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ACQUISITION OF GPS SIGNALS FOR HIGH ALTITUDE SPACECRAFT NAVIGATION: EXPLORING ISSUES IN VISIBILITY AND LINK ANALYSIS

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17 Apr 1998

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ABSTRACT

The Global Positioning System has far exceeded expectations in its short lifetime, and the scientific and engineering community has churned up application after application for expanding and exploiting the system. Satellite navigation using GPS has become a hot new area of GPS applications, with several satellites making use of new commercially available GPS spaceborne receivers. It appears that GPS, originally intended for terrestrial and near-terrestrial users, is well suited to providing accurate coverage for low earth orbiting satellites below the GPS constellation.

Using GPS to navigate satellites above the GPS constellation altitude poses many problems. Coverage is far more limited and signal strength drops sharply for high altitude users. Spacecraft above the GPS satellite have limited availability of the GPS signal. Possibilities to still exploit the signal do exist, including making use of the GPS cross-link ranging signal, capturing GPS signals from across the globe that are not blocked by the Earth, and using mathematical filters to determine a navigation solution when visibility is limited.

Signal strength is also a major area of concern. Higher gain antennas will be required for high altitude spacecraft in order to achieve an acceptable carrier to noise density ratio required to receive the GPS signal.

This paper will explore these options and describe the potential benefits of exploiting this capability: decreased costs of ground operations, increased accuracy, as well as increased satellite autonomy. I will also present the possible application of GPS time tags to missile warning data from Defense Support Program satellites as well.

INTRODUCTION

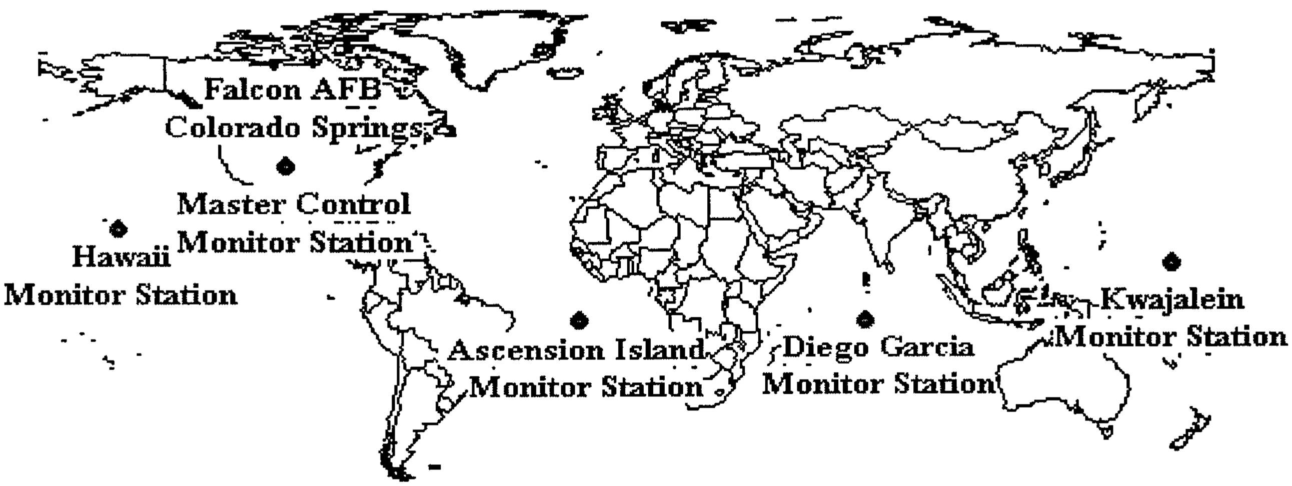
The Global Positioning System has revolutionized the science of navigation, setting new standards for navigation in terms of global coverage, timeliness, positioning and velocity accuracy. Originally intended for the military, GPS navigation has spawned a host of applications, including many the system was not originally designed for. Space system navigation using the GPS has become a rapidly growing field. Chao reports on the merits of using GPS for space navigation. Space qualified GPS receivers are relatively light (about 15 lb@ = 6.804 kg). They are inexpensive, about $100,00 - $300,000 apiece, including the requirements to space qualify the receiver and the antenna. Cost per unit will drop as the technology progresses from a research and experimentation phase to a more commercial phase. Using GPS for space navigation can also provide greater orbit determination accuracy (within 20 – 100 meters) over traditional systems. [9]

The possibilities and problems of using GPS to navigate high altitude spacecraft will be discussed in this paper. Currently, applying GPS to high altitude spacecraft navigation is in an experimentation and research phase. Scientists at the Aerospace Corporation in El Segundo California have researched the possibility of using GPS to aid in navigating spacecraft in geosynchronous transfer orbits. The Falcon Gold mission, a joint science mission between UCCS and the United States Air Force Academy, demonstrated the ability to acquire the GPS signal from a geosynchronous transfer orbit. These research projects and others will be discussed in this paper as well as some important questions. How would widening the GPS antenna beamwidth improve visibility for satellites at GEO? How could a Kalman filter be used to provide a navigation solution for a GEO satellite when an inadequate number of GPS satellite broadcasts can be received?

BACKGROUND

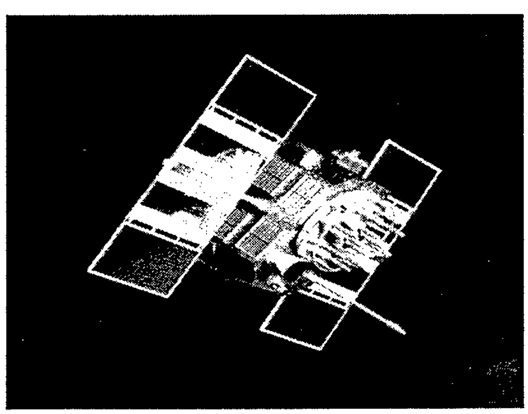
The NAVSTAR GPS (Navigation System with Timing and Ranging) evolved out of the Air Force's System 621B prototype for a space based navigation system. In the 1970's Phase I of the program occurred. This was the concept validation phase when the first prototype satellites were made. By 1979 full scale development was underway, and the first GPS Block I satellites were launched. Phase III of the program began in 1985, when full scale production and deployment of the GPS Block II constellation began. In December of 1993, the Air Force declared initial operational capability (IOC) of the GPS constellation, consisting of 24 fully functional Block I and Block II satellites properly positioned in the orbit.

The Global Positioning System consists of three segments: Control, Space, and User. The Control Segment includes the Master Control Station (MCS) located at Falcon Air Force Base just east of Colorado Springs, CO as well as the Air Force Satellite Control Network (AFSCN). The AFSCN is a global network of ground antennas the Air Force uses to transmit and receive messages to and from the satellites. Unmanned monitoring stations collect range, azimuth, and elevation data on each of the satellites and transmit that information to the Master Control Station over the Defense Secure Communication Satellite system (DSCS) Personnel in the 2nd Space Operations Squadron of the Air Force's 50th Space Wing use the site tracking data to maintain precise ephemerides on all GPS satellites. 2nd SOPS is also responsible for maintaining the GPS clock accuracy, a key element in navigation. A picture of the GPS Control Segment follows: [1]



Global Positioning System (GPS) Master Control and Monitor Station Network

The GPS Space Segment is composed of 24 satellites orbiting the earth in near circular orbits. Six orbital planes evenly spaced 60 degrees divide the constellation. Four GPS satellites occupy each plane. This constellation design provides excellent coverage for ground users. The satellites have an inclination of roughly 55 degrees, and orbit the earth at an altitude of about 20,200 km. At this altitude the satellites have a twelve hour orbit.



More characteristics of the GPS Block II satellites are provided in the following table. [2]

Manufacturer	Rockwell
Mass	900 kg
Size	5 m wide with solar panels extended
Design Life	7.5 years
Unit Cost	$40 million $100 million including launch
Total System Cost to Date	$10 billion

Each GPS space vehicle is equipped with four atomic clocks (two rubidium and two cesium) to measure time with high precision. These clocks weigh hundreds of kg's each and cost as much as $200,000. Accurate timing is crucial because the GPS satellites transmit the time each broadcast is sent from the satellite. It is important that the timing of both the clocks on the space vehicles and the receivers be as closely synchronized as possible, not always practical since receivers typically use less precise quartz clocks. Receivers must know the time the transmission was sent to compare with the time the transmission was received in order to determine a range to the satellite.

The satellites broadcast their signal in a wide beam with the center of the beam pointed towards nadir, the center of the Earth. The satellites broadcast on two frequencies, the L1 carrier, 1.5752 GHz, and the L2 carrier, 1.2276 GHz. The satellites modulate a unique Pseudo Random Noise (PRN) code using Bi-Phase Shift Keying on each carrier frequency. This means the GPS carrier signals are varied by changing their phase, the up and down positions of the waves, back and forth at a regular programmed rate and interval unique to each satellite. This enables a receiver to identify the satellite from which it receives a transmission. [3] Two different PRN codes are modulated on each carrier frequency, the Coarse/Acquisition (C/A) code and the Precise (P) code.

The C/A code provides the GPS Standard Positioning Service. It consists of a sequence of 1,023 bi-phase modulations of the carrier wave, or a "Chip rate" of 1.023 MHz. The P Code is similar to the C/A code, but the Chip rate is greater by several orders of magnitude. GPS receivers know the unique codes broadcast by each satellite. By comparing a received code to a known code, the GPS receiver is able to determine the time it took for the signal to travel from GPS satellite to the receiver. Multiplying travel time of the signal by the speed of light ($3 * 10^6$ m/s) yields the pseudorange to the satellite, biased because the clocks on the receiver and the GPS spacecraft are not synchronized. [2] Specifications for accuracy for both the Standard Positioning Service and the Precise Positioning Service follow in this table: [2]

	Horizontal accuracy	Vertical Accuracy	Time Accuracy
Standard Positioning Service	100 m	156 m	167 nano seconds
Precise Positioning Service	17.8 m	27.7 m	100 nano seconds

The GPS satellites broadcast their ephemerides, including position, velocity, and orbital information. Terrestrial receivers obtain this data and can calculate pseudoranges to at least four GPS satellites. The receiver software forms four independent equations to solve explicitly for four unknown variables (three components of the user position vector plus the receiver clock offset from GPS time). Geometrically this is equivalent to determining the receivers position by mathematically solving for the intersection of 4 spheres with the centers of each sphere at the GPS satellite. The intersection of these spheres is the position of the receiver. [9] This allows the receiver to determine a navigation solution for the user's position. Receivers can determine the user's velocity by solving for the doppler shift in frequency resulting from the relative motion of the GPS spacecraft and the user. Since the GPS satellite velocity vectors are known and transmitted in the NAV-msg, the receiver can solve for the user's velocity. [2]

The GPS Navigation Message (NAV-msg) contains all the necessary data a receiver needs to calculate the range to a particular satellite. The GPS NAV-msg consists of time tagged data bits marking the time the GPS began transmitting its message. Twenty five data frames compose the whole NAV-msg that the satellites transmit over a 12.5 minute period. The GPS space vehicles transmit each data frame consisting of 1500 bits every 30 seconds. Thus the GPS space vehicles transmit at a rate of 50 bits per second, or 50 Hz. Included in the NAV-msg are satellite clock corrections and precise space vehicle ephemeris (position and velocity vectors as well as orbital elements). The ephemeris data can be used to describe the GPS satellite orbit for very short sections of its orbit. The ephemeris for each satellite needs to be updated continuously by the Air Force. [1] Outdated ephemerides would yield inaccurate navigation solutions for users.

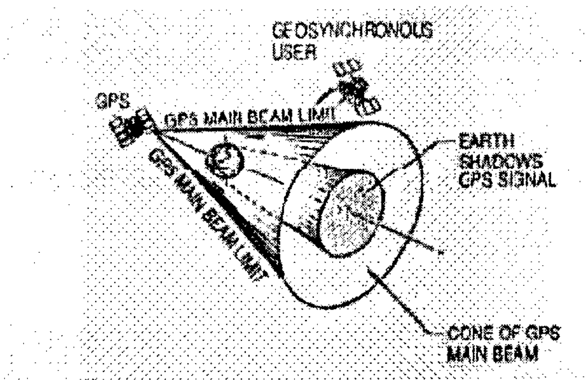
SPACECRAFT NAVIGATION USING GPS

GPS provides excellent coverage for users on or near the surface of the earth, as the system was designed to do. From these locations, users can expect to obtain ranges from the required number of satellites in order to determine a navigation solution. Furthermore, the geometry of the received satellites is favorable for greater positioning accuracy. Users on or near the surface of the earth can expect low Dilution of Precision (DOP) factors, indicating a good spacing of GPS satellites from which the receiver obtained transmissions. Users on or near the surface of the earth can also expect a quality transmission. Hand held GPS receivers with low gain antennas are perfectly sufficient for fulfilling the link requirements and obtaining a good carrier to noise density (C/No) ratio to obtain a quality transmission.

GPS satellite visibility characteristics are also favorable for low earth orbit (LEO) satellites. Applying GPS navigation to LEO satellites has already been demonstrated in several missions. For example, the GPS/MET mission in one test produced a receiver navigation solution accuracy of 46m. The DARPASAT mission demonstrated a GPS position accuracy of 350 m. The Wake Shield Facility-02 mission produced a solution accuracy of 62.6 m with a least squares fit of an orbit to the navigation solution. [10] The conditions that make GPS a viable method of LEO navigation are not as optimal for users at higher altitudes. Potential users of GPS at high altitudes must solve problems in both of these issues, visibility and link requirements, in order to make use of GPS.

GPS VISIBILITY TO GEO USERS

The GPS antenna broadcasting system is composed of twelve helical broadcasting elements pointing directly to the center of the earth (nadir). This antenna system provides adequate signal strength within a cone starting from the GPS satellite and extending to nadir. Lester reports a usable cone with a half angle of 21.4 degrees. [4] Scientists from the Aerospace Corporation report a usable half angle beamwidth of 22



degrees of Block IIA satellites and only 20 degrees for Block IIR. [5] Users at altitudes above the GPS constellation can only make use of the antenna radiation coming from GPS satellites across the globe. The earth blocks 27.8 degrees of the GPS broadcast, allowing only a 7.5 degree conical annulus of GPS signal which can be obtained from users on the other side of the earth. [4] This has drastic effects on the number of GPS satellites visible to a user at GEO. Please note the appenidix has to scale drawings of the GPS to GEO constellation geometry.

A user at GEO will almost never have at least four GPS satellites in view from across the globe, thus requiring a mathematical filter such as a Kalman filter to propagate previously received data in time forward until at least four GPS receptions have been made sequentially.

GPS TO GEO LINK CONSIDERATIONS

Receiving an acceptable carrier to noise density ratio (C/No) is another problem for GEO users.. Source [5] reports that the C/No of the L1 carrier decreases by 17.14 dB from the worst case C/No for an Earth user and the cross-link ranging signal received at geosynchronous altitude decreases 6.74 dB beyond the farthest intra-GPS constellation link. These additional losses must be corrected by higher gains in the GPS receiver antenna.

Lester reports the best possible transmit antenna gain of 13.4 dB for a GPS transmission and a worst case possible transmit antenna gain of 1.4 dB for a GPS transmission. Using the Space Mission Analysis and Design software, Lester calculated that a receiving antenna aboard a geosynchronous satellite would have to have a gain of 9.23 dB in order to achieve the required C/No to receive the GPS transmission. To produce this 9.23 dB, Lester states a parabolic dish or a special helical antenna will be required. Again using SMAD, he calculates a 9.23 dB antenna with a beamwidth of 57.7 degrees will satisfy the link requirements, and also be able to receive GPS satellites in view with a maximum angular separation of 26.6 degrees. [4]

The receiver Lester endorses is the GPS receiver for the Phase-3D satellite developed by NASA's Goddard Flight Center and the Radio Amateur Satellite Corporation (AMSAT). This receiver uses a unique antenna design developed at Jet Propulsion Laboratory. The antenna is called "Helibowl", and theoretically provides a 10dB gain. The design consists of a two-turn cavity-backed helix. The cavity for the helix is a stainless-steel bowl with a diameter of 20cm, and the two turn helix is attached to the center of the mixing bowl. The parabolic bowl focuses radio signals on to the two turn helix. Unfortunately, the AMSAT homepage on the Internet currently lists the Phase-3D GPS project as defunct, and so apparently the receiver and antenna design failed to live up to expectations.

EXPLOITING THE GPS CROSS-LINK

One possible method to improve the number of GPS satellite transmissions received for a high altitude spacecraft is to make use of the GPS cross-link ranging signal. This is another example of exploiting a capability for an application it was not originally intended for. GPS cross-link capability first became available with the GPS Block IIR replacement satellites. The purpose of the cross link was to allow the GPS satellites to continually range to one another, thus eventually allowing the satellites to maintain their own ephemerides autonomously.

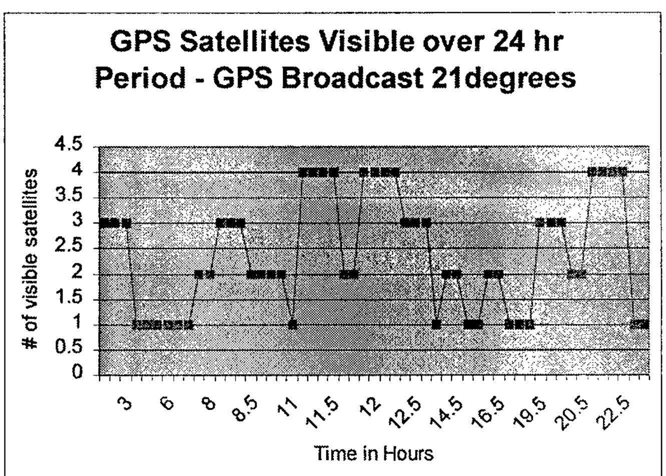
The GPS cross-link frequency of 280-300 MHz is much lower than the L1 carrier frequency. The GPS cross-link also is broadcast with a wider, hemispherical beamwidth. This wider beam is the result of a lower gain antenna, and therefore the transmission power of the cross-link has been boosted up to 100 Watts (as opposed to 20 Watts for the L1 transmission) to ensure enough signal strength. Source [5] reports the cross-link signal is available to users at all altitudes on the nadir side of the GPS satellites, and graphs from source [5] indicate that GPS satellite visibility can reach as high as 20 or more visible satellites to a GEO user!

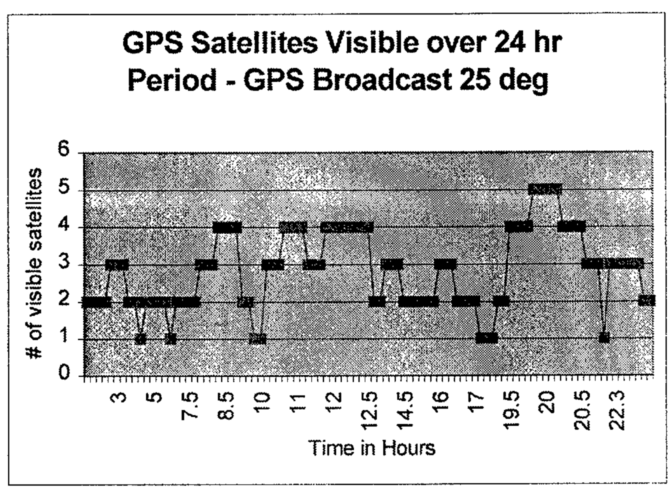
Another benefit of exploiting the cross-link is that the technology, hardware, and receiving software required to receive the GPS cross-link has already been developed, since the GPS satellites themselves already make use of the cross-link. The most severe constraint to applying this technology for GEO users is the additional propagation path length and resulting decrease in signal strength. Higher gains in the receiving equipment of GPS receivers at GEO must account for the loss in signal strength.

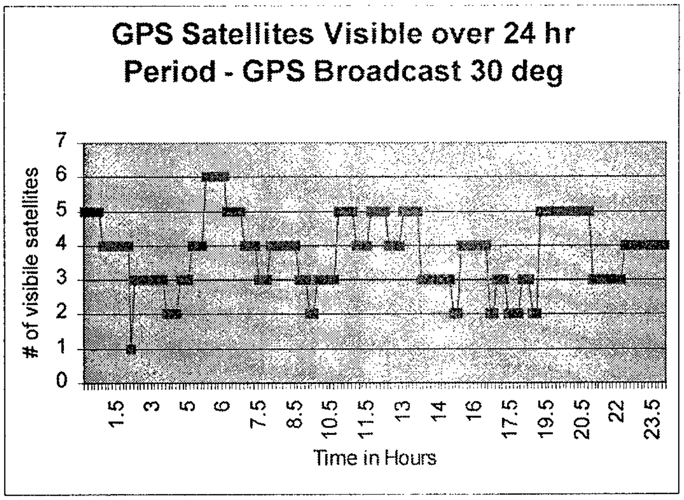
WIDENING THE GPS BROADCAST

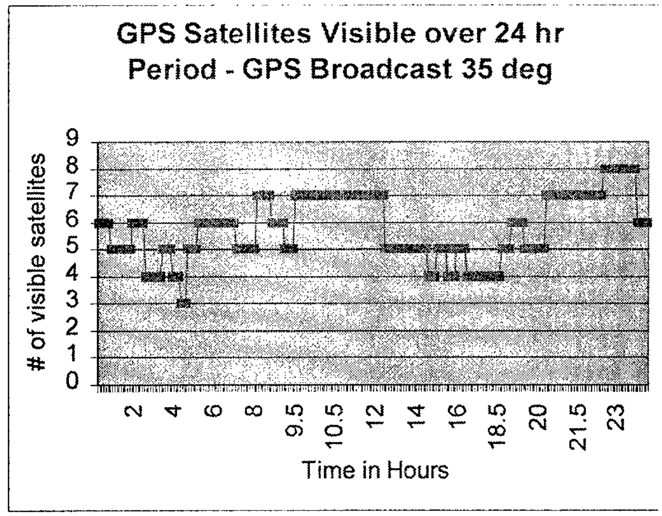
Another possible way to increase the availability of the GPS signal to GEO users is to widen the GPS transmission beam. This is the area of my independent research – I am not aware of any other attempts to explore this possibility. I used the Satellite Missile and Analysis Tool (SMAT) at the Space Warfare Center at Falcon Air Force Base to compute GPS visibility to a Navy UHFO geostationary satellite. SMAT is a comprehensive 3 dimensional animated visual modeling tool that can be used for analysis on orbiting satellites and ballistic missiles. SMAT uses Simplified General Perturbations theory (SGP) to propagate satellites in orbit. It is owned and controlled by Air Force Space Command/Space Warfare Center/Analysis and Engineering and was validated on October 1, 1997.

I varied the usable GPS antenna beamwidth to see how this affected visibility. Currently geostationary satellites will only be in position to receive one to three GPS transmissions at any given point in time, with many periods of zero visibility. It is still possible for a GEO satellite to navigate using GPS with this current setup, but special mathematical modeling and filtering techniques would be required by the receiver.









If the usable GPS half cone transmit antenna beam on each of the GPS satellites in the constellation was widened to 35 degrees, it appears we would have at least four GPS broadcast transmission visible to a GEO satellite at almost any given time. Obtaining four GPS transmissions would enable the GEO satellite to navigate with GPS in real time. Widening the usable GPS antenna beam further would yield even greater improvements in constellation visibility. Increasing this visibility would also improve the geometry of the GPS satellites received by the GEO user. GPS transmissions received would be more widely spaced apart, reducing the Dilution of Precision factor and thus improving navigation solution accuracy.

Widening the GPS broadcast transmission beam would have implications on the radio link between the GPS satellite and the user satellite, however. A wider beam would have a lower gain. Along with the increased propagation path length from the GPS satellite across the earth to a GEO satellite, increases in the GPS transmit power or the GEO satellite's receive antenna gain must be made to ensure a quality transmission.

Propagation path lengths can be found by simple right triangle geometry. The appendix contains the geometric calculations for determining the propagation path lengths from a GPS satellite to a user GEO satellite. Two propagation path lengths are calculated for each case of usable antenna beamwidth. The first path length is for the best case of making maximum use of the GPS antenna gain (Gt). This case occurs when the GPS signal skirts above the earth with an additional 1000 km added to account for the earth's ionosphere. We do not consider GPS transmissions that would propagate through the ionosphere because that would increase error in pseudo range determination. The second case occurs when the GEO satellite is situated at the point where there is the widest angle to make use of the GPS transmit antenna beamwidth. Propagation path lengths for best case Gt and worst case Gt are summarized in this table:

Usable GPS antenna beamwidth	Best Case Gt propagation path length	Worst Case Gt propagation path length
21	67153 km	65984 km
25	67153 km	64841 km
30	67153 km	63126 km
35	67153 km	61180 km
40	67153 km	59003 km
45	67153 km	56619 km

Link budget analysis was performed on the scenarios with different transmission beamwidths. Current GPS receiver technology requires a received carrier to noise density ratio (C/No) of about 31 dB. Receive antenna gains on the GEO satellite needed to fulfill this requirement were calculated. A summary of those results follows:

Usable GPS antenna beamwidth	Required Gain of Receive antenna	Required diameter of the receive antenna
21 deg	8.26858 dB	0.271746 m
25 deg	11.1962 dB	0.380666 m
30 deg	12.54837 dB	0.444788 m
35 deg	13.61382 dB	0.502835 m
40 deg	14.45909 dB	0.554229 m
45 deg	15.1239 dB	0.598314 m

The usable GPS antenna beamwidth of 21 degrees is the narrowest beam on any given satellite in the current GPS constellation. [5] I arbitrarily selected the modifications to the usable transmit antenna half cone beamwidth. The gain of these wider beam GPS transmit antennas was modeled using an equation in Space Mission Analysis and Design. [13] This equation, $G = 44.3 - 10 * LOG(theta_x, theta_y)$, is used to calculate the gain of a noncircular antenna. $Theta_x$ and $theta_y$ are half power beamwidths along the major and minor axis of the antenna. This equation tended to bias the transmit antenna gains lower than what was expected. Although not perfect, the model suffices and the bias actually provides extra link margin. To model the drop off in transmit antenna gain for the wider beams, I modeled the antenna gain patterns to drop off analagously to how they drop off for the current transmit antenna of the GPS antennas. The peak transmit antenna gain is currently 13.4 dB, and the lowest usable antenna gain is 1.4 dB. The lowest usable antenna gain is approximately 6.31 % of the peak transmit antenna gain. This percentage was applied to the lower gains of the wider beams.

The receive antenna gain required for the geostationary satellite's GPS receiver was calculated by rearranging the equation for C/No. Solving for receive antenna gain yielded

$$Gr = C/No - EIRP - Ls - La - 228.6 + 10 * LOG(Ts) \quad [13]$$

C/No is an input parameter equal to 31 dB for current GPS receiver technology. EIRP is Effective Isotropic Radiated Power, equal to the Transmit Power + Gain of Transmit Antenna (both input parameters). Ls is Free Space Loss, accounting for the loss in signal strength as the transmission propagates through space. Note that Ls increases by as much as 8 dB as the propagation path increases from GPS satellite to earth to GPS satellite to GEO satellite. Ls increases because there is about an extra 45,000 km of space that a signal must propagate through before a GPS signal reaches a GEO satellite! La is equal to atmospheric loss, set equal to 0 dB since signal propagated through the earth's atmosphere is ignored for this link analysis. Ts is an estimate for system noise temperature, this is rather small because we are dealing with a satellite to satellite link, in general not as great as an earth to satellite or satellite to earth link. [4]

As the receive antenna gain increases its half power beamwidth decreases. The antenna becomes more directional and focused. For the satellite at GEO, it must be able to view all GPS broadcasts that can see the satellite, or equivalently all GPS satellites within the maximum angular separation given by their transmission beams. The half power beamwidth of the receive antennas was calculated using SMAD eq 13-17:

$$\text{Half power beamwidth} = 21 / (f_{GHz} * D)$$

I found that the widest GPS transmission possible would be 40 degrees in order for the receive antenna to be able to view all possible transmissions.

To estimate the size and mass of such a circular, parabolic receive antenna, I referred to Space Mission Analysis and Design and found an analagous antenna system. A FLTSATCOM satellite uses a fixed parabolic antenna with a reported gain of between 16-19 dB and a beamwidth of 18 degrees. [13] These are fairly comparable performance characteristics to the required antenna for a GEO satellite to make use of the GPS signal. This antenna has a mass of 3.9 kg, and a diameter of 0.7 m. The antennas required for a GEO satellite to make use of the GPS signal have lower gain and wider beamwidth requirements. Smaller antennas than the one FLTSATCOM uses could be used for this application. The mass of 3.9 kg and and antenna diameter of 0.7 m are significant, but certainly manageable into large, complicated satellites such as the Defense Support Program (mass = 2386 kg, size = 10 m high x 7 m wide).

Making use of a Quad Helix antenna may reduce mass and size parameters further. The Intelsat-V satellite uses a Quad Helix antenna specifically designed for use in the L Band frequency. This antenna performs similarly to the FLTSATCOM parabolic antenna, with a gain of between 16-19 dB and a beamwidth of 18 degrees. However it has a mass of only 1.8 kg and dimensions of 0.4 x 0.4 x 0.47 meters. SMAD even recommends the helix antenna for lower-gain, wide beam applications. SMAD also states that the helix antenna is also easier to mount on a satellite structure. [13]

One area of concern was how widening the usable GPS antenna beam would affect GPS signal strength for terrestrial users. Obviously, a modification to the GPS that would adversely affect the users the system was originally designed for is unacceptable. Widening the GPS antenna beam would decrease the Gain of the transmit antenna, therefore reducing effective isotropic radiated power, potentially making the link unusable. One way around this problem would be to increase the GPS satellite's transmission power. This would require larger solar arrays to generate this power and hence increased size, cost, and weight to the satellite. But does enough margin exist in the current link between GPS satellites and terrestrial users to accommodate a wider beam? I did an analysis of the link between the GPS satellites and terrestrial users and found that in all cases of widening the GPS transmission beamwidth, a C/No of 31 dB-Hz or greater was achievable. The summary of this link analysis follows in the table below:

Usable GPS transmit antenna beamwidth	Worst Case C/No received by terrestrial user
21	45.96682
25	42.88742
30	41.3038
35	36.96486
40	38.80502
45	37.78197

The worst case propagation path length was calculated using the case where the GPS broadcast barely skirts the limb of the earth. This propagation path length was determined to be equal to 25566 km. The link budget was recalculated using a 3 dB receive antenna gain, typical of the low receive antenna gain of many commercial GPS receivers. Although there is definitely a noticeable drop off in received C/No for terrestrial users, we are still above the required threshold for GPS receiver technology.

The conclusion is that widening the GPS antenna gain would result in decreased signal strength for terrestrial users, but the existing link margin built into the system can compensate for the decreased signal. Terrestrial users would not be adversely affected by this modification.

After hearing about some interference issues and how the GPS signal is often easily blocked by such factors as trees and electrical interference, I decided that it would be best to not suggest a modification to the existing communication system that would decrease the signal strength for terrestrial users. Using the current system as a starting point, I calculated how much increase in transmit power would be required for each of the wider transmission beams to maintain a C/No ratio of 46 dB Hz for a worst case terrestrial users. I found that the resulting increase in transmission power required is actually not too bad, as the following table shows:

GPS Usable antenna half cone beamwidth – (deg)	P_t Increase required to maintain terrestrial signal strength – (W)	Solar Array Size Increase Required – (m^2)
21	0	0
25	2.0477	0.0108
30	2.9486	0.0155
35	4.0134	0.0211
40	5.242	0.0276
45	6.6344	0.0349

Solar array size increase required was calculated using an estimated power output of 190 W/m^2 for Silicon solar cells. [13] These transmission power increases and solar array size increases are quite manageable given the current GPS satellite setup, which produces an estimate 1100 Watts.

To conclude, I have calculated that by widening the usable GPS antenna half cone beamwidth to 35 degrees or more over the current 21 degrees, a satellite in geostationary orbit will be able to view 4 or more GPS satellite transmissions, and thus be able to navigate in real time just as any terrestrial user would. Transmission power increase from the GPS satellite to maintain terrestrial user signal strength is manageable. If the usable GPS antenna half cone beamwidth were widened to 45 degrees, the required antenna gain on the receiver would be so great that the narrower beam would not be able to view all possible GPS transmissions. My final system improvement recommendation is as follows:

- Widen the usable GPS half cone transmission to 40 degrees
- Increase the Transmit Power from the GPS satellites to 30.36086 W from the current 25.12 W
- Use a GPS receiver with a receive antenna gain of 14.46 dB. A circular antenna with such a gain would be about .6 meters in diameter

With the GPS Block IIF satellites currently under construction and with future navigation systems being designed, perhaps these modifications could be considered to allow real time navigation of GEO satellites using GPS.

SPACEBORNE GPS RECEIVER TECHNOLOGY

GPS receivers for spaceborne applications face unique challenges not presented to terrestrial receivers. For one thing, the relative motion of two different spacecraft is much greater than the relative motion between a GPS satellite and a terrestrial received doppler frequency signals for a LEO spacecraft vary by as much as 100kHz. By contrast, received doppler frequency signals for a terrestrial user vary by one tenth of that amount. Spaceborne GPS receivers determine a navigation solution similar to how terrestrial GPS receivers do, by measuring pseudoranges and carrier phase observations from each GPS satellite in view. This navigation solution, consisting of the satellite's position and velocity vectors, is called the satellite's state vector. The receiver's software must be able to convert this state vector into a determination of the satellite's orbit. [10]

The basic process of determining the navigation solution will now be described. First the receiver's antenna collects all GPS signals available and the receiver receives the GPS data. Using the GPS satellite ephemerides from the NAV-msg of each GPS satellite transmission received, the GPS receiver calculates the state vectors of each of the GPS satellites. The receiver's software code must propagate the state vector and the state vector error covariance matrix using analytical or numerical integration. An Extended Kalman Filter is used to predict the states between two updates. Then the EKF is used to update the state vector, and using astrodynamics algorithms, the navigation solution for the satellite can be determined. [10]

REVIEW OF FALCON GOLD

The Falcon Gold mission was a joint effort between the University of Colorado, Colorado Springs and the United States Air Force Academy. The goal was to determine if GPS signals could be received from across the earth at high altitudes. Falcon Gold piggybacked aboard an Atlas rocket launching a military communications satellite (DSCS) on October 24, 1997. The Atlas launched from Space Launch Complex 36A at Cape Canaveral Air Station in Florida. Data was received and decoded at two ground stations during the course of the mission, a station at the US Naval Academy in Annapolis, MD, as well as a UCCS/USAFA ground station in Boulder, CO. The ground station in Colorado had some difficulties due to a surprise fall blizzard, but overall Falcon Gold data was obtained until November 9 when battery power died and the spacecraft could not transmit any longer. [6]

Falcon Gold used the Tidget sensor developed by NAVSYS to receive the GPS signal. The Tidget sensor was selected for its low mass, size, power requirements, high reliability, and cost effectiveness. NAVSYS reports that Tidget provides position accuracy of less than 10 meters. Tidget uses a sparse sampling technique to take a short snapshot (40 milliseconds of data for Falcon Gold) of GPS raw data into a digital buffer and then transmits that data back to the processing and control center. [7]

After the Tidget sensor received the raw GPS data and sent it to Falcon Gold's computer, the spacecraft transmitted data using a standard AX.25 modulation protocol at a frequency of 400.475 MHz. Falcon Gold used Tekk KS-960 transmitters to modulate the digital data stream from a Kantronics KPC-9612 terminal node controller. Captain

Brian Mork of the Air Force Academy wanted to get as many people around the world to listen in and aid in the post processing of this data. [6]

Scientists from the Aerospace Corporation aided in the processing of this data. Preliminary results show that GPS data was received and the receiver was able to determine pseudo-ranges to several GPS satellites over the course of its mission, as the following chart shows: [14] The range from Falcon Gold to GPS satellite is consistent with the expected ranges one would expect from apogee of geosynchronous transfer orbit where Falcon Gold would receive GPS signals from across the globe.

Table 1 Summary of Intermediate Falcon-Gold Data (On November 7 and 8, 1997)

Frame No.	OK Segments	UTC hr:mn:s	GPS half cone angle in deg	Falcon-gold to GPS range in km	Remarks (GPS Sat)
3634	38	15:44:59	20.0, 14.7	64278.1, 65256.4	#4, #5
3637	40	16:01:08	19.7	63711.5	#4
3650	40	17:11:05	17.3, 18.3	58910.8, 58496.4	#15, #21
3653	41	17:27:13	15.2, 17.3	57760.4, 57108.3	#21, #27
3656	40	17:43:22	14.6, 16.4	55556.4, 55473.4	#5, #27
3659	38	17:59:30	16.1, 15.7	53183.7, 53593.7	#5, #27
3996[@]	38	00:12:46	18.5	59919.5	#3

BENEFITS OF APPLYING GPS TO HIGH ALTITUDE SPACECRAFT NAVIGATION

Two studies by scientists at the Aerospace Corporation demonstrate the advantages of applying GPS to high altitude users. The Aerospace Corporation has focused on studies of applying GPS to space systems in transfer orbits from low earth orbit to geosynchronous altitude. They have used a Kalman filter to obtain the navigation solution for space systems in transfer orbit due to the low visibility of GPS satellites to users at high altitudes. They update their Kalman filter by propagating previous state vectors in time and improving them as new pseudo-range and pseudo-range measurements become available to a GEO user. Their filter had 33 states. Six states consisted of a users earth centered inertial position and velocity vectors, 24 bias states for estimating GPS bias errors, and two bias states for user clock bias and drift. They modeled their space system through its transfer orbit and found a worst case positioning error of 2500 meters. They also conclude that by making a wise choice of launch epoch, GPS constellation can be improved for a space system in GEO transfer orbit, thus improving navigational accuracy. [8]

Another study by the Aerospace Corporation provides even more encouraging results. This study again measures the possible benefits navigating a space system in transfer orbit with GPS has over navigating with an inertial measurement unit. In this study, the scientists from Aerospace have done a covariance analysis with a Kalman filter modeling of 60 INS error states. They then expanded the simulation by processing GPS L1 p-code pseudorange measurements into a dual frequency user. The dual frequency receiver allows for ionospheric correction. Again they report that as the space system ascends higher into transfer orbit, GPS constellation visibility worsens and navigation error rises. However they report that augmenting the INS with GPS can reduce the payload insertion error by more than two errors of magnitude. From a worst case navigation error of 10,000 ft 1 sigma for navigating with the INS only, GPS can decrease the error down to 150 ft within 1 sigma of the true value. Using the GPS crosslink ranging signal improves accuracy further. Simulation shows that navigation error can be decreased to as low as 20 ft 1 sigma over a transfer orbit trajectory. [5]

Besides improving navigational accuracy, GPS navigation of high altitude space systems can save time and money as well. Currently satellites in geostationary orbit must be maintained in a tight 'box' in their orbital slot. This requires time intensive and costly ground control operations. Currently satellite ranging is done with C-band and SGLS (S-band Space Ground Link System) [5] The Air Force also uses the AFSCN to aid in orbit determination. The 1st Space Operations Squadron of Falcon AFB's 50th Space Wing schedules approximately 450 minutes each day of AFSCN antenna time solely for DSP orbit determination, to attain an accuracy of 200-400 m 1 sigma. GPS navigation could provide the same or better accuracy autonomously, saving time and money. Reducing dependence on overseas stations is another benefit as well.

Other benefits and unique applications for applying GPS to GEO satellites exist as well. Capt. Derek Sebalj of Air Force Space Command brought to my attention a problem with the Defense Support Program (DSP) and its capability to detect missile launches. DSP was originally designed to be able to detect ICBM and other nuclear missile launches against the United States. As the world political climate has changed, the focus on DSP is to improve its ability to detect smaller Theater Ballistic Missiles that are widely proliferating throughout the world.

DSP has time association errors that prevent it from accurately determining the exact time when a ballistic missile was launched. These time association errors are the result of the time it takes for signals to travel through space to the DSP satellite located in the geostationary ring.

DSP data is then transmitted down to several ground stations located around the world, and also up to military communication satellites. The cumulative effect of all these "hops" is that the exact time when the DSP detected the missile launch is lost. Although the time error may be as low as several hundred milliseconds, the errors translate into several kilometers when analysts try to determine the missile's launch location.

If an accurate time tag could be applied to the DSP's launch detection report, then intelligence analysts could more accurately analyze when and where the missile was launched from. Where can you find a more accurate timing mechanism than from the atomic clocks aboard the GPS satellites? Integrating GPS data into this application could be very beneficial to defense. Dr. Ed Tagliaferri from the Aerospace Corporation has done some research into this problem and has commented that is a shame to not exploit the technology and capability that we currently have.

CONCLUSION:

The area of GPS applications is expanding rapidly, and includes many applications the system was not originally intended for. This paper has shown how while the GPS constellation is not optimally designed for spacecraft at high altitudes. The use of GPS for high altitude spacecraft is severely hampered by visibility and communications link constraints. However, engineering solutions do exist that can alleviate some of these issues and make GPS use for high altitude spacecraft navigation feasible.

Currently the GPS Block IIF satellites are being designed and built. It may not be too late to make modifications to them to allow greater exploitation of their navigation capabilities of high altitude spacecraft. Modeling and research into this field have demonstrated the potential merits of this application. Further research will determine if it is cost effective and worthwhile to do so.

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APPENDIX

This Appendix contains technical details on the geometric analysis in calculating the propagation path lengths from GPS satellites to GEO satellites, as well as Link Budgets for the various configurations of GPS / GEO links.

Propagation Path Length Calculations –

GPS beamwidth = 21°

7378 km

26610 km

42241 km

$\mathbb{P}_{GPS} = 12\ \text{hrs} = 43200\ \text{s} \Rightarrow a_{GPS} = \left(\left(\frac{\mathbb{P}}{2\pi}\right)^2 \mu\right)^{\frac{1}{3}} = 26610\ \text{km}$

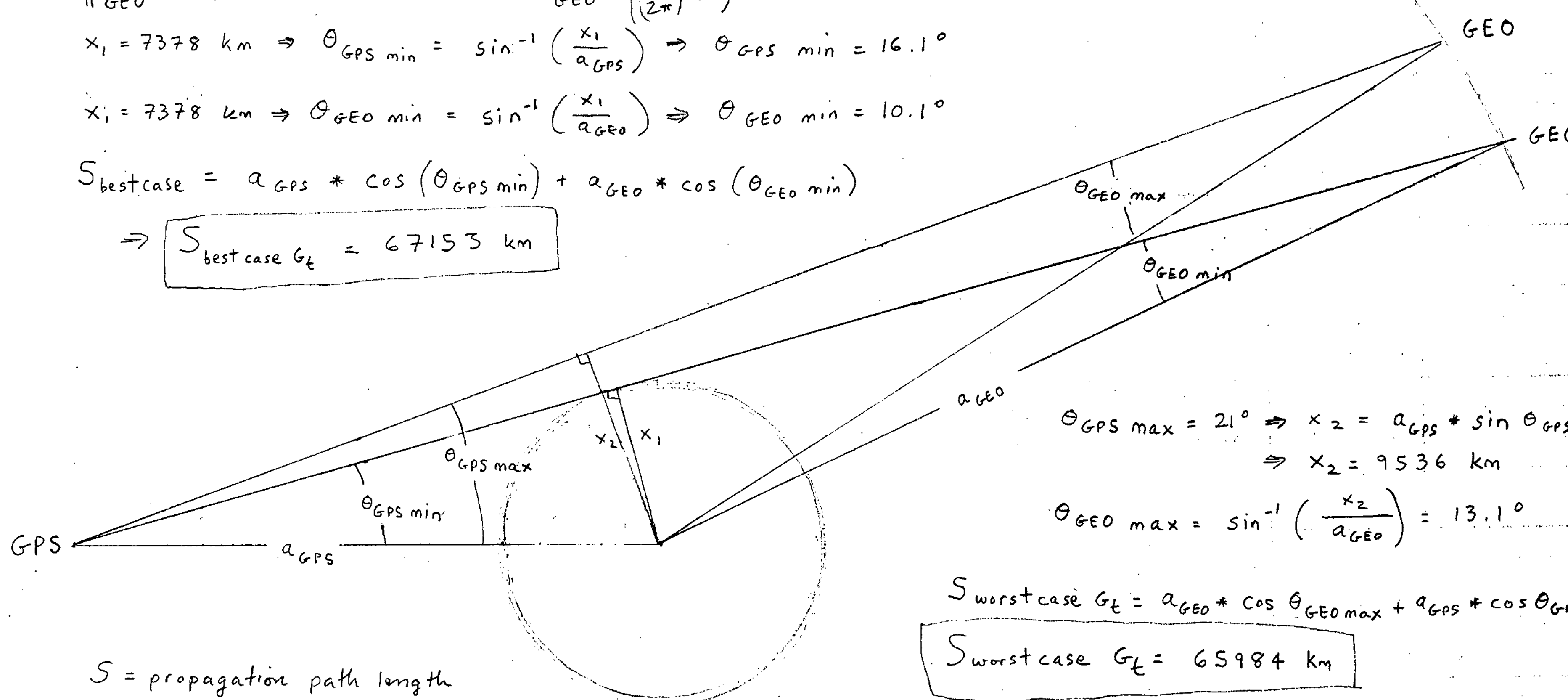
$\mathbb{P}_{GEO} = 24\ \text{hrs} = 86400\ \text{s} \Rightarrow a_{GEO} = \left(\left(\frac{\mathbb{P}}{2\pi}\right)^2 \mu\right)^{\frac{1}{3}} = 42241\ \text{km}$

$x_1 = 7378\ \text{km} \Rightarrow \theta_{GPS\ min} = \sin^{-1}\left(\frac{x_1}{a_{GPS}}\right) \Rightarrow \theta_{GPS\ min} = 16.1°$

$x_1 = 7378\ \text{km} \Rightarrow \theta_{GEO\ min} = \sin^{-1}\left(\frac{x_1}{a_{GEO}}\right) \Rightarrow \theta_{GEO\ min} = 10.1°$

$S_{best case} = a_{GPS} * \cos(\theta_{GPS\ min}) + a_{GEO} * \cos(\theta_{GEO\ min})$

$\Rightarrow \boxed{S_{best\ case\ G_t} = 67153\ \text{km}}$



$\theta_{GPS\ max} = 21° \Rightarrow x_2 = a_{GPS} * \sin \theta_{GPS\ max}$

$\Rightarrow x_2 = 9536\ \text{km}$

$\theta_{GEO\ max} = \sin^{-1}\left(\frac{x_2}{a_{GEO}}\right) \doteq 13.1°$

$S_{worst case\ G_t} = a_{GEO} * \cos \theta_{GEO\ max} + a_{GPS} * \cos \theta_{GPS\ max}$

$\boxed{S_{worst case\ G_t} = 65984\ \text{km}}$

S = propagation path length from GPS satellite to GEO satellite

Propagation Path Length Calculations -

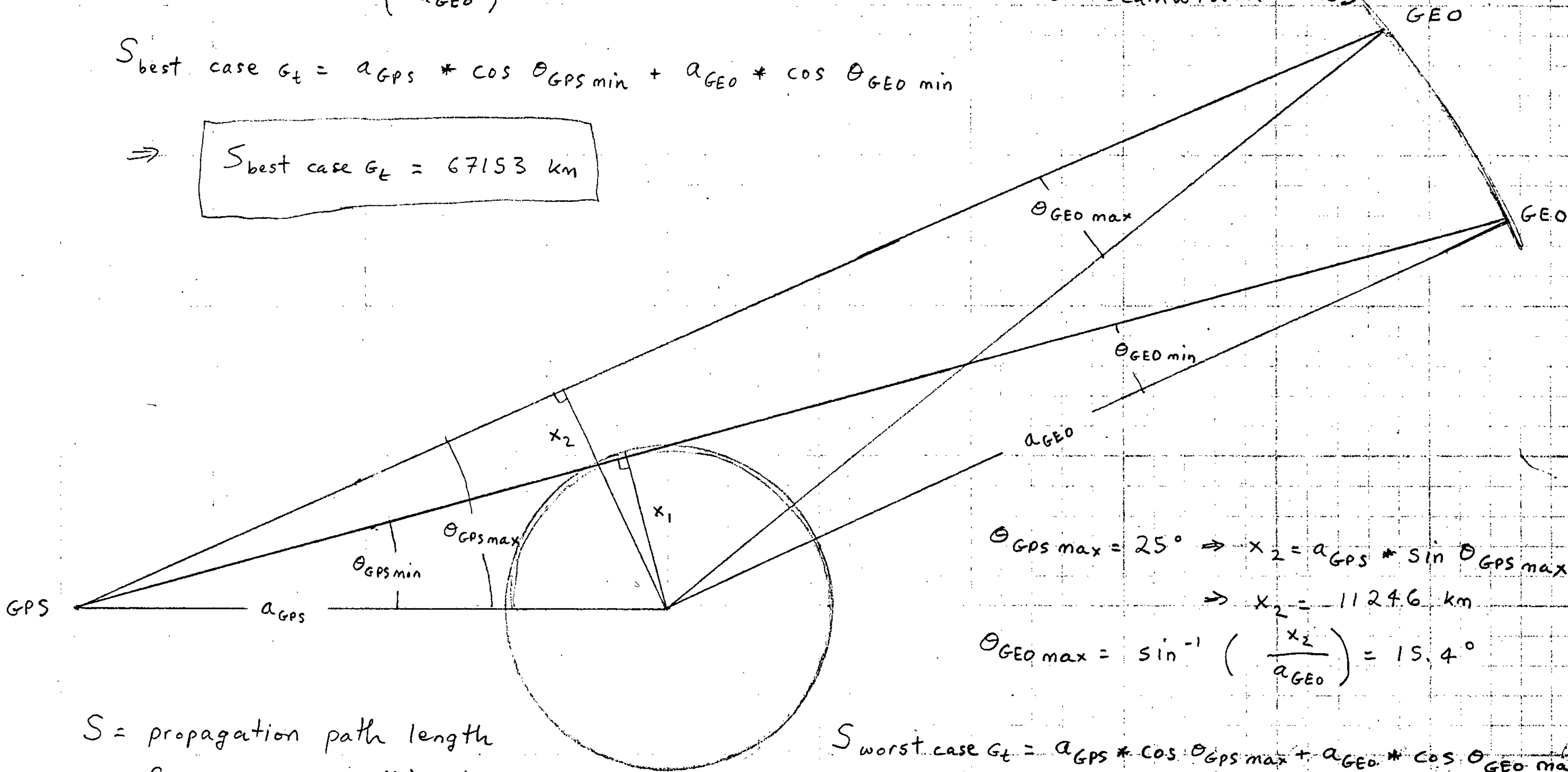
GPS beamwidth = 25°

$x_1 = 7378$ km $\Rightarrow \theta_{GPS\,min} = \sin^{-1}\left(\frac{x_1}{a_{GPS}}\right) = 16.1°$

$\theta_{GEO\,min} = \sin^{-1}\left(\frac{x_1}{a_{GEO}}\right) \Rightarrow \theta_{GEO\,min} = 10.1°$

$S_{best\ case\ G_t} = a_{GPS} * \cos\theta_{GPS\,min} + a_{GEO} * \cos\theta_{GEO\,min}$

$\Rightarrow \boxed{S_{best\ case\ G_t} = 67153 \text{ km}}$



$\theta_{GPS\,max} = 25° \Rightarrow x_2 = a_{GPS} * \sin\theta_{GPS\,max}$

$\Rightarrow x_2 = 11246$ km

$\theta_{GEO\,max} = \sin^{-1}\left(\frac{x_2}{a_{GEO}}\right) = 15.4°$

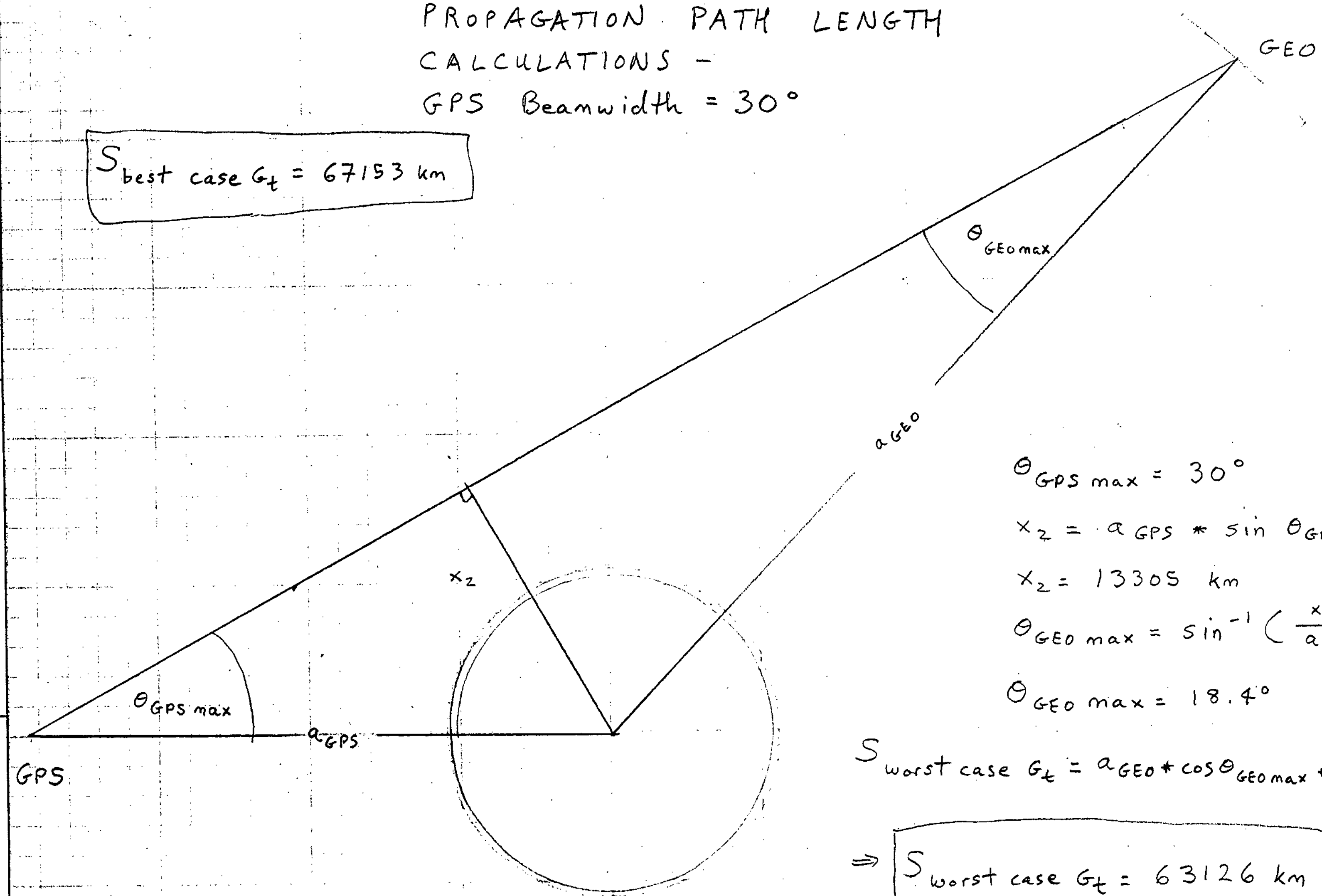
S = propagation path length from GPS satellite to GEO satellite

$S_{worst\ case\ G_t} = a_{GPS} * \cos\theta_{GPS\,max} + a_{GEO} * \cos\theta_{GEO\,max}$

$\Rightarrow \boxed{S_{worst\ case\ G_t} = 64841 \text{ km}}$

PROPAGATION PATH LENGTH
CALCULATIONS –
GPS Beamwidth = 30°

$\boxed{S_{\text{best case } G_t} = 67153 \text{ km}}$



$\theta_{GPS\,max} = 30°$

$x_2 = a_{GPS} * \sin \theta_{GPS\,max}$

$x_2 = 13305$ km

$\theta_{GEO\,max} = \sin^{-1}\left(\frac{x_2}{a_{GEO}}\right)$

$\theta_{GEO\,max} = 18.4°$

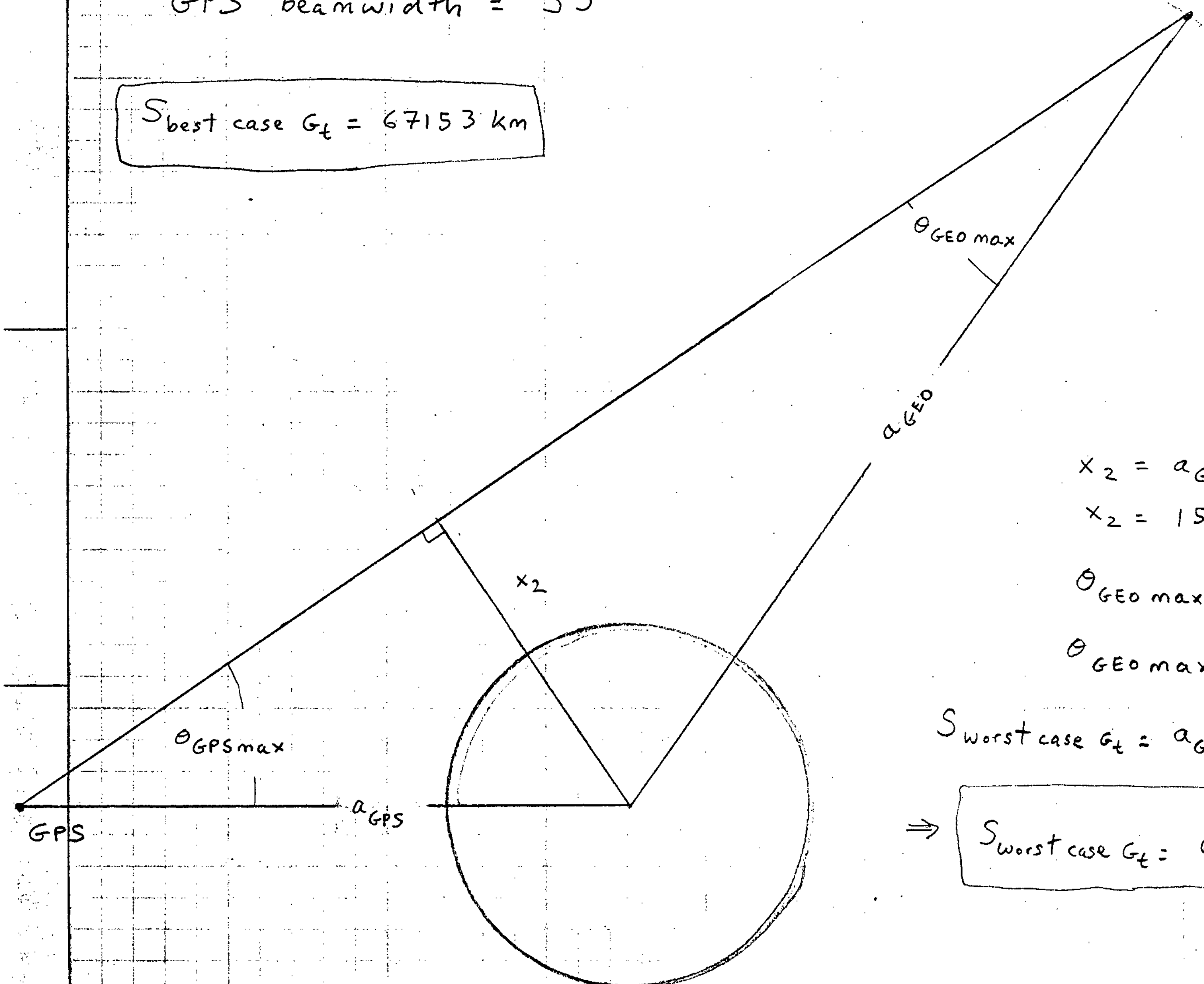
$S_{\text{worst case } G_t} = a_{GEO} * \cos\theta_{GEO\,max} + a_{GPS} * \cos\theta_{GPS\,max}$

$\Rightarrow \boxed{S_{\text{worst case } G_t} = 63126 \text{ km}}$

Propagation Path Length Calculations

GPS beamwidth = 35°

$\boxed{S_{\text{best case } G_t} = 67153 \text{ km}}$



$x_2 = a_{GPS} * \sin \theta_{GPS\,max}$

$x_2 = 15263$ km

$\theta_{GEO\,max} = \sin^{-1}\left(\frac{x_2}{a_{GEO}}\right)$

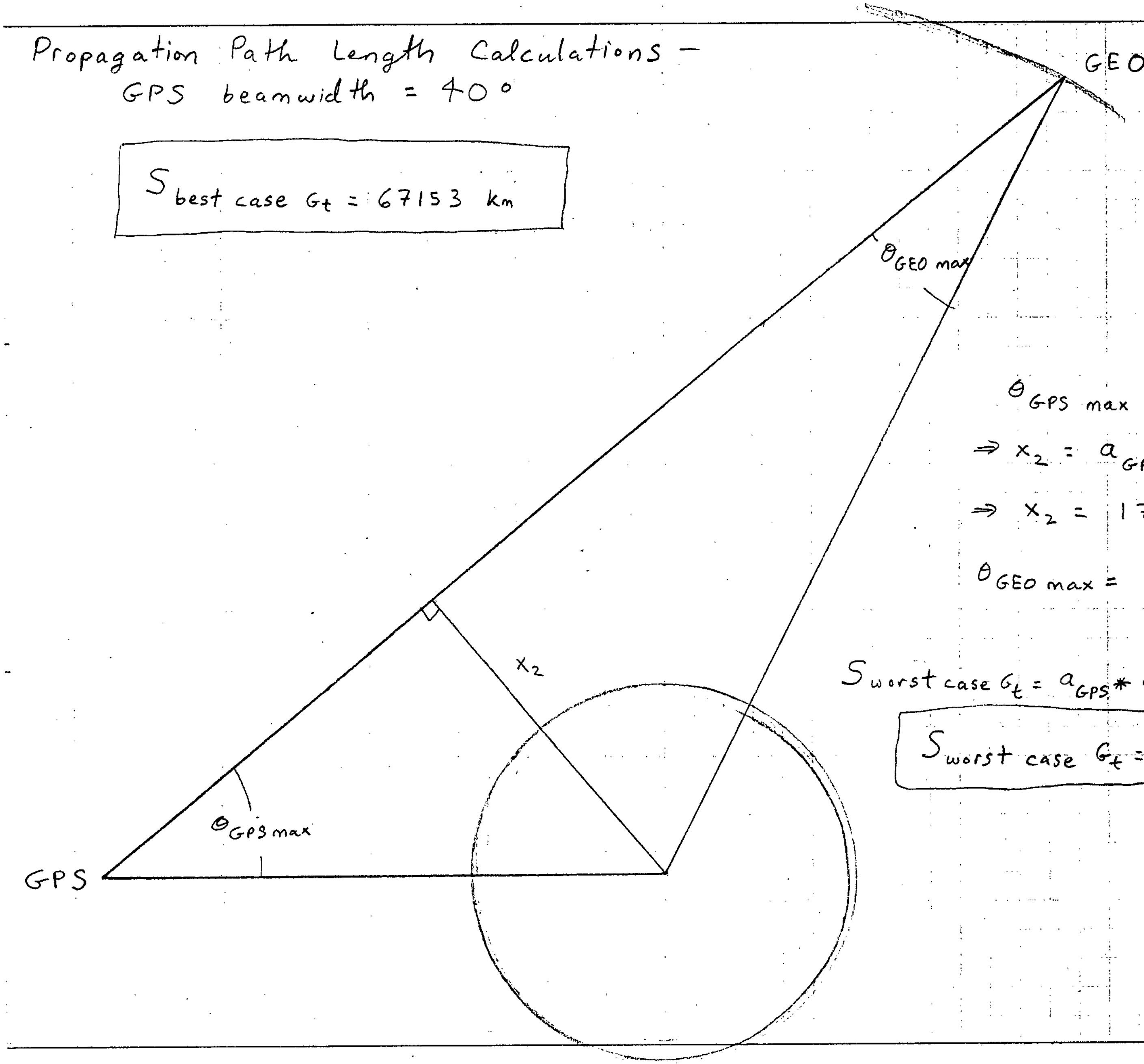
$\theta_{GEO\,max} = 21.2°$

$S_{\text{worst case } G_t} = a_{GPS} * \cos \theta_{GPS\,max} + a_{GEO} * \cos \theta_{GEO\,max}$

$\Rightarrow \boxed{S_{\text{worst case } G_t} = 61180 \text{ km}}$

Propagation Path Length Calculations –
GPS beamwidth = 40°

$$\boxed{S_{\text{best case } G_t} = 67153 \text{ km}}$$



$\theta_{GPS\ max} = 40°$

$\Rightarrow x_2 = a_{GPS} * \sin \theta_{GPS\ max}$

$\Rightarrow x_2 = 17105$ km

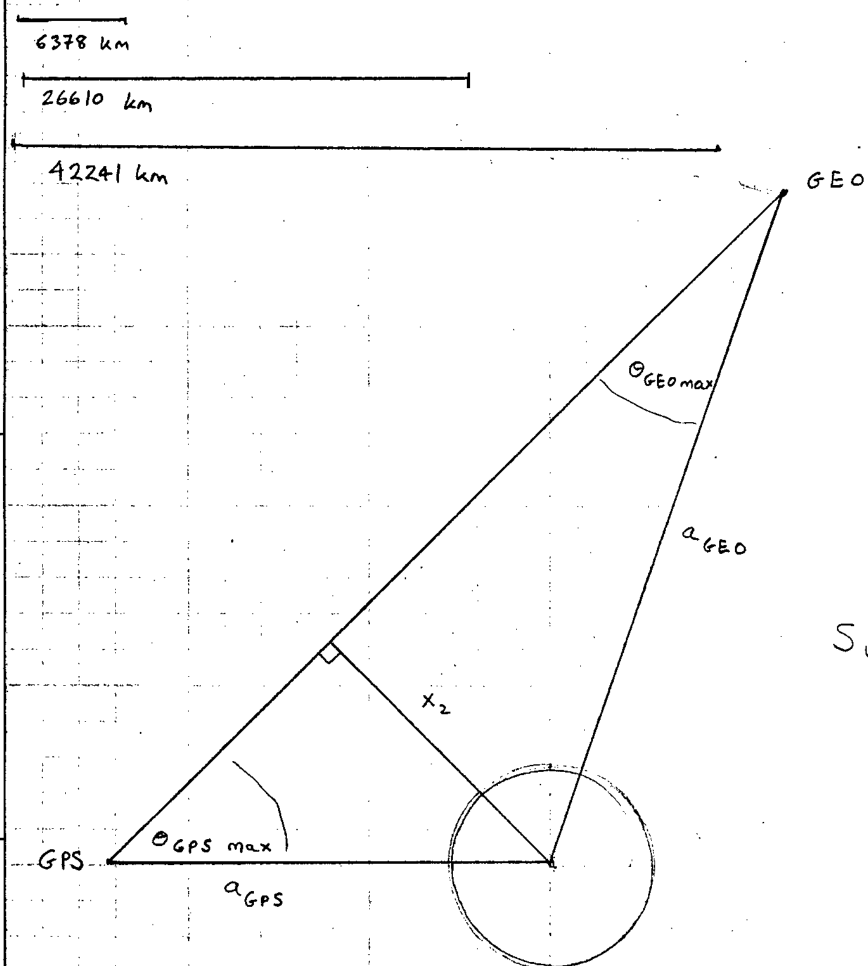
$\theta_{GEO\ max} = \sin^{-1}\left(\frac{x_2}{a_{GEO}}\right) = 23.9°$

$S_{\text{worst case } G_t} = a_{GPS} * \cos \theta_{GPS\,max} + a_{GEO} * \cos \theta_{GEO\ max}$

$$\boxed{S_{\text{worst case } G_t} = 59003 \text{ km}}$$

Propagation Path Length Calculations

GPS beamwidth = 45°



$$\boxed{S_{best\ case\ G_t} = 67153 \text{ km}}$$

$$x_2 = a_{GPS} * \sin(\theta_{GPS\,max}) = 18816 \text{ km}$$

$$\theta_{GEO\,max} = \sin^{-1}\left(\frac{x_2}{a_{GEO}}\right) = 26.5°$$

$$S_{worst\ case\ G_t} : a_{GPS} * (\cos \theta_{GPS\,max}) + a_{GEO} * \cos(\theta_{GEO\,max})$$

$$\Rightarrow \boxed{S_{worst\ case\ G_t} = 56619 \text{ km}}$$

GPS signal to GEO link Budgets - CASE 1 : Transmission beamwidth of 21 degrees

Parameter	Units	Symbol	Best Case	Worst Case
frequency	GHz	f	1.57542	1.57542
Tx Power	W	Pt	25.11886	25.11886
Tx Power	dB W	Pt	14	14
Tx Line Loss	dB W	Ll	-1	-1
Tx Ant Beamwidth	deg	bw	21	21
Tx Ant Gain	Gt	dB	13.4	1.4
EIRP	dBW	EIRP	26.4	14.4
Prop Path Length	km	S	67153	65984
Free Space Loss	dB	Ls	-192.939	-192.787
Prop and Pol Loss	dB	La	0	0
System Noise Temp	deg K	Ts	560	560
Data Rate	bps	R	50	50
C/No (required)	dB-Hz	C/No	31	31
Eb/No (with Margin)	dBf	Eb/No	14.0103	14.0103
Rc Ant Gain Req	dB	Gr	-3.57888	8.26858
C/No (calculated)	dB-Hz	C/No	31	31

Free space loss from GPS to earth		-184.551
C/No at Earth (worst case)	C/No earth	45.96682

model Gt = 44.3-10*log(beamwidth**2) (SMAD p. 524)
so Gt = 44.3 - 10*log(50*50) = 10.3206

model min Gt = .0630957344 of peak Gt

GPS signal to GEO link Budgets - CASE 1 : Transmission beamwidth of 25 degrees

Parameter	Units	Symbol	Best Case	Worst Case
frequency	GHz	f	1.57542	1.57542
Tx Power	W	Pt	25.11886	25.11886
Tx Power	dB W	Pt	14	14
Tx Line Loss	dB W	Ll	-1	-1
Tx Ant Beamwidth	deg	bw	25	25
Tx Ant Gain	Gt	dB	10.3206	-1.6794
EIRP	dBW	EIRP	23.3206	11.3206
Prop Path Length	km	S	67153	64841
Free Space Loss	dB	Ls	-192.939	-192.635
Prop and Pol Loss	dB	La	0	0
System Noise Temp	deg K	Ts	560	560
Data Rate	bps	R	50	50
C/No (with Margin)	dB-Hz	C/No	31	31
Eb/No (with Margin)	dBf	Eb/No	14.0103	14.0103
Rc Ant Gain Req	dB	Gr	-0.49948	11.1962
C/No (calculated)	dB-Hz	C/No	31	31

Free space loss from GPS to earth		-184.551
C/No at Earth (worst case)	C/No earth	42.88742

Pt increase to maintain
C/No of 46 dB at earth

model Gt = 44.3-10*log(beamwidth**2) (SMAD p. 524)
so Gt = 44.3 - 10*log(50*50) = 10.3206

model min Gt = .0630957344 of peak Gt

GPS signal to GEO link Budgets - CASE 1 : Transmission beamwidth of 30 degrees

Parameter	Units	Symbol	Best Case	Worst Case
frequency	GHz	f	1.57542	1.57542
Tx Power	W	Pt	25.11886	25.11886
Tx Power	dB W	Pt	14	14
Tx Line Loss	dB W	Ll	-1	-1
Tx Ant Beamwidth	deg	bw	30	30
Tx Ant Gain	Gt	dB	8.736975	-3.263025
EIRP	dBW	EIRP	21.73697	9.736974
Prop Path Length	km	S	67153	63126
Free Space Loss	dB	Ls	-192.9392	-192.4021
Prop and Pol Loss	dB	La	0	0
System Noise Temp	deg K	Ts	560	560
Data Rate	bps	R	50	50
C/No (with Margin)	dB-Hz	C/No	31	31
Eb/No (with Margin)	dBf	Eb/No	14.0103	14.0103
Rc Ant Gain Req	dB	Gr	1.084141	12.547
C/No (calculated)	dB-Hz	C/No	31	31

Free space loss from GPS to earth		-184.5513
C/No at Earth (worst case)	C/No earth	41.3038

model Gt = 44.3-10*log(beamwidth**2) (SMAD p. 524)
so Gt = 44.3 - 10*log(50*50) = 10.3206

model min Gt = .0630957344 of peak Gt

GPS signal to GEO link Budgets - CASE 1 : Transmission beamwidth of 35 degrees

Parameter	Units	Symbol	Best Case	Worst Case
frequency	GHz	f	1.57542	1.57542
Tx Power	W	Pt	25.11886	25.11886
Tx Power	dB W	Pt	14	14
Tx Line Loss	dB W	Ll	-1	-1
Tx Ant Beamwidth	deg	bw	35	35
Tx Ant Gain	Gt	dB	7.398039	-4.601961
EIRP	dBW	EIRP	20.39804	8.398038
Prop Path Length	km	S	67153	61180
Free Space Loss	dB	Ls	-192.9392	-192.1301
Prop and Pol Loss	dB	La	0	0
System Noise Temp	deg K	Ts	560	560
Data Rate	bps	R	50	50
C/No (with Margin)	dB-Hz	C/No	31	31
Eb/No (with Margin)	dBf	Eb/No	14.0103	14.0103
Rc Ant Gain Req	dB	Gr	2.423077	13.61396
C/No (calculated)	dB-Hz	C/No	31	31

Free space loss from GPS to earth			-184.5513
C/No at Earth (worst case)		C/No earth	39.96486

model Gt = 44.3-10*log(beamwidth**2) (SMAD p. 524)
so Gt = 44.3 - 10*log(50*50) = 10.3206

model min Gt = .0630957344 of peak Gt

GPS signal to GEO link Budgets - CASE 1 : Transmission beamwidth of 40 degrees

Parameter	Units	Symbol	Best Case	Worst Case
frequency	GHz	f	1.57542	1.57542
Tx Power	W	Pt	25.11886	25.11886
Tx Power	dB W	Pt	14	14
Tx Line Loss	dB W	Ll	-1	-1
Tx Ant Beamwidth	deg	bw	40	40
Tx Ant Gain	Gt	dB	6.2382	-5.7618
EIRP	dBW	EIRP	19.2382	7.2382
Prop Path Length	km	S	67153	59003
Free Space Loss	dB	Ls	-192.9392	-191.8154
Prop and Pol Loss	dB	La	0	0
System Noise Temp	deg K	Ts	560	560
Data Rate	bps	R	50	50
C/No (with Margin)	dB-Hz	C/No	31	31
Eb/No (with Margin)	dBf	Eb/No	14.0103	14.0103
Rc Ant Gain Req	dB	Gr	3.582916	14.45909
C/No (calculated)	dB-Hz	C/No	31	31

Free space loss from GPS to earth		-184.5513
C/No at Earth (worst case)	C/No earth	38.80502

model Gt = 44.3-10*log(beamwidth**2) (SMAD p. 524)
so Gt = 44.3 - 10*log(50*50) = 10.3206

model min Gt = .0630957344 of peak Gt

GPS signal to GEO link Budgets - CASE 1 : Transmission beamwidth of 45 degrees

Parameter	Units	Symbol	Best Case	Worst Case
frequency	GHz	f	1.57542	1.57542
Tx Power	W	Pt	25.11886	25.11886
Tx Power	dB W	Pt	14	14
Tx Line Loss	dB W	Ll	-1	-1
Tx Ant Beamwidth	deg	bw	45	45
Tx Ant Gain	Gt	dB	5.21515	-6.78485
EIRP	dBW	EIRP	18.21515	6.215149
Prop Path Length	km	S	67153	56619
Free Space Loss	dB	Ls	-192.9392	-191.4572
Prop and Pol Loss	dB	La	0	0
System Noise Temp	deg K	Ts	560	560
Data Rate	bps	R	50	50
C/No (with Margin)	dB-Hz	C/No	31	31
Eb/No (with Margin)	dBf	Eb/No	14.0103	14.0103
Rc Ant Gain Req	dB	Gr	4.605967	15.1239
C/No (calculated)	dB-Hz	C/No	31	31

Free space loss from GPS to earth		-184.5513
C/No at Earth (worst case)	C/No earth	37.78197

model Gt = 44.3-10*log(beamwidth**2) (SMAD p. 524)
so Gt = 44.3 - 10*log(50*50) = 10.3206

model min Gt = .0630957344 of peak Gt

GPS to GEO commlink summary

GPS antenna beamwidth	Gt max	Gt min	Gr req		Dr req	BW req
21	13.4	1.4	8.26858	6.712094	0.271746	13.3
25	10.3206	-1.6794	11.1962	13.17104	0.380666	15.4
30	8.736975	-3.26303	12.54837	17.98196	0.444788	18.4
35	7.398039	-4.60196	13.61382	22.98169	0.502835	21.2
40	6.2382	-5.7618	14.45909	27.91959	0.554229	23.9
45	5.21515	-6.78485	15.1239	32.53794	0.598314	26.5

	BW calc
Dr required - antenna diameter required for adequate antenna gain	
Calculated from SMAD - p. 521 Dr = ((Gr*wavelength**2)/(pi**2*efficiency))	49.06539
Assume antenna efficiency = .55	35.0263
Use wavelength for L2 frequency (worst case analysis) lower frequency =	29.97684
higher wavelength = larger Dr)	26.51632
So this communication setup will allow for reception of both L1 and L2 signals	24.05746
BW required is the half angle beamwidth required to obtain GPS signal with the	22.28483

maximum possible angular separation (min Gt)

BW calc is calculated from SMAD eq 13-17

According to these results, widening the usable GPS antenna beam to a half angle of 40 degrees is the widest possible solution

The calculated half power beamwidth of the receive antenna for the 45 degree solution would be too narrow to make use of the widest angular separation of GPS broadcasts

GPS half cone antenna BW	Pt increase to maintain Earth C/No	
21	0	0
25	3.11258	2.0477
30	4.6962	2.9486
35	6.03514	4.0134
40	7.19498	5.242
45	8.21803	6.6344