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THESIS

TYPE INFERENCE WITH OVERLOADING USING AN ATTRIBUTE GRAMMAR

by
Bruce James Bull
March, 1994

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94 7 19 067

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time reviewing instructions, searching existing data sources gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE March 1994	3. REPORT TYPE AND DATES COVERED Master's Thesis

4. TITLE AND SUBTITLE Type Inference With Overloading Using an Attribute Grammar (U)	5. FUNDING NUMBERS
6. AUTHOR(S) Bull, Bruce James	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/ MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/ MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the United States Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.	12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

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We implement an interactive type checker for the polymorphic type system of *ML* with overloading. The implementation was produced automatically from an attribute grammar using the Synthesizer Generator, an attribute evaluator generator. Type inference then is accomplished via attribute evaluation so that if the evaluation is done incrementally, then type inference becomes incremental as well.

14. SUBJECT TERMS Overloading, Polymorphism, Type Inference, Attribute Grammar, SynGen, Incremental, Constrained Type Schemes, Constraint Set Satisfiability	15. NUMBER OF PAGES 51
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT Unlimited

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89) Prescribed by ANSI Std. 239-18

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TYPE INFERENCE WITH OVERLOADING USING AN ATTRIBUTE GRAMMAR

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN COMPUTER SCIENCE

from the

NAVAL POSTGRADUATE SCHOOL

March, 1994

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ABSTRACT

Interactive programming environments for languages offer many advantages over traditional batch-oriented ones, such as immediate static analysis. One form of analysis is type checking, yet type checking in this setting for languages with common features like overloading has received little attention.

We implement an interactive type checker for the polymorphic type system of *ML* with overloading. The implementation was produced automatically from an attribute grammar using the Synthesizer Generator, an attribute evaluator generator. Type inference then is accomplished via attribute evaluation so that if the evaluation is done incrementally, then type inference becomes incremental as well.

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ACKNOWLEDGMENTS

I owe an immense debt of gratitude to Dr. Dennis Volpano for offering timely and valuable assistance and insight into every aspect of my thesis. His tireless pursuit of excellence and high ideals have been an inspiring example. He has made my efforts at the Naval Postgraduate School both rewarding and memorable.

I wish to also thank Dr. Craig Rasmussen for lending his time and expertise in ensuring the correctness of this thesis. His ability to make difficult concepts understandable and genuine concern have greatly benefited many students at NPS, including myself.

Most of all, I would like to express my gratitude to my family. To my wife Tacie; for her encouragement, thoughtfullness, and untiring patience. To my children; for providing reminders of what is truly important in life.

I. INTRODUCTION

In this thesis, we assume the reader is familiar with basic type theory and its associated notational conventions. We also assume a general familiarity with the concepts and notation of the lambda-calculus. A comprehensive presentation of these concepts can be found in the texts of Thompson [Tho91] and Gunter [Gun92].

The advantages of interactive programming environments to increase programmer effectiveness and maximize utilization of system resources are significant. For example, during program development, extensive context-sensitive type checking is a valuable tool. The immediate recognition of type errors at this stage could yield vast improvements to the quality and reliability of today's software products. Valuable system resources would be preserved through decreased waste due to unnecessary re-compilations. Perhaps more significantly, the advantages of providing an environment where programmers can focus on the fundamental aspects of a problem with a much higher degree of continuity are clear.

The study of type inference is integral to this effort. Though significant advances have been made in this research area, further work needs to be done. This thesis considers a suitable type system for implementing a polymorphic programming language with overloading. Utilizing this type system, an implementation is produced that performs incremental type inference in an interactive environment.

One can argue that ***system ML*** represents the current state of the art in type systems. It is a polymorphic type system but prohibits the use of overloading. Yet the need for overloading in programming languages is well known. Current imperative languages, such as *Ada* and *C++*, and even the functional language *standard ML* , allow an identifier to represent different types but the resulting programs merely contain monomorphic instances overloaded on an identifiers name. A process called *overloading resolution* is required to assign a particular type to an identifier based on its context. Consider the following expressions, where + is defined over integers and reals:

$$\begin{array}{ll} (a) & 1+2 \\ (b) & 1.0+2.0 \end{array}$$

What is the type of +? We only know that it can have the type $int \rightarrow int \rightarrow int$ or the type $real \rightarrow real \rightarrow real$. But we can reliably assign neither of these types to + without first examining its context. In a polymorphic language, like *ML* , we can assign + the type $\forall \alpha . \alpha \rightarrow \alpha \rightarrow \alpha$ but this results in + having too many types. On the other hand, if we assign + the type $real \rightarrow real \rightarrow real$ we preclude its use in expression (a). We will examine these issues in more detail in Chapter II.

What is needed is a means to express a type for + which encompasses all of its possible types and no more. We can do this with the use of *constrained type schemes.* We can then assign to any occurrence of +, regardless of its context, the type $\forall \alpha\ with(+ : \alpha \rightarrow \alpha \rightarrow \alpha) . \alpha \rightarrow \alpha \rightarrow \alpha$. This means that + can assume any finite type $\alpha \rightarrow \alpha \rightarrow \alpha$, with α instantiated to any particular type for which + is defined.

Using the concept of constrained type schemes, an extension to *system ML* has been developed incorporating overloading called ML_o. The associated type inference algorithm W_o infers principal types for expressions in ML_o. It turns out that, unless we place restrictions on the kinds of overloadings we can express using constrained type schemes, typability in ML_o is undecidable. In Chapter IV we consider a form of overloading called *parametric overloading* which makes typability in ML_o decidable and present an algorithm which determines satisfiability of constraints with respect to a parametric assumption set.

A. IMPLEMENTING W_o

W_o performs batch type inference. In this respect, it is unsuitable for direct incorporation into a useful interactive programming environment. What is needed is an incremental approach to type inference which will provide immediate feedback to the programmer when type errors are encountered.

One might attempt to rewrite W_o to achieve incremental type inference. Our approach is to utilize the formalism of attribute grammars to express W_o. In this setting type inference is performed via attribute evaluation. As expressions are input a corresponding change is reflected in the attribution. If we are able to perform attribute evaluation incrementally, type inference can also be done incrementally. Furthermore, it is implicit in the formalism.

We present an implementation of W_o utilizing an attribute grammar in SSL, the language of the Synthesizer Generator of Grammatech. It is an attribute evaluator generator that takes as input a set of attribute equations and returns as output an attribute evaluator, or in our case, a type-checker. By utilizing the Synthesizer Generator for our implementation we are not only able to produce an attribute evaluator, but one in which attribute evaluation is done incrementally. As a result, we are able to achieve both attribute evaluation and type inference in an incremental setting. Chapters IV and V discuss details of the implementation and the algorithms used.

II. TYPE SYSTEMS

The concept of type systems in programming languages deals with a set of rules which, when applied to terms of a language, produce types for those terms. The notion of types in programming languages has been given steadily increasing importance over the past several years. It is clear that languages with rich type classes offer programmers more flexibility in modeling real-world objects. Yet, there remains a significant lack of consensus as to what types are. As consensus in this area is critical to the successful application of type theory to practical implementations of new programming environments, this chapter outlines the most important aspects of type systems and their application to this thesis.

A. WHAT IS A TYPE?

When discussing types, there exists a tendency to confuse the distinction between implementation issues and the underlying nature of types in general. Actual machines, for example, provide relatively few types (i.e. integers, floating-point numbers, pointers, etc...). The implementation of types in a high-level language, while posing some very real problems in the area of compiler design, should remain distinct from a discussion of type correctness in the higher context of the meaning of types. With reference to implementation issues, referred to as Reductionist type correctness, Smith states:

> The key issue is how to protect the representation from misuse. [Smi91]

In this thesis, we will not concern ourselves with the reductionist view of types. Rather, we will view a type as an algebra, a set of values and operations such that the set is closed under these operations. For example, type *int* is the set of integers together with the usual arithmetic operations, but the set of natural numbers and the predecessor operation do not form a type. This view gives us a fundamental basis from which to discuss the meaning and usage of types in programming languages unencumbered by implementation issues. Operations of an algebra are axiomatized, providing then a semantics that one can use to reason about programs in which they occur. In order to use the axioms, however, it may be necessary to restrict the types of certain program arguments to the algebras in question. For example, if we are to prove that a function adds 1 to its argument then we might wish to fix the type of its argument to *int*, say. For some programs, though, reasoning can proceed without fixing argument types. Such programs are called *polymorphic*.

B. POLYMORPHISM

Polymorphic means to have *many forms*. With respect to programming languages, this refers to programs or terms which have many types, or can operate on values of many types. Perhaps more intuitively, we can state that the purpose of polymorphism is to allow programs which use a single name to operate on many different types of inputs and, perhaps, produce different types of output.

We will first be concerned with a form of polymorphism called *parametric polymorphism*, where polymorphic entities can be described by a universally quantified formula with all quantification at the outermost level (e.g. $\forall\alpha.\alpha \rightarrow \alpha$). In Figure 2.1, we give an example of a function, *length*, defined in a generic polymorphic programming language. We can ascribe to *length* type $\forall\alpha.\, list(\alpha) \rightarrow int$. It's meaning is a function which given a list computes its length.

Languages which do not support polymorphism put unnecessary restrictions on the use of a function. Consider the Pascal program in Figure 2.2. Procedure *min* has the type: $int \rightarrow int \rightarrow int$. Yet there is nothing inherent in *min* which depends

```
function length(x)
{
    if not null(x) then
      1 + length(tail(x))
    else
      0
}
```

Figure 2.1: Polymorphic *length* function

on *integer*. Replacing *integer* with *char* would yield a correct Pascal program with meaning corresponding to the lexicographic ordering of characters.

It is not uncommon for the claim to be made that *Ada* is a polymorphic programming language, as in [ASU86]. One might argue that it is, but really only weakly so. Through the use of generics, one can define a template for representing what *appears* to be a polymorphic function. In the example of Figure 2.3, one might wish to ascribe the type $\forall\alpha.\alpha \rightarrow \alpha \rightarrow \alpha$ to the Ada function *min* within the generic package *MIN_PKG*. This would indicate that *min* is defined over all instantiations of α, including *int* and *char*. This is obviously not the case, for a generic package cannot be used directly in Ada. It must first be instantiated for a particular type so that it can be properly *type checked.* Though the language provides constructs for expressing polymorphism, the resulting compiled program merely contains monomorphic instances of the function overloaded on the identifier *min*. Research into providing polymorphism in an imperative language is ongoing [Car87].

```
procedure min(x,y : integer);

begin
  if x < y then
    return(x)
  else
    return(y)
end
```

Figure 2.2: Pascal *min* function

```
generic
type ITEM is private;
  with function "<"(left,right : ITEM)
                      return BOOLEAN is <>;
package MIN_PKG is
  function min(x,y : ITEM) return ITEM is
  begin
    if x < y then
      return(x)
  else
      return(y)
  end min;
end MIN_PKG;
```

Figure 2.3: Ada generic *min* function

It is clear that parametric polymorphism is a desirable property of practical programming languages. Yet, in practice, situations arise where parametric polymorphism alone cannot provide us with the means to express certain types adequately. Consider a polymorphic type for *min* in Figure 2.2. Clearly it is meaningful for multiple types. However, if we ascribe the type $\forall\alpha.\,\alpha \rightarrow \alpha \rightarrow \alpha$ to *min*, terms without meaning, such as $min(true, false)$, become typable. It can be seen that *min* depends on "<" being defined over its parameters. What is needed is the ability to restrict use of *min* to input types whose values are partially ordered. In other words, we need to be able to *overload* "<" so that *min* is polymorphic yet bounded in the types of arguments to which it can be applied, a form of ***bounded polymorphism.***

C. OVERLOADING

The common view of overloading is stated as follows:

An overloaded symbol is one that has ***different meanings*** depending on its ***context*** [ASU86] (emphasis added).

This process of determining the meaning of an expression by examining its context is called ***overloading resolution***. This is, in fact, the usual way of treating overloaded symbols in a program; demanding that the local context of an overloaded symbol determine a *particular* overloading to be used at each occurrence. This kind of treatment is used even in the polymorphic language *ML* . In fact, any overloading that *requires* overloading resolution to determine its meaning is termed an *incoherent overloading* and gives rise to potential semantic ambiguity. For example

$$(2.1)\ \left\{ \begin{array}{l} * : real \rightarrow real \rightarrow real, \\ * : \forall\alpha.matrix(\alpha) \rightarrow matrix(\alpha) \rightarrow matrix(\alpha) \end{array} \right\}$$

is an incoherent overloading of the operator $*$ where $*$ stands for real multiplication and matrix multiplication.

Consider a term $\lambda x.\lambda y.\, x * y$. We can infer two different types for it: $real \rightarrow real \rightarrow real$ and $\forall\alpha.\, matrix(\alpha) \rightarrow matrix(\alpha) \rightarrow matrix(\alpha)$. We must apply the process of overloading resolution to determine the meaning of the term.

A more desirable form of overloading, called *coherent overloading*, arises when an overloading is constructed in such a way that its various instances share a common semantics. In this case, overloading resolution is not required to ascribe a unique meaning to terms. It's meaning is uniquely determined from an inspection of the axioms for the operators occurring in a term. For example, suppose $*$ is commutative. We can readily see that our overloading in (2.1) is incoherent. For, although we can derive from (2.1) that $\lambda x.\lambda y.\, x * y$ has type $real \rightarrow real \rightarrow real$ and $\forall\alpha.\, matrix(\alpha) \rightarrow matrix(\alpha) \rightarrow matrix(\alpha)$, matrix multiplication is not commutative. If we replace our second assumption on $*$ with $* : int \rightarrow int \rightarrow int$ with the meaning of integer multiplication, the overloading now becomes coherent for both integer and real multiplication are commutative. So we know that regardless of the types of x and y, the function is commutative.

As will be seen, while our implementation of W_o does not prohibit the introduction of incoherent overloadings, our assumption is that all overloadings *are* coherent. If this assumption is invalid with respect to a particular overloading, types will still be correctly inferred for expressions involving that overloading. However, the guarantee that the meaning of such an expression is uniquely determined is lost.

Surprisingly, it is common in current languages to introduce incoherent overloadings regardless of the potential for semantic ambiguities. In Ada, for example, the operator "/" is overloaded with different meanings of integer and floating-point division.

Overloadings allowed in most languages, including: *Ada*, *C++* and standard *ML* , are restricted to being finite. In the ML_o type system this restriction is lifted. For example, we can represent an infinite overloading over lists under equality as: $\forall\alpha$ **with** $= : \alpha \rightarrow \alpha \rightarrow \mathit{bool} \,.\, \mathit{list}(\alpha) \rightarrow \mathit{list}(\alpha) \rightarrow \mathit{bool}$. In this case, if $=$ has an instance at $\tau \rightarrow \tau \rightarrow \mathit{bool}$, then it also has an instance at $\mathit{list}(\tau) \rightarrow \mathit{list}(\tau) \rightarrow \mathit{bool}$.

III. THE *ML* TYPE SYSTEM AND OVERLOADING

In this chapter we consider an extension of a Curry-style typed lambda calculus (Λ_0) with *type schemes* called *System ML* . As mentioned previ a type scheme represents *parametric polymorphism*, implying that all quantifi n must be outermost, or *shallow*. Research aimed at removing this restriction is described in [Lei83, McC84, KT90].

A free identifier may be denoted as having infinitely many types via a type scheme. For instance, the primitive LISP operation *hd* may be given the type: $\forall\alpha.\,\mathbf{seq}(\alpha) \rightarrow \alpha$ which would indicate that for any choice of α, say τ, *hd* has the type: $\mathbf{seq}(\tau) \rightarrow \tau$.

System ML preserves the property of principal types; every typable term has a principal type, one that is more general than any other type derivable for the term. For instance, the term $\lambda f.\lambda x.fx$, f and x occurring free, would have as principal type $\forall\alpha.\forall\beta.(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta)$. This is regarded as the most general typing for this expression. This means that any type whatsoever of $\lambda f.\lambda x.fx$ can be derived from the type $\forall\alpha.\forall\beta.(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \beta)$ by suitably instantiating α and β; formally, we say that all the types of $\lambda f.\lambda x.fx$ are *instances* of the principal type. The existence of principal types means that a type inference algorithm will always compute a unique "best" type for a program.

In order to retain principal types, lambda abstraction in *System ML* , as in Λ_0, is monomorphic. This means that lambda-bound identifiers within a λ-expression cannot be assigned multiple types. Consider the expression $(\lambda x.x(\lambda y.y))\lambda z.z$. This expression is typable in *System ML* with principal type $\forall\alpha.\alpha \rightarrow \alpha$. This conforms to the restriction on lambda abstraction since x, while being able to assume infinitely many values, has polymorphic type $\forall\alpha.\alpha \rightarrow \alpha$. The restriction is manifest when an attempt at self-application is made within a λ-expression. For example, a term such as $(\lambda y.\, yy)\lambda x.\, x$ is illegal in *System ML* . Here, y must be able to assume two different types; $(\alpha \rightarrow \alpha)$ and α for some particular α. This results in the term $\lambda y.\, yy$ having type $\forall\alpha.(\forall\beta.\beta \rightarrow \beta) \rightarrow \alpha$, which is not a *principal* type.

In order to allow free identifiers denoting polymorphic values to be assigned multiple types, one uses the *let* construct. The above expression can then be represented as **let** $y = \lambda x.\, x$ **in** $y\, y$. This involves no inner quantification, since each instance of y is replaced with $\lambda x.\, x$ in determining the type for yy.

System ML , like Λ_0, has a decidable typability problem. In other words, if a type exists for a program (there may be more than one), the type inference algorithm will be able to infer a correct type for it. Conversely, if a type does not exist, the algorithm is capable of making that determination. *System ML* is also widely accepted and has been incorporated into mainstream languages like Standard *ML* [HMM86] and Miranda [Tur86]. Yet, an obvious and practical limitation exists in *System ML* that prohibits overloading by restricting the number of assumptions per identifier in a type assumption set to at most one. Milner himself makes the comment in his 1978 paper [Mil78] that allowing more than one assumption is desirable.

An extension to the *ML* type system has been developed called ML_b[VoS91]. It retains principal types and allows overloading. Deviations from *System ML* include the introduction of ***constrained type schemes*** and modifications to the type instantiation and generalization rules. Many extensions of *System ML* have been proposed to incorporate overloading. Among these are the systems of [Kae88, CDO91, Smi91, Kae92, Jon92] and those related to the development of the functional programming language Haskell [WaB89, CHO92, NiP93]. All of these type systems share the notion of a constrained type scheme in various forms. A critique of these type systems is given in [Vol93b].

A. *ML_0*

Given a set of type variables ($\alpha, \beta, \gamma, \ldots$) and a set of type constructors *(int, real, bool, list,...)* of various arities, the set of unquantified types is defined by:

$$\tau ::= \alpha \mid \tau \rightarrow \tau \mid \chi(\tau_1, \ldots, \tau_n)$$

The set of quantified types or *type schemes*, then, is defined by

$$\sigma ::= \forall(\alpha_1, \ldots, \alpha_n) \textbf{\textit{ with }} (x_1 : \tau_1, \ldots, x_m : \tau_m).\ \tau,$$

where $\alpha_1, \ldots, \alpha_n$ is the set of ***quantified variables*** of σ, $x_1 : \tau_1, \ldots, x_m : \tau_m$ is the set of ***constraints*** on σ, and τ is the ***body*** of σ. If there are no quantified variables, the "$\forall$" may be omitted. If there are no constraints, the "***with***" may be omitted. In our terminology, σ will always be reserved to represent a type scheme, $\overline{\alpha}$ denotes an abbreviation for $\alpha_1, \ldots, \alpha_n$ and C will be used to represent a list of constraints. The most general form of a type scheme is then:

$$\sigma ::= \forall\ \overline{\alpha} \textbf{\textit{ with }} C.\ \tau$$

A ***substitution*** is a set of replacements for type variables applied simultaneously to all type variables. For example:

$$[\alpha_1, , \ldots, \alpha_n := \tau_1, \ldots, \tau_n]$$

is a substitution where all of the α_i's are distinct. The substitution is applied to a type τ by simultaneously replacing all of the α_i's with the corresponding τ_i's. The application of substitution S to type τ will be denoted by τS.

Two new type assignment rules, ($\forall$-intro) and ($\forall$-elim), are given in Figure 3.1; these represent extensions to System *ML* developed to accommodate overloading. It should also be noted that if the constraint list C is empty, these two extensions are identical to type generalization and instantiation in system *ML* [Mil78, DaM82].

$$\begin{array}{ll}
(\text{hypoth}) & A \vdash x : \sigma, \text{ if } x : \sigma \in A \\[1em]
(\rightarrow\text{-intro}) & \dfrac{A_x \cup \{x : \tau\} \vdash M : \tau'}{A \vdash \lambda x.M : \tau \rightarrow \tau'} \\[1em]
(\rightarrow\text{-elim}) & \dfrac{A \vdash M : \tau \rightarrow \tau', \;\; A \vdash N : \tau}{A \vdash (M\,N) : \tau'} \\[1em]
(\text{let}) & \dfrac{A \vdash M : \sigma, \;\; A_x \cup \{x : \sigma\} \vdash N : \tau}{A \vdash \textbf{let } x = M \textbf{ in } N : \tau} \\[1em]
(\forall\text{-intro}) & \dfrac{A \cup C \vdash M : \tau', \;\; A \vdash C[\bar{\alpha} := \bar{\tau}]}{A \vdash M : \forall \bar{\alpha} \textbf{ with } C \,.\, \tau'} \quad (\bar{\alpha} \text{ not free in } A) \\[1em]
(\forall\text{-elim}) & \dfrac{A \vdash M : \forall \bar{\alpha} \textbf{ with } C \,.\, \tau', \;\; A \vdash C[\bar{\alpha} := \bar{\tau}]}{A \vdash M : \tau'[\bar{\alpha} := \bar{\tau}]}
\end{array}$$

Figure 3.1: System ML_o

Consider a term $M = \lambda x.\,\lambda y.\,((x * x) = y)$ which contains free identifiers $*$ and $=$, and the following assumption set.

$$A = \left\{ \begin{array}{l} * : \mathit{real} \rightarrow \mathit{real} \rightarrow \mathit{real}, \\ * : \mathit{int} \rightarrow \mathit{int} \rightarrow \mathit{int}, \\ = : \mathit{int} \rightarrow \mathit{int} \rightarrow \mathit{bool}, \\ = : \forall \alpha \textbf{ with } * : \alpha \rightarrow \alpha \rightarrow \alpha, = : \alpha \rightarrow \alpha \rightarrow \mathit{bool} \,.\, \mathit{list}(\alpha) \rightarrow \mathit{list}(\alpha) \rightarrow \mathit{bool} \end{array} \right\}$$

Here we show a derivation of

$$A \vdash \lambda x.\lambda y.((x * x) = y) : \forall \alpha \textbf{ with } * : \alpha \rightarrow \alpha \rightarrow \alpha, =: \alpha \rightarrow \alpha \rightarrow \mathit{bool} \,.\, \alpha \rightarrow \alpha \rightarrow \mathit{bool}$$

in ML_o.

$$\begin{array}{lll}
(1) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \cup \{x : \alpha\} \cup \{y : \alpha\} \vdash x : \alpha & (\text{hypoth}) \\
(2) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \cup \{x : \alpha\} \cup \{y : \alpha\} \vdash * : \alpha \rightarrow \alpha \rightarrow \alpha & (\text{hypoth}) \\
(3) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \cup \{x : \alpha\} \cup \{y : \alpha\} \vdash (*x) : \alpha \rightarrow \alpha & (\rightarrow\text{-elim}) \\
(4) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \cup \{x : \alpha\} \cup \{y : \alpha\} \vdash (x * x) : \alpha & (\rightarrow\text{-elim}) \\
(5) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \cup \{x : \alpha\} \cup \{y : \alpha\} \vdash y : \alpha & (\text{hypoth}) \\
(6) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \cup \{x : \alpha\} \cup \{y : \alpha\} \vdash =: \alpha \rightarrow \alpha \rightarrow \mathit{bool} & (\text{hypoth}) \\
(7) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \cup \{x : \alpha\} \cup \{y : \alpha\} \vdash = (x * x) : \alpha \rightarrow \mathit{bool} & (\text{hypoth}) \\
(8) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \cup \{x : \alpha\} \cup \{y : \alpha\} \vdash (x * x) = y : \mathit{bool} & (\text{hypoth}) \\
(9) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \cup \{x : \alpha\} \vdash \lambda y.((x * x) = y) : \alpha \rightarrow \mathit{bool} & (\rightarrow\text{-intro}) \\
(10) & A \cup \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\} \vdash \lambda x.\lambda y.((x * x) = y) : \alpha \rightarrow \alpha \rightarrow \mathit{bool} & (\rightarrow\text{-intro}) \\
(11) & A \vdash \{* : \alpha \rightarrow \alpha \rightarrow \alpha\} \cup \{=: \alpha \rightarrow \alpha \rightarrow \mathit{bool}\}[\alpha := \mathit{int}\} & (\text{hypoth}) \\
(12) & A \vdash \lambda x.\lambda y.((x * x) = y) : \forall \alpha \textbf{ with } * : \alpha \rightarrow \alpha \rightarrow \alpha, =: \alpha \rightarrow \alpha \rightarrow \mathit{bool} \,.\, \alpha \rightarrow \alpha \rightarrow \mathit{bool} & (\forall\text{-intro})
\end{array}$$

We are required to introduce assumptions about $*$ and $=$ in our derivation in order to arrive at a type for M. However, for our derivation to succeed, we need to be able to discharge those assumptions via the $\forall$-intro rule. This ensures that M can be derived from only our initial assumption set A. The $\forall$-intro rule deviates from generalization in *system ML* in that it requires that the constraint set C be derivable from the initial assumption set A. This ensures that C is *satisfiable* with respect to A. In our derivation, we can see from (11) that *satisfiability* is achieved by substituting *int* for α. In general, there can be more than one finite type which satisfies this requirement. For example, if $=$ were defined for reals, both *int* and *real* could be used in our substitution for α. Conversely, it is not always possible to achieve satisfiability. For instance, if we removed the second assumption on $*$ from A, our derivation would end at (10). There would be no single substitution for α which could satisfy the overlapping constraint requirements in (11) and we would conclude that M is untypable with respect to A.

This requirement for satisfiability of constraint sets ensures that the type system ML_o is sound. It is interesting to compare ML_o to a similar extension to *system ML* proposed by Kaes [Kae88] based on *type kinds*, where a *type kind* is a universe of types over which a type variable may be quantified. It proposed a restricted form of overloading which is generally the same restriction adopted by ML_o. However, this type system turns out to be unsound in that it does not enforce satisfiability of constraint sets as outlined above. This results in terms with multiple non-overlapping constraints being deemed typable in some instances. In the last example of the previous paragraph, for instance, the term M would be deemed typable. On the other hand, the similar work of [CDO91], in an effort to relax the restrictions on overloading in Kaes type system, enforces satisfiability and hence remains sound.

We have shown, by example, the process required to determine the typability of a term in ML_o. This process can be described as a modification to the concept, used in *system ML* , of *strong type inference* [Tiu90]. Formally, *strong type inference* says that a term M is typable with respect to an assumption set B if $A \vdash M : \sigma$ is derivable for some type σ and $B \subseteq A$. This criterion turns out to be less restrictive than required in the presence of overloading. We are free, under *strong type inference*, to choose any assumption set A which contains B. Returning to our derivation, it can be seen that, in step (11), we would have the freedom to introduce any new assumptions we required in order to satisfy typability under *strong type inference*, resulting in untypable terms being deemed typable. *Strong type inference* relies on the premise that assumption sets may contain at most one assumption per identifier. This premise, of course, does not hold in ML_o. We then can view typability in ML_o as being that of *strong type inference* with the requirement that $B = A$. In other words, $B \vdash M : \sigma$ must be derivable for some type σ.

IV. TYPE INFERENCE IN SYSTEM ML_o

An algorithm, based on W of *system ML* , has been developed for ML_o named W_o [Smi91]. In this chapter, we will discuss type inference utilizing W_o, which is given in Figure 4.1.

W_o infers *principal* types for typable expressions in ML_o , failing on untypable expressions. Given assumption set A and expression e, $W_o(A, e)$ returns (S, B, τ). S is a substitution such that $AS \cup B \vdash e : \tau$ is derivable. B represents a set of constraints on A, which describe dependencies associated with overloaded identifiers occurring in e, needed to arrive at a type for e. W_o, unlike W, utilizes the *least common generalization* (*LCG*) of an identifier overloaded in A. This concept, along with the function $close(A, B, \tau)$ and $unify(\tau, \tau')$, we will examine in some detail in this chapter.

The *LCG* of an overloaded identifier can, perhaps, be best described by beginning with an example. Consider the identifier $*$, overloaded in A with the assumptions $* : int \rightarrow int \rightarrow int$, $* : real \rightarrow real \rightarrow real$ and $* : int \rightarrow real \rightarrow real$. We can see that all of these assumptions have in common second and third arguments which are identical. There is no common ground in their structure with respect to their first arguments. We can describe their common structure by the use of two quantified type variables, one for the first argument and another for the remaining two. We would then assign as the *LCG* of $*$, $\forall\alpha, \beta . \alpha \rightarrow \beta \rightarrow \beta$.

More formally, we can say that a *common generalization* of some set of finite types $\tau_1, \ldots, \tau_n$ is τ if we can apply some set of substitutions $S_1, \ldots, S_n$ such that $\forall i . \tau S_i = \tau_i$. We further say that τ is a *least* common generalization if, for any other generalization τ' of τ, there exists a substitution S such that $\tau' S = \tau$. We can

```
$W_o(A, e)$ is defined by cases:

$e$ is $x$
    if $x$ is overloaded in $A$ with $LCG\ \forall\bar{\alpha}.\tau$,
        return $([\,], \{x : \tau S\}, \tau S)$
        where $S = [\bar{\alpha} := \bar{\beta}]$ and $\bar{\beta}$ are new
    else if $(x : \forall\bar{\alpha}\ \textbf{with}\ C\,.\,\tau) \in A$,
        return $([\,], CS, \tau S)$
        where $S = [\bar{\alpha} := \bar{\beta}]$ and $\bar{\beta}$ are new
    else $\mathit{fail}$.

$e$ is $\lambda x.M$
    let $(S, B, \tau) = W_o(A_x \cup \{x : \alpha\}, M)$ where $\alpha$ is new
    return $(S, B, \alpha S \rightarrow \tau)$.

$e$ is $M\,N$
    let $(S, B, \tau) = W_o(A, M)$
    let $(S', B', \tau') = W_o(AS, N)$
    let $S'' = \mathit{unify}(\tau S', \tau' \rightarrow \alpha)$ where $\alpha$ is new
    return $(SS'S'', BS'S'' \cup B'S'', \alpha S'')$.

$e$ is $\textbf{let}\ x = M\ \textbf{in}\ N$
    let $(S, B, \tau) = W_o(A, M)$
    let $(B', \sigma) = \mathit{close}(AS, B, \tau)$
    let $(S', B'', \tau') = W_o(A_x S \cup \{x : \sigma\}, N)$
    return $(SS', B'S' \cup B'', \tau')$.
```

Figure 4.1: Algorithm W_o

extend this principle to constrained type schemes by applying the concept over the *bodies* of each constrained type scheme. Least common generalizations are discussed in [Rey70], which gives an algorithm for computing them.

Function *unify* of W_o performs first-order unification of terms in expressions. In essence, $unify(\tau', \tau'')$ returns a substitution S such that $\tau' S = \tau'' S$, and fails if no such substitution exists. Formal discussions of unification are given by Knight and Robinson in [Rob65, Kni89].

Function *close* of W_o takes as input (A, B, τ) and returns a constrained type scheme for τ. This is accomplished, essentially, by applying the ($\forall$-intro) rule of ML_o to τ. Function *satisfy* within *close* checks for satisfiability of B with respect to A. The issue of satisfiability turns out to be one of the more interesting problems in the ML_o type system. We will discuss this problem, therefore, in detail later in this chapter. Actually, there is latitude in how one computes the *closure* of a type in W_o. A basic algorithm for *close* is given by Smith [Smi91] which is sufficient in supporting his *soundness* and *completeness* proofs of W_o, but leaves the critical issue of satisfiability somewhat unresolved. Our implementation of W_o uses an algorithm developed by Volpano which incrementally determines satisfiability as an expression is being constructed [Vol93a]. This approach allows us to detect certain type errors, with respect to constraints, earlier than the alternative approach of delaying satisfiability checks until the complete expression has been type checked.

We reproduce Volpano's algorithm for $close(A, B, \tau)$ here for the sake of completeness:

1. Let V be the set of all finite types in B. For any two types τ_1 and τ_2 in V, define an undirected edge (τ_1, τ_2) if types τ_1 and τ_2 share a type variable, and let E be the set of all such edges.

2. Let B' be the set of all constraints $x : \tau'$ in B for which there is no type τ'' such that τ'' contains a variable free in A and there is a path from τ' to τ'' in (V, E).

3. If B' is unsatisfiable under A then *fail*.
4. Let C be the set of all constraints $x : \tau'$ in B for which there is a type τ'' such that τ and τ'' share a type variable and there is a path from τ' to τ'' in (V, E).
5. Return $(B - B', \forall\bar{\alpha}$ **with** $C . \tau)$, where $\bar{\alpha}$ are the type variables free in C or τ but not A.

In steps (1) and (2) we define a graph which connects constraints in B which share a type variable, and extract types from B which do not overlap on a type variable. Set B' then contains all of the constraints in B which can be eliminated, provided they are collectively satisfiable with respect to A. If we assume as we do in our implementation, that the initial assumption set cannot contain free type variables, then in the final call to *close* we are guaranteed that all constraints in B will be discharged. This approach allows us to perform satisfiability checks in an incremental manner. We do not eliminate a constraint from B if it requires us to instantiate a type variable to some finite type; a subsequent term in the expression may *require* instantiation of that type variable, in which case we need to be able to ensure that previous overloading dependencies are satisfied. Consider the example, slightly modified, from [Vol93a]. If we have assumption set:

$$A = \left\{ \begin{array}{l} b \ : \mathit{bool} \\ + \ : \mathit{int} \rightarrow \mathit{int} \rightarrow \mathit{int}, \\ + \ : \mathit{int} \rightarrow \mathit{real} \rightarrow \mathit{real}, \\ = : \forall\alpha.\alpha \rightarrow \alpha \rightarrow \mathit{bool} \end{array} \right\}$$

recognizing that $+$ has *LCG* $\forall\alpha.\alpha \rightarrow \alpha \rightarrow \alpha$, say we have the partial expression

$$\lambda x.\,\textbf{let}\ y = \lambda z.\mathit{pair}(z + z, z + x)\ \textbf{in} < \mathit{exp} > .$$

where $<\mathit{exp}>$ represents a placeholder. W_o, in the process of computing a type for y, makes the call,

$$\mathit{close}(A \cup \{x : \alpha\},\ B,\ \gamma \rightarrow (\gamma \times \alpha)),$$

where B is the constraint set,

$$B = \{+ : \gamma \rightarrow \gamma \rightarrow \gamma,\ + : \gamma \rightarrow \alpha \rightarrow \alpha\}.$$

Function *close* determines that B' is empty, since all constraints in B share a type variable, γ. So B is determined satisfiable, and close leaves B intact in returning

$$(B,\ \forall\gamma \textbf{ with } B\,.\,\gamma \rightarrow (\gamma \times \alpha)).$$

Now, suppose we replace <*exp*> with the term $x = b$. This determines the type of x to be *bool*. W_o now makes its final call to *close* for the entire λ-expression as

$$close(A,\ B,\ bool \rightarrow bool)$$

where

$$B = \{+ : \gamma \rightarrow \gamma \rightarrow \gamma,\ + : \gamma \rightarrow bool \rightarrow bool\}.$$

In our final call to close, since our initial assumption set contains no free type variables, step (2) of the algorithm discharges all assumptions from B. This final call, then, fails since the second constraint on $+$ is unsatisfiable. In the previous call to close, if we had discharged the constraints on $+$ by including them in B', satisfiability would be decided by instantiation of γ to *int* and α to *real*. As a result, the final call to close would succeed, causing an untypable expression to be deemed typable.

A. PARAMETRIC OVERLOADING AND SATISFIABILITY

Typability in ML_0 is Turing reducible to the problem of deciding whether a set of constraints is satisfiable with respect to a given set of type assumptions. Through the use of constrained type schemes, we can be very expressive in representing overloadings. It turns out that, unless we restrict our representations to certain kinds of

$$
\begin{array}{l}
/ : int \to int \to real \\
/ : real \to real \to real \\
+ : int \to int \to int \\
+ : real \to real \to real \\
+ : \forall\alpha \textbf{ with } + : \alpha \to \alpha \to \alpha . \\
\qquad list(\alpha) \to list(\alpha) \to list(\alpha) \\
avg : \forall\alpha \textbf{ with } + : \alpha \to \alpha \to \alpha,\ / : \alpha \to \alpha \to real . \\
\qquad list(\alpha) \to real \\
avg : \forall\alpha \textbf{ with } + : \alpha \to \alpha \to \alpha,\ / : \alpha \to \alpha \to real . \\
\qquad set(\alpha) \to real
\end{array}
$$

Figure 4.2: Infinite and recursive overloadings

overloadings, the problem of constraint-set satisfiability, and therefore typability, in ML_o is undecidable [Smi91].

Consider the assumption set in Figure 4.2. We can see that the assumptions on *avg* and + contain ***infinite overloadings***, e.g., + can assume a finite type, say $list(list(\ldots(list(int))))$. Note also the occurrences of *recursive* overloadings, where the satisfiability of the constraint set depends on the assumption itself. A *mutually recursive* overloading would result if we added a constraint involving *avg* to the third assumption on +.

Constraint-set satisfiability remains undecidable in the presence of mutual recursion and/or straight recursion without restrictions [Vol94a]. We should therefore explore suitable bounds on recursion which make our satisfiability problem decidable. We can see that recursion is a natural occurrence in practice through our example in Figure 4.2. For this reason, while it makes constraint-set satisfiability decidable, forbidding recursion entirely is unacceptable.

Various approaches have been examined. Smith gives a restriction called *overloading by constructors* which makes constraint-set satisfiability decidable in polynomial time [Smi91]. But it disallows constraints on an overloaded identifier x involving y where $x \neq y$. This would prohibit the overloading on *avg* in Figure 4.2. Another restriction, similar to that proposed by Kaes [Kae88] and adopted by Haskell [Has89], is called *parametric overloading.*

Parametric overloading is a more practical form of overloading which allows naturally recursive overloading like that of Figure 4.2 and makes constraint-set satisfiability decidable. This is the form that we adopt in this thesis.

Parametric overloading makes use of the concept of the *least common generalization* of finite types discussed earlier. We give a formal definition here from [Vol94b].

Definition A..1 *Parametric assumption sets* are defined inductively.

The empty set is parametric.

If A is parametric with no assumption for x and σ is a constrained type scheme $\forall\alpha$ **with** $C\,.\,\tau$ such that for each $z : \rho \in C$, z is overloaded in A and ρ is a generic instance of its LCG then $A \cup \{x : \sigma\}$ is parametric.

If A is parametric with no assumption for x and B is the set

$$\left\{ \begin{array}{l} x : \forall\bar{\gamma}_1 \ \mathbf{with}\ C_1\,.\,\tau[\alpha := \chi_1(\bar{\gamma}_1)] \\ \vdots \\ x : \forall\bar{\gamma}_n \ \mathbf{with}\ C_n\,.\,\tau[\alpha := \chi_n(\bar{\gamma}_n)] \end{array} \right\}$$

such that

- x has LCG $\forall\alpha.\ \tau$,
- $\chi_i \neq \chi_j$ for $i \neq j$ (where χ's are type constructors of various arities), and
- $z : \rho \in C_i$ implies that z has LCG $\forall\pi.\ \rho$, for some $\pi \in \bar{\gamma}_i$, and either z is overloaded in A or $z = x$,

then $A \cup B$ is parametric.

Note that we can only specify constraints which involve an overloaded identifier; constraints involving finite types or even polymorphic types are not allowed under our definition. Though there are instances where this limits the practical use of parametric overloading, this restriction is generally not a limiting factor in practice. Smith has considered approaches to relaxing this particular restriction for type checking a language with subtyping and overloading [Smi91, Smi93]. This thesis, however, considers overloading only. We can also make the observation that an identifier x parametrically overloaded in A can always be characterized by an LCG which has only one quantified variable. This gives us a practical view of the restrictions we are talking about.

$$\Sigma = \left[\begin{array}{llll} \Sigma_0 : & int, & real, & bool \\ \Sigma_1 : & list, & ref & \\ \Sigma_2 : & map, & pair & \end{array} \right]$$

Figure 4.3: Type constructors of various arities

We can characterize parametric overloadings as a regular forest of trees [Vol94b]. These regular forests can be generated by a class of context-free grammars called regular tree grammars [GeS84]. If A is parametric then every overloaded identifier x in A has an LCG of the form $\forall\alpha.\tau$ and the set of finite types π to which α can be instantiated, meaning $A \vdash x : \tau[\alpha := \pi]$ is derivable, form a regular tree language or forest.

B. SATISFIABILITY ALGORITHM

The determination of constraint-set satisfiability, which is computed by the function *satisfiable*(A, C), takes the assumption set A and the constraint set C as inputs. For any parametric assumption set A, we can construct for every overloaded identifier x a regular tree grammar G_x such that if x has LCG $\forall\alpha.\tau$ then for any variable-free finite type τ', we can derive $A \vdash x : \tau[\alpha := \tau']$ if an only if $\tau' \in L(G_x)$, where $L(G_x)$ represents the regular tree language generated by G_x. In this context, we need only parse τ' with respect to $L(G_x)$ to determine whether constraint $x : \tau[\alpha := \tau']$ is satisfiable with respect to A.

An algorithm for satisfiability has been developed based on the property that regular forests are effectively closed under intersection [Vol94b]. Our implementation of W_o uses this algorithm. Consider an example using the parametric assumption set of Figure 4.2 and the type constructors in Figure 4.3, which includes constructors of arity-0,1 and 2.

We can see that /, + and *avg* are overloaded in A with respective *LCG*'s: $\forall\alpha.\,\alpha \rightarrow \alpha \rightarrow \mathit{real}$, $\forall\alpha.\,\alpha \rightarrow \alpha \rightarrow \alpha$ and $\forall\alpha.\,\alpha \rightarrow \mathit{real}$. Our first task is to construct a dependency graph of assumptions in A; if an assumption on x contains a constraint on y we need to produce the grammar of y before we produce x's grammar. We then can proceed to create regular tree grammars for each overloaded identifier in A based on dependencies. We see that *avg* depends on / and + in A so we must compute the grammar for *avg* last.

Since identifiers may be overloaded recursively, as in our example, we will represent occurrences of an identifier x in its own constraint list with the start symbol for G_x. In the case of constrained type schemes with multiple constraints, as occurs in *avg*, we will represent this as a new non-terminal. This non-terminal will define new productions for the grammar which result from the computed intersection of the constraints. Given a constraint set which contains a constraint on x and a constraint on y there intersection is computed as $L(G_x) \cap L(G_y)$.

We represent the type constructors in Σ as a grammar G_Σ. We can then take advantage of the fact that $L(G_\Sigma) \cap L(G_x) = L(G_x)$ for any overloaded identifier x as we construct our grammars for A. We therefore obtain the following set of grammars for our example assumption set A:

$$\left\{ \begin{array}{lllllll} G_\Sigma : & S = & \mathit{int} & \mid \; \mathit{real} & \mid \; \mathit{bool} & \mid \; \mathit{list}(S) & \mid \\ & & \mathit{ref}(S) & \mid \; S \rightarrow S & \mid \; \mathit{pair}(S,S) & & \\ G_/ : & A = & \mathit{int} & \mid \; \mathit{real} & & & \\ G_+ : & B = & \mathit{int} & \mid \; \mathit{real} & \mid \; \mathit{list}(B) & & \\ G_{avg} : & C = & \mathit{list}(D) & \mid \; \mathit{set}(D) & & & \\ & D = & \mathit{int} & \mid \; \mathit{real} & & & \end{array} \right\}$$

where the non-terminal D represents $L(G_/) \cap L(G_+)$.

In our *batch* implementation of W_o, where we do not allow occurrences of free type variables in the initial assumption set, we can create the set of regular tree grammars once and reuse the representation. This was the approach we adopted in our implementation.

We can now determine satisfiability of a constraint set C with relation to an assumption set B by parsing each constraint in C, of the form $id : \tau$, with respect to the grammars computed for B i.e. if τ parses with respect to $L(G_{id})$ for each constraint in C then C is satisfiable. It is possible, though, that we may encounter overlapping constraints in C. In this case we must first compute the intersections on any overlapping constraints before parsing those that don't overlap. If the computed intersection is empty then C is unsatisfiable. An intersection is empty if there exists no common type constructor of arity-0 between constraints. For example, grammar G below represents an empty intersection.

$$G = \mathit{list}(G) \mid \mathit{ref}(G)$$

This algorithm is exponential in the number of forests input, but this is very likely the best we can do for the problem has been shown NP-complete [Vol94b]. The use of our implementation of W_o should provide valuable insight into determining whether the NP lower bound for constraint set satisfiability is a practical limitation.

V. IMPLEMENTATION OF W_o

As we have shown, algorithm W_o has been developed to infer the most general type of a term given suitable forms of overloading. We envision an interactive programming environment in which incomplete expressions are type checked (may have placeholder terms) and can be subsequently updated, perhaps requiring new types to be inferred.

In this setting, W_o is unsuitable because it is not *incremental.* If a function, say f, is computed on input x, then on input change Δ, we say that the computation of $f(x+\Delta)$ is incremental if $f(x+\Delta)$ is computed from only $f(x)$ and Δ. Although our implementation does not type check definitions, it nonetheless exhibits incremental type re-computation at the expression level, as we will show.

In efforts to develop an incremental approach to type inference, we might attempt to re-write W_o. We have, however chosen an approach which makes use of a formalism, namely *attribute grammars*, for achieving incremental type re-computation. Utilizing this formalism we foresee our implementation not only providing a means to validate and explore bounds on the problem of type inference in the presence of overloading, but also as a step towards integrating incremental algorithms for on-line type inference and those for overloading.

A. THE ROLE OF ATTRIBUTE GRAMMARS

Updating expressions affords an opportunity to re-use previous type computation. The attribute grammar formalism provides a framework in which type re-computation is identified with attribute re-evaluation. So if attribute re-evaluation is done incrementally then type re-computation is incremental as well and furthermore it is implicit in the formalism.

Using an attribute grammar we can specify the syntax of a language via a context-free grammar. Nodes of parse trees are annotated with attributes that are prescribed by a set of attribute equations given as part of the attribute grammar. If a parse tree is edited then attributes of the tree are re-computed using the equations so that a consistent attribution is maintained. Re-computing the attributes is implicit and is done by the attribute evaluator.

The productions of the context-free grammar for type inference in ML_o which we have developed for our implementation are given in Figure 5.1. Non-terminals are represented in upper case while terminals are in lower case. Terminals in productions that begin with *Null* represent placeholder terms which have universal type $\forall\alpha.\alpha$.

Attributes are distinguished as either *synthesized* or *inherited*. Synthesized attributes occur on the left-hand side of attribute equations; inherited attributes occur on the right-hand side. In other words, in one case attributes are propagated up (synthesized) in the parse tree and in the other they are propagated down (inherited) in the parse tree. Figure 5.2 shows the inherited (AI) and synthesized (AS) attributes associated with the productions of Figure 5.1.

To implement W_o, we define attribute equations, which create dependencies between attribute values. As the derivation tree is updated these dependencies determine what part of the tree is affected and where *selective re-computation*, via the attribute equations, needs to be done in order to re-establish consistent *attribute values* throughout the tree. The set of attribute equations in Figure 5.3 then defines the dependencies required in each attributed production from Figure 5.1 to implement W_o. Functional support, indicated by italics, is simplified and represented by descriptive function names. The attributes S and B of EXP are precisely those terms returned by W_o as discussed in Chapter IV.

(1)	TOPLEVEL	→ ASSUMPTIONSET EXPLIST
(2)	TOPLEVEL	→ NullPrgrm
(3)	ASSUMPTIONSET	→ ASSUMPTIONLIST
(4)	ASSUMPTIONSET	→ NullAssumptions
(5)	ASSUMPTIONLIST_1	→ ASSUMPTION ASSUMPTIONLIST_2
(6)	ASSUMPTIONLIST	→ NullAssumption
(7)	ASSUMPTION	→ ID TYPESCHEMELIST
(8)	ID	→ id
(9)	ID	→ NullId
(10)	TYPESCHEMELIST_1	→ TYPESCHEME TYPESCHEMELIST_2
(11)	TYPESCHEMELIST	→ NullTypeSchemeList
(12)	TYPESCHEME	→ TYPEVARLIST CONSTRAINTLIST TYPEEXP
(13)	TYPESCHEME	→ NullTypeScheme
(14)	TYPEVARLIST_1	→ QUANTTYPEVAR TYPEVARLIST_2
(15)	TYPEVARLIST	→ NullTypeVarList
(16)	QUANTTYPEVAR	→ TypeVar
(17)	QUANTTYPEVAR	→ NullTypeVar
(18)	CONSTRAINTLIST_1	→ CONSTRAINT CONSTRAINTLIST_2
(19)	CONSTRAINTLIST	→ NullConstraintList
(20)	CONSTRAINT	→ ID TYPEEXP
(21)	CONSTRAINT	→ NullConstraint
(22)	TYPEEXP	→ UniversalType
(23)	TYPEEXP	→ Int
(24)	TYPEEXP	→ Real
(25)	TYPEEXP	→ Bool
(26)	TYPEEXP	→ TypeVar
(27)	TYPEEXP	→ NullType
(28)	TYPEEXP_1	→ Map(TYPEEXP_2 TYPEEXP_3)
(29)	TYPEEXP_1	→ Pair(TYPEEXP_2 TYPEEXP_3)
(30)	TYPEEXP_1	→ List(TYPEEXP_2)
(31)	TYPEEXP_1	→ Seq(TYPEEXP_2)
(32)	TYPEEXP_1	→ Ref(TYPEEXP_2)
(33)	EXPLIST_1	→ EXP EXPLIST_2
(34)	EXPLIST	→ NullExpression
(35)	EXP	→ ID
(36)	EXP_1	→ EXP_2 EXP_3
(37)	EXP_1	→ λ ID.EXP_2
(38)	EXP_1	→ let ID = EXP_2 in EXP_3
(39)	EXP	→ NullExp

Figure 5.1: Context-free grammar for ML_0 type inference

$AI_{TOPLEVEL} = \{\}$	$AS_{TOPLEVEL} = \{\}$
$AI_{ASSUMPTIONSET} = \{\}$	$AS_{ASSUMPTIONSET} = \{\text{typeEnv}\}$
$AI_{ASSUMPTIONLIST} = \{\}$	$AS_{ASSUMPTIONLIST} = \{\text{typeEnv}\}$
$AI_{ASSUMPTION} = \{\}$	$AS_{ASSUMPTION} = \{\text{typeEnv}\}$
$AI_{ID} = \{\}$	$AS_{ID} = \{\text{name}\}$
$AI_{EXPLIST} = \{\text{typeEnv,typeGrammar}\}$	$AS_{EXPLIST} = \{\}$
$AI_{EXP} = \{\text{typeEnv,typeGrammar}\}$	$AS_{EXP} = \{\text{S,B,typeAssignment}\}$

Figure 5.2: Inherited and synthesized attributes of implementation grammar

To illustrate how incremental type recomputation is achieved via incremental attribute evaluation, consider Figure 5.4. Here we have a partial derivation tree annotated with a dependence graph showing the propagation of attributes in the tree. For simplicity, we have chosen one inherited attribute and one synthesized attribute. The inherited attribute A represents an assumption set. The synthesized attribute T is a constrained type scheme representing the type of an expression at each node of the tree. Figure 5.4 represents the partial derivation tree for the expression $pr\,(x, \lambda y . \lambda z .\ y\ z)$, where pr is of type $\forall \alpha, \beta . \alpha \rightarrow \beta \rightarrow pair(\alpha, \beta)$. Suppose the expression rooted at node n_3 is updated. We can see that T at node n_2 now must be recomputed but notice that no change has been made to the expression rooted at node n_4, which therefore need not be retypechecked. In practice, this can result in significant savings as the tree whose root is n_4 can be arbitrarily large.

1. The Synthesizer Generator Platform

An attribute evaluator generator takes as input a set of attribute equations, such as those in Figure 5.3, for a set of terms and outputs an attribute evaluator that takes a term and annotates it with an attribution as prescribed by the equations. There are attribute evaluator generators available today that not only output an attribute evaluator but output one that evaluates attributes incrementally. One such generator is GrammaTech's Synthesizer Generator (SynGen).

(1) EXPLIST.typeEnv = *InitialEnv*() @ ASSUMPTIONSET.typeEnv
EXPLIST.typeGrammar = *ComputeGrammar*(ASSUMPTIONSET.typeEnv)

(3) ASSUMPTIONSET.typeEnv = ASSUMPTIONLIST.typeEnv
(4) ASSUMPTIONSET.typeEnv = NullTypeEnv

(5) ASSUMPTIONLIST$_1$.typeEnv = *ConcatEnv*((ASSUMPTION.name, ASSUMPTION.type),
ASSUMPTIONLIST$_2$.typeEnv)

(6) ASSUMPTIONLIST.typeEnv = NullTypeEnv

(7) ASSUMPTION.name = ID.name
ASSUMPTION.type = TYPESCHEMELIST

(8) ID.name = id
(9) ID.name = "undeclared"

(33) EXP.typeEnv = EXPLIST$_1$.typeEnv
EXP.typeGrammar = EXPLIST$_1$.typeGrammar
EXPLIST$_2$.typeEnv = EXPLIST$_1$.typeEnv
EXPLIST$_2$.typeGrammar = EXPLIST$_1$.typeGrammar

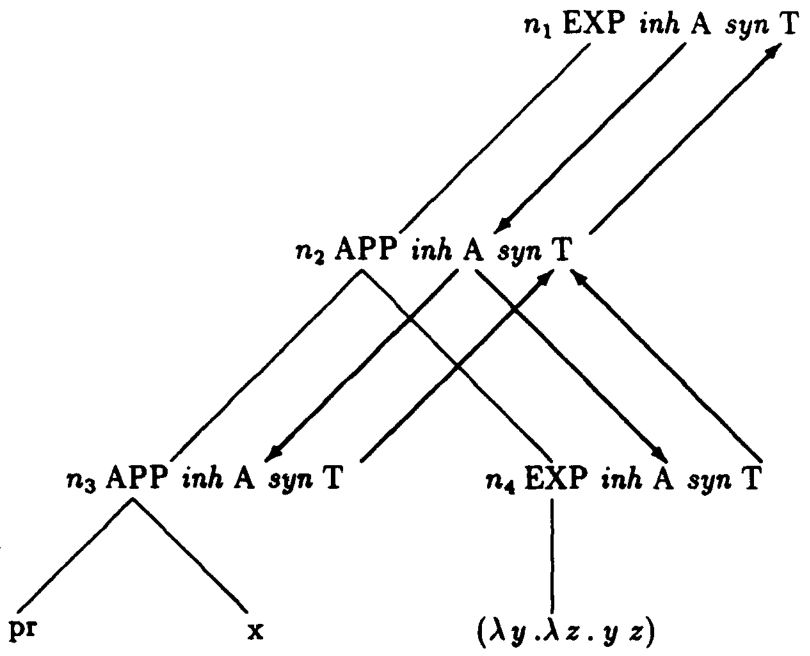
(35) EXP.typeAssignment = *ComputeType*(ID.name, EXP.typeEnv)
(36) EXP$_1$.S = **let V =** (*Unify*(EXP$_3$.S EXP$_2$.typeAssignment),
(EXP$_3$.typeAssignment $\rightarrow$ *NewVar*(**beta**))) **in**
V (EXP$_3$.S EXP$_2$.S)
EXP$_1$.typeAssignment = **V beta**
EXP$_1$.B = (**V** (EXP$_3$.S EXP$_2$.B)) @
(**V** EXP$_3$.B)
EXP$_2$.typeEnv = EXP$_1$.typeEnv
EXP$_3$.typeEnv = EXP$_2$.S EXP$_1$.typeEnv
EXP$_2$.typeGrammar = EXP$_1$.typeGrammar
EXP$_3$.typeGrammar = EXP$_1$.typeGrammar

(37) EXP$_1$.typeAssignment = (EXP$_2$.S *NewVar*(**beta**)) $\rightarrow$ EXP$_2$.typeAssignment
EXP$_1$.S = EXP$_2$.S
EXP$_1$.B = EXP$_2$.B
EXP$_2$.typeEnv = *ConcatEnv*((ID.name, **beta**), EXP$_1$.typeEnv)
EXP$_2$.typeGrammar = EXP$_1$.typeGrammar

(38) **let** (**B'**, σ) = *Close*((EXP$_2$.S EXP$_1$.typeEnv), EXP$_2$.B,
EXP$_2$.typeAssignment, EXP$_1$.typeGrammar)
EXP$_1$.typeAssignment = EXP$_3$.typeAssignment
EXP$_1$.S = (EXP$_3$.S EXP$_2$.S)
EXP$_1$.B = (EXP$_3$.S **B'**) @ EXP$_3$.B
EXP$_2$.typeEnv = EXP$_1$.typeEnv
EXP$_3$.typeEnv = *ConcatEnv*((ID.name, σ), (EXP$_2$.S EXP$_1$.typeEnv))
EXP$_2$.typeGrammar = EXP$_1$.typeGrammar
EXP$_3$.typeGrammar = EXP$_1$.typeGrammar

(39) EXP.typeAssignment = *NewVar*(**beta**)
EXP.S = NullSubst
EXP.B = NullConstraintList

Figure 5.3: Attribute equations for ML_o type inference



n_1 EXP *inh* A *syn* T
n_2 APP *inh* A *syn* T
n_3 APP *inh* A *syn* T
n_4 EXP *inh* A *syn* T
pr
x
$(\lambda y . \lambda z . y\ z)$

Figure 5.4: A partial derivation tree and dependence graph for $pr\,(x, \lambda y . \lambda z . y\ z)$

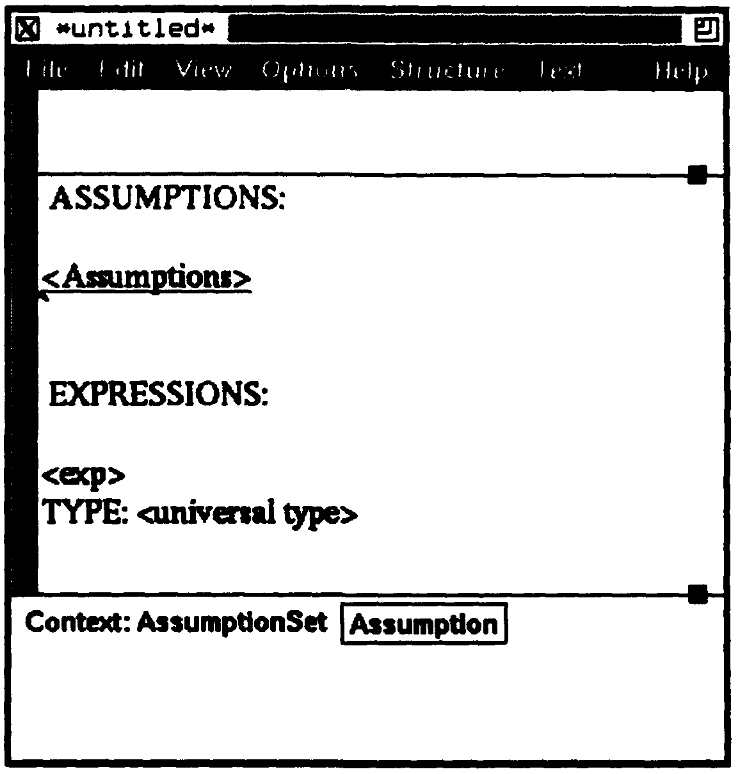
We have developed our implementation utilizing SynGen for several reasons. We are able to get a comprehensive and visually appealing X-windows interface with relative ease. Utilizing Syngen also fits nicely with our opinion of attribute grammars as being a desirable approach to achieving incremental type inference. Furthermore, since we profit by the incremental algorithms embedded in SynGen, new advances in this area, which may well be incorporated in future versions, will directly enhance our implementation.

The incremental algorithms used in SynGen rely heavily on the concept of *ordered attribute grammars* which were introduced in [Kas80]. The ordered attribute grammars are a subclass of the noncircular attribute grammars. Though SynGen can accept attribute grammars which are not ordered, it prohibits circular attribute grammars.

The language of SynGen is *SSL*. Every (useful) SSL specification has three major declaration areas: *Abstract syntax* which defines a set of grammar rules, *Attribution* which annotates the grammar with attributes and describes their dependencies, and *Unparsing* which defines display formats for terms, identifies selectable productions of the grammar and annotates which productions are editable. For our implementation, Figure 5.1 represents the Abstract syntax and Figures 5.2 and 5.3 represent the Attribution.

2. The Implementation

We demonstrate our implementation through an annotated sequence of actual X-windows display screens generated by our type checker. Figure 5.5 shows an initial screen with placeholders for an assumption set entry, where we define extensions to an Initial Environment, and an expression. The currently selected term, corresponding to the *ASSUMPTIONSET* production of our grammar, is underlined. Note that the type inferred for the placeholder term <exp> is <universal type>.



ASSUMPTIONS:

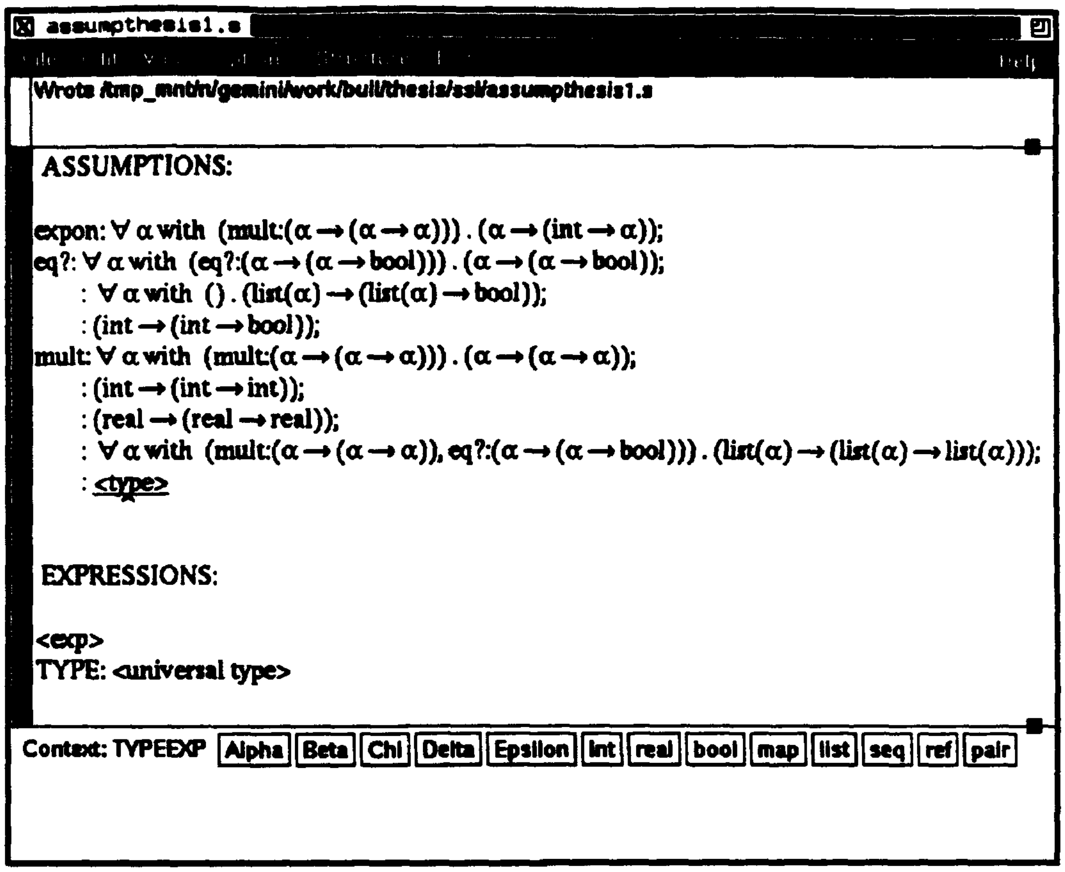
<Assumptions>

EXPRESSIONS:

<exp>
TYPE: <universal type>

Context: AssumptionSet Assumption

Figure 5.5: Implementation initial screen.



```
assumpthesis1.s
Wrote /tmp_mnt/n/gemini/work/bull/thesis/ssl/assumpthesis1.s

 ASSUMPTIONS:

expon: ∀ α with (mult:(α → (α → α))) . (α → (int → α));
eq?: ∀ α with (eq?:(α → (α → bool))) . (α → (α → bool));
     : ∀ α with () . (list(α) → (list(α) → bool));
     : (int → (int → bool));
mult: ∀ α with (mult:(α → (α → α))) . (α → (α → α));
     : (int → (int → int));
     : (real → (real → real));
     : ∀ α with (mult:(α → (α → α)), eq?:(α → (α → bool))) . (list(α) → (list(α) → list(α)));
     : <type>


 EXPRESSIONS:

<exp>
TYPE: <universal type>

Context: TYPEEXP [Alpha] [Beta] [Chi] [Delta] [Epsilon] [int] [real] [bool] [map] [list] [seq] [ref] [pair]
```

Figure 5.6: An assumption set defined.

We have entered an assumption set in Figure 5.6 with three overloaded identifiers. The first type scheme for each identifier, without the constraints, must represent the *LCG* of that identifier. The implementation currently does not compute the *LCG* and so it must be provided by the user. Note the terms enclosed in boxes at the bottom of the screen. These are called *transforms*. With the placeholder for *TYPEEXP* selected, we may select a transform with the mouse and replace the selected placeholder term with a term associated with the transform. This provides an alternative means to enter terms without the need for the user to remember the appropriate syntax. Users may also enter terms directly as long as the term being edited is defined in the *unparsing* rules.

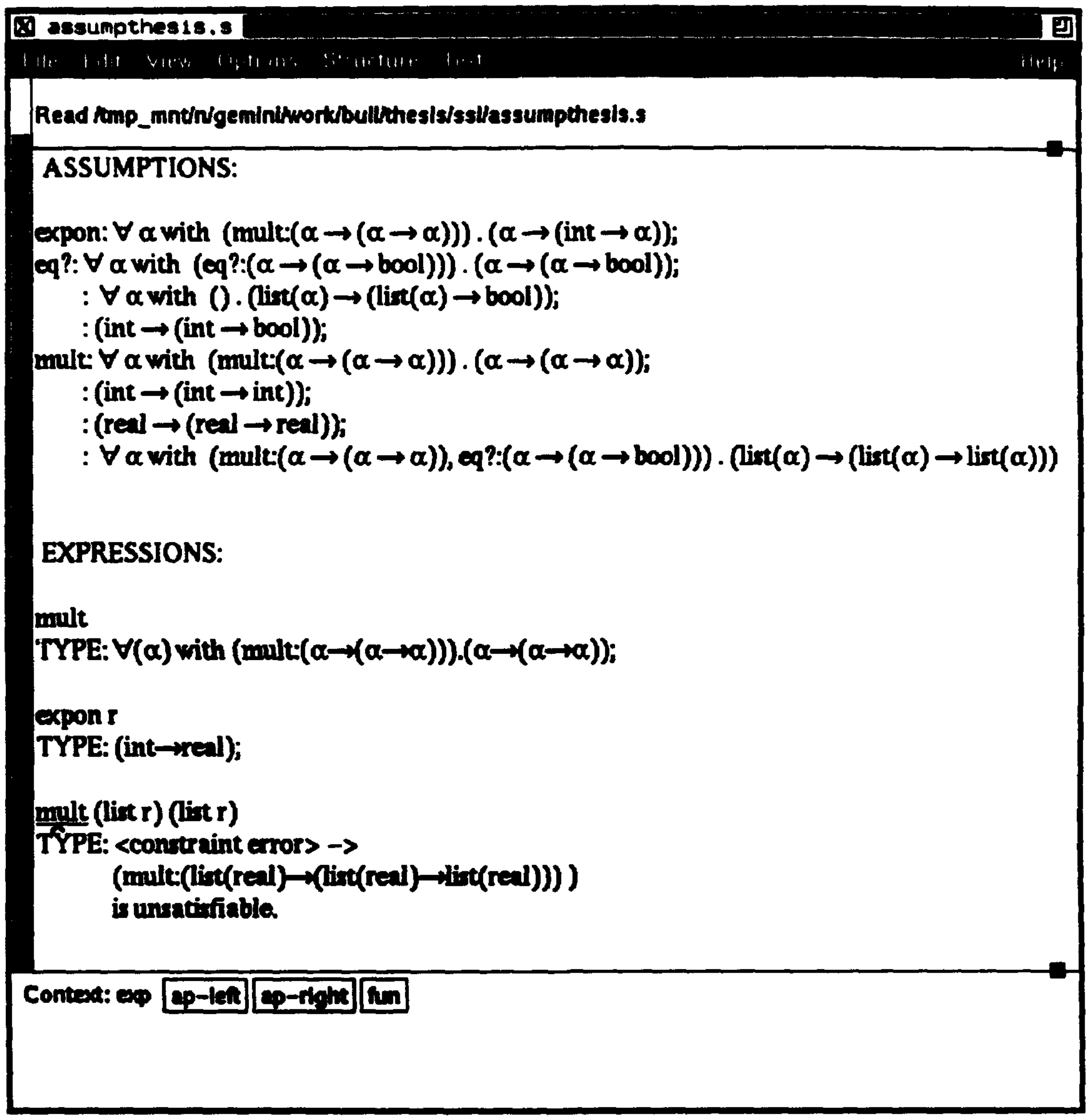
In Figure 5.7 we have entered three expressions whose types have been inferred. The type of our first expression is represented by a constrained type scheme; it is the most general type we can give to it and we can be no more specific without more information. In the second expression, where r : *real* is defined in the initial environment, we see that, since *mult* is defined over reals, applying *expon* to r satisfies the constraint on *expon* and we are able to infer a finite type for the expression. An unsatisfiable constraint has been encountered in our final expression. This is a result of the multiple constraints on *mult* and *eq?* in the third assumption of *mult*. We can see that the grammar for *mult* is:

$$\left\{ \begin{array}{lll} G_{mult}: & S = & int \mid real \mid list(U) \\ & U = & int \mid list(U) \end{array} \right\}$$

which clearly does not derive *list*(*real*).

It is also possible to directly examine the attributes of the parse tree at any point in the execution. This functionality, though mainly useful for debugging, can provide a means to investigate aspects of the implementation from a lower level viewpoint. For example, one might wish to examine a representation of the regular tree grammars produced for overloaded identifiers in a given assumption set. This can be done by examining the attribute *typeGrammar* at any *EXP* node of the parse tree. For instance, Figure 5.8 shows the regular tree grammars computed for the assumption set of Figure 5.7. Note that we have chosen to represent the start symbol of grammar G_{id} as *id*, for each overloaded identifier *id* in the assumption set. In addition, $id_1\_*\_id_2$ was chosen to represent $L(G_{id_1}) \cap L(G_{id_2})$.

We have given a brief overview, through examples, of the X-windows interface and general functionality of our implementation of W_o with parametric overloading. By examining instances of type inference in ML_o, in the setting of our implementation, we have endeavored to provide the reader with a clearer understanding of concepts discussed more formally in previous chapters.

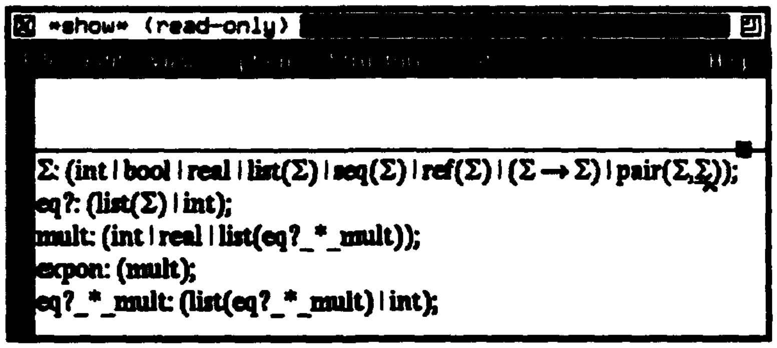


Read /tmp_mnt/n/gemini/work/bull/thesis/ssl/assumpthesis.s

```
ASSUMPTIONS:

expon: ∀ α with (mult:(α → (α → α))) . (α → (int → α));
eq?: ∀ α with (eq?:(α → (α → bool))) . (α → (α → bool));
    : ∀ α with () . (list(α) → (list(α) → bool));
    : (int → (int → bool));
mult: ∀ α with (mult:(α → (α → α))) . (α → (α → α));
    : (int → (int → int));
    : (real → (real → real));
    : ∀ α with (mult:(α → (α → α)), eq?:(α → (α → bool))) . (list(α) → (list(α) → list(α)))


EXPRESSIONS:

mult
TYPE: ∀(α) with (mult:(α→(α→α))).(α→(α→α));

expon r
TYPE: (int→real);

mult (list r) (list r)
TYPE: <constraint error> ->
        (mult:(list(real)→(list(real)→list(real))) )
        is unsatisfiable.
```

Context: exp

Figure 5.7: Type inference of three expressions.



```
Σ: (int | bool | real | list(Σ) | seq(Σ) | ref(Σ) | (Σ → Σ) | pair(Σ,Σ));
eq?: (list(Σ) | int);
mult: (int | real | list(eq?_*_mult));
expon: (mult);
eq?_*_mult: (list(eq?_*_mult) | int);
```

Figure 5.8: Representation of attribute *typeGrammar*

VI. CONCLUSIONS

We have considered the problem of type inference in an extension to the type system *ML* called ML_o. The type system ML_o is a formalism which is more suitable for implementing future languages by virtue of its incorporation of global overloading. Yet this increased functionality introduces new problems in developing algorithms which make typability decidable. Without restrictions on the types of overloadings and the structure of constraint sets, typability in ML_o is undecidable. Typability in ML_o is Turing reducible to the problem of determining if a set of constraints is satisfiable with respect to a given set of assumptions. If assumption sets are restricted to parametric overloadings the problem of constraint set satisfiability is NP-complete.

The type inference algorithm W_o with parametric overloading has been implemented utilizing the formalism of attribute grammars with GrammaTech's Synthesizer Generator. It performs type inference on expressions in an interactive environment. Type inference is performed incrementally so that the types of partial expressions can be inferred and efficiency of re-computation in the presence of updates is enhanced. Consequently, immediate feedback is provided to the user as expressions are entered and updated.

Our implementation will be used to examine the practical bounds on the problem of constraint-set satisfiability. It will also represent a significant tool for exploring the limits of bounded polymorphism, or overloading, in programming languages. Can we devise new forms of overloading which are more flexible than parametric overloading yet retain a decidable satisfiability problem?

A. FUTURE WORK

This thesis will serve as a basis for further research aimed at ultimately developing a type discipline for a class of implicitly-typed imperative programming languages with subtypes. overloading and polymorphism. A more immediate goal is to merge our implementation of W_o with an *SSL* implementation that performs on-line type inference utilizing the type inference algorithm W of *ML* .

On-line type inference allows the introduction of new global definitions as a program is produced. This differs from our batch implementation, where we have assumptions about types of free ids available to each expression in the form of an assumption set. The incorporation of overloading in an on-line implementation will be the subject of the next step in this research effort. This will produce an interactive environment where global definitions, perhaps overloaded, may be introduced at any point in the program. Types of all dependent terms are then recomputed as a result of these new definitions.

LIST OF REFERENCES

[ASU86] Aho, A., Sethi, R. and Ullman, J.: Compilers: Principles, Techniques, and Tools, *Addison-Wesley Publishing Company*, 1986.

[Car87] Cardelli, L.: Basic polymorphic typechecking, *Science of Computer Programming*, **8**, pp. 147–172, 1987.

[CHO92] Chen, K., Hudak, P. and Odersky, M.: Parametric Type Classes, *Proc. 7th ACM Conf. on Lisp and Functional Programming*, pp. 170–181, 1992.

[CDO91] Cormack, G. Duggan, D. and Ophel, J.: Decidable Type Reconstruction with Recursive Overloading (Extended Abstract), Department of Computer Science, University of Waterloo, 1991.

[DaM82] Damas, L. and Milner, R.: Principal Type Schemes for Functional Programs, *Proc. 9th ACM Symposium on Principles of Programming Languages*, pp. 207–212, 1982.

[GeS84] Gecseg, F. and Steinby M.: Tree Automata, Akademiai Kiado, Budapest Hungary, 1984.

[Gun92] Gunter, C.: Semantics of Programming Languages, Structures and Techniques, The MIT Press, 1992.

[Has89] Report on the Programming Language Haskell, Version 1.0, April 1990.

[HMM86] Harper, R., MacQueen, D. and Milner, R.: Standard ML. Technical Report ECS–LFCS–86–2, Department of Computer Science, University of Edinburgh, 1986.

[Jon92] Jones, M.: A theory of qualified types, *Proc. 4th European Symposium on Programming, LNCS* **582**, Springer-Verlag, pp. 287–306, 1992.

[Kae88] Kaes, S.: Parametric Overloading in Polymorphic Programming Languages, *Proc. 2nd European Symposium on Programming, LNCS* **300**, Springer-Verlag, pp. 131–144, 1988.

[Kae92] Kaes, S.: Type Inference in the Presence of Overloading, Subtyping, and Recursive Types, *Proc. 7th ACM Conf. on Lisp and Functional Programming*, pp. 193–204, 1992.

[Kas80] Kastens, U.: Ordered attribute grammars, *Acta Inf.*, **13,3**: pp. 229–256, 1980.

[Kni89] Knight, K.: Unification: A multidisciplinary survey. *ACM Computing Surveys*, **21(1)**: pp. 93–124, March 1989.

[KT90] Kfoury, A. and Tiuryn, J., Type reconstruction in finite-rank fragments of the polymorphic λ-calculus. *Fifth IEEE Symposium on Logic in Computer Science*, pp. 2–11, 1990.

[Lei83] Leivant, D.: Polymorphic Type Inference, *10th ACM Symposium on Principles of Programming Languages*, pp. 88–98, Austin, Texas, 1983.

[McC84] McCracken, N.: The Typechecking of Programs with Implicit Type Structure, *Semantics of Data Types LNCS*, **173**, pp. 301–315, 1984.

[Mil78] Milner, R.: A Theory of Type Polymorphism in Programming, *J. of Computer and System Sciences*, **17**, pp. 348–375, 1978.

[NiP93] Nipkow, T. and Prehofer, C.: Type Checking Type Classes, *Proc. 20th ACM Symposium on Principles of Programming Languages*, pp. 409–418, 1993.

[Rey70] Reynolds, J.C.: Transformational Systems and the Algebraic Structure of Atomic Formulas, *Machine Intelligence*, **5**, pp. 135–151, 1970.

[Rob65] Robinson, J.A.: A machine-oriented logic based on the resolution principle, *Journal of ACM*, **12:1**, pp. 23–41, 1965.

[Smi91] Smith, G.S.: Polymorphic Type Inference for Languages with Overloading and Subtyping, Ph.D. Thesis, Department of Computer Science, Cornell University, Technical Report 91-1230, 1991.

[Smi93] Smith, G.S.: Polymorphic Type Inference with Overloading and Subtyping, *Proc. TAPSOFT '93, LNCS* **668**, Springer-Verlag, pp. 671–685, 1993.

[Tiu90] Tiuryn, J.: Type Inference Problems: A Survey, *Proc. Mathematical Foundations of Computer Science, LNCS* **452**, Springer-Verlag, pp. 105–120, 1990.

[Tho91] Thompson, S.: Type Theory and Functional Programming, Addison-Wesley Publishing Company, 1991.

[Tur86] Turner, D.: An Overview of Miranda, *ACM SIGPLAN Notices*,**21** pp. 156–166, 1986.

[VoS91] Volpano, D.M. and Smith, G.S.: On the Complexity of *ML* Typability with Overloading, *Proc. 5th Conf. on Functional Programming Languages and Computer Architecture*, *LNCS* **523**, Springer-Verlag, pp. 15–28, 1991.

[Vol94a] Volpano, D.M.: Haskell-Style Overloading is NP-hard, *Proc. 5th IEEE Int'l Conf. on Computer Languages*, Toulouse, France, to appear, May 1994.

[Vol94b] Volpano, D.M.: Parametric Overloading and the Computational Complexity of Satisfiability, *submitted for publication.*

[Vol93a] Volpano, D.M.: Basic Polymorphic Type Checking with Overloading, *unpublished manuscript.*

[Vol93b] Volpano, D.M.: A Critique of Type Systems for Global Overloading, *Naval Postgraduate School Technical Report NPSCS-94-002.*

[WaB89] Wadler, P. and Blott, S.: How to make *ad-hoc* polymorphism less *ad-hoc*, *Proc. 16th ACM Symposium on Principles of Programming Languages*, pp. 60–76, 1989.

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