NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 368

A NEW CHART FOR ESTIMATING THE ABSOLUTE CEILING OF AN AIRPLANE

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By WALTER S. DIEHL

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AERONAUTICAL SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Symbol	Unit	Symbol
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of one kilogram	kg	weight of one pound	lb.
Power	P	kg/m/s		horsepower	hp
Speed		km/hr	k. p. h.	mi./hr.	m. p. h.
		m/s	m. p. s.	ft./sec.	f. p. s.

2. GENERAL SYMBOLS, ETC.

W, Weight, $=mg$

g, Standard acceleration of gravity $=9.80665$ m/s$^2=32.1740$ ft./sec.^{2}

m, Mass, $=\frac{W}{g}$

ρ, Density (mass per unit volume).

Standard density of dry air, 0.12497 (kg-m^{-4} s^2) at 15° C and 760 mm $=0.002378$ (lb.-ft.$^{-4}$ sec.^{2}).

Specific weight of "standard" air, 1.2255 kg/m$^3=0.07651$ lb./ft.^{3}

mk^2, Moment of inertia (indicate axis of the radius of gyration, k, by proper subscript).

S, Area.

S_w, Wing area, etc.

G, Gap.

b, Span.

c, Chord length.

b/c, Aspect ratio.

f, Distance from C. G. to elevator hinge.

μ, Coefficient of viscosity.

3. AERODYNAMICAL SYMBOLS

V, True air speed.

q, Dynamic (or impact) pressure $=\frac{1}{2}\rho V^2$

L, Lift, absolute coefficient $C_L=\frac{L}{qS}$

D, Drag, absolute coefficient $C_D=\frac{D}{qS}$

C, Cross-wind force, absolute coefficient $C_C=\frac{C}{qS}$

R, Resultant force. (Note that these coefficients are twice as large as the old coefficients L_C, D_C.)

i_w, Angle of setting of wings (relative to thrust line).

i_t, Angle of stabilizer setting with reference to thrust line.

γ, Dihedral angle.

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension.

e. g., for a model airfoil 3 in. chord, 100 mi./hr. normal pressure, 0° C: 255,000 and at 15° C., 230,000;

or for a model of 10 cm chord 40 m/s, corresponding numbers are 299,000 and 270,000.

C_p, Center of pressure coefficient (ratio of distance of C. P. from leading edge to chord length).

β, Angle of stabilizer setting with reference to lower wing, $=(i_t-i_w)$.

α, Angle of attack.

ϵ, Angle of downwash.

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By WALTER S. DIEHL,
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REPORT No. 368

A NEW CHART FOR ESTIMATING THE ABSOLUTE CEILING OF AN AIRPLANE

By Walter S. Diehl

SUMMARY

This report, which was prepared for publication by the National Advisory Committee for Aeronautics, is concerned with the derivation of a chart for estimating the absolute ceiling of an airplane. This chart may be used in conjunction with the usual curves of power required and power available as an accurate substitute for extended calculation, or it may be used in the estimation of absolute ceiling when power curves are not available.

INTRODUCTION

The absolute ceiling of an airplane is the greatest altitude that can be attained under specified conditions of loading and power. It is the altitude at which the maximum rate of climb is zero. At the absolute ceiling the curves of power available and power required are tangent and flight is possible at only one air speed or angle of attack.

There are many methods for calculating the absolute ceiling with considerable accuracy or estimating it to a close average approximation. One widely used method is to calculate the rate of climb at three or more altitudes and plot these rates of climb against altitude. Since the rate of climb decreases almost linearly with altitude, a curve may readily be drawn through the calculated points and extrapolated to the altitude axis where the intercept gives the absolute ceiling. The calculations required are comparatively simple but rather extensive.

It has been shown by the writer (Chapter VII, Reference 1) that the absolute ceiling may be obtained from the intersection of two special but easily determined curves drawn on the usual plot of power available and power required against air speed. It now appears that the calculations required in this method can be still further simplified and the absolute ceiling obtained directly from the ratio of minimum thrust power required to maximum thrust power available and the ratio of maximum speed to minimum speed. These ratios may be determined either from the calculated power curves or from a direct estimation. Since the estimation of these ratios can be made to a comparatively high degree of accuracy, it follows that a similar degree of accuracy is obtained in the estimation of absolute ceiling. Extended application of this method in routine performance calculations has indicated that for all practical purposes it is fully equivalent to a complete calculation. The use of the ratio of minimum thrust power required to maximum thrust power available in the determination of absolute ceiling is very old. Hofmann (Reference 2) employed such a method in 1913 and implied by footnote reference that it was used as early as 1909 or 1910. Warner (References 3 and 4) gives the absolute ceiling in terms of this ratio. The present method, while making use of the power ratio, differs from the previous methods in that the final ceiling chart given herein is essentially a condensed general solution for an exact graphical

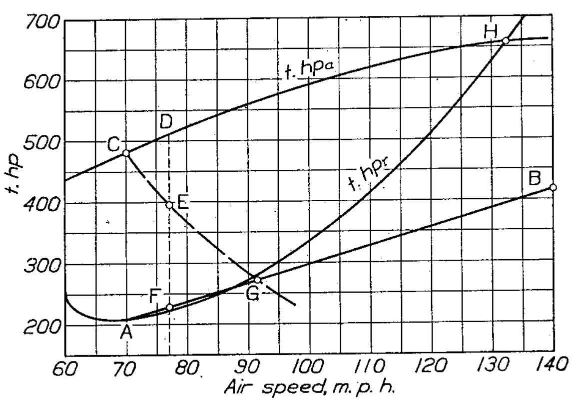

FIGURE 1

method, rather than an approximation based on the obvious and well-recognized importance of the power ratio.

CALCULATION OF ABSOLUTE CEILING

The author has shown elsewhere (Reference 1) that the usual sea level curves of power available and power required plotted against air speed may be used in conjunction with a simple construction to obtain the absolute ceiling. This construction, illustrated in Figure 1, is based on the assumption that at the absolute ceiling the curve of power available is tangent to the curve of power required at or very near to the minimum power required. If a climb is started at sea level at the angle of attack for minimum power required and this angle of attack is maintained up to the absolute ceiling, the best rate of climb will not be obtained at low altitudes but the final altitude obtained will not be affected. Since the angle of attack is constant the power required and the air speed will both vary directly as the square root of the inverse density ratio $\sqrt{\rho_o/\rho}$. When $\sqrt{\rho_o/\rho}=2$, both power required and air speed will be twice as great as at sea level. In other words, the minimum power required will lie on the straight line AB which is so drawn that the power and the speed at point B have twice

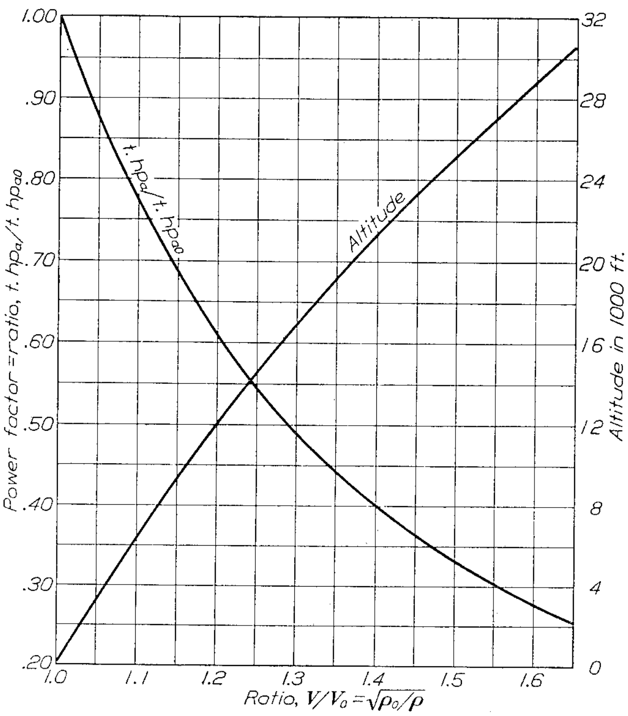

FIGURE 2.—Absolute ceiling chart

their values at point A. The multiple 2 is merely a matter of convenience; any desired factor may be used.

The curve CEG on Figure 1 is the corresponding variation in power available, determined as follows. When $\sqrt{\rho_o/\rho}=\frac{V}{V_o}=1.10$ the minimum power required is at the point F. At this speed at sea level the power available is at D. Now $\sqrt{\frac{\rho_o}{\rho}}=1.10$ corresponds to an altitude of 6,368 feet at which the power of an average engine is 77.6 per cent of its sea level value, as shown on Figure 2 or Table 1. Multiplying the power at D by the power factor 0.776 gives the point E, which is the power available, corresponding to the minimum power required at F. This process is repeated at several values of $\sqrt{\rho_o/\rho}=\frac{V}{V_o}$ and a curve CEG is drawn through the points so determined. The intersection of this curve with the line AB at the point G gives the value of the speed V at the absolute ceiling. The ratio of this speed to the speed for minimum power required at sea level gives $\sqrt{\frac{\rho_o}{\rho}}=\frac{V}{V_o}$ from which the absolute ceiling is determined. In the example shown on Figure 1, the speed at absolute ceiling, given by the intersection at G is $V=91$ m. p. h. The sea level speed for minimum power at point A is $V_o=70$ m. p. h. Hence $\frac{V}{V_o}=\frac{91.0}{70.0}=1.30$ and from Figure 2 the altitude corresponding to $\sqrt{\frac{\rho_o}{\rho}}=\frac{V}{V_o}=1.30$ is 16,864 ft.

These calculations are simplified by the use of a chart such as Figure 2, on which power factors and altitudes are plotted against $\sqrt{\frac{\rho_o}{\rho}}=\frac{V}{V_o}$. This power factor is the ratio of the thrust power at altitude to the thrust power at sea level at constant true air speed. It includes any effect due to change in r. p. m. with altitude at constant air speed.

The particular curve of power factors shown on Figure 2 applies to the average airplane engine without supercharging. A similar curve may be obtained for any particular engine from the performance in a full throttle climb at constant true air speed by the use of the method described in Chapter VI of Reference 1.

DERIVATION OF THE CEILING CHART

A study of the method just outlined has led to the development of a chart which greatly reduces the calculations required for the determination of absolute ceiling. This chart also provides a most satisfactory method of estimating absolute ceiling. The diagrammatic sketch, Figure 3, shows the curves on which the chart is based. The minimum power required and the

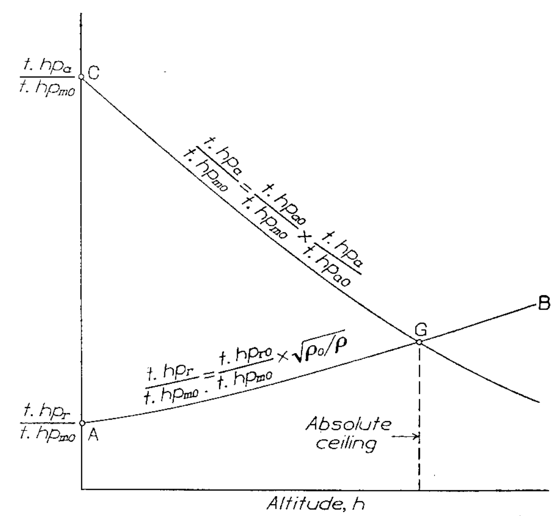

FIGURE 3

power available expressed as ratios of the maximum sea level thrust power available are plotted against altitude. The curves AB and CG on Figure 3 are equivalent to the similarly lettered curves on Figure 1. The ratio of the powers at C and H on Figure 1 determines the point C on Figure 3. In a similar manner the ratio of the powers at A and H on Figure 1 determines the point A on Figure 3. Each point on the two curves is in terms of the maximum thrust horsepower available and the plot is against altitude directly instead of speed as in Figure 1. It is assumed, as is generally the case, that the maximum thrust horsepower available is that at maximum speed.

This method of plotting may be expanded into a set of general curves by the use of the general curve of t.hp. given on Figure 4. An analysis of the power curves for a great number of airplanes shows that the average air speed V at which the curves of power available and power required are tangent at the absolute ceiling is 17 per cent greater than the stalling speed, V_s, that is, $V_o=1.17V_s$, or $V=1.17V_s\sqrt{\frac{\rho_o}{\rho}}$. Consequently for any given ratio of stalling speed, V_s, to maximum speed, V_m, the ratio at sea level of $t.hp_{ao}$ to $t.hp_{mo}$ may be read directly from Figure 4, at the point $1.17\frac{V_s}{V_m}$. As the altitude is increased, this speed becomes $1.17\frac{V_s}{V_m}\sqrt{\frac{\rho_o}{\rho}}$ or $1.17\frac{V_s}{V_m}\times\frac{V}{V_o}$, and the corresponding sea level value of the ratio $t.hp_{ao}/t.hp_{mo}$ increases accordingly. For any given altitude, or corresponding value of $\sqrt{\frac{\rho_o}{\rho}}$, the value of $t.hp_a/t.hp_{mo}$ is obtained by multiplying $t.hp_{ao}/t.hp_{mo}$ by the power factor for the altitude in question. The calculations for $\frac{V_m}{V_s}=2.4$ or $\frac{V_s}{V_m}=0.4167$ are given in Table I to illustrate the method.

factors, or variation of power with altitude as given in column 5 of Table I or Figure 2, gives the values of $t.hp_a/t.hp_{mo}$ in Table III.

The calculation of $t.hp_r/t.hp_{mo}$ is less complicated. Any initial value of $t.hp_{ro}/t.hp_{mo}$ is multiplied by $\sqrt{\frac{\rho_o}{\rho}}$ to obtain $t.hp_r/t.hp_{mo}$. Any convenient series

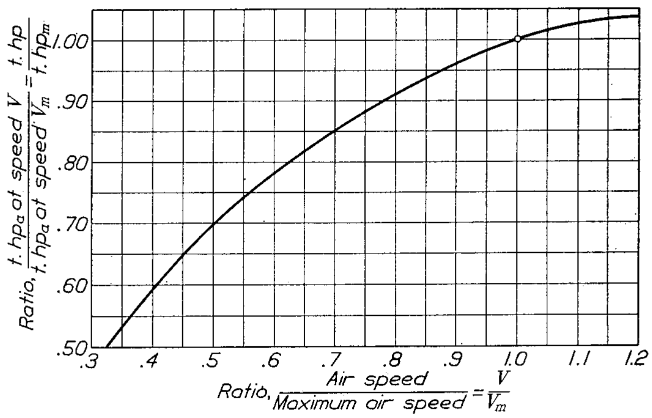

FIGURE 4.—General curve for thrust horsepower against air speed

of initial values, such as those used in Table IV, may be used.

The data from Tables III and IV may now be plotted as in Figure 5. The absolute ceiling is determined by the intersection of given curves of $t.hp_r/t.hp_{mo}$ and $t.hp_a/t.hp_{mo}$. Since the latter is a function of V_m/V_s, all that is required to determine the absolute

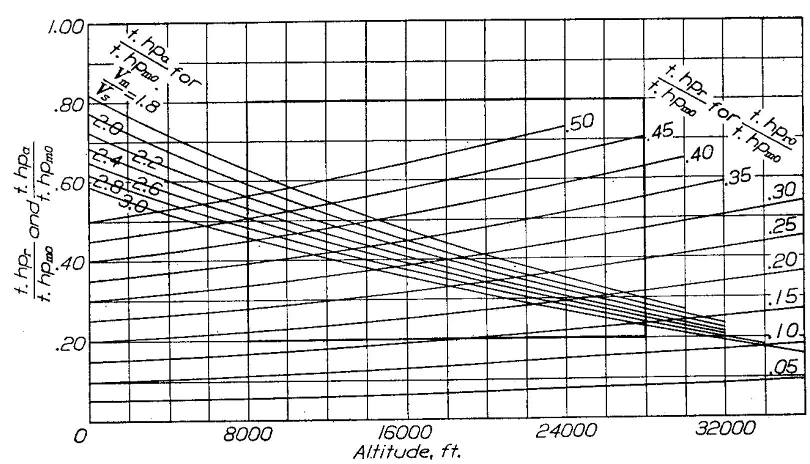

FIGURE 5.—Absolute ceiling chart

These calculations can be carried through for a series of values of V_m/V_s giving the values of $t.hp_{ao}/t.hp_{mo}$ in Table II. If these values in Table II are multiplied by the power factors at each altitude, one obtains the values of $t.hp_a/t.hp_{mo}$; that is, the power available at altitude in terms of the maximum power available at sea level. The values in Table II apply to any engine not operated at the "peak" of its power curve. Consequently, they may be used to construct a ceiling chart for any engine having a known variation of power with altitude. Using the average power

ceiling is the maximum speed, minimum speed, minimum horsepower required, and maximum horsepower available.

Since the absolute ceiling normally falls between 8,000 feet and 28,000 feet, the section indicated by the rectangle on Figure 5 has been redrawn to a larger scale in Figure 6 and additional curves interpolated for convenience in reading.

To use the chart find the sea level values of the ratio of maximum speed to minimum speed V_m/V_s and the ratio of minimum thrust horsepower required to maximum thrust horsepower available (min. t.hp$_{ro}$/max. t.hp$_{ao}$). The intersection of the two curves corresponding to these values determines the absolute ceiling.

For example; refer to Figure 1.

Maximum speed, $V_m=132.3$,

Stalling speed, $V_s=60.0$ $\quad \therefore \frac{V_m}{V_s}=2.205$,

Minimum t.hp required, t.hp$_{ro}=208$,

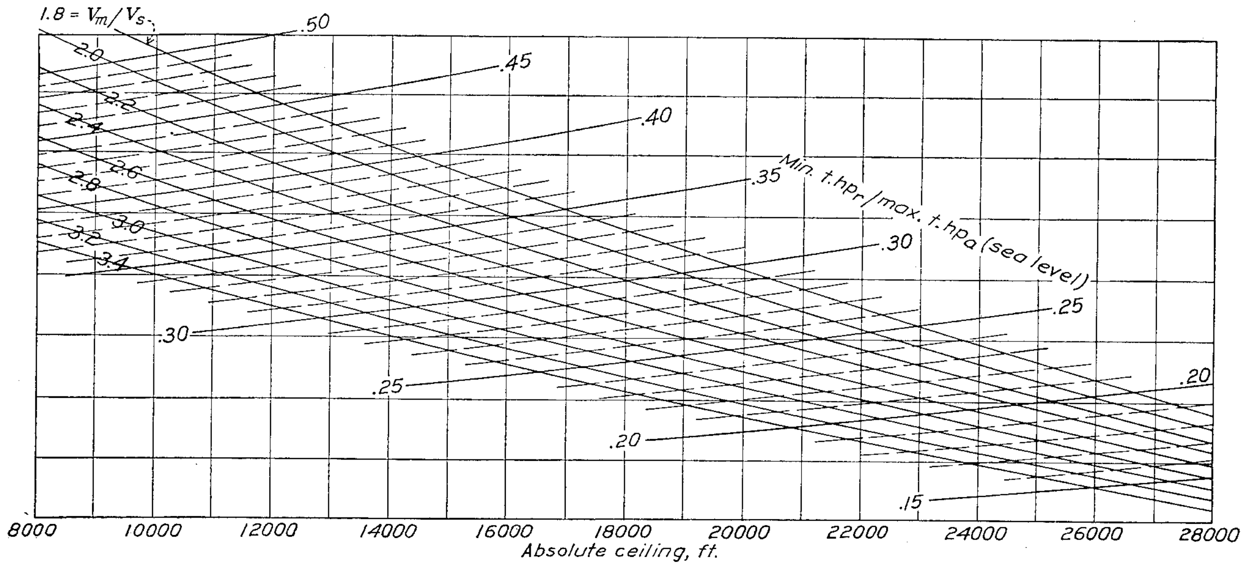

FIGURE 6.—Absolute ceiling chart

Maximum t.hp available, t.hp$_{ao}=657$,

Minimum t.hp$_{ro}$/max. t.hp$_{ao}=208/657=0.317$.

From Figure 6 for $\frac{V_m}{V_s}=2.205$ and (min. t.hp$_{ro}$/max. t.hp$_{ao}$)$=0.317$, the absolute ceiling is found to be 16,900 feet. This value checks very closely with that

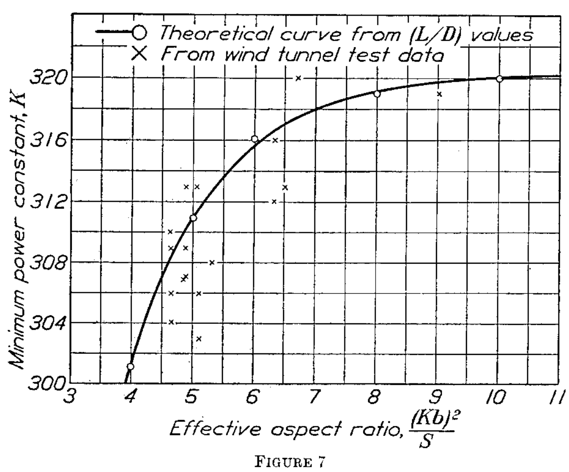

FIGURE 7

found from the ratio of $\sqrt{\frac{\rho_o}{\rho}}$ given by the construction on Figure 1.

USE OF CHART FOR ESTIMATING ABSOLUTE CEILING

Absolute ceiling is determined on Figure 6 by the ratios of $\frac{V_m}{V_s}$ and t.hp$_{ro}$/t.hp$_{mo}$. Since these ratios may be estimated with reasonable accuracy, the chart may be used to estimate absolute ceiling.

It has been shown (Reference 5) that the ratio of maximum speed V_m to stalling speed V_s, is given by

$$\frac{V_m}{V_s}=\frac{K\left(\eta_m\cdot\frac{L}{D}\right)^{1/3}}{\left(V_s\cdot\frac{W}{b.hp}\right)^{1/3}} \tag{1}$$

where η_m is the maximum propeller efficiency L/D the maximum ratio of lift to drag, W the gross weight in pounds, b.hp the maximum brake horsepower, V_s the stalling speed, and K an approximately constant coefficient having an average value of 10.2. The variation of K and max. (L/D) with aspect ratio and parasite is given in Reference 1.

The horsepower required for horizontal flight is

$$\text{t.hp}_r=\frac{W.V_c}{375\left(\frac{L}{D}\right)} \tag{2}$$

Where V_c is the air speed in m. p. h. and (L/D) is the ratio of lift to drag corresponding to the speed V_c. To determine the minimum value of t.hp$_r$ by estimation, it is desirable that relations be found between V_s and V_c and between (L/D) and the maximum value of (L/D). This can be done either directly or indirectly. A direct method is to write equation (2) in the form

$$\text{t.hp}_{ro}=\frac{W\times K_1V_s}{375\times K_2\left(\frac{L}{D}\right)_{\text{max.}}}=\frac{WV_s}{k\left(\frac{L}{D}\right)_{\text{max.}}} \tag{3}$$

and determine the value of k from calculated power curves. An indirect method is to determine the relation between $(L/D)_{\text{max.}}$ and the value of (L/D) at $V_c=1.17\ V_s$. Both of these methods have been employed. k shows a small but definite variation with effective aspect ratio as indicated on Figure 7.

With the direct method, values of k were determined from wind tunnel test data for about 20 airplanes. With the indirect method, the calculated variation of L/D with speed range and aspect ratio, gives a series of points which determine the curve shown on Figure 7. This curve may be used in estimates where the extra accuracy is required, but for all practical purposes it is sufficient to assume $k=310$.

Since the maximum thrust horsepower is

$$\text{t.hp}_{mo}=\eta_m \text{ b.hp} \tag{4}$$

it follows that

$$\frac{\text{t.hp}_{ro}}{\text{t.hp}_{mo}}=\frac{\left(\frac{W}{\text{b.hp}}\right)\cdot V_s}{k\eta_m\left(\frac{L}{D}\right)_m} \tag{5}$$

With equations 1 and 5, the ceiling chart, Figure 6, may be used to estimate absolute ceiling. This method has proved highly satisfactory for routine estimation.

As an approximate method for airplanes with supercharged engines the initial conditions may be taken at the critical altitude and the calculations carried through as for a normal engine. The absolute ceiling so obtained is the altitude referred to the critical altitude as zero, and the true absolute ceiling is obtained by adding the critical altitude. For example, if the critical altitude is 15,000 feet and upon calculations based on initial conditions at 15,000 feet (i. e., speeds, power required, and power available) the ceiling is found to be 20,000 feet, this indicates that the airplane can climb to an altitude 20,000 feet higher than the critical altitude for the engine, or that the absolute ceiling is 35,000 feet. Comparisons with the limited test data available for supercharged engines indicates that this approximation gives satisfactory results.

Bureau of Aeronautics,
Navy Department, *July, 1930*.

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Table I.—*Example of calculation of* $\text{t.hp}_a/\text{t.hp}_{mo}$

$\frac{V_m}{V_s}=2.40$ $\frac{V_s}{V_m}=0.4167$

$\frac{V}{V_o}=\sqrt{\frac{\rho_o}{\rho}}$	Altitude, h feet	$\frac{V}{V_m}=1\cdot17\ \frac{V_s}{V_m}\cdot\frac{V}{V_o}$	$\frac{\text{t.hp}_{ao}}{\text{t.hp}_{mo}}$	Power factor F	$\frac{\text{t.hp}_a}{\text{t.hp}_{mo}}$ $\frac{\text{t.hp}_{ao}}{\text{t.hp}_{mo}}\times F$
1.00	0	0.488	0.685	1.000	0.685
1.05	3,296	.512	.708	.876	.620
1.10	6,368	.536	.730	.776	.567
1.15	9,242	.561	.751	.688	.517
1.20	11,938	.585	.771	.612	.472
1.25	14,474	.609	.789	.546	.431
1.30	16,864	.634	.807	.490	.395
1.35	19,124	.658	.825	.443	.366
1.40	21,263	.682	.842	.402	.338
1.50	25,228	.731	.872	.332	.289
1.60	28,812	.780	.893	.278	.248

Table II.—*Calculated values of* $\frac{\text{t.hp}_{ao}}{\text{t.hp}_{mo}}$

$\frac{V}{V_o}=\sqrt{\frac{\rho_o}{\rho}}$	Altitude, h feet	$\text{t.hp}_{ao}/\text{t.hp}_{mo}$								
		$\frac{V_m}{V_s}=1.8$	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4
1.00	0	0.818	0.771	0.726	0.685	0.649	0.616	0.585	0.558	0.531
1.05	3,296	.842	.792	.749	.708	.672	.638	.606	.577	.552
1.10	6,368	.862	.813	.771	.730	.697	.658	.628	.598	.574
1.15	9,242	.881	.835	.792	.751	.713	.679	.647	.617	.593
1.20	11,938	.893	.853	.810	.771	.733	.700	.667	.638	.613
1.25	14,474	.916	.872	.829	.789	.752	.718	.686	.658	.632
1.30	16,864	.932	.889	.847	.807	.771	.736	.704	.677	.651
1.35	19,124	.948	.903	.864	.825	.788	.758	.722	.694	.669
1.40	21,263	.963	.919	.879	.842	.804	.771	.738	.710	.685
1.50	25,228	------	.948	.908	.872	.836	.802	.771	.740	.715
1.60	28,812	------	------	.936	.893	.865	.832	.800	.765	.740

Table III.—*Variation of* $\text{t.hp}_a/\text{t.hp}_{mo}$ *with altitude and speed ratio*

$\frac{V}{V_o}=\sqrt{\frac{\rho_o}{\rho}}$	Altitude h feet	$\text{t.hp}_a/\text{t.hp}_{mo}$								
		$\frac{V_m}{V_s}=1.8$	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4
1.00	0	0.818	0.771	0.726	0.685	0.649	0.616	0.585	0.558	0.531
1.05	3,296	.737	.693	.656	.620	.588	.559	.530	.506	.484
1.10	6,368	.668	.632	.599	.567	.542	.511	.488	.465	.446
1.15	9,242	.606	.575	.545	.517	.491	.468	.445	.425	.408
1.20	11,938	.546	.522	.495	.472	.448	.428	.408	.391	.376
1.25	14,474	.500	.476	.453	.431	.411	.392	.374	.359	.344
1.30	16,864	.456	.436	.415	.395	.378	.361	.345	.332	.320
1.35	19,124	.421	.401	.383	.366	.350	.336	.320	.308	.297
1.40	21,263	.387	.369	.353	.338	.323	.310	.297	.285	.275
1.50	25,228	------	.315	.301	.289	.277	.266	.254	.245	.236
1.60	28,812	------	------	.260	.248	.240	.231	.223	.215	.209

Note.—These values of $\text{t.hp}_a/\text{t.hp}_{mo}$ are based on the power factors given in column 5 of Table I.

Table IV.—*Variation of* $\text{t.hp}_r/\text{t.hp}_{mo}$ *with altitude*

$\frac{V}{V_o}=\sqrt{\frac{\rho_o}{\rho}}$	Altitude h feet	Values of $\text{t.hp}_r/\text{t.hp}_{mo}$								
1.00	0	0.500	0.450	0.400	0.350	0.300	0.250	0.200	0.150	0.100
1.05	3,296	.525	.472	.420	.368	.315	.262	.210	.157	.105
1.10	6,368	.550	.495	.440	.385	.330	.275	.220	.165	.110
1.15	9,242	.575	.518	.460	.402	.345	.288	.230	.172	.115
1.20	11,938	.600	.540	.480	.420	.360	.300	.240	.180	.120
1.25	14,474	.625	.562	.500	.438	.375	.312	.250	.187	.125
1.30	16,864	.650	.545	.520	.455	.390	.325	.260	.195	.130
1.35	19,124	.675	.608	.540	.472	.405	.338	.270	.202	.135
1.40	21,263	.700	.630	.560	.490	.420	.350	.280	.210	.140
1.50	25,228	.750	.675	.600	.525	.450	.375	.300	.225	.150
1.60	28,812	------	------	.640	.560	.480	.400	.320	.240	.160

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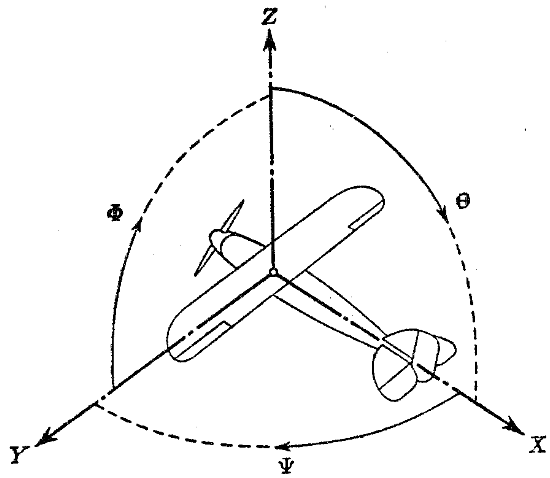

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	rolling	L	$Y \longrightarrow Z$	roll	Φ	u	p
Lateral	Y	Y	pitching	M	$Z \longrightarrow X$	pitch	Θ	v	q
Normal	Z	Z	yawing	N	$X \longrightarrow Y$	yaw	Ψ	w	r

Absolute coefficients of moment

$$C_L=\frac{L}{qbS} \qquad C_M=\frac{M}{qcS} \qquad C_N=\frac{N}{qfS}$$

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter.
p_e, Effective pitch.
p_g, Mean geometric pitch.
p_s, Standard pitch.
p_v, Zero thrust.
p_a, Zero torque.
p/D, Pitch ratio.
V', Inflow velocity.
V_s, Slip stream velocity.

T, Thrust.
Q, Torque.
P, Power.
(If "coefficients" are introduced all units used must be consistent.)
η, Efficiency $=T\,V/P$.
n, Revolutions per sec., r. p. s.
N, Revolutions per minute, r. p. m.
Φ, Effective helix angle $=\tan^{-1}\left(\frac{V}{2\pi rn}\right)$

5. NUMERICAL RELATIONS

1 hp = 76.04 kg/m/s = 550 lb./ft./sec.
1 kg/m/s = 0.01315 hp
1 mi./hr. = 0.44704 m/s
1 m/s = 2.23693 mi./hr.

1 lb. = 0.4535924277 kg
1 kg = 2.2046224 lb.
1 mi. = 1609.35 m = 5280 ft.
1 m = 3.2808333 ft.