(12)

INCREASING THE EFFICIENCY OF SIMULATED ANNEALING SEARCH BY LEARNING TO RECOGNIZE (UN)PROMISING RUNS *

Yoichiro Nakakuki †and Norman M. Sadeh

CMU-RI-TR-94-30

The Robotics Institute
Carnegie Mellon University
Pittsburgh, PA 15213-3891

September 1994

*This research was supported, in part, by the Advanced Research Projects Agency under contract F30602-91-C-0016 and, in part, by an industrial grant from NEC. This paper appears in the proceedings of the Twelfth National Conference on Artificial Intelligence (AAAI-94), pp. 1316-1322.

†This research was carried out while the first author was a visiting scientist at Carnegie Mellon's Robotics Institute. His current address is: NEC C&C Research Laboratories, 4-1-1 Miyazaki, Miyamae-ku, Kawasaki 216, JAPAN.

Abstract

Simulated Annealing (SA) procedures can potentially yield near-optimal solutions to many difficult combinatorial optimization problems, though often at the expense of intensive computational efforts. The single most significant source of inefficiency in SA search is the inherent stochasticity of the procedure, typically requiring that the procedure be rerun a large number of times before a near-optimal solution is found. This paper describes a mechanism that attempts to learn the structure of the search space over multiple SA runs on a given problem. Specifically, probability distributions are dynamically updated over multiple runs to estimate at different checkpoints how promising a SA run appears to be. Based on this mechanism, two types of criteria are developed that aim at increasing search efficiency: (1) a *cutoff criterion* used to determine when to abandon unpromising runs and (2) *restart criteria* used to determine whether to start a fresh SA run or restart search in the middle of an earlier run. Experimental results obtained on a class of complex job shop scheduling problems show (1) that SA can produce high quality solutions for this class of problems, if run a large number of times, and (2) that our learning mechanism can significantly reduce the computation time required to find high quality solutions to these problems. The results further indicate that, the closer one wants to be to the optimum, the larger the speedups.

1 Introduction

Simulated Annealing (SA) is a general-purpose search procedure that generalizes iterative improvement approaches to combinatorial optimization by sometimes accepting transitions to lower quality solutions to avoid getting trapped in local minima [8, 1]. SA procedures have been successfully applied to a variety of combinatorial optimization problems, including Traveling Salesman Problems [1], Graph Partitioning Problems [6], Graph Coloring Problems [7], Vehicle Routing Problems [14], Design of Integrated Circuits, Minimum Makespan Scheduling Problems [9, 13, 19] as well as other complex scheduling problems [23], often producing near-optimal solutions, though at the expense of intensive computational efforts.

The single most significant source of inefficiency in SA search is the inherent stochasticity of the procedure, typically requiring that the procedure be rerun a large number of times before a near-optimal solution is found. Glover et al. developed a set of "Tabu" mechanisms that attempt to increase the efficiency of SA and other neighborhood search procedures by maintaining a selective history of search states encountered earlier during the *same run* [4]. This history is then used to dynamically derive "tabu restrictions" or "aspirations", that guide search, preventing it, for instance, from revisiting areas of the search space it just explored. This paper describes a complementary mechanism that attempts to learn the structure of the search space *over multiple runs* of SA on a given problem. Specifically, we introduce a mechanism that attempts to predict how (un)promising a SA run is likely to be, based on probability distributions that are refined ("learned") over multiple runs. The distributions, which are built at different checkpoints, each corresponding to a different value of the temperature parameter used in the procedure, approximate the cost reductions that one can expect if the SA run is continued below these temperatures. Two types of criteria are developed that aim at increasing search efficiency by exploiting these distributions:

- *A Cutoff Criterion*: This criterion is used to detect runs that are unlikely to result in an improvement of the best solution found so far and, hence, should be abandoned;
- *Restart Criteria*: When completing a run or abandoning an unpromising one, these criteria help determine whether to start a fresh SA run or restart search in the middle of an earlier promising run.

The techniques presented in this paper have been applied to a class of complex job shop scheduling problems first described in [18]. Problems in this class require scheduling a set of jobs that each need to be completed by a possibly different due date. The objective is to minimize the sum of tardiness and inventory costs incurred by all the jobs. This class of problems is known to be NP-complete and is representative of a large number of actual scheduling problems, including Just-In-Time factory scheduling problems [18, 17]. Experimental results indicate (1) that SA can produce high quality solutions for this class of problems, if run a large number of times, and (2) that our learning mechanism can yield significant reductions in computation time. The results further indicate that, the closer one wants to be to the optimum, the larger the speedups.

The balance of this paper is organized as follows. Section 2 quickly reviews fundamentals of SA search. Section 3 analyzes the behavior of typical SA runs and introduces a mechanism that aims at learning to recognize (un)promising runs on a given problem, using the concept of *Expected Cost Improvement Distributions* ($ECID$). In Section 4, we use $ECID$ distributions to develop a *cutoff criterion* to determine when to abandon unpromising runs. Section 5 presents three *restart criteria* based $ECID$ distributions. Experiments obtained on a set of benchmark job shop scheduling problems with tardiness and inventory costs are reported in Section 6. A summary is provided in Section 7.

2 Simulated Annealing Search

Figure 1 outlines the main steps of a SA procedure designed to find a solution $x \in S$ that minimizes a real-valued function, $cost(x)$. The procedure starts from an initial solution x_0 (randomly drawn from S) and iteratively moves to other neighboring solutions, as determined by a neighborhood function, $neighbor(x)$, while remembering the best solution found so far (denoted by s). Typically, the procedure only moves to neighboring solutions that are better than the current one. However, the probability of moving from a solution x to an inferior solution x' is greater than zero, thereby allowing the procedure to escape from local minima. $rand()$ is a function that randomly draws a number from a uniform distribution on the interval $[0, 1]$. The so-called temperature, T, of the procedure is a parameter controlling the probability of accepting a transition to a lower quality solution. It is initially set at a high value, T_0, thereby

frequently allowing such transitions. If, after N iterations, the best solution found by the procedure has not improved, the temperature parameter T is decremented by a factor α $(0 < \alpha < 1)$. One motivation for progressively lowering the temperature is to obtain convergence. Additionally, as the procedure slowly moves towards globally better solutions, accepting transitions to lower quality solutions becomes increasingly less attractive. When the temperature drops below a preset level T_1, the procedure stops and s is returned (not shown in Figure 1).

```
T = T0; x = x0 (∈ S); min = ∞;
while (T > T1)  {
      for i = 1, N  {
            x' = neighbor(x);
            if (cost(x') < cost(x))  x = x';
            else if (rand() < exp{(cost(x) − cost(x'))/T})  x = x';
            if(cost(x) < min)  min = cost(x), s = x;
      }
      if (Min was not modified in the above loop)  T = T * α;
}
```

Fig. 1 Basic Simulated Annealing Procedure.

Fig. 2 depicts the cost distribution of the best solutions returned by 300 SA runs on a typical combinatorial optimization problem – a job shop scheduling problem from a set of benchmarks to be described in Section 6.

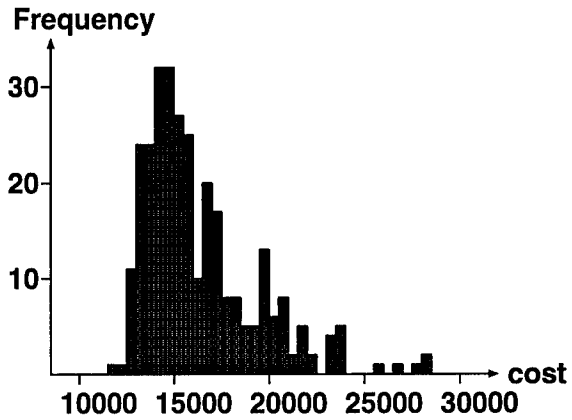

Fig. 2 Cost Distribution of the Best Solutions Found by 300 SA Runs.

The optimal solution for this problem is believed to have a cost around $11,500$ – the value in itself is of no importance here. Figure 2 indicates that, if run a large

number of times, SA is likely to eventually find an optimal solution to this problem. It also shows that, in many runs, SA gets trapped in local minima with costs much higher than the global minimum. For instance, 60% of the runs produce solutions with a cost at least 30% above the global minimum. This suggests that, if rather than completing all these unsuccessful runs, one could somehow predict when a run is likely to lead to a highly sub-optimal solution and abandon it, the efficiency of SA could be greatly enhanced. The following section further analyzes the behavior of typical SA runs and proposes a mechanism which, given a problem, aims at learning to recognize (un)promising SA runs.

3 Learning To Recognize (Un)promising SA Runs

Figure 3 depicts the behavior of a SA procedure on two different scheduling problems (from the set of benchmarks used in Section 6). For each problem, the figure depicts five SA runs, plotting the cost of the best solution, s, as the temperature of the procedure is progressively lowered – temperatures are shown in log scale, which is almost equivalent to computation time in linear scale. SA behaves very differently on these two problems. For instance, in Problem #1, the range of final solutions is relatively narrow, while in Problem #2 it is much wider. Another differentiating factor is the behavior of the procedure at low temperatures. It seems that for Problem #1, the quality of a run can already be estimated quite accurately at $T = 50$ (e.g. the best run at $T = 50$ remains best at lower temperatures), while this is less so for Problem #2.

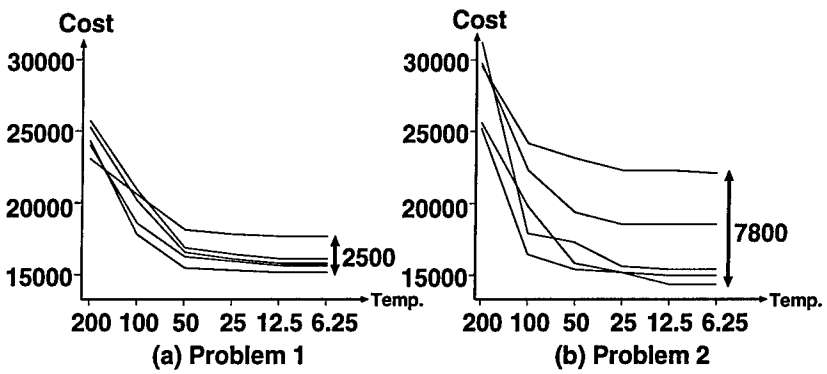

Fig. 3 Cost reductions in five SA runs on two different problems.

Clearly, such properties are not intrinsic to a problem itself. They could change if a different neighborhood structure or a different cooling profile was selected, as these parameters can affect the types of local optima encountered by the procedure and the chance that the procedure extricates itself from these local optima below a given temperature. While, in general, it may be impossible to find a SA procedure that reliably

converges to near-optimal solutions on a wide class of problems, we can try to design adaptive SA procedures which, given a problem, can learn to recognize (un)promising runs and improve their performance over time. Below, we present a mechanism, which, given a problem, attempts to "learn" at different checkpoint temperatures the distribution of cost improvements that one can hope to achieve by continuing search below these temperatures.

Specifically, we postulate that, given a problem and a checkpoint temperature $T = t$, the distribution of the cost improvement that is likely to be achieved by continuing a run below t can be approximated by a normal distribution. Using performance data gathered over earlier runs on a same problem, it is possible to approximate these *Expected Cost Improvement Distributions* ($ECID$) for a set C of checkpoint temperatures and use these distributions to identify (un)promising runs.

Formally, given a combinatorial optimization problem and a SA procedure for that problem, we define c_i^t as the cost of the best solution, s, at check point t in the i-th run and c_i^0 as the cost of the best solution obtained at temperature $T = T_1$ in the i-th execution. When the $(n+1)$-st run reaches a checkpoint temperature t, the $ECID$ below t is approximated as a normal distribution $N[\mu_n^t, \sigma_n^t]$, whose average, μ_n^t, and standard deviation, σ_n^t, are given by:

$$\mu_n^t = \frac{\sum_{i=1}^{n}(c_i^t - c_i^0)}{n}, \quad \sigma_n^t = \sqrt{\frac{\sum_{i=1}^{n}\{(c_i^t - c_i^0) - \mu_n^t\}^2}{n-1}}$$

By incrementally refining these estimators over multiple runs, this mechanism can in essence "learn" to recognize (un)promising SA runs. The following sections successively describe a cutoff criterion and three restart criteria based on $ECID$ distributions.

4 A Cutoff Criterion

Suppose that, in a sixth run on Problem #1, the best solution obtained at checkpoint $T = 100$ is solution **A** – Figure 4(a). At this checkpoint, the distribution of c_6^0 – the cost of the best solution that will have been found if the run is completed – can be approximated by the normal distribution $N[c_6^{100} - \mu_5^{100},\ \sigma_5^{100}]$. This distribution, represented in Fig. 4(a), suggests that, if continued, the current run has a good chance of improving the current best solution, **x**. Suppose that based on this analysis, the run continues until the next checkpoint, $T = 50$, and that the best solution found by the run when it reaches that temperature is **A'**. At this point, a new distribution of c_6^0 can be computed to check how the run is doing. This distribution, $N[c_6^{50} - \mu_5^{50},\ \sigma_5^{50}]$ is shown in Figure 4(b). It appears much less promising than the one at $T = 100$. Now, the chances of improving the current best solution, **x**, appear remote: it probably does

not make sense to continue this run.

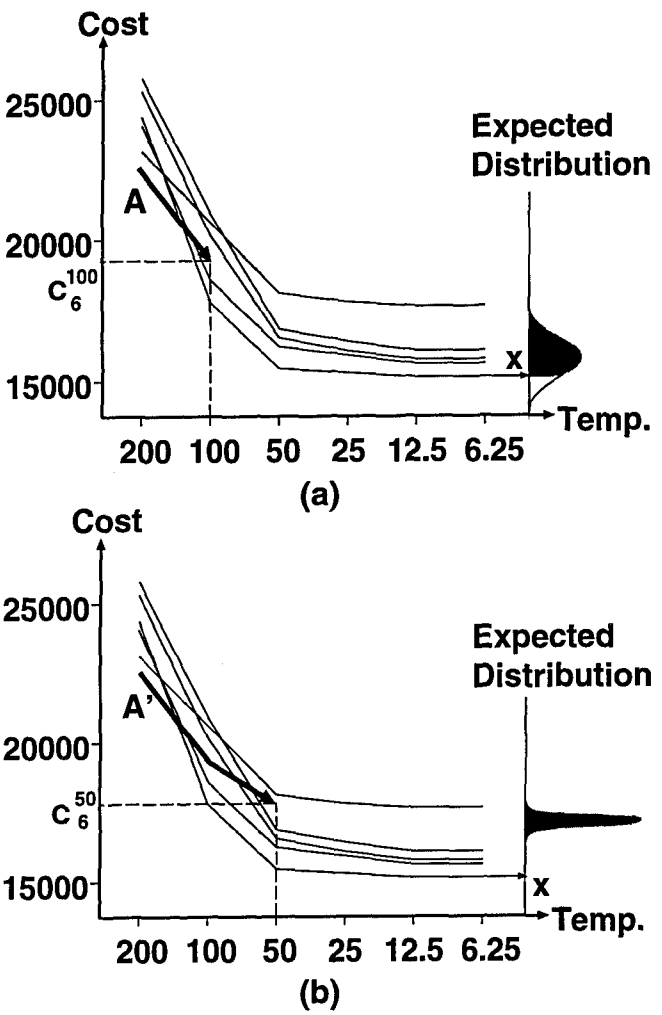

Fig. 4 Expected Cost Improvement Distributions at T=100 and T=50.

Formally, when the $(n+1)$-st run reaches a checkpoint temperature t, a cutoff criterion is used to determine whether or not to continue this run. In the study reported in Section 6, we use a cutoff criterion of the form:

$$\frac{(c_{n+1}^t - \mu_n^t) - x_n}{\sigma_n^t} > threshold$$

where x_n is the cost of the best solution found during the previous n runs and *threshold* is a threshold value. If the inequality holds, the current run is abandoned. For example, if $threshold = 3$ (the value used in our experiments) and the cutoff inequality holds at a given checkpoint temperature t, the probability of improving x_n by continuing the run below t is expected to be less than 1% [2].

5 Three Restart Criteria

Whenever a run is completed or abandoned, two options are available: either start a fresh new annealing run *or*, instead, restart an earlier (promising) run, using a different sequence of random numbers ("reannealing"). In total, if reannealing is constrained to start from one of the checkpoint temperatures, there are up to $n \cdot |C| + 1$ possible options, where n is the number of earlier runs and $|C|$ the number of checkpoints in set C. Below, we describe three "restart criteria" that aim at selecting among these options so as to maximize the chances of quickly converging to a near-optimal solution.

5.1 Maximum Cost Reduction Rate Criterion

When considering several points from which to restart search, two factors need to be taken into account: (1) the likelihood that restarting search from a given point will lead to an improvement of the current best solution and (2) the time that it will take to complete a run from that point. Restarting from a low temperature will generally bring about moderate solution improvements, if any, while requiring little CPU time. Starting fresh new runs or restarting from higher temperatures can lead to more significant improvements, though less consistently and at the cost of more CPU time. In general, the cost improvements that can be expected from different temperatures will vary from one problem to another, as illustrated in Figure 3 (and as formalized by $ECID$ distributions).

A natural restart criterion is one that picks the restart point expected to maximize the rate at which the cost of the current best solution will improve. For each restart candidate O_k (fresh annealing or reannealing), this can be approximated as the expected cost reduction (in the best solution), if search is restarted from O_k, divided by the expected CPU time required to complete a run from that restart point. Below, we use $R(O_k)$ to denote this approximation of the expected cost reduction rate, if search is restarted from O_k:

$$R(O_k) = \frac{expected\text{-}reduction(O_k)}{expected\text{-}CPU(O_k)}$$

where $expected\text{-}reduction(O_k)$ is the expected cost reduction at the end of a run starting from O_k and $expected\text{-}CPU(O_k)$ is the CPU time that this run is expected to require. $expected\text{-}CPU(O_k)$ can be approximated as the average time required to complete earier runs from O_k's temperature. $expected\text{-}reduction(O_k)$ can be evaluated using $ECID$ distributions, as detailed below.

Given a reannealing point O_k at checkpoint temperature t and n earlier SA runs completed from t or above, $expected\text{-}reduction(O_k)$ can be approximated as:

$$expected\text{-}reduction(O_k) = \int_{LB}^{x_n} \{P_{nk}^t(x) \cdot (x_n - x)\} dx$$

where $P_{nk}^t(x)$ is the density function of the normal distribution $N[c_k - \mu_n^t, \sigma_n^t]$, c_k is the

cost of O_k's best solution[1], x_n is the cost of the best solution obtained over the first n runs, and LB is a lower-bound on the optimal solution[2]

Similarly, if O_k is a fresh SA run, $expected\text{-}reduction(O_k)$ can be approximated as:

$$expected\text{-}reduction(O_k) = \int_{LB}^{x_n} \{P_n(x) \cdot (x_n - x)\} dx$$

where $P_n(x)$ is the density function of the normal distribution $N[\mu_n^0, \sigma_n^0]$, with

$$\mu_n^0 = \frac{\sum_{i=1}^{n} c_i^0}{n}, \quad \sigma_n^0 = \sqrt{\frac{\sum_{i=1}^{n} \{c_i^0 - \mu_n^0\}^2}{n-1}}.$$

5.2 Randomized Criterion

One possible problem with the above criterion is its high sensitivity to possible inaccuracies in approximations of $ECID$ distributions. This can be a problem when the number of earlier runs is still small. When inaccurate $ECID$ distributions lead the criterion to choose a poor restart point, the procedure may take a long time before it improves the quality of the current best solution. In the meantime, it may keep on coming back to the same poor restart point. For this reason, it is tempting to use a randomized version of the criterion. One such variation involves randomly picking from a set of promising restart points, $H = \{O_l | R(O_l) > \beta \cdot Max\{R(O_k)\}\}$, while assuming that each element in H has the same probability, $1/|H|$, of being selected. β is a constant whose value is between 0 and 1.

5.3 Hybrid Criterion

A third alternative involves keeping some level of stochasticity in the restart criterion, while ensuring that more promising restart points have a higher chance of being selected. This is done by selecting restart points in H according to a Boltzmann distribution that assigns to each element $O_l \in H$ a probability

$$p(O_l) = \frac{exp(R(O_l)/\tau)}{\sum_{O_k \in H} exp(R(O_k)/\tau))}$$

Here, τ is a positive constant. If τ is very large, this method becomes equivalent to the randomized criterion described in subsection 5.2. If$\tau \approx 0$, this criterion becomes similar to the criterion of subsection 5.1. A similar distribution is used in the Q-learning algorithm described in [21].

[1]To be consistent, if O_k correponds to the i-th SA run, $c_k = c_i^t$, as defined in Section 3.

[2]In the experiments reported in this paper, LB was simply set to 0.

6 Performance Evaluation

6.1 The Job Shop Scheduling Problem with Tardiness and Inventory Costs

To evaluate performance of our cutoff and restart criteria, we consider a set of complex job shop scheduling problems first introduced in [18]. The problems assume a factory, in which a set of jobs, $J = \{j_1, j_2, \cdots, j_n\}$, has to be scheduled on a set of resources, $RES = \{R_1, R_2, \cdots, R_m\}$. Each job requires performing a set of operations $O^l = \{O^l_1, O^l_2, \cdots O^l_{n_l}\}$ and, ideally, should be completed by a given due date, dd_l, for delivery to a customer. Precedence constraints specify a complete order in which operations in each job have to be performed. By convention, it is assumed that operation O^l_i has to be completed before operation O^l_{i+1} can start ($i = 1, 2, \cdots, n_l - 1$). Each operation O^l_i has a deterministic duration du^l_i and requires a resource $R^l_i \in RES$. Resources cannot be assigned to more than one operation at a time. The problem is to find a feasible schedule that minimizes the sum of tardiness and inventory costs of all the jobs ("Just-In-Time" objective). This problem is known to be NP-complete [18] and is representative of a large number of actual factory scheduling problems where the objective is to meet customer demand in a timely yet cost effective manner. Additional details on this model can be found in [18].

Experimental results reported below suggest that a good neighborhood function for this problem can be obtained by randomly applying one of the following three operators to the current schedule[3]:

- SHIFT-RIGHT: randomly select a "right-shiftable" operation and increase its start time by one time unit[4].
- SHIFT-LEFT (mirror image of SHIFT-RIGHT): randomly select a "left-shiftable" operation and decrease its start time by one time unit.
- EXCHANGE: randomly select a pair of adjacent operations on a given resource and permute the order in which they are processed by that resource. Specifically, given two consecutive operations, A and B on a resource R, with A preceding B in the current solution, the exchange operator sets the new start time of B to the old start time of A and the new end time of A to the old end time of B [5].

In our experiments, the probability of picking the EXCHANGE operator was empirically set to 3/7 while the probabilities of picking SHIFT-RIGHT or SHIFT-LEFT were

[3] In the scheduling jargon, the Just-In-Time objective considered in this study is known to be irregular[10]. Prior applications of SA to job shop scheduling have only considered regular objectives such as Minimum Makespan. It can be shown that the neighborhoods used in these earlier studies are not adequate to deal with irregular objectives such as the one considered here [16].

[4] An operation is said to be "right(left)-shiftable" if its start time can be increased (decreased) by one time unit without overlapping with another operation.

[5] In our implementation, exchanging two operations is allowed even if a precedence constraint is violated in the process. Precedence constraint violations are handled using large artificial costs that force the SA procedure to quickly get rid of them [16].

each set to 2/7. Additionally, the values of parameters in the SA procedure (see Figure 1) were set as follows: $T_0 = 700$, $T_1 = 6.25$, $N = 200,000$ and $\alpha = 0.85$.

The performance of this SA procedure has been evaluated in a comparison against 39 combinations of well-regarded dispatch rules and release policies previously used to assess the performance of the Sched-Star [11] and Micro-Boss [18, 17] systems on a set of 40 benchmark problems similar to the ones described in [18]. The 40 benchmarks consisted of 8 problem sets obtained by adjusting three parameters to cover a wide range of scheduling condition: an average due date parameter (tight versus loose average due date), a due date range parameter (narrow versus wide range of due dates), and a parameter controlling the number of major bottlenecks (in this case one or two). For each parameter combination, a set of 5 scheduling problems was randomly generated, thereby resulting in a total of 40 problems. Each problem involved 20 jobs and 5 resources for a total of 100 operations. On average, when compared against the best solution found on each problem by the 39 combinations of dispatch rules and release policies, SA reduced schedule cost by 15% (average over 10 SA runs). When comparing the best solution obtained in 10 SA runs against the best solution obtained on each problem by the 39 combinations of dispatch rules and release policies, SA produced schedules that were 34% better. However, while running all 39 combinations of dispatch rules and release policies takes a few CPU seconds on a problem, a single SA run takes about 3 minutes on a DECstation 5000/200 running C. Additional details on these experiments can be found in [16].

6.2 Empirical Evaluation of Cutoff and Restart Criteria

We now turn to the evaluation of the cutoff and restart criteria presented in this paper and compare the performance of five variations of the SA procedure presented in 6.1:

- **N-SA**: regular SA, as described in 6.1 (no learning).
- **P-SA**: SA with cutoff criterion.
- **B-SA**: SA with cutoff and Maximum Cost Reduction Rate restart criteria.
- **R-SA**: SA with cutoff and randomized restart criteria ($\beta = 0.5$).
- **H-SA**: SA with cutoff and hybrid restart criteria ($\beta = 0.5$ and $\tau = 1$).

When running P-SA, B-SA, R-SA, and H-SA, the cutoff and/or restart criteria were only activated after 5 complete SA runs to allow for the construction of meaningful $ECID$ distributions. All four of these procedures used the same set of checkpoints, $C = \{200, 100, 50, 25, 12.5\}$.

The five procedures were compared on the same 40 benchmark problems described in subsection 6.1[6]. Each SA procedure was run for 2 hours on each benchmark problem.

[6]At the present time, only a subset of the problems in each of the 8 problem sets have been completed. Complete results will be presented in the final version of the paper.

Furthermore, to eliminate possible noise effects, each two-hour experiment was repeated a total of 15 times. The results presented here were obtained by averaging performance of these 15 runs of each procedure on each problem.

Fig. 5 depicts the performance of the five SA procedures on a typical benchmark problem. The first 15 minutes are not represented, as they correspond to the first 5 runs when the cutoff and restart criteria have not yet been activated.

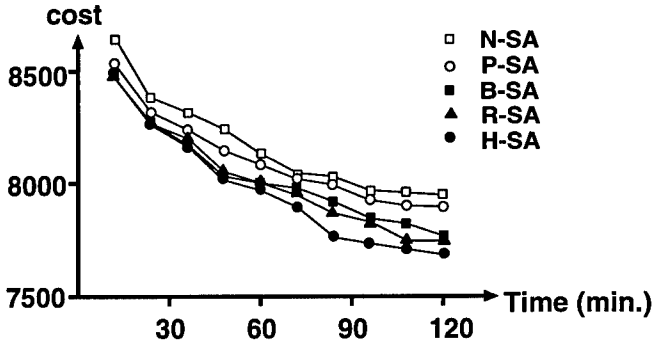

Fig. 5 Improvement of the best solution over time.

The figure shows that throughout its run, N-SA was dominated by the other four procedures. It also indicates that both the cutoff criterion and the restart criteria contributed to this performance improvement. Among the three restart criteria, H-SA appears to perform best. Figure 5 further suggests that the restart criterion in H-SA improves performance through the entire run, as the gap between H-SA and N-SA widens over time. These observations are confirmed by results obtained on the 8 problem sets of the study, as depicted in Figure 6. Fig. 6(a) shows the average cost reductions yielded by P-SA, B-SA, R-SA and H-SA over N-SA at the end of the two-hour runs. Figure 6(b) gives the average reduction in the CPU time required by each of these four procedures to find a solution of equal or better quality than the best solution found by N-SA in two hours. It can be seen that H-SA requires between 30%

and 70% less CPU time than N-SA.

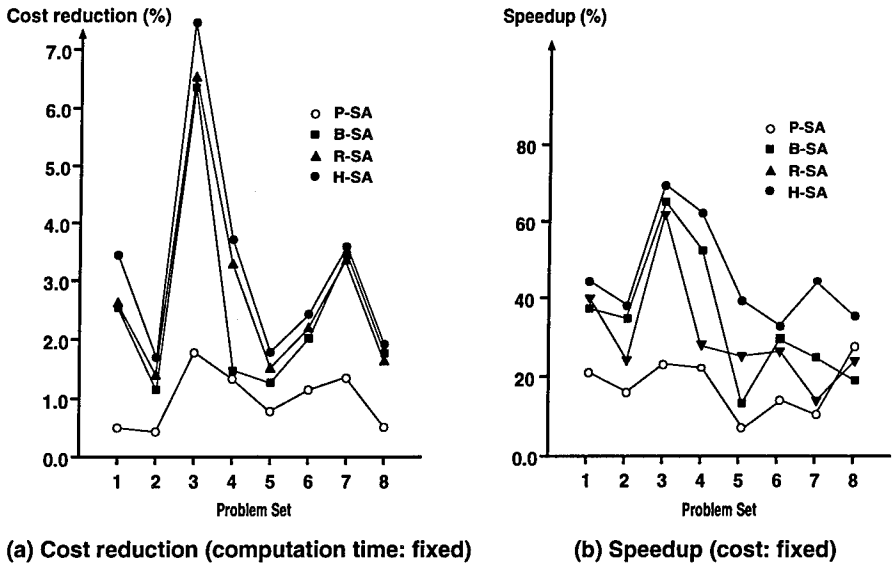

Fig. 6 Empirical comparison.

A finer analysis indicates that performance improvements produced by our cutoff and restart criteria increase as one requires higher quality solutions. Figure 7, compares the average CPU time of each of the five procedures as the required quality of solutions is increased. While all five procedures take about as long to find a solution with cost below 9000 or 8800, the time required to find a solution below 8500 varies significantly (e.g. H-SA can find such a solution in 3500 seconds while N-SA requires close to 10,000 seconds).

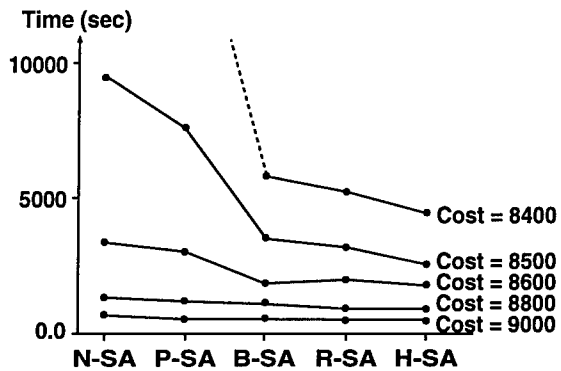

Fig. 7 Speedups as a function of required solution quality.

As already indicated in Section 5, the difference in performance between B-SA, R-SA and H-SA suggests that a deterministic use of $ECID$ distributions to decide where to restart search can be tricky, as these distributions may not be accurate, especially when only a small number of runs has been completed. By injecting non-determinism in the restart criterion, R-SA and H-SA ensure that the procedure will not always

restart from the same point. The procedure is forced to sample a wider area and in the process gets a chance to refine $ECID$ distributions. From this point of view, B-SA is a procedure that places more emphasis on using existing knowledge of the search space than acquiring new one, while R-SA places more emphasis on learning and less on exploiting already acquired information. H-SA appears to provide the best balance between these two requirements.

Finally, it should be obvious that the CPU time and memory overheads of our cutoff and restart criteria are very moderate. All in all, in our experiments, the CPU time required to learn $ECID$ distributions and apply the cutoff and restart criteria was well under 1% of total CPU time.

7 Summary

In summary, we have developed a mechanism that learns to recognize (un)promising SA runs by refining "Expected Cost Improvement Distributions" ($ECID$s) over multiple SA runs, and have developed search cutoff and restart criteria that exploit these distributions. These mechanisms can be applied to any SA procedure and have been validated on complex job shop scheduling problems with tardiness and inventory costs, where they have been shown to dramatically reduce the computational requirements of a competitive SA procedure. Experiments presented in this paper further indicate that the closer one seeks to be to the optimum, the larger the speedups.

References

[1] Cerny, V., "Thermodynamical Approach to the Traveling Salesman Problem: An Efficient Simulation Algorithm," *J. Opt. Theory Appl.,* Vol. 45, pp. 41–51, 1985.

[2] Beyer, W.H. "CRC Standard Mathematical Tables, 28th Edition," CRC Press, Inc., Boca Raton, Florida, 1987.

[3] Garey, M. R. and Johnson, D. S. "Computers and Intractability: A Guide to the Theory of NP-Completeness," Freeman and Co., 1979

[4] Glover, F. and Laguna M. "Tabu Search," To appear as a Chapter in *Modern Heuristic Techniques for Combinatorial Problems*, June 1992

[5] Graves, S. C. "A Review of Production Scheduling," *Operations Research* Vol. 29 no. 4, pp. 646–675, 1981

[6] Johnson, D. S., Aragon, C. R., McGeoch, L. A. and Schevon, C. "Optimization by Simulated Annealing: Experimental Evaluation; Part I, Graph Partitioning" *Operations Research* Vol. 37 no. 6, pp. 865–892, 1989

[7] Johnson, D. S., Aragon, C. R., McGeoch, L. A. and Schevon, C. "Optimization by Simulated Annealing: Experimental Evaluation; Part II, Graph Coloring and Number Partitioning" *Operations Research* Vol. 39 no. 3, pp. 378–406, 1991

[8] Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P., "Optimization by Simulated Annealing," *Science,* Vol. 220, pp. 671–680, 1983.

[9] Matsuo H., C.J. Suh, and R.S. Sullivan "A Controlled Search Simulated Annealing Method for the General Jobshop Scheduling Problem" Technical Report, "Department of Management, The University of Texas at Austin Austin, TX, Working Paper, 1988. 1992.

[10] Morton, T.E. and Pentico, D.W. "Heuristic Scheduling Systems," *Wiley Series in Engineering and Technology Management,* 1993

[11] Morton, T.E. "SCHED-STAR: A Price-Based Shop Scheduling Module," *Journal of Manufacturing and Operations Management,* pp. 131–181, 1988.

[12] Nakakuki,Y. and Sadeh,N. "Increasing the Efficiency of Simulated Annealing Search by Learning to Recognize (Un)Promising Runs," *Proceedings of the Twelfth National Conference on Artificial Intelligence,* To appear, 1994.

[13] Osman,I.H., and Potts, C.N., "Simulated Annealing for Permutation Flow-Shop Scheduling," *OMEGA Int. J. of Mgmt Sci.,* Vol. 17, pp. 551–557, 1989.

[14] Osman, I.H. "Meta-Strategy Simulated Annealing and Tabu Search Algorithms for the Vehicle Routing Problem" Technical Report, Institute of Mathematics and Statistics, University of Kent, Canterbury, Kent CT2 7NF, UK 1992.

[15] Palay, A. "Searching With Probabilities" PhD thesis, Department of Computer Science, Carnegie Mellon University, Pittsburgh, PA 15213, July 1983.

[16] Sadeh, N. and Nakakuki Y. "Focused Simulated Annealing Search: An Application to Job Shop Scheduling" Technical Report, The Robotics Institute, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213 1994.

[17] Sadeh, N. "Micro-Opportunistic Scheduling: The Micro-Boss Factory scheduler" Morgan Kaufmann Publishers, 1993.

[18] Sadeh, N. "Look-ahead Techniques for Micro-opportunistic Job Shop Scheduling" PhD thesis, School of Computer Science, Carnegie Mellon University, Pittsburgh, PA 15213, March 1991.

[19] Van Laarhoven, P.J., Aarts, E.H.L., and J.K. Lenstra "Job Shop Scheduling by Simulated Annealing," *Operations Research,* Vol. 40, No. 1, pp. 113–125, 1992.

[20] Vepsalainen,A.P.J. and Morton, T.E. "Priority Rules for Job Shops with Weighted Tardiness Costs," *Management Science,* Vol. 33, No. 8, pp. 1035–1047, 1987.

[21] Watkins, C. J. C. M., "Learning with Delayed Rewards" PhD thesis, Cambridge University, Psychology Department, 1989.

[22] Wefald, E.H. and Russel, S.J. "Adaptive Learning of Decision-Theoretic Search Control Knowledge" *Sixth International Workshop on Machine Learning*, Ithaca, NY, 1989.

[23] Zweben, M., Davis E., Daun B., Deale M., "Rescheduling with Iterative Repair" Technical Report FIA-92-15, NASA Ames Research Center, Artificial Intelligence Research Branch, Moffett Field, CA 94025 April, 1992.