

**A SUMMARY REPORT**  
**VISIONS OF NONLINEAR SCIENCE AND  
TECHNOLOGY IN THE 21ST CENTURY**

submitted to

**OFFICE OF NAVAL RESEARCH**

by

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For a joint decision from their Steering Committees, the 4th IEEE International Workshop on Cellular Neural Networks and Applications (CNNA-96) and the 4th International Workshop on Nonlinear Dynamics of Electronic Systems (NDES'96), were both held in Sevilla, Spain, on June 24-25, and June 27-27, respectively.

Although these two workshops have a different focus, and were held independently in the past on different dates and at different locations, there is in fact a large intersection among the audience of these two workshops, both were attracted to a common desire to exploit the applications of nonlinear science and high technology to the design of intelligent, compact, and real-time nonlinear systems, including secure and portable communication systems, as well as robots and bionic based systems, such as cellular neural networks. In order to enhance the cross fertilization between the audience from these two workshops, and to increase the participation from members of the IEEE Circuits and Systems Society to these predominantly European activities, it was decided to strengthen this linkage by organizing a **Distinguished Plenary Lecture** session (during June 26, 1996) entitled *Visions Of Nonlinear Science In The 21st Century*.

The organizers' plan was to invite a few eminent scientists, who have made major contributions in different areas of nonlinear science or technology, to deliver a formal lecture on their visions of the great challenges and possible payoffs from nonlinear science, mathematics and technology, for the 21st century. We believe that the thoughtful ideas and positions to be advocated by these distinguished lecturers will help to identify important but tractable research directions in nonlinear science, and to provide leaderships and inspirations to future generations of researchers for solving many fundamental problems from such multi-disciplinary areas as neural networks, artificial intelligence, nanotechnology, nonlinear dynamics, nonlinear mathematics, self organization, complexity, artificial life, etc.

A main objective was to peer into the dawn of the next century and attempt to identify in this workshop some of the most fundamental and significant problems from nonlinear science, mathematics, and high technology which are tractable, and economically feasible, and whose solutions are likely to have major impacts in industry, as well as economy, in the 21st century, and beyond. We believe that such major impacts in science could be no less than the discovery of the ubiquitous phenomenon of chaos, and that such major impacts in technology could be no less than those which resulted from the invention of the transistor nearly half a century ago. The visions will be aimed for the next half century for research areas which are likely to be revolutionary in nature, yet solvable within the next five decades. Blue sky ideas whose solutions are clearly unrealistic within this time span will not be addressed in this workshop.

Eleven lectures (see the attached program) were offered to an audience primarily formed by attendants of both CNNA-96 and NDES-96, plus a number of people who only participated in this seminar on "*Visions Of Nonlinear Science In The 21st Century*". A summary from each presentation is included below.

The number of attendees was 246. A lunch was offered to the audience as well as two coffee breaks served by the staff from Hotel Al-Andalus, the place where the seminar was given.

## **SUMMERY OF THE PRESENTATION**

### **"Nonlinear Science And The Laws Of Nature"**

Ilya Prigogine

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In recent years a radical change of perspectives has been witnessed in science following the realization that large classes of systems may exhibit abrupt transitions, a multiplicity of states, coherent structures or a seemingly erratic motion characterized by unpredictability often referred to as deterministic chaos. Classical science emphasized stability and equilibrium; now we see instabilities, fluctuations and evolutionary trends in a variety of areas ranging from atomic and molecular physics through fluid mechanics, chemistry and biology to large scale systems of relevance in environmental and economic sciences. Concepts such as "dissipative structures" and "self-organization" have become quite popular. Distance from equilibrium and therefore the arrow of time plays an essential role in these processes, somewhat like temperature in equilibrium physics.

When we lower the temperature we have in succession various states of matter. In non-equilibrium physics and chemistry, when we change the distance from equilibrium the observed behavior is even more varied. How can these findings be interpreted from the point of view of the basic laws of physics ? These questions are at the heart of our present description of nature.

### **Control And Applications Of Chaos**

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and

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This paper describes a procedure for stabilizing a desirable chaotic orbit embedded in a chaotic attractor of dissipative dynamical systems by using small feedback control. The key observation is that certain chaotic orbits may correspond to a desirable system performance. By carefully selecting such an orbit, and then applying small feedback control to stabilize a trajectory from a random initial condition around the target chaotic orbit, desirable system performance can be achieved. As applications, three examples are considered:

- (1) synchronization of chaotic systems
- (2) conversion of transient chaos into sustained chaos
- (3) controlling symbolic dynamics for communication

The first and third problems are potentially relevant to communications in engineering, and the solution of the second problem can be applied to electrical power systems to avoid catastrophic event such as the voltage collapse.

## Nonlinear Numerics

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The objectives and some basic methods of numerical bifurcation analysis are described. Several computational examples are used to illustrate the power as well as the limitations of these techniques. Directions in which further algorithmic and software development is desirable are also discussed.

The principal objective of numerical bifurcation analysis, as opposed to numerical simulation, is to compute continuum families of solutions to well-defined operator equations. Such computational results give a deeper understanding of the solution behavior, stability, multiplicity, and bifurcations, and they often provide direct links with the underlying mathematical theories.

The basic numerical method is that of continuation, the language in which algorithms are expressed and analyzed is that of functional analysis, and the fundamental theoretical tool is the implicit function theorem (IFT). For ordinary differential equations, the numerical algorithms have attained a high degree of reliability. For introductions see [Rheinboldt, 1986; Seydel, 1994], tutorial articles are in [Doedel *et al.*, 1991], and for a comprehensive literature overview see [Allgower & George]. Here we highlight some representative basic algorithms used in the numerical analysis of nonlinear equations. The emphasis is on ordinary differential equations (ODEs) for which there now exist several software packages in which these techniques have been implemented. We describe computational results for four different problems, namely, a singular perturbation problem, a problem with "bursting" phenomena (Plant's model), a problem with discontinuities (Chua's circuit), and a problem with homoclinic bifurcations (coupled Josephson junctions). These examples are used to illustrate the power as well as the limitations of the numerical techniques.

We conclude by describing a selection of problems which we think merit future effort, since resulting algorithms and software would find many applications. Some current efforts in these directions are indicated.

## Experimental Nonlinear Physics

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In his essay "Tractatus Logico-Philosophicus" Ludwig Wittgenstein writes:

"The world is all that is the case". A physicist will easily agree but may feel unhappy about the amount of information given. But are there better solutions? Yes, there are. For instance, when a physicist at the turn of the century (1900) would have been asked, in view of the unification of mechanism and thermodynamics, the answer may have been: "The world is statistical".

Ten years later, around 1910, impressed by the results of Einstein, the same physicist may have answered: "The world is relative". Twenty years later, around 1930, when looking at the achievements of quantum mechanics, the still alive physicist may have added: "The world is quantal".

What to add today, near the turn of the next century? Is there something left in fundamental science to be discovered besides statistics, relativity, and quantality? If not, this would be strange and even disappointing. To be sure, physics has not yet come to an end, and we see the next fundamental step in science prospering, that is nonlinearity. Nonlinearity is fundamental for all processes in nature, except perhaps quantum physics [Heisenberg, 1967]. However, for long times it has not been recognized as a fundamental fact. Why? It seems to have been the general opinion that no universal laws may be found for nonlinear phenomena leaving them as a set of individual independent problems. Then nothing can be learned for a specific nonlinear problem from solving the other ones. This situation has changed drastically in the last decade.

Nonlinearity now is recognized as being fundamental in almost any area of physics, notably hydrodynamics, optics, acoustics, and extends to chemistry, biology, ecology. Thus, today, a physicist will add: "The world is nonlinear", and put this sentence on an equal footing with the three previous ones.

The basic hypothesis of a physicist (not long ago called philosopher) is that the natural phenomena surrounding us are governed by laws and that a language can be invented to pin down these laws. We call that basic research, and it worked well so far for the benefit of mankind, although it has been a long way from ancient times to arrive at this insight. For instance, Descartes worked hard to give hints how to proceed in science to make progress. But why do we want to understand nature? One reason is our intellectual curiosity. The second reason is that we can make use of the laws for better mastering our life, for instance by building houses, beating enemies with improved arms, moving faster than with our legs, transmitting information, etc. This works because with every law detected we gain the ability to predict the behavior of the respective system.

When we collect more and more laws of nature, will we, in the end, be able to predict the future? The answer is no, and the reason is nonlinearity. Even simple deterministic laws

may have such complicated dynamics that a prediction over longer periods of time is impossible. Thus determinism does not imply (practical) predictability. And what do we do facing this insight? We now do experiments to gain insight into our limitations set by nonlinearity. This review is an attempt to at least partially follow the ways experimentalists have gone and will go further to elucidate the nature of unpredictability.

## Nonlinear Science - The Impact Of Biology

A. V. Holden

Nonlinear science has primarily developed from applications of mathematics to physics. The biological sciences are emerging as the dominant growth points of science and technology, and biological systems are characterized by being information dense, spatially extended, organized in interacting hierarchies, and rich in diversity. These characteristics, linked with an increase in available computing power and accessible memory, may lead to a nonlinear science of complicated interacting systems that will link different types of mathematical object within a framework of algebraic models of computing systems. Examples, drawn from current work on intracellular, cellular, tissue, organ, and integrative physiology of an individual, are outlined within the theory of synchronous concurrent algorithms. Possible directions in population dynamics and applications to ecosystem management are outlined.

In this chapter I am outlining the future development of nonlinear science in the 21st century. Since historians continually re-interpret and revise our view of the past, any predictions of the future from a current perspective of nonlinear science and its development must be uncertain. A common feature of nonlinear systems is that their behaviors can be unexpected, counterintuitive and surprising; homogeneous systems can self-organize; simple systems can generate complex behavior and long-term predictability may be impossible. What is to be expected from nonlinear science is the new and unexpected.

Nonlinear science is a recent and premature concept, and its birth can perhaps be identified with the opening of the Center for Nonlinear Studies at Los Alamos, or the informal circulation of Nonlinear Science Abstracts by Joe Ford in 1977, or the start of publication of *Physica-D* Nonlinear phenomena in 1980. Its conception was not a single event, more a coming together from different roots, chaos from Poincare's celestial mechanics, Cartwright's studies on forced oscillators and Lorenz's three variable ordinary differential equation model; solitons from Scott Russel's observations by the Glasgow-Edinburgh canal, and Fermi, Pasta and Ulam's early computations at Los Alamos on a nonlinear vibrating string. It is premature as most of nonlinear science is still mathematics, rather than science, and where the mathematics is applied it is usually to problems from imaginary physics, rather than laboratory or real-world problems.

The institutionalization of nonlinear science, with its workshops, initiatives, programs, journals, monographs and textbooks, has already begun. The development of a nonlinear science, as distinct from a menagerie of nonlinear novelties from different disciplines, is underway. Its future growth may be by a gradual accretion of processes and mechanisms, or by metamorphoses, barely staying afloat on a sea of shifting paradigms.

## Visions Of Cnns In The 21st Century

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This paper presents a generalized definition of CNN which covers a much broader terrain outside of engineering and brain science where it was originally intended. Henceforth, the term CNN may be interpreted as "Cellular Neural network" if applied in the more general setting encompassing many disciplines, including biology, chemistry, ecology, economics, mathematics, physics, social sciences, etc. In particular, by simply specifying the "cells", almost all well-known nonlinear partial differential equations can be modeled by a CNN. Some of these equations are FitzHugh-Nagumo Equation, Brusselator, Oregonator, Meinhardt-Gierer Equation, Sine-Gordon Equation, Toda Lattice, Volterra-Lotka Equation, etc.

From the practical (e.g. image processing) applications viewpoint, more than 75% of all current CNN's can be explicitly and rigorously specified by a "minimal CNN truth table" containing only 512 binary bits (for 3x3 neighborhoods). This important class of CNN's is henceforth called "locally Boolean" and one fundamental result of this paper is a proof that any locally Boolean CNN can be realized by using only linearly-separable CNN's as building blocks, and can be trivially implemented in real time via the CNN universal chip. As a corollary, this theorem proves that the CNN is in fact a universal Turing machine capable of implementing any algorithm, and a universal constructor which in principle is capable of "self reproduction" in the von Neumann sense, and hence is capable of creating "artificial life".

The highlight of this paper is the presentation of a truly fundamental physical principle, called the "CNN Local Activity Principle", which provides a unified, precise, and easily testable mathematical criterion for the existence and explanation of many macroscopic phenomena and principles currently referred to in the literature under many different names, including dissipative structures, synergetics, slaving principle, collective, cooperative, competitive, far-from-equilibrium, emergent phenomena, edge of chaos etc. This unifying theorem proves that all of the above seemingly distinct phenomena and principles are in fact equivalent in the sense that they are manifestations of the "CNN local activity principle" from different perspectives.

### **Some Historical Aspects Of Nonlinear Dynamics Possible Trends For The Future**

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Dynamics is a concise term referring to the study of time-evolving processes, and the corresponding system of equations, which describes this evolution, is called a dynamic system. A complete survey of results, obtained in this too wide field, is impossible, even

limiting the matter to systems described by ordinary differential equations (continuous dynamics) and maps (discrete dynamics), as done in this paper.

Initially, the theory of Dynamic Systems builds up itself on the foundations of the Poincare (1878-1900), Liapunov (1893), Birkhoff's (1908-1944) results, and those concerning the point mappings, iterations, recurrences, obtained at the end of the 19th and the beginning of the 20th century, by Koenigs, Lemery, Lattes, Hadamard. After it has had its most spectacular and organized growth within the framework of two Soviet schools of thought (1920 till now): the Mandelstham-Andronov's school (Moseou-Gorki), and the Krylov-Bogoliubov's school (Kiev). Very strong interactions between the theoretical researches and the practical implications in the physical, or engineering systems, are the reason of the success of these two schools. They have led the Poineare-Liapunov-Birkhoff's methods to the highest development in what they called theory of nonlinear oscillations, the most important component of the theory of dynamical systems. In dynamics, these two schools occupied incontestably the first place, which was admitted by some of the most famous American mathematicians [for example, cf. J. P. Lasalle and S. Lefschetz, *J. of Math Anal. and Appl.*, Vol. 2, 1961, pp. 467-499].

From 1960, and specially since 1975 with the explosive growth of the researches in the chaotic dynamics field (irregular oscillations of deterministic origin), and with the translation of some Soviet results in the western countries, the study of dynamic systems has become a subject in vogue out of USSR, with more and more papers concerning all the scientific disciplines. Nevertheless, in spite of a practical motivation often announced by the authors, a large part of these papers concerns "abstract dynamic systems", and are devoted to nonessential generalizations without any interest for understanding a typical dynamic behavior, or for practical purposes. In opposition with the motivation of study of "concrete dynamic systems", purpose of the Mandelstham-Andronov-Krylov-Bogoliubov's schools, from 1970 an increasing number of pure mathematicians have developed a purely "abstract approach" of dynamics without direct utility for applied fields.

This paper is limited to "concrete dynamic systems", the field of which is considered as constituted by two sets of results. The first one is related to the study of problems directly suggested by practice (Physics, Engineering,...). The second set concerns the study of equations, not directly tied with practice, but having the lowest dimension, and the simplest structure, which permits to isolate in the purest form a "mathematical phenomenon", by eliminating the "parasitic effects" of a more complicated structure.

## **Nonlinear Physics: Integrability, Chaos And Beyond**

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Integrability and chaos are two of the main concepts associated with nonlinear physical systems which have revolutionized our understanding of them. Highly stable exponentially localized solitons are often associated with many of the important integrable nonlinear systems while notions which are sensitively dependent on initial conditions are associated with chaotic systems. Besides dramatically raising our perception of many natural phenomena, these concepts are opening up new vistas of applications and unfolding



technologies: Optical soliton based information technology, magneto electronics, controlling and synchronization of chaos and secure communications, to name a few.

These developments have raised further new interesting questions and potentialities. We present a particular view of some of the challenging problems and payoffs ahead in the next few decades by tracing the early historical events, summarizing the revolutionary era of 1950-70 when many important new ideas including solitons and chaos were realized and reviewing the current status. Important open problems both at the basic and applied level are discussed.

## **Nonlinear Computation**

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Nonlinear Computation will provide a framework for the computational science and engineering of the future. Topics such as bifurcation and chaos, and methods such as continuation and branch switching, and features such as stability and sensitivity are basic ingredients in the attempt to understand our world. Nonlinear computation is indispensable when we try to predict and simulate the dynamics of states in the biological systems we consist of, in the ecological and economical systems we live in, and in the technical systems we make use of the term "nonlinear computation" is a short form for "computation of something that is nonlinear." But since "nonlinear" is the normal state and linearity is only found in specific local situations, the term nonlinear computation in its proper sense may be meaningless. This title rather reflects a historical change in that the traditional assumption of linearity is no longer required.

This paper essentially consists of three parts. The first part (Section 1) explains what nonlinear computation means. The second part (Sections 2 through 5) presents a survey on nonlinear phenomena, and on the methods of nonlinear computation. The final part (Sections 6 and 7) will speculate on visions and trends for the next decades.

## **Visions Of Synergetics**

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Synergetics is an interdisciplinary field of research. It deals with open systems that are composed of many individual parts that interact with each other and that can form spatial, temporal, or functional structures by self organization. The research goal of synergetics is three-fold:

1. Are there general principles of self-organization?
2. Are there analogies in the behavior of self-organizing systems?

### 3 Can new devices be constructed because of the results 1 and 2?

From a mathematical point of view, synergetics deals with nonlinear partial stochastic differential equations and studies their solutions close to those points, where the solutions change their behavior qualitatively. As I will show in my article, synergetics in its present form is based on the concepts of stability and instability, control parameters order parameters, and the slaving principle. The slaving principle allows one to compress the information that is necessary to describe complex systems into few order parameters. This is possible if systems are close to their instability points. But it appears that the order parameter concept is also applicable to situations away from such instabilities. At the level of the order parameter equations, profound analogies between otherwise quite different systems become visible. This allows one to realize the same process (for instance dealing with information) on quite different material substrates. The order parameter concept and the slaving principle are explained and their extension to discrete noisy maps and to delay equations are mentioned.

These results can be applied to pattern formation in fluids, lasers, semiconductors, plasmas, and other fields. A section is devoted to the analysis of spatio-temporal patterns in terms of order parameters and the slaving principle. It is shown how the concepts of synergetics can be utilized to devise a new type of computer for pattern recognition. In connection with preprocessing, it can recognize patterns that are shifted, scaled or rotated in space and that are deformed. It can recognize scenes and also facial expressions as well as movement patterns. The learning procedure is briefly outlined. Because of the analogy principle of synergetics this computer allows for hardware realizations by means of semiconductors and lasers. Decision making by humans or machines is interpreted by means of an analogy with pattern recognition. Further sections are devoted to recognition of dynamic processes and learning by machines. In this author's opinion synergetics will find important applications to medicine, for instance in the analysis of MEG and EEG patterns and to the development of devices with brain like functions. Future tasks of synergetics will be the application of the order parameter concept and the slaving principle to the integration of specialized computers or computer algorithms, for instance for the recognition of faces, movement patterns, and so on, into a computer network for scene analysis and decision making. This will hold also for complicated production processes, and so on. Generally speaking, the potentialities of synergetics are based on its self-organization principles.

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