect of changing wave directions. The problem of changing wave heights throughout the year does effect longshore diffusion, however, due to diffusivity, K, being proportional to $H^{5/2}$ in Equation 12 and Equation 13.

The first step in accounting for the random nature of waves was to input an average wave height for each of the four seasons of the year. Simply using these quarterly averages every year did not provide much variability in wave conditions however. Therefore, Equation 25 from Chapter 4 of the *Shore Protection Manual (1984)* was used to provide a probability distribution of wave heights during each quarter as:

$$Hs = \frac{0.61\,\overline{H_s} - \overline{H_s}\ln(1-P)}{1.61} \tag{25}$$

H_s = the varying quarterly wave height
H_s (mean) = the given average quarterly wave height
P = probability of occurrence of wave height

This expression is useful since the probability distribution in each quarter depends only on the mean wave height in each season. The random probability P is found using a random number generation to select a uniformly-distributed random number between 0 and 1 to determine the probability of occurrence. A different random number is selected each quarter (season) and this randomly changing probability produces a different wave height used for diffusion in each quarter. The probability density function for this wave height distribution is shown in Figure 15. With this procedure, simulation over a long-term N-year period would produce N values of H_s for each season. The average of these would approach the mean H_s if N is a large enough sample.

However, problems arose when the varying wave heights were entered into the basic diffusion solution. The equations, as developed by Walton and Chiu (1979), were based on constant wave heights and constant diffusivity K. Therefore, some adjustment to the initial model was needed to use the analytical solution with varying wave heights. In order to modify Equation 14, it is first recognized that the quantity $(Kt)^{1/2}$ provides a characteristic unit of length in the diffusion solution based on diffusivity K and the total time t. To accommodate random wave heights, the diffusivity term Kt was replaced by a cumulative term $\Sigma K\Delta t$, with Δt being the time step used for analysis of one quarter or 0.25 years, and with the summation from the initial time (t=0) to current time of interest where $t = \Sigma\ \Delta t$. This cumulative term took into account what had previously happened to the project yet still allowed for the varying wave heights to produce their individual effects on the project's shape. Therefore, the diffusion solution in Equation 14 now became:

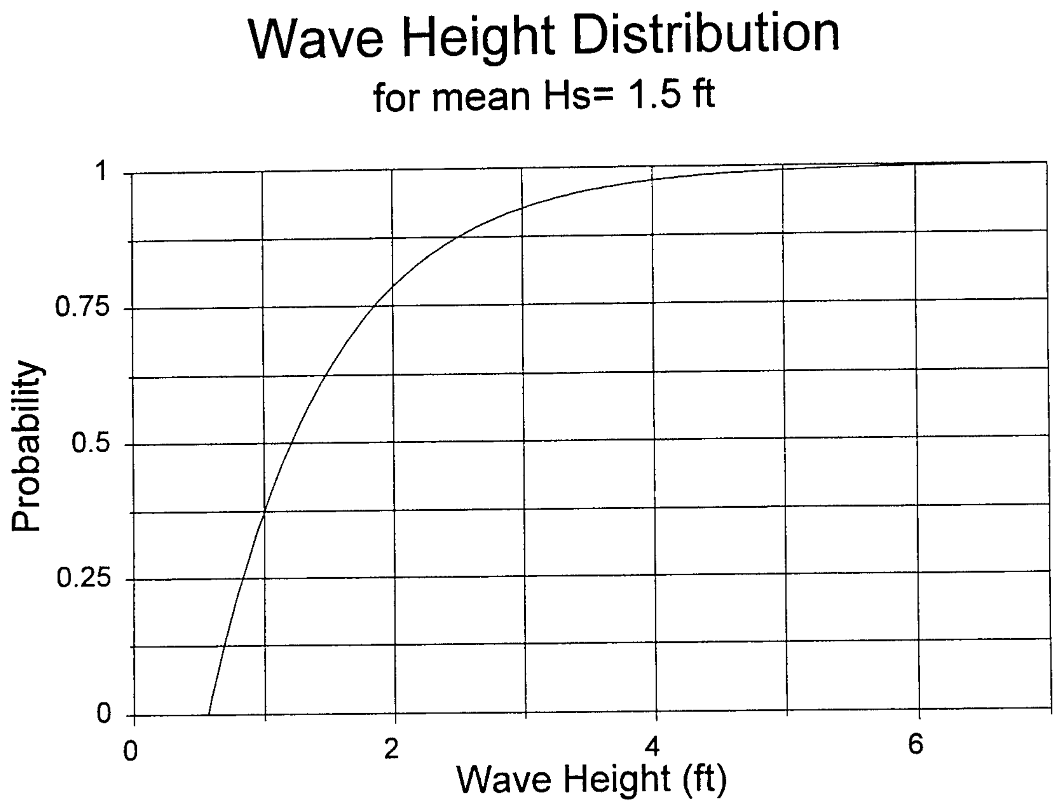


Figure 15. Probability Density Function for wave height distribution

$$x = \frac{X}{2}\left[erf\left(\left(\frac{a}{2\sqrt{\Sigma K_n \Delta t}}\right)\left(1-\frac{y}{a}\right)\right) + erf\left(\left(\frac{a}{2\sqrt{\Sigma K_n \Delta t}}\right)\left(1+\frac{y}{a}\right)\right)\right] \tag{26}$$

$K_n\Delta t$ = the cumulative sum of individual diffusivities K_n * time step Δt
with K_n based on randomly selected seasonal wave height H_n

This improved model now allows for the more realistic simulation of varying wave heights while still allowing the use of an analytical solution of the diffusion equation.

In terms of the general shape of the beach throughout time, the new probabilistic solution matches the earlier solution of Walton and Chiu quite well. As shown in Figure 16 the shape is similar to that in Figure 8. One important result arising from the use of the variable wave heights is the effect that one quarter of large wave heights can have on the longshore diffusion. As Figure 17 shows, when K_n is large for one quarter, $K\Delta t$ in that season may be larger than in several seasons of smaller wave heights. Thus, one period of large wave heights will effect the shoreline position much more than several years of calm in which the shoreline remains relatively constant. Over time, although the accelerated erosion due to stormy seasons can persist for several years, eventually, the solution begins to agree with that obtained by using an average value of K.

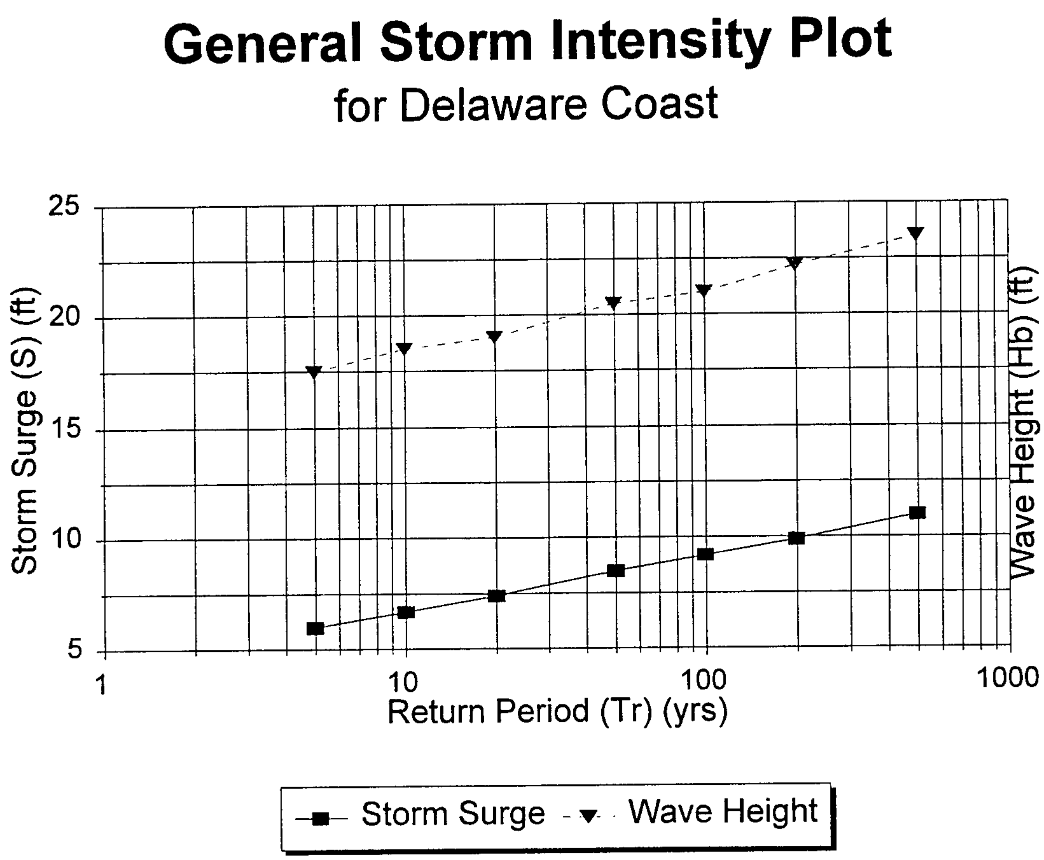


Figure 16. General storm intensity plot for Delaware Coast

Time (t) (yrs)	Quarterly Wave Heights (Hb)	Longshore Diffusivity (Kdeltat) mi^2	Cumulative Diffusivity
25.00	1.05	0.010	3.216
25.25	2.01	0.052	3.268
25.50	0.95	0.008	3.276
25.75	0.45	0.001	3.277
26.00	1.17	0.013	3.290
26.25	1.64	0.031	3.321
26.50	2.29	0.072	3.393
26.75	1.11	0.012	3.405
27.00	1.41	0.021	3.426
27.25	1.95	0.048	3.474
27.50	1.47	0.024	3.497
27.75	1.06	0.010	3.508
28.00	0.58	0.002	3.510
28.25	5.89	0.758	4.268
28.50	1.72	0.035	4.303
28.75	1.30	0.017	4.321
29.00	1.41	0.021	4.342
29.25	0.84	0.006	4.348
29.50	0.53	0.002	4.350
29.75	2.55	0.094	4.443
30.00	1.08	0.011	4.454

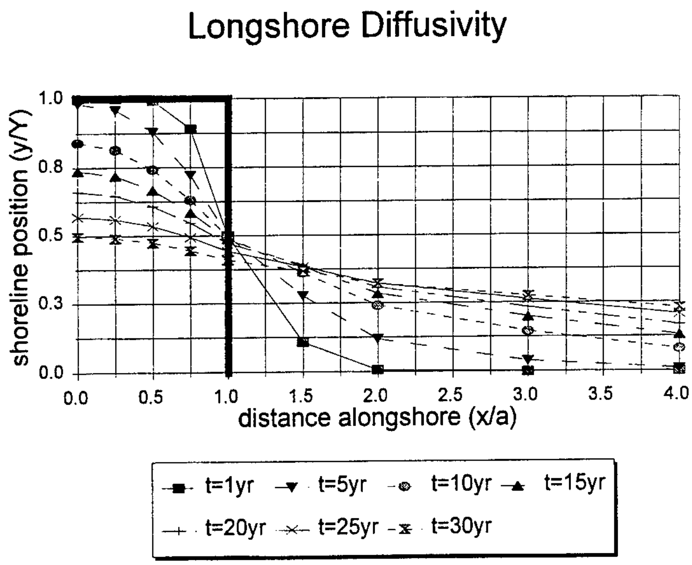
Longshore Diffusion Model showing the affect of one stormy season

Figure 17. Longshore diffusion model showing the effect of one stormy season

Background Erosion Plus Longshore Diffusion

Next, the spreadsheet model combines the effects of historical "background" beach recession with longshore diffusion to produce a net chronic erosion trend for a beachfill project. The importance of this chronic erosion model is that it can show the overall erosion of the beach from year to year in the absence of major storms. This model could be used to determine the renourishment interval at which a project will need to have sand replaced to account for chronic erosion. Renourishment is not something that can be eliminated, rather it is a necessary aspect of dealing with a dynamic environment like the beach.

Figure 18 shows the importance of combining these two erosion mechanisms in order to determine the chronic erosion of an area. When longshore diffusion is considered alone, the true effect of the erosion is minimized. However, with the addition of background erosion, the beachfill project erodes much faster. This total chronic erosion is used to determine the time at which a project needs to be renourished based on long-term erosion processes. This does not, however, account for the impact of severe storm as these will be added in the next section.



Erosion Model Inputs

Beach slope (m)	0.077
Berm Height (B)	6
Dune Height (D)	10.5
Berm width (W)	145
Dune Width (Wd)	25

Sediment Parameter (A)	0.225
Nourishment Length (l), mile	4
Original Berm Width (Wo)	25
Construction Width (Wc)	187
Construction Slope (mc)	0.066667

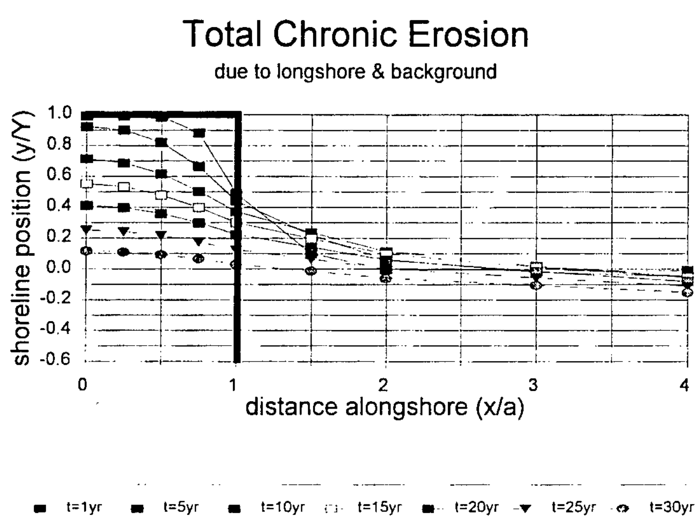


Figure 18. Comparison of longshore diffusion with combined background and longshore erosion

Storm Occurrence Model

The model developed for storm erosion can determine the volume eroded as well as the linear distance that a beach berm or dune has been eroded in a storm. In order to implement the model, it is necessary to input the beach characteristics of the area as well as the storm surge, storm duration, and wave heights for various storms of different return periods as shown in Figure 12.

It is impossible to predict exactly when a powerful storm will impact an area. Therefore, this model has been developed to probabilistically look at the effects of storms on a beach. Storms intensities are usually defined by the return period (T_r) of the storm. This return period describes historically how often, on average, a storm of a certain magnitude hits an area. For example, a 100-year storm should only impact an area once every 100 years on average. Return periods are also related to storm probability, P, by the following relationship:

$$P = \frac{1}{T_r} \tag{27}$$

so that this 100-year storm would have a probability of occurrence of 0.01 in any year. For this study, the intensity of a storm is determined by the two major inputs into the Kriebel-Dean erosion model, the storm surge, S, and the breaking wave height, H_b, of the storm. The area in which the project being studied is located will determine the exact values of these two inputs. A historical study of an area yields a plot such as Figure 16 which shows how storm surge and breaking wave height increase as the return period of a storm increases. The levels read off this plot will be used in this model to determine the intensity of a specific storm.

Not all storms are the same however. They vary not only in their intensity but in the duration for which they impact an area. The two major classes of storms that this study deals with are hurricanes and northeasters or winter storms. Hurricanes are extremely powerful storms that last for short durations. They persist for 12 to 24 hours typically and have an erosion time scale term β of approximately 10.5 (Kriebel and Dean, 1993). In addition, they usually only occur during the hurricane season which roughly corresponds to the period from July through September. Winter storms are not as powerful as hurricanes, but these gales last for much longer times of 48 to 72 hours. They usually have a time scale parameter of $\beta \approx 0.90$ (Kriebel and Dean, 1993). These storms can occur any time during the period from October to March. Even though the winter storms do not have the power of a hurricane, they often produce more erosion due to the longer time which the storm stresses the beach.

Smaller storms may cause some beach retreat, but usually the beach is able to recover because sand is not moved seaward past the depth of closure. The larger, more powerful storms tend to drive sand out to sea where once it passes the depth of closure, the sand will not be able to return accrete on the beach due to natural wave motions. With this model it will be possible to analyze the effects of not only different storms, but also differing beach conditions such as berm and dune height as well as sand sizes in order to determine if certain areas or designs are better suited for beach nourishment.

In this study the following assumptions are made with regard to storm occurrence in any year:

1st quarter -- Winter storms only
2nd quarter-- No storms
3rd quarter -- Hurricanes only
4th quarter -- Winter storms only

The assumption that no storms will occur during the second quarter of the year based on the fact that this time of the year is usually devoid of large storms. In addition, the duration of a storm, T_d, will be varied so as to match hurricanes with a $\beta = 10.5$ and winter storms with a $\beta = 0.90$. In addition, the model uses separate storm intensity distributions for hurricanes and winter storms. This allows for the differing effects of the longer but weaker winter storms and the short, powerful hurricanes to be accurately modeled.

In order to determine the intensity of a storm that may occur during each quarter in which a storm is possible, the model uses a random number generator to pick a random number that corresponds to the probability of occurrence of a storm. This probability is then converted into the return period of the storm using Equation 27. This model uses the assumption that storms with return periods of five years and less will not have a lasting effect on the beachfill project, and thus these storms will not be considered in this study. Any erosion caused by these smaller storms will be small enough that the beach will naturally repair itself unless a larger storm hits the project. The other assumption that the model uses is that only one major storm may impact an area per quarter. This assumption could easily be eliminated in any future study so as to allow two or more storms to impact an area in any season.

Once the return period of the storm is determined, the model calculates the storm surge and breaking wave height that matches that storm. The model takes the probability curve as shown in Figure 16 and matches it to a best-fit curve in the form of

$$S = z(T_r - 5)^2 + S_5 \tag{28}$$

S = the storm surge for a specific intensity
z = coefficient used to change shape of curve to fit probability plot
S_5 = storm surge at the 5 year return period

This equation could also be used for breaking wave height by substituting H_b for S. Figure 19 provides an example of this curve fit. Even though the curves do not match exactly at higher return periods, the most important section, the lower return periods, match well. The lower return periods are more important because this is where the majority of return periods will be plotted.

Once the storm surge and breaking wave height values are probabilistically determined, they can be entered into the Kriebel-Dean storm erosion model along with the appropriate β value based on the type of storm. From this, the maximum erosion associated with each storm can be determined including the retreat of the berm or dune, R. This probabilistic storm model will be used along with the economic model discussed next in order to help determine the renourishment interval necessary for a project. In general, renourishment is assumed to be required if the dune erodes more than 20 feet at a point three-quarters of the way from the center of the project.

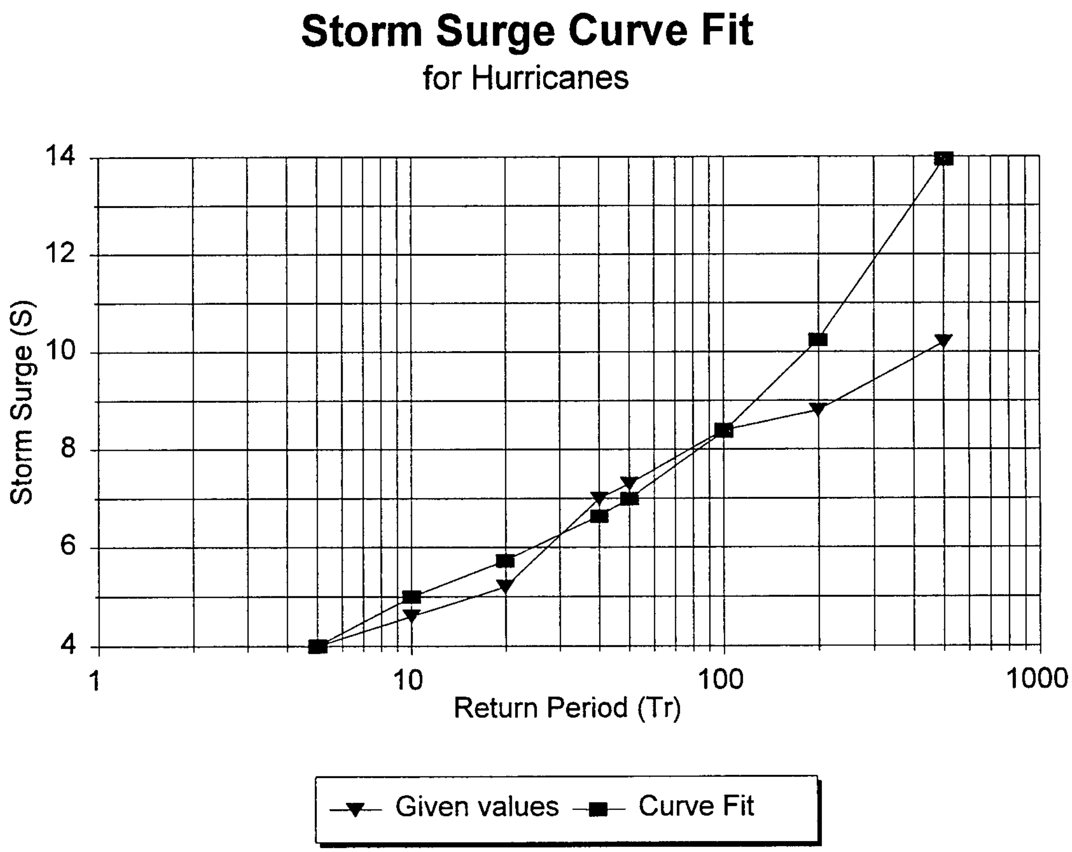


Figure 19. Curve fit for storm intensities in a hurricane

ECONOMIC ANALYSIS MODEL

When a proposed project is being analyzed to determine its predicted effectiveness, the project's actual performance is not the only factor that is considered. The life cycle costs of a project must also be analyzed in order to determine the feasibility of a project. In the political realities of today, a project is often accepted or rejected not on its expected performance, but rather on its cost. Thus, cost is often the determining factor in deciding whether funding will be approved for a beach nourishment project. When analyzing the life cycle cost of such a project, just as when determining its erosion characteristics, there is not just one cost that must be considered and analyzed, but a combination of many.

For an erosion control project, the most widely used method of analyzing costs is to express the costs as an equivalent uniform annual cost or EUAC. This method combines all costs throughout the life cycle of a project and amortizes these costs into an equal annual cost. This equivalent annual cost can then be easily compared with other nourishment projects or other erosion control alternatives to determine the most cost-effective protection method. Total costs expressed as an EUAC allow a community to see how much a project will cost per year so it can determine if such an amount can be allocated from its budget and thus determine if the project is economically feasible. In order to calculate an EUAC or any other expression of life cycle cost, two major inputs are necessary. These are the design life of the project and the discount rate.

The design life of a project is the term for which the costs are amortized. The model has been developed so that the design life can be altered. However, "for Federal projects, project life is established by the Congressional authorization for the project and is usually 50 years" (Camfield, 1993). This study adopts this guideline and establish 50 years as the design life of any project.

In 1972 the Office of Management and Budget set 10% as the government opportunity cost based on competing for available funds with private industry (Newman and Johnson 1995). This discount rate is to be used for public works projects including beach nourishment and erosion control. This model will use the 10% discount rate to determine the time value of money as it is amortized throughout the project life.

This project will analyze two different methods of determining the total cost of such a project. The first will be the standard method used by the Army Corps of Engineers. The second will use the storm occurrence spreadsheet model to develop a new method of determining the renourishment and storm repair costs. This study will compare results from both of these models in order to evaluate the reliability of present Army Corps methods of determining the renourishment interval based on projected resistance of a project to erosion

In order to determine the total EUAC of a beach nourishment and erosion control project, it is necessary to determine three different costs; the initial cost, the renourishment cost, and the storm repair cost. The traditional Corps of Engineers model defines the total EUAC of a project as:

$$EUAC_T = EUAC_{CR} + EUAC_R + EUAC_S$$

(Newman and Johnson, 1995) (29)

$EUAC_T$ = Total EUAC of the project
$EUAC_{CR}$ = Capital recovery of the initial cost
$EUAC_R$ = Annual renourishment cost
$EUAC_S$ = Annual storm repair cost

The dynamic model developed in this project defines the total EUAC of a project as:

$$EUAC_T = EUAC_{CR} + EUAC_{R+S}$$

(Newman and Johnson, 1995) (30)

$EUAC_{R+S}$ = Combined annual renourishment and storm cost

These three costs will next be developed further so as to illustrate how they are determined and entered into the spreadsheet model.

Initial Cost

The initial cost of a project is simply the cost of actually placing the initial volume of sand onto the beach. Thus, the first step in determining this cost is to determine the initial volume of sand to be placed on the beach. The spreadsheet model does this by combining the construction volume associated with widening the beach berm with any sand placed onto the dune. The required total volume of sand is then multiplied by the cost of placing sand onto the project site. For purposes of this study the placement of sand on the project will be assumed to cost \$7.00 / yd^3 as was used in studies carried out along the Delaware Coast (Army Corps Philadelphia District, 1995).

This initial cost is determined as a present cost since it is calculated at the beginning of the project life. The present value is then converted to an equivalent uniform capital recovery cost ($EUAC_{CR}$):

$$EUAC_{CR} = P_o\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = P_o\left(\frac{A}{P}, i, n\right)$$

(Newman and Johnson 1995) (31)

i = discount rate = 10%
n = design life of project
P_o= Present value initial cost of the project

Figure 20 shows this amortization graphically. From this plot, it is possible to see how the present cost of the beachfill is hypothetically spread out over the design life of the project.

The same method of determining the $EUAC_{CR}$ due to initial cost is used in both the Corps of Engineers model and the dynamic model developed in this project except for one

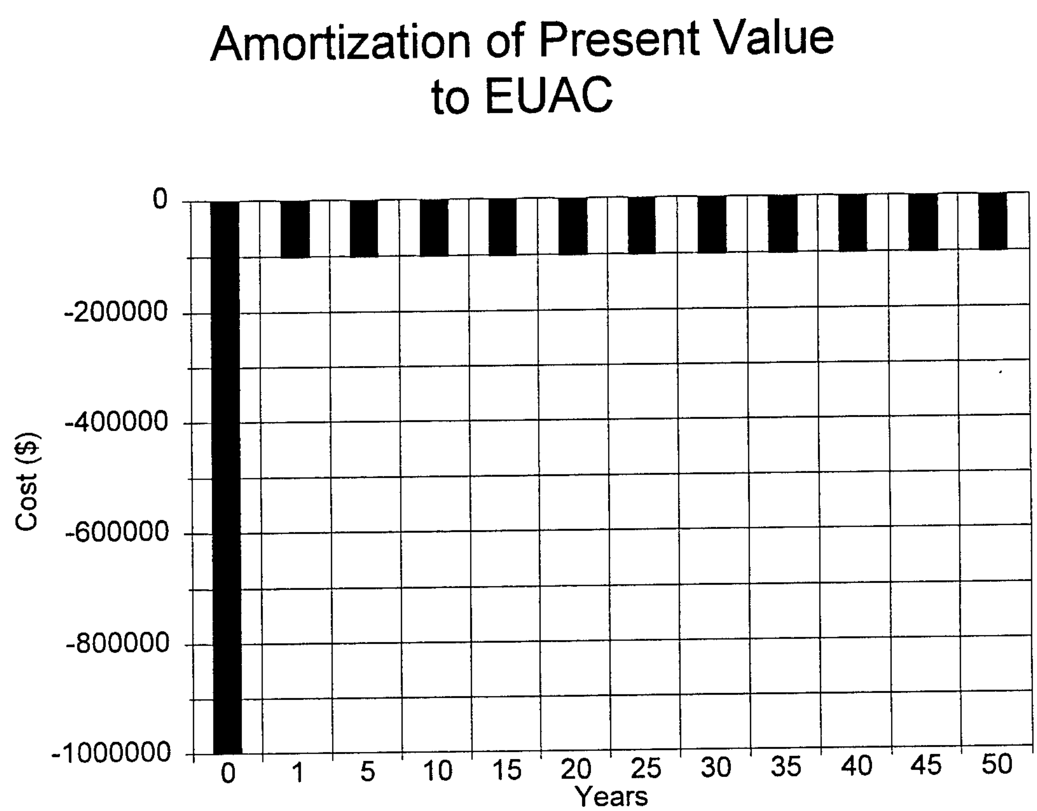


Figure 20. Amortization of present value into annual cost

difference. The Corps of Engineers builds their projects with a small amount of advance nourishment built into the design. This advance nourishment is equal to the amount of background erosion expected in the area during the renourishment interval. For example, in an area with a mean historical erosion rate of 2 feet/yr, a project that was designed with a renourishment interval of 5 years would add 10 feet of berm width to the project in order to account for this background erosion. This assumption does not include any reference to the random nature of the historical erosion. The result of this advance nourishment is to increase the initial cost of a project as the renourishment interval increases.

Thus, by knowing the initial volume of sand placed on the project, it is possible to compute the equivalent annual cost of constructing the project. This initial cost will then be combined with the renourishment and storm rehabilitation costs to determine the total cost of the project.

Renourishment Cost

Due to the dynamic nature of beaches, the initial project that is placed down will not remain in its original position. As the beach begins to erode, the storm protection qualities of the beach will also erode. Therefore, at some point it is necessary to renourish a project. Renourishment is the process of replacing sand that has been lost to erosion. Renourishment is similar to the actual placing of sand in that it will be done in the same manner. The volume of sand used, however, will be less. An important question that has been debated is the interval at which a project needs to be renourished.

The present method as used by the Army Corps of Engineers stipulates that when a project is designed, some baseline must be set as the minimum storm protection level that a project will always have. Without this, there can be no quantitative decision as to the renourishment interval of a project. It is not enough just to notice that the size of the initial project has dropped off substantially. It is important to remember that the initial construction width of a project will conform to the beach profile in the first few months after being laid down. The model developed for this project monitors the evolution of the beach throughout time, thus making it possible to determine when a beach has reached its minimum storm protection level.

For this analysis of the Army Corps' static methods, the beach will be renourished at an interval that provides the lowest project costs to the Corps. In order to determine this interval, the Corps' method will analyze the cost of the projects based on differing return periods. The initial cost will increase slightly with increased renourishment interval. In addition, the renourishment cost of the project will change as different renourishment intervals are altered. Once these costs have been determined, the overall project costs will be compared in order to determine which renourishment interval produces the lowest total cost. This value then becomes the design interval for the project.

The cost of placing the sand during renourishment is a future cost that will occur at a time determined by the renourishment criterion noted above. The design life of a project also comes into effect here as it determines how many renourishments may be necessary for a project. The economic model takes these separate projected future costs and individually brings them back into a present cost in the baseline year using the equation:

$$P = F_n\ (1+i)^{-n} = F_n(\frac{P}{F}),i,n)$$

(Newman and Johnson, 1995) (32)

P = present value
F_n = future cost of renourishment
i = discount rate= 10%
n = year that the renourishment occurs

This equation is used for each separate renourishment during the design life of the project. Then these individual renourishment costs are summed in order to produce the total present value of the project as shown in Figure 21. This present value is analogous to the present value initial cost of the project. Therefore, the present value renourishment cost can be converted into an EUAC by using Equation 30.

The dynamic probabilistic renourishment interval method developed for this project determination begins with the original project. The storm occurrence model is then run to determine when the first major projected storm occurs. This storm is then tested in the storm erosion model to determine if any dune erosion would occur. If the dune does not erode, or

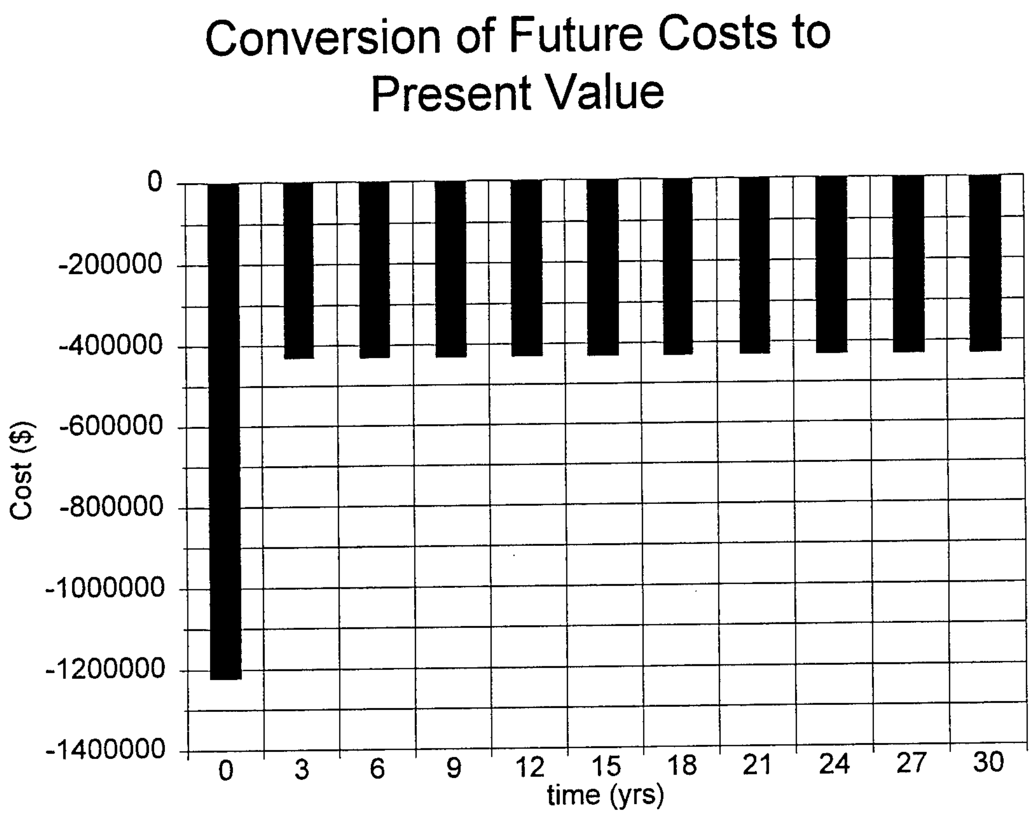


Figure 21. Future values brought back to a present value

if less than 20 feet of dune erosion occurs, then the next storm in the model is used with the shoreline position from before as the starting point for the project. However, if more than 20 feet of dune erosion occurs, then it is assumed that the project will be rebuilt just after the time the storm occurred and the renourishment volume is based on the volume of sand that was eroded from the beach profile. The rebuilt project is now restored back to it original dimensions. The project is then analyzed again using the same method until a large enough sample of renourishment times and volumes has been collected. This method produces some simulations in which the beach has to be rebuilt within the first quarter that is was built. In addition, there are some simulations in which the beach lasts for over fifteen years. These varying times are then analyzed to produce a mean renourishment interval as well as the standard deviation of the renourishment interval. A similar procedure is also used for the volume of sand needed for renourishment.

Once the mean renourishment interval has been determined, the same methods as described earlier are used to determine the EUAC of renourishment costs for the project. This new approach combines the effect of chronic erosion with storm probabilities. In a later application it will be compared with the previous existing Corps of Engineers method in order to analyze any differences that develop. This new approach will also be useful in providing an answer to Pilkey's argument that the Army Corps plans renourishment projects at too long of an interval, thus overestimating the performance of their beachfill projects.

Storm Rehabilitation Cost - Traditional Approach

In the traditional approach used by the Corps of Engineers, the storm rehabilitation is determined in a different way. The first step in developing the Corps' model is to determine the return periods that will be studied. For this model the return periods 5, 10, 20, 50, 100, 200, and 500 years are analyzed. These return periods are then converted to their frequency of occurrence which is given by Equation 27. The model as shown in Figure 22 then determines the interval between each discrete frequency.

The next input into the model is the volume and thus cost of rebuilding the beachfill project for each return period. This is determined by using the storm erosion model for each storm intensity desired. These rehabilitation costs are then averaged between two successive return periods. This average is multiplied by its corresponding frequency interval in order to determine an interval annual cost. These interval costs are summed to determine the total equivalent annual storm rehabilitation cost ($EUAC_S$).

This method of determining the cost of rebuilding the project takes into account the probabilistic nature of storm damage without relying on time-stepping simulations as is used in the spreadsheet model. The equivalent annual cost represents a value that corresponds to the amount of money that should be set aside annually in order to prepare for a possible storm. If no large storm requiring renourishment occurs, then hypothetically, this fund should be allowed to continue to grow so that any later contingencies may be met. Even though in reality this money is not set aside properly, for this model the assumption is made that money is allocated every year for this rehabilitation cost.

Storm probabilities and their effect on the cost of a beachfill project are a difficult

Plan 1 for S Bethany Beach w/ 2 yr spacing						
Storm recurrence	Storm probability	Frequency Interval	Volume eroded	Reconstruction costs	Cost Average	Interval Damage
3	0.33		0	$0		
		0.133			$2,169,339	$289,245
5	0.2		619,811	$4,338,677		
		0.100			$4,847,360	$484,736
10	0.1		765,149	$5,356,043		
		0.050			$5,799,003	$289,950
20	0.05		891,709	$6,241,963		
		0.030			$7,117,390	$213,522
50	0.02		1,141,831	$7,992,817		
		0.010			$8,475,376	$84,754
100	0.01		1,279,705	$8,957,935		
		0.005			$9,417,912	$47,090
200	0.005		1,411,127	$9,877,889		
		0.003			$10,672,963	$32,019
500	0.002		1,638,291	$11,468,037		
		0.002			$11,468,037	$22,936
MAX	0.0		1,638,291	$11,468,037		
		0.000			$5,734,019	$0

Annual Storm Damage
$1,464,251

Figure 22. Example of Army Corps storm probability model

question. Thus, this study will look at both models in order to determine any disparity between the two. This analysis should provide part of the answer to the Pilkey-Houston debate mentioned earlier.

SENSITIVITY ANALYSIS OF BEACHFILL PARAMETERS

As the equations developed earlier for beachfill performance indicate, there are a wide variety of parameters that influence beach response. These parameters include natural site-specific variables such as the sediment size of a project as well as designer controlled variables such as the dune height and berm width of a project. Optimization of these parameters is necessary in order to plan a project that is economically feasible while still providing adequate storm protection. The spreadsheet model developed in this project allows for the variation of these parameters so that each one's individual effect on the project can be analyzed. From this sensitivity analysis it will be possible to provide general guidelines for beachfill design that take into account the effect of various parameters associated with the project.

This analysis will look into the effect of various parameters on both the chronic or progressive erosion characteristics of a beachfill as well as the beachfill's storm erosion response. These two erosion responses have different controlling parameters. In order to properly design a beachfill project, the parameters which control the different responses must be optimized. This optimization must also be sensitive to the economic costs of the project. Often, a design that would provide excellent storm protection will simply be too costly for a community to consider. Therefore, proper optimization of a project must include a compromise between beachfill performance and cost.

This study will first analyze the effect of various parameters on the chronic erosion response of a project. This response includes the combination of both longshore and background erosion that act continually on a project. The progressive response is characterized by Equation 26 which depends on the longshore diffusivity term K described in Equation 13. After the chronic erosion response has been simulated, the storm erosion of a beach will be analyzed. This storm response is based on the Kriebel-Dean erosion model as defined by Equations 16 through 24. In order to analyze the overall performance of a project, the following design parameters will be adjusted in order to determine their effect on a project:

Beachfill project length (L)
Berm width (W)
Berm height (B)
Dune height (D)

Once these adjustable variables have been analyzed, a number of environmental variables which depend on the location of a project will be investigated. These location parameters include:

Beach slope (m)
Sediment size (A)
Wave climate

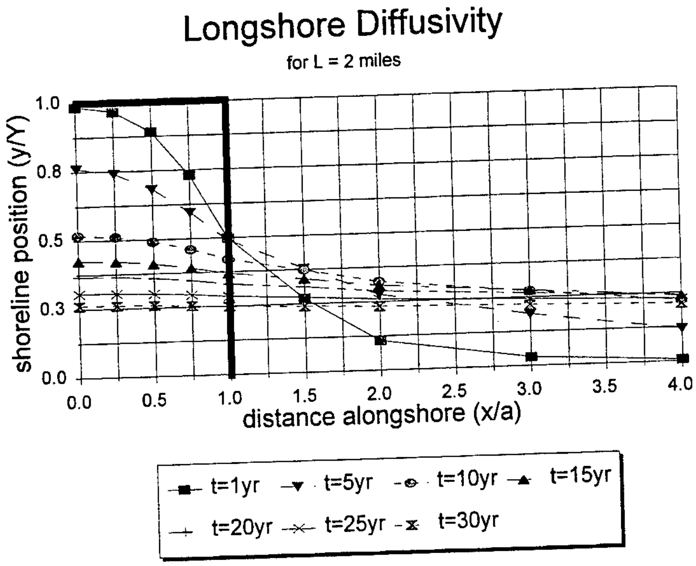
The goal of this portion of the study is to determine if certain variables in the chronic erosion and storm responses make a beachfill project either advantageous or unfeasible to build. This study will show which parameters have a controlling effect on both the performance and the cost of a beachfill project.

Beachfill Project Length (L)

Adjusting the length of a beachfill project in the longshore direction has a number of important effects on the response of a beach. First of all, increasing the length of a project will result in an increase in the initial as well as renourishment costs of a project. This occurs because the longer project length increases the volume of sand that is required for an area. In addition, the storm rehabilitatition costs will also increase as the increased length of the project will increase the volume of sand eroded by a storm.

However, the length of a project has an inverse effect on the longshore diffusion response of the beachfill with longer projects suffering relatively less from longshore diffusion. Shorter projects of under 5 miles in overall length have a definite loss of beach width in the center of the project as shown in Figure 23. As time progresses this narrow beach begins to flatten out as the wave action attempts to eliminate the bulge in the shoreline due to the beachfill project. However, as the length of the project increases up through 10 miles, the response begins to remain much more stable in the center of the project. Here the only real loss of berm width occurs on the project endpoints. Even this loss remains stable at approximately 50% of the original length over a long time period. As Figure 23 shows, this type of response with a stable center continues as the project length increases to over 10 miles. With the longer projects, wave action will require extremely long periods of time to eliminate a project through longshore diffusion.

When background erosion is analyzed along with the longshore diffusion, the general effect is an increase in the erosion of the beach. The general shape of the response does not change, however, the rate at which the beach erodes does change. The background erosion will have this same effect on all of the beachfill design parameters. This increased erosion rate for a shorter project as shown in Figure 24 results in the project completely eroding in



Erosion Model Inputs

Beach slope (m)	0.080
Berm Height (B)	6
Dune Height (D)	6
Berm width (W)	150
Dune Width (Wd)	25

Sediment Parameter (A)	0.25
Nourishment Length (l), mile	[illegible]
Original Berm Width (Wo)	25
Construction Width (Wc)	173
Construction Slope (mc)	0.066667

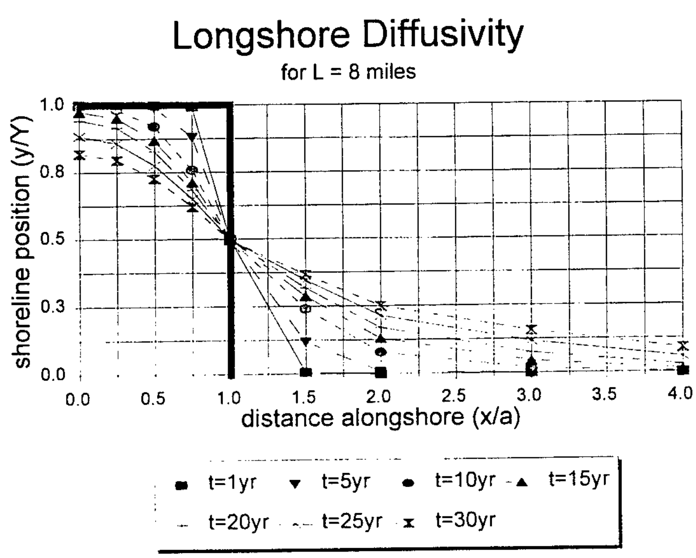
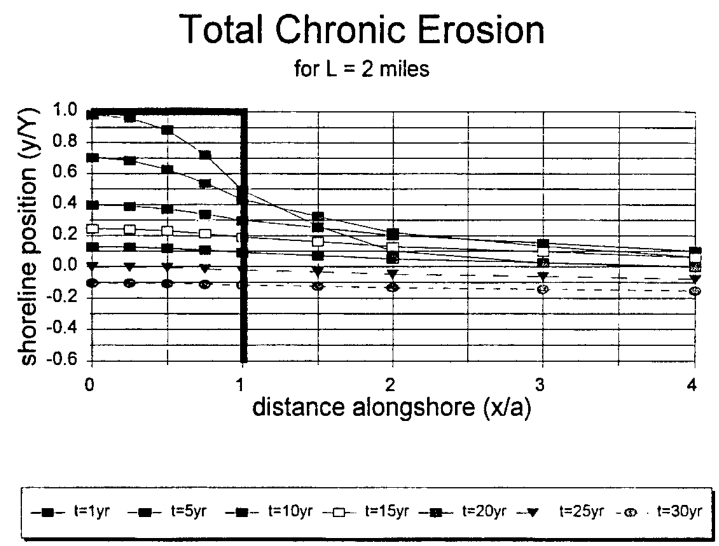


Figure 23. Longshore diffusion for varying project lengths



Erosion Model Inputs

Beach slope (m)	0.080
Berm Height (B)	6
Dune Height (D)	6
Berm width (W)	150
Dune Width (Wd)	25

Sediment Parameter (A)	0.25
Nourishment Length (l), mile	
Original Berm Width (Wo)	25
Construction Width (Wc)	173
Construction Slope (mc)	0.066667

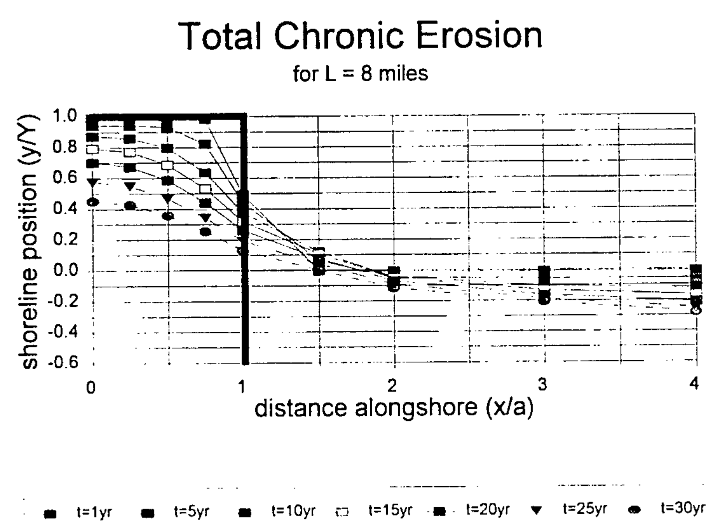


Figure 24. Total erosion graph for varying project lengths

a relatively short period of time. Longer projects also have increased erosion, however, these projects are able to retain more of their storm protection beach since longshore spreading losses are not as severe.

The slower long-term or progressive erosion that occurs on a longer beach also provides greater storm damage reduction associated with the project length. The longer projects will be able to withstand a storm that occurs a number of years after construction simply because the longer project will have more of its original beach width present than a shorter project will. Thus, the smaller projects do not provide the degree of storm protection for an area as a larger project does.

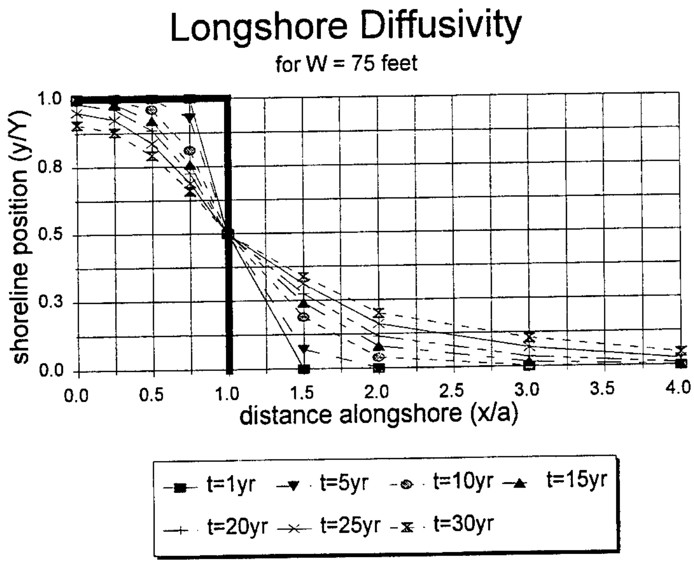
One method which would be beneficial in improving storm protection for a specific site would be to build the project wider than the area under concern. Then, when increased erosion occurs at the project endpoints, the area under concern will be behind the wider portion of the beach and not the sacrificial endpoints. In order for this to occur, adjacent communities would have to agree if the projects crossed geographic boundary lines.

Therefore, the general effect of increasing the length of a project is to increase the beachfill's resistance to erosion. Economically, there are negatives to both large and small projects. Large projects have the obvious problem of increased costs both initially and when renourishment occurs. However, the larger project will not have to be renourished as frequently as a smaller project will, thus providing better storm protection. These less frequent renourishments, while larger in size, may still be less costly than a smaller project which will require much more rapid renourishing due to the fact that the beachfill erodes quickly. One method that could be used to determine an optimal length for the project would be to run the model for varying lengths in order to determine one with a minimum EUAC. Often, this parameter geographically does not have the option of being altered. Therefore, the rapid erosion and lessened storm protection potential of smaller projects must be considered and may make such projects economically unfeasible to a community.

Berm Width (W)

The berm width of a project is the major design factor in providing storm protection. When analyzed in terms of longshore diffusion alone, there is no change in the nondimensional erosion plot as is shown in Figure 25. When the background erosion is factored in as is done in Figure 26 the wider berms provide much better chronic erosion protection. The increased width of the berm allows more sand to erode before the erosion reaches the dune. Thus, a wider project will not need as frequent nourishing as a narrower project because beachfill longevity is proportional to berm width.

The major benefit associated with increasing the berm width is in the area of storm protection. The wider berm has more sand to sacrifice to the energy of a storm before the dune begins to erode. For example, a beachfill project with a berm width of 200 feet will have 131 feet of berm erosion, as shown in Figure 27, when a 50-year storm hits the project. However, the same storm impacting a project with a berm width of only 150 feet will result in the complete erosion of the 150 foot berm plus complete erosion of the protective dune. Thus, the smaller project places the structures behind the beachfill in much greater danger of wave damage as this damaged dune will not provide adequate protection for the inland areas.



Erosion Model Inputs

Beach slope (m)	0.080
Berm Height (B)	6
Dune Height (D)	6
Berm width (W)	[illegible]
Dune Width (Wd)	25

Sediment Parameter (A)	0.25
Nourishment Length (l), mile	10
Original Berm Width (Wo)	25
Construction Width (Wc)	173
Construction Slope (mc)	0.066667

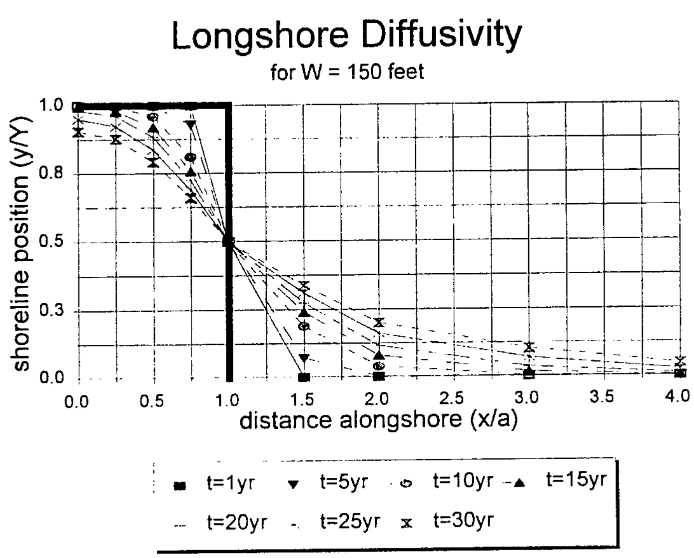
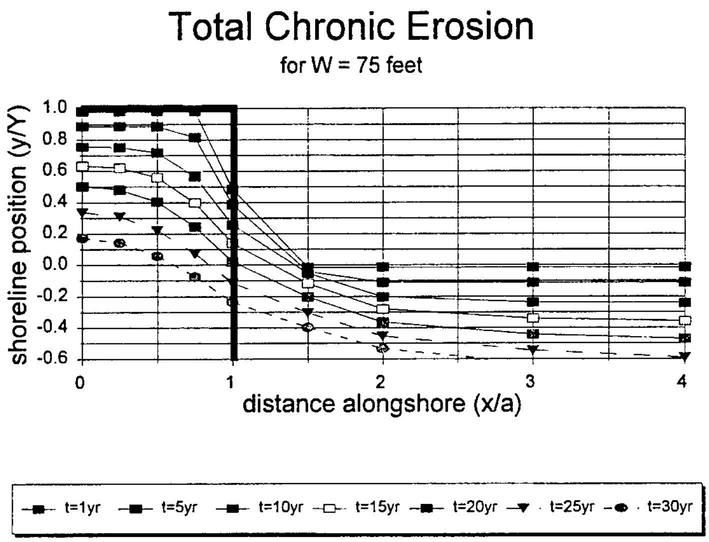


Figure 25. Longshore diffusion for varying berm width (W)



Erosion Model Inputs

Beach slope (m)	0.080
Berm Height (B)	6
Dune Height (D)	6
Berm width (W)	[illegible]
Dune Width (Wd)	25

Sediment Parameter (A)	0.25
Nourishment Length (l), mile	10
Original Berm Width (Wo)	25
Construction Width (Wc)	173
Construction Slope (mc)	0.066667

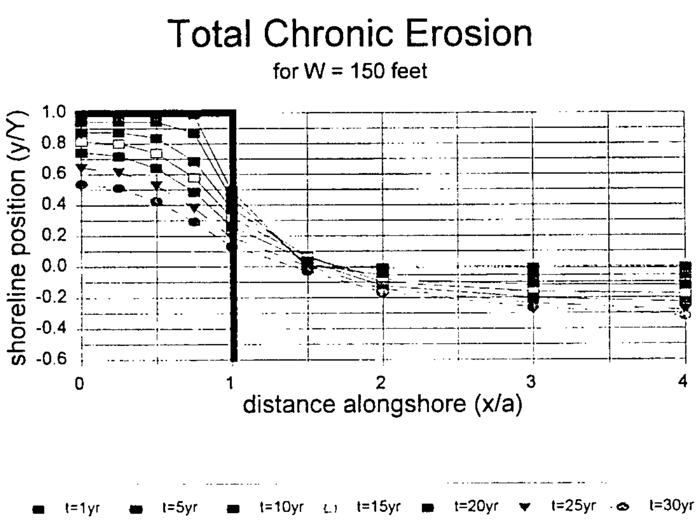


Figure 26. Total erosion graph for varying berm width (W)

Base Profile

Inputs		Tr	Rfinal	Type
W=	150	5	72.5	B
B=	8	10	87.80	B
Area=	Delaware	20	3.20	D
D=	8	50	29.50	D
m=	0.067	100	48.30	D
A=	0.25	200	66.7	D
		500	93.5	D

Vary width W

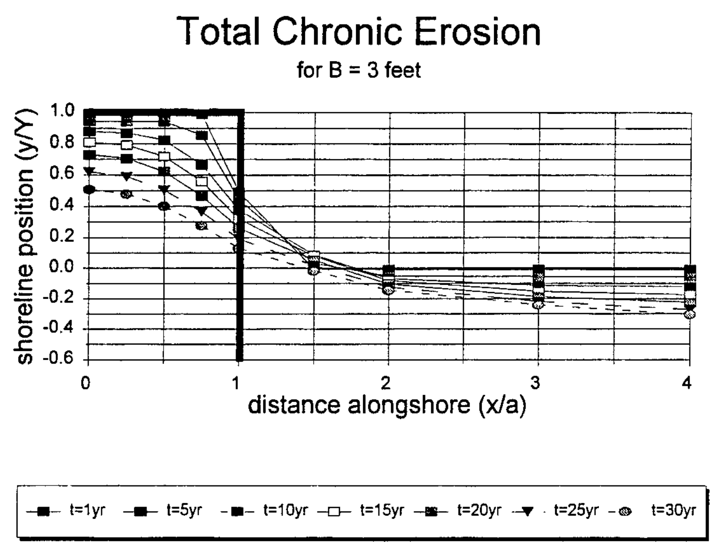
Inputs		Tr	Rfinal	Type
W=	200	5	72.5	B
B=	8	10	87.70	B
Area=	Delaware	20	103.00	B
D=	8	50	131.00	B
m=	0.067	100	14.60	D
A=	0.25	200	32.7	D
		500	58.2	D

Figure 27. Storm result table for varying berm width (W)

Just as increasing the length of a project will increase the volume of sand required for construction, the longer the berm width the more expensive a project will be. For each additional foot of berm width must add a volume of sand equal to 1* (B + d_c) * L to the project. This expense will have to be analyzed against the less frequent renourishing along with the increased storm protection that will develop with the project in order to determine an optimal berm width for a region.

Berm Height (B)

When analyzed for chronic erosion alone, the berm height of a beachfill project does not change the project's response as is shown in Figure 28. This is a result of model assumption that the beach erodes from the top of the berm out to the depth of closure. However the berm height of a project does have a controlling influence over the storm response of the project. Increasing the height of the berm reduces the degree of erosion that will occur due to a storm. This occurs for the same reason the wider berm width is more effective; because the larger berm has more sand to sacrifice to the wave energy before the dune erodes. Figure 29 shows an example of this effect. In this example the doubling of the berm height allows the beach to withstand the impact of up to a 50-year storm without damaging the dune as opposed to only being able to withstand a 20-year storm with the smaller berm. However, the height of the berm does not limit erosion to the degree that some other parameters do. There is an increase in storm protection with a larger berm, but not an extraordinarily large increase as can be explained by looking at Equation 18 which reveals the limited influence of berm height in the storm erosion response of the project.



Erosion Model Inputs

Beach slope (m)	0.080
Berm Height (B)	[illegible]
Dune Height (D)	6
Berm width (W)	150
Dune Width (Wd)	25

Sediment Parameter (A)	0.25
Nourishment Length (l), mile	10
Original Berm Width (Wo)	25
Construction Width (Wc)	173
Construction Slope (mc)	0.066667

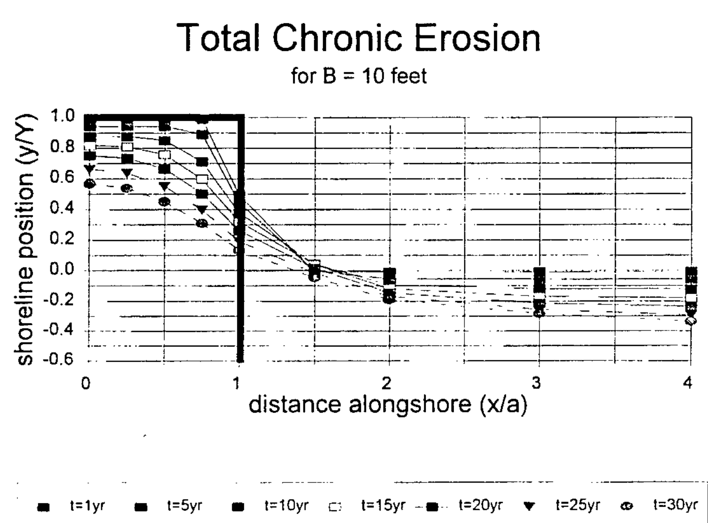


Figure 28. Total erosion graph for varying berm height (B)

Base Profile

Inputs		Tr	Rfinal	Type
W=	150	5	72.5	B
B=	8	10	87.80	B
Area=	Delaware	20	3.20	D
D=	8	50	29.50	D
m=	0.067	100	48.30	D
A=	0.25	200	66.7	D
		500	93.5	D

Vary berm B

Inputs		Tr	Rfinal	Type
W=	150	5	59.8	B
B=	15	10	72.80	B
Area=	Delaware	20	85.60	B
D=	8	50	4.10	D
m=	0.067	100	19.40	D
A=	0.25	200	35.1	D
		500	58.2	D

Figure 29. Storm results for varying berm height (B)

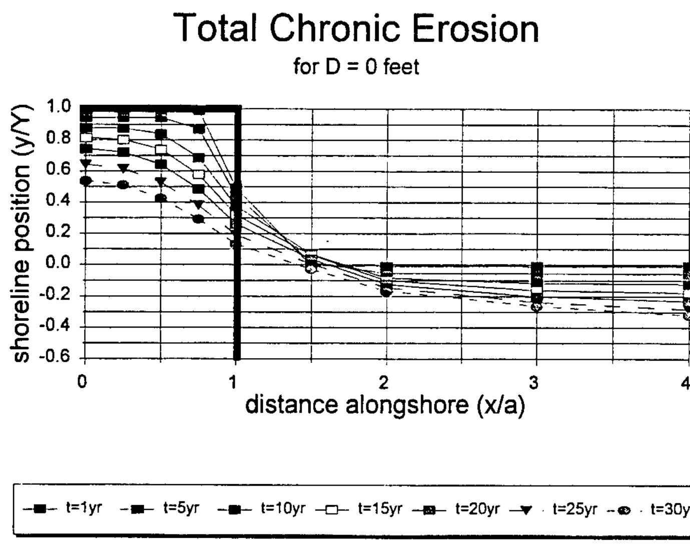
The berm height does have important economic considerations however. This has the effect of increasing the cost of the project. Therefore, the increased cost of the project must be analyzed against the increase in storm protection in order to decide if a larger berm is necessary for a region.

Dune Height (D)

Since longshore diffusion and transport occur along the shoreline, these processes deal with sand in the berm. The dune is assumed to be untouched by this progressive erosion. Figure 30 verifies this by showing how varying dune heights have no impact on the chronic erosion characteristics of the beach.

In the area of storm protection, however, the dune height of a project has an effect, particularly on the most severe storms. As can be seen in Figure 31, the increase in dune height has no real effect on the storm response for lower intensity storms that do not completely erode the berm. When the erosion reaches the dune, the larger dune height slows the erosion so that less dune erosion occurs with the larger dune. This again occurs because there is more sand to yield to the incoming waves. However, when two dune heights are compared, the larger dune has an extremely limited effect on the overall storm response. As shown in Figure 31, the dune still erodes with a storm of the same intensity, the erosion is just slightly less severe. This limited effect of the dune can be shown from Equation 17 in which the dune in not a controlling variable.

As has been shown before with the berm, increasing the height of the dune will increase the volume of sand placed on the beachfill. The larger the berm height, the more



Erosion Model Inputs

Beach slope (m)	0.080
Berm Height (B)	6
Dune Height (D)	[illegible]
Berm width (W)	150
Dune Width (Wd)	25

Sediment Parameter (A)	0.25
Nourishment Length (l), mile	10
Original Berm Width (Wo)	25
Construction Width (Wc)	173
Construction Slope (mc)	0.066667

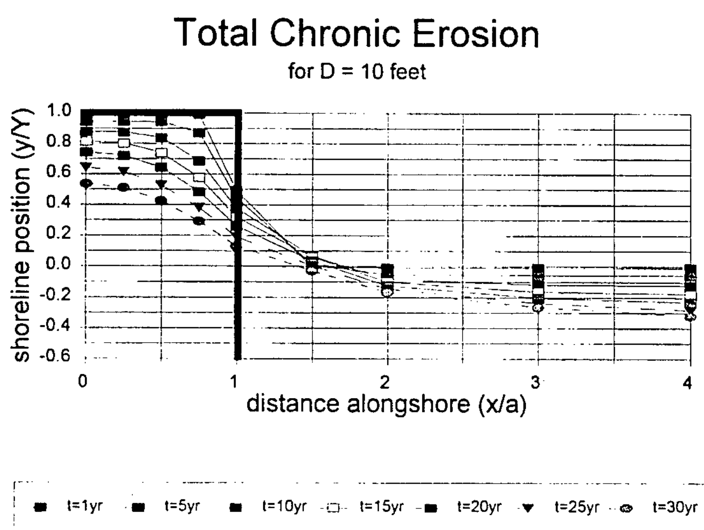


Figure 30. Total erosion graph for varying dune heights (D)

Base Profile

Inputs		Tr	Rfinal	Type
W=	150	5	72.5	B
B=	8	10	87.80	B
Area=	Delaware	20	3.20	D
D=	8	50	29.50	D
m=	0.067	100	48.30	D
A=	0.25	200	66.7	D
		500	93.5	D

Vary dune D

Inputs		Tr	Rfinal	Type
W=	150	5	59.9	B
B=	8	10	72.80	B
Area=	Delaware	20	2.70	D
D=	15	50	25.40	D
m=	0.067	100	40.50	D
A=	0.25	200	56	D
		500	78.8	D

Figure 31. Storm results table for varying dune height (D)

sand must be placed on a project as can be shown by the following equation:

$$Vol\ dune = W_d * D + \frac{D^2}{m_c}$$

W_d = Dune width (32)
D = Dune height
m_c = Dune slope

Thus, the cost of the project will increase. From this analysis, an extremely large dune does not provide exceptional storm protection. The dune does add some benefit, however, this additional protection must be looked at closely in regard to the increased cost that it requires in order to determine if the larger dune is justified.

Beach slope (m)

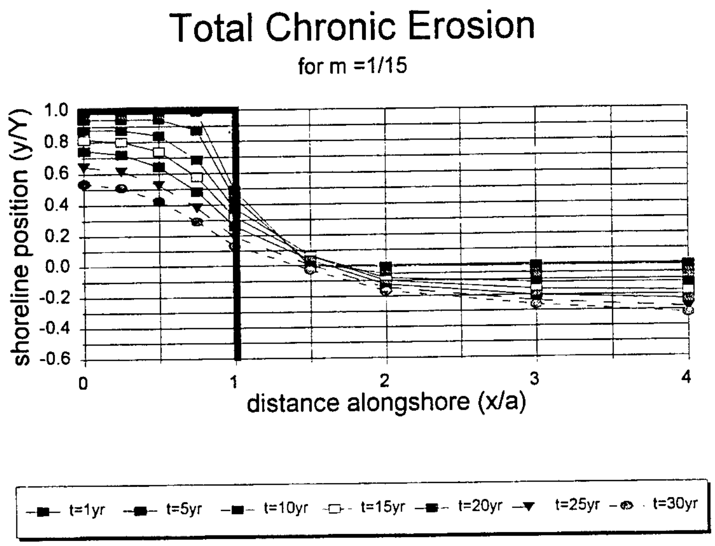
The initial slope of the beach is a parameter over which the designer has limited control since it is controlled by sediment size and wave climate in the area. This slope can change throughout the course of the year as it is often steeper in the summer months and flatter as the more destructive winter waves hit the shoreline. This model, however, assumes a constant slope throughout the life of a project. The use of this model to analyze the effect of beach slope on chronic erosion shows that there is no change in the erosion rate due to differing slopes. Figure 32 shows how the beach slope parameter does not have any control over the chronic erosion response of the beach.

However, the slope of the beach in an area does have a major impact on the storm erosion response. The flatter the slope of the beach, the more resistant the beach is to erosion. On a flat beach, the effect of gravity is not as pronounced when the off rushing wave pulls sand out to sea . The sand on the steeper beaches simply rolls back into the sea with more ease, resulting in more erosion of the beach. For instance, if all other parameters are held steady, a beach with a steeper slope of 1/15 will have dune erosion with only a 20 year storm. However, a flatter beach with a slope of 1/25, as shown in Figure 33, will not have dune erosion until a 200 year storm impacts the area. Therefore, the slope of the beach in an area can determine if beachfill would be a viable alternative for the region. Areas with flat slopes would make excellent candidates for a nourishment project. This analysis does not eliminate areas that may have steeper natural slopes, however, the storm rehabilitation costs of such a region will be more severe. The choice of whether to construct a beach nourishment project on an area with a naturally steep slope must be analyzed with the increased storm damage that such a project will encounter in mind. The storm rehabilitation costs to a region may simply be so much as to make a project economically unfeasible.

Sediment Size (A)

The size of the sand in a particular region is not a parameter over which a designer has any control. However, it is possible to fill a project with a different sized sand than the native sand in order to improve the durability of a project. This study uses general sediment sizes for different regions of the country to investigate the effect that different sediment sizes have on the beach's performance.

According to sediment size data found in the *Shore Protection Manual* (Army Corps,



Erosion Model Inputs

Beach slope (m)	
Berm Height (B)	6
Dune Height (D)	6
Berm width (W)	150
Dune Width (Wd)	25

Sediment Parameter (A)	0.25
Nourishment Length (l), mile	10
Original Berm Width (Wo)	25
Construction Width (Wc)	173
Construction Slope (mc)	0.066667

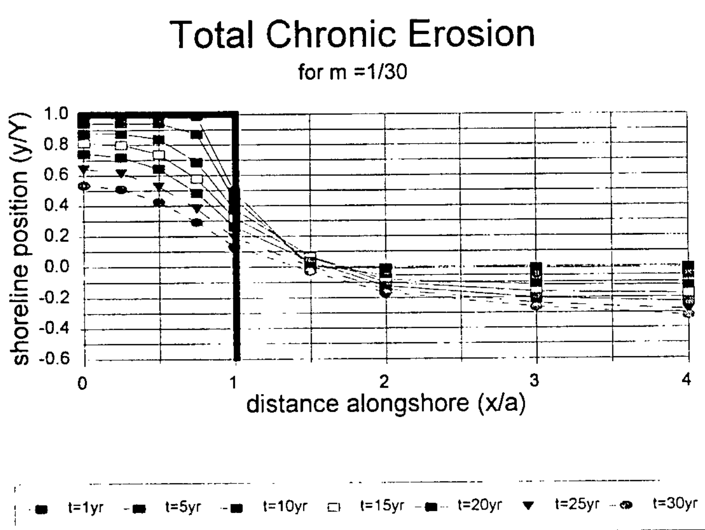


Figure 32. Total erosion graph for varying beach slopes (m)

Base Profile

Inputs		Tr	Rfinal	Type
W=	150	5	72.5	B
B=	8	10	87.80	B
Area=	Delaware	20	3.20	D
D=	8	50	29.50	D
m=	0.067	100	48.30	D
A=	0.25	200	66.7	D
		500	93.5	D

Vary slope m

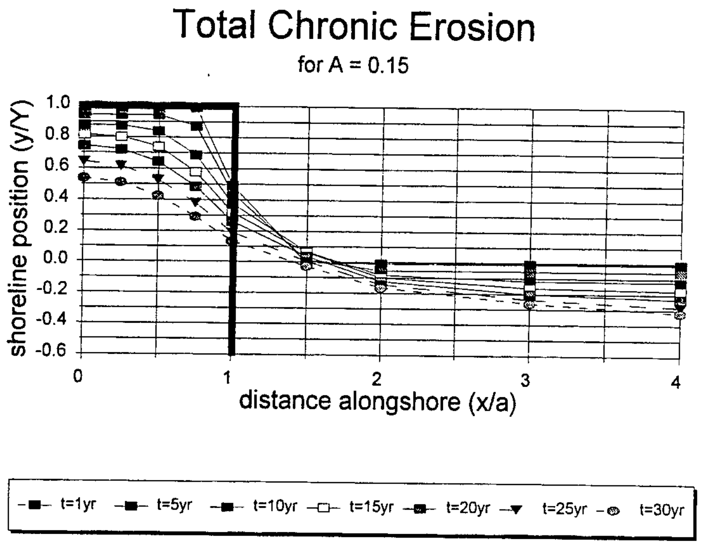
Inputs		Tr	Rfinal	Type
W=	150	5	43.3	B
B=	8	10	53.80	B
Area=	Delaware	20	64.10	B
D=	8	50	84.40	B
m=	0.040	100	97.30	B
A=	0.25	200	9	D
		500	28.5	D

Figure 33. Storm results table for varying beach slope (m)

1984), most beaches on the East Coast of the United States have sediment size parameters, A, that range from 0.20 to 0.28 $ft^{1/3}$. The Gulf Coast begins to show much finer sand with an A value of close to $0.15ft^{1/3}$ while the Pacific Coast has coarser sand in the vicinity of $0.30ft^{1/3}$ for an A parameter . When these varying sediment sizes were analyzed for chronic erosion, there were no changes in the beachfill due to sediment size alone. Figure 34 shows the uniformity of the longshore response of the beach with size parameters from different regions of the country.

However, when storm erosion is analyzed, the sediment size has a large effect. Figure 35 illustrates how larger sediment sizes result in far less erosion of the beachfill project. As the finer sands found in the Gulf Coast are tested, the project begins to erode quickly with even a low intensity storm. The smaller sediment erodes much more quickly because the smaller grains have less mass to hold them in place against the onrushing water. The larger grains, on the other hand, are heavier and thus more resistant to motion.

Therefore, if a beachfill project uses sand that matches the native sand, it may be economically impossible to build a project in an area such as the Gulf Coast which has fine native sands. One solution to this problem would be to bring in fill sand of a coarser nature from an offsite dredge area. This project will not delve into the specific details of matching sediment sizes. However, the *Shore Protection Manual* does contain charts which illustrate the impact of differing sized native and fill sands. The one problem with this solution is the additional cost of importing the sand depending on the location of the site. The best possible areas for a beachfill are those with offshore dredge sites available which match coarse native sand or are coarser than the native sand. The size of the sand available for a beachfill project



Erosion Model Inputs

Beach slope (m)	0.080
Berm Height (B)	6
Dune Height (D)	6
Berm width (W)	150
Dune Width (Wd)	25

Sediment Parameter (A)	
Nourishment Length (l), mile	10
Original Berm Width (Wo)	25
Construction Width (Wc)	173
Construction Slope (mc)	0.066667

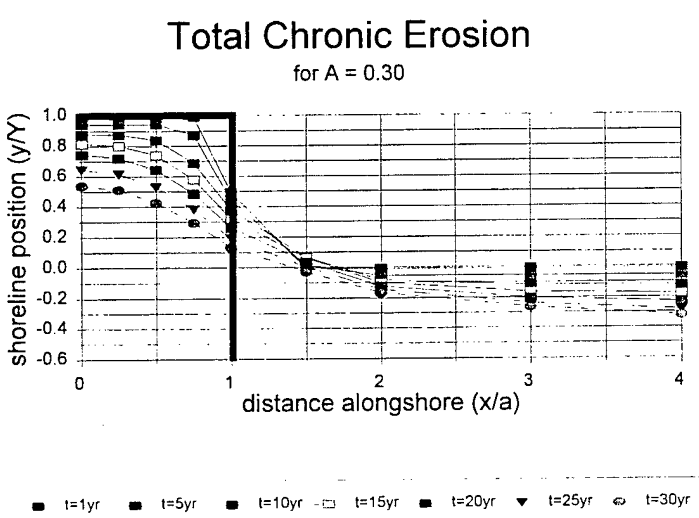


Figure 34. Total erosion graph for varying sediment size

Base Profile

Inputs		Tr	Rfinal	Type
W=	150	5	72.5	B
B=	8	10	87.80	B
Area=	Delaware	20	3.20	D
D=	8	50	29.50	D
m=	0.067	100	48.30	D
A=	0.25	200	66.7	D
		500	93.5	D

Vary sediment size A

Inputs		Tr	Rfinal	Type
W=	150	5	114.2	D
B=	8	10	152.40	D
Area=	Delaware	20	193.80	D
D=	8	50	268.60	D
m=	0.067	100		
A=	0.15	200		
		500		

Figure 35. Storm results table for varying sediment size

must be a major consideration when designing a project. Improper use of fine sand will result in the loss of a project after one small, low-intensity storm.

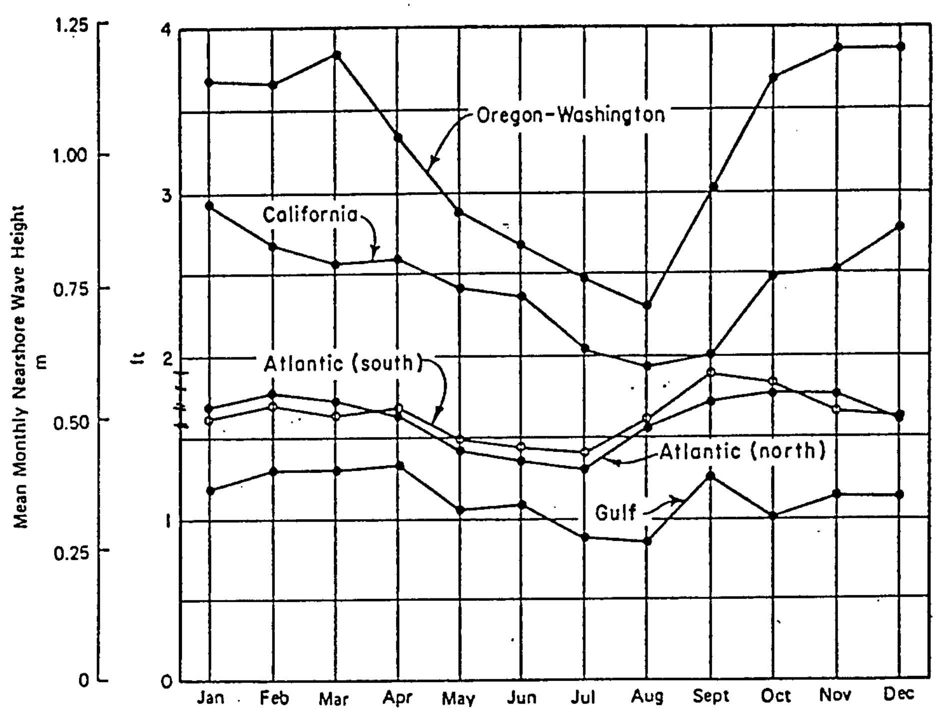
Background Erosion Rate

Areas with extremely high background erosion rates may not be suitable for beach nourishment if no coastal structures are used to alter the longshore transport in an area. If an area has a low historical erosion rate, a beachfill project would be an excellent alternative to consider since the project will not be eroded as quickly as it would in other areas. Overall, the background erosion environment that a region has a major impact in determining the renourishment interval and thus the feasibility of a project.

Wave Climate

The final environmental variable that was studied in this project was the wave climate in which a beachfill project is located. The wave climate refers to the average wave conditions that an area faces. The general mean monthly wave conditions that were used for this study came from the *Shore Protection Manual* (1984) and is shown in Figure 36. Using these long-term wave statistics, it was possible to use the spreadsheet model to determine if the wave climate severely effected the beaches performance.

This study found that areas subjected to more severe wave action face faster longshore spreading than lower wave energy sites. The basis for this observation is found in the diffusivity term K as developed in Equation 16. In this equation the diffusivity is related to wave height as $K \sim H_b^{5/2}$. Therefore, the locations with higher average wave



(Army Corps, 1984)

Figure 36. Mean wave heights for various locations

heights must have a more rapid spreading of the beach due to longshore diffusion. In fact an area with an average wave height just 10% larger that another area would have a diffusivity $(1.1)^{5/2}$ or approximately 1.27 times greater .

The wave climate effects the storm response of a beach only if it produces more intense storm conditions. Also, if the beach is eroded at a quicker pace due to increased spreading, the beachfill project will be more susceptible to damage by storms. Therefore, the project will have to be rebuilt at more frequent intervals in order to maintain an adequate level of storm protection for an area. This more frequent renourishing will again drive the total cost of the project upwards. Thus, the optimal location for a beachfill project would be in an area of lower wave energy. The other alternative would be to combine beachfill with a structural measure such as an offshore breakwater that is designed to limit the wave energy hitting the beach.

Sensitivity Analysis Conclusion

In order to properly design a beachfill project, the combined effect of all these parameters must be considered. No one parameter can determine if a project provides adequate protection or is simply too costly to construct. A community considering beach nourishment must consider various project designs in order to determine a design that meets pre-determined levels of storm protection while staying within economic boundaries determined by funds available for a shore protection project.

Table 1 below provides a summary of the parameter sensitivity analysis performed in this project. The results listed were all determined using the spreadsheet model as developed in this project. This table provides a basic guideline for beachfill design. From these determinations, it is possible to make an initial assumption as to how a project will perform when influenced by progressive and storm related erosion. Once this initial comparison has been made, the most promising projects may be analyzed using the spreadsheet model developed for this study. This model can provide an accurate simulation of the future performance of the beach nourishment project, thus becoming a useful tool for an engineer designing a beach nourishment project.

Effect of Various Beach Nourishment Parameters			
Parameter	*Chronic Erosion*	*Storm Response*	*Economic*
Fill Length (L)	Longer Fill more stable	No effect	Longer fill more expensive
Berm Width (W)	Wider provides more protection	Wider provides more protection	Wider fill more expensive
Berm Height (B)	No effect	Higher berm more protection	Higher berm more expensive
Dune Height (D)	No effect	Higher dune more protection	Higher dune more expensive
Beach slope (m)	No effect	Flatter slope decreased erosion	Flatter slope more expensive
Sediment Size (A)	No effect	Larger sediment decreased erosion	Larger sediment less maint. costs
Erosion Rate	Higher rate increased erosion	No direct effect	Higher more expensive
Wave Climate	Severe climate, increased erosion	Severe storms, more erosion	Severe climate, more expensive

Table 1

MODEL TEST

The final step in this analysis is to simulate a number of hypothetical beach nourishment projects in order to analyze the differences between the Army Corps of Engineer's model and the probabilistic spreadsheet model as described before. From this analysis, it will be possible to determine if there are any drastic differences in the results produced by each model. This will provide an answer to the Pilkey-Houston debate that has been discussed before.

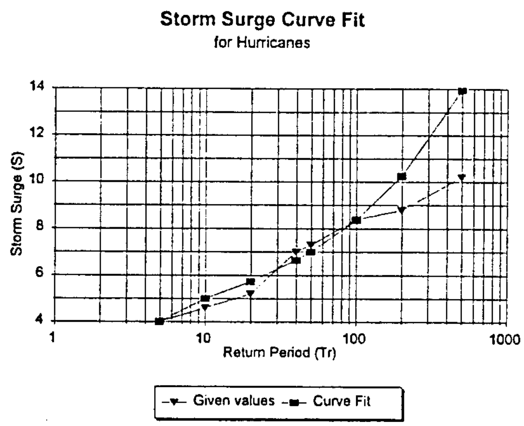
For this simulation, one prototypical site along the coast of Delaware was selected for analysis. This beachfill project was entered into the spreadsheet model using both methods of renourishment interval determination. The results from this simulation were then tabulated into the economic model in order to provide a means of comparison between the two models. Table 2 describes the project analyzed in this study.

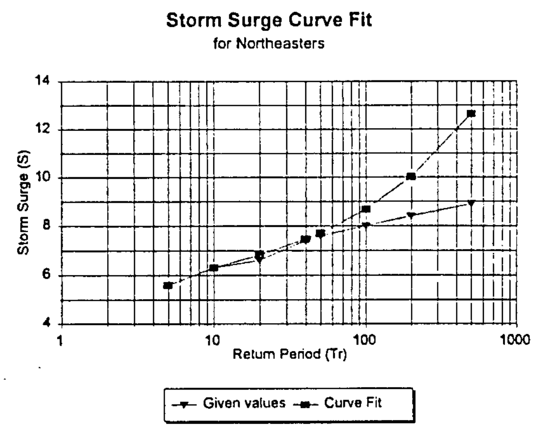
Variables for Delaware Projects Analyzed					
Variable	Unit	S. Bethany	Variable	Unit	S. Bethany
m	-	1:12.5	L	miles	3.5
B	ft	7.0	Wi	ft	25
D	ft	7.5	Wd	ft	148
W	ft	120			

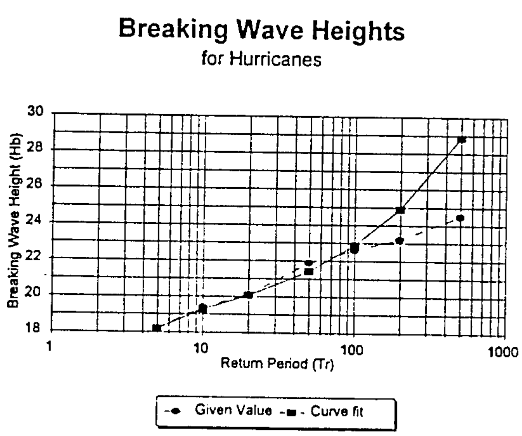
TABLE 2

In addition to the beachfill design variables shown, the background erosion and wave climate in the area had to be inputted into the spreadsheet model. For this study the same background erosion of 1.75 ft/yr with a standard deviation of 0.55ft/yr were used for the project. This provided a relatively mild erosion climate for the study which does not have a realistic probability of accretion or severe erosion. The wave climate that was used for the monthly wave heights came from the *Shore Protection Manual* (1984) as shown in Figure 36. The values for the South Atlantic were used to model the wave climate on the Atlantic coast of Delaware. The final environmental input that was used in this analysis was the storm conditions for the area. This project used the same storm reoccurrence curves to model the storm surge and wave height for a storm of given intensity. Figure 37 shows the distributions used in this study for both winter storms and hurricanes.

This project was then analyzed using both the spreadsheet dynamic model and the classic Army Corps of Engineers steady-state model. Again, the major differences in these two models are that the spreadsheet model combines the storm and chronic erosion conditions together into the determination of renourishment interval. The Corps' method, on the other hand, is driven by the lowest cost alternative among possible renourishment intervals.







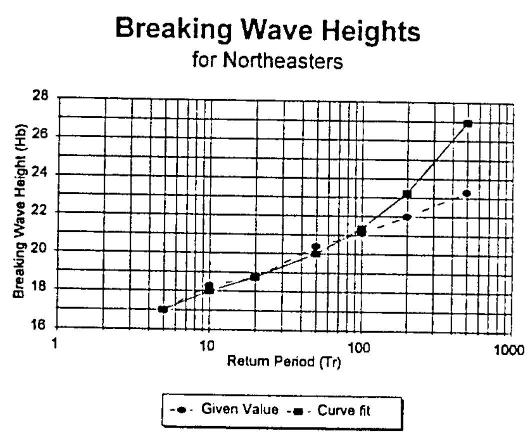


Figure 37. Curve fit for storm intensities

Simulation - South Bethany Beach

Using the beach project parameters described in Table 2, the project found at South Bethany Beach Delaware was analyzed using both the dynamic spreadsheet model developed in this project and the traditional steady-state Army Corps of Engineers model usually used in beachfill design.

First of all, the project was analyzed using the spreadsheet model. The initial step in this analysis was to run the model through the storm occurrence model fifty times in order to produce a group of values for renourishment interval and renourishment volume. The collection of these values is shown in bar chart form in Figure 38. As can be seen in this graph, there were a number of times when the project was required to be rebuilt within the first year after construction. Most of the renourishment intervals determined by the model fell within the one to three year value. Calculating the mean for this sampling of intervals determined that the renourishment interval for this project would be two and a half years. Therefore, the spreadsheet model predicted that the project would be renourished every three years with a renourishment volume of 700,000 yd^3 every time.

The use of this spreadsheet model combined the effects of background erosion, longshore spreading, and storm erosion into one analysis that thus determined the performance of the project based on all. Next, this same project was analyzed using the Army Corps of Engineers model. This model does not consider longshore diffusion for the area. Therefore, any long-term erosion would simply be the background erosion in the area. In addition, this model considers storms in a weighed probability fashion vice analyzing them as individual storm events.

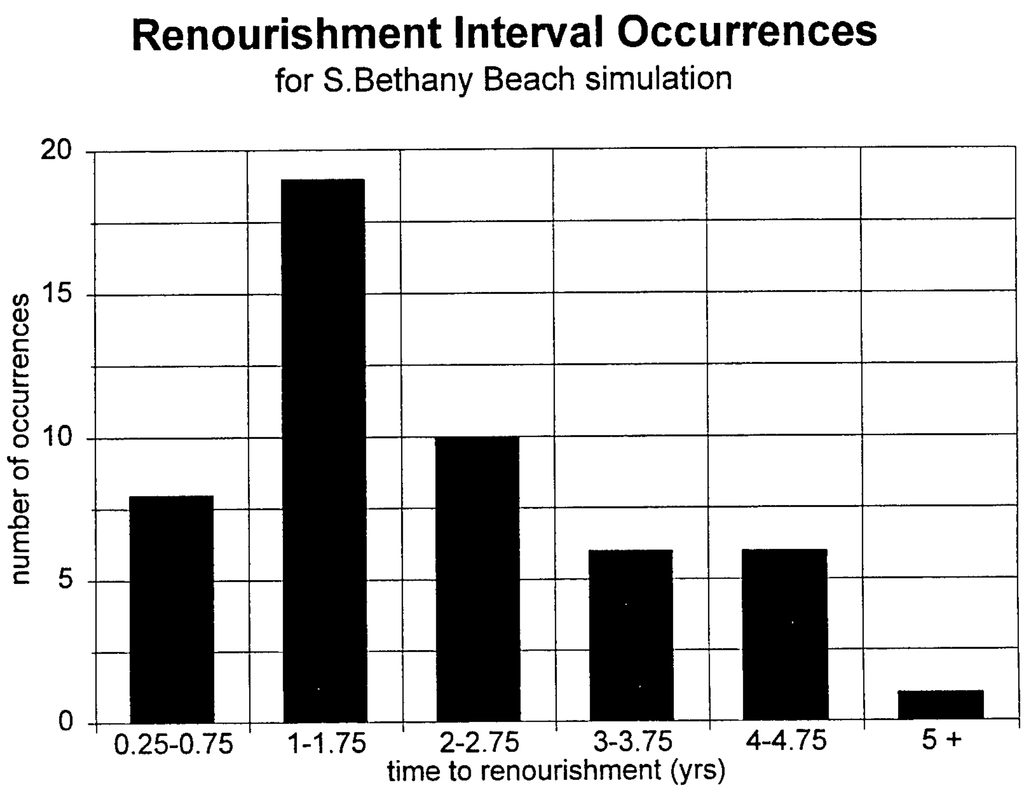


Figure 38. Histogram of renourishment intervals for S. Bethany Beach, Delaware

In order to determine the renourishment interval that would be used for the project, the total EUAC for the project was calculated for renourishment intervals spanning from two to ten years. Since the Corps method does not consider longshore spreading, all advance nourishment and renourishment determinations are based solely on the background erosion in the area. Figure 39 shows how the EUAC curve changes only slightly through these different renourishment intervals. There is a slight upward trend in the $EUAC_{CR}$ as well as the $EUAC_{R}$. The storm damage calculation remained stable for the entire watch. Analysis of this data showed that the most cost effective solution would be to renourish the project every two years with a volume of 38,000 yd^3 of sand. This is far less than that predicted using the spreadsheet model.

The differential in volumes predicted by the two models is in large part due to the fact that the Corps' methods, while still analytically looking into storm probabilities, does not account in the renourishment volume for the sand that would have to be replaced after a major storm. Instead, this cost is found as the storm probability cost for the Corps' method of determining renourishment interval. When the models are each put together as in Figure 40, it is possible to see how the costs, while in different subheadings within the model, are nearly the same. In fact, the values as calculated by the dynamic probability model are higher than the Corps' method. In large part this is due to the fact that the spreadsheet model considers all forms of beach erosion while the Corps neglects longshore diffusion..

From this analysis it can be seen that the dynamic spreadsheet model considers all forms of beach erosion. It doing so it is possible to produce a more realistic simulation of

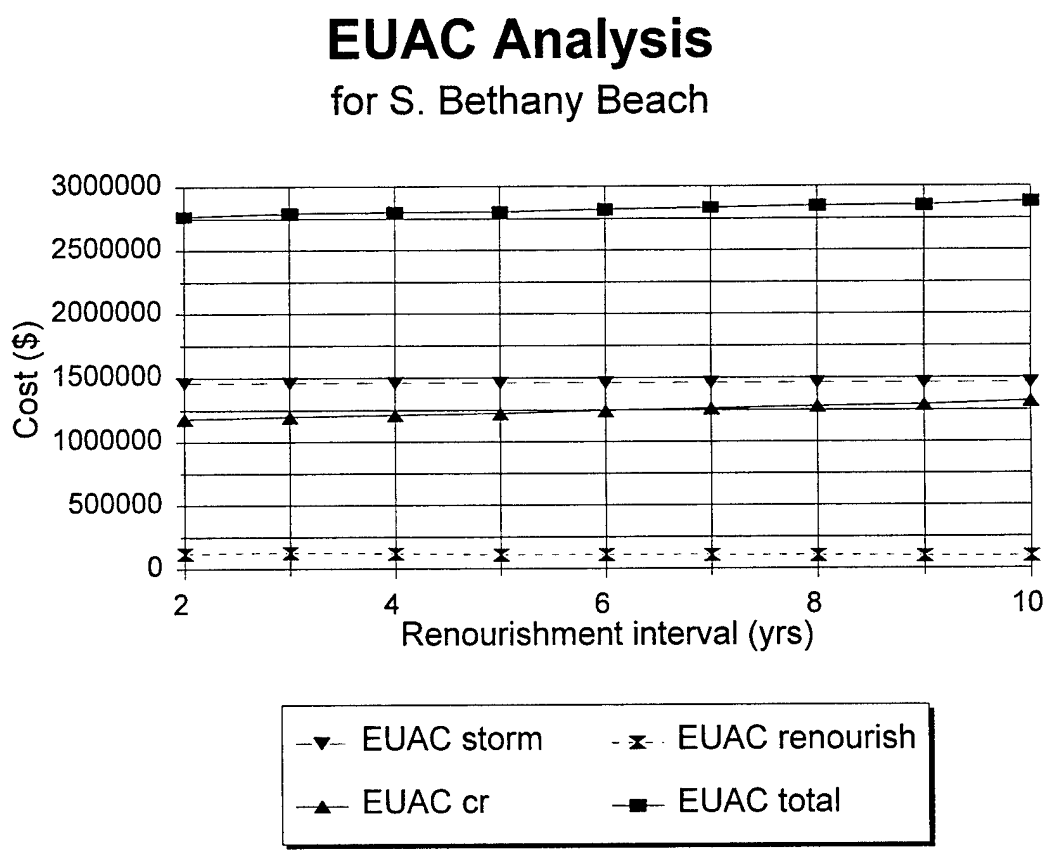


Figure 39. EUAC graph for determining renourishment interval

Plan 2 for S Bethany Beach w/ 3 yr spacing							
Storm recurrence	Storm probability	Frequency Interval	Volume eroded	Reconstruction costs	Cost Average	Interval Damage	Annual Storm Damage $1,464,251
3	0.33		0	$0			
		0.133			$2,169,339	$289,245	EUAC of renourishment costs $129,089
5	0.2		619,811	$4,338,677			
		0.100			$4,847,360	$484,736	
10	0.1		765,149	$5,356,043			EUAC of initial costs $1,199,830
		0.050			$5,799,003	$289,950	
20	0.05		891,709	$6,241,963			
		0.030			$7,117,390	$213,522	EUAC total $2,793,171
50	0.02		1,141,831	$7,992,817			
		0.010			$8,475,376	$84,754	PWC total $27,693,767
100	0.01		1,279,705	$8,957,935			
		0.005			$9,417,912	$47,090	
200	0.005		1,411,127	$9,877,889			
		0.003			$10,672,963	$32,019	
500	0.002		1,638,291	$11,468,037			
		0.002			$11,468,037	$22,936	
MAX	0.0		1,638,291	$11,468,037			
		0.000			$5,734,019	$0	
Volume added:		1,638,291	yd^3	Renourishment interval:		3	years
Initial cost:		$11,896,094		Renourishment volume:		61,151	yd^3 erosion only
Advance volume:		61,151	yd^3	Renourishment cost:		$428,057	

Plan 5 for South Bethany Beach		using storm reoccurrence model methods					
							EUAC of combined R + S $1,816,954
							EUAC of initial costs $1,156,657
							EUAC total $2,973,611
							PWC total $29,482,798
Volume added:		1,638,291	yd^3	Renourishment interval:		2.5	years
Initial cost:		$11,468,037		Renourishment volume:		700,000	yd^3 storm + erosion
				Renourishment cost:		$4,900,000	

Figure 40. Economic data comparing two methods of renourishment interval

produce the expected performance of the beachfill project. Overall, the spreadsheet model did results in terms of nourishment interval that were extremely close to those predicted by the Corps except in the area of renourishment volume. In this way the Corps has grossly under designed by only considering background erosion to be the major cause of chronic erosion on a beach. A model such as the one developed here offers a more realistic look at the overall response of the beach and therefore does not base an important consideration such as renourishment interval on one variable.

CONCLUSIONS

This goal of this project was to develop a spreadsheet model through which the response of beach nourishments projects could be simulated. To this end the project modeled the statistical variability inherent in the beach system. This model gives a realistic analysis of the performance of a beachfill project from its initial construction template up to the first major renourishment that is needed throughout its life cycle in order to maintain the storm protection capacity of the beach.

However, the spreadsheet model does not provide an exact representation of the evolution of a project. The assumptions made in the various equations used throughout the model as well as the accuracy of the historical data inputs could both result in possible sources of error from an actual project. Thus, when using the model, it is important to remember that the calculated eroded volumes and distances are not exact determinations, but rather they are close approximations that provide a general idea as to the evolution of the project due to varying environmental forces.

The other main goal in developing this model was to provide quantitative answers to the debate led by Professor Orrin Pilkey and Dr. James Houston over beach nourishment and the manner in which projects are designed. The major focus of this debate was on whether current methods used by the Army Corps of Engineers to design beachfill projects were resulting in under designed projects with inflated predicted performance levels.

The major statement that Pilkey made in this regard was to state that beachfill projects erode at a much faster rate than the native beach on which they are placed (Pilkey

1988). His determination was not based on diligent scientific study, but rather on broad personal observation. Through the use of this model it has become evident that Pilkey is both right and wrong in making his statement.

First of all, beach nourishment projects do erode more quickly than the natural area around them. This is due to the added erosion process of longshore diffusion that is not present for a natural beach. This only effects a nourishment project because the beachfill is placed out into the water such that it creates a perturbation in the shoreline. The effect of longshore diffusion is to eliminate this disturbance and thus spread out the project. Therefore, any erosion due to longshore spreading is in fact an increase in the erosion rate for a nourishment project as compared to a natural beach.

However, the other major observation that Pilkey bases his statement on seems to be the loss of beach width which occurs during the equilibration period in which the project conforms to the natural beach contours in the area. This evolution from the construction template to the design profile results in a loss of visible beach width. Pilkey has based most of his observations on the original construction width instead of the designed width which the project will have after equilibrium. This seems to be an unfair way to judge project performance, since this period of equilibration cannot be avoided and since it is accounted for in engineering design.

Therefore, Pilkey is both correct and incorrect in his statement. The increased erosion due to longshore spreading is a factor which can be designed for as has been done in this model. It does decrease the overall effectiveness of a project, but not to the degree that Pilkey claims.

The other major contention that Pilkey made was that the Army Corps of Engineers does not renourish projects at a frequent enough interval or plans to renourish at a longer interval than occurs in practice. The case study of a hypothetical project in Delaware was designed to provide an answer to this question. From that analysis it became evident that the Corps was designing at the correct renourishment interval, in fact that interval matched the value chosen by Pilkey for that region of the country. However, the use of the dynamic spreadsheet model developed for this study showed that the volume of sand used in renourishment was often too low. Therefore, from the use of statistical models which simulated beachfill performance, there may be merit in Dr. Pilkey's assertion. In addition, the spreadsheet model suggests a greater volume of sand may be needed for renourishment than is predicted by the Corps' method.

Even though a final answer as to the correct manner of designing beach nourishment cannot be given, it has become evident that no one side in this debate is entirely correct. Both sides have valid arguments which must be looked at with realistic modeling, not just unscientific observation.

In addition to providing an answer to part of this debate, this study looked into the parameters involved in beach renourishment in order to determine the optimal conditions under which a project should be constructed. These results are summarized in Table 1 and include both environmental and design variables. When looking at the location at which beachfill may be successful, project sites should meet a number of important criteria. The area should have a low to moderate background erosion rate, generally coarse sand, long project length if possible, and a low wave climate. Even though these general guidelines cannot always be met, they will provide the best possible project performance.

One area which this project did not consider was the use of beachfill in combination with structural alternatives. Structural alternatives can provide a beach nourishment project with support that is needed to make up for poor project location. For example, offshore breakwaters can limit the wave climate in which a project is built, groins and jetties can provide permanent boundaries that limit the effects of spreading, and seawalls can protect sand from any interaction with the wave climate. When designing a beachfill project in an area usually unsuitable for nourishment, the use of structural alternatives can greatly enhance the performance and thus storm protection abilities of a project.

Finally, beach nourishment is a way in which man attempts to mimic nature and in so doing protect other structures from storm damage as well as providing a recreational area. This method of shoreline protection is aesthetically pleasing and generally beneficial to a community. Communities should consider beach nourishment as an investment in their future. These projects can provide both storm protection and increase the recreational use of the beach. It is important to remember that the use of realistic models can enhance the design of these projects so that they produce the maximum benefit for the area under concern. There is no one design that can work for all locations. Thus, different designs must be evaluated for each individual area so that an optimal design that provides the necessary storm protection and is economically feasible is finally built. Models such as the one developed here are essential in this optimization process.

BIBLIOGRAPHY

Bodge, Kevin, "Damage Benefits and Cost Sharing for Shore Protection Projects," Shore and Beach, Vol. 59, No. 2, April 1991.

Camfield, Fred. "Different Views of Beachfill Performance" Shore and Beach, Vol. 61, No. 4, October 1993.

Dean, Robert, "Realistic Economic Benefits from Beach Nourishment" 21st Coastal Engineering Conference, Costa del Sol-Malaga, Spain, June 1988.

Dean, Robert and Chul-Hee Yoo, "Beach-Nourishment Performance Predictions," Journal of Waterway, Port, Coastal, and Ocean Engineering, Vol. 118, No. 6, November/December, 1992.

Greenberg, Michael, Foundations of Applied Mathematics, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1978.

Houston, James. "Beachfill Performance" Shore and Beach, Vol. 59, No. 3, July 1991.

Kriebel, David L., EN462C Class Notes, United States Naval Academy, Department of Naval Architecture, Ocean, and Marine Engineering, 1995.

Kriebel, David L. and Robert Dean, "Convolution Method for Time-Dependent Beach-Profile Response," Journal of Waterway, Port, Coastal, and Ocean Engineering, Vol. 119, No. 2, March/April 1993.

Newman, Donald G. and Bruce Johnson, Engineering Economic Analysis, Engineering Press, Inc., San Jose, CA, 1995.

Pelnard-Considere, R., "Essai de Theorie de l'Evolution des Formes de Rivage en Plages de Sable et de Galet," 4th Journees de l'Hydraulique, Les Energies de la Mer, Question III, Rapport No. 1, 1956.

Pilkey, Orrin, "A 'Thumbnail Method for Beach Communities: Estimation of Long-Term Beach Replenishment Requirements," Shore and Beach, Vol. 56, No. 3, July 1988.

Sudar, Anne and Joan Pope, Ted Hillyer, and John Crum. "Shore Protection Projects of the US Army Corps of Engineers" Shore and Beach, April 1995.

US Army Corps of Engineers Philadelphia District, Rehobeth Beach/Dewey Beach Interim Feasibility Study, April 1995.

US Army Corps of Engineers, Shore Protection Manual, Coastal Engineering Research Center, Vicksburg, MS, 1984.

Walton, Todd and T.Y. Chiu, "A Review of Analytical Techniques to Solve the Sand Transport Equation and some Simplified Solutions." Proceeding of the ASCE Conference on Coastal Structures '79, 1979.