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Solving the "Inverse" Problem in Terrain Modeling

Joseph K. Wald

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Joseph K. Wald

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U.S. Army Research Laboratory
ATTN: AMSRL-WT-WE
Aberdeen Proving Ground, MD 21005-5066

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In the computer simulation of combat over a specific geographical area, the terrain is typically represented using sets of isolated, measured terrain altitudes. The "inverse" problem is to construct in closed form a smooth surface function that matches these measured values to within a prescribed tolerance. If such a surface could be defined, then at any point on the simulated battlefield, both the terrain altitude and the slope of the terrain in any direction could be obtained simply by evaluating the surface function and its derivatives. Such a terrain description would allow all calculations related to terrain, from line-of-sight to troop movement, to be done more simply and at any desired resolution. In this report, we show how to solve the inverse problem.

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1. Introduction: Statement of the Problem

In the computer simulation of combat over a specific geographical area, the terrain is typically represented using finite sets of isolated data points that we will call digitized terrain altitudes (DTAs). These DTAs are obtained by measurement, either direct or indirect, and are generally arranged in a rectangular grid. A 100-meter grid spacing has been typical for many combat models, although the current trend is toward somewhat smaller grid sizes, especially for applications in the area of distributed interactive simulation (DIS). In any case, when a terrain altitude is needed at an (x, y)-coordinate in between grid points, it must be approximated using the surrounding DTAs. Moreover, since the DTAs are isolated points, there is no direct method for obtaining terrain slope information even at the DTA coordinates.

The "inverse" problem is to construct **in closed form** a "smooth" surface $z = T(x, y)$ (with the properties that $T(x, y)$ is continuous everywhere and has continuous partial derivatives $T_x(x, y)$ and $T_y(x, y)$ almost everywhere [i.e., everywhere except a set of measure zero]) that "matches" the DTAs to within a prescribed tolerance, δ (i.e., for each DTA point (x, y, d), $|T(x, y) - d| \leq \delta$). If such a function could be defined, then at any point on the battlefield both the terrain altitude and the slope of the terrain in any direction could be obtained simply by evaluating $T(x, y)$, $T_x(x, y)$, and $T_y(x, y)$ at the desired (x, y)-coordinate.

The benefits of such a terrain description in combat simulation are obvious. All calculations related to terrain, from line-of-sight calculation to troop movement, could be done more simply and at **any desired resolution.** In this report, we show how to construct a function $T(x, y)$ with all of the above properties.

Once $T(x, y)$ has been constructed, it is possible to add realistic "micro-terrain" to the surface. We indicate how this can be accomplished. The critical tool in this process is the Variable Resolution Terrain (VRT) model.

2. The VRT Model

The VRT model, developed at the U.S. Army Ballistic Research Laboratory (now the U.S. Army Research Laboratory) in the early 1990s, is a model that represents basic topography as a continuous surface, capable of being viewed at any desired resolution.[1] This surface is the superposition of individual terrain features or "hills," each of which is

[1] Wald, Joseph K. and Patterson, Carolyn J. "*A Variable Resolution Terrain Model for Combat Simulation.*" BRL-TR-3374, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD, July 1992.

described by a closed form mathematical function that is continuous everywhere and has continuous partial derivatives almost everywhere. While examples of many different types of terrain have been constructed using the VRT model, they are all "generic" in the sense that they were developed from basic principles and do not represent any particular piece of Earth topography. For generic terrain creation, the distribution of the sizes of the hills is based on the idea of self-similarity (i.e., invariance with respect to scale) which is embodied in the power law

$$D = Ks^{-2}, \tag{2.1}$$

in which s is a dimensionless scale factor associated with a hill, K is a constant that depends on terrain type, and D is the areal density of hills as a function of s. The choice of exponent in this power law ensures self-similarity in the density of terrain features. Integration of this power law produces the cumulative distribution function:

$$\int_{t = s_{\min}}^{t = s} Kt^{-2} dt = K(1 / s_{\min} - 1 / s). \tag{2.2}$$

In practice, there is a range of scales, $[s_{\min}, s_{\max}]$, for which the power law holds. Thus, to build each hill, we need only draw a uniform random number u from the interval $(0, 1 - [s_{\min} / s_{\max}])$, with the hill scale factor being $s = s_{\min} / (1 - u)$. The location of the hill is randomly chosen in the desired area. The complete terrain surface is defined by the superposition of all of the hills, i.e.,

$$T(x, y) = \sum_{k=1}^{N} f_k(x, y). \tag{2.3}$$

The form of a single hill function, $f_k(x, y)$, is given by:

$$f_k(x, y) = s_k\, h_k exp(- \left\{ \frac{1}{s_k\, \rho_k} \left[q_k(x, y) \right]^{\frac{1}{2}} \right\}^{\sigma_k}), \tag{2.4a}$$

where

$$q_k(x, y) = a_1(x - \xi_k)^2 - a_2(x - \xi_k)(y - \eta_k) + a_3(y - \eta_k)^2, \tag{2.4b}$$

$$a_1 = \varepsilon_k - \left(\varepsilon_k - \frac{1}{\varepsilon_k} \right) \cos^2 \lambda_k, \tag{2.4c}$$

$$a_2 = \left(\varepsilon_k - \frac{1}{\varepsilon_k} \right) \sin 2\lambda_k, \tag{2.4d}$$

and

$$a_3 = \varepsilon_k - \left(\varepsilon_k - \frac{1}{\varepsilon_k}\right)\sin^2 \lambda_k. \tag{2.4e}$$

This formulation is a revision of the original hill function definition.[2] Varying the parameters $\xi_k, \eta_k, h_k, \rho_k, \varepsilon_k, \lambda_k, \sigma_k$, and s_k produces hills in a variety of sizes and shapes. Figure 1 shows a simple "3D wire mesh" of a VRT surface generated by this procedure.

3. Constructing T(x, y)

The first step in the process of "fitting" a smooth surface to a set of DTAs consists of detrending the DTAs by fitting a plane through them and subtracting the height of this plane from each of the DTAs. One way to define this plane (but not the only way) is via a least-squares procedure. This produces "residual" DTAs.

Next the highest residual DTA (x, y, d) is found (or the lowest, if the largest DTA happens to have a negative value) and we define a hill whose center is at or near (x, y) and whose height is equal to or close to d. An iterative loop is used to find the best values of the parameters in the construction of this hill. [Although, we use equation 2.4 to define the hill, any function with the flexibility of equation 2.4 and the same smoothness properties could be used.] Here, the best hill is the one that leaves the smallest modified residual DTAs in the neighborhood of (x, y) when the hill is subtracted from the residual DTAs. This neighborhood is user-definable and will, in general, depend upon terrain type. This hill is subtracted from each of the residual DTAs to create a new set of residual DTAs.

We now proceed to find the new highest residual DTA and repeat the above hill fitting process until all of the residual DTAs are smaller in absolute value than δ. The smooth surface $T(x, y)$ is then defined to be the sum of the fitting plane and all of the fitting hills. By the method of construction, $T(x, y)$ is guaranteed to satisfy all of the conditions of the inverse problem.

To illustrate the terrain fitting process, we selected a 1 kilometer square of terrain located near Denver, Colorado. The terrain data were provided by the U.S. Army Topographic Engineering Center.[3] The grid spacing for the DTAs is 5 meters, resulting in a total of 40,401 DTAs in the set. Figure 2 contains a contour map of the sample DTAs.

[2] op. cit.

[3] Ray Norvelle, U.S. Army Topographic Engineering Center. Private communication, March, 1994.

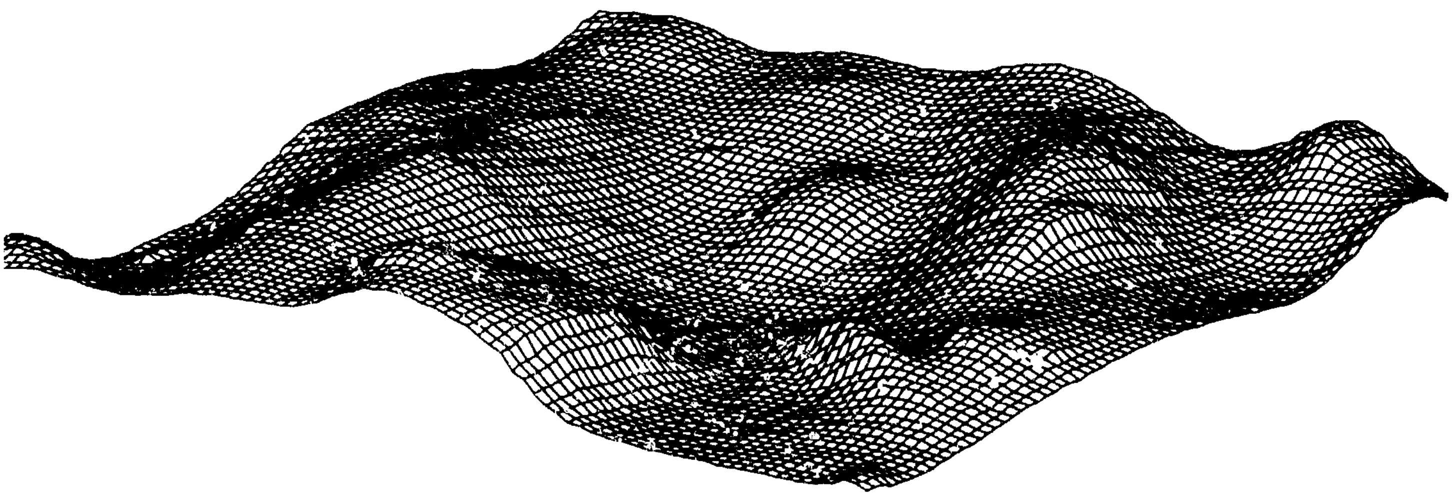

Figure 1. Sample VRT Surface.

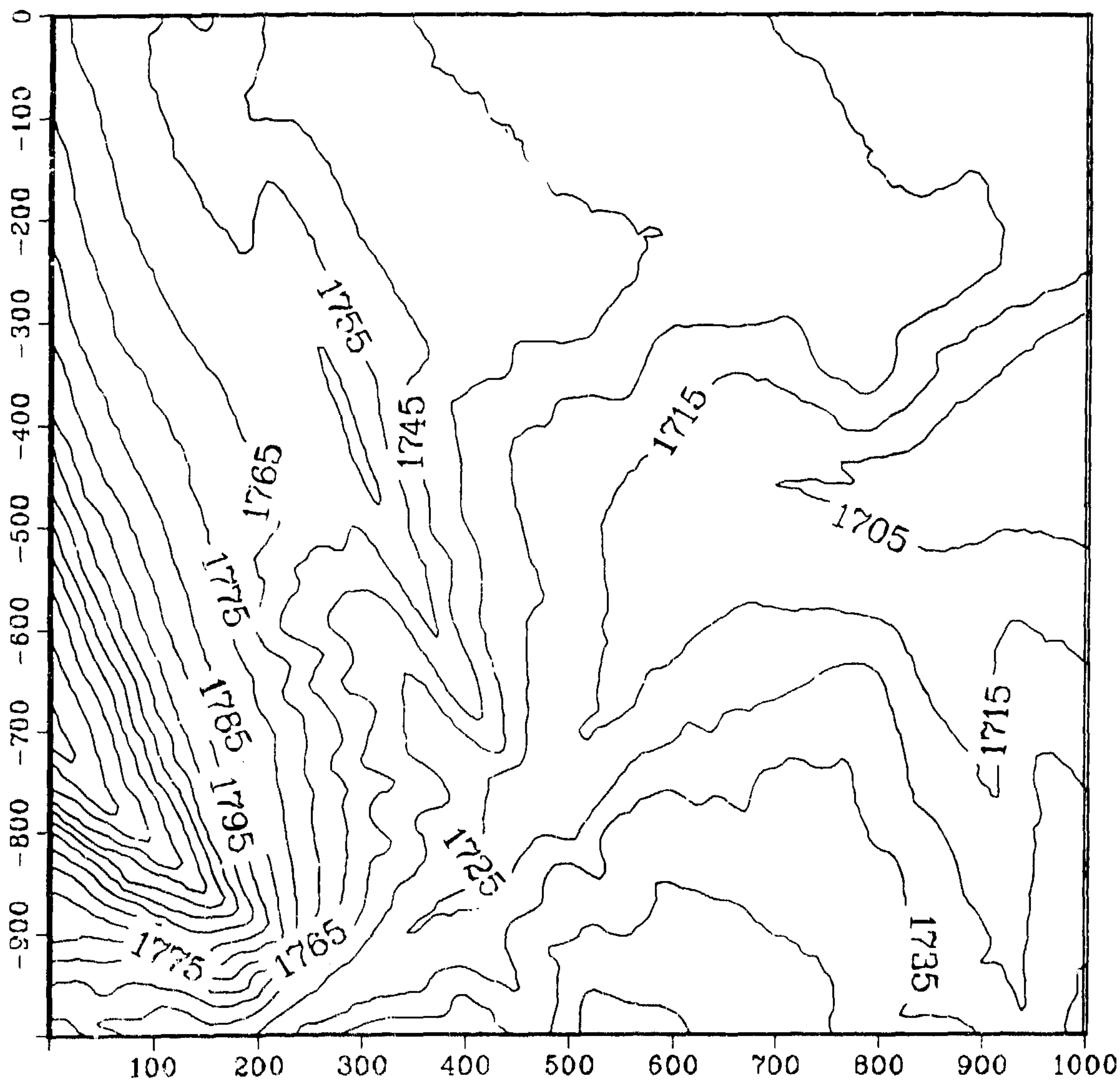

Figure 2. Sample Contour Map: DTAs.

We proceeded to construct three smooth surfaces, which we denote by $T_5(x, y)$, $T_2(x, y)$, and $T_1(x, y)$ corresponding, respectively, to $\delta = 5$, $\delta = 2$, and $\delta = 1$. The contour maps of these surfaces are shown in Figures 3, 4, and 5.

The increasing quality of the fit as δ decreases is evident from the contour maps. However, the best value of delta for the fitting process need not always be the smallest. Rather, it should be the **largest value consistent with the user's intended application.** This is due to the fact that as δ decreases, the computational burden, both in constructing $T(x, y)$ and in using it, increases. Table 1 contains the computational statistics for the construction of $T_5(x, y)$, $T_2(x, y)$, and $T_1(x, y)$. The computations were done on a CRAY Y-MP.

Table 1. Computation Statistics for Three Smooth Surfaces.

Smooth Surface	Maximum Deviation (*meters*)	Average Deviation (*meters*)	Number of hills	Computation Time (*seconds*)
$T_5(x, y)$	5	2.01	630	48
$T_2(x, y)$	2	0.88	3799	236
$T_1(x, y)$	1	0.34	17,960	1079

If $T(x, y)$ is to be precomputed and then used in a simulation, the difference in computation times in the creation of the smooth surfaces will probably not be too important. However, since the computational burden of using $T(x, y)$ is directly proportional to the number of hills comprising the surface, "real time" applications may be quite sensitive to this factor.

It should be pointed out that although $T(x, y)$ was constructed using the VRT hill format (equation 2.4), the strict self-similarity of the "ideal" VRT surface was not used. While we are free to determine for ourselves the distribution of terrain features when building generic terrain, when we construct a $T(x, y)$ to match a given set of DTAs, the distribution of terrain features must conform to the peculiarities of that particular piece of terrain. Thus, the best we can hope for is approximate self-similarity. Even this may not exist if, for example, the set of DTAs under consideration includes two or more different types of terrain.

However, when it comes to adding micro-terrain to $T(x, y)$, we are, in effect, creating a miniature set of generic terrain. Here, the full power of the VRT theory can be used. In general terms, the empirical distribution of the VRT parameters comprising

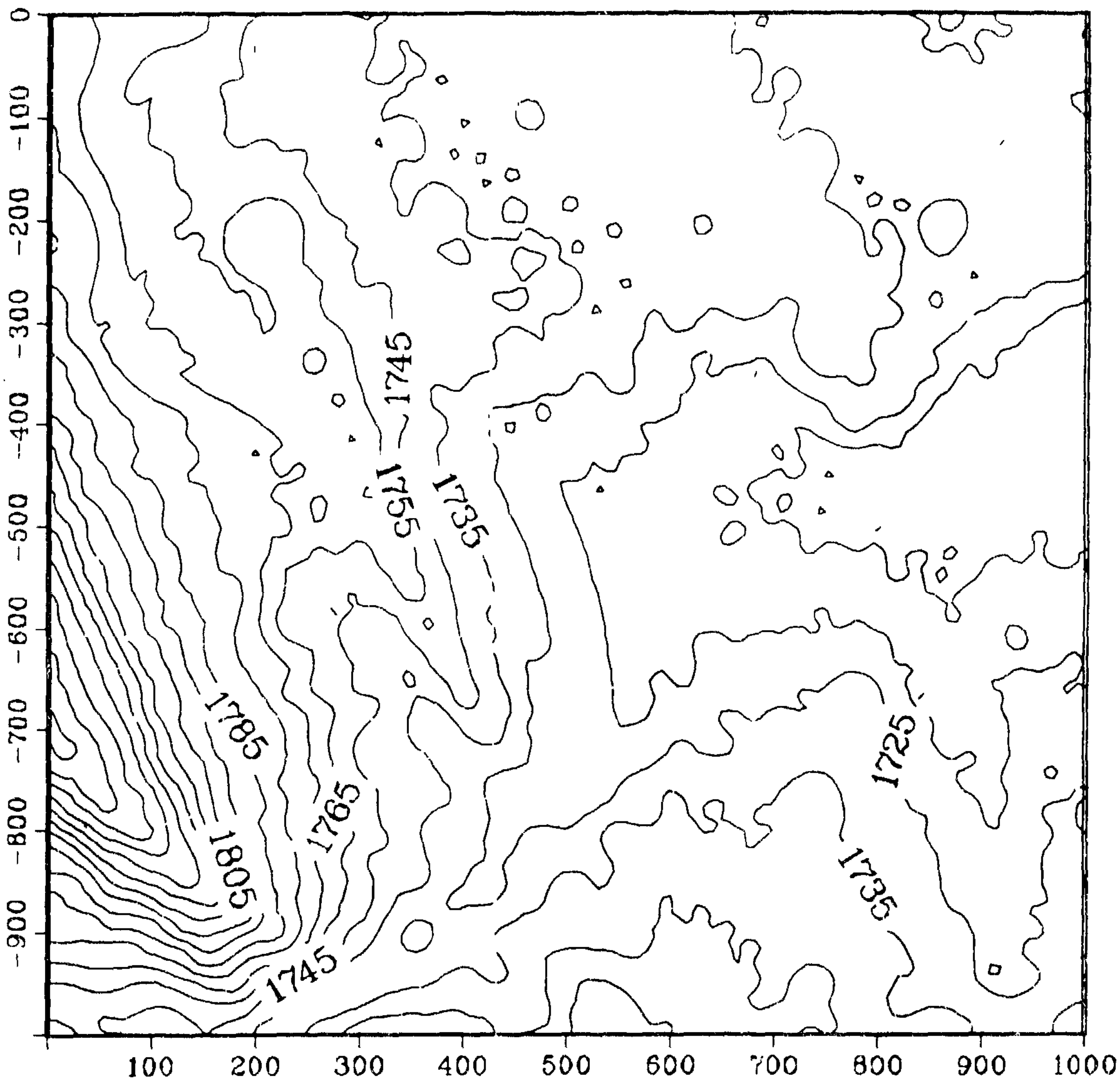

Figure 3. Sample Contour Map: δ=5.

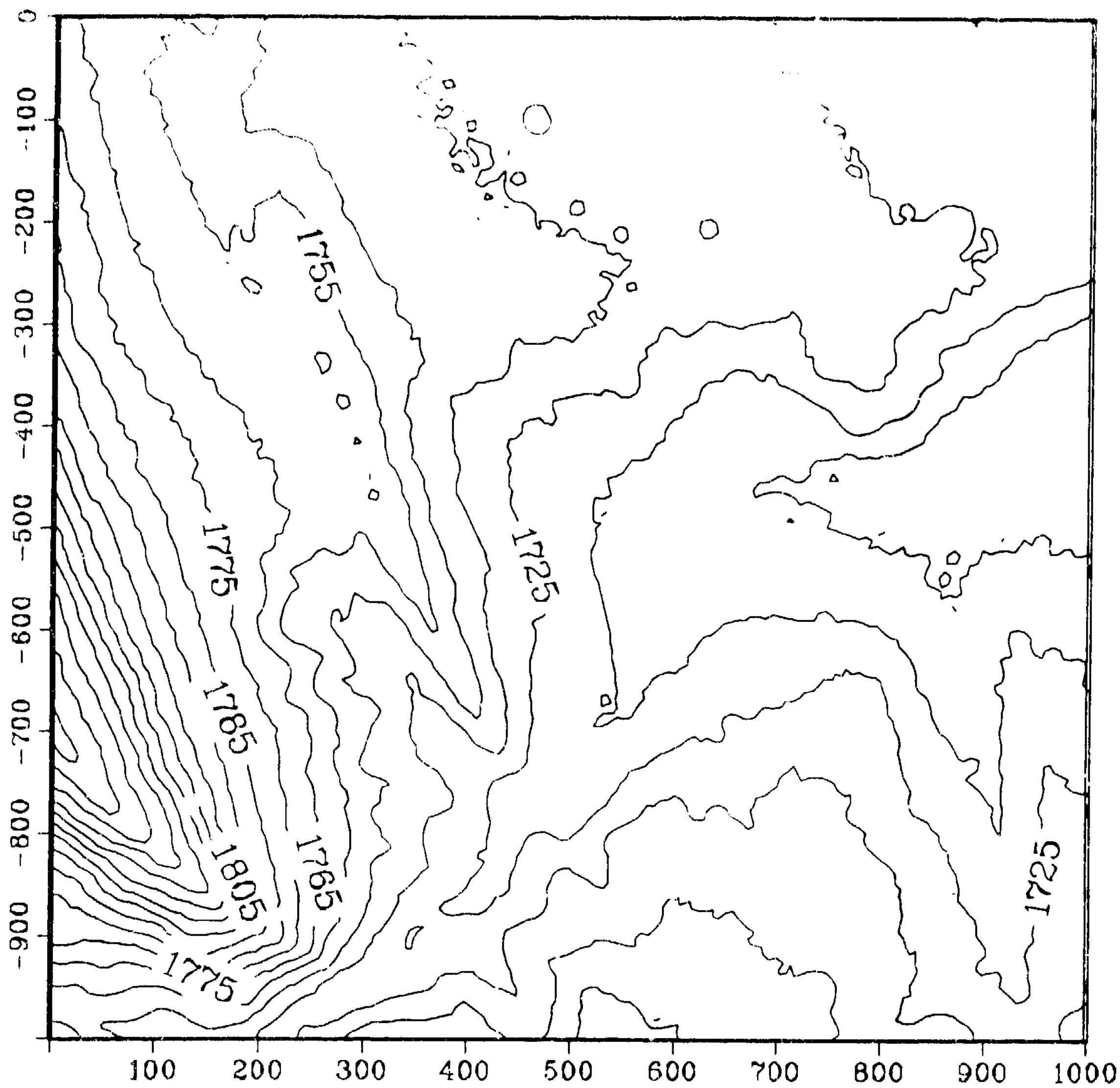

Figure 4. Sample Contour Map: δ=2.

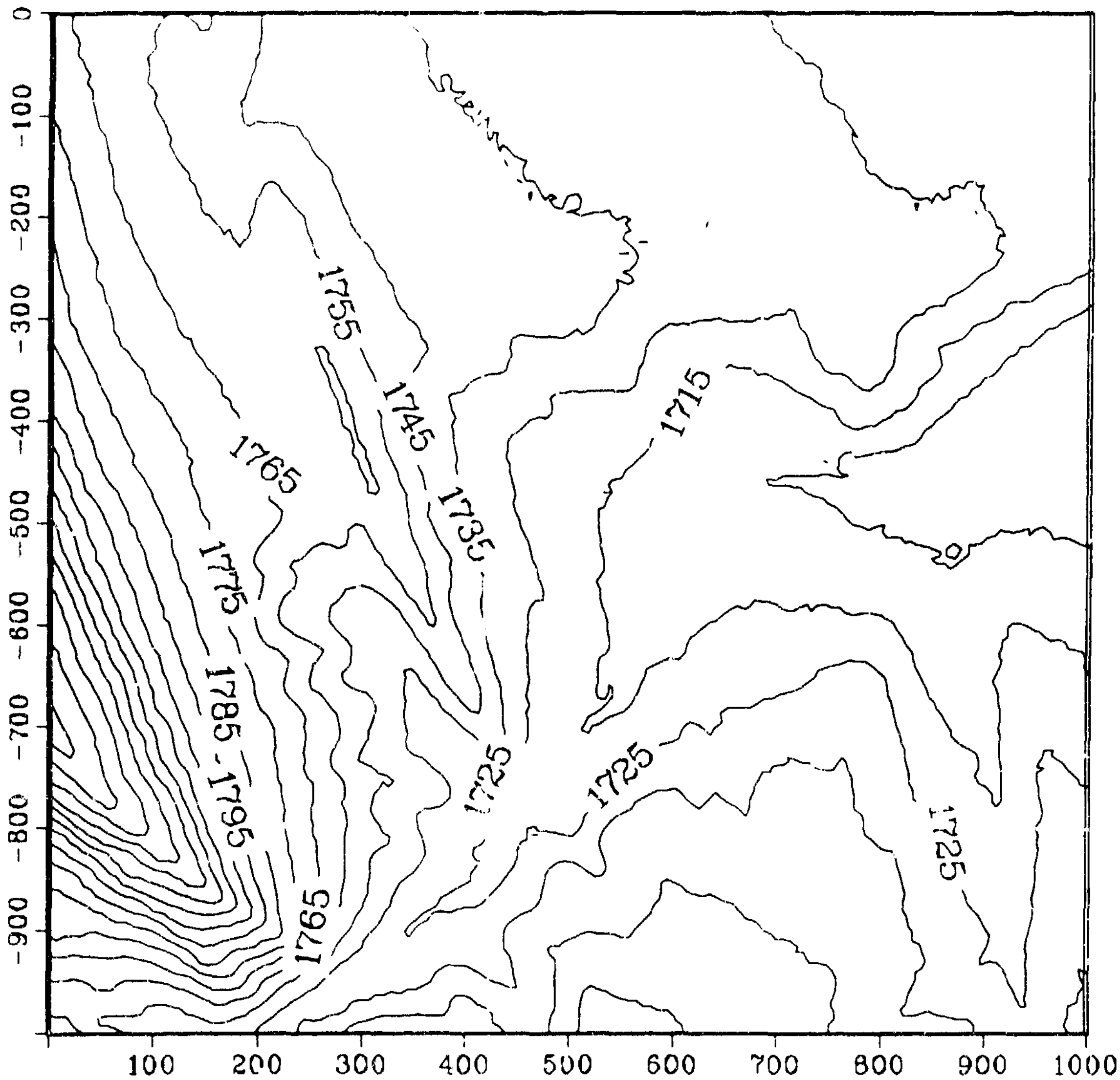

Figure 5. Sample Contour Map: δ=1.

$T(x, y)$ can be analyzed and used to randomly generate small terrain features, whose scale distribution is governed by equation 2.1.

4. Future Work: Optimization of the Algorithm

It is for just such applications that optimization of the algorithm will be important. Here, optimization means creating $T(x, y)$ (for a given δ) using the fewest hills possible. Unfortunately, for a given set of DTAs, we will not know what that minimum number of hills is. However, there are some techniques we can apply in the surface construction process that will tend to reduce the number of hills. One method is to increase the number of iterations in the process of selecting the parameters for each hill. Early experiments indicate that this does, in fact, yield a reduction in the number of hills, although it comes with an increase in calculation time.

Another idea is to increase the variability of the hill parameters in the iteration loop. This will allow us to explore more varied hill shapes and, presumably, find a better fit. However, the rate of convergence will be reduced, making it necessary to increase the number of iterations per hill to find that better fit. We will have to find the proper balance between increased parameter variability and number of iterations.

It is also possible to make the iterative loop process adaptive. By this we mean that the rate of improvement of fit during the iteration loop will be monitored, and the iteration automatically terminated when the rate of improvement falls below a specified threshhold value.

Finally, instead of treating the hill fitting process locally (i.e., by fitting the hill at the site of single highest residual DTA), we could look at the process on a "regional" basis. Here, we would do a global statistical analysis of the residual DTAs, find the largest "coherent" topographical feature, and try to find a hill to fit the whole thing. If this could be done, it would clearly speed up the entire process. The open question is how to define the largest "coherent" topographical feature.

Also, additional work is necessary to fully develop the capability to superimpose micro-terrain on $T(x, y)$.

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