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CHAPTER 11
CRUISE PERFORMANCE THEORY

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11.1 OVERVIEW AND BASIC ASSUMPTIONS

This chapter examines the theory and flight tests required to determine cruise data presented in aircraft flight manuals. Aircraft cruise performance is dependent upon the combination of airplane aerodynamics and engine characteristics. Basic aerodynamic and engine theory applied to cruise testing are covered. Aerodynamic forces acting on the aircraft, i.e., lift and drag, and engine thrust and fuel flow are presented as functions of easily measured parameters. Engine and aerodynamic functions are then combined to complete the analysis. The end result provides a method by which engine and airplane cruise performance characteristics may be determined with minimum flight testing. The data obtained from the flight tests are used to determine cruise performance charts and tables presented in the flight manual, and to determine cruise specification compliance and military utility.

The basic assumption of cruise theory is that during cruise the aircraft maintains level, unaccelerated flight. When the aircraft is in level, unaccelerated flight, the sum of the forces acting upon it equals zero. Assuming the thrust acts along the direction of flight (or differences in direction of thrust due to engine installation angle and aircraft angle of attack are negligible), the lift force, L, is equal to the aircraft weight, W, and the net thrust, F_n, is equal to the aircraft drag, D, as shown in Figure 11.1.

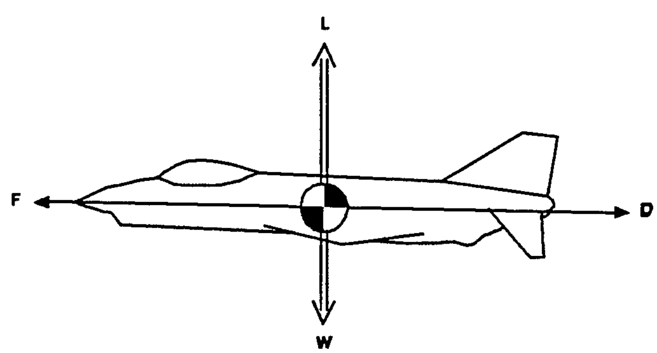


Figure 11.1 Level Unaccelerated Flight

11.2 LIFT AND DRAG FUNCTIONAL RELATIONSHIPS

The first step in the development of a method to test and analyze cruise performance is to use the aerodynamic lift and drag equations to derive relationships for lift coefficient, C_L, and drag coefficient, C_D, as functions of cruise parameters (aircraft gross weight, pressure altitude, and speed). These, in conjunction with flight test data and the engine thrust model, are used to plot drag polars.

Lift can be written as

$$L = \frac{\rho_a V_t^2 S C_L}{2} \tag{11.1}$$

In unaccelerated, level flight, lift equals weight, therefore

$$L = W = \frac{\rho_a V_t^2 S C_L}{2} \tag{11.2}$$

Recalling the definition of Mach number, M,

$$M = \frac{V_t}{a} \tag{11.3}$$

Therefore

$$V_t^2 = M^2 a^2 \tag{11.4}$$

From the Perfect Gas law

$$P_a = \rho_a R T_a$$

we know

$$\frac{P_a}{\rho_a} = R T_a \tag{11.5}$$

Substituting Equation 11.5 into the definition of speed of sound we obtain

$$a = \sqrt{\gamma R T_a} = \sqrt{\frac{P_a}{\rho_a} \gamma} \tag{11.6}$$

or

$$a^2 = \gamma \frac{P_a}{\rho_a} \qquad (11.7)$$

Now Equation 11.4 becomes

$$V_t^2 = M^2 a^2 = M^2 \gamma \frac{P_a}{\rho_a} \qquad (11.8)$$

Substituting this result into Equation 11.2 we obtain

$$W = \frac{\rho_a V_t^2 S C_L}{2} = \frac{\rho_a S C_L}{2} \left(M^2 \gamma \frac{P_a}{\rho_a} \right)$$

or

$$W = \frac{S C_L M^2 \gamma P_a}{2} \qquad (11.9)$$

Multiplying Equation 11.9 by $\frac{P_{a_{SL}}}{P_{a_{SL}}}$, we obtain

$$W = \frac{S C_L M^2 \gamma P_a P_{a_{SL}}}{2 P_{a_{SL}}} \qquad (11.10)$$

By definition, $\delta = \frac{P_a}{P_{a_{SL}}}$; therefore, Equation 11.10 becomes

$$W = \frac{S C_L M^2 \gamma \delta P_{a_{SL}}}{2} \qquad (11.11)$$

Solving Equation 11.11 for the lift coefficient

$$C_L = \frac{2W}{\gamma \delta P_{a_{SL}} M^2 S} = \frac{2 \left(\frac{W}{\delta} \right)}{\gamma P_{a_{SL}} M^2 S}$$

For a given aircraft and standard sea level conditions, γ, $P_{a_{SL}}$, and S are constants.

Substituting for the values of γ and P_{SL}

$$C_L = \frac{\left(\frac{W}{\delta}\right)}{1481\ M^2\ S} \tag{11.12}$$

This shows that

$$C_L = f\left(\frac{W}{\delta},\ M\right) \tag{11.13}$$

From equation 11.12, lift coefficients can be easily determined for stabilized points flown at known values of W/δ and M.

Drag can be written as

$$D = \frac{\rho_a\ V_t^2\ S\ C_D}{2} \tag{11.14}$$

By analyzing this equation as we did the lift equation, we obtain

$$C_D = \frac{\frac{D}{\delta}}{1481\ M^2\ S} \tag{11.15}$$

From the unaccelerated, level flight assumption, thrust equals drag. Therefore

$$\frac{D}{\delta} = \frac{F_n}{\delta} \tag{11.16}$$

and

$$C_D = \frac{F_n/\delta}{1481\ M^2\ S} \tag{11.17}$$

The contractor-provided engine thrust model gives F_n as a function of an appropriate engine parameter, such as engine speed, engine pressure ratio, or throttle angle, measured during stable points. Equation 11.17, coupled with the engine thrust model and flight test data from stable points, is used to compute drag coefficients. The coefficients of lift and drag from Equations 11.12 and 11.17 are plotted to construct the drag polar as shown in Figure 11.2. This drag polar represents the aerodynamic characteristics of an aircraft and is extensively used to develop performance data presented in the flight manual.

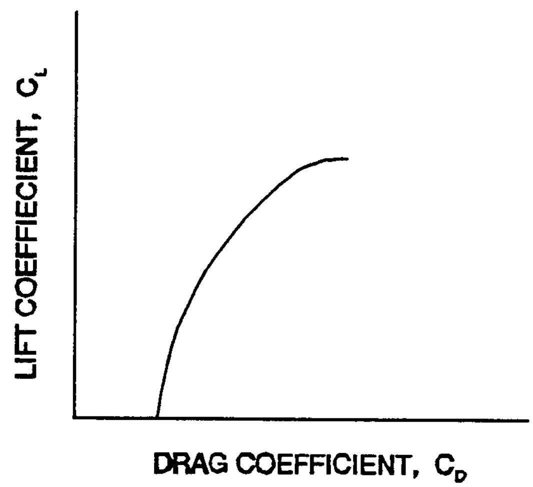


Figure 11.2 Drag Polar

Equation 11.15 can be rewritten

$$\frac{D}{\delta} = f\ (C_D, M) \qquad (11.18)$$

The total drag coefficient, C_D, is the sum of the parasite drag coefficient, C_{D_P}, the induced drag coefficient, C_{D_i}, and the Mach drag coefficient, C_{D_M}. Throughout the incompressible flow speed regime, where cruise range and endurance are optimized, Mach drag is negligible. Therefore,

$$C_D = C_{D_P} + C_{D_i} \qquad (11.19)$$

The parasite drag coefficient is constant for a given aircraft configuration. The induced drag coefficient was defined in aerodynamic theory as

$$C_{D_i} = \frac{C_L^2}{\pi\, AR\, e} \qquad (11.20)$$

For a given aircraft, the aspect ratio, AR, and Oswald's efficiency factor, e, are constants. Therefore,

$$C_{D_i} = f\,(C_L) \tag{11.21}$$

and,

$$C_D = C_{D_p} + C_{D_i} = f\,(C_L) \tag{11.22}$$

Substituting $C_L = f\,(W/\delta, M)$ from Equation 11.13 into Equation 11.22

$$C_D = f\left(\frac{W}{\delta}, M\right) \tag{11.23}$$

From Equations 11.17 and 11.23

$$\frac{D}{\delta} = \frac{F_n}{\delta} = f\left(\frac{W}{\delta}, M\right) \tag{11.24}$$

11.3 ENGINE PARAMETER FUNCTIONAL RELATIONSHIPS

The objective of cruise testing is to determine the fuel flow for any set of unaccelerated, level cruise flight conditions. Functional relationships between fuel flow, flight condition (speed and altitude), and some easily measured engine parameter (engine speed, for example), will next be derived. That engine parameter, in turn, will be used with a known engine thrust model to determine thrust, and therefore drag, at each set of flight conditions and measured fuel flows.

Relationships for engine parameters, i.e., thrust, fuel flow, and engine speed, can be developed using the Buckingham π technique of dimensional analysis.

The variables which affect thrust include true airspeed, ambient temperature, ambient pressure, engine speed, nozzle area, viscosity, and engine component efficiencies. Thus, we can write

$$F_n = f\,(V_t, T_a, P_a, N, A, \mu, \eta_I, \eta_C, \eta_B, \eta_t, \eta_n)$$

Since the component efficiencies are primarily functions of V_t, T_a, P_a, N, and A, and μ is primarily a function of T_a and P_a, we can simplify the above equation and express thrust as a function of its prime variables

$$F_n = f\ (V_t,\ T_a,\ P_a,\ N,\ A) \qquad (11.25)$$

This relationship consists of n = 6 variables, which can each be expressed in terms of k = 3 fundamental dimensions mass, length, and time, as shown in Table 11.1.

Variable	Units	Dimensions
net thrust, F_n	lbf	mlt^{-2}
fuel flow, $\dot{w}_f$	lbf/sec	mlt^{-3}
true airspeed, V_t	ft/sec	lt^{-1}
ambient temperature, T_a	°K	l^2t^{-2}
ambient pressure, P_a	lbf/ft²	$ml^{-1}t^{-2}$
engine speed, N	rad/sec	t^{-1}
nozzle area, A	ft²	l^2

TABLE 11.1 PRIME VARIABLES AND THEIR DIMENSIONS

Buckingham's π theorem states that we can express these variables in terms of n - k = 3 independent non-dimensional parameters.

A linear algebraic analysis of the dimensions associated with equation 11.25 follows: If we choose 3 non-dimensional parameters $\pi_1,\ \pi_2,\ \pi_3$ such that

$$\pi^1 = P_a^{a_1}\ T_a^{a_2}\ A^{a_3}\ F_n$$

$$\pi^2 = P_a^{b_1}\ T_a^{b_1}\ A^{b_3}\ N$$

$$\pi^3 = P_a^{c_1}\ T_a^{c_2}\ A^{c_3}\ V_t$$

then replacing the variables with their fundamental dimensions of mass, length, and time, and setting the units of each non-dimensional parameter equal to one, in the form $m^0l^0t^0$

$$\pi^1 \quad : \quad [ml^{-1}t^{-2}]^{a_1}\,[l^2t^{-2}]^{a_2}\,[l^2]^{a_3}\,[mlt^{-2}] \quad = \quad [m^0l^0t^0]$$

$$\pi^2 \quad : \quad [ml^{-1}t^{-2}]^{b_1}\,[l^2t^{-2}]^{b_2}\,[l^2]^{b_3}\,[t^{-1}] \quad = \quad [m^0l^0t^0]$$

$$\pi^3 \quad : \quad [ml^{-1}t^{-2}]^{c_1}\,[l^2t^{-2}]^{c_2}\,[l^2]^{c_3}\,[lt^{-1}] \quad = \quad [m^0l^0t^0]$$

Since mass, length, and time are independent dimensions, we can equate exponents. For the π^1 equation:

$$m \; : \quad a_1 \qquad\qquad\qquad + 1 = 0$$

$$l \; : \quad -a_1 + 2a_2 + 2a_3 + 1 = 0$$

Solving the three equations for the three unknowns a_1, a_2, a_3

$$a_1 = -1, \qquad a_2 = 0, \qquad a_3 = -1$$

and therefore

$$\pi_1 = P_a^{-1}\,T_a^0\,A^{-1}\,F_n = \frac{F_n}{P_a\,A}$$

A similar analysis for b_1 , b_2 , b_3 and c_1 , c_2 , c_3 from the π_2 and π_3 equations shows

$$\pi_2 = P_a^0\,T_a^{-1/2}\,A^{1/2}\,N = \frac{N\sqrt{A}}{\sqrt{T_a}}$$

$$\pi_3 = P_a^0\,T_a^{-1/2}\,A^0\,V_t = \frac{V_t}{\sqrt{T_a}}$$

Therefore, we can rewrite equation 11.25 in terms of the three non-dimensional parameters π_1 , π_2 , π_3

$$\frac{F_n}{P_a\,A} = f\left(\frac{N\sqrt{A}}{\sqrt{T_a}}\,,\,\frac{V_t}{\sqrt{T_a}}\right) \tag{11.26}$$

Since nozzle area is approximately constant at cruise power settings, we can eliminate A from equation 11.26. From equation 11.3, $M = V_t / a$. As shown in equation 11.6

$$a = f(\sqrt{T_a})$$

Therefore

$$M = f\left(\frac{V_t}{\sqrt{T_a}}\right)$$

Also, since δ is a function only of P_a , and θ is a function only of T_a , we can rewrite equation 11.26 in the dimensional form

$$\frac{F_n}{\delta} = f\left(\frac{N}{\sqrt{\theta}}, M\right) \tag{11.27}$$

This functional relationship states that the corrected thrust parameter (F_n/δ) is a function of the Mach and the corrected engine speed parameter $(N/\sqrt{\theta})$.

A relationship for fuel flow similar to equation 11.27 can be obtained by a Buckingham π analysis of the variables which affect fuel flow

$$\dot{W}_f = f\ (V_t,\ T_a,\ P_a,\ N,\ A) \tag{11.25}$$

to arrive at a functional relationship for the corrected fuel flow

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(\frac{N}{\sqrt{\theta}}, M\right) \tag{11.29}$$

These functional relationships between corrected thrust, corrected fuel flow, corrected engine speed, and Mach number allow us to combine the six-variable equations for thrust and fuel flow into simpler, three-variable equations. These equations easily account for variations in temperature and pressure, are applicable throughout the cruise envelope, and will more readily integrate with the lift and drag functional relationships developed in the previous section. They are also of the form in which an engine thrust model, explained in the next section, is provided.

11.4 ENGINE THRUST MODEL

Accurate thrust data can be obtained by flight testing an aircraft and measuring the thrust at various airspeeds, altitudes, temperatures and power settings. In-flight measurement of thrust data is desirable, but aircraft engines frequently are not adequately instrumented to provide all the engine parameters necessary to obtain thrust. The ground static test is a cheaper and more frequently used method to obtain thrust data. Figure 11.3 is a plot of net thrust versus Mach for an engine at 100% RPM. The dotted lines represent specific fuel consumption, C; the solid lines represent altitude. Figure 11.4 is a plot of net thrust versus Mach number for the same engine at 95% RPM.

Note the thrust variations between the 100% and 95% RPM plots. Many plots similar to Figure 11.3 and 11.4 are necessary to completely describe the thrust characteristics of the engine throughout the Mach and altitude range required. Data can be crossplotted to obtain Figure 11.5, which is a plot of engine thrust at all altitudes and Mach.

Note that these data are presented in the form of the functional relationships developed earlier

$$\frac{F_n}{\delta} = f\left(\frac{N}{\sqrt{\theta}}, M\right) \tag{11.27}$$

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(\frac{N}{\sqrt{\theta}}, M\right) \tag{11.29}$$

The engine manufacturer normally supplies this thrust model in the form of an engine-airframe thrust deck. With this model, the aircraft lift and drag characteristics can be determined during flight testing. Both the engine model and the derived drag polars are then used to correct perfomance flight test data and to predict overall aircraft performance characteristics.

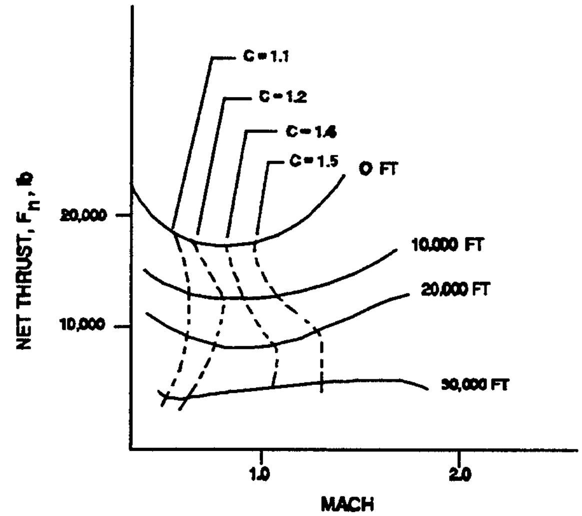


Figure 11.3 Engine Thrust Curve, 100% RPM

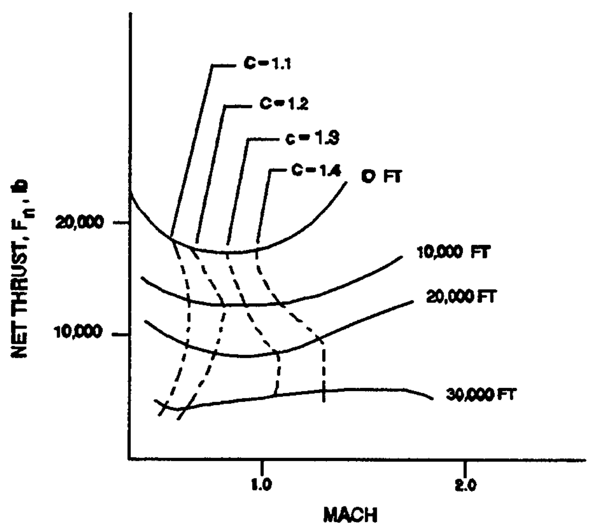


Figure 11.4 Engine Thrust Curve, 95% RPM

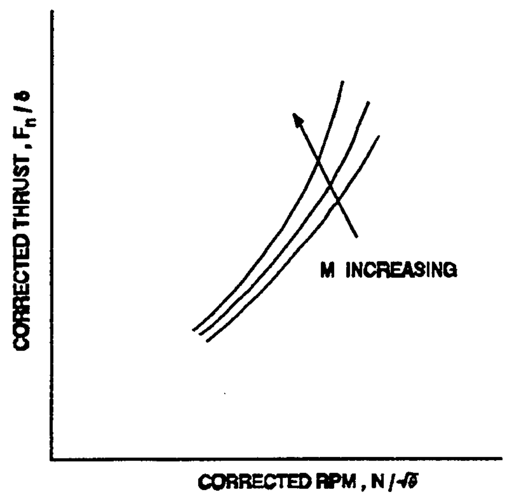


Figure 11.5 Corrected Engine Thrust Curve

11.5 ENGINE-AIRPLANE FUNCTIONAL COMBINATIONS

In previous paragraphs the following aircraft and engine performance relationships were shown.

$$\frac{D}{\delta} = f\left(\frac{W}{\delta}, M\right) \tag{11.24}$$

$$\frac{F_n}{\delta} = f\left(\frac{N}{\sqrt{\theta}}, M\right) \tag{11.27}$$

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(\frac{N}{\sqrt{\theta}}, M\right) \tag{11.29}$$

In this paragraph these relationships will be combined. The thrust model in conjunction with the drag polar are used to determine all the aircraft's performance characteristics.

In steady flight, thrust equals drag. Therefore, Equation 11.27 can be written

$$\frac{D}{\delta} = f\left(\frac{N}{\sqrt{\theta}}, M\right)$$

Substituting this result for D/δ into Equation 11.24, we obtain

$$f\left(\frac{N}{\sqrt{\theta}}, M\right) = f\left(\frac{W}{\delta}, M\right)$$

A further reduction produces

$$M = f\left(\frac{W}{\delta}, \frac{N}{\sqrt{\theta}}\right) \tag{11.30}$$

The functional relationship shown in Equation 11.30 is extremely important and leads directly to a method in which we can flight test an aircraft to obtain the cruise performance data presented in the flight manual. It is the basis for the "Speed Power Flight Test Technique."

One method to solve this functional relationship is to fly a constant W/δ and vary M, obtaining $N/\sqrt{\theta}$ required. The test pilot preplans what pressure altitude, δ, he should fly for each fuel weight in order to maintain a constant W/δ. While flying a constant W/δ profile and stabilizing at various Mach numbers, the test pilot records indicated airspeed, engine speed, temperature, altitude, and fuel quantity at start of the time interval, fuel quantity at end of the time interval, and the time interval. Data obtained in this flight test can be plotted in many different forms. Flight test data obtained from speed power tests are directly used to plot the functional relationship given in Equation 11.24 as shown in Figure 11.6

$$\frac{F_n}{\delta} = f\left(\frac{W}{\delta}, M\right) \tag{11.24}$$

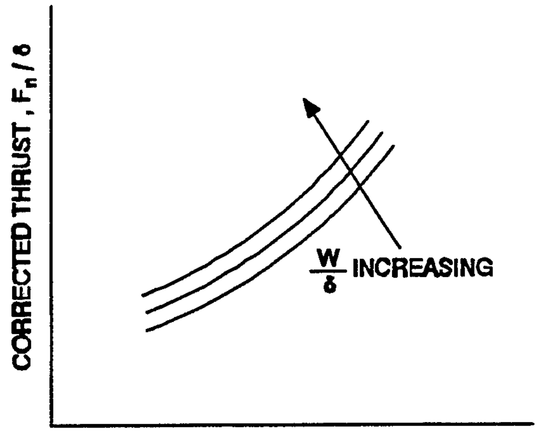


Figure 11.6 Corrected Thrust

Substituting Equation 11.30 into Equation 11.29 we obtain

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(\frac{N}{\sqrt{\theta}}, \frac{W}{\delta}\right) \tag{11.31}$$

Flight test data obtained from speed power tests are directly used to plot this functional relationship as shown in Figure 11.7. Parts (a) and (b) of Figure 11.7 are for different engine/airframe combinations; thus they are considerably different.

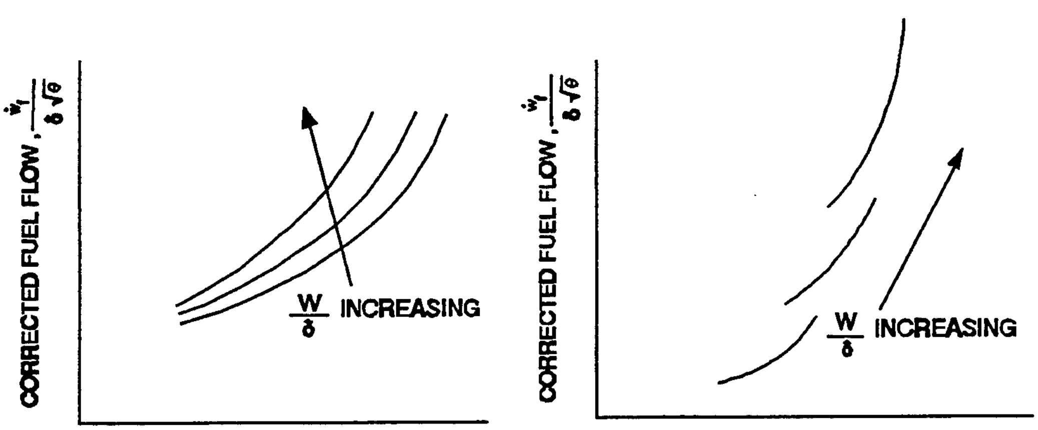


Figure 11.7 Corrected Fuel Flow

11.6 ENDURANCE , JET AIRCRAFT

Endurance , E, is defined as

$$E = \int dt \tag{11.32}$$

Fuel flow , $\dot{w}_f$, can be defined as the time rate of change of aircraft gross weight

$$\dot{w}_f = - \frac{dW}{dt} \quad \frac{lb}{hr} \tag{11.33}$$

The negative sign indicates the gross weight decreases with time.

Turbojet specific fuel consumption, C, is defined as

$$C = \frac{\dot{w}_f}{F_n} \tag{11.34}$$

Therefore

$$\dot{w}_f = CF_n \tag{11.35}$$

Substituting Equation 11.33 into Equation 11.35 for $\dot{w}_f$

$$- \frac{dW}{dt} = CF_n$$

and therefore

$$dt = - \frac{dW}{CF_n} \tag{11.36}$$

Substituting Equation 11.36 into Equation 11.32 for dt and integrating from an initial gross weight, W_i, to a final gross weight, W_f, yields

$$E = - \int_{W_i}^{W_f} \frac{dW}{CF_n}$$

Multiplying by W/W

$$E = - \int_{W_i}^{W_f} \frac{dW}{CF_n} \frac{W}{W}$$

Reversing the limits of integration to change the sign yields

$$E = \int_{W_f}^{W_i} \frac{dW}{CF_n} \frac{W}{W}$$

In unaccelerated, level flight, F_n = D and L = W. Therefore,

$$E = \int_{W_f}^{W_i} \frac{dW}{CD}\left(\frac{W}{W}\right) = \int_{W_f}^{W_i} \frac{1}{C}\left(\frac{L}{D}\right)\frac{dW}{W}$$

Assuming L/D and specific fuel consumption are constant

$$E = \left(\frac{1}{C}\right)\left(\frac{L}{D}\right) \ln \frac{W_i}{W_f} = \left(\frac{1}{C}\right)\left(\frac{C_L}{C_D}\right) \ln \frac{W_i}{W_f} \qquad (11.37)$$

Equation 11.37 illustrates that in order to obtain maximum endurance at a particular altitude, the jet powered aircraft must fly at a speed where C_L/C_D is maximum. The drag coefficient equation must be examined to determine where this speed occurs.

$$C_D = C_{D_P} + \frac{C_L^2}{\pi\ AR\ e}$$

Replacing $\frac{1}{\pi\ AR\ e}$ by the constant K

$$C_D = C_{D_P} + K\ C_L^2$$

Dividing both sides of the equation by C_L yields

$$\frac{C_D}{C_L} = \frac{C_{D_P} + K\ C_L^2}{C_L}$$

Differentiating with respect to C_L and equating the result to zero

$$K\ C_L^2 = C_{D_P}$$

Therefore

$$C_{D_i} = C_{D_P} \qquad (11.38)$$

The second derivative of C_D / C_L with respect to C_L is positive; therefore, C_D/C_L is a minimum when C_{D_i} = C_{D_p} . Therefore, C_L/C_D maximum occurs where induced drag equal parasite drag. From Equations 11.37 and 11.38 it is apparent that maximum endurance can be obtained by flying at a speed where C_L/C_D or L/D is maximum. This is shown in Figure 11.8.

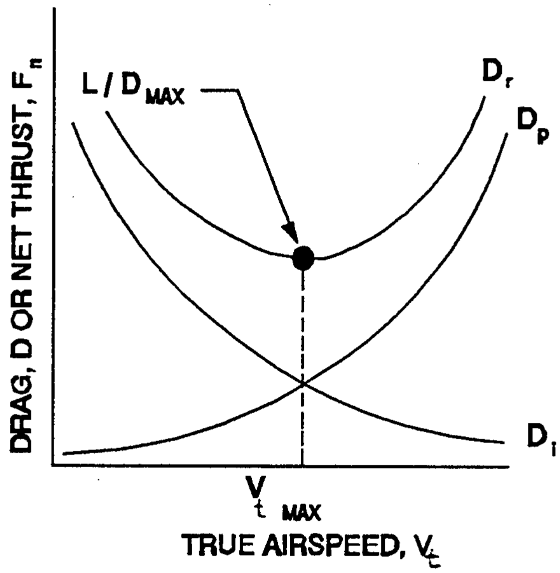


Figure 11.8 Drag or Net Thrust Required

Equation 11.37 appears to indicate that a jet powered aircraft could loiter equally well at all altitudes, provided the engine and intake duct efficiency did not change with altitude and specific fuel consumption, C, remained constant.

In practice, this is not a true statement, because specific fuel consumption is dependent on thrust developed, internal component efficiencies, ambient air temperature, true airspeed, and ambient air pressure, and may change appreciably with increased altitude. As altitude increases, higher engine speeds are required to maintain a true airspeed commensurate with the minimum drag point, and engine efficiency improves with an increase in RPM, so endurance will generally increase with an increase in altitude.

11.7 RANGE, JET AIRCRAFT

Range is defined as

$$R = \int dS = \int Vdt \tag{11.39}$$

Substituting Equation 11.36 for dt into Equation 11.39

$$dt = \frac{dW}{CF_n} \tag{11.36}$$

$$R = -\int_{W_i}^{W_f} V \frac{dW}{CF_n}$$

Multiplying by W/W

$$R = -\int_{W_i}^{W_f} V \frac{dW}{W} \frac{W}{CF_n}$$

For unaccelerated, level flight, L = W, and $F_n = D$

$$R = -\int_{W_i}^{W_f} V \frac{1}{C} \frac{L}{D} \frac{dW}{W}$$

Reversing the limits of integration to change the sign yields

$$R = \int_{W_f}^{W_i} \frac{1}{C} V \frac{L}{D} \frac{dW}{W} \tag{11.40}$$

This is the general range equation. In order to maximize range from Equation 11.40, we must maximize VL/D, minimize specific fuel consumption, and have a large fuel fraction W_i / W_f.

Solving the lift equation for true airspeed

$$V_t = \sqrt{\frac{2L}{\rho_a C_L S}}$$

For steady state flight, L = W

$$V_t = \sqrt{\frac{2W}{\rho_a C_L S}}$$

Substituting this result for V_t into Equation 11.40,

$$R = \int_{W_f}^{W_i} \left(\frac{2W}{\rho_a C_L S}\right)^{1/2} \frac{1}{C} \frac{L}{D} \frac{dW}{W}$$

Since L/D = C_L/C_D,

$$R = \int_{W_f}^{W_i} \left(\frac{2W}{\rho_a C_L S}\right)^{1/2} \frac{1}{C} \frac{C_L}{C_D} \frac{dW}{W}$$

Simplifying

$$R = \int_{W_f}^{W_i} \left(\frac{2}{\rho_a S}\right)^{1/2} \frac{1}{C} \left(\frac{C_L^{1/2}}{C_D}\right) \frac{dW}{W^{1/2}}$$

Assuming a constant altitude, constant angle of attack profile, and constant specific fuel consumption,

$$R = \left(\frac{2}{\rho_a S}\right)^{1/2} \frac{1}{C} \frac{C_L^{1/2}}{C_D} \int_{W_f}^{W_i} W^{-1/2}\, dW$$

Integrating,

$$R = 2\left(\frac{2}{\rho_a S}\right)^{1/2} \frac{1}{C} \frac{C_L^{1/2}}{C_D} \left[\sqrt{W_i} - \sqrt{W_f}\right] \qquad (11.41)$$

Examination of this equation gives the following conclusions for maximum range:

1. Increasing altitude (decreasing ρ_a) will increase the range.

2. Increasing altitude (decreasing C) will also increase the range, up to the altitude where optimum engine efficiency is reached. Above that point, specific fuel consumption begins to increase, reducing range. This effect overwhelms any further increase from decreasing air density.

3. The aircraft should be flown at a speed where $C_L^{1/2}/C_D$ is maximum.

The point where $C_L^{1/2}/C_D$ is a maximum should be examined. The drag coefficient is

$$C_D = C_{D_p} + KC_L^2$$

Thus

$$\frac{C_L^{1/2}}{C_D} = \frac{C_L^{1/2}}{C_{D_p} + KC_L^2}$$

Differentiating with respect to C_L and equating the result to zero:

$$\frac{d}{dC_L}\left(\frac{C_L^{1/2}}{C_{D_p}+KC_L^2}\right) = \frac{\left(C_{D_p}+KC_L^2\right)1/2\,C_L^{-1/2} - C_L^{1/2}\left(2KC_L\right)}{\left(C_{D_p}+KC_L^2\right)} = 0$$

so

$$\left(C_{D_p} + KC_L^2\right) 1/2\,C_L^{-1/2} - 2KC_L^{3/2} = 0$$

Solving for C_{D_p}

$$C_{D_p} = 3\,KC_L^2 \qquad (11.81)$$

or

$$C_{D_p} = 3\,C_{D_i} \qquad (11.42)$$

The second derivative of $C_L^{1/2}/C_D$ with respect to C_L is negative. Therefore $C_L^{1/2}/C_D$ is maximum when $C_{D_p} = 3\,C_{D_i}$

Cruise flight for maximum range conditions should be conducted so that the maximum number of miles can be flown with the minimum amount of fuel. To determine increment of range obtainable from each increment of fuel burned, we must define a new parameter.

Specific range, SR, is defined as

$$SR = \frac{dR}{dW} \tag{11.43}$$

Multiplying by dt/dt

$$SR = \frac{dR}{dW}\frac{dt}{dt} = \frac{dR/dt}{dW/dt} = \frac{V_t}{\dot{W}_f} \ (NAMPP) \tag{11.44}$$

where NAMPP is nautical air miles per pound of fuel.

Figure 11.9 is a classic drag, or thrust required, curve. The vertical axis also corresponds to fuel flow.

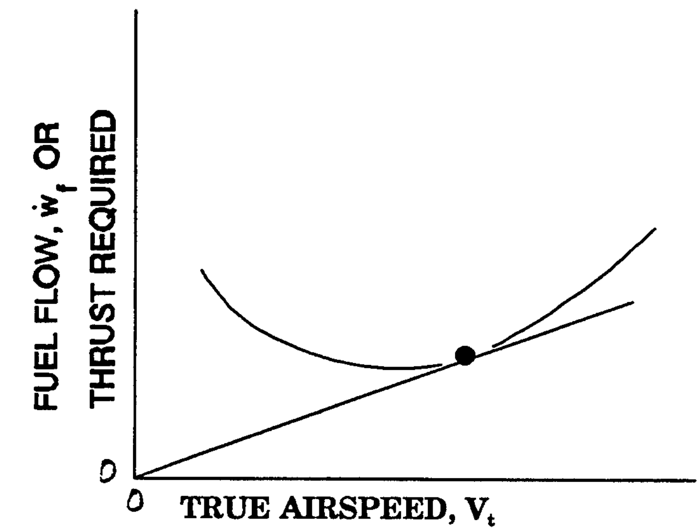


Figure 11.9 Airspeed for Maximum Range

It is apparent that to maximize specific range, the aircraft must be flown at the point where $\frac{V_t}{\dot{W}_f}$, is maximized, which is the tangent point of a line from the origin to the drag curve. At this point, $\frac{F_{n_x}}{V_t}$ is minimized. Since $F_{n_x} = D$,

$$\frac{F_{n_x}}{V_t} = \frac{D}{V_t}$$

Substituting the drag equation for D

$$\frac{F_{n_x}}{V_t} = \frac{1}{2} \rho_a V_t C_D S$$

Solving the lift equation for V_t and substituting

$$\frac{F_{n_x}}{V_t} = \left(\frac{\rho_a W S}{2}\right)^{1/2} \left(\frac{C_D}{C_L^{1/2}}\right) \qquad (11.45)$$

Flying at this tangent point, minimum $\frac{F_{n_x}}{V_t}$, results in maximizing $C_L^{1/2}/C_D$, and thus maximizing specific range.

11.8 THE CRUISE CLIMB

It is well known that the specific range of a jet airplane increases with increasing altitude. The reason for this may be seen from an examination of Equation 11.41, which shows that range varies inversely as the square root of the density, so that as long as C, the specific fuel consumption, does not increase markedly, a continuous gain in range is experienced as altitude increases.

Actually, up to the stratosphere, C tends to decrease for most engines, so that greater gains in range are obtained than would be found if C were assumed constant. Moreover, at low altitudes, inefficient part throttle operation further increases the obtainable C, producing an additional decrease in range. At high altitudes (above 35,000 to 40,000 ft) C starts to increase so that present test data reveal a "leveling off" in range for stratospheric conditions. As airplanes fly higher, this leveling off should result in an optimum best range altitude for any given gross weight, above which, decreases in range will be encountered.

Because increases in altitude result in increases in specific range, it may be reasoned that a gradual climb should increase overall range, provided that the climb were made at close to the optimum aerodynamic speed for best range and close to the engine throttle setting for best thrust specific fuel consumption. This thinking leads at once to the concept of the cruise climb.

The cruise climb amounts first to setting the airplane to fly at the optimum range Mach at a given value of W/δ. Then, as fuel is used up and W decreases, allow a gradual climb so that the ratio of W/δ is kept constant as the Mach is also held constant. This amounts to flight at the optimum Mach for the selected W/δ value.

Flight test has established that the cruise climb procedure results in improved range performance over that obtained by flight at constant altitude (varying W/δ and M). A plot of maximum specific range in nautical miles per pound of fuel as a function of gross weight and W/δ is shown in Figure 11.10.

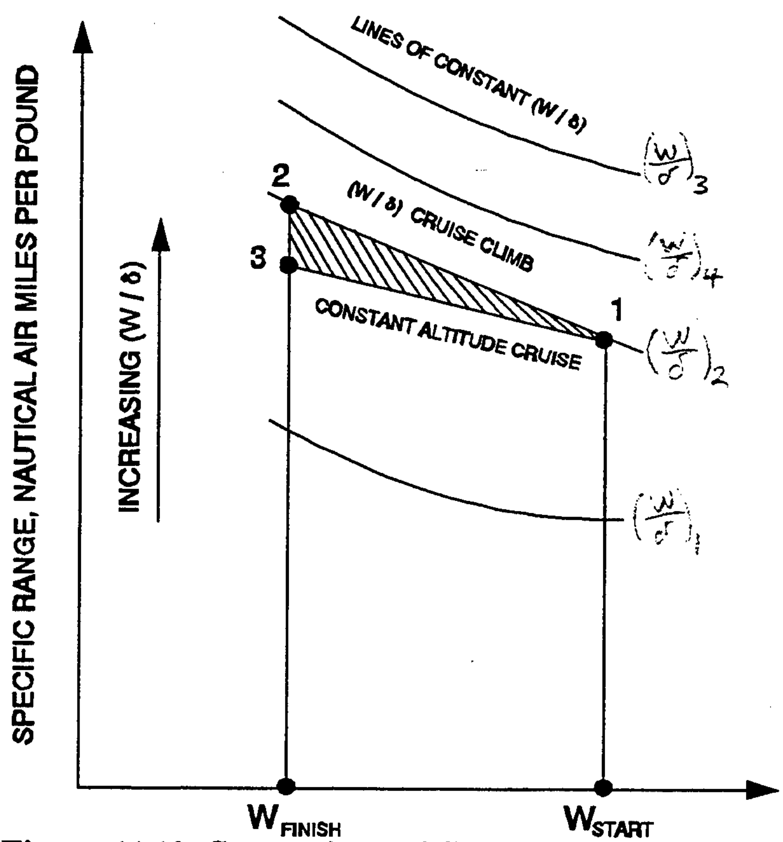


Figure 11.10 Comparison of Cruise Climb and Constant Altitude Flight

In practice, the cruise climb is accomplished by starting out at some value of W/δ, and establishing the optimum Mach for that W/δ. Given a schedule of weight versus altitude presented in some convenient form, the pilot then climbs the airplane so as to maintain a constant W/δ and M as the gross weight decreases. This means that he flies from Point (1) to (2) of Figure 11.10 with the net range being given by the area under the curve.

In this same figure, flight at constant altitude is illustrated by the line between (1) and (3), with the increase in range of the cruise climb shown as the shaded area bounded by the Lines 1-2, 2-3, 3-1.

We have assumed that the fuel consumption data obtained in level flight will be adequate to describe conditions in the cruise climb even though the cruise climb does not represent level flight conditions. Experiments have demonstrated that this assumption is satisfactory, at least for present day airplanes, for the simple reason that the climb rates are quite small and produce negligible errors. Accordingly, cruise climb fuel consumption characteristics may legitimately be computed on the basis of data obtained during level flight runs.

The maximum range profile is developed from specific range data obtained on speed power test flights. To develop the maximum range profile, an additional parameter needs to be defined.

Range factor is defined as

$$RF = (SR)\,(W) \tag{11.46}$$

Recalling that

$$R = \int V\,dt \tag{11.39}$$

$$SR = \frac{V}{\dot{W}_f} \tag{11.44}$$

$$\dot{W}_f = -\frac{dW}{dt} \tag{11.33}$$

then

$$R = -\int SR \ dW \tag{11.47}$$

Multiplying by W/W yields

$$R = -\int (SR)\,(W)\,\frac{dW}{W}$$

and therefore,

$$R = -\int RF \,\frac{dW}{W} \tag{11.48}$$

For a constant Mach, constant W/δ cruise climb, if range factor can be proven to be constant, it will be easy to integrate the range equation, Equation 11.48. The following analysis will prove range factor is indeed a constant during a constant Mach, constant W/δ cruise climb. Remember Equation 11.30, the basis of the speed power flight test technique

$$M = f\left(\frac{W}{\delta},\ \frac{N}{\sqrt{\theta}}\right) \tag{11.30}$$

Therefore,

$$\frac{N}{\sqrt{\theta}} = f\left(\frac{W}{\delta},\ M\right) \tag{11.49}$$

The maximum range profile is flown at a constant W/δ and M. Therefore $N\sqrt{\theta}$ is constant. As previously discussed,

$$\frac{\dot{w}_f}{\delta\,\sqrt{\theta}} = f\left(\frac{N}{\sqrt{\theta}},\ \frac{W}{\delta}\right) \tag{11.31}$$

For a constant $N/\sqrt{\theta}$ and W/δ, corrected fuel flow $\dot{w}_f/(\delta\,\sqrt{\theta})$ is constant

$$\frac{\dot{w}_f}{\delta\sqrt{\theta}} = K_1 \tag{11.50}$$

Mach is defined $M = \frac{V_t}{a}$. Therefore

$$M = \frac{V_t}{\sqrt{\gamma \; R \; T_a}}$$

Multiplying by $\frac{\sqrt{T_{SL}}}{\sqrt{T_{SL}}}$

$$M = \frac{V_t}{\sqrt{\gamma R T_a}} \; \frac{\sqrt{T_{SL}}}{\sqrt{T_{SL}}}$$

Since $\theta = T_a/T_{SL}$

$$M = \frac{1}{\sqrt{\gamma R T_{SL}}} \left(\frac{V_t}{\sqrt{\theta}} \right)$$

or

$$M = K_2 \; \frac{V_t}{\sqrt{\theta}}$$

Since Mach is constant

$$V_t = K_3 \; \sqrt{\theta} \qquad (11.51)$$

Specific range is

$$SR = \frac{V_t}{\dot{w}_f} \qquad (11.44)$$

Substituting Equation 11.50 and 11.51 for V_t and $\dot{w}_f$ yields

$$SR = \frac{K_3 \sqrt{\theta}}{K_1 \delta \sqrt{\theta}}$$

or

$$SR = \frac{K_4}{\delta} \qquad (11.52)$$

Substituting Equation 11.52 into the range factor definition, R = (SR)(W), yields

$$RF = K_4 \frac{W}{\delta} \tag{11.53}$$

Since W/δ is constant, range factor is constant and Equation 11.48 becomes

$$R = - \int RF \frac{dW}{W} = - RF \int \frac{dW}{W}$$

Integrating from initial gross weight (W_i) to final gross weight (W_f),

$$R = - RF \int_{W_i}^{W_f} \frac{dW}{W}$$

Reversing the limits of integration to change the sign

$$R = RF \int_{W_f}^{W_i} \frac{dW}{W}$$

Integrating

$$R = RF \ln \frac{W_i}{W_f} \tag{11.54}$$

Figure 11.11 shows specific range as a function of Mach for four different constant W/δ conditions, i = 1, 2, 3, 4. Associated with each $(W/\delta)_i$ is a maximum specific range and an optimum Mach where that maximum specific range is achieved.

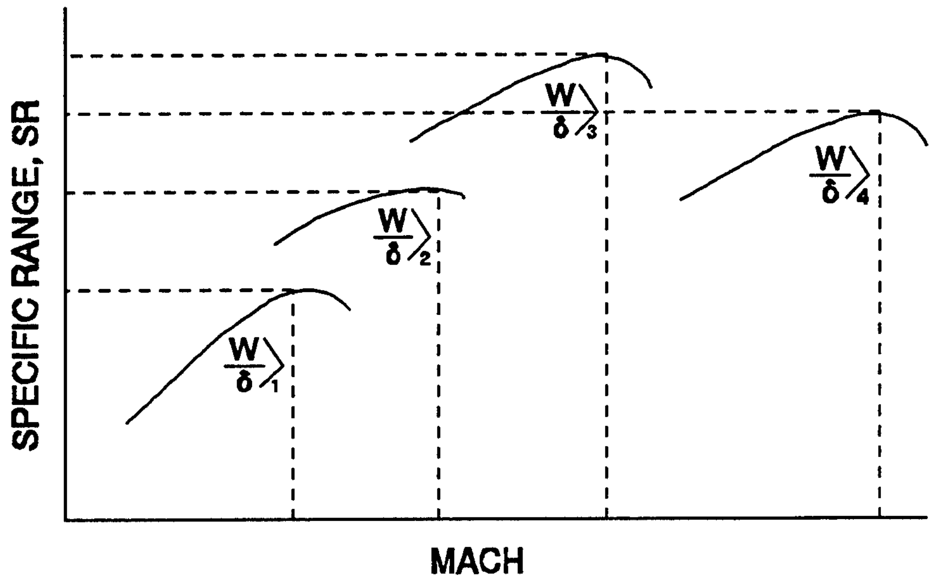


Figure 11.11 Specific Range

From the speed power test data the range factor for each maximum specific range is computed using

$$RF_i = SR_i \; W_i \tag{11.55}$$

where W_i is the standard weight used to compute the particular W/δ. These data are plotted versus W/δ as shown in Figure 11.12. Mach for each corresponding maximum specific range are also plotted.

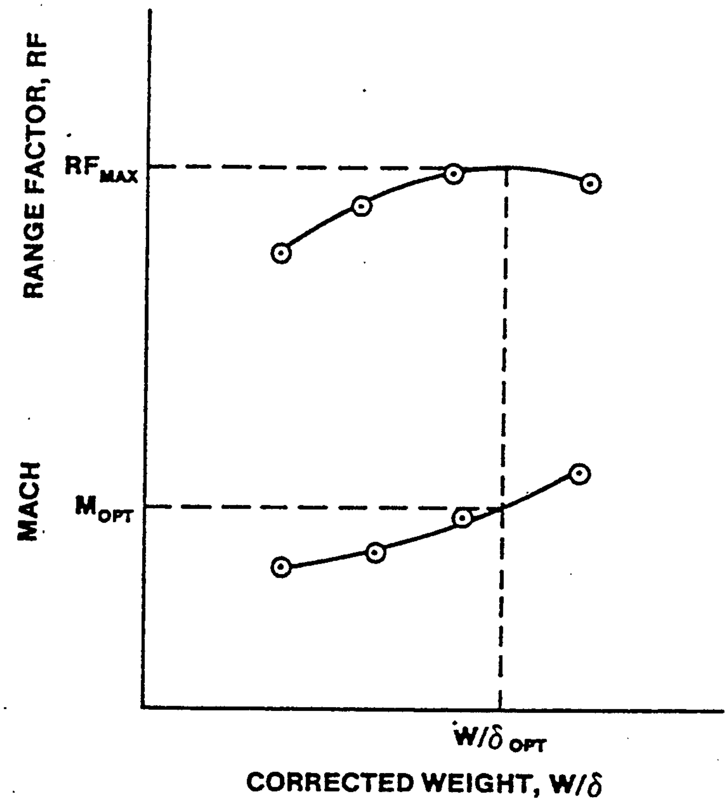


Figure 11.12 Determination of Maximum Range Factor, Optimum Weight-to-Pressure Ratio, and Optimum Mach For Any Given Range Factor

The maximum range of the aircraft is obtained by flying a constant Mach, constant W/δ cruise climb at the optimum W/δ and associated Mach from Figure 11.12.

It is important to understand that Figure 11.12 is good for all altitudes, gross weights and associated W/δ's. However, the range factor curve may have gross weight breakouts if the ratio of fuel weight to maximum gross weight is extremely large. The maximum range for any given W/δ can only be attained by flying the Mach number associated with the maximum specific range for the W/δ as shown in Figure 11.12.

11.9 DRAG POLAR DETERMINATION

Another important outcome of the speed power flight test is the determination of the aircraft's drag polar. Recalling that

$$C_L = \frac{W/\delta}{1481 M^2 S} \tag{11.12}$$

and

$$C_D = \frac{D/\delta}{1481 M^2 S} = \frac{F_n/\delta}{1481 M^2 S} \tag{11.15}$$

Knowing the test W/δ and M, C_L can be calculated from Equation 11.12.

From the functional relationship

$$\frac{N}{\sqrt{\theta}} = f\left(M, \frac{W}{\delta}\right) \tag{11.49}$$

$N/\sqrt{\theta}$ can be determined. Using $N/\sqrt{\theta}$ and the functional relationship

$$\frac{F_n}{\delta} = f\left(\frac{N}{\sqrt{\delta}}, M\right) \tag{11.27}$$

C_D can be calculated from Equation 11.15.

The aircraft drag polar can be determined by plotting C_L as a function of C_D from the speed power flight test data.

In summary, cruise performance of a jet aircraft is determined by using the speed power flight test method. The data can be analyzed to determine maximum specific range and endurance fuel flow and associated Mach numbers. Further analysis results in determination of the optimum W/δ and Mach number to fly to obtain the maximum range of the aircraft.

11.10 VARIABLE GEOMETRY AND DUAL ROTOR ENGINES

Previous sections of this chapter were restricted to constant geometry, single rotor engines. In this section some of the complications encountered with variable geometry and dual rotor engines are analyzed. In defining the performance characteristics of a variable geometry engine, it is advantageous to directly measure thrust. If thrust is directly measured, then the following functional relationship is still valid

$$\frac{F_n}{\delta} = f\left(M, \frac{W}{\delta}\right) \tag{11.24}$$

This relationship combined with the constant geometry engine analysis gives

$$M = f\left(\frac{W}{\delta}, \frac{N}{\sqrt{\theta}}\right) \tag{11.30}$$

If thrust is not directly measured, then the following equations must be used:

For variable geometry

$$M = f\left(\frac{W}{\delta}, \frac{N}{\sqrt{\theta}}, \Delta\right) \tag{11.56}$$

For dual rotor

$$M = f\left(\frac{W}{\delta}, \frac{N_1}{\sqrt{\theta}}, \frac{N_2}{\sqrt{\theta}}\right) \tag{11.57}$$

where Δ is the ratio of the variable area to some reference area; N_1 , N_2 are the two rotor speeds of a dual rotor engine. Engine speeds in a dual rotor engine are physically more difficult to measure and are less directly a function of thrust output. However, engine pressure ratio, EPR, is a direct measure of the thrust output of the engine. EPR is defined

$$EPR = \frac{P_{T_{10}}}{P_{T_2}} \tag{11.58}$$

$P_{T_{10}}$ and P_{T_2} are the total pressures at the exhaust nozzle and the compressor face. When EPR is used, it replaces the $N/\sqrt{\theta}$ term used throughout this chapter.

11.11 PROPELLER-DRIVEN AIRCRAFT CRUISE THEORY

The cruise performance of a propeller-driven aircraft can be determined from engine horsepower curves and cruise flight test data. The data reduction equations and constant altitude flight test technique will be briefly discussed in this section.

Up to this point, only drag and thrust have been considered, but in the case of the propeller-driven aircraft, it is more convenient to consider the aircraft performance in terms of power. Power is defined as the time rate of doing work.

$$Power = \frac{work}{time} = \frac{F\,d}{t}$$

Since distance, d, divided by time, t, is true airspeed, V_t, power may be expressed

$$Power = F_n\,V_t$$

Horsepower, HP, is the unit of power most commonly used and is defined

$$HP = 33{,}000\,\frac{ft\text{-}lbf}{\text{min}} = 550\,\frac{ft\text{-}lbf}{\text{sec}}$$

When the velocity is expressed in ft/sec, horsepower is expressed

$$HP = \frac{F_n V_t}{550}$$

Assuming unaccelerated, level flight (F_n = D), HP must be expressed as thrust horsepower required, THP_r.

$$THP_r = \frac{D\,V_t}{550} \tag{11.59}$$

The drag equation may be written

$$D = C_{D_p}\,\frac{\rho_a\,V_t^2\,S}{2} + \frac{2\,L^2}{\rho_a\,V_t^2\,S\,\pi\,AR\,e} \tag{11.60}$$

Substituting Equation 11.60 into Equation 11.59,

$$THP_r = \frac{DV}{550} = C_{D_P}\frac{\rho_a V_t^3 S}{1100} + \frac{L^2}{275\,\rho_a V_t S \pi AR e}$$

For unaccelerated, level flight, L = W,

$$THP_r = \frac{C_{D_P}\rho_a V_t^3 S}{1100} + \frac{W^2}{275\,\rho_a V_t S \pi\, AR\, e}$$

Substituting $V_t = \frac{V_e}{\sqrt{\sigma}}$

$$THP_r = \frac{C_{D_P}\rho_a V_e^3 S}{1100\sigma^{3/2}} + \frac{W^2\sqrt{\sigma}}{275\,\rho_a V_e S \pi\, AR\, e}$$

Multiplying by $\frac{\rho_{SL}}{\rho_{SL}}$

$$THP_r = \frac{C_{D_P}\rho_a \rho_{SL} V_e^3 S}{1100\sigma^{3/2}\rho_{SL}} + \frac{W^2 \rho_{SL}\sqrt{\sigma}}{275\,\rho_a \rho_{SL} V_e S \pi\, AR\, e}$$

Since $\sigma = \rho_a/\rho_{SL}$

$$THP_r = \frac{C_{D_P}\rho_{SL} V_e^3 S}{1100\sqrt{\sigma}} + \frac{W^2}{275\sqrt{\sigma}\,\rho_{SL} V_e S \pi\, AR\, e}$$

Multiplying both sides of the equation by $\sqrt{\sigma}$

$$\sqrt{\sigma}\, THP_r = \frac{C_{D_P}\rho_{SL} V_e^3 S}{1100} + \frac{W^2}{275\,\rho_{SL} V_e S \pi\, AR\, e}$$

Dividing both sides of the equation by $W^{3/2}$,

$$\frac{\sqrt{\sigma}\, THP_r}{W^{3/2}} = \frac{C_{D_P}\rho_{SL} V_e^3 S}{1100\, W^{3/2}} + \frac{W^{1/2}}{275\,\rho_{SL} V_e S \pi\, AR\, e} \qquad (11.61)$$

The W in Equation 11.61 is the test weight, W_t. Eliminating weight as a variable by multiplying both sides of Equation 11.61 by $W_s^{3/2}$ (the standard weight),

$$\frac{\sqrt{\sigma}\, THP_I}{\left(\frac{W_t}{W_s}\right)^{3/2}} = \frac{C_{D_P}\rho_{SL} S}{1100}\left[\frac{V_e}{\left(\frac{W_t}{W_s}\right)^{1/2}}\right]^3 + \frac{W_s^2}{275\,\rho_{SL}\left[\frac{V_e}{\left(\frac{W_t}{W_s}\right)^{1/2}}\right] S\,\pi\, AR\, e} \tag{11.62}$$

Let

$$K_1 = \frac{C_{D_P}\,\rho_{SL}\,S}{1100}$$

$$K_2 = \frac{1}{275\,\rho_{SL}\,S\,\pi\,AR\,e}$$

Substituting these constants into Equation 11.62,

$$\frac{\sqrt{\sigma}\, THP_I}{\left(\frac{W_t}{W_s}\right)^{3/2}} = K_1\left[\frac{V_e}{\left(\frac{W_t}{W_s}\right)^{1/2}}\right]^3 + \frac{K_2\, W_s^2}{\left[\frac{V_e}{\left(\frac{W_t}{W_s}\right)^{1/2}}\right]} \tag{11.63}$$

To simplify Equation 11.63, substitute

$$P_{iw} = \frac{\sqrt{\sigma}\; THP_I}{\left(\frac{W_t}{W_s}\right)^{3/2}} \tag{11.64}$$

$$V_{iw} = \left[\frac{V_e}{\left(\frac{W_t}{W_s}\right)^{1/2}}\right] \tag{11.65}$$

where P_{iw} is power required, corrected to a standard weight, and V_{iw} is equivalent airspeed corrected to a standard weight.

Now equation 11.63 becomes

$$P_{iw} = K_1 \ (V_{iw})^3 + \frac{K_2 \ W_S^2}{V_{iw}} \qquad (11.66)$$

Figure 11.13 is a plot of the power required for level flight of a propeller driven aircraft at all altitudes, temperatures, and weights. These data can be obtained from level flight test data in conjunction with engine horsepower data.

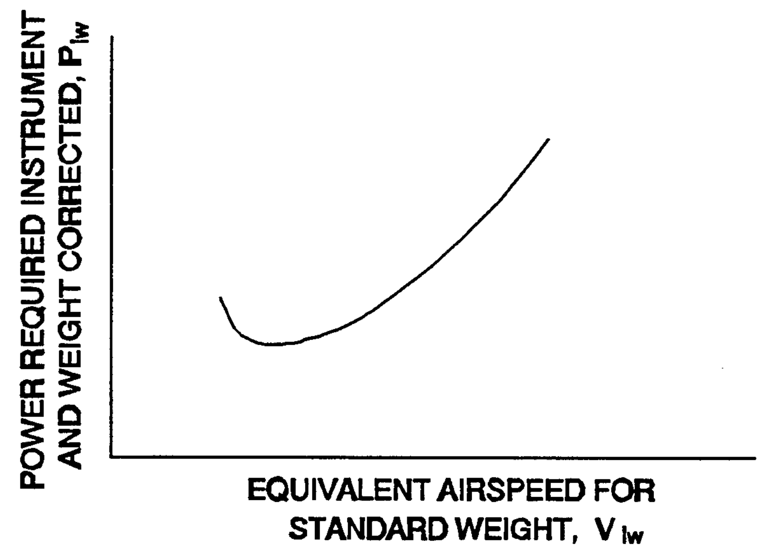


Figure 11.13 Power Required for Level Flight

Figure 11.14 is a typical plot of propeller engine horsepower data. The Continental 0-470-M engine data are presented in this figure.

The constant altitude flight test technique is used to determine cruise data for the propeller-driven aircraft. Steady state points are flown at different airspeeds throughout the flight envelope. Airspeed, V_i, temperature, T_i, altitude, H_i, manifold pressure, engine speed, and fuel weight are recorded at each stabilized point. Brake horsepower, BHP, is determined from engine horsepower curves similar to Figure 11.14 with the manifold pressure and engine speed recorded from each stable point.

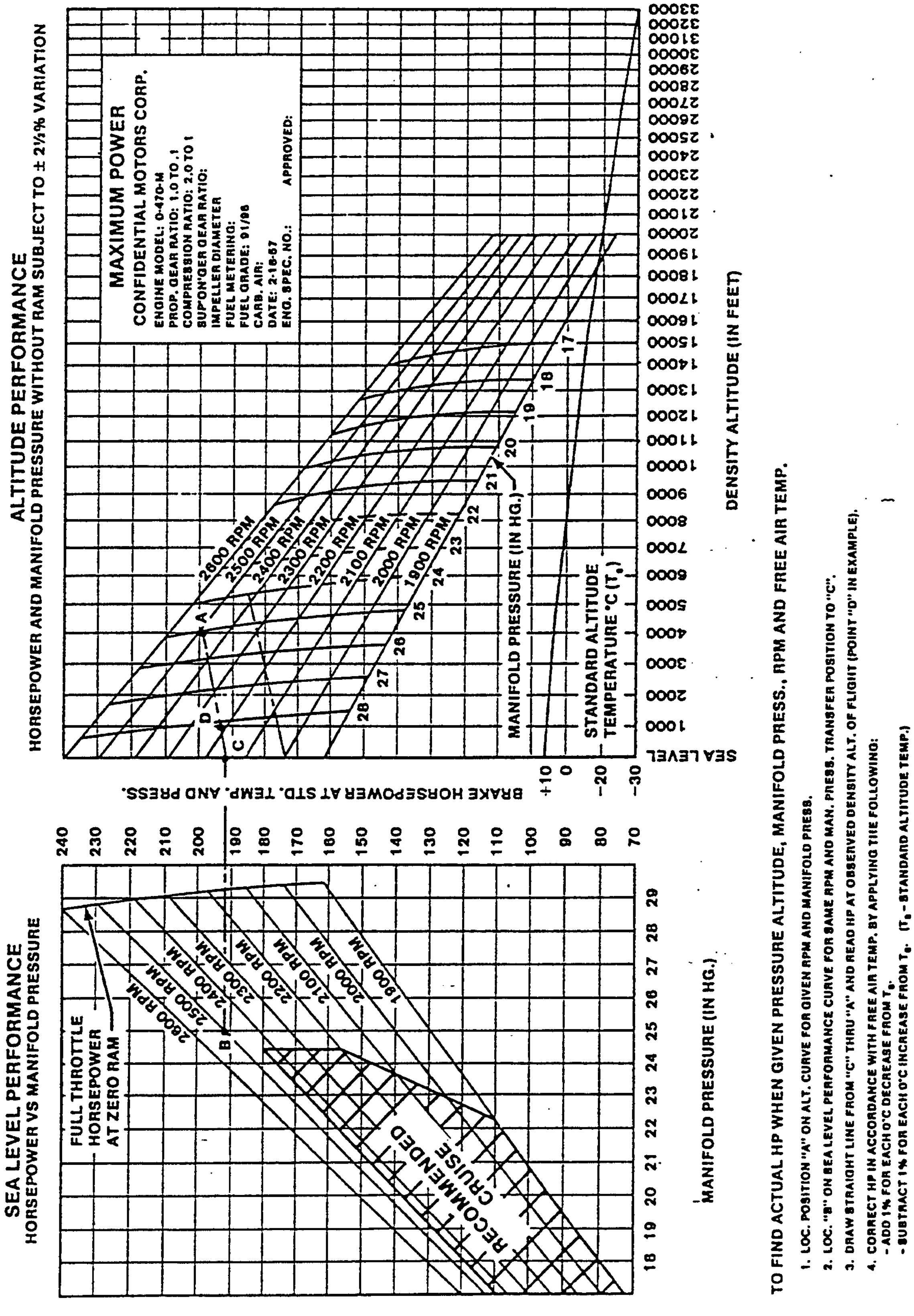


FIGURE 11.14 ENGINE HORSEPOWER CURVE

Propeller efficiency, η_p, is defined

$$\eta_P = \frac{THP}{BHP} \tag{11.67}$$

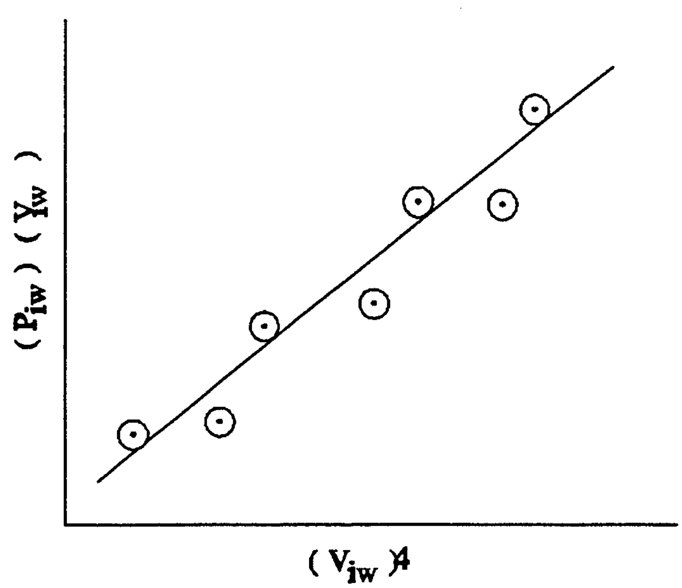
The propeller efficiency is obtained from wind tunnel data. Propeller efficiencies normally vary from 0.50 at stall airspeed to 0.80 at cruising airspeed. P_{iw} is calculated from the wind tunnel propeller efficiency, the BHP from the engine horsepower chart, test altitude, H_i, test weight, W_t, and the standard weight, W_s.

$$P_{iw} = \frac{\sqrt{\sigma}\ BHP\ \eta_P}{\left(\frac{W_t}{W_s}\right)^{3/2}} \tag{11.68}$$

Equivalent airspeed, V_e, is calculated with V_i, H_i, and T_i. V_{iw} is calculated using

$$V_{iw} = \left[\frac{V_e}{\left(\frac{W_t}{W_s}\right)^{1/2}}\right] \tag{11.69}$$

Data are obtained at different airspeeds at constant altitudes throughout the flight envelope. Data scatter is reduced by plotting (P_{iw}) (V_{iw}) versus $(V_{iw})^4$ as in Figure 11.15. A straight line can be drawn through the data points.



Points are crossplotted from Figure 11.15 to obtain Figure 11.16, the BHP required for level flight.

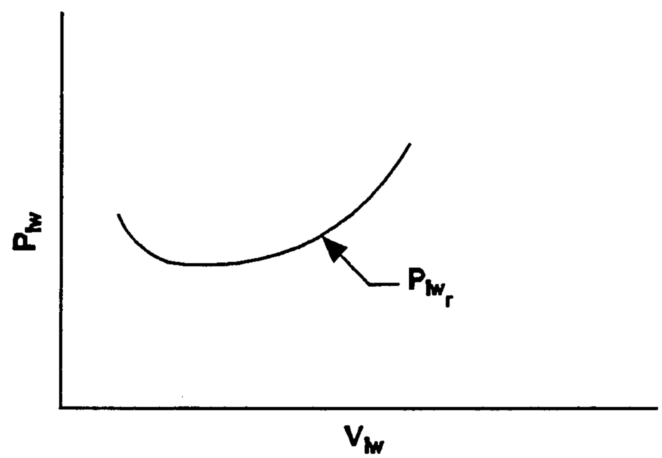


Figure 11.16 Power Required for Level Flight

A drag polar can also be constructed from level flight cruise data. From Equation 11.59

$$D = \frac{(THP_r)\ (550)}{V_t}$$

From Equation 11.64

$$THP_r = \frac{P_{iw}\left(\frac{W_t}{W_s}\right)^{3/2}}{\sqrt{\sigma}}$$

Substituting for THP_r

$$D = \frac{P_{iw}\left(\frac{W_t}{W_s}\right)^{3/2} 550}{V_t\sqrt{\sigma}}$$

Since $V_e = V_t\sqrt{\sigma}$

$$D = \frac{P_{iw}\left(\frac{W_t}{W_s}\right)^{3/2} 550}{V_e} \tag{11.70}$$

Solving the drag equation for the drag coefficient

$$C_D = \frac{2D}{\rho_{SL}\, V_e^2\, S}$$

Substituting Equation 11.70 for D

$$C_D = \frac{1100\ P_{iw}\left(\frac{W_t}{W_s}\right)^{3/2}}{\rho_{SL}\, V_e^3\, S} \tag{11.71}$$

From Equation 11.69

$$\left(\frac{W_t}{W_s}\right)^{1/2} = \frac{V_e}{V_{iw}}$$

or

$$\left(\frac{W_t}{W_s}\right)^{3/2} = \frac{V_e^{\,3}}{V_{iw}^{\,3}}$$

Substituting this result into Equation 11.71

$$C_D = \frac{1100\ P_{iw}}{\rho_{SL}\, S\, V_{iw}^{\,3}} \tag{11.72}$$

Solving the lift equation for the lift coefficient

$$C_L = \frac{2L}{\rho_{SL} V_e^2 S}$$

For a level, unaccelerated flight L = W, or

$$C_L = \frac{2\ W_t}{\rho_{SL}\ V_e^2\ S} \tag{11.73}$$

From Equation 11.69

$$W_t = \frac{V_e^2\ W_S}{V_{iw}^2}$$

Substituting this result into Equation 11.73

$$C_L = \frac{2\ W_S}{\rho_{SL}\ S\ V_{iw}^2} \tag{11.74}$$

Using Equation 11.72 for C_D and Equation 11.74 for C_L, we can plot a drag polar, Figure 11.2, from level flight cruise data.

11.12 PROPELLER-DRIVEN AIRCRAFT ENDURANCE AND RANGE

For propeller-driven aircraft, we will define brake specific fuel consumption as the fuel flow per brake horsepower developed

$$C = -\frac{dW/dt}{BHP} \tag{11.75}$$

The computation of both maximum endurance and maximum range airspeeds for a propeller driven aircraft can be simplified if the brake specific fuel consumption, C, and the propeller efficiency, η_p, are assumed constant. This is generally true at the speeds associated with maximum range and endurance.

11.12.1 RANGE

Recalling that range is defined as

$$R = \int ds = \int V dt \tag{11.39}$$

Solving Equation 11.75 for dt

$$dt = -\frac{dW}{BHP\ C}$$

Substituting this result into the range equation, Equation 11.39

$$R = -\int_{W_i}^{W_f} \frac{V_t\ dW}{BHP\ C}$$

From Equations 11.67 and 11.59

$$BHP = \frac{THP}{\eta_P} = \frac{D\ V_t}{550\ \eta_P} \tag{11.76}$$

Substituting for BHP,

$$R = -\int_{W_i}^{W_f} \frac{\eta_P\ 550\ V_t\ dW}{D\ V_t\ C}$$

Cancelling V_t / V_t

$$R = -\int_{W_i}^{W_f} \frac{\eta_P\ 550\ dW}{D\ C}$$

Reversing the limits of integration to change the sign

$$R = \int_{W_f}^{W_i} \frac{\eta_P\ 550\ dW}{D\ C}$$

Assuming brake specific fuel consumption and propeller efficiency to be constant

$$R = \frac{\eta_P\ 550}{C} \int_{W_f}^{W_i} \frac{dW}{D}$$

Multiplying by W/W

$$R = \frac{\eta_P \; 550}{C} \int_{W_f}^{W_i} \frac{W \, dW}{D \; W}$$

For unaccelerated, level flight L = W, or

$$R = \frac{\eta_P \; 550}{C} \int_{W_f}^{W_i} \frac{C_L}{C_D} \frac{dW}{W}$$

Assuming a constant angle of attack,

$$R = \frac{\eta_P \; 550}{C} \left(\frac{C_L}{C_D}\right) \int_{W_f}^{W_i} \frac{dW}{W}$$

Integrating

$$R = \frac{\eta_P \; 550}{C} \left(\frac{C_L}{C_D}\right) \ln \frac{W_i}{W_f} \qquad (11.77)$$

Equation 11.77 indicates that for maximum range:

1. Fly where η_p / C is a maximum.
2. Fly at C_L/C_D maximum.

Note that the altitude does not appear in the range equation for a reciprocating engine. However, C will decrease with an increase in altitude, causing a corresponding increase in range.

C_L/C_D maximum occurs when parasite drag equals induced drag. This occurs at the tangent to the P_{iw} versus V_{iw} curve as depicted in Figure 11.17. Therefore, the airspeed for maximum range is available from level flight performance data discussed in the range section.

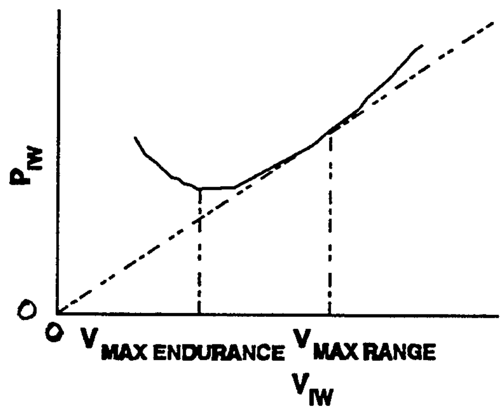


Figure 11.17 Determination of Maximum Endurance and Maximum Range Airspeeds

11.12.2 ENDURANCE

For maximum endurance an aircraft should fly at an airspeed to minimize dW/dt. Assuming specific fuel consumption is constant, this airspeed occurs at minimum BHP. Assuming propeller efficiency is constant, the airspeed for maximum endurance occurs where P_{iw} is minimum, as shown on Figure 11.17. Therefore, flight test results (P_{iw} versus V_{iw}) provide the maximum endurance airspeed.

Endurance is defined:

$$E = \int dt \qquad (11.32)$$

Solving Equation 11.75 for dt and substituting

$$E = -\int \frac{dW}{(BHP)\quad C}$$

Recalling that

$$BHP = \frac{DV_t}{550 \quad \eta_P}, \tag{11.76}$$

$$E = -\int_{W_i}^{W_f} \frac{\eta_P \, 550 \; dW}{DV_t C}$$

Since $D = \dfrac{W}{L/D}$,

$$E = -\int_{W_i}^{W_f} \frac{\eta_p \, 550}{V_t C} \left(\frac{L}{D}\right) \frac{dW}{W}$$

Solving the lift equation for true airspeed

$$V = \left[\frac{2W}{C_L \, \rho \, S}\right]^{1/2}$$

Substituting

$$E = -\int_{W_i}^{W_f} \frac{\eta_P \, 550}{C} \left(\frac{L}{D}\right) \left(\frac{C_L \rho S}{2W}\right)^{1/2} \frac{dW}{W}$$

Assuming brake specific horsepower and propeller efficiency are constant

$$E = -\frac{\eta_P \, 550}{C} \frac{C_L^{\;3/2}}{C_D} \frac{\sqrt{\rho \; S}}{2} \int_{W_i}^{W_f} \frac{dW}{W^{3/2}}$$

Integrating

$$E = \frac{\eta_P 550}{C} \frac{C_L^{3/2}}{C_D} \sqrt{2\rho S} \left[\frac{1}{\sqrt{W_f}} - \frac{1}{\sqrt{W_i}}\right] \qquad (11.78)$$

Equation 11.78 shows that maximum endurance should be achieved by flying at low altitude at a speed where $C_L^{3/2}/C_D$ is maximized. The velocity for maximum endurance shown in Figure 11.17 occurs where induced drag equals three times parasite drag. The derivation is similar to the derivation of Equation 11.42.

In conclusion, we fly stable points in a propeller-driven aircraft to obtain a power required for level flight curve and drag polar. The maximum endurance and maximum range airspeeds are obtained from the power required for level flight curve.

11.13 CRUISE PERFORMANCE TESTING

The speed power flight test technique is a common method used to obtain the cruise performance of an aircraft. This method allows determination of both maximum endurance and maximum range and considerably reduces the number of flight test sorties required.

The speed power flight test involves gathering fuel flow data at various altitudes, gross weights, and airspeeds that sufficiently define the operating envelope of the aircraft. These data are generally presented as shown in Figure 11.18.

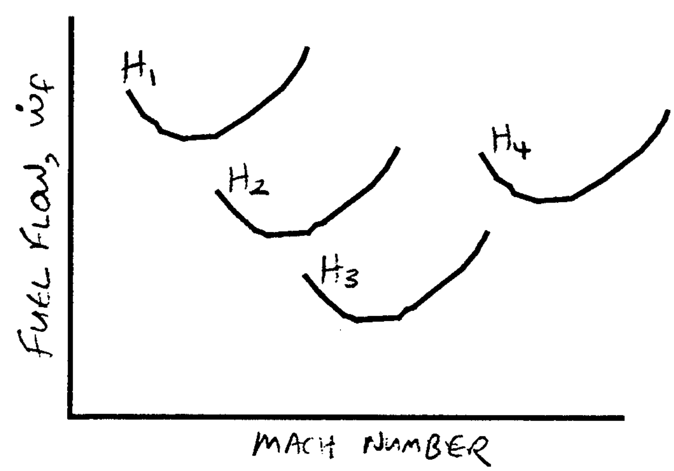


Figure 11.18 Standard Fuel Flow

Each of the curves depicted in Figure 11.18 represents one altitude and one gross weight and therefore one W/δ. It is important to note that these curves do not represent all altitude and gross weight combinations that result in the particular W/δ of the curve, but are restricted to one altitude and one gross weight. As an example, an aircraft weighing 100,000 lb at an altitude of 18,000 ft has the same W/δ as a 200,000 lb aircraft at sea level. However, it should be obvious that the fuel flow at 18,000 ft will be much less than that at sea level, resulting in a different fuel flow versus Mach curve. When the specific range is multiplied by the aircraft's weight, however, the range factor will be the same in both cases.

Since it is not realistic to consider taking data at only one altitude and one gross weight due to fuel consumption, the data must be collected within a reasonable tolerance and then standardized to the altitude and gross weight of interest. As a rule, if the W/δ of the test is held within ± 2% of the standard W/δ and the altitude is within ± 2,000 ft of the standard altitude, this functional relationship will hold true.

Recalling Equation 11.29 and Equation 11.30 from the dimensional analysis of the fuel flow parameter, we have

$$\frac{\dot{W}_f}{\delta\sqrt{\theta}} = f\left(\frac{N}{\sqrt{\theta}}, \frac{W}{\delta}\right) = f\left(\frac{W}{\delta}, M\right)$$

With the data from the speed power flight test, this functional relationship can be determined and plotted as shown in Figure 11.19

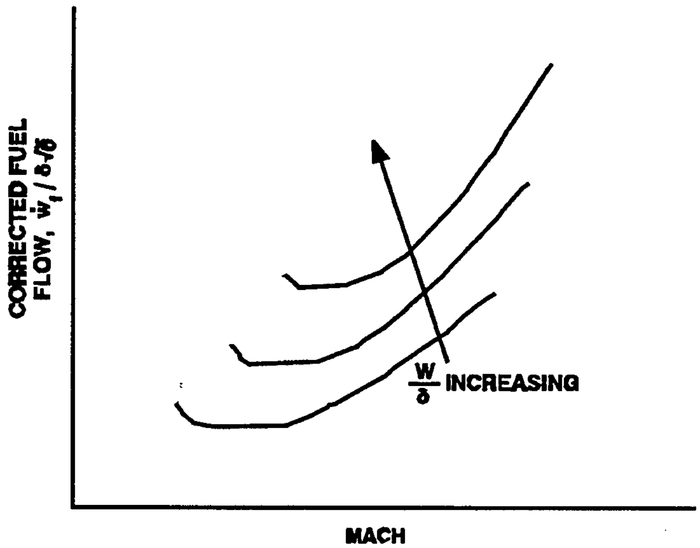


Figure 11.19 Corrected Fuel Flow

This functional relationship states that for a given corrected RPM, $N/\sqrt{\theta}$, and corrected weight, W/δ, there is only one corrected fuel flow $\dot{w}_f / \delta \sqrt{\theta}$. If W/δ is held constant during the flight test, then

$$\left(\frac{\dot{w}_f}{\delta \sqrt{\theta}}\right)_{test} = \left(\frac{\dot{w}_f}{\delta \sqrt{\theta}}\right)_{standard}$$

for a given $N/\sqrt{\theta}$ and therefore

$$(\dot{w}_f)_s = \left(\frac{\dot{w}_f}{\delta \sqrt{\theta}}\right)_t (\delta \sqrt{\theta})_s$$

Since

$$M = f\left(\frac{N}{\sqrt{\theta}}, \frac{W}{\delta}\right)$$

for the measured $N/\sqrt{\theta}$ and W/δ, then $M_t = M_s$. This is the method by which the standard fuel flow versus Mach graph is obtained.

From the $\dot{w}_f/\delta\sqrt{\theta}$ versus Mach plot, the maximum endurance airspeed can be determined for any given W/δ by picking the point of minimum fuel flow. This particular plot does not readily indicate the altitude effects and whether climbing will increase endurance (decrease fuel flow). However, maximum endurance is the point of minimum fuel flow, and the effect of climbing is evident from Figure 11.18, the fuel flow versus Mach plot.

Recalling the definition of specific range,

$$SR = \frac{V_t}{\dot{w}_f} \tag{11.44}$$

the maximum specific range for a given corrected weight to pressure ratio, W/δ, can be calculated from the fuel flow versus Mach plot. It is the point of tangency of a line drawn from the origin as shown in Figure 11.20.

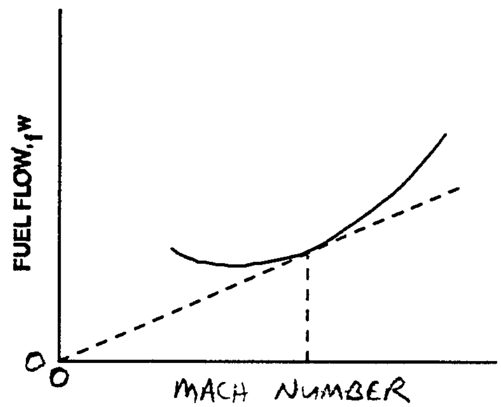


Figure 11.20 Maximum Range Mach

Since true airspeed is proportional to Mach number

$$\frac{\dot{W}_f}{M} \propto \frac{\dot{W}_f}{V_t} = \frac{1}{SR}$$

From this equation it can be seen that maximum specific range occurs where $\dot{W}_f/M$ is a minimum.

Specific range can be plotted from the fuel flow versus Mach data. A typical set of curves is shown in Figure 11.21.

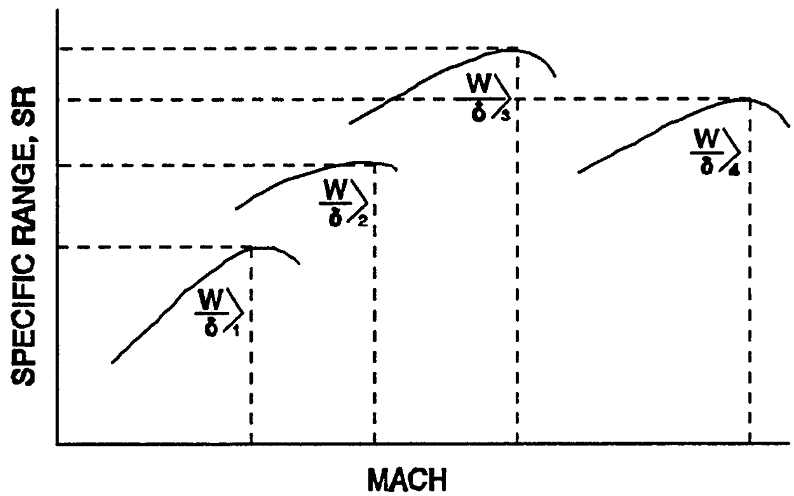


Figure 11.21 Specific Range Curves

The same information obtained from the fuel flow versus Mach curves can also be obtained from the specific range versus Mach curves.

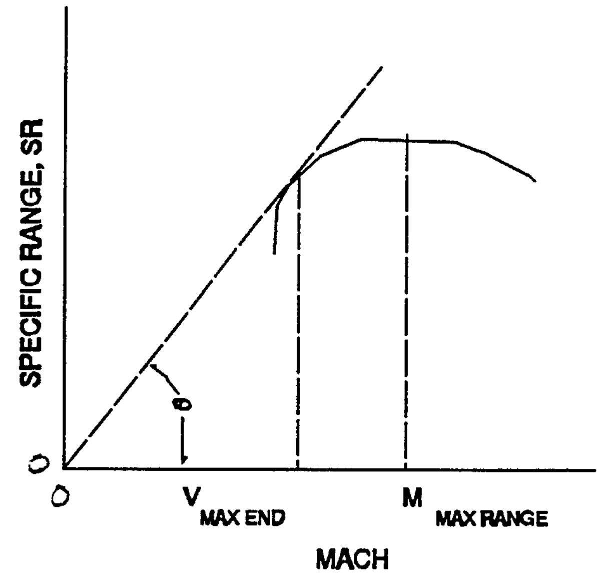


Figure 11.22 Specific Range, One Altitude

Referring to Figure 11.22, it can be seen that maximum specific range occurs at the peak of the curve. Maximum endurance occurs at the tangency point of a line drawn from the origin, since

$$\frac{SR}{M} = \frac{V_t/\dot{W}_f}{M} \propto \frac{V_t/\dot{W}_f}{V_t} = \frac{1}{\dot{W}_f}$$

From this relationship it can be seen that minimum fuel flow occurs where SR/M is maximized.

Taking the peaks of the specific range curves (the points of maximum specific range) and multiplying by the standard weight, the range factor versus W/δ curve can be generated. This curve is good for all altitudes and all gross weights.

$$RF = (SR)\,(W) \tag{11.46}$$

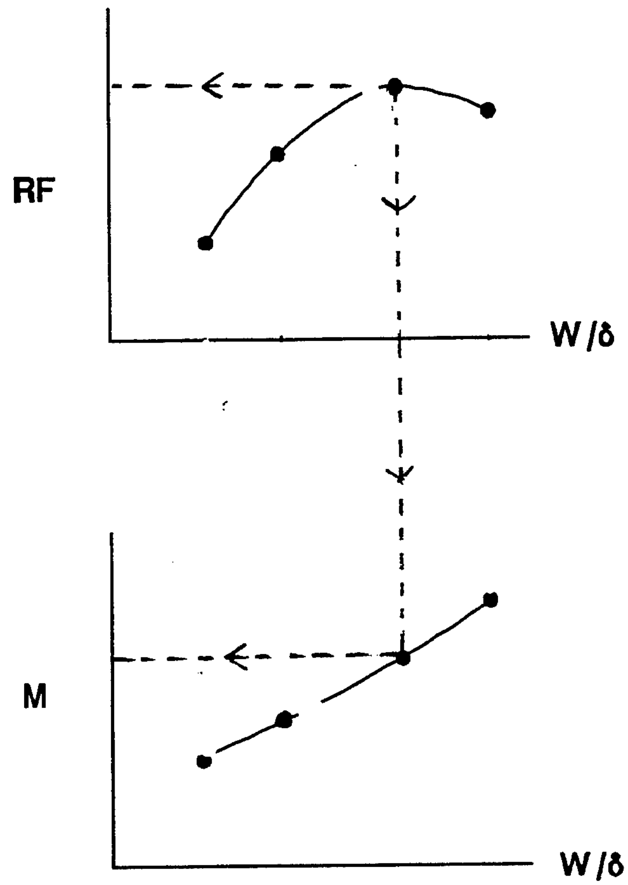


Figure 11.23 Range Factor, All Altitudes and Weights

From the peak of the range factor curve, the maximum range factor, and thus best range, can be determined, using Equation 11.54.

$$R_{BEST} = RF_{MAX} \ln \frac{W_i}{W_f} \qquad (11.54)$$

The altitude to fly to achieve best range is determined by finding δ for the optimum W/δ and test W. By plotting maximum range Mach from the peaks of the specific range curves for each W/δ, the Mach for best range can also be determined, as shown in Figure 11.23.

PROBLEMS

11.1 Define and write symbols for:

Weight-to-pressure ratio

Specific range

Corrected thrust parameter

Corrected fuel flow parameter

Corrected drag parameter

Range factor

Specific fuel consumption

Corrected engine speed parameter

11.2 The design lift coefficient of the T-38A for cruise is 0.28. If design optimum cruise Mach is 0.88 and the aircraft wing area is 170 ft^2, estimate the optimum cruise weight-to-pressure ratio.

11.3 For the T-38A design cruise condition in Problem 11.2, the T-38A parasite drag coefficient is 0.15, the aircraft efficiency factor is 0.79, and the aspect ratio is 3.75. What is the initial corrected thrust parameter F_n/δ required for cruise? How does corrected thrust parameter change during cruise climb?

HINT:

$$\frac{D}{\delta} = 1481\, C_{D_P} S M^2 + \frac{n^2}{1481\, S \pi AR e}\left(\frac{W}{\delta}\right)^2 \frac{1}{M^2}$$

11.4 During a speed power test point at 360 kts, the fuel flow was 1090 lbs/hr. What is the specific range for this aircraft?

11.5 An aircraft in-flight is attaining a specific range of 0.33 NAMPP at a gross weight of 14,000 lbs. What is its range factor?

11.6 How far will the aircraft in Problem 11.5 cruise on 4,000 lbs of fuel at the same range factor if its end cruise gross weight is 10,000 lbs? How is this accomplished?

11.7 An aircraft was flown on a constant W/δ profile of 60,000 lbs. The aircraft standard weight was 17,820 lbs. On one speed-power point stabilized at 30,300 ft, the fuel flow was measured to be 2,000 lbs/hr at 96% RPM. The ambient temperature while stabilized was measured to be 225.75°K. What is the standard fuel flow and RPM?

HINT: Use Appendix A, Performance Handbook, FTC-TIH-79-1, for atmospheric data.

11.8 Show that straight lines through the origin of a plot of SR versus true airspeed represent lines of constant fuel flow.

11.9 Given the following equations:

$$E_1 = \frac{1}{C}\left(\frac{C_L}{C_D}\right)\ln\frac{W_i}{W_f} \qquad E_6 = \left(\frac{\dot{W}_f}{\delta\sqrt{\theta}}\right)_t$$

$$E_2 = C_{D_P} + \frac{C_L^{\ 2}}{\pi\ AR\ e}$$

$$E_3 = RF\ln\frac{W_i}{W_f} \qquad E_7 = F\left(\frac{N}{\sqrt{\theta}},\ M\right)$$

$$E_4 = 2\left(\frac{2}{\rho S}\right)^{1/2}\left(\frac{C_L^{\ 1/2}}{C_D}\right)\left(\frac{1}{C}\right)\left[\sqrt{W_i} - \sqrt{W_f}\right]$$

$$E_5 = \left(\frac{N}{\sqrt{\theta}}\right)_t$$

Answer the following questions:

A. Equation ______ is the general range equation, turbojet or propeller.

B. Equation ______ is an endurance equation developed from aerodynamic analysis for turbojet aircraft.

C. Equation ______ is the equal to standard day corrected RPM parameter.

D. Equation ______ is the drag polar equation.

E. Equation ______ is used to calculate range available from a given fuel load at a given range factor.

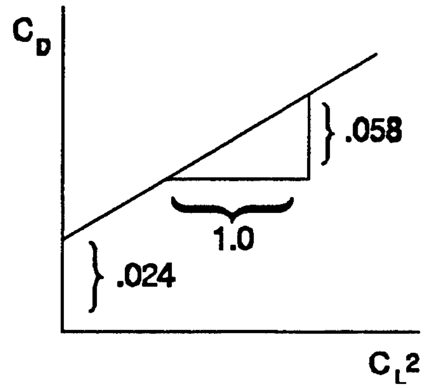
F. Equation ______ is used to determine an aircraft's thrust deck.

G. Equation ______ is a range equation developed from aerodynamic analysis for turbojet aircraft.

H. Equation ______ is used for determining standard day corrected fuel flow parameter.

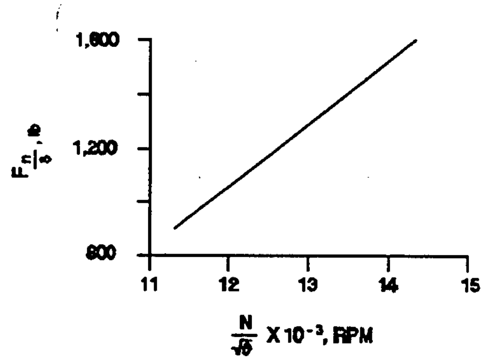
I. Equation ______ is used for determining standard day range factor from flight test range mission data.

11.10 A. The manufacturer's estimated drag polar of a YAT-37D aircraft is presented below. The aircraft reference wing are is 184 ft^2 . Using the equation developed in class for corrected drag parameter (repeated below), estimate D/δ for a speed power point flown at W/δ of 16,168 lbs (weight is 6,000 lbs, altitude is 25,000 ft) stabilized at Mach 0.4.



$$\frac{D}{\delta} = 1481\, C_{D_P}\, S\, M^2 + \frac{n^2}{1481\, S\, \pi\, ARe}\left(\frac{W}{\delta}\right)^2 \frac{1}{M^2}$$

11.10 B. At the same speed power point described in Problem 4A, the ambient temperature was determined to be 233 deg K and engine RPM, N, was measured as 11,700 RPM. Using the manufacturer's furnished chart below, what is the engine corrected thrust parameter? Does this agree well with the results of problem 4A?



11.11 The following questions apply to the YAT-37D with the drag polar and aircraft data given in Problem 4A.

A. Estimate the aircraft's L/D_{max}. What is the significance of this point?

B. If the aircraft weighs 6,000 lbs, what equivalent velocity should be flown to obtain maximum L/D?

C. Estimate the aircraft's maximum value of $C_L^{1/2} / C_D$. What would this value be used for?

D. If the aircraft weighs 6,000 lbs, what equivalent velocity should be flown to obtain maximum $C_L^{1/2} / C_D$? What is the significance of this velocity?

E. What is the maximum range L/D for constant altitude cruise?

11.12 The following problem is based on data from the T-38A Category II Performance Test, FTC-TDR-63-27, Nov 63, AFFTC, Edwards AFB, CA.

A. The contractor initially estimates that maximum range will result from a constant 0.88 Mach cruise at a constant W/δ of 54,000 lbs. Since you have three speed power missions available to determine optimum cruise, you elect to fly the three missions whose results are tabulated below:

W_s (lbs)	Altitude (H_c) ft	Maximum SR	Mach for Max. SR
10,094	36,000	0.357	0.87
9,990	40,000	0.380	0.88
9,805	45,500	0.388	0.89

Does the speed power data verify the contractor's prediction?

B. The contractor revises his estimate and now predicts that maximum range will result from a constant 0.88 Mach cruise at W/δ of 58,000 lbs. You elect to fly a ferry range mission at these cruise conditions. During the climb nearing cruise altitude, you notice you have burned 1,006 lbs of fuel. You estimate that it will take 60 lbs to stabilize at your initial start cruise altitude. Using the weight and fuel data given below, at what altitude should you begin cruise?

C. After flying your constant W/δ profile for one hour and twelve minutes, you have to terminate cruise because of an emergency with a fuel reading of 980 lbs. Data reduction shows that the cruise climb was flown at a constant T_a of -76°F, and at a constant 0.88 Mach. If the aircraft traveled 75 nam in the climb to cruise altitude, what was the total test range?

D. What was the test day range factor?

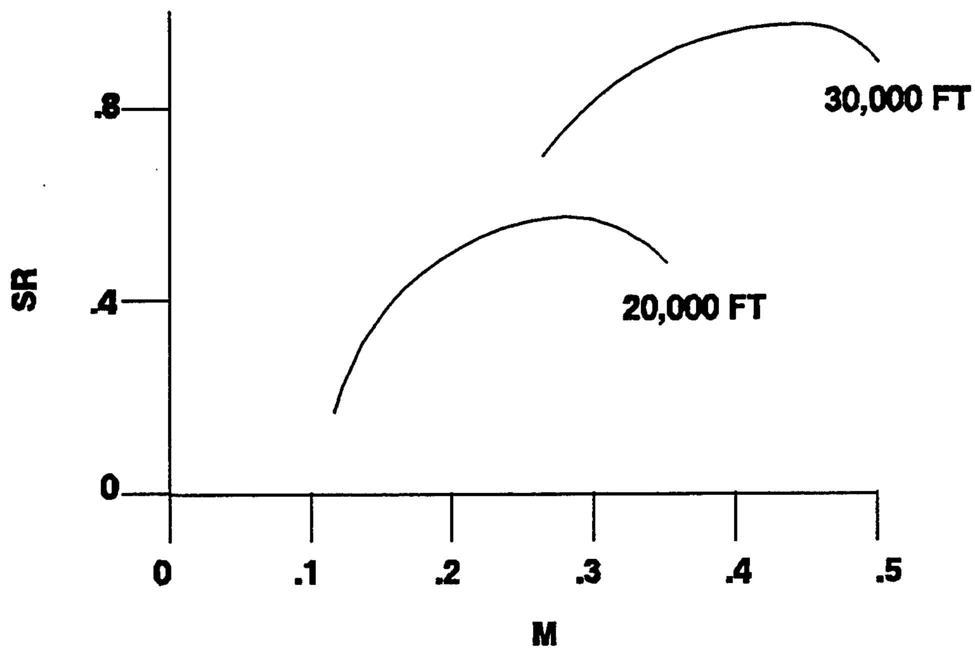
E. What was the standard day range factor?

F. Estimate what the total range of the test mission would have been if the emergency had not occurred and you could have continued cruise to your MIL-C-5011A fuel reserve.

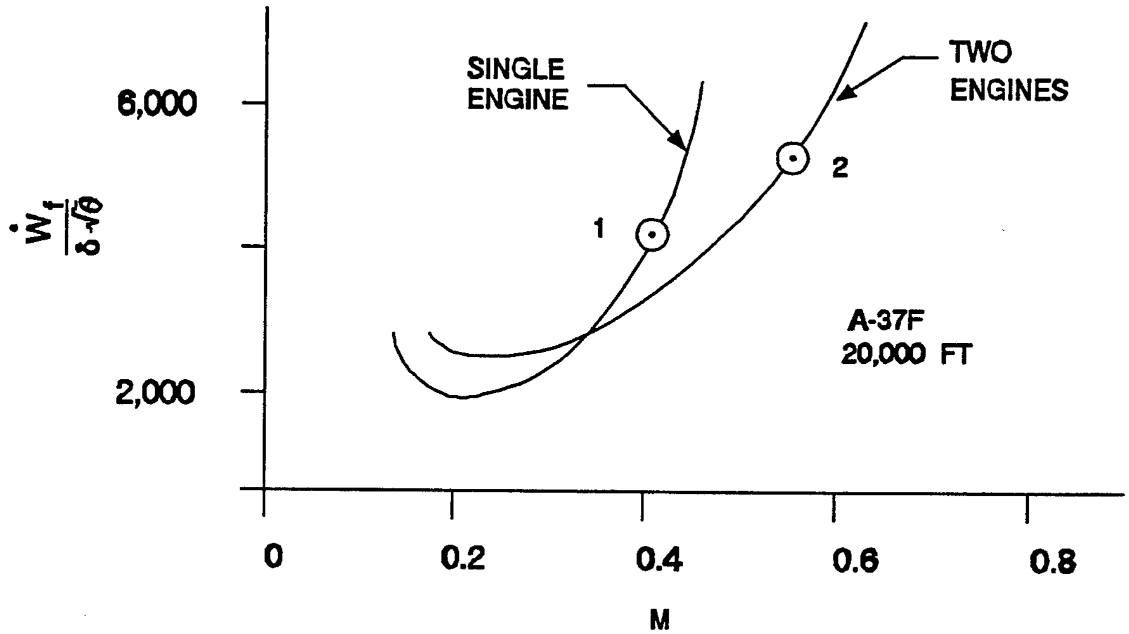
G. Does the test day range factor verify your speed power data?

H. If you had one more speed power mission to fly to verify maximum cruise range, what test conditions would you pick?

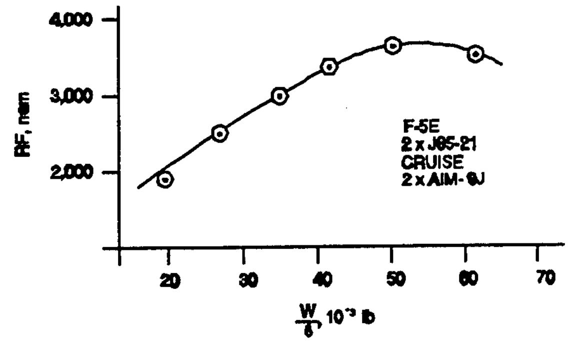
11.13 Given the flight test data below, which of the two altitudes would you choose for max endurance holding? Why? (Assume you're already at the selected altitude when you establish max endurance; i.e., ignore fuel required to climb/descend to holding altitude).

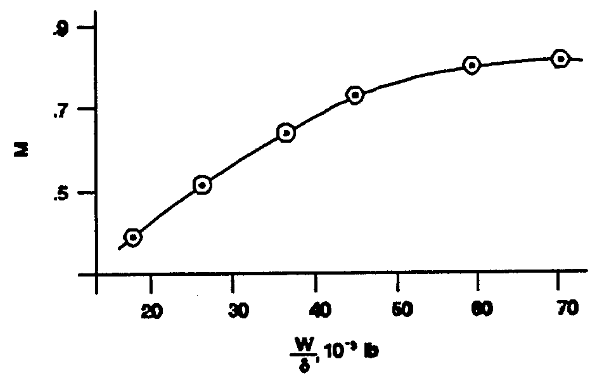


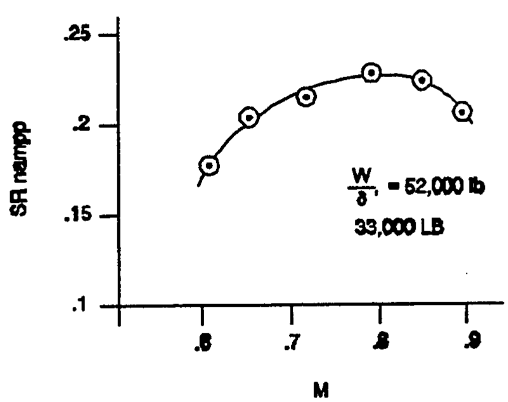
11.14 Given the flight test data below, does Point 1 or 2 give the best range? Explain.



11.15 (a) The contractor has provided the plots in Figure 1 of range factor and cruise Mach versus weight-pressure ratio based on his initial flight test data. You have just flown the speed power mission plotted in Figure 2. Does your data agree with the contractor's? Explain briefly.


4,000
3,000
2,000
RF, nam
F-5E
2 x J85-21
CRUISE
2 x AIM-9J
20
30
40
50
60
70
W/δ, 10⁻³ lb


.9
.7
.5
M
20
30
40
50
60
70
W/δ, 10⁻³ lb


.25
.2
.15
.1
SR nampp
W/δ = 52,000 lb
33,000 LB
.6
.7
.8
.9
M

(b) Using the contractor data in Part A and the fuel and weight data below, estimate the maximum cruise distance available.

Gross weight at engine start	15,745
Gross weight at start cruise	14,618
Total usable fuel	4,375
MIL-C-5011A fuel reserve	952

(c) Based on the contractor data in Parts A & B, what altitude and Mach would you plan to stabilize at to start cruise for maximum range?

(d) You were held at the end of the runway for 20 minutes and did not start cruise until 2820 lbs remaining. You also landed at your alternate, terminating your cruise climb 45 min after level off with 1400 lbs. Given a constant temperature of -58°F, what was the standard day range factor?

(e) Does your results confirm the contractor's estimates? Explain briefing.

(f) You are carrying an AIM-9J with a special lens limited to .8 Mach. Assuming a start cruise weight of 14,781 lbs, at what altitude should you plan to level off for best range?

11.16 Given the test data below, plot the aircraft drag polar.

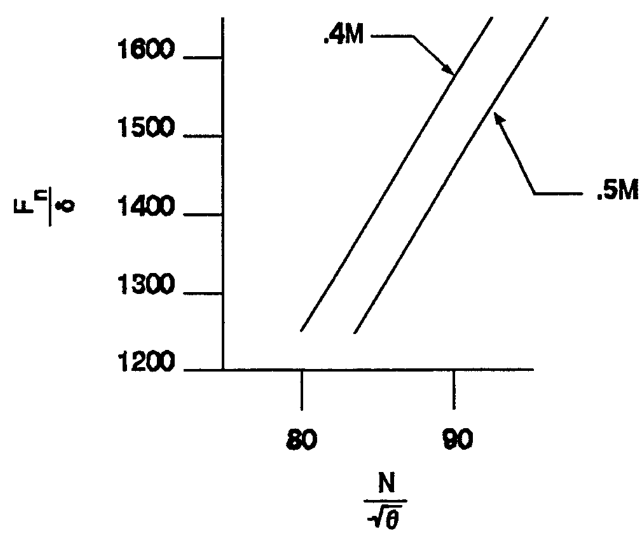
$$\frac{W}{\delta} = 20{,}000$$

$$AR = 18.0$$

$$S = 175$$

$$At\ M = 0.4,\ N/\sqrt{\theta} = 80$$

$$At\ M = 0.5,\ N/\sqrt{\theta} = 90$$



ANSWERS

11.2 54,590

11.3 30,880

11.4 .33

11.5 4620

11.6 1550 mi

11.7 $W_{f_s} = 1815$

$N_s = 87\%$

11.10
A. 1,394 lb
B. 1,400 lb

11.11
A. 13.4
B. 207 ft/sec
C. 19
D. 272 ft/sec
E. 11.6

11.12
B. 39,000 ft
C. 675 nam
D. 3,909
F. 747 nam