Bibliographic Review of Nearshore Wave Models

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Abstract

The aim of this investigation was to review the different types of nearshore wave models currently available and to make recommendations as to which were most suitable for modelling nearshore wave conditions. The models were first placed into one of two broad classes. The first was those that could be applied in a small amount of time (for example a few minutes) either by quick calculations or the usage of nomograms. The second type was those that were computer resource intensive and were more likely to be executed in the laboratory.

For the first type of nearshore wave model, those deemed worthwhile included the Krylov, Strekalov and Tsyplukhin (1976) model and the model by Thornton and Guza (1983). For the second type of wave model, SWAN (Simulating WAves Nearshore) and REF/DIF_S (REFraction and DIFfraction Spectral) were recommended. All of these models have been tested against laboratory or field data with good comparisons made and all have particular qualities and attributes which make them useful.
Bibliographic Review of Nearshore Wave Models

Executive Summary

There are a number of nearshore characteristics that can have profound effect upon the success of amphibious operations. Such characteristics include wave amplitudes, wave propagation angles, longshore and cross-shore velocities, sand transport rates (an indicator of turbidity) and beach profile changes. Hence, it was envisaged that a set of reports outlining the most appropriate methods (or models) for calculating such parameters would be of great benefit to those planning amphibious operations.

The aim of this investigation was to firstly determine the different types of nearshore wave models currently available. The aim was to then review their attributes (for example: (1) the equations and assumptions utilised, (2) any validation exercises performed and their findings) and to then use the review to make recommendations on which models were most suitable for modelling nearshore wave conditions. Before such an investigation took place, however, it was envisaged that this review would be most beneficial if the wave models were placed into one of two broad classes. The first was those that could be applied out in the field, either by small calculations or rules of thumb, whilst the second was those that were computer based (Laptop or PC).

For the first type of nearshore wave model, those that were deemed worthwhile included the Krylov, Strekalov and Tsyplukhin (1976) model and the model by Thornton and Guza (1983). The first of these is an empirical model which uses the fetch and wind speeds to forecast the significant wave height and wave period in water of finite depth. It has the advantages that the wind speed, direction and water depth can vary along the fetch and that it can be applied to complicated shorelines. The model by Thornton and Guza meanwhile is solely a surf zone model which predicts the decay of significant wave height over complex bathymetries.

For the second type of wave model, two were deemed recommendable. The first is called SWAN (Simulating WAves Nearshore) and models the propagation of two-dimensional (frequency-direction) wave spectra over complex bathymetries. It includes processes such as wave growth, refraction and wave breaking (white-capping and depth induced breaking). The second model is called REF/DIF_S and like SWAN models the propagation of two dimensional wave spectra over complex bathymetries whilst taking into account refraction, bottom friction, depth induced breaking and wave-current interactions. Unlike SWAN however, it can also model diffraction.
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1. Introduction

1.1 Stimulus for Report:

There are a number of nearshore characteristics that can have profound effect upon the success of amphibious operations, or military operations occurring within the coastal zone. Such nearshore characteristics include wave amplitudes, wave propagation angles, longshore and cross-shore current speeds, sand transport rates (an indicator of turbidity) and beach profile changes. Hence, it was envisaged that a set of reports outlining the most appropriate methods (or models) for calculating such parameters would be of great benefit to those planning amphibious operations, this being the stimulus for this report.

1.2 Structure of Report:

This report is directed towards describing pertinent wave models that can be used to forecast the wave conditions within the nearshore zone. Pertinent background information regarding wind generated surface waves will firstly be outlined. Then, the different types of nearshore wave models available are summarised with an emphasis on placing each model into one of two groups: simple models, which are likely to be of more use in the field, and then more computer based models. This report concludes by outlining the most appropriate wave models based upon the advantages, disadvantages and validation procedures of those described.

Note: It will be assumed throughout this report that surface wind conditions, deep-water wave characteristics, tidal action, shelf current regimes, and bathymetry characteristics are readily available for any nearshore wave modelling.

1.3 Background Information:

The coastal zone (which in this report is considered as the interface between the land and any body of water) is a very dynamic and responsive environment, with numerous forces present and each with its own temporal and spatial variability. Such forces include wind action, wave action, tidal effects, shelf circulation regimes and nearshore circulation regimes (for example longshore and cross-shore). On average the strongest force along the coastline is that associated with the action of the surface waves (Gross, 1985; Komar, 1983).

Surface waves (or gravity waves) are generated by the effect of the overlying winds upon capillary waves, with capillary waves being generated by the turbulent pressure fluctuations within the atmosphere (Massel, 1996). For any region where surface waves are being generated, such waves are sometimes collectively called "sea" (Soulsby, 1997) with the amplitude of the waves generated being dependent upon three parameters. These are: (1) wind force (or wind speed), (2) the duration, which is the time interval
over which the wind has been blowing at a constant value and (3) the fetch, which is the
distance over which the wind has been blowing in one direction (Bearman, 1989). Figure 1 illustrates these concepts.

![Figure 1](image)

Figure 1 A schematic diagram illustrating the dependence of wave generation upon wind speed (U), duration (D) and fetch (F). Modified from Komar (1983).

Considering the situation of a steady wind blowing over an infinite fetch, at some point in time (for example duration) the waves generated will stop growing. Such a time occurs when the input of energy from the wind is balanced by the release of energy from the waves by them breaking, with the production of "white caps" being an indicator of such a point. When such a situation is reached the wave field (or sea) is called a "fully developed sea", for obvious reasons, with the term "un-developed sea" applicable before such a point is reached. Once surface waves leave the region from which they formed, they are called swell, with such waves having well defined amplitudes and directions (Soulsby, 1997).

Due to surface waves forming as a result of the wind field, which is itself fluctuating about a mean value, the surface waves, instead of being single monochromatic waves, have a spectrum of wave heights, frequencies and wavelengths. Because of this, such waves are sometimes called irregular, random or natural waves (Soulsby, 1997), with studies showing that surface wave characteristics have well defined spectra. For example, it has been observed that fully developed seas have frequency spectra that can be described by the Pierson-Moskowitz (PM) spectrum, whilst in areas where surface waves are fetch-limited, wave frequency spectrum are described by the JONSWAP (Joint North Sea Wave Project [Hasselman et al., 1973] (as cited by Massel (1996)) spectrum. Figure 2 illustrates the characteristics of these two spectra. Another type of wave spectrum is the Rayleigh Spectrum. This spectrum describes surface wave amplitude spectra, from the open ocean to the surf zone, although for the surf zone some criteria must be included for depth-induced wave breaking and non-linear changes to the wave profile. The same spectrum describes wave period distributions, and it is even
applicable within the surf zone. This is because the period of any wave does not change once it has formed (Lakhan and Trenhaile, 1989).

(a) 

(b) 

Figure 2. (a) The JONSWAP spectrum for a developing wave field of various fetch lengths (X). (b) A comparison of the JONSWAP and PM spectra for a wind speed of 20 m/s\(^1\) and fetch values of 200 kms and infinity, respectively. Both diagrams taken from Massel (1996).

One of the problems caused by surface waves being irregular or having a spectrum of wave heights and frequencies is: how can they be easily described and how can formulations be applied to them which require only one wave height? In trying to overcome this problem, numerous parameters have been formulated (see Massel (1989) for a listing) to describe the wave height spectrum. One of the most frequently used is the significant wave height (denoted by \(H_s\)). This parameter was introduced by Sverdrup and Munk, (1947) (as cited by Massel, 1996) and is defined as the average of the highest one third of the waves. It is used because it has been shown that \(H_s\) roughly corresponds to a visual estimate of the mean wave height by observers at sea (Komar, 1983). Another commonly used wave height parameter is the root mean square wave height (\(H_{\text{rms}}\)), with this parameter and \(H_s\) related by the expression \(H_s = \sqrt{2H_{\text{rms}}}\) (Massel, 1996). Figure 3 shows the relative position of these two parameters in a spectrum of wave heights.

With the wave heights in particular being a spectrum of values it could be intuitively envisaged that the sea surface takes on a chaotic appearance. However, this is generally not the case, with surface waves sometimes interfering to produce waves of larger wavelengths called wave groups (see Fig 4). The importance of wave groups is that their propagation speed (\(C_g\)) is the speed at which wave energy travels (Pond and Pickard, 1983) and is related to the individual wave speed (or wave celerity) (\(C\)) by the following relationship.

\[
C_g = \frac{C}{2 \left[ 1 + \frac{2kh}{\sinh(2kh)} \right]} \tag{1}
\]

2. \( k \) is the wavenumber = \( \frac{2\pi}{\lambda} \)
3. \( \lambda \) is the wavelength
4. \( C \) is the individual wave speed = \( \frac{g}{k} \tanh(kh) \)
5. \( g \) is the acceleration due to gravity (9.8ms\(^{-2}\))

![Figure 3](image)

**Figure 3** A schematic diagram of a Rayleigh distribution showing the relative location of the parameters \( H_{rms} \) and \( H_s \). Modified from Komar (1983).

![Figure 4](image)

**Figure 4** An example of how two surface waves can interact to produce a second wave (or wave group) of larger wavelength.

As surface waves propagate toward a shoreline, the changes in bathymetry causes waves to be affected by two processes, called refraction and diffraction. Refraction results from the dependence of wave speed upon depth and causes the wave front to be bent or refracted towards shallower areas (see Fig#5). This characteristic is analogous to the bending of light rays as they pass through media of differing refractive indices and, like this type of refraction, can be described by Snells Law (eqn.#2).

\[
\frac{\sin(\psi_s)}{C_s} = \frac{\sin(\psi_d)}{C_d} \quad [2]
\]
Where:

(1) $\psi_d$ and $\psi_s$ are the angles of incidence and refraction, respectively (measured from the normal) for a wave front transversing from a region of deep to shallower bathymetry.

(2) $C_d$ and $C_s$ are the speeds of propagation of the wave front in the two different regions.

By using the fact that in shallow regions the speed of propagation is given by $\sqrt{gh}$, where $g$ is the acceleration due to gravity and $h$ the depth, then Snells Law can be simplified to the following more applicable form:

$$\psi_s = \sin^{-1}\left(\frac{h_s}{h_d}\right)$$

[3]

Figure 5. (a) A schematic diagram illustrating the phenomenon of refraction when applied to a wave front traversing from a region of deep to shallower water (Taken from Beer (1983)), with (b) showing an example of the same phenomenon at a beach. (Taken from McCormick and Thiruvathukal (1981))
Diffraction, meanwhile, is the process by which wave energy spreads laterally orthogonal to the propagation direction (Massel, 1996) and occurs when waves encounter obstacles (island, shoals, groynes and man-made structures) whose radius of curvature is comparable to or less than the wavelength of the incident waves (LeBlond and Mysak, 1978) (see Fig#6(a)). Like refraction, diffraction can also occur to electromagnetic waves (see Fig#6(b)).

Figure#6 Two examples of diffraction (a) for surface waves past a groyne. Note how wave energy is diffracted into the shadow zone behind the groyne. (Taken from Ippen, 1966). (b) for electromagnetic waves past a razor blade. (Taken from Giancoli, 1989).

At some point as a wave propagates towards a coastline, the drag caused by the wave feeling the bottom will result in the top of the wave moving forward at a greater speed than the bottom. At this point the wave will become hydrostatically unstable and break, with the wave’s momentum being transformed into "plunging"-vertical motion as well as a strong horizontal motion. The momentum flux of this horizontal motion has been given the term radiation stress (Soulsby, 1997) with the horizontal motion being able to continue all the way up until it has reached the top stages of the beach face. Here it is termed swash, with the term backwash applied to the swash that runs back down the beach face (see Fig#7).

Figure#7 A schematic diagram showing the formation of swash and backwash due to waves breaking on the beach face, with the loop trajectories that sediment particles undergo because of these two events also illustrated. Taken from Komar (1976).
2. Nearshore Wave Models

2.1 Introduction:

As noted in section 1.2, nearshore wave models will be placed into one of two groups: (1) simple models which can be run quickly or used quickly (for example a few minutes), (2) and computer intensive models. The historical development of wave models as well as the different types will firstly be discussed.

Historically, Sverdrup and Munk (1947) had the honour of developing the first wave prediction model, with this model developed to forecast nearshore wave heights for World War II amphibious operations (Cardone, 1974). The model Sverdrup and Munk developed also introduced the now well known terms, significant wave height and significant wave period. As such, the model was termed the significant wave method, with the model predicting these two parameters from a knowledge of the wind speed, fetch and duration. The technique employed was normally one of a nomogram.

The wave model Sverdrup and Munk developed, including some later revisions (see Bretschneider (1952 and 1958) and Wilson (1955)) (as cited by Cardone, 1974) represents one of the most widely used wave models, with such a distribution stemming from its ease of usage. This model however, began to lose its appeal in 1978 when it was replaced by a more sophisticated type of model. The reasons for this replacement is that firstly, incorrect theory used to develop the model resulted in it being not very accurate. The second reason was that the model failed to provide a framework for improvement (Cardone, 1974).

The type of model that replaced the Sverdrup and Munk model was the spectral wave energy model (sometimes called wave action if current effects are included). This type of model has its origins back in the 1950s with Gelci et-al (1956) (as cited by Cardone, 1974) and employs the conservation of energy through the radiation transfer equation (eqn.4). In deep-water with no ambient currents this equation is formulated as:

\[ \frac{\partial \psi}{\partial t} + C_g \cdot \nabla \psi = \text{Sources} + \text{Sinks} = S_{in} + S_{nl} + S_{ds} + S_{bot} \]  

\[ \text{[4]} \] (Ewing and Hague, 1993)

where:

1. \( \psi \) is the directional wave energy spectrum and is a function of the space and time (t) co-ordinates, as well as frequency and direction.
2. \( \nabla \) is the gradient operator,
3. \( C_g \) is the (deep-water) wave group speed,
4. \( S_{in} \) represents the input of energy from the wind field,
5. $S_{nl}$ represents the non-linear transfer of energy from high to low frequencies. It is an
important parameter for nearshore wave modelling but only of relevance when a
wave field is growing.

6. $S_{ds}$ represents the dissipation due to wave breaking. Such breaking can either be
away from coastlines, in which case it is called “white capping” or it can be induced
by shallow topography.

7. $S_{bot}$, like $S_{ds}$, is a sink of energy and represents such processes as bottom friction and
refraction.

The first investigations of this type of modelling centred upon understanding and
formulating the individual components, with the first models tackling the somewhat
tsimpler offshore regions. Once spectral models had been successfully applied to
offshore locations then focus shifted towards coastal and nearshore areas. For these
areas, however, such shallow water phenomena as friction, refraction and depth induced
breaking needed to be included.

In addition to nearshore spectral wave models, there are four other types of models
which have been used successfully to forecast waves in the nearshore. These are the
elliptic “mild slope” model, Boussinesq models, simulation models and empirical
models.

The mild slope equation is so-called because it can only be applied to regions where the
bottom has a mild slope (< 0.1), and is widely used within the scientific community.
These types of wave models are based on the momentum equations and unlike wave
spectral models, can take into account diffraction and reflection. However, they cannot
model wave generation.

The Boussinesq models model the propagation of swell and wind waves from deep to
shallow water and incorporate the processes of refraction, diffraction, shoaling,
reflection, wave generation, wave grouping and non-linear wave to wave interactions
(http://www.dhi.dk/software/mike21/mike21sw/m21bw/m21bw.htm). They are
sometimes called phase-resolving models and are normally used in engineering studies
where wave conditions within small coastal regions are required. This is because they
can only be applied in regions where the depth to wavelength ratios are less than 0.22
(Madsen et-al, 1991), with the restriction to small regions arising because they require
over five data points per wavelength to be implemented (private communication Dr
Roland Ris, 1998).

The first Boussinesq model was developed by Boussinesq (1872) (as cited by
Dingemans, 1997) for flat bathymetries. Since 1872 there have been numerous methods
used to generate the Boussinesq equations (which commonly include a continuity of
mass equation with two linear momentum equations) with a common assumption in
each derivation being that the waves are long with respect to the local depth.
Nowadays, Boussinesq models can be applied to variable bathymetries with some
attention directed towards improving their treatment of frequency dispersion. Note that
the equations representing Boussinesq models should not be confused with the equations of motion which have incorporated Boussinesq's assumptions regarding negligible horizontal density variations.

Simulation models use the observation that as waves propagate from offshore to nearshore areas their characteristics continuously change, with some investigators using the depth to wave length ratio to bin the continuous changes into four different subdivisions, each with its own set of formulations for wave characteristics. These subdivisions, from offshore to shallower areas, are: (1) sinusoidal or Airy, (2) Stokes (3) cnoidal and then (4) solitary. The only limitation with such models is that discontinuities develop at the boundaries of the different wave regions.

Empirical models, like the Sverdrup and Munk model, predict the significant wave height from values such as wind speed, fetch, duration, and depth, with such models arising from dimensional analysis. They can be thought of as basic models suitable for quick calculations, for example when calculations need to be made at, or on arriving at, a beach.

2.2 Simple Models:

In this report simple nearshore wave models can be thought of as models which can be used in a relatively short time frame (for example several minutes) by observers whose background is not nearshore modelling using either hand held calculators or nomograms.

2.2.1 Empirical Models

2.2.1.1 Revised Shore Protection Manual Model

The revised shore protection model (RSPM) is an empirical model which permits the significant wave height \( H_s \) and mean wave period \( T \) to be determined in water of finite depth from a knowledge of the depth \( h \), wind speed \( U \) and fetch \( X \). It was formulated by Hurdle and Strive (1989) after it had been shown that the shore protection model (SPM) does not approach its deep-water formulations as depth increases. The equations representing this model are as outlined below with it felt that the model would be best utilised if the equations were represented in the form of a nomogram.

\[
\frac{gH_s}{U^2} = 0.25 \tanh \left[ 0.6 \left( \frac{h}{gU^2} \right)^{0.75} \right] \tanh \left[ 0.4 \left( \frac{4.3 \times 10^{-5} gX}{U^2} \right) \right] \left[ \tanh \left( 0.6 \left( \frac{h}{gU^2} \right)^{0.75} \right) \right] \]

[5]
where:
g is the acceleration due to gravity (9.8 ms\(^{-2}\))

As empirical models are generated using observed data, there is little to gain from comparing them to real data, other than to see under what conditions they behave well. Unfortunately, such a test could not be found within the literature.

2.2.1.2 Krylov, Strekalov and Tsyplukhin Model

The empirical model developed by Krylov, Strekalov and Tsyplukhin (1976), like the empirical RSPM, permits the mean wave height and period (\(\tilde{H}\) and \(\tilde{T}\)) to be calculated using the fetch, wind speed and depth values in conjunction with the following equations. Also like the RSPM model it is foreseen that the Krylov et-al model would be implemented in the form of a nomogram.

\[
\frac{g\tilde{H}}{U^2} = 0.16 \left[ 1 - \left( 1 + 6.1 \times 10^{-3} \left( \frac{gX}{U^2} \right)^{0.379} \right)^{-2} \right] \tanh \left[ 0.625 \left( \frac{gh}{U^2} \right)^{0.625} \left( 1 - \left( 1 + 6.1 \times 10^{-3} \left( \frac{gX}{U^2} \right)^{0.379} \right)^{-2} \right) \right] \tag{7}
\]

\[
\frac{g\tilde{T}}{U} = 19.478 \left( \frac{gH}{U^2} \right)^{0.625} \tag{8}
\]

These two equations were developed by using wave data from numerous (non-referenced) water basins (Krylov et al, 1976). Such an approach, however, assumes that the wind and wave directions coincide or equally that the waves do not disperse directionally. In applying the above equations it is also assumed that the wind speed, direction and depth do not vary along the fetch.

As such assumptions are physically unrealistic, Krylov et al in 1976 developed a procedure which accounts for waves dispersing directionally. In particular they assumed that waves disperse according to the law \(\cos^2(\theta)\) and that the wave energy observed at any point is the superposition of the energies from various directions. This last statement, in conjunction with the dispersion law, was used by Krylov to generate the following equation:
\[ \overline{H}^2 = \frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \overline{H}_i^2(u, x = (\theta)) \cos^2 \theta \, d\theta \quad [9] \]

which in practical form reads:

\[ \overline{H}^2 = \frac{2}{\pi} \sum_{i=1}^{n} \overline{H}_i^2(u, X_i) \cos^2 \theta \Delta\theta \quad [10] \]

where:
1. \( \overline{H} \) is the mean wave height at the location being considered
2. \( \overline{H}_i \) is the mean wave height of the waves originating from the \( i^{th} \) sector
3. \( \theta \) is the wave propagation angle (measured as a departure from the wind direction).
4. \( \Delta\theta \) is the subsector size, which depends upon the variability of the coastline and the required accuracy of the wave characteristics.
5. \( X \) is the fetch.
6. The overbars represent averages in time.

As an example of how this last equation can be implemented, assume that there is a point P in a small basin of constant depth for which the average wave height needs to be known (see Fig#8). It can be seen from this figure that because of the waves spreading directionally all wave energy propagating towards P from each of the seven subsectors should be accounted for in estimating the average wave height at P.

Figure#8 A schematic diagram showing the dissection of an 180° arc into 7 subsectors. The wind direction is as shown. Taken from Massel (1996).

This is achieved by firstly analysing the coastline shape and creating subsectors of equal angular width (in this case 22.5°) such that the coastline is well represented. The fetch from each point along the coastline to point P projected into the direction of the wind velocity vector (termed effective fetch) is then used in conjunction with the directional energy spreading function (\( \cos^2 \theta \)) and eqn(#7) to determine the energy originating from each sector. These values are then placed into eqn(#10) to obtain the mean wave characteristics.
In addition to Krylov et al developing a procedure which accounts for waves originating from different directions, Krylov also developed a procedure which permitted the wind speed, direction and water depth to vary along the fetch. The procedure developed is somewhat more complicated than that above and will not be described here: Suffice to say however, that it involves using the same equations as for the above example and involves representing the variable fields (wind speed, direction and depth) by stepwise changes.

The empirical model developed by Krylov et al and more importantly the two procedures accounting for directional wave energy spreading and variable wind speed, direction and/or depth, would be very valuable tools for the quick evaluation of the wave characteristics in the nearshore.

2.2.2 Surf Zone Models

2.2.2.1 Simplest:

The strong energy dissipation in the surf zone and the associated decrease of the wave height towards the shore produces a gradient in the radiation stress. This gradient is balanced in the steady state by the mean water surface sloping, with the mean water level initially being lower than the still water level just after breaking (the wave set-down) and then being higher (wave set-up) as the broken waves approach the beach face (see Fig.9).

![Figure 9](https://example.com/fig9.png)

Figure 9 A schematic diagram illustrating the phenomenon of wave set-up and set-down. Taken from Fredsoe and Deigaard (1992).

One of the most basic surf zone models for predicting the wave height ($H$) variation and maximum wave set-up ($\Delta D_{\text{max}}$) (and which is the starting point for other surf zone models) makes the assumptions:

- that the wave height (after breaking) is directly proportional to depth. Such an assumption is called the self similarity law (Battjes and Jansen, 1978).
- that the slope of the surface is balanced by the cross-shore variation of the radiation stress, and
- that waves can be described by linear shallow water wave theory.
The equations representing this model are:

\[ H = KD \]  \[11\]

\[ \rho g D \frac{d(\Delta D)}{dx} = -\frac{dS_{xx}}{dx} \]  \[12\]

\[ \Delta D_{\text{max}} = \frac{3K}{8} H_b \]  \[13\]

where:
1. \( D \) is the height of the mean water level (m) at a distance \( x \) from the shoreline (see Fig#10).
2. \( \Delta D \) is the departure of the mean water surface from the still water surface.
3. \( K \) is a constant of proportionality (dimensionless).
4. \( H_b \) is the wave height at breaking.
5. \( S_{xx} \) is the radiation stress (or momentum flux) propagating normally towards the coastline (Fredsoe and Deigaard, 1992).

Figure#10 A schematic diagram illustrating the relationship between the various parameters used in the above formulations. Taken from Fredsoe and Deigaard (1992).

As this model is a very simplified case of the real situation one might suspect that it has some major restrictions. All of these arise due to the usage of the self-similarity law and include:

- it can only be applied to beaches of monotonically increasing depth profiles in the seaward direction.
- the self similarity law forces the dissipation during breaking to be too dependent upon the bottom slope (Battjes and Jansen, 1978b) (as cited by Battjes and Jansen, 1978)
2.2.2.2 Battjes and Jansen (1978)

The aim of the investigation by Battjes and Jansen was to develop a model which could predict the variation of the wave height across the surf zone for irregular waves and which was based on a more physically sound formulation than the self-similarity formulation used above. The direction taken was to use the equation describing the conservation of energy flux (eqn. 14), with such an approach thus permitting the model to be applied to beaches with non-monotonic profiles and, unlike the self-similarity law, also permitted (in theory) other dissipation terms to be included. For the model, the variation of the mean sea level was predicted using the conservation of momentum equation (eqn. 12).

The assumptions made in deriving the model were the following:

- the waves can be modelled by linear wave theory
- wave height distributions (breaking and nonbreaking) have a Rayleigh distribution, which is truncated so that there are no waves with heights exceeding a value $H_{\text{max}}$ (which occurs when the waves break), and
- wave dissipation during breaking is described by bore formulations (propagating hydraulic jumps).

Such assumptions were then used to rearrange the energy flux balance eqn (13) which was then integrated in the shoreward direction to predict the cross-shore variation of the significant wave height.

\[
\frac{\partial P_x}{\partial x} + D = 0 \quad [14]
\]

where:
1. $P_x$ is the energy flux ($= E C_g$)
2. $E$ is the wave energy
3. $C_g$ is the group speed
4. $D$ is the dissipation of energy (in this formulation it represents the amount of energy lost due to breaking)

In an effort to test the model, a wave tank was firstly constructed at the Delft University of Technology and set up for two different bathymetry regimes. The first was a plane beach of constant (but non-zero) slope, whilst the second was a beach with a longshore bar. Figure#11 is a schematic showing the layout of the wave tank used.

By observing the wave characteristics in the wave tank for the two different bathymetry profiles and then comparing such results with those predicted from the model, it was observed that the model predicted the variation in the sea surface height and significant
wave height quite well. Figure 12 shows the close comparison observed by Battjes and Jansen between modelled and measured values for the plane beach case.

Figure 11 The layout of the wave tank used in testing Battjes and Jansen's surf zone model. (Taken from Battjes and Jansen (1978)).

Figure 12 A comparison between measured and modelled significant wave heights ($H_s$) and mean water level ($n$) for a plane beach for two different deep water wave steepnesses (1% and 3%). Taken from Battjes and Jansen (1978).

The one disadvantage this model has, as noted by Battjes and Jansen, is that although there is good agreement between the predicted and measured $H_s$ this does not mean that the underlying parameterized probability density functions (pdfs) have been parameterized correctly.

2.2.2.3 Thornton and Guza (1983)

The model developed by Thornton and Guza (1983) describes the transformation of the broken wave height distributions over complex (non-monotonic) bathymetries. It was stimulated by Thornton and Guza's view that the forcing of broken waves to have heights which were proportional to the depth by Battjes and Jansen (1978) (and not their unbroken height) was a poor approach. Such a view was supported by observations at
Torres Pine Beach, California in 1978, which showed that wave pdf (which included non-breaking as well as breaking waves) were well described by Rayleigh distributions. As the approach taken by Battjes and Jansen (1978) permitted irregular waves over complex bathymetries to be modelled then Thornton and Guza kept their approach, but replaced the pdf used by an empirically derived broken wave Rayleigh probability density function. Such a formulation permits the amount of energy dissipated by breaking waves to be estimated, with the empirically derived broken wave height pdf \( P_b(H) \) taking the form described by eqn.15

\[
P_b(H) = W(H)P(H) \quad [15]
\]

where:

1. \( P(H) \) is a Rayleigh distribution for wave height (broken and unbroken waves)
2. \[
W(H) = \left( \frac{H_{rms}}{\gamma h} \right)^{2/3} \left[ 1 - \exp \left( \frac{H}{\gamma h} \right)^2 \right] \quad [16]
\]
3. \( \gamma \) is a constant (determined from field data).
4. \( h \) is the depth.

Figure#13 shows an example of using the above formulation to estimate the broken wave pdf and how it compares with field data.

Figure#13 An example showing the ability of the above formulation in predicting the broken wave pdf (hashed region). The data used was taken at Soldiers Beach, California on August 24 1981. Modified from Thornton and Guza (1983).

The model developed can be applied two ways. The first and easiest, is to simply calculate \( H_{rms} \) using the expression

\[
H_{rms} = a^{1/3}h^{9/10} \sqrt{1 - h^{2/3} \left( \frac{1}{h_{o}^{2/3}} - \frac{a}{y^{2/3}} \right)} \quad [17]
\]

where:

1. \( h \) is the depth
2. \( h_{o} \) is the depth at the offshore boundary (this should be located at the break point)
3. $y_d$ is the offshore distance at the offshore boundary.
4. $a$ is a constant, dependent upon the bed slope, mean frequency, and the intensity of the breaking.

**Note:** Such an expression can only be applied in shallow water and was obtained after applying the shallow water assumptions ($C_g = C = \sqrt{gh}$ and $h < \frac{A}{20}$).

If the shallow water assumptions are not obeyed at any beach, then the model needs to be applied by integrating the energy flux equation from an offshore position shoreward.

The model developed by Thornton and Guza (1983) was tested in two ways. Firstly it was tested against the laboratory data of Battjes and Jansen (1978) and then against field data obtained at Torres Pine Beach, California. The laboratory data of Battjes and Jansen was for irregular waves on a beach slope of 1:20. Figure 14(a) shows the model results and how they compare with the laboratory data. For the wave data from Torres Pine Beach, two days were chosen to illustrate the models prediction ability. These were November 10 and 20, 1978 with Figure 14(b,c) showing the comparisons between the model and field data.

**Figure 14** Three diagrams showing the comparison between the model predictions (solid lines) of $H_{rms}$ in the offshore direction ($X$) against (a) the laboratory data ($\times$) of Battjes and Jansen (1978) and (b,c) the field data ($\times$, ⊙) from Torres Pine Beach, California (Thornton and Guza, 1983). $H_d$ is the deep water wave height.
From Fig#14 it can be clearly seen that the model works very well in predicting the changes in $H_{\text{rms}}$ as the shoreline is approached.

Regarding disadvantages of the model, one would be that the model does not account for wave setup. Like the model of Battjes and Jansen (1978) however, the setup can be predicted using the conservation of momentum equation.

### 2.2.2.4 Svendsen (1984)

The model developed by Svendsen was for regular waves over simple bathymetries and was restricted to within the inner surf zone. This restriction to the inner surf zone, and not the whole surf zone, was needed as the model did not include breaking waves. In addition to being restricted to the inner surf zone it also represented broken waves by rollers and not bores. As can be seen from Fig#15 such a formulation greatly improved the forecasting capabilities of this model (Svendsen, 1984).

![Diagram](image)

Figure#15 Two diagrams showing Svendsen’s model output of normalized wave height and wave set-up against real data for a plane sloping beach. (Taken from Fredsoe and Deigaard, 1992).

### 2.2.3 Recommendations for simple models:

By evaluating and comparing the attributes of the wave models listed above, it was thought that the model by Thornton and Guza (1983) and that by Krylov, Strekalov and Tsyplukhin (1976) could be recommended for usage. The model by Thornton and Guza was chosen because it can predict the characteristics of irregular waves over complex (non-monotonic) bathymetries using well founded physical approaches. The model by
Krylov was chosen because it has a simple procedure which can account for variable bathymetry, wind speed and direction as well as wave directional spreading.

2.3 Complex Models:

There were four kinds of complex (but accurate) nearshore wave models found within the literature. These included: (1) the evolution of the frequency-direction spectra, (2) conservation of momentum models, (3) Boussinesq models, and (4) simulation models. The characteristics and attributes of these four types of models will be outlined in the following section. In addition, any validation or testing procedures performed on the models will also be outlined.

2.3.1 Spectral Models:

2.3.1.1 HISWA:

HISWA (HIndcasting Shallow water Waves) is a second-generation wave spectral model developed at the Delft University of Technology for the modelling of short crested waves under stationary conditions. In particular HISWA can be used to hindcast wave fields in coastal and nearshore areas or predict wave conditions inshore and offshore. The processes HISWA includes are: shoaling, refraction, wave generation by winds, wave breaking (which includes offshore “white capping” and depth induced breaking) and wave blocking due to opposing currents. HISWA, however, does not include Fresnel diffraction and reflection of waves, and as such should be used cautiously in regions of complex bathymetry (Holthuijsen et-al, 1989).

In regards to the accuracy of HISWA, Holthuijsen et-al performed a very extensive validation procedure upon HISWA compared to most other wave models. In particular they employed three tests: In sequential order these included comparing the model results for: (1) a plane sloping beach with a shear current against analytical solutions from linear wave theory (2) a submerged breakwater against laboratory data and, (3) a region of the Rhine estuary (the Haringvliet) against wave data from moored buoys.

In addition, HISWA has also been observed to successfully predict the wave conditions in Botany Bay, NSW, after there was a need for such information for the second parallel runway at Kingsford Smith Airport, Sydney (Willoughby, Trindade and Foster, 1995). Dr Geoffrey List of the United States Geological Society has also used HISWA successfully (personal communication G List, 1998) to model wave heights in Lake Pontchartrain in the United States. Lake Pontchartrain is a shallow lake with wave height data measured with the usage of bottom pressure sensors placed at three locations (see Fig#16). Comparisons between model output and wave measurements were very good (see Fig#17) with the three correlation coefficients at locations B, C and D being 0.65, 0.80 and 0.73, respectively.
Figure 16: Bathymetry layout of Lake Pontchartrain in the United States and the location of three wave sensors used to provide *in situ* data. (Personal communication Dr Jeffrey List, 1998)

Figure 17: Comparisons between modelled (grey) and measured (black) wave heights at three stations (see Fig 16) in Lake Pontchartrain.
Despite the success HISWA has had (and it has been used now operationally for over 10 years, personal communication Dr Roland Ris, 1998), it does have some disadvantages. These include: (1) not being able to model the propagation of multimodal spectrums (such as would be expected with the occurrence of a wind-sea and a swell), (2) computational grids need to be orientated in the mean wave propagation direction, and (3) wave propagation is limited to a sector of approximately 120 degrees (Holthuijsen, Booij and Ris, 1993).

2.3.1.2 SWAN:

SWAN (Simulating WAves Nearshore) is a non-stationary spectral wave model which was developed to overcome the above inadequacies of HISWA. In particular, SWAN models random, short crested waves in coastal and nearshore areas, and has optional pre-defined frequency spectrum constraints. The processes SWAN can model include: wave generation by winds, shoaling, refraction (due to currents and variable depths), breaking (white capping and depth induced), bottom friction, non-linear wave-wave interactions and frequency shifting due to currents. SWAN, however, like its predecessor HISWA, does not include diffraction and reflection (Holthuijsen et-al, 1993).

In testing the wave propagation scheme employed in SWAN, two simulations were utilised, with a third comparison performed against real data. For the two simulations, the first included using SWAN to model the propagation of a sinusoidal, long crested wave normally onto a plane beach. In this simulation of SWAN no friction or depth induced breaking were used. This was because such a situation enabled the results from SWAN to be compared with an analytical solution from linear theory.

The second analytical test involved using SWAN to model the refraction of short crested waves around the tip of a barrier island, although in this case SWAN was run with friction and depth induced breaking. The results obtained from SWAN were then compared with the results obtained from a conventional ray tracing model. Figure#18 shows such a comparison.

Note: From the point of view of outlining viable wave models, ray-tracing models were not included in this group because they have two major disadvantages. Firstly, they have the inherent problem of supplying data only along wave orthogonals and not at equally spaced grid points, as is normally required by numerical models (Martinez and Harbaugh, 1993). Secondly, in shallow regions such models are sometimes difficult to interpret because they can generate caustics (regions where wave orthogonal cross) (Holthuijsen et-al, 1993).
The results obtained using SWAN in the two simulations showed very good agreement with those predicted from linear theory and ray tracing, respectively. For the third evaluation of SWAN, however, which involved comparing model results with buoy data taken in a branch of the Rhine delta, the same could not be said. This was because, although SWAN modelled the significant wave height distribution very well, there was a low correlation between the measured and modelled mean wave periods. This disparity between the model results and buoy data was believed to be due to the presence of non-linear wave-wave interactions, which have been incorporated into new versions of SWAN. Figure 19 shows the results obtained from SWAN and eight buoys in a branch of the Rhine delta.

Figure 18 (a) Significant wave height distribution obtained from SWAN for wave propagation around a barrier island whose 3 m and 10 m isobaths are labelled. (b) Wave orthogonal distributions obtained from a conventional ray tracing model for the same barrier island bathymetry scenario. Both diagrams taken from Holthuijsen et-al, 1993.

Figure 19 Comparing the model results from SWAN with buoy data in a branch of the Rhine delta. Taken from Holthuijsen et-al, 1993.
2.3.2 Conservation of Momentum Models:

2.3.2.1 REF/DIF_S

REF/DIF_S is a parabolic model for the propagation of weakly non-linear waves (called Stoke's waves) across variable bathymetry and includes the effects of shoaling, REFraction, DIFfraction, wave-current interactions, and energy dissipation. REF/DIF_S was developed from the monochromatic model REF/DIF_1 and allows for the simultaneous computation of two-dimensional wave Spectra at each grid point. Such a procedure could be performed with REF/DIF_1 but it would be computationally expensive. REF/DIF_S also includes a statistical wave breaking model (Kirby and Tuba Ozkan, 1994).

REF/DIF_1 was developed by Kirby and Dalrymple (1983, 1985) by altering Radder's (1979) linear parabolic refraction-diffraction model to include weakly non-linear waves. Radder's parabolic model was generated from the elliptic mild slope equation (hereafter termed MSE) by splitting the velocity potential into forward and backscattered components, and has two major advantages over the elliptic MSE. These are that it does not require boundary conditions at the down-wave end of the grid and secondly, because of its parabolicity, efficient solution techniques are available in finite difference form (Kirby and Tuba Ozkan, 1994). The elliptic MSE, in terms of the wave amplitude (A), and the REF/DIF_1 equation read as follows:

MSE

\[ \nabla_h \cdot (Cg \nabla_h A) + \omega^2 \frac{C_g}{C} A = 0 \]  \[18\]  Kirby and Dalrymple (1983)

REF/DIF_1

\[ 2kCCgAx + 2(k-k_0)A + i(kCCg)A + (CCgA_y) - kCCg|A|^2 A = 0 \]  \[19\]

where:
1. subscripts x and y refer to partial differentials,
2. \(k_0\) is a reference wave number dependent upon bathymetry,
3. A is the wave amplitude (wave height (H)/2),
4. K is a nonlinear parameter defined as \(k^3 CD/C_g\) and where D is given by,
5. \(D = (\cosh 4kh + 8 - 2 \tanh^2 kh)/8 \sinh^4 kh\)  (Martin, Dalrymple and Miller, 1987).

As REF/DIF_S is the model being described here then model verification/testing for its predecessor REF/DIF_1 will not be outlined. Suffice to say that the verification procedures are extensively outlined in Kirby and Dalrymple (1984) and Martin et-al
(1987) with the latter showing that REF/DIF_1 models wave propagation very well in regions where refraction and diffraction are prominent.

Through reading the “Documentation and User Manual” for REF/DIF_S version 1.1, two examples were found that were designed to test REF/DIF_S. The first of these was conducted by Mase and Kirby (1992) and considered the shoaling of unidirectional waves on a plane sloping beach. The second example was conducted by Vincent and Briggs (1989) and considered the behaviour of multi-directional waves over a submerged shoal. Both investigations centred upon setting up wave tanks with simplistic bathymetries and comparing the results obtained with those from REF/DIF_S.

The test performed by Mase and Kirby (1992) firstly involved setting up a wave tank with a constant depth of 47 cm over a length of 10 m, and which then shoaled with a slope of 1:20. A wave paddle was then used to generate waves with a PM frequency spectrum, and twelve wave gauges were placed in the shoaling region to detect the changes of the wave amplitudes. Such data served two purposes. Firstly, the wave data from the first wave gauge (which was at the base of the shoaling region) was used as input to REF/DIF_S, with the wave gauge data from the other gauges compared with the model output. Figure#20 is a schematic cross-section showing the layout of the wave tank.

Figure#20 A schematic cross-section of the wave tank used by Mase and Kirby (1992). See text for details.

By performing the experiment and comparing the significant wave height data from the model with that from the wave gauges, it was observed that REF/DIF_S modelled both the changes in the significant wave height and the location of the breaking point, very well. The model also predicted the decay of the amplitude of the broken waves although close to the simulated shoreline the model output did deviate from the gauge data. This discrepancy was caused by the breaking waves producing a set-up, which is not parameterised in REF/DIF_S. Figure#21 shows the comparison between the models results and wave gauge data.
With it being established that REF/DIF_S could successfully model uni-directional waves, the next step was to test it for multidirectional waves. Vincent and Briggs (1989), like Mase and Kirby (1992), performed such tests using wave data from a wave tank, with the bathymetry of the wave tank being an elliptical shoal superimposed on a constant bathymetry background, and with a directional spectral wave generator used to generate the waves. Figure 22 is a diagram showing the top view layout of the wave tank.

In the tests conducted three different situations were chosen. The first two of these centred upon non-breaking low amplitude waves of narrow and then broad
directionality, respectively. For the third test, larger amplitude waves were considered and were used to investigate the model's ability to model breaking waves. Table#1 outlines the wave characteristics used in each of the three tests.

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Case Number</th>
<th>Tp (sec)</th>
<th>H1/3 (cm)</th>
<th>σm (°)</th>
</tr>
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<tbody>
<tr>
<td>Non-breaking</td>
<td>1</td>
<td>1.3</td>
<td>2.54</td>
<td>10</td>
</tr>
<tr>
<td>Non-breaking</td>
<td>2</td>
<td>1.3</td>
<td>2.54</td>
<td>30</td>
</tr>
<tr>
<td>Non-breaking</td>
<td>3</td>
<td>1.3</td>
<td>19.00</td>
<td>30</td>
</tr>
</tbody>
</table>

Table#1 Wave characteristics used for each of the three tests by Vincent and Briggs (1989). σm represents the directional or azimuthal spread of the waves.

By comparing the model results for case#1 and #2 with wave tank data (see Fig#23) it can be seen that there is a general agreement between the model and wave gauge data. Such results show that the coefficient of the breaking term stays small and does not dissipate significant energy from the spectrum. For the third and final test case where breaking did occur, a comparison between the wave gauge data and model output (Fig#23) shows that the model partially predicts the defocussing effect behind the shoal that arises because of the non-linear effects. It can be observed however, that the model does not predict the wave gauge result of the waves tendency to recover their initial significant wave height. It was thought this discrepancy with the model arose because the breaking wave scheme employed does not take into account multi-directional waves.

Figure#23 The REF/DIF_S output of significant wave height compared with wave gauge data for the first wave condition specified in table #1. All data was for transect#4 (x=12.2 m) in Fig#22.
Figure#23 The REF/DIF_S output of significant wave height compared with wave gauge data for the two last wave conditions specified in table #1. All data was for transect#4 (x=12.2 m) in Fig#22.

Regarding any disadvantages associated with REF/DIF_S, these are that over five data points are needed per wavelength to resolve a wave. With such a high data density REF/DIF_S is normally applied to relative small regions (of the order of one kilometre). Also, REF/DIF_S, like REF/DIF_1, does not model growing wave fields.

2.3.2.2 REFRAC:

REFRAC (short for refraction) was a wave model developed in the 1980s to model the propagation of waves over finite depths, and includes the processes of refraction, wave
and current interactions and depth induced breaking. REFRAC can be used over large areas (on the order of several hundred kilometres). This is because it requires only a few data points per wavelength, as compared to REFDIF, which requires five or more. REFRAC, however, does not include the effects of diffraction, reflection and non-linear wave-wave interactions.

The origin of REFRAC stems from Kirby's (1984) hyperbolic MSE, with REFRAC based on two principles. The first is the irrotationality of the wave number vector and permits the propagation direction to be computed. This principle is written in vectorial form as:

$$\nabla \times \mathbf{K} = 0 \quad [20]$$

where:

$$\mathbf{K}$$ is the wave number vector.

The second principle is the conservation of wave action (which is conserved in the presence of ambient currents and not wave energy). This law permits the computation of wave height over a grid and is expressed as the following inner product:

$$\nabla \cdot \mathbf{B} = 0 \quad [21]$$

where:

1. $$\mathbf{B}$$ is termed the wave action density and defined by,

$$\mathbf{B} = \frac{E}{\omega} (U + C_{gx}) + \frac{E}{\omega} (V + C_{gy}) \quad [22]$$

2. $$E$$ is the wave energy.
3. $$\omega$$ is the angular frequency.
4. $$U$$ and $$V$$ are the ambient current speeds in the x and y directions, respectively.
5. $$C_{gx}$$ and $$C_{gy}$$ are the group wave speeds in the x and y directions, respectively.

In addition to the above two principles, the non-linear dispersion relationship for $$\omega$$ is also utilised in REFRAC. This relationship is given by the following equation.

$$\omega^2 = gk \tanh(\frac{k(\omega)}{2}) \left(1 + \frac{f_1(kA)^2}{D}\right) \quad [23]$$

where

1. $$D$$ is defined the same way the “D” in the REF/DIF_1 equation (#19) is defined.
2. $$f_1$$ and $$f_2$$ are parameters which permit the magnitude of $$\omega$$ to vary smoothly from deep to shallow water and are defined as.
3. \( f_1 = \tanh^5 k h \)

4. \( f_2 = \left[ \frac{k h}{\sinh(k h)} \right]^4 \)

The modelling capabilities of REFRAC were tested using wave height data obtained in autumn of 1982 from ten wave rider buoys located off Haringvliet sluice in Holland (see Fig#24). The wave data was split into three time segments and compared with the model output in each case. The data segments generated were labelled as “normal”, “stormy” and “swell dominated” with the “normal” used to calibrate the model before running it for the non-idealised cases (Dingemans, 1983).

![Figure 24](image)

Figure 24 The location of ten wave rider buoys which were used to collect data in the autumn of 1982. (Modified from Martin, Dalrymple and Miller, 1987).

When such a comparison was performed it was observed that for the “normal”, “stormy” and “swell” conditions, the model’s output and wave buoy data were in good agreement. There was a general inconsistency however, between the model and measured wave heights over and behind the inshore shoal. It was believed this occurred because of the existence of processes associated with the shoal which are not adequately
formulated in the model, speculated to be partial breaking and non-linear wave attenuation (Martin et al., 1987).

2.3.3 Boussinesq Models

The model which will be described here was developed by Madsen and Sorensen (1992) and was chosen because it has been endorsed by, and is commercially available from, Delft University of Technology; a leading wave modelling university. This model costs 188,100 DKK ($40,203.30AUD 18/5/99) (with a pre-processor module) and models the propagation of wind waves and swell from deep to shallow water over variable bathymetry taking into account refraction, diffraction, shoaling, partial reflection and non-linear wave-wave interactions: Its continuity and x momentum equations are:

Continuity Equation:

\[ p \frac{\partial \eta}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \] \[ \text{[24]} \]

x Momentum Equation:

\[ p \frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \left( \frac{U^2}{d} \right) + p^2 g d \frac{\partial \eta}{\partial x} + p^2 U \left[ \alpha + \beta \frac{U^2 + V^2}{d} \right] + \frac{g U \sqrt{U^2 + V^2}}{C^2 d^2} + p \psi_1 = 0 \] \[ \text{[25]} \]

where:

(1) \( \psi_1 = \left( \beta + \frac{1}{3} \right) m^2 (U_{xx} + V_{xy}) - pBgm(\eta_{xx} + \eta_{xy}) - mm\left( \frac{1}{3} U_{xx} + \frac{1}{6} V_{xx} + pBgm(2\eta_{xx} + \eta_{yy}) \right) \)

(2) \( p \) is the porosity of the water column (used for modelling partial reflection from, and transmission through, jetties and breakwaters)
(3) \( \eta \) is the free surface elevation
(4) \( m \) is the mean water depth
(5) \( d \) is the total water depth (\( = \eta + m \))
(6) \( U \) and \( V \) are the horizontal volume transports (m³s⁻¹) in the x and y directions, respectively.
(7) \( C \) is the Chezy resistance number (m¹/²s⁻¹)
(8) \( \alpha \) is the resistance coefficient for laminar flow in porous media
(9) \( \beta \) is the resistance coefficient for turbulent flow in porous media
(10) \( B \) is the linear dispersion factor = \( \frac{1}{15} \)

This model was tested for two hypothetical bathymetries. Firstly, a bathymetry of (non-zero) constant slope was used to test the linear shoaling properties of the model, with the refraction and diffraction ability of the model then tested by modelling the propagation of waves over a semi-circular shoal.
For the first test, the model output was compared with that generated from Stokes second order theory. For the second, Whalin's (1971) experimental results for waves over a semi-circular shoal were used to test the model's refraction and diffraction capabilities. In each of the tests, the model predictions were very good (Madsen and Sorensen, 1992).

As with most models, Boussinesq models do have disadvantages. One of the biggest is that they require over five data points per wavelength to resolve the waves and as such can only be applied to relatively small coastal regions. Another disadvantage (if it could be called that) is that Boussinesq models are limited to shallow depth ranges. As an example, the model by Madsen and Sorensen can only be applied to regions where the depth to deep water wave length ratios are less than 0.5 (see www.dhi.dk/software/mike21/).

2.3.4 Simulation Models:

2.3.4.1 WAVE

WAVE is a model which was developed by Ebersole and Dalrymple (1979) to predict the changes that occur to waves due to refraction, wave-current interaction, wave setup and set-down, lateral mixing and wind effects, as waves propagate towards a shoreline. WAVE was also developed to calculate wave induced nearshore circulations and is sold in conjunction with a morphology model called SED-SIM (short for sedimentary simulations). The wave induced circulation schemes employed in WAVE and the morphology model SED-SIM will be outlined in later reports.

In WAVE the assumption is firstly made that deep-water waves are represented by Airy or sinusoidal waves and that shallow water waves are represented by Stokes waves. Such an assumption is a good approximation, as it has been known since the 1840s that deep water waves are sinusoidal in shape and change to that of a Stokes wave as they interact with shallow water. Figure#25 shows the difference in the shapes between Airy and Stokes waves. These approximations are then applied to differential equations representing the aforementioned processes and to the equations of continuity of mass and momentum so that the changes that occur to the waves as they propagate towards shallower regions can be modelled.

WAVE has been calibrated for particular situations, and its sensitivity and performance tested. Such investigations were performed by Martinez (1987b) and Martinez and Harbaugh (1989), however, these investigations only reveal WAVE's performance in estimating nearshore currents and not how well they predict nearshore wave characteristics. As such no conclusions could be drawn regarding WAVE's ability to model nearshore wave characteristics.
The model developed by Lakhan (1989) was developed to simulate the propagation of regular waves towards a shoreline and included depth-induced breaking. Lakhan achieved this by modelling the shoreward propagation of waves using four different wave theories. The four wave theories, from offshore to nearshore, were Airy, Stokes, Cnoidal and Solitary, with the depth to wavelength ratios corresponding to each as quoted below:

Airy, or Linear or Sinusoidal \((d/\lambda > 0.5)\) is appropriate for deep-water waves.

Stokes, or second order, waves \((0.5 > d/\lambda > 0.1)\) describes waves of a transitional depth.

Cnoidal waves \((d/\lambda < 0.1)\) is ideal for describing waves in shallow water.

Solitary Waves \((d/\lambda < 0.1)\) is applicable for shallow water waves closest to the shore.

where:

(1)\(d\) is depth,
(2)\(\lambda\) is the wavelength.

The model developed by Lakhan, however, is a very simple model, and does not include refraction, diffraction, reflection, wind generation, white-capping and wave-current interactions. As such, the view could be taken that such a model is too primitive for inclusion here. It was included, however, to illustrate the spectrum of nearshore wave models which exist. This model also has the disadvantage that at the boundaries between the different types of wave theories discontinuities form in the wave characteristics.

Figure 25 A schematic diagram showing the difference in wave profile for Airy (sinusoidal) waves and Stokes waves. Taken from Martinez and Harbaugh (1993).
Due to this model being of such simplicity it has a small distribution within the scientific community, and as such investigations concerning testing and validation are few. The only validation attempts performed were by Lakhan (1989) himself, with Lakhan performing this by analysing model output for an offshore wave height of 0.51 m and a period of 5.48 seconds (see Fig#26). From this figure it can be seen that "when deepwater Airy waves enter into the Stokes’ wave range there is an initial increase in the height of the wave accompanied by shoreward decreases in wave celerity and wave length. As expected, the wave steepness ratios also progressively increase with shoreward propagation". Hence, according to Lakhan, "the model correctly predicts wave height variations which are similar to those of natural shallow water waves very close to the shore". In this way Lakhan qualitatively validated his model in terms of the general behaviour of real waves. Even though the model reportedly reproduces the expected behaviour of surface waves, and would be a useful model in the absence of other more sophisticated models, field measurements necessary to fully validate the model are lacking.

**RESULTS OF ITERATION 1**

**WAVE PERIOD = 4.58 SECONDS**  
**INITIAL DEEPWATER WAVE HEIGHT = 0.51 METRES**

<table>
<thead>
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<th>DISTANCE</th>
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<th>LENGTH</th>
<th>CELERITY</th>
<th>STEEPNESS</th>
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<td>0.510</td>
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<td>7.148</td>
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</table>

Figure#26 Results obtained using Lakhan’s simulation model for a wave with offshore wave period and wave height of 5.48 seconds and 0.51 m, respectively. (Taken from Lakhan, 1989).
2.3.5 Recommendations for complex models:

Through reading the available literature regarding nearshore wave models it was quickly observed that there was no single model which can model all the processes which affect surface waves. What is available are models which model particular processes very well. Because of this, the need here was to choose the best models which model certain processes very well, with further research needed to evaluate which parts of the northern Australian coastline each model can be applied.

The models chosen for recommendation were SWAN and REF/DIF_S. Both of these models are free of charge, with SWAN particularly suitable for modelling growing waves and including such processes as refraction, shoaling, white capping, depth-induced wave breaking and bottom friction. Where waves are no longer growing, the model REF/DIF_S is more suitable. This model, like SWAN, includes shoaling, refraction and dissipation, but unlike SWAN also models diffraction.

3. Summary and Discussion

In this report nearshore wave models found within the literature were firstly placed into one of two broad classes with their attributes and validation procedures then analysed. The two broad classes were: (1) simple models which could be used relatively quickly in a matter of minutes through the usage of nomograms or quick calculations and (2) complex models which could only be used through the usage of computers.

The nearshore wave models found within the literature and the processes they model have been summarised in the following two tables:

Simple Nearshore Wave Models:

Note: All require as input wind speed, fetch and bathymetry.

<table>
<thead>
<tr>
<th>Name or Reference (type)</th>
<th>Prognostic Parameters</th>
<th>Input parameters whose spatial derivatives can be included in the models</th>
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<tr>
<td>RSPM (nearshore)</td>
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<tr>
<td>Krylov, Strekalov and Tsyplukhin (1976) (nearshore)</td>
<td>H_s and T_p</td>
<td>Fetch, wind speed and depth</td>
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<td>Fredsoe and Deigaard (1992) (Surf Zone)</td>
<td>Broken wave height for regular waves and maximum set-up</td>
<td>depth</td>
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<tr>
<td>Battjes and Jansen (1978) (Surf Zone)</td>
<td>Broken wave height for irregular waves</td>
<td>depth</td>
</tr>
<tr>
<td>Thornton and Guza (1983) (Surf Zone)</td>
<td>Broken wave height for irregular waves</td>
<td>depth</td>
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</table>
Svendsen (1984) (Surf Zone) | Broken wave height for regular waves | depth
---|---|---

Complex Nearshore Wave Models:

<table>
<thead>
<tr>
<th>Name</th>
<th>Wave Growth</th>
<th>Diffraction</th>
<th>Refraction</th>
<th>Wave-Current Interactions</th>
<th>Shoaling</th>
<th>Reflection</th>
<th>Wave breaking</th>
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<td>Yes</td>
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4. Acknowledgments

First and foremost I would like to thank my Task Manager Dr Phil Mulhearn and the librarians at DSTO, Pyrmont for making available to me many helpful books and journal articles. Also thanked for providing helpful advice is Professor Ian Young (University of Adelaide), Associate Processor Andy Short (University of Sydney), Dr Roland Ris (WL | Delft Hydraulics), Dr Jeffrey H. List (U.S. Geological Survey) and Dr Graham Warren (Australian Bureau of Meteorology).

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The aim of this investigation was to review the different types of nearshore wave models currently available and to make recommendations as to which were most suitable for modelling nearshore wave conditions. The models were first placed into one of two broad classes. The first was those that could be applied in a small amount of time (for example a few minutes) either by quick calculations or the usage of nomograms. The second type was those that were computer resource intensive and were more likely to be executed in the laboratory.

For the first type of nearshore wave model, those deemed worthwhile included the Krylov, Strekalov and Tsypulkin (1976) model and the model by Thornton and Guza (1983). For the second type of wave model, SWAN (Simulating WAves Nearshore) and REF/DIF_S (REFraction and DIFfraction Spectral) were recommended. All of these models have been tested against laboratory or field data with good comparisons made and all have particular qualities and attributes which make them useful.