GEOPHYSICAL TESTS FOR INTERMEDIATE-RANGE FORCES

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The suggestion in 1986 of a possible "fifth force" in Nature led to considerable experimental and theoretical effort to detect such a force, and deviations from Newtonian gravity. Geophysical experiments play an important role in the detection of such a force, because the natural scale of geophysical experiment covers a range that is not readily accessible via other methods. This work describes several investigations which search for the presence of non-Newtonian gravity. These include a new tower experiment, and an analysis of exponential models of non-Newtonian gravity. In the course of carrying out the tower experiment problems were encountered in working with the Global Positioning System, and these are described in detail. As a result of this work we can say that support for the validity of Newtonian gravity over geophysical scales has increased.
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GEOPHYSICAL TESTS FOR INTERMEDIATE-RANGE FORCES

1. INTRODUCTION

The suggestion in 1986 of a possible "fifth force" in Nature led to considerable experimental and theoretical effort to detect such a force. Evidence for the presence of such a force could come from apparent deviations from the predictions of Newtonian gravity. We can characterize this new force by writing the effective gravitational potential between point masses $m_1$ and $m_2$ in the form.

$$V(r) = \frac{-Gm_1m_2}{r}(1 + \alpha e^{-r/\lambda}) \equiv V_N(r) + V_S(r)$$  \hspace{1cm} (1)

Here $r = |\vec{r}_1 - \vec{r}_2|$ is the separation of the masses, $\lambda = h/mc$ is the Compton wavelength of the particle responsible for the new interaction, and $\alpha$ is a dimensionless constant which characterizes the overall strength of this interaction relative to gravity. The results of any experimental search for non-Newtonian gravity can thus be expressed in terms of the strength parameter $\alpha$ and the range $\lambda$. For technical reasons a given experiment can set sensitive limits only on values of $\lambda$ comparable to the natural size of the experimental system. For distance scales of order 10m - 10 km geophysical experiments using towers, boreholes, etc. provide the natural distance scale, and this observation has motivated much of the recent interest in non-Newtonian gravity. At the same time, the fundamental unity of physics demands that geophysical experiments be integrated along with other tests for non-Newtonian physics. Thus our efforts have been directed both at geophysical tests as well as at other tests for evidence of non-Newtonian gravity.
Since the work done here has been published in various journals and conference proceedings, we will simply summarize the topics covered in this Introduction, and refer to the full papers in the Appendices for complete details.

2. Geophysical Tests for Non-Newtonian Gravity

A. Finite-Size Effects in Eötvös-Type Experiments [E. Fischbach and C. Talmadge in “New and Exotic Phenomena”, Proceedings of the XXVth Rencontre de Moriond, Les Arcs, France, 20-27 January 1990, edited by O. Fackler and J. Tran Thanh Van (Editions Frontières, Gif-Sur-Yvette, 1990) pp. 187-196]. This paper investigates the possibility that all non-Newtonian effects are fundamentally composition-independent, but that composition-dependent effects may arise due to the fact that $\nabla^2 V_5(r) \neq 0$ for $V_5(r)$ given in Eq.(1). This paper is reprinted Appendix A.


This paper discusses the possibility that the systematic effect uncovered in 1986 by Fischbach et al. in the Eötvös data could represent the coupling of a new field to a novel spin-dependent change. This model has specific implications for geophysical experiments. This paper is reprinted in Appendix B.

C. Exponential Models of non-Newtonian Gravity [E. Fischbach, C.
The non-Newtonian potential in Eq.(1) has the typical Yukawa form, which gives the spatial variation \((1/r)e^{-r/\lambda}\). However, in some models the spatial variation of the potential is given by an experimental \(\sim (1/\lambda)e^{-r/\lambda}\). This changes the phenomenology of the putative fifth force in a significant way, which we describe in detail in Appendix C.


This paper presents a short overview and review of the fifth force as an introduction to the first ever bibliography on the subject of new weak forces. The bibliography contains 813 entries by 825 authors, and is presented in Appendix D.


The original Eötvös experiment provided much of the motivation for the current interest in searches for non-Newtonian gravity. In this paper we provide additional historical details on this experiment, which remains one of the few which gives any evidence for the possibility of a new force. This paper constitutes Appendix E.


This review was solicited by the Editor of Nature, and summarizes the status of searches for the putative "fifth force" as of 1992. It presents the
exclusion plots which summarize what had been learned up to that point. This paper is presented in Appendix F.


The pioneering work of the Air Force Geophysics Laboratory on gravity measurements carried out on towers, has been pursued by a new tower experiment using the WABG tower in Inverness, Mississippi. The first results from this experiment are given in Appendix G.


In the course of carrying out the tower experiment, we noticed that the measurement of ground elevations using the Global Positioning Satellite Systems could not always be carried out in close proximity to active television towers. In this paper we discuss this problem in detail, and describe our efforts to understand its origins. This paper is given in Appendix H.
FINITE-SIZE EFFECTS IN EÖTVÖS-TYPE EXPERIMENTS*

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in New and Exotic Phenomena
Proceedings of the Xth Moriond Workshop

ABSTRACT

We discuss a new class of experiments in which searches for composition-dependent deviations from Newtonian gravity can be used to set limits on composition-independent effects as well. These experiments utilize the observation that objects with different charge or mass distributions, will couple differently to the (non-vanishing) Laplacian of a non-Newtonian potential.

* Work supported in part by the United States Air Force Geophysics Laboratory and by the United States Department of Energy
I. INTRODUCTION

Current searches for deviations from the predictions of Newtonian gravity,\textsuperscript{1-9} have thus far focussed primarily on two classes of experiments: These are the \textit{composition-independent} experiments, which look specifically for a breakdown of the 1/r\textsuperscript{2} force law, and the \textit{composition-dependent} experiments which compare the accelerations of chemically-different test masses towards a common source. Both classes of experiments can be understood in terms of the conventional parametrization of the potential energy \( V(r) \) for two point masses \( i \) and \( j \), in which a "fifth force" contribution \( V_5(r) \) is added to the Newtonian term \( V_N(r) \) to give\textsuperscript{3}

\[
V(r) = V_N(r) + V_5(r) = -G_\infty \frac{m_i m_j}{r} \pm f^2 \frac{Q_i Q_j e^{-r/\lambda}}{r}.
\]  

(1)

Here \( G_\infty \) is the Newtonian gravitational constant, \( m_i(m_j) \) is the mass of \( i(j) \), \( \lambda \) is the range of the fifth force, and \( Q_i(Q_j) \) is the corresponding charge which determines the fifth force coupling. The overall strength is set by the constant \( f \), which is the fifth force analog of the electric charge \( e \), and the \( \pm \) sign reflects the possibility that the contribution from \( V_5 \) can be attractive or repulsive. Using (1) we can write \( V(r) \) in the form\textsuperscript{3}

\[
V(r) = -G_\infty \frac{m_i m_j}{r} (1 \pm \alpha_{ij} e^{-r/\lambda}),
\]

\[
\alpha_{ij} = -\left( \frac{Q_i}{\mu_i} \right) \left( \frac{Q_j}{\mu_j} \right) \xi, \quad \xi = \frac{f^2}{G_\infty m_H^2}.
\]  

(2)

In Eq.(2) \( \mu_i = m_i/m_H \), where \( m_H = m(1 H^1) \), and \( \xi \) gives the strength of \( V_5 \) relative to gravity. Differentiating Eq.(2) we can write the force \( \vec{F}(r) \) in the form

\[
\vec{F}(r) = -\nabla V(r) = \frac{-G(r) m_i m_j \hat{r}}{r^2},
\]

\[
G(r) = G_\infty [1 \pm \alpha_{ij}(1 + r/\lambda)e^{-r/\lambda}].
\]  

(3)

The deviation of \( G(r) \) from \( G_\infty \) is an indication of the presence of a non-Newtonian coupling, and we see from Eq.(3) that such deviations can be detected in two ways: A dependence of the effective strength \( G(r) \) on the separation \( r \) of the test masses would reflect a breakdown of the expected 1/r\textsuperscript{2} law of Newtonian gravity. Secondly the constant \( \alpha_{ij} \) in (3) is in general different for each pair \( i,j \) of materials, and hence the effective strength \( G(r) \) depends on the \textit{compositions} of the interacting test masses. Experiments which look for a dependence of \( G(r) \) on \( r \), and on \( \alpha_{ij} \), are denoted respectively as composition-independent, and composition-dependent, searches for non-Newtonian gravity.

In what follows we point to the existence of a new class of experiments which complements those described above, and which are distinguished by the fact that they depend on the \textit{finite size} of the test masses relative to the range \( \lambda \). The principle behind these experiments was first described by Stacey,\textsuperscript{10} and is a consequence of the fact that \( \nabla^2 V_5 \neq 0 \).
DERIVATION OF THE FINITE-SIZE EFFECT

To understand the origin of the "finite-size effect" consider the interaction energy $W$ of a test object whose charge distribution is $\rho(\vec{r})$, with the potential $\Phi(\vec{r})$ of an external field,

$$W = \int d^3r \rho(\vec{r}) \Phi(\vec{r}).$$  \hspace{1cm} (4)

$\Phi(\vec{r})$ can be expanded about the center-of-mass of the test object ($\vec{r} = 0$) and, retaining the first few terms, we have

$$W = \int d^3r \rho(\vec{r})[\Phi(0) + \vec{r} \cdot \vec{\nabla} \Phi(0) + \frac{1}{2} \sum_{i,j} x_i x_j \partial_i \partial_j \Phi(0) + \cdots],$$  \hspace{1cm} (5)

where $\partial_i \equiv \partial/\partial x^i$, and $(\vec{r})_i = x_i$. If $\Phi(\vec{r})$ arises from a massless field (so that $\nabla^2 \Phi(\vec{r}) = 0$), then the usual multipole expansion for $W$ is obtained by adding to Eq.(5) the term $-\frac{1}{4} |\vec{r}|^2 \delta_{ij} \nabla^2 \Phi(0)$, which allows the third term in (5) to be expressed in terms of the quadrupole moment tensor $Q_{ij}$,

$$Q_{ij} = \frac{1}{6} \int d^3r \rho(\vec{r})[3x_i x_j - |\vec{r}|^2 \delta_{ij}].$$  \hspace{1cm} (6)

However, in searching for deviations from Newtonian gravity we are probing specifically for interactions for which $\nabla^2 \Phi(\vec{r}) \neq 0$. It follows that if we wish to expand $W$ in terms of the same multipole moments that arise in the massless case, then we must add back the term proportional to $\nabla^2 \Phi(0)$, so that the leading rotationally-invariant contribution to $W$ now has the form

$$W = \int d^3r \rho(\vec{r})[1 + \frac{1}{6} |\vec{r}|^2 \nabla^2 + \cdots] \Phi(0).$$  \hspace{1cm} (7)

Since the term proportional to $|\vec{r}|^2$ is independent of the orientation of the test object, it will behave as an additional contribution to its mass. To see what effect this term has consider the case where $\Phi(\vec{r})$ is given by a Yukawa,

$$\Phi(\vec{r}) = \frac{\alpha G M}{|\vec{r} - \vec{r}'|} \exp(-|\vec{r} - \vec{r}'|/\lambda),$$  \hspace{1cm} (8)

corresponding to a point source of mass $M$ and strength $\alpha G$ located at $\vec{r}'$. Since $\Phi(\vec{r})$ is a solution of the time-independent Klein-Gordon equation, it follows that

$$\nabla^2 \Phi(\vec{r}) = (1/\lambda^2) \Phi(\vec{r}),$$  \hspace{1cm} (9)

and hence

$$W = \int d^3r \rho(\vec{r})[1 + \frac{1}{6} \frac{|\vec{r}|^2}{\lambda^2} + \cdots] \Phi(0) = m[1 + \frac{1}{6} \frac{(R^2)}{\lambda^2}] \Phi(0) + \cdots.$$  \hspace{1cm} (10)
Here $m$ is the mass of the test object, $\langle R^2 \rangle \equiv (1/m) \int d^3 r \rho(r) |\vec{r}|^2$ is its mean-square-charge radius, and $\cdots$ denotes the remaining terms from (5) which depend on higher derivatives of $\Phi(\vec{r})$. We see from (10) that for a Yukawa potential the leading correction to the standard multipole formula, arising from the fact that $\lambda$ is finite, has the effect of multiplying the (inertial) mass $m$ by the expression in [ ] in (10). As we now show, this correction leads to an apparent violation of the Equivalence Principle.

Let $\vec{F}_1$ denote the total force acting on a test mass $m_1$ in the combined presence of the Earth's acceleration field $\vec{g}(\vec{r})$ and $\vec{\nabla}\Phi(\vec{r})$,

$$\vec{F}_1 = m_1 \vec{g} - m_1(1 + \kappa_1) \vec{\nabla}\Phi,$$

(11)

where $\kappa_1 = \langle R^2 \rangle_1 / 6 \lambda^2$. The acceleration difference of objects 1 and 2 along the direction $\hat{n}$, $\Delta a = (\vec{a}_1 - \vec{a}_2) \cdot \hat{n}$, is then given by

$$\frac{\Delta a}{\hat{n} \cdot \hat{n}} = \Delta \kappa \left( \frac{- \vec{\nabla}\Phi \cdot \hat{n}}{\vec{g} \cdot \hat{n}} \right),$$

(12)

where $\Delta \kappa = \kappa_1 - \kappa_2$, and $\vec{a} = (\vec{a}_1 + \vec{a}_2)/2$. It follows from (12) that two objects with different values of $\langle R^2 \rangle$ will experience different accelerations in the presence of $\Phi$, irrespective of whether or not they have the same composition. $\Delta \kappa$ will almost always be different from zero in an Eötvös experiment, where the accelerations of two objects having the same mass but different compositions are compared. For the current generation of Eötvös experiments, where test samples have not only the same mass but the same external dimensions as well, $\rho(\vec{r})$ must be different for the two test masses, and hence $\Delta \kappa$ is necessarily different from zero.

The finite-size contribution in Eq.(7), which applies to any matter distribution $\rho(\vec{r})$ interacting with an arbitrary potential $\Phi(\vec{r})$, generalizes a result originally derived by Stacey$^{3,10}$ for a spherical mass in the field of a point Yukawa source. To establish the connection between (7) and Stacey's work, we consider (as he did) the experiment of Thieberger$^{11}$ in which the differential acceleration between a spherical copper shell and the water it displaced was measured. For a uniform sphere of radius $R$ we have

$$\langle R^2 \rangle = \frac{3}{5} R^2,$$

(13a)

while for a shell with outer (inner) radius $R_2 (R_1)$ the corresponding result is

$$\langle R^2 \rangle = \frac{3}{5} \left( \frac{R_2^5 - R_1^5}{R_2^5 - R_1^5} \right).$$

(13b)

From Eqs.(13a,b) and (10) we find immediately that the (anomalous) acceleration difference between the shell and the water is proportional to

$$\Delta \kappa = \kappa_{\text{sphere}} - \kappa_{\text{shell}} = \frac{-R_1^3}{10 \lambda^2 R_2} \left( 1 - \frac{R_1^2}{R_2^2} \right) \left( 1 - \frac{R_1^3}{R_2^3} \right)^{1/3}.$$
which is what Stacey found. In addition to generalizing Stacey’s result, Eq. (7) also demonstrates that the leading non-Newtonian corrections to the multipole formula can be obtained directly, without having to start from an exact analytic result as Stacey did. This means that the non-Newtonian contribution can be calculated simply for test masses of arbitrary shape, where an analytic expression would be difficult (if not impossible) to come by.

It follows from the preceding discussion that any experiment which measures the acceleration of a test object \( a \), will be sensitive at some level to the finite-size anomaly \( \kappa_a \). This effect will be the dominant signal for a non-Newtonian force when the mean-square charge radius of an object is comparable to the distance scale over which the non-Newtonian potential is varying, which for a Yukawa occurs when \( \langle R^2 \rangle^{1/2} \approx \lambda \). This observation suggests that the finite-size effect may offer a practical means of adapting Eötvös experiments to carry out high-precision searches for composition-independent non-Newtonian gravity over distances of several centimeters, which is the characteristic size of the masses that are typically used. More significantly, by appropriately redesigning the test masses, it may be possible to achieve a far greater sensitivity for composition-independent short-range forces than is currently possible by other means.

**CONNECTION TO EXPERIMENT**

**a) Short-Range Composition-Independent Experiments**

At present, the most sensitive composition-independent experiments at short ranges are those of Hoskins, et al.\(^{12}\) (2 - 105 cm), and Chen et al.\(^{13}\) (5 - 9 cm). Hoskins et al. in fact carried out two separate experiments which respectively covered the intervals 2 - 5 cm and 5 - 105 cm. Although various clever methods were used to enhance any possible non-Newtonian signal, none of these measurements is a null experiment in the same sense that the Eötvös experiment is. For this reason an Eötvös experiment utilizing the finite-size effect would possess a number of advantages over existing experiments, such as minimizing the sensitivity to source inhomogeneities, and to the source-detector separation. More importantly, since the finite-size effect depends on the Laplacian of the non-Newtonian force, rather than on the force itself, it may be more sensitive than existing techniques to certain types of couplings.

For the special case of a point Yukawa source, we see from Eq. (9) that \( \nabla^2 \Phi(\vec{r}) \) has the same functional dependence on \( \vec{r} \) as does \( \Phi(\vec{r}) \) itself, but this is not generally the case. For a test mass in the presence of an arbitrary force \( \vec{f}(r) \) we have,

\[
\nabla^2[\vec{\tau} \vec{f}(r)] = -2\vec{\tau} \frac{f(r)}{r^2} + 2[\vec{\nabla} f(r) \cdot \vec{\nabla}] \vec{\tau} + \vec{\tau} \nabla^2 f(r),
\]

where we have used the identity \( \nabla^2 \vec{\tau} = -2\vec{\tau}/r^2 \) in the first term. We see from (15) that the functional form of \( \nabla^2[\vec{\tau} \vec{f}] \) can be quite different from that of \( \vec{\tau} \vec{f} \) itself, and that this difference could be significant for small \( r \). Consider the case of two nearly cancelling Yukawas, which
in the appropriate limit can be approximated by an exponential coupling, 1

\[ \Phi(r) = V_E(r) = \alpha_E \frac{e^{-r/\lambda}}{\lambda}, \]  

(16)

\[ \vec{F}_E(r) = -\vec{\nabla} V_E(r) = \vec{\alpha}_E \frac{e^{-r/\lambda}}{\lambda^2}. \]  

(17)

The experiments of Hoskins, et al.,12 and Chen, et al.,13 search for \( \vec{F}_E \) directly, and hence are sensitive to the factor \( \exp(-r/\lambda) \). By contrast, the finite-size effect is determined by

\[ \left\langle R^2 \right\rangle \nabla^2 \vec{F}_E(r) = \vec{\alpha}_E \frac{\left\langle R^2 \right\rangle \alpha_E e^{-r/\lambda}}{6 \lambda^2} \left( 2 + \frac{2r - r^2}{\lambda} \right), \]  

(18)

and hence is sensitive to a different radial function. It is easy to demonstrate that the experiments in Refs. (12 and 13) would have very limited sensitivity to certain combinations of elementary Yukawas, whereas the finite-size effect would be quite sensitive, and vice versa. Thus an Eötvös experiment utilizing the finite-size effect, would complement existing measurements at short ranges by virtue of their different systematics. The same observation applies to the Laplacian detector of Paik et al.,14 whose greatest sensitivity will be to forces of somewhat longer range.

b) The Eötvös Experiment

Since \( \left\langle R^2 \right\rangle \) can be calculated in a straightforward way for any test mass, it is natural to apply the preceding analysis to the original experiment of Eötvös, Pekár, and Fekete (EPF)15 to see whether there is any suggestion of a correlation with \( \left\langle R^2 \right\rangle \) in their data. The torsion balances used by Eötvös, Pekár, and Fekete (EPF) in performing their experiments were originally designed to measure gravity gradients, which was the purpose of the set of experiments performed by Eötvös in the mountains of Hungary15. For this reason, the centers-of-mass of the test bodies used were separated by a vertical distance of about 21 cm, which had the unfortunate side effect of making Eötvös' apparatus significantly more sensitive to vertical gravity gradients than to anomalous accelerations. To cancel out the effects of gravity gradients, EPF combined the results of measurements made with the torsion balance oriented in different directions. The \( \text{H}_2\text{O-Cu} \) comparison, for example, was carried out by measuring the difference in the deflection of the torsion bar when it was oriented North-South versus South-North, and East-West versus West-East. This was done for both \( \text{H}_2\text{O} \) compared to Pt (where the Pt was loaded at the upper position of the torsion balance), and for Cu compared to Pt. The effects due to the gravity gradients could then be eliminated by taking the appropriate difference of the deflections measured using \( \text{H}_2\text{O-Pt} \) versus those measured using Cu-Pt.

In the case of certain comparisons (e.g., magnalium-Pt or snakewood-Pt), the sample could be machined to the desired specifications, and hence did not need to be stored in a
container. Where this was not possible (e.g., H₂O-Cu or Asbestos-Cu), the sample was placed in a brass vial, and the outside dimensions and masses for the various containers are given in Ref. 15. Since the densities of the samples could estimated from the composition of the respective samples, there is sufficient information to allow the separate determination of the holding volume of the brass vials, and the volume of each sample. In many cases, the volume of the sample turned out to be smaller that the volume of the brass vial, and in this case the model for brass vial+sample shown in Fig. 1 was used. If we assume that the thickness of the wall of the vials, \( t \), was the same for all samples contained in a vial, and was equal to that of the end caps, then we obtain \( t \approx 0.025 \) cm. For a solid cylinder of radius \( R \) and length \( L \) we have

\[
\langle R^2 \rangle = \frac{1}{2} R^2 + \frac{1}{12} L^2, \tag{19}
\]

where \( \langle R^2 \rangle \) is measured relative to the center of mass. For the composite sample given by Fig. 1,

\[
\langle R^2 \rangle = \frac{\pi}{2m_t} \left\{ \rho_s L_s R_1^2 \left[ R_1^2 + \frac{1}{6} L_1^2 + \frac{1}{2} h^2 \right] + \rho_c \left[ (L_2 R_2^5 - L_1 R_1^5) \
+ \frac{1}{6} (L_2^3 R_2^4 - L_1^3 R_1^4) + \frac{1}{2} h^2 (L_2 R_2^2 - L_1 R_1^2) \right] \right\}, \tag{20}
\]

where \( m_t \) is the total mass of the brass vial + sample, \( L_s = m_s/(\pi R_1^2) \), and \( m_s \) is the mass of the sample. Using Eqs. (19) and (20), we then obtained estimates for \( \langle R^2 \rangle \) for each sample,
Table I: Estimated values of \((R^2)\) for the various samples used in the EPF experiment. Here \((R^2)\) is given in units of cm².

<table>
<thead>
<tr>
<th>Sample I</th>
<th>((R^2))</th>
<th>Sample II</th>
<th>((R^2))</th>
<th>(\Delta(R^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnesium</td>
<td>12.0</td>
<td>Pt</td>
<td>3.0</td>
<td>8.9</td>
</tr>
<tr>
<td>Snakewood</td>
<td>48.1</td>
<td>Pt</td>
<td>3.0</td>
<td>45.1</td>
</tr>
<tr>
<td>Cu</td>
<td>3.5</td>
<td>Pt</td>
<td>3.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Ag-Fe-SO₄</td>
<td>17.3</td>
<td>Ag-Fe-SO₄</td>
<td>17.2</td>
<td>0.0</td>
</tr>
<tr>
<td>(\text{H}_2\text{O})</td>
<td>15.5</td>
<td>Cu</td>
<td>3.6</td>
<td>11.9</td>
</tr>
<tr>
<td>&quot;</td>
<td>15.8</td>
<td>Cu</td>
<td>3.5</td>
<td>12.3</td>
</tr>
<tr>
<td>(\text{CuSO}_4\cdot 5\text{H}_2\text{O})</td>
<td>46.4</td>
<td>Cu</td>
<td>5.5</td>
<td>40.9</td>
</tr>
<tr>
<td>&quot;</td>
<td>49.8</td>
<td>Cu</td>
<td>5.5</td>
<td>44.3</td>
</tr>
<tr>
<td>(\text{CuSO}_4) Solution</td>
<td>27.0</td>
<td>Cu</td>
<td>5.5</td>
<td>21.4</td>
</tr>
<tr>
<td>&quot;</td>
<td>26.7</td>
<td>Cu</td>
<td>5.5</td>
<td>21.2</td>
</tr>
<tr>
<td>Asbestos</td>
<td>41.1</td>
<td>Cu</td>
<td>5.5</td>
<td>35.6</td>
</tr>
<tr>
<td>&quot;</td>
<td>40.5</td>
<td>Cu</td>
<td>5.5</td>
<td>35.0</td>
</tr>
<tr>
<td>Tallow</td>
<td>19.4</td>
<td>Cu</td>
<td>5.5</td>
<td>13.9</td>
</tr>
<tr>
<td>&quot;</td>
<td>19.1</td>
<td>Cu</td>
<td>5.5</td>
<td>13.6</td>
</tr>
</tbody>
</table>

and \(\Delta(R^2)\) for each pair, and our results are shown in Table I.

These values of \(\Delta(R^2)\) were fitted against the acceleration differences \(\Delta \kappa = \Delta a / \bar{n} \cdot \bar{g}\) measured by EPF using a model of the form

\[
\Delta \kappa = a \Delta(R^2) + b, \tag{21}
\]

and the results are shown in Fig. 2 (solid curve). It evident from Fig. 2 that Eq. (21) is a very poor model of the EPF data. More quantitatively, we find \(\chi^2 = 43\) for 7 degrees of freedom for this fit. Fitting to only the "Method III" data points (c.f., Refs. 3 or 15), a somewhat better fit is obtained (dashed line in Fig. 2). However, the resulting line fails to pass through the origin, and hence must also be rejected on physical grounds.

**SUMMARY**

The principal results of our paper are presented in Eqs. (7)-(12). They indicate that, irrespective of the functional form of the non-Newtonian potential, an apparent composition-dependent effect will arise in an Eötvös-type experiment by virtue of the fact that \(\nabla^2 \Phi(\vec{r}) \neq 0\). Failure to detect such an effect, at some level, is thus an (almost) model-independent test for the presence of a non-Newtonian interaction. In practice this test will be most sensitive, and hence most useful, as a null test for forces which vary significantly in space over a distance scale comparable to the size of the test masses. We note in passing that in the presence of a non-Newtonian coupling, the finite-size effect produces an analog of the Nordvedt effect. This
Figure 2: Results of a fit of $\Delta \langle R^2 \rangle$ from Table I versus $\Delta \kappa$ obtained by Eötvös, Pekár, and Fekete. The solid curve represents a fit to all of the data points, and the dashed line represents a fit to only the “Method III” data points. Both fits fail to give a satisfactory result, as is discussed in the text.

will be discussed elsewhere, as will be the details of possible laboratory experiments utilizing the finite-size effect.

ACKNOWLEDGMENTS

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REFERENCES


10 The details of Stacey's argument are given in Ref. 3 above, pp. 65-67.


IS THE EÖTVÖS EXPERIMENT SENSITIVE TO SPIN?

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Recently, Hall and Armbruster have introduced a phenomenological spin-dependent charge, which they have suggested may account for the nonzero acceleration differences reported in the Eötvös experiment. They discuss three possible explanations for the origin of this charge. This paper examines specifically one of those explanations, namely, that this charge is evidence for a “fifth force” in Nature. In doing so, we explore the implications of fifth force models based on this charge, and study the question of whether this charge can be derived from a more general theory.

INTRODUCTION

There is considerable experimental and theoretical interest at present in the possibility of deviations from the predictions of Newtonian gravity. Part of the motivation for this renewed interest was a reanalysis of the classic experiment of Eötvös, Pekár, and Fekete (EPF) which uncovered in the EPF
data a correlation suggesting the existence of a new intermediate-range coupling, \( V_5 \), to baryon number \( B \). The potential energy \( V(r) \) for two point masses in the presence of the "fifth force" potential energy \( V_5(r) \) and the Newtonian gravitational potential energy \( V_N(r) \) is then given by \(^2\)

\[
V(r) = V_N(r) + V_5(r) = -G_\infty \frac{m_im_j}{r} + f^2 \frac{B_iB_j}{r}e^{-r/\lambda},
\]

where \( m_i \) and \( B_i \) are the mass and baryon number of \( i \), respectively, \( G_\infty \) is the Newtonian gravitational constant, \( f \) is the coupling strength of the new interaction (the analog of the electric charge \( e \) for electromagnetism), and \( \lambda \) is the range of the proposed new force. If the masses are expressed in units of \( m(H^1) = 1.00782519(8)u \), so that \( m_i = \mu_i m(H^1) \), then Eq. (1) can be written in the form \(^2\)

\[
V(r) = -G_\infty \frac{m_im_j}{r} \left( 1 + \alpha_{ij}e^{-r/\lambda} \right),
\]

\[
\alpha_{ij} = -\frac{B_iB_j}{\mu_i\mu_j} \xi, \quad \xi = \frac{f^2}{G_\infty m_H^2}.
\]

Since \( \alpha_{ij} \) depends on the compositions of \( i \) and \( j \) through the ratios \( B_i/\mu_i \) and \( B_j/\mu_j \), the coupling in (2) leads to an apparent violation of the Weak Equivalence Principle (WEP), which postulates the equivalence of gravitational and acceleration effects. The acceleration difference \( \Delta \alpha_{jj'} \) of two objects \( j \) and \( j' \) towards \( i \) would then be directly proportional to

\[
\alpha_{ij} - \alpha_{ij'} = -\xi \frac{B_i}{\mu_i} \left( \frac{B_j}{\mu_j} - \frac{B_{j'}}{\mu_{j'}} \right) \equiv -\xi \frac{B_i}{\mu_i} \Delta \left( \frac{B}{\mu} \right)_{jj'},
\]

and it was this specific correlation between \( \Delta(B/\mu)_{jj'} \) and the EPF data for \( \Delta \alpha_{jj'} \) which stimulated discussion of a "fifth force."

The model defined by Eqs. (1)-(3) has been discussed extensively in the literature\(^1,2,4-9\). It has been noted in Refs. 1 and 2 that \( B/\mu \) varies across the Periodic Table in a way that is quite different from that of other proposed charges, such as lepton number \( L \). That \( B/\mu \) is not (even approximately) a monotonic function of atomic number \( Z \), may help to explain why attempts to understand the correlation in the Eötvös data in terms of conventional physics\(^2,10,11\) thus far have been unsuccessful. Experiments aimed at reproducing the EPF results have also been unsuccessful, however, and by now there can be little doubt that
the model of $V_5$ given in Eqs. (1)-(3) is incompatible with existing data. Various attempts to generalize this model have thus far failed to explain how the correlation in the EPF data could be compatible with the results of modern experiments. The implications of these experiments is that the Eötvös experiment is flawed, yet no convincing model has been put forward to date which can account for the original EPF data in conventional terms. We refer the interested reader to the discussions in Refs. 2, 12, and 13 for further consideration of these points.

Before turning to the question of the possible relevance of spin in the EPF experiment, it is worth emphasizing that both the modern repetitions of the EPF experiment and the related Galileo (free-fall) experiment\cite{14,15} have been designed to optimize their sensitivity to the coupling in Eqs. (1)-(3), or to certain alternatives to it. These experiments may not be equally sensitive to a spin-dependent charge, and thus to a spin-dependent "fifth force" $V_{5S}$.

PHENOMENOLOGY OF A SPIN-DEPENDENT CHARGE

Recently Hall and Armbruster (HA) have noted\cite{16} that a correlation similar to that found in Ref. 1 arises in the Eötvös data, if the baryon number $B$ of a nucleus is replaced by a charge $Q$ defined by

$$Q = M\delta; \quad \delta = \begin{cases} 1 & \text{for } J > 0, \\ 0 & \text{for } J = 0, \end{cases}$$

where $M$ is the mass of the nucleus, and $J$ is its nuclear spin. (Note that we use the notation $J$ rather than the more conventional $I$ to avoid any possible confusion with the nuclear isospin $I$, which was another suggestion for $Q$.) For elements with more than one isotope, the total $Q$ is obtained in the usual way\cite{2} by weighting contributions from each isotope according to its isotopic abundance $r_k$:

$$Q = \sum_k M_k \delta_k r_k.$$  \hspace{1cm} (5)

Before returning to the question of the physical significance of $Q$, we present in Table I the values of $Q/\mu$ for the natural elements, and in the Table II the values $\Delta(Q/\mu)$ for the EPF samples, where for samples $i$ and $j$, $\Delta(Q/\mu) = \ldots$
Table 1: Average value of $Q/\mu$ using Eq. (4) for the first 92 elements of the Periodic Table.

<table>
<thead>
<tr>
<th>Element</th>
<th>$Q/\mu$</th>
<th>Element</th>
<th>$Q/\mu$</th>
<th>Element</th>
<th>$Q/\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>1.00000</td>
<td>Germanium</td>
<td>0.44976</td>
<td>Europium</td>
<td>1.00000</td>
</tr>
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<td>Helium</td>
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<td>Arsenic</td>
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<td>Gadolinium</td>
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</tr>
<tr>
<td>Lithium</td>
<td>1.00000</td>
<td>Selenium</td>
<td>0.07381</td>
<td>Terbium</td>
<td>1.00000</td>
</tr>
<tr>
<td>Beryllium</td>
<td>1.00000</td>
<td>Bromine</td>
<td>1.00000</td>
<td>Dysprosium</td>
<td>0.43735</td>
</tr>
<tr>
<td>Boron</td>
<td>1.00000</td>
<td>Krypton</td>
<td>0.11427</td>
<td>Holmium</td>
<td>1.00000</td>
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<tr>
<td>Carbon</td>
<td>0.01198</td>
<td>Rubidium</td>
<td>1.00000</td>
<td>Erbium</td>
<td>0.22896</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1.00000</td>
<td>Strontium</td>
<td>0.06963</td>
<td>Thulium</td>
<td>1.00000</td>
</tr>
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<td>Oxygen</td>
<td>0.00040</td>
<td>Yttrium</td>
<td>1.00000</td>
<td>Ytterbium</td>
<td>0.30259</td>
</tr>
<tr>
<td>Fluorine</td>
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<td>Zirconium</td>
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<td>Lutetium</td>
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<tr>
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<td>Niobium</td>
<td>1.00000</td>
<td>Hafnium</td>
<td>0.32121</td>
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<td>Sodium</td>
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<td>Molybdenum</td>
<td>0.25119</td>
<td>Tantalum</td>
<td>1.00000</td>
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<tr>
<td>Magnesium</td>
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<td>Technetium*</td>
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<td>Tungsten</td>
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<tr>
<td>Aluminum</td>
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<td>Ruthenium</td>
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<td>Rhenium</td>
<td>1.00000</td>
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<tr>
<td>Silicon</td>
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<td>Rhodium</td>
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<td>Osmium</td>
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<tr>
<td>Phosphorus</td>
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<td>Palladium</td>
<td>0.21909</td>
<td>Iridium</td>
<td>1.00000</td>
</tr>
<tr>
<td>Sulfur</td>
<td>0.00782</td>
<td>Silver</td>
<td>1.00000</td>
<td>Platinum</td>
<td>0.33779</td>
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<tr>
<td>Chlorine</td>
<td>1.00000</td>
<td>Cadmium</td>
<td>0.24890</td>
<td>Gold</td>
<td>1.00000</td>
</tr>
<tr>
<td>Argon</td>
<td>0.00000</td>
<td>Indium</td>
<td>1.00000</td>
<td>Mercury</td>
<td>0.29946</td>
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<tr>
<td>Potassium</td>
<td>1.00000</td>
<td>Tin</td>
<td>0.16424</td>
<td>Thallium</td>
<td>1.00000</td>
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<tr>
<td>Calcium</td>
<td>0.00155</td>
<td>Antimony</td>
<td>1.00000</td>
<td>Lead</td>
<td>0.22578</td>
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<tr>
<td>Scandium</td>
<td>1.00000</td>
<td>Tellurium</td>
<td>0.07679</td>
<td>Bismuth</td>
<td>1.00000</td>
</tr>
<tr>
<td>Titanium</td>
<td>0.12772</td>
<td>Iodine</td>
<td>1.00000</td>
<td>Polonium*</td>
<td>1.00000</td>
</tr>
<tr>
<td>Vanadium</td>
<td>1.00000</td>
<td>Xenon</td>
<td>0.47074</td>
<td>Astatine*</td>
<td>1.00000</td>
</tr>
<tr>
<td>Chromium</td>
<td>0.09683</td>
<td>Cesium</td>
<td>1.00000</td>
<td>Radon*</td>
<td>0.00000</td>
</tr>
<tr>
<td>Manganese</td>
<td>1.00000</td>
<td>Barium</td>
<td>0.17759</td>
<td>Francium*</td>
<td>0.50106</td>
</tr>
<tr>
<td>Iron</td>
<td>0.02233</td>
<td>Lanthanum</td>
<td>1.00000</td>
<td>Radium*</td>
<td>0.00000</td>
</tr>
<tr>
<td>Cobalt</td>
<td>1.00000</td>
<td>Cerium</td>
<td>0.00000</td>
<td>Actinium*</td>
<td>1.00000</td>
</tr>
<tr>
<td>Nickel</td>
<td>0.01235</td>
<td>Praseodymium</td>
<td>1.00000</td>
<td>Thorium*</td>
<td>0.00000</td>
</tr>
<tr>
<td>Copper</td>
<td>1.00000</td>
<td>Neodymium</td>
<td>0.20396</td>
<td>Protactinium*</td>
<td>1.00000</td>
</tr>
<tr>
<td>Zinc</td>
<td>0.04207</td>
<td>Promethium*</td>
<td>1.00000</td>
<td>Uranium</td>
<td>0.00711</td>
</tr>
<tr>
<td>Gallium</td>
<td>1.00000</td>
<td>Samarium</td>
<td>0.28324</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*No stable isotopes

$(Q/\mu)_i - (Q/\mu)_j$. These are plotted against the acceleration differences $(\Delta \kappa$ in the EPF notation) in Fig. 1. Fitting these data to a straight line,

$$\Delta \kappa = \gamma \Delta (Q/\mu) + \delta,$$

(6)
Table II: Calculated values of $\Delta(Q/\mu)$ using Eq. (4) for the EPF samples versus the quoted EPF values of $\Delta \kappa$. The misprint in the sign quoted by EPF for the RaBr$_2$-Pt datum has been corrected, as discussed in Ref. 2.

<table>
<thead>
<tr>
<th>Samples</th>
<th>Legend</th>
<th>$\Delta(Q/\mu)$</th>
<th>$10^6\Delta \kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asbestos-Cu</td>
<td>a</td>
<td>-0.568</td>
<td>-2±1</td>
</tr>
<tr>
<td>H$_2$O-Cu</td>
<td>b</td>
<td>-0.611</td>
<td>-5±1</td>
</tr>
<tr>
<td>Tallow-Cu</td>
<td>c</td>
<td>-0.493</td>
<td>-3±1</td>
</tr>
<tr>
<td>CuSO$_4$(sol'n)-Cu</td>
<td>d</td>
<td>-0.505</td>
<td>-4±1</td>
</tr>
<tr>
<td>CuSO$_4$·5H$_2$O-Cu</td>
<td>e</td>
<td>-0.441</td>
<td>-3±1</td>
</tr>
<tr>
<td>Snakewood-Pt</td>
<td>f</td>
<td>-0.274</td>
<td>-1±2</td>
</tr>
<tr>
<td>Ag-Fe-SO$_4$</td>
<td>g</td>
<td>+0.000</td>
<td>+0±1</td>
</tr>
<tr>
<td>RaBr$_2$-Pt</td>
<td>h</td>
<td>+0.343</td>
<td>+1±2</td>
</tr>
<tr>
<td>Magnalium-Pt</td>
<td>i</td>
<td>+0.578</td>
<td>+4±1</td>
</tr>
<tr>
<td>Cu-Pt</td>
<td>j</td>
<td>+0.668</td>
<td>+4±2</td>
</tr>
</tbody>
</table>

![Figure 1](image)

**Fig. 1** Plot of $\Delta(Q/\mu)$ obtained from Eq. (4) versus $\Delta \kappa$, using the data from Table II. The straight line is the result of a least squares fit to these data, as discussed in the text. The labels a–j are defined in Table II.

we find

$$\gamma = (6.5 \pm 0.8) \times 10^{-9}, \quad \delta = (-0.1 \pm 4.1) \times 10^{-10},$$

$$\chi^2 = 5.1 \text{ (8 degrees of freedom).} \quad (7)$$

The quality of this fit is very similar to that obtained originally in Ref. 1 for a
coupling to $B$: Using the results of Table V in Ref. 2, the corresponding values of $\gamma$ and $\delta$ are:

$$
\gamma = (5.08 \pm 0.61) \times 10^{-6}, \quad \delta = (0.87 \pm 0.59) \times 10^{-9},
$$

$$
\chi^2 = 5.2 \text{ (8 degrees of freedom)}. \quad (8)
$$

HA note in Ref. 16 that the effect of $J = 1/2$ nuclei may be removed (by setting $\delta = 0$ for $J = 1/2$ nuclei in Eq. (4)) without significantly affecting the correlation in Fig. 1. Further, the EPF data do not discriminate between other variants of the charge $Q$ (such as replacing $\delta$ by $[J(J + 1)]^{1/2}$), as these variants also evidence a correlation with the EPF data.\textsuperscript{16} Even the extreme assumption that $Q = 0$ for all elements but Cu and Al leads to a correlation.\textsuperscript{16} While these variants may or may not be significant, we consider here only the charge $Q$ given in Eq. (4).

MODELS OF THE SPIN-DEPENDENT CHARGE $Q$

In this section we discuss preliminary models aimed at deriving a spin-dependent interaction from a more fundamental theory. This is desirable partly because one must also know the spatial-dependence of whatever coupling leads to Eq. (4), in order to design appropriate experiments. This is especially important for composition-independent experiments, which directly measure the spatial variation of $V(r)$, as we discuss below.

One of the challenges in constructing a model of the EPF data based on the spin-dependent charge $Q$, is that the EPF samples were presumably unpolarized. It is reasonable to assume that $\langle \vec{J} \rangle \cong 0$ for the samples as well as for the source, although bulk matter may, however, have a small net polarization due to the Earth's magnetic field $\vec{B}_\oplus$. At room temperature ($T = 300^\circ K$), however, the polarization $P$ is expected to be of order\textsuperscript{17}

$$
P \approx \frac{\mu_n B_\oplus}{k_B T} \cong 1 \times 10^{-10} B_\oplus \text{ (Gauss)}, \quad (9)
$$

where $\mu_n$ is the nuclear magneton, and $k_B$ is the Boltzmann constant. For $B_\oplus \cong 1 \text{ Gauss}$ this gives $P \cong 1 \times 10^{-10}$. Since any effect would depend on the
product $P_s P_d$ for the source and detector, experiments sensitive to $P_s P_d$ would be suppressed by a factor of order $10^{-20}$ relative to those with $P_s \approx P_d \approx 1$. This means that the effective strength of the "gravitational" interaction between spin-polarized samples (for which $P \approx 1$) should be $\approx 10^{20}$ stronger than the strength of the claimed signal in the EPF experiment, which we may take to be of order $(10^{-2} G_{\infty} - 10^{-8} G_{\infty})$. This conclusion is significantly at variance with the results of recent experiments on spin-polarized samples which set upper limits on the strength of a spin-dependent gravitational coupling to electron spin at the level of $\approx 10^{-3} G_{\infty}$. These experiments can also be used to set limits on a coupling to nuclear spin, because the polarized electrons produce a net hyperfine field at the nucleus, which in turn polarizes the nucleus. Since this field is much larger than $B_\oplus$, it would be difficult to understand how an effect could have shown up (incidentally) in the EPF experiment, but not in those of Ref. 18.

The implication of the preceding discussion is that if spin is indeed relevant in the EPF experiment, it must be so in a way that does not depend on the net polarization of the nuclei in the test masses. One way this can happen is if the spin were to contribute a term to the nuclear mass-energy, and that term coupled anomalously to gravity. If $M_I$ and $M_G$ denote the inertial and gravitational masses of a nucleus, then the content of the WEP is that $M_I = M_G$. Suppose, however, that a particular contribution $\epsilon_\alpha$ of the nuclear mass-energy coupled anomalously to gravity and violated the WEP so that $M_I \neq M_G$. We can then define a parameter $\eta_\alpha$,

$$\frac{M_G}{M_I} \equiv 1 + \eta_\alpha \left( \frac{\epsilon_\alpha}{M_I} \right) \equiv 1 + \kappa_\alpha, \quad (10)$$

which measures the strength of the anomalous gravitational coupling of $\epsilon_\alpha$. The parametrization in (10) has been widely used in the literature to set limits on the coupling of various possible energy terms in the nucleus using the Eötvös experiment. Consider, for example, the acceleration of nucleus 1 towards the Earth in the presence of (10):

$$M_{I1} a_1 = -\frac{G_{\infty} M_\oplus M_{G1}}{R_\oplus^2} = -\frac{G_{\infty} M_\oplus M_{I1}}{R_\oplus^2} (1 + \kappa_\alpha)$$

$$a_1 = -\frac{G_{\infty} M_\oplus}{R_\oplus^2} (1 + \kappa_\alpha) \equiv -g_\oplus (1 + \kappa_\alpha). \quad (11)$$
It follows that
\[
\frac{a_1 - a_2}{g_\Theta} = \kappa_a^2 - \kappa_{a1} = \eta_a \left[ \left( \frac{\epsilon_a}{M_I} \right)_1 - \left( \frac{\epsilon_a}{M_I} \right)_2 \right].
\] (12)

Since \((\epsilon_a/M_I)\) is in general different for dissimilar nuclei, it follows that Eq. (12) can be used to set constraints on \(\eta_a\). In terms of this picture, the problem of justifying the form of \(Q\) in Eq. (4) reduces to finding an energy term in the nucleus which depends on the product \(M\delta\), or some expression similar to it.

To illustrate how a dependence of some energy \(\epsilon_o\) on \(M\delta\) could come about, we consider for illustrative purposes the gravitational contribution to the spin-orbit energy \(\epsilon_{SO}\) of a single nucleon outside a filled or partially-filled shell. This is given by
\[
\epsilon_{SO} = \frac{3}{2} \frac{G_\infty M}{mc^2 r^3} \mathbf{L} \cdot \mathbf{S},
\] (13)

where \(M\) is the mass of the shell, \(m\) is the nucleon mass, \(\mathbf{L}\) and \(\mathbf{S}\) are the nucleon orbital and spin angular momenta, and \(r\) is the distance from the nucleon to the center of the shell. The contribution from \(\epsilon_{SO}\) appears to have the property of being proportional to \(M \mathbf{L} \cdot \mathbf{S} \propto \frac{1}{2} M(J^2 - L^2 - S^2)\), and hence vanishes when there are no unpaired nucleons, as does the charge \(M\delta\) in (4). Since nuclei have an approximately constant density \(\rho\), we can set \(M \propto \frac{4}{3} \pi r^3 \rho\), however, so that
\[
\epsilon_{SO} \propto \frac{2\pi \rho G_\infty \mathbf{L} \cdot \mathbf{S}}{mc^2}.
\] (14)

This contribution to the nuclear energy does not grow with \(M\) as desired, and thus is not a candidate for the charge of Eq. (4). A priori, there is an even more serious objection to this toy model, quite apart from the obvious problem of the magnitude of \(\epsilon_{SO}\). This has to do with the fact that the same (gravitational) interaction which couples the valence nucleon to the filled-shell (core), also couples the core nucleons to one another. Thus if the spin-orbit gravitational energy coupled in some anomalous way to gravity, the spin-independent energy would as well. This might well lead to a violation of the WEP, but it would be predominantly a spin-independent effect, which is not what we are seeking. Evidently, similar arguments apply mutatis mutandis for other contributions to the total spin-orbit interaction. Since any contribution to the energy from the exchange of bosons in the normal spin-parity series \((J^P = 0^+, 1^-, 2^+, \ldots)\) will lead to the
same problem, the best hope for constructing a model of the spin-dependent charge along these lines appears to arise from the exchange of bosons in the abnormal parity series, \( J^P = 0^-, 1^+, 2^- \ldots \)

Consider, for example, the potential arising from the exchange of a massive axial-vector \((J^P = 1^+\) field,
\[
V_A(r) = g_A^2 \sigma_1 \cdot \sigma_2 \frac{e^{-r/\lambda_A}}{r},
\]
where \( \sigma_1 \) and \( \sigma_2 \) are the spins of the interacting nucleons, \( \lambda_A = \hbar/m_Ac \) is the Compton wavelength of the \( 1^+ \) quantum, and \( g_A \) is an appropriate coupling constant. The interaction in Eq. (15) describes not only the coupling of the valence nucleon to a nucleon in the core, but also the interaction of the core nucleons among themselves. In the latter case \( \sum_j \sigma_j \cdot \sigma_k \) may average to zero for the core nucleons, depending on the details of the interaction and on the form of the core wavefunction. Due to exchange (Pauli) effects, this is not necessarily the case for \( \sigma_1 \cdot \sigma_k \), where \( 1 \) denotes a valence nucleon and \( k \) a nucleon in the core. What this means is that to \( \mathcal{O}(g_A^3) \) there may be no contribution from \( V_A \) to the energy of a nucleus, except for possible contributions from valence nucleons interacting with the core. If we then suppose that this term has an anomalous coupling to gravity, or perhaps to another long-range gravity-like field, then we have at least the outline of a model which could give rise to some of the qualitative features of the spin-dependent charge in Eq. (4).

To summarize, the proposed mechanism for modeling the spin-dependent charge as a fifth force is based on an anomalous coupling of the spin-dependent energy arising from \( V_A \) in Eq. (15). This energy term could have the property of vanishing for \( J = 0 \) nuclei, while at the same time being nonzero for nuclei with a valence nucleon outside a core. In this case, the total energy should scale with the size of the core, and these features may qualitatively simulate the behavior of \( Q = M\delta \) (or one of the alternatives to \( Q \) which also explains the EPF data). We emphasize that considerable effort will be required before we will know whether this or any other specific model actually works. Nonetheless, the preceding arguments indicate that a spin-dependent charge, such as that proposed in Ref. 16, may in fact have a field-theoretic justification.
IMPLICATIONS OF A SPIN-DEPENDENT CHARGE

We outline in this section some of the phenomenological implications of the spin-dependent charge in Eq. (4). Eq. (3) can be generalized by noting that whatever the specific form of the charge $Q$ that determines the strength of $V_{ss}$, the composition-dependent acceleration difference will always be proportional to the product

$$\xi \left( \frac{Q}{\mu} \right)_{\text{source}} \Delta \left( \frac{Q}{\mu} \right)_{\text{detector}} \equiv \xi q_s \Delta q_d. \quad (16)$$

Here "detector" refers to the two masses whose accelerations are being compared as they accelerate toward a third object which is defined as the "source." The product $S = q_s \Delta q_d$, termed the "sensitivity function" by Adelberger, et al., is one of the factors that determines the sensitivity of a given experiment to the putative fifth force. Another factor is the source integral $\tilde{F}$, which depends in turn on the the $r$-dependence of $V_{ss}$. Since we do not as yet have even a rudimentary phenomenological theory which predicts the $r$-dependence of $V_{ss}$, we will limit ourselves to what can be learned from $Q$ alone.

**Experiments with Large Geophysical Sources**

This category includes the experiments of Adelberger, et al., Bizzeti, et al., Boynton, et al., Fitch, et al., Kuroda and Mio, Niebauer, et al. and Thieberger. These experiments have in common the feature that their source is predominantly SiO$_2$ for which $q_s \equiv 0$. This means that for the class of models we are considering, the relative sensitivity of the different experiments would depend crucially on the presence at different sites of "impurities" with $q_s \neq 0$. The same remark would apply to experiments carried out at sites rich in CaCO$_3$ (e.g., limestone), which is another common mineral. Further, for variants of the model in which the spin charge $Q = 0$ for hydrogen (as in the model discussed in the previous section), the presence or absence of water in the source is irrelevant. In this context it is interesting to note that if $Q = 0$ for hydrogen then the constraints imposed on $V_{ss}$ by the solar Eötvös experiments are reduced by a factor of $\sim 10^3$ from their quoted values.
Experiments with Laboratory Sources

A number of experiments with laboratory sources have been carried out which may set interesting limits on $V_{ij}$, once the distance dependence is known. Laboratory experiments with Pb sources, however, would have very limited sensitivity to the spin charge discussed here, as $q_{Pb} \equiv 0$ using the charge of Eq. (4). It is interesting to note that the implications of the spin charge model are similar to those of the isospin model (related to $V_5$) of Boynton, et al. and Adelberger, et al., with one critical difference. In the latter model $Q = N - Z = I_z$, where $N$ and $Z$ denote the numbers of neutrons and protons in the sample. Since $(I_z/\mu)$ is relatively large for Pb, experiments with laboratory sources of Pb can set stringent limits on the strength of a coupling to $I_z$. For a summary of recent laboratory results, see Fischbach and Talmadge. Unlike the case of isospin, the spin-dependent charge of Eq. (4) is non-negative. This means that there can be no cancellations as occur for $I_z$ between the contributions from water (for which $I_z/\mu \approx -0.112$) and the slightly positive $I_z/\mu$ of mineral impurities.

Pumped Lake Eötvös Experiments

Bennett has carried out an Eötvös experiment using a pumped water facility as his source. This experiment can set interesting limits on a coupling to $I_z$ or to $Q$ in (4). The EPF data, however, are also consistent with the assignment $\delta = 0$ for $J < 1$ (and the toy model above does not contain a coupling to hydrogen). In this case the Bennett experiment would be insensitive to $Q$, which is then compatible with his null result.

Tower, Mine/Borehole and Lake Experiments

In the notation of Eq. (9) these composition-independent experiments are sensitive to the product $q_s q_d$ where $q_d$ is the charge of the standard mass in the gravimeter. Typically measurements are carried out with similar gravimeters, so any substantial differences among experiments would arise from the source charge $q_s$. At present there is no compelling evidence for deviations from Newtonian gravity in any of these experiments, but since lingering anomalies remain, measurements are continuing. It would be difficult at this point to use the spin-charge model to analyze these experiments, since at the present stage
the model makes no statement about the spatial variation of $V_{5S}(r)$ to which these experiments are sensitive.

SUMMARY AND CONCLUSIONS

The phenomenological spin-dependent charge in Eq. (4) introduced by Hall and Armbruster\textsuperscript{16} provides an alternative to the original hypercharge model in Refs. 1 and 2 as an explanation of the EPF results. In this paper, we have explored the specific possibility of modeling this charge as that of a spin-dependent "fifth force" $V_{5S}$. Although the spin-dependent charge of Eq. (4) as originally proposed does not appear to arise in this context from a more fundamental theory, some variants of a model based on the charge of Eq. (4) may have a field-theoretic basis, as we have discussed. The success of a spin-dependent charge in accounting for the EPF data raises the question of whether there are variables other than $B$ or $Q$ in (4) which could explain the EPF data. If so, then we might be tempted to ask whether these data have any deep physical significance, if they can be accounted for by such seemingly different variables. On the other hand, among the many physical variables which have been tried,\textsuperscript{2,36} $B$ and $Q$ are the only two discovered to date which give the correct correlation. Since both involve non-classical variables, and may be related to each other at a deeper level, it is possible that they may be the signal for new physics. Although it is still too early to assess the physical significance of the spin-dependent charge, or the success in modeling it as a charge of a spin-dependent fifth force, it is clear that the correlation noted in Ref. 16 raises new and interesting possibilities that are worth pursuing.

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21. We thank Mark Haugan and Steve Wallace for discussions on this point.


Exponential models of non-Newtonian gravity

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We show that in certain classes of theories the spatial variation of the non-Newtonian potential would be dominantly an exponential rather than a Yukawa potential, and we compare the phenomenological interpretation of the existing data in the exponential and Yukawa models. We also show that generalized forms of the exponential potential can arise naturally from simple mass spectra. Although such models cannot reconcile all of the existing data on non-Newtonian gravity, they have novel properties that can be directly studied experimentally.

I. INTRODUCTION

A number of experiments are presently underway to search for both composition-independent and composition-dependent deviations from Newtonian gravity.\(^1\) To date there is no compelling evidence for the existence of such deviations, although some unexplained experimental anomalies remain to be fully understood.\(^5\) The accumulation of a large number of null results can be used to infer stringent constraints on various theories which would attribute any departures from Newtonian gravity to the existence of new forces. The non-Newtonian effects expected in such theories are conventionally described in terms of a modified expression for the potential energy \(V(r)\) of two point masses \(m_{1,2}\) separated by a distance \(r\):

\[
V(r) = \frac{-G_m m_1 m_2}{r} (1 + \alpha e^{-r/\lambda}) \equiv V_N(r) + V'(r) .
\]

(1.1)

Here \(G_m\) is the Newtonian constant of gravity, and the parameters \(\alpha\) and \(\lambda\) respectively, give the range and its strength relative to gravity. Also, \(V'(r)\) describes the correction to the effective gravitational potential arising from the particular non-Newtonian interaction we are considering (which in this case is a Yukawa). The functional form of the Yukawa contribution \(V'(r) = V_Y(r)\) in Eq. (1.1) is suggested by models in which \(V_Y(r)\) arises from the exchange of a single new quantum with mass \(m_y = \lambda^{-1}\). If the source of this quantum is a charge \(Q = B \cos \theta + I_2 \sin \theta\), where \(B = N + Z\) is baryon number, and \(I_2 = N - Z\) is isospin, then

\[
\alpha = -\xi (Q_1/\mu_1) (Q_2/\mu_2) , \quad \mu_{1,2} = m_{1,2}/m_H .
\]

(1.2)

Here \(\xi = f^2/G_m m_H^2\) gives the coupling strength in terms of the unit of charge \(f\), and \(m_H = m_1 m_2 / m_H\). For later purposes it is instructive to exhibit the expression for the force \(F(r)\) implied by (1.1):

\[
F(r) = -\nabla V(r) = \frac{-G_m m_1 m_2}{r^2} \left[ 1 + \alpha (1 + r/\lambda) e^{-r/\lambda} \right] - \frac{G(r)m_1 m_2}{r^2} \equiv \nabla V_N(r) + \nabla V'(r) .
\]

(1.3a)

Each experimental limit defines a contour in the \(\alpha-\lambda\) plane which specifies the region in the plane excluded by that experiment. A summary of recent limits can be found in Refs. 9–11.

To date the Yukawa model in Eqs. (1.1)–(1.3) has been the most widely studied framework for introducing non-Newtonian effects. However, there are no compelling reasons to believe that the phenomenological non-Newtonian coupling must have the simple form suggested by Eq. (1.3), and since the theoretical implications of the experimental data are quite different in other models, a number of alternatives to Eq. (1.3) have also been examined. For example, Moffat\(^12\) has studied the experimental consequences of his non-symmetric gravity theory, in which the non-Newtonian contribution to the force varies as \(r^{-2}\), rather than that expected from a Yukawa potential. In addition, various authors have considered some of the implications of a model with two (nearly) canceling Yukawa potentials.\(^13\)–\(^15\) The latter model is the starting point of the present paper, whose objective is to demonstrate that there is an interesting (and hitherto unexplored) limiting case of two canceling Yukawa potentials, in which the resulting potential can be represented as an approximate exponential.\(^15\) By analyzing this regime in terms of a single exponential, we demonstrate that such a coupling leads to novel phenomenology. Moreover, by representing this interaction directly as an exponential we avoid the inevitable computational errors which arise when the contributions from two (nearly) canceling Yukawa potentials are evaluated numerically, as is discussed below. One of the purposes of this analysis is to compare in detail the phenomenological implications of the existing data in the Yukawa and exponential models, in order to establish the extent to which the relative sensitivities of different experiments change in the two models.

In Sec. II below we obtain the exponential potential as the limiting case of two canceling Yukawa potentials, and discuss models in which the necessary cancellations can take place. The phenomenological implications of the exponential model are discussed in Sec. III, and our conclusions are summarized in Sec. IV. Some mathematical details of the exponential model are given in the Appendices.
II. ORIGIN OF THE EXPONENTIAL POTENTIAL

As we have noted in Sec. I, the exponential potential arises as the limiting case of two (nearly) canceling Yukawa potentials. In this section, we demonstrate this explicitly, and discuss at the same time models in which the necessary cancellations could come about.

Consider the interaction of two nucleons arising from the exchange of fields \( \phi_a \) and \( \phi_b \) which interfere destructively, so that \( V'(r) = V_a(r) - V_b(r) \). Denote the masses of these fields by \( m_{a,b} \) and their couplings to \( Q \) by \( f_{a,b} \). If the masses and coupling constants are related in such a way that

\[
m_a = m_b[1 + O(\epsilon)], \quad f_a = f_b[1 + O(\delta \epsilon)],
\]

where \( \epsilon, \delta \ll 1 \), then it is straightforward to show that the leading contribution to the potential \( V'(r) = V_a(r) - V_b(r) \) depends exponentially on \( r \) (see Appendix A):

\[
V'(r) \approx V_E(r) = f^2 Q_1 Q_2 \left[ \frac{\Delta \lambda}{\lambda} \frac{e^{-r/\lambda}}{\lambda} \right],
\]

\[
F_E(r) = -\nabla V_E(r) = f^2 Q_1 Q_2 \left[ \frac{\Delta \lambda}{\lambda} \frac{e^{-r/\lambda}}{\lambda^2} \right],
\]

\[
A_E = \frac{F_E(r)}{m_1} \equiv \xi \left[ \frac{\Delta \lambda}{\lambda} \frac{Q_1}{\mu_1} \frac{Q_2}{\mu_2} \right] J_E(r, \lambda).
\]

Here \( \Delta \lambda = \lambda_a - \lambda_b = 1/m_a - 1/m_b, f \equiv f_b, \) and \( J_E \) is the field strength for a point source \( m_2 \) in the exponential model. It follows from the preceding discussion that we can view \( V_E(r) \) as a simple parametrization of the limiting case of two nearly canceling Yukawa potentials, which has novel phenomenological implications, as we discuss below.

We turn next to discuss some representative models of non-Newtonian forces which illustrate how an exponential potential could arise in specific theories. As we have already noted, the possibility of two nearly canceling Yukawa potentials has been investigated by a number of authors, including Goldman, Hughes, and Nieto (GHN). These authors and others note that in simple phenomenological theories the exchange of a vector \( J^P = 1^- \) leads to a repulsive force, whereas scalar \( J^P = 0^+ \) exchange gives an attractive force. If the vector and scalar fields were in turn related by some higher symmetry, such as supersymmetry, then the masses and couplings of these fields might be sufficiently close for a substantial cancellation to take place. However, in order to arrive at an exponential potential the masses and coupling constants must satisfy Eq. (2.1) and whether this can happen in such theories is not known.

A less obvious, but potentially more interesting, mechanism for producing an exponential potential is to consider the cancellation between two scalar fields, rather than between a scalar and a vector field as above. Ordinarily the exchange of a scalar field coupling to a simple charge such as baryon number or lepton number gives rise to an attractive force as we have noted previously. However, several models of weak gravititylike forces have been proposed recently in which a scalar field couples to a "composite" charge, i.e., one which is a linear combination of more elementary charges. For example, in the theory of Peccei, Solà, and Wetterich (PSW), scalar exchange produces a coupling between nuclei 1 and 2 of the form

\[
V'(r) = -G \frac{f^2 Q_1 Q_2}{r^2},
\]

\[
Q = \left( 1 - \frac{x \sigma}{m_H} \right) M \left( 1 - x \sigma \right) B - \frac{1}{2} \delta I_z.
\]

Here \( f \) is a coupling constant, and \( \sigma, \delta, \) and \( x \) are dynamical parameters which we define below. The novel feature of \( Q \) in (2.5b) is that the charge in the PSW model is a linear combination of the mass \( M \), baryon number \( B \), and isospin \( I_z \) of a nucleus, and hence the product \( Q_1 Q_2 \) in Eq. (2.5a) contains cross terms among \( M, B, \) and \( I_z \). For later purposes it is instructive to understand how the expression for \( Q \) in Eq. (2.5b) comes about. A scalar field can couple to the nucleons in the nucleus through two natural scalar operators. These are \( T^{j}_j(x) \), which is the trace of the full energy-momentum tensor, and \( \Theta^j_j(x) \), which is the anomalous trace of the energy-momentum tensor. The latter operator is proportional to the divergence of the dilatation current \( J^{j}(x) \), which is not conserved at the quantum level due to the presence of anomalies, i.e.,

\[
\partial J^{j}(x) = \sqrt{|g_{ab}(x)|} \Theta^{j}(x),
\]

where \( g_{ab}(x) \) is the metric tensor. The matrix elements of \( T^{j}_j(x) \) and \( \Theta^j_j(x) \) for a nuclear state \( |N\rangle \) are related as follows:

\[
\langle N| T^{j}_j|N\rangle = M,
\]

\[
\langle N| \Theta^j_j|N\rangle = \langle N| m_u u + m_d d|N\rangle
\]

\[
= M - \sigma B - \frac{1}{2} \delta I_z + \frac{x \sigma}{m_H} \epsilon B.
\]

Here \( m_u \) (\( m_d \)) is the mass of the \( u \) (\( d \)) quark, and \( x \ll 1 \) is a parameter which represents the fraction of the time that the operator \( (m_u u + m_d d) \) contributes to the binding energy \( \epsilon_B \) rather than to \( \sigma \) or \( \delta \). Theoretical arguments suggest that the term proportional to \( x \) can be neglected, and if we temporarily assume that \( \delta = 0 \) as well, then \( \langle \Theta^j_j \rangle \) in Eq. (2.7b) can be written as

\[
\langle N| \Theta^j_j|N\rangle \approx M \left( 1 - \frac{\sigma}{m_H} \right) \left[ \frac{B}{\mu} \right],
\]

where we have written \( M \equiv \mu m_H \). One can form an infinite number of charges by taking linear combinations of \( \langle \Theta^j_j \rangle \) and \( \langle T^{j}_j \rangle \), and from Eqs. (2.7) and (2.8) these charges will have the form

\[
Q(y) = M \left( 1 - y \left[ \frac{B}{\mu} \right] \right),
\]

\[
Q = \alpha \langle \Theta^j_j \rangle + \beta \langle T^{j}_j \rangle,
\]

where

\[
Q = a_n \langle \Theta^j_j \rangle + a_T \langle T^{j}_j \rangle,
\]

\[
Q = a_n \langle \Theta^j_j \rangle + a_T \langle T^{j}_j \rangle.
\]
and where the "mixing parameter" \( y \) is given by

\[
y = \frac{a_y}{a_y + a_T} \left( \frac{\sigma}{m_H} \right) .
\]

Consider now the interaction of test masses 1 and 2 in the presence of the potential that would arise from charges \( Q_a \) and \( Q_b \) in Eq. (2.9) corresponding to mixing parameters \( y_a \) and \( y_b \):

\[
V'(r) = -G_\infty \frac{\mu_1 m_1}{r} \left[ 1 - y_a \frac{B}{\mu} \right] \left[ 1 - y_a \frac{B}{\mu} \right] - G_\infty \frac{\mu_2 m_2}{r} \left[ 1 - y_b \frac{B}{\mu} \right] \left[ 1 - y_b \frac{B}{\mu} \right] ,
\]

At this stage, we can reinstate the contribution from \( I_z \) by letting \( B/\mu \rightarrow B/\mu + (\delta/2\sigma) I_z/\mu \equiv q \). The potential energy in Eq. (2.10) can then be rewritten in the form

\[
V'(r) = -G_\infty \frac{\mu_1 m_1}{r} \left( e^{-r/\lambda_a} + e^{-r/\lambda_b} - G_\infty \frac{\mu_2 m_2}{r} \left[ -(q_1 + q_2)(y_a e^{-r/\lambda_a} + y_b e^{-r/\lambda_b}) + q_1 q_2 (y_a^2 e^{-2r/\lambda_a} + y_b^2 e^{-2r/\lambda_b}) \right] ,
\]

where \( \delta = B/\mu \) and \( \tilde{\lambda} = L/\mu \). As before we can regroup the terms in (2.14) to give

\[
V'(r) = \delta_1 \delta_2 \left[ f_a e^{-r/\lambda_a} + f_b e^{-r/\lambda_b} \right] + (\delta_1 \tilde{\lambda}_2 + \delta_2 \tilde{\lambda}_1) \left[ f_a^2 z_a e^{-r/\lambda_a} + f_b^2 z_b e^{-r/\lambda_b} \right] + \tilde{\lambda}_1 \tilde{\lambda}_2 \left[ f_a^2 z_a e^{-r/\lambda_a} + f_b^2 z_b e^{-r/\lambda_b} \right] .
\]

Since \( z_a \) and \( z_b \) can have opposite signs, the term proportional to \( \delta_1 \delta_2 z_a z_b \) can give rise to an exponential potential for appropriate values of \( f_a^2 \) and \( f_b^2 \). We note that here, as in the PSW model, the exponential arises from the cross terms that are characteristic of scalar exchange in such models.

### III. PHENOMENOLOGY IN THE EXPONENTIAL MODEL

We discuss in this section the mechanisms by means of which the exponential potential changes the phenomenological interpretation of the existing composition-dependent experiments. Since the present status of composition-independent searches for non-Newtonian gravity is somewhat unsettled, the implications of the exponential model for such experiments will be considered elsewhere. Starting with Eq. (2.3) the sum of the Newtonian and non-Newtonian contributions can be written in a form analogous to Eq. (1.3), but with \( G(r) \) now having the form

\[
G(r) = G_E(r) = G_\infty \left[ 1 - \frac{\Delta \mu}{\mu} \frac{Q_1}{\mu_1} - \frac{Q_2}{\mu_2} \right] \left[ \frac{r}{\lambda} \right]^2 e^{-r/\lambda} .
\]

Comparing Eqs. (1.3) and (3.1) we see that in both cases the non-Newtonian effects vanish for \( r \rightarrow \infty \) as expected. However, for the exponential force, the non-Newtonian effects become negligible as \( r \rightarrow 0 \), in contrast with the
Yukawa case where they do not. It follows from Eq. (3.1) that the exponential force leads to a suppression of the non-Newtonian contributions from sources located at separations $r < \lambda$ from the detector. Thus a laboratory experiment utilizing as a source a sphere of radius $R$ would be reduced in sensitivity by a factor of order $(R / \lambda)^2$ compared to what would be expected from a Yukawa potential. For typical values of $R$ and $\lambda$, e.g., $R = 1$ m and $\lambda = 1000$ m, we have $(R / \lambda)^2 = 10^{-6}$. Since the $O(\delta)$ correction term in Eq. (2.1) becomes important when $(r / \lambda)^2 \lesssim \delta$ (see Appendix A), an additional Yukawa contribution can arise whose strength is of order $f^2(\delta)$, and which does not vanish as $r \to 0$. A reasonable upper bound on $\delta$ is $\alpha_{\text{em}} / \pi \approx 2 \times 10^{-3}$, which is the largest small parameter that naturally appears in perturbative theories. It follows that laboratory experiments are suppressed by a factor of either $(R / \lambda)^2$ or $\delta$, depending on the values of $\lambda$ and $\delta$ and on the dimensions of the source ($\sim R$). Since constraints on a possible coupling to $I_2$ come primarily from laboratory experiments, there are no significant constraints on the strength $\xi_2$ of such a coupling for $\lambda \gtrsim 10$ m in the exponential model.

For an experiment utilizing a large source $(R / \lambda \gtrsim \delta)$, the exponential contribution dominates. Experiments with large man-made sources have been carried out by Thieberger,22 and by Bennett,23 but the most stringent tests of the exponential model come from the cliff or hillside experiments of Thieberger,24 Adelberger et al.,25 Boynton et al.,26 Fitch et al.,27 and Bizzeti et al.28 To evaluate the intrinsic strengths $F(r, \lambda)$ of the various geophysical sources in the exponential model, one can either directly integrate the point source expression given in (2.4), or else start from the corresponding results for $F(r, \lambda)$ for the Yukawa model (if known) and use

$$F(r, \lambda) = \lambda \frac{\partial F_y(r, \lambda)}{\partial \lambda}. \quad (3.2)$$

To study the differences between the exponential and Yukawa models for large sources, we have analyzed the experiments of Adelberger et al.,25 (Seattle), and Boynton et al.26 (Mt. Index), which have the greatest intrinsic sensitivities among the hillside experiments. The surface terrain surrounding each site was modeled in a series of grids of different scales, which varied in size from 50 m x 50 m to 500 m x 500 m depending on the topography and on the distance from the experimental site. The grids contained 5653 squares at the Seattle site, and 5246 squares at the Index site, corresponding to a total area of approximately 30 km x 30 km surrounding each experiment. For each square, the maximum and minimum elevations were recorded, and these were used to estimate the error arising from the discretization of the topography. Our results, which were obtained from Eqs. (B15)-(B22), are shown in Figs. 1(a) and 1(b). These figures exhibit the north and east components of the limiting case of several canceling Yukawa potentials.

![FIG. 1. Field strengths for the Index and Seattle sites, in units of mgal = 10^{-5} m s^{-2}. At each site, the north and east components are plotted separately. (a) Yukawa case [F_y(\lambda)], and (b) exponential case [F_e(\lambda)].](image-url)
where the $x_n$ are appropriate constants. We see from (3.4) that for $n \geq 1$ the deviation of $G^{(n)}(r)$ from the Newtonian value $G_\infty$ vanishes not only at $r=0$ and $r \to \infty$, but also at $r = n \lambda$. This is a novel feature of such an interaction, which has interesting experimental consequences. Since $G^{(n)}(r)$ has extrema at $r = (1 + n \pm \sqrt{1 + n^2}) \lambda$, and changes sign in going from $r < n \lambda$ to $r > n \lambda$, it follows that even an experiment with a large geophysical source might fail to detect the presence of $V^{(n)}(r)$, due to fortuitous cancellations in the source contributions (see below).

It is clear from the preceding discussion that if the non-Newtonian force were described by the potential in Eq. (3.3) with $n \geq 1$, our view of the current experimental situation could be quite different from what it is in the conventional Yukawa formalism. It is thus interesting to note that the sum of a small number of primitive Yukawa terms can in fact lead to an expression for $G(r)$ having the property that $\Delta G(r) = [G(r) - G_\infty]$ vanishes at some intermediate value of $r$. As an example, consider the four-component potential

$$V'(r) = f^2 Q_1 Q_2 \left[ \frac{e^{-m_0 r}}{r} - \frac{e^{-m_1 r}}{r} \right],$$

(3.5)

where $m_0 = m(1 - \epsilon/2)$, $m_1 = m(1 + \epsilon/2)$, $m_2 = m(1 + \kappa)(1 + \epsilon/2)$, $m_3 = m(1 + \kappa)(1 - \epsilon/2)$, with $\epsilon, \kappa \ll 1$. Expanding (3.5) in $\epsilon$ and $\kappa$, and setting $\lambda = 1/m$, we find for the force

$$F'(r) \approx -7f^2 Q_1 Q_2 \epsilon \kappa (2 - r/\lambda) e^{-r/\lambda}/\lambda^2.$$  

(3.6)

It follows from Eqs. (3.5) and (3.6) that since $F'(r = 2\lambda) = 0$, the vanishing of $F'$ at intermediate $r$ could conceivably arise from a relatively simple mass spectrum. To quantitatively study the phenomenological implications of forces which vanish at intermediate $r$, consider $V^{(1)}$ in Eq. (3.3), and the four-component potential in Eq. (3.5). We calculate $|F|$ for an observer on the axis of a cylindrical source (length $= \text{radius} = R$) at one end cap, and compare these results to the corresponding Newtonian, Yukawa, and exponential contributions (see Appendix B and Fig. 2). We see that for the cylindrical source all of the generalized exponential models are suppressed at small $x = R/\lambda$. At large $x$, however, all finite-range models (including the Yukawa) converge to the same limit. This implies that when the characteristic size is much greater than $\lambda$, the interpretation of large scale experiments, such as the Galileo free-fall experiments, is essentially model independent. Most interestingly, the $n=1$ results show that in some models an observer adjacent to a macroscopic source could fail to detect the presence of a non-Newtonian force (in this case for $\lambda = 2.9$), even though $V^{(1)}(r) \neq 0$. The fact that this does not occur in the four-component model further underscores the sensitivity of the final results to both the nature of the source and to the detailed model of $V'(r)$.

### IV. SUMMARY

To summarize, we have shown that the exponential potentials in Eqs. (2.3) and (3.3) can arise from simple models, and that the phenomenological implications of such couplings can be quite different from those of the conventional Yukawa. Although the exponential model cannot reconcile the existing (conflicting) data, it is clear that the comparison of experimental results using different sources (both geophysical and laboratory) is highly model dependent. Hence by analyzing experimental data in terms of both a Yukawa and an exponential, we are in effect exploring the implications of a much broader class of interesting models.

### ACKNOWLEDGMENTS

We are indebted to Donald Eckhardt, Harry Kloor, and Daniel Sudarsky for valuable discussions. We wish to acknowledge the support of the U.S. Department of Energy under Contract No. DE-AC02-76ER01428, and the Air Force Geophysics Laboratory under Contract No. F192628-90-K-0010.

### APPENDIX A: EXPANSION OF TWO YUKAWA POTENTIALS

We present in this appendix some additional details of the exponential model. \(^{11}\) As noted in Sec. II, the exponential model arises as the limiting case of two nearly canceling Yukawa potentials, when the masses and coupling constants satisfy Eq. (2.1). To clarify the relation between Eq. (2.1) and the exponential model, we write the non-Newtonian potential energy $V'(r)$ arising from the sum of two Yukawa contributions in the form

\[ V'(r) = \sum_{n=1}^{\infty} \frac{G_{n+1}}{r^{n+1}} - \frac{G_{n+2}}{r^{n+2}} - \frac{G_{n+1}}{r^{n+1}}. \]
EXPONENTIAL MODELS OF NON-NEWTONIAN GRAVITY

\[ V'(r) = -\frac{G_x m_1 m_2}{r} \begin{vmatrix} Q_1 & Q_2 \\ \mu_1 & \mu_2 \end{vmatrix} \]

\[ \times (-\xi_a e^{-r/\lambda} + \xi_b e^{-r/\lambda}) , \]  
(A1)

where by hypothesis the two contributions are assumed to interfere destructively. To incorporate the content of Eq. (2.1) we first simplify our notation by writing \( \lambda(\epsilon) \rightarrow \epsilon \) and \( O(\epsilon^2) \rightarrow \epsilon^2 \), and we then define

\[ \xi_a = \xi(1 + \epsilon b / 2) , \quad \xi_a = (\xi_a + \xi_b) / 2 \]

\[ \xi_b = \xi(1 - \epsilon b / 2) , \quad \epsilon b = (\xi_a - \xi_b) / \xi \]

\[ \lambda_a = \lambda(1 + \epsilon / 2) , \quad \lambda = (\lambda_a + \lambda_b) / 2 \]

\[ \lambda_b = \lambda(1 - \epsilon / 2) , \quad \epsilon = (\lambda_a - \lambda_b) / \lambda \]

\[ q_i = Q_i / \mu_i , \quad i = 1, 2 \]

Assuming \( \delta, \epsilon << 1 \) as before, we find, upon expanding Eq. (A1),

\[ \Delta a = -\frac{1}{m_1 m_2} \nabla V'(r) \]

\[ = -\xi q_2 \Delta D [\delta F_Y(r, \lambda) + F_E(r, \lambda)] , \]  
(B1)

where \( S \) and \( D \) refer to the source and detector, respectively, and

\[ F_Y(r, \lambda) = \int_{\text{body}} d^3 r' G_x \rho(r') \frac{r - r'}{|r - r'|^2} \]

\[ \times \left[ 1 + \frac{|r - r'|}{\lambda} \right] e^{-|r - r'| / \lambda} , \]  
(B2)

\[ F_E(r, \lambda) = \frac{\partial F_Y(r, \lambda)}{\partial \lambda} \]

\[ = \int_{\text{body}} d^3 r' G_x \rho(r') \frac{r - r'}{|r - r'|} e^{-|r - r'| / \lambda} . \]  
(B3)

Using Eq. (B3), we can obtain closed-form expressions for \( F_E(r, \lambda) \) when closed-form expressions for \( F_Y(r, \lambda) \) already exist. In the examples we consider below, the mass distribution is assumed to be chemically homogeneous and of constant density \( \rho \).

(i) Point mass. We have

\[ F_Y(r, \lambda) = \hat{\delta} G_x M \left( 1 + \frac{r}{\lambda} \right) e^{-r / \lambda} , \]  
(B4)

where \( M \) is the source mass, and \( r \) is the distance from the source to the detector. Hence

APPENDIX B: CLOSED-FORM SOLUTIONS

In this appendix, we present closed-form solutions to the non-Newtonian fields for both the Yukawa and exponential models. \( ^{23} \) We begin by defining these quantities in terms of the differential acceleration between two test masses. From Eq. (A1), we have

\[ \Delta a = -\frac{1}{m_1 m_2} \nabla V'(r) \]

\[ = -\xi q_2 \Delta D [\delta F_Y(r, \lambda) + F_E(r, \lambda)] , \]  
(B1)

where \( S \) and \( D \) refer to the source and detector, respectively, and

\[ F_Y(r, \lambda) = \int_{\text{body}} d^3 r' G_x \rho(r') \frac{r - r'}{|r - r'|^2} \]

\[ \times \left[ 1 + \frac{|r - r'|}{\lambda} \right] e^{-|r - r'| / \lambda} , \]  
(B2)

\[ F_E(r, \lambda) = \frac{\partial F_Y(r, \lambda)}{\partial \lambda} \]

\[ = \int_{\text{body}} d^3 r' G_x \rho(r') \frac{r - r'}{|r - r'|} e^{-|r - r'| / \lambda} . \]  
(B3)

Even for \( \epsilon = 1/\lambda \) and \( n = 2 \), at the value of \( r / \lambda \) where \( s_n = s_n \) \( \exp() \) is on the order of \( 10^{-22} \); for \( \epsilon = \frac{1}{10\lambda} \), \( \exp() \) is on the order of \( 10^{-22} \). The analogous terms in the exponential decrease even more rapidly for \( n = 0 \). We conclude that the phenomenology will be dominated by the \( n = -1 \) term for \( r / \lambda \leq \delta \), and by the \( n = 0 \) term for all other values of \( r / \lambda \) for which \( V'(r) \) is non-negligible. \( V'(r) \) is then given to the required accuracy by

\[ V'(r) = -\xi G_x m_1 m_2 q_1 q_2 \left( \delta \frac{e^{-r/\lambda}}{r} + \epsilon \frac{e^{-r/\lambda}}{\lambda} \right) . \]  
(A8)
\[ \mathcal{F}_E(r, \lambda) = \hat{G} \frac{M e^{-r/\lambda}}{\lambda^2}, \]  

(E5)

in agreement with Eq. (2.3). In comparing Eqs. (2.4) and (E5), we note that for \( r \ll \lambda \) the exponential force is suppressed relative to the Yukawa force, since the point mass is always (effectively) a distance \( \lambda \) away from the source. This is the origin of the short-distance suppression of the exponential force which we have discussed previously.

(ii) Spherical mass distribution. We consider the case of a test object located a distance \( r \) from the center of mass of a spherical mass distribution of uniform density with mass \( M \) and radius \( R \). For \( r \geq R \) we have\(^1\)

\[ \mathcal{F}_Y(r, \lambda) = \hat{G} \frac{M}{r} \left[ 1 + \frac{r}{\lambda} \right] e^{-r/\lambda} \Phi \left( \frac{r}{\lambda} \right), \]  

(E6)

\[ \Phi(x) = \frac{3}{x^3} (x \cosh x - \sinh x). \]  

(B7)

Since

\[ \lambda \frac{\partial \Phi(R/\lambda)}{\partial \lambda} = 3 \Phi(R/\lambda) - \frac{3 \sinh(R/\lambda)}{R/\lambda}, \]  

(E8)

we find

\[ \mathcal{F}_E(r, \lambda) = \hat{G} \frac{M}{R} \left[ 1 + \frac{R}{\lambda} \right] e^{R/\lambda} \Phi \left( \frac{R}{\lambda} \right) \]

\[ - \hat{G} \frac{M}{R} \left[ 1 + \frac{R}{\lambda} \right] \frac{3 \sinh(R/\lambda)}{R/\lambda}. \]  

(E9)

For \( r \leq R \),

\[ \mathcal{F}_Y(r, \lambda) = \hat{G} \frac{M}{R} \left[ 1 + \frac{R}{\lambda} \right] e^{R/\lambda} \Phi \left( \frac{R}{\lambda} \right), \]  

(B10)

and

\[ \mathcal{F}_E(r, \lambda) = \hat{G} \frac{M}{R} \left[ 1 + \frac{R}{\lambda} \right] e^{R/\lambda} \Phi \left( \frac{R}{\lambda} \right) \]

\[ - \hat{G} \frac{M}{R} \left[ 1 + \frac{R}{\lambda} \right] \frac{3 \sinh(R/\lambda)}{R/\lambda}. \]  

(B11)

(iii) Cylindrical mass distribution. We consider here the case of a test object located along the symmetry axis of a cylinder of uniform density \( \rho \), radius \( R \), and length \( L \). We also choose \( z = 0 \) to occur at one end and define \( z \) to increase away from that end. We then have

\[ \mathcal{F}_Y(r, \lambda) = 2\pi \rho G \lambda \left[ (\sqrt{R^2 + z^2}/\lambda + 1) e^{-R^2 + z^2}/\lambda \right. \]

\[ \left. - (|z|/\lambda + 1) e^{-R^2/\lambda} \right] z = z + L, \]  

(B12)

and

\[ \mathcal{F}_E(r, \lambda) = 2\pi \rho G \lambda \left[ (\sqrt{R^2 + z^2}/\lambda + 1) e^{-R^2 + z^2}/\lambda \right. \]

\[ \left. - (|z|/\lambda + 1) e^{-R^2/\lambda} \right] z = z + L. \]  

(B13)

For purposes of comparison, the gravitational acceleration \( g \) for this configuration is given by

\[ g(r) = - 2\pi \rho G \lambda \left( \sqrt{R^2 + z^2} - |z| \right) z = z + L. \]  

(B14)

(iv) Parallelipiped mass distribution. We finally consider the case for a parallelipiped whose edges are aligned along the \( \hat{x}, \hat{y}, \) and \( \hat{z} \) axes, and whose extent is given by vertices located at \((x_0, y_0, z_0)\) and \((x_1, y_1, z_1)\), with \( x_0 < x_1, y_0 < y_1, z_0 < z_1 \). (The more general case can be obtained from the results presented here by three-dimensional rotations about the appropriate axes.) Then

\[ \mathcal{F}_Y(\lambda) = \frac{\pi}{2} \rho G \lambda \sum_{i,j,k} (-1)^i j k \left[ \hat{\Phi}_{Q}(\Delta x_i, \Delta y_j, \Delta z_k; \lambda) + \hat{\Phi}_{Q}(\Delta y_j, \Delta z_k, \Delta x_i; \lambda) + \hat{\Phi}_{Q}(\Delta z_k, \Delta x_i, \Delta y_j; \lambda) \right], \]  

(B15)

where \( \Delta x_i = x_i - x_{i-1}, \ldots \), and \( \hat{\Phi}_{Q}(x, y, z; \lambda) \) is given by

\[ \Phi_{Q}(x, y, z; \lambda) = \Phi_{Q}(x, \lambda, y; \theta_0) + \Phi_{Q}(x, \lambda, z; \theta'_0) \quad (y < 0, z < 0) \]

(B16a)

\[ = \Phi_{Q}(x, \lambda, y; \theta_0) - \Phi_{Q}(x, \lambda, z; \theta'_0) \quad (y < 0, z > 0) \]

(B16b)

\[ = \Phi_{Q}(x, \lambda, z; \theta'_0) - \Phi_{Q}(x, \lambda, y; \theta_0) \quad (y > 0, z < 0) \]

(B16c)

\[ = 4 e^{-x/\lambda} - \Phi_{Q}(x, \lambda, y; \theta_0) - \Phi_{Q}(x, \lambda, z; \theta'_0) \quad (y > 0, z > 0) \]

(B16d)

and where

\[ \theta_0 = \frac{\pi}{2} - \theta_0, \]  

\[ \theta'_0 = |\arctan(z/y)|. \]  

(B17)

\[ \Phi_{Q}(\alpha, \beta; \theta) = \frac{2}{\pi} \int_0^{\alpha/2} d\theta' \exp(-\sqrt{\alpha^2 + \beta^2 \sec^2 \theta'}). \]  

(B19)
\[ \phi_i(\alpha, \beta) = \phi_i(\alpha, \beta; 0), \quad (B20) \]
\[ \phi_i'(\alpha, \beta; \theta) = 2 \phi_i(\alpha, \beta) - \phi_i(\alpha, \beta; \theta), \quad (B21) \]

The solution for \( \mathcal{F}_p(\lambda) \) for a parallelepiped can be obtained trivially from this equation, by defining

\[ \phi_i'(\alpha, \beta; \theta) = \phi_i(\alpha, \beta; \theta) \quad \text{everywhere in Eqs.} \quad (B16)-(B21). \]

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17An example of the symmetry-breaking pattern in Eq. (2.1) is provided by the Ademollo-Gatto theorem for flavor SU(3), which corresponds to \( \delta = \varepsilon \). See M. Ademollo and R. Gatto, Phys. Rev. Lett. 13, 264 (1964).


21See Refs. 2–5 for references to various laboratory Eötvös experiments.


31C. Talmadge and E. Fischbach, in 5th Force—Neutrino Physics (Ref. 3), p. 413.
Non-Newtonian Gravity and New Weak Forces: an Index of Measurements and Theory
Non-Newtonian Gravity and New Weak Forces: an Index of Measurements and Theory


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Non-Newtonian Gravity and New Weak Forces: an Index of Measurements and Theory


Abstract. The precise measurement of weak effects plays a pivotal role in metrology and in the determination of the fundamental constants. Hence, the possibility of new weak forces, and the related question of non-Newtonian behaviour of the gravitational force, have been of special interest to both measurement scientists and those involved in precise tests of physical laws. To date there is no compelling evidence for any deviations from the predictions of Newtonian gravity in the nonrelativistic weak-field regime. A significant literature on this question has developed over the past few years, and a host of experiments and theoretical scenarios have been discussed. Moreover, a very close relationship exists between the experimental methodologies used to determine the absolute value of the Newtonian gravitational constant \( G \), and those employed in searches for new weak forces and for breakdowns in the inverse-square law of gravity. We have therefore prepared a new index of measurements of such effects, using the original bibliographic work of Gillies as a starting point, but also including citations to the appropriate theoretical papers in the field. The focus of the present version of the index is then studies of the "fifth force", measurements of gravitational effects on antimatter, searches for a spin-component in the gravitational force, and related phenomena.

1. Introduction

During the past several years interest in the possibility of deviations from the predictions of Newtonian gravity has increased significantly, motivated by both theoretical and experimental considerations. On the theoretical side, a number of workers, most notably Fujii [1] and Scherk [2], stimulated interest in this question by demonstrating that various models suggest the existence of new weak inter-mediate-range forces coexisting with gravity. These forces share with gravity the property of acting over macroscopic distances but, unlike gravity, their effects are negligible beyond some characteristic distance \( \lambda \) (called the range) from a source. As we discuss in more detail below, the combined effect of such a finite-range force and gravity is a net gravity-like force whose overall strength depends on the separation \( r = |\vec{r}_i - \vec{r}_j| \) of two test masses \( i \) and \( j \), and thus leads to deviations from the expected \( 1/r^2 \) force law.

Stimulated by these ideas, experimental studies were undertaken in the 1970s and 1980s to search for deviations from the \( 1/r^2 \) law. A detailed history of these efforts is contained in a number of recent reviews [3, 4, 5] to which we refer the interested reader. The accumulation of a number of experimental and theoretical results eventually led Fischbach et al. [6] to reanalyze the classic experiment of Eötvös, Pékár, and Fekete [7] (EPF) which compared the accelerations of different pairs of materials to the Earth. Fischbach et al. observed that the experimental results of EPF could be interpreted as suggesting the existence of a new weak intermediate-range force, whose presence would lead to apparent deviations from the predictions of Newtonian gravity.

Since the publication of [6] a large number of experimental and theoretical papers have been written which explore in various ways the possible existence of new weak gravity-like forces. At present there is no compelling evidence for any deviations from the predictions of Newtonian gravity in the nonrelativistic regime (i.e. for systems where general relativistic effects are negligible). The present bibliography is an attempt to collect in a single place all the relevant literature on such forces. To understand the relationships among the various papers, as well as the criteria we have used in compiling this bibliography, the following simplified phenomenological introduction may be useful. The presence of a single new field of mass \( m \) gives rise to an additional Yukawa potential proportional to \( \exp(-r/\lambda) \) \( r \), where \( \lambda = \hbar/mc \). This potential modifies the Newtonian interaction so that the new potential describing the interaction of masses \( m_i \) and \( m_j \) assumes the form

\[
V(r) = -G\infty \frac{m_i m_j}{r} (1 + \alpha e^{-r/\lambda}).
\]
Here $G_{\infty}$ is the Newtonian constant of gravity (in the limit $r \to \infty$), and $\alpha$ is a constant which characterizes the strength of the new interaction relative to gravity. Differentiating (1) we find for the corresponding force,

$$F(r) = -\nabla V(r) = -\frac{G_{\infty}}{r^2} \frac{m_i m_j}{r} \left[ 1 + \alpha \left( 1 + r/\lambda \right) e^{-r/\lambda} \right]$$

$$= -G(r) \frac{m_i m_j}{r^2} e^{-r/\lambda}, \hspace{1cm} (2a)$$

$$G(r) = G_{\infty} \left[ 1 + \alpha \left( 1 + r/\lambda \right) e^{-r/\lambda} \right]. \hspace{1cm} (2b)$$

We see from (2) that the effect of the new interaction is to replace the Newtonian constant $G_{\infty}$ by the function $G(r)$ in (2b). It follows that tests of the constancy of $G(r)$ as a function of $r$, i.e., tests of deviations from a simple $1/r^2$ force law, are also direct tests of the presence of new couplings.

In the form of (2), which is that suggested by Fujii [1] and others, $\alpha$ is a universal constant determined by the coupling strength of some new quantum to matter. The work of Fischbach et al. [6] focuses attention on the fact that in many theories $\alpha$ is not a universal constant, but depends instead on the compositions of the test masses $i$ and $j$. In the original "fifth force" model of [6], $\alpha$ is given by

$$\alpha \equiv \alpha_{ij} = -\xi \left( \frac{B_i}{\mu_i} \right) \left( \frac{B_j}{\mu_j} \right). \hspace{1cm} (3)$$

where $\xi = f^2/G_{\infty}$, $m_i^j$ expresses the strength of the new interaction in terms of a new constant $f$, which is the analog for this interaction of what the electric charge $e$ is for electromagnetism. Here $B_i$ denotes the baryon number (the number of neutrons and protons) in the sample $i$, and $\mu_i = m_i/m_p$, where $m_p$ is the mass of atomic hydrogen. Since $(B_i/\mu_i)$ varies from one material to another, $\alpha_{ij}$ is a function of the compositions of the interacting materials. It follows that the accelerations of two samples $j$ and $j'$ towards a common source $i$ depend on the (generally unequal) constants $\alpha_{ij}$ and $\alpha_{j'i}$. If $i$ denotes the Earth, then the accelerations of $j$ and $j'$ towards the Earth depend on their compositions, and this is what the Eötvös experiments set out to measure.

It follows from the preceding discussion that the presence of a new intermediate-range weak force can be detected either through the modification of the usual inverse-square law for the net force, and/or through a composition-dependence of the net acceleration. (The latter effect is often referred to as a violation of the Weak Equivalence Principle or of the universality of free-fall). Although in principle both of these effects are always present, in practice experiments are usually designed to isolate one or the other of these manifestations of non-Newtonian gravity. When appropriate we have therefore classified the papers in this bibliography according to whether they deal primarily with tests for composition-dependent effects (denoted by CD), or with tests of the inverse-square law (IS). Forces which give rise to composition-dependent effects in bulk matter will generally affect matter and antimatter differently, and references on antimatter are denoted by AM. In this category we have also included, for obvious reasons, papers dealing with the $K^0-\bar{K}^0$ system. New forces can also manifest themselves via spin-dependent couplings, and papers on this topic are denoted by SD. We have classified papers depending on whether they are primarily experimental (E), theoretical (T), or phenomenological (P), the final label being one we use to characterize theoretical papers analyzing data. Where a paper quotes a new experimental result or a theoretical limit, we have denoted this by a $+$ sign. Summaries of the existing data, along with descriptions of the techniques that have been used, are given in [4] and [5].

In addition to these primary classifications, we have also noted whether an experiment utilizes a laboratory source (L), a geophysical source — e.g. a cliff (G), a lake (L.K), or an astrophysical source (A). For theoretical papers presenting models of non-Newtonian effects, we have denoted by AG those in which the model is essentially an alternative theory of gravity, and by FI those for which the starting point is a new fundamental interaction such as the "fifth force". In addition, theoretical or experimental papers dealing with high-energy or elementary particle systems are denoted by H. Finally, REV denotes a review paper, and INT indicates an introductory or elementary exposition. We recognize that these distinctions are sometimes subtle and not always well-defined, and hence the labels attached to each paper are only meant to supply a general indication of its contents.

We have been guided by a number of criteria in selecting the papers included in this bibliography. As explained above, our focus has been on the search for new weak forces, particularly the hypothesized "fifth force" first proposed in 1986. We have thus included virtually all papers published on this subject since January 1986. In selecting papers published prior to 1986, we have limited ourselves to those which can be viewed as the direct antecedents of the current searches for non-Newtonian gravity, such as the seminal papers by Fujii. With very few exceptions we have thus excluded papers already cited in the comprehensive bibliography by Gillies [8] dealing with the dependence of $G$ on various external influences (temperature, shielding, etc.) which can also be interpreted as tests for non-Newtonian gravity. Moreover, given the focus of our effort, we have chosen not to include papers aimed primarily at redeterminations of the absolute value of $G$, or the other classical effects discussed by Gillies, as these will be the topic of a separate updated report. It is thus our intention that the present work be viewed as complementing and updating that of Gillies which, coincidentally, was completed just as interest in the "fifth force" was beginning. We have also excluded the area of gravitational effects on quantum systems, most notably the important work of Colella, Overhauser and Werner [9], because the extensive literature that has grown up in this area genuinely warrants an index of its own. Finally, we have not included papers dealing with the deviations from Newtonian gravity implied by general relativity, which are covered in excellent reviews by Will [10], or papers relating to the time variation of $G$.

We acknowledge that our choices for the papers to be included in this bibliography are necessarily somewhat arbitrary. We also recognize that there are papers which
should properly be included, but of which we are simply unaware. We therefore apologize in advance to all our colleagues whose papers we have inadvertently omitted. It is our intention to update this listing, and we therefore request that any suggestions for papers to be included be brought to the attention of the authors. Finally we wish to thank Sam Aronson and Lisa Schwan for their help in compiling this bibliography.

Selected References


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### Key to Paper Categories

#### General categories

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#### Qualifiers

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<tr>
<td>LK</td>
<td>Lake</td>
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<tr>
<td>+</td>
<td>New (i.e. original) result or limit</td>
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674. Speake C. C., Quinn T. J., “A search for composition-dependent gravity using a beam balance", 245


705. Sudarsky D., "Bounds on oscillating physics from high energy experiments", In Progress in Electro-


R. W. Hellings), National Aeronautics and Space Administration, 1989, 137-140. Topics: CD,E,A


**Author Index**

The following is a cross-index of the papers of all authors who appear in the bibliography, where the articles are listed by article number as they appear in the bibliography. Boldface numbers indicate papers for which the cited individual is “first author”.

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Topics Index

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Addendum

Since submitting the original version of our bibliography, we have received numerous suggestions for additional papers to be included, and most of these have been incorporated into the preceding text. However, we have also decided to include in proof a number of papers related to spin-dependent effects, and these are listed below. Although they could not be cross-referenced in the index, we hope that their inclusion here will serve to make this bibliography more useful.


In addition, there are several very recent papers which we include here for the sake of completeness.

ONE HUNDRED YEARS OF THE EÖTVÖS EXPERIMENT*

L. Bod³, E. Fischbach²
G. Marx¹ and Maria Náray-Ziegler³

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² Department of Physics, Purdue University, W. Lafayette IN, USA
³ Central Research Institute for Physics, Budapest, Hungary

(Received 31 August 1990)

Roland Eötvös' classic experiment concerning the proportionality of inertial and gravitating masses, performed at first in 1889, has again become the focus of scientific interest in the 1980's, due to the possibility of the existence of a Fifth Force, as proposed by Fischbach and coworkers. The publication of Eötvös, Pekár and Fekete omitted various details of their experiment which may be relevant for the re-interpretation of their results. The aim of this report is to fill in some of these details, and to discuss the impact of the Eötvös experiment on modern research.

"Ars longa, vita brevis"

Inspired by the beauty of the Newtonian system, Baron Roland von Eötvös experimentally investigated the proportionality of inertial and gravitating masses in 1889, and reported his results in the Proceedings of the Hungarian Academy in 1890 [1]. In this work he improved Bessell's accuracy 1/60 000 to 1/20 000 000. This was a short report of 3 pages. Inspired by this achievement, the Royal Scientific Society of Göttingen in 1906 offered a prize (see Appendix I) for the following task:

"A very sensitive method was given by Eötvös to make a comparison between the inertia and gravity of matter. Considering this and the new development of electrodynamics as well as the discovery of radioactive substances, Newton's law concerning the proportionality of inertia and gravitation is to be proved as extensively as possible."

Eötvös began a series of investigations with his co-workers Pekár and Fekete in the years 1906-1909. This included data taking through approximately 4000 hours. Eötvös personally reported his results at the 16th International Geodesic Conference in London in 1909 [2], quoting an achieved accuracy of 1/100 000 000. The complete work of Eötvös, Pekár, Fekete was submitted to the Beneke Foundation in 1909 [3]. Its motto was "Ars longa, vita brevis" (the art lasts long, life lasts short), which is indeed a true characterization of the fate of Eötvös' work. The evaluation of C. Runge, Dean of the Faculty of Science in Göttingen [4] says that Eötvös has quoted an accuracy of 1/200 000 000, but since the submitted text does not include

* Dedicated to Prof. J. Csikai on his 60th birthday
the real theoretical discussion of the observational data, the Faculty recommends only a reduced prize for this work (3400 German Marks instead of 4500 Marks). In Appendix I we reprint the text of this evaluation.

Shortly thereafter the First World War came. Roland Eötvös died in 1919, and the detailed description of the experiments performed in 1906-1909 was published by his assistants Pekár and Fekete only in 1922 [5]. This is the text known, cited, and translated by the international scientific community worldwide. When the collected works of Roland Eötvös were published by the Hungarian Academy of Sciences [6], the editor (Eötvös' former student P. Selényi) included some additions in parentheses [...] in the reprinted Eötvös-Pekár-Fekete paper [5], taken from the original manuscript [3]. The original Beneke-prize manuscript was lost somewhere in the hands of the heirs to Pekár and Selényi. The more complete text taken from the Volume [6] has been reprinted in English in Budapest in 1963 [5].

The Eötvös experiment was repeated by J. Renner (a former student of Eötvös, and a physics teacher in the famous Lutheran High School in Budapest where among others J. von Neumann and E. P. Wigner studied). The results of Renner's experiment were published in Hungarian [7] (with a German abstract, reprinted in Appendix II). Renner claimed an empirical accuracy of $1/2 \times 10^{-9}$ to $1/5 \times 10^{-9}$.

About Dicke's criticisms

Acknowledging the basic role played by the connection between inertial mass and gravitating mass in General Relativity, P. G. Roll, R. Krotkov and R. H. Dicke carried out a new experiment, using modern technology, and achieved an accuracy of $1/100 \times 10^{-9}$ [8]. Dicke and co-workers were able to increase the sensitivity compared to Eötvös in part by measuring the accelerations of their test masses to the Sun, rather than to the Earth as Eötvös had done. In such an experiment any signal arising from the difference between gravitational and inertial mass would have the same 24-hour periodicity as the Earth's rotation. The advantage of such an approach from an experimental point of view is that it allows such a signal to be discriminated from background perturbations, without disturbing the torsion fibre. Of course, one must be careful to exclude other perturbations which will have the same 24-hour period. In fact Eötvös, Pekár and Fekete were the first to compare the accelerations of different materials to the Sun, and for platinum versus mangalium they quote a fractional difference of $\delta \times 10^{-9}$. However, since no error is quoted and few other details of their analysis are presented, it is difficult to know precisely how the sensitivity of this part of their experiment compared with that of their more extensive work measuring accelerations to the Earth.

In analyzing the Eötvös results, Dicke expressed his polite doubts about the accuracy claimed by Eötvös' assistants. Among his concerns were the following:

1. Dicke was suspicious about the perturbation of air motion created by temperature differences. The present authors think that Eötvös' team was quite careful in this respect. The observations were performed in a shaded closed
Fig. 1. Torsion balance used in Eötvös' measurements
Fig. 2. Sketch of the torsion balance

room, in a double-walled tent (the space between the two walls was stuffed with sea-weed), with a torsion balance protected by triple copper coating with air space between (Figs 1, 2 and the letter of J. Renner to R. H. Dicke, due to the encouragement of one of us (G.M.) as reprinted in Appendix III.) Further evidence for the concern of Eötvös and co-workers over thermal effects is reflected in the fact that they affixed thermometers to the torsion balance at various locations. (The double-arm balance, used in their third method, had three thermometers: One along each arm, and one near the torsion fibre.) The question of possible thermal effects was raised by Dicke as an alternative to the Fifth Force to explain the correlations in the Eötvös data noted by Fischbach and co-workers. This question has been discussed in detail in [12], where it is noted that the main objection to such an explanation of the Eötvös results is that the thermal effects would have to be of constant sign and magnitude during approximately 4000 hours of data taking, spread out
over several years, which seems unlikely.

2. Dicke was also concerned about the poorly defined gravitational perturbation, caused by the observer himself. This potential source of error was well known to Eötvös. While the vibrations of the torsion pendulum were damped, the observer was far away. When the balance had come to rest, the observer came running and made the reading, before the pendulum (of period of 40 minutes) had time to swing out [9].

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Fig. 3. Data sheet from the R. Eötvös bequest

3. Unfortunately, the statistical evaluation of the empirical data cannot be fully reconstructed in the case of the Eötvös–Pekár–Fekete experiment, as the details are not given in any of their publications. (In the bequest of Eötvös, kept in the Library of the Hungarian Academy of Sciences, sheets with laboratory data readings can be found, e.g. Fig. 3, but they do not contain sufficient
information to allow the statistical evaluation to be repeated.) In the Hungarian publication of J. Renner [7] more details can be found. It can be seen, as pointed out by Dicke [8], that Renner tried to eliminate the influence of environmental changes by interpolation in time. The numbers obtained by interpolation are not statistically independent, but Renner treated them as if they were. For this reason the statistical errors might be larger by a factor of 3 than claimed by Renner. It should be noted that this factor is 2.4 as shown by Király [19]. Taking this source of error into account, Renner's data lay suspiciously near to zero, concerning the difference of the two masses. Therefore Dicke and others have suggested that Renner's conclusions cannot be relied upon. Renner learned this statistical technique from Eötvös' team. The inaccuracy quoted by Pekár in his paper [5] originated from observational errors and statistical errors in a ratio unknown to us; it could be that the accuracy of $10^{-4}$ [2], given by Eötvös personally is the reliable one. This is in any case a marvellous achievement, and the curious trends, noted by Fischbach and co-workers seem to survive. (See the next Section.)

It should be emphasized that the analyses of both Dicke and Fischbach agree that the errors quoted by Eötvös, Pekár and Fekete are consistent with the statistical scatter of their data. Moreover, the confidence level of the best fit of Fischbach et al to the Eötvös data, viewed as suggesting a Fifth Force, is 86%, which is perfectly reasonable.

Let us quote Eötvös himself [5]: “Ars longa, vita brevis. The admonition of this old saying motivates the authors of this paper to compile the results of their investigation and to submit them to the judgement of a high scientific Aeropag. Methods of observation refine and improve naturally in the course of observation, and hence no mortal could close his work if without cease would follow the otherwise laudable impetus to replace the useful by the even better.”

The hypothesis of the Fifth Force

Eötvös' experiment is one of the last pearls of the grand epoch of classical physics. At the end of their investigations [5] Eötvös, Pekár, and Fekete studied how far the proportionality of inertia and gravity is valid in case of radioactive materials. (This was already in the era of $E = mc^2$.) The proportionality was verified for a 0.1 g sample of RaBr$_2$ with an accuracy of 1/2 000 000.

In the following decades, the investigation of the structure of matter called attention to other possible forces of Nature beyond the long-ranged gravity and electricity, and beyond the short-ranged nuclear and weak interactions. According to the quantum theory the range of the force ($r$) is related to the mass ($m$) of the quanta of the transmitting field by the quantum law $r = h/mc$, where $h$ is Planck's constant and $c$ is the speed of light.

The infinite length of the gravitational and electric field lines is logically connected to the absolute conservation laws of mass (energy) and charge. If there is any further exact or approximate conservation law (e.g. the conservation of the...
baryonic charge $B$, discovered by E. P. Wigner [10], it may be that a further unknown field exists which may transmit a Fifth Force. But if the rest mass of the field quanta were exactly zero (as in the case of photons), hot bodies would radiate these quanta as well, in contradiction to thermodynamical experience. Thus one may hypothesize that the field quanta should have a small but nonvanishing rest mass ($m$), consequently the transmitted Fifth Force would have a long but finite range ($z_0 = h/mc$). This gives rise to a composition-dependent “action at a distance” between two massive bodies, where the interaction energy is the sum of the gravity and the Fifth Force contributions:

$$V(z) = -Gmm'/z + FBB' \exp(-z/z_0)/z.$$  

For astronomical distances only Newtonian gravity contributes,

$$V(z) = -Gmm'/z \text{ if } z \gg z_0.$$  

For laboratory distances, however, one may experience an “effective gravity”

$$V(z) = -G_{eff}mm'/z \text{ if } z \ll z_0,$$

with an effective gravitational constant

$$G_{eff} = G - F(B/m)(B'/m'),$$

which may differ from the (astronomical) Newtonian gravitational constant $G$. If $B$ is the baryon number (protons plus neutrons) in the atom, and if $M$ is the mass of the atom in Hydrogen atom mass units $m_H$, that is $M = m/m_H$, then

$$G_{eff} = G[1 - a(B/M)(B'/M')]$$

with $a = F/Gm_H^2$. In the case of hydrogen $B/M \equiv 1$, whereas for carbon $B/M = 1.00782$, for copper $B/M = 1.00895$, and for lead $B/M = 1.00794$. Hence the effective gravitational constant, manifesting itself over laboratory scales may be composition dependent. This idea can be checked by comparing the empirical value of $G$ on astronomical and laboratory scale, and by testing its composition-dependence. Eötvös’ experiments contributed to both. By plotting (see in the work of P. Király [19]) the measured ratio inertial mass/gravitating mass with respect to $B/M$ (Fig. 4), from the results published by Eötvös–Pékár–Fekete [5], Fischbach and co-workers concluded that the slope of the resulting line was $(5.65 \pm 0.7) \times 10^{-6}$, which differs from the expected value of zero by several standard deviations [11]. If the Fifth Force really exists with a range of 100 m, say, the composition-dependence must be due to the action of nearby mass distribution.

The Fifth Force hypothesis has had a double effect: It has encouraged a series of modern experiments, and it has increased the interest in details of the original
Eötvös–Pekár–Fekete experiment [5] (which seemed to indicate a positive effect, full lines in Fig. 4; even at increased statistical error, indicated by dashed lines in the Figure), and in the environment of the Renner experiment [7] (apparently suggesting a zero result).

![Graph](image)

**Fig. 4.** Plot of $\Delta(a/g)$ as a function of $\Delta(B/M)$ [19]. In EPF measurement (thick lines): 1. tallow – Cu; 2. water – Cu; 3. CuSO$_4$ solution – Cu; 4. CuSO crystals – Cu; 5. Asbestos – Cu; 6. snakewood – Pt; 7. AgSO$_4$ and FeSO$_4$ (before and after the reaction); 8. mangalium – Pt; 9. Cu – Pt. In Renner’s measurement (thin lines): a) paraffin – brass; b) NH$_4$F – Cu; c) Bi – brass; d) Pt – brass; e) glass – brass; f) Mn – Cu alloy – Cu. (The dotted lines include the increased statistical scatter, as indicated in the text.)

The authors of the present report do not intend to enter the field of controversies related to the Fifth Force, the interested readers may find references in the review papers [e.g. 12]. Our only goal is to supply information about the environments of the two classic Hungarian experiments. But when doing so, let us keep in mind what Nieto, Goldman and Hughes wrote [13]: “neither the concept of baryon number, nor the mass defect existed at that time. Without these concepts, Eötvös could have spent considerable time and effort in a fruitless attempt to find out why the scatter in his data points was larger than his error estimates. We can easily sympathize and imagine the gnawing feeling that something was wrong, or that something very important was being missed.”

**Eötvös’ Laboratory revisited**

The Faculty of Science of the University of Budapest (carrying the name of Roland von Eötvös since 1950) is located in the downtown of Budapest, East of the Danube. The river follows a geological break: Its West shore abounds in steep hills...
Fig. 5. Geological cross-section between Gellért Hill and Eötvös University in Budapest
Fig. 6. Building of the Physics Institute from the South
(CaCO₃, MgCO₃), its East shore is flat (mostly wet sand deposit of the river). The East–West asymmetry is the dominating geological feature (Fig. 5).

Eotvos designed and built the building of the Physics Institute in the 1880's, his laboratory is now the Department of Atomic Physics (Puskin utca 5). According to the Eotvos–Pekar–Fekete paper [6, page 328] the laboratory room where the Eotvos–Pekar–Fekete experiment was carried out looks South with two windows on the ground floor; opposite to it there are tall buildings [6, page 328]. (Recollections made two decades later [13] contradict this original paper [5] but are compatible with the site of Renner's experiment, therefore this hint should be probably disregarded). J. Barnóthy joined the Institute 5 years after the departure of Eotvos, and he firmly locates the site of the Eotvos–Pekar–Fekete experiment is a small annex at the SW end of the building [14] (E on Figs 6 and 7), which now houses neutron generators. At Eotvos' time there was no building to the West. To the SW there was a temporary hole that was dug for future construction, to the East there is the huge complex of the Physics Institute with a strong concrete tower, about 20 meters NE (Fig. 8). Below the experimental room there was no cellar but only soil, above it there was no floor.

In contrast to the highly asymmetric location of the Eotvos–Pekar–Fekete site, the Renner experiment was performed 25 years later, probably in the geophysical laboratory, on the North side, in the middle of the ground floor of the Institute of Physics building (about where the computer room is now). This location of the geophysical laboratory is given G. Barta [15] indicated by R in Fig. 7. Below this room there was a cellar, above it one additional floor, which means that the Renner site is located rather symmetrically (up-down, E–W) in the building. According to Talmadge et al [16], the asymmetric location of the Eotvos site may be the source of a Fifth Force, explaining the positive (composition-dependent) outcome (Fig. 8). The symmetric position of the Renner site R (compensated Fifth Force) may explain the zero (composition-independent) outcome (Fig. 8). The explanation works if the Fifth Force exists with a range of 10–50 meters. These conclusions have to be checked by modern experiments by observers who are ready to learn patience from Baron Roland von Eotvos. (G. Barta, who is presently repeating the original Eotvos Experiment, speaks about 2 days of waiting before one single reading of the equilibrium position of the torsion balance [17]). Eotvos selected the best (most linear and most sensitive) Tungsten wires which hung with weight for several years, to get rid of any distorting internal tension. Eotvos demanded thousands of hours of patient unbiased observations from his assistants. Let us conclude this report from the past with Eotvos' message [5]: "The authors bow to the fate of human limitations and leave it to future times and future workers to further elaborate those observations which they themselves believe upon mature experience to be able to still improve."

Impact of the Eotvos experiment

The recent revival of interest in the possibility of non-Newtonian gravity, which followed the reanalysis of the Eotvos experiment by Fischbach and co-work-
ers [11,12] owes much to the perception that the experiments of Eötvös, Pekár, and Fekete were carefully done and hence deserve to be taken seriously. The widespread favorable view of this series of experiments is due in part to the detailed description of their experiment contained in the published literature, and in part to other details of their experiment which we know of from personal communications [9,14,15,17], and from aspects of their methodology that we can infer from various sources. The following are two additional examples of some of the details of their experiment which were not described in their paper. The torsion balances used in the experiment were mounted on stone piers (approximately one meter on a side) which were sunk deep into the ground. The purpose of these piers was to provide a stable shock-free platform for the sensitive balances, and a number of these are still visible today at the Atomic Physics Institute. Another interesting example deals with their

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comparison of the reactants before and after the chemical reaction

$$\text{Ag}_2\text{SO}_4 + 2\text{FeSO}_4 \rightarrow 2\text{Ag} + \text{Fe}_2(\text{SO}_4)_3.$$  

Since the Ag produced by this reaction precipitates out of the liquid, the center-of-mass of the initial reactants would not coincide with that of the final products. If the difference of the centers of mass were not corrected for, then it would couple to local gravity gradients and produce a large (but spurious) signal which could simulate a violation of Equivalence Principle [12]. In fact Eötvös and co-workers found that the accelerations of the reactants before and after the chemical reaction were the same, which is what we expect in all theories. This indicates that the authors were evidently careful to correct for this effect, although the details of their methodology are not provided.

It has now been approximately five years since the classic work of Eötvös, Pekár and Fekete stimulated interest in the possibility of a fifth force. During this period numerous experiments have been carried out, and many are still under way. To date these experiments have not confirmed the original suggestion of a fifth

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force, as inferred from the Eötvös data by Fischbach and co-workers [12]. However, neither has any g. ap pinpointed an error in the Eötvös experiment which could be the source of their suggestive data. Since all of the recent experiments differ from the original Eötvös experiment in various ways, the possibility remains that there is some theoretical model in which a subtle aspect of the original experiment which we have heretofore overlooked could explain why those authors saw an effect while the more recent ones do not. The significance of the Eötvös experiment is that it will continue to be a stimulus for new ideas, such as the recent suggestion [18] that spin may have played a role in the original work. However the search for new gravity-like forces turns out, it is clear that the Eötvös experiment has played a fundamental role in shaping our understanding of gravity and other possible forces in Nature.

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Nachrichten
von der
Königlichen Gesellschaft der Wissenschaften
gt Göttingen.

Geschäftliche Mitteilungen.
1909. Heft 1.
Benekeische Preisstatung.

Auf die im Jahre 1906 gestellte Preisaufgabe:
"Von Eötvös wurde eine sehr empfindliche Methode angegeben, Trägheit und Gravität der Materie zu vergleichen. Mit Rücksicht hierauf und im Hinblick auf die neueren Entwicklungen der Elektrodynamik sowie auf die Entdeckung der radioaktiven Substanzen ist das Newtonsche Gesetz der Proportionalität von Trägheit und Gravität möglichst weitgehend zu prüfen."

ist eine Bewerbungsschrift mit dem Titel:
"Beiträge zum Gesetz der Proportionalität von Trägheit und Gravität",
und dem Motto:
"Ars longa, vita brevis"
ingelaufen.

Um zu einem Urteil der Bewerbungsschrift zu gelangen, scheint es wichtig, die Gesichtspunkte zu beachten, welche die Gesellschaft bei der Stellung der Preisfrage leiteten.

Das von Newton entdeckte und nach ihm benannte Gesetz der Allgemeinen Gravitation, welches die Erscheinungen der materiellen Welt in ihrer Gesamtheit umfaßt, spricht einige Sätze aus, die überaus merkwürdig sind, die man aber trotzdem nicht hervorzuheben pflegt: 1) Die Anziehung wird gar nicht beeinflußt durch die physikalische Beschaffenheit der Materie, sondern wird allein durch die "Trägheit" bestimmt. Mit dieser Trägheit sind die Fernwirkungen proportional, so daß man kurz den Satz formulieren kann: Das Verhältnis von Gravität und Trägheit ist für alle materiellen Teile unveränderlich und für alle gleich groß. Da allen materiellen Teilen eine unveränderliche Trägheit anzuhaften scheint, so wäre zu folgern, daß auch die Gravität eine unveränderliche Eigenschaft der Materie ist. 2) Die Fernwirkung irgend zweier materieller Teile wird durch die Anwesen-
Benenesche Preisstiftung

heit der übrigen materiellen Teile nicht beeinflußt. — Ein Teilchen im Innern der Erde und eines inmitten des Sonnenballes, ziehen hiernach einander gerade so an, als ob die von dem Erdkörper und dem Sonnenkörper gebotenen materiellen Mäntel gar nichts vorhanden wären. 3) Die Fernwirkung hängt allein von der jeweiligen gegenseitigen Lage der materiellen Körper, nicht von ihrem Bewegungszustand ab. Es scheint hiernach die Gravitation sich in unendlicher Geschwindigkeit auszubreiten.

Man hat sich an den Gedanken der unbeschränkten Gültigkeit des Newtonschen Gesetzes so sehr gewöhnt, daß das Gefühl für die Merkwürdigkeit der hervorgehobenen Sätze fast verloren gegangen ist.


Diesen Erwägungen weiter nachgehend, welche zu der Frage führen, wie die Materie in das physikalische Weltbild einzuordnen ist, schien es der Fakultät besonders wichtig, dem Satz von der Proportionalität der Trägheit und der Gravität erneut die Aufmerksamkeit zu schenken, und insbesondere erschien die denkbar
schärfste experimentelle Prüfung dringend erwünscht. Dies war der Anlaß zur Stellung der Preisaufgabe für 1909. —

Die Bewerbungsschrift mit dem Motto „Ars longa, vita brevis“ geht auf die theoretischen Erwägungen, an die erinnert wurde, gar nicht ein, berührt sie nicht einmal. Damit ist klar, daß einem wesentlichen Teil der Wünsche, welche die Fakultät bei Stellung der Preisaufgabe leiteten, nicht Rechnung getragen wird. Dafür wird die ganze Kraft auf die Ausführung der experimentellen Untersuchung verwendet, und es wird auch gezeigt, wie man unter Verwertung bekannter Erfahrungen über die Erscheinung bei Ebbe und Flut weitere wertvolle Folgerungen für das hier zur Behandlung stehende Problem ziehen kann. —

Es werden ohne eine Änderung die durch Etvős konstruierten Apparate benutzt. Dem Studium der Fehlerquellen wird eine große Aufmerksamkeit geschenkt, sodaß die Beobachtungen einen hohen Grad der Zuverlässigkeit erhalten. Die gewonnenen Resultate sind so wertvoll, daß es die Fakultät mit Genugtuung begrüßen darf, durch Stellung der Preisaufgabe, zu den Beobachtungen Anlaß gegeben zu haben. —


Das Endresultat der ganzen Arbeit wird so ausgesprochen:

Wir haben eine Reihe von Beobachtungen angestellt, die an Genauigkeit alle vorangehenden übertrafen, doch konnten wir in keinem Falle eine bemerkbare Abweichung von dem Gesetz der Proportionalität von Trägheit und Gravitation entdecken. 

Die Verfasser bemerken zu Anfang ihres Berichtes: Mit Rücksicht auf die Kürze der Zeit, die uns für die genauere Durchsicht unserer Arbeit zur Verfügung stand, bitten wir für eventuell vorkommende Schreibfehler und das Wesen der Resultate nicht beeinträchtigende Rechenfehler um Nachsicht. So mag denn nicht viel Gewicht darauf gelegt werden, daß in der Tat bei der Beurteilung der Flutwirkung ihre Darlegungen mehrfach Verbesserungen und Vervollständigungen bedürfen.

Es ist gewiß, daß die Verfasser der Preisarbeit in sehr wesentlichen Punkten den Erwartungen der Fakultät nicht entsprochen haben, und es muß auch bemerkt werden, daß in Einzelheiten die Ausführungen nicht anerkannt werden können. Trotzdem aber bringt die Arbeit höchsten wertvollen Resultate, indem sie als Grundlage für alle theoretischen Spekulationen den außerordentlich weitgehenden Gültigkeitsbereich der Newton'schen Gesetze zeigt. Die Fakultät steht darum nicht an, der Arbeit den vollen Preis zu erteilen.


Die philosophische Fakultät.
Der Dekan:
C. Runge.
EXPERIMENTELLE UNTERSUCHUNGEN
ÜBER DIE PROPORTIONALITÄT VON GRAVITÄT
UND TRÄGHEIT.

Von J. RENNER, (Budapest.)

Als Fortsetzung der von Baron R. Eötvös, D. Pekáry, und
E. Fekete noch im Jahre 1908 unternommenen, im nächsten
Jahre mit dem ersten Preis aus der Benecke’schen Stiftung von
der Universität Göttingen preisgekrönten, aber erst im Jahre
1922 veröffentlichten Untersuchungen wurden vom Verfasser
weitere Beobachtungen mit der Absicht ausgeführt, um die Ge-
nauigkeit der Drehwagemethode womöglich zu erhöhen. Dies
gelang einerseits durch die geeignete Wahl der Drehwage und
der vorsichtigen Torsiondraht, anderseits durch die vollkom-
mene Beseitigung der störenden Einflüsse der Temperaturschwan-
kungen.

Zur Beobachtung wurden solche Stoffe gewählt, welche bei
den obengenannten Untersuchungen nicht vorkamen. Die be-
kannte Eötvös’sche Methode, welche Beobachtungen in der
Meridianstellung und in der ostwestlichen Azimutstellung be-
nützt, wurde dadurch erweitert, dass Beobachtungen in noch
vier anderen symmetrischen Azimutstellungen zur Berechnung
des Einflusses der Massenverschiedenheiten herangesogen wur-
den. Dieses letztere Verfahren war auch dazu geeignet, um die
störende Wirkung des erdmagnetischen Feldes genau in Rech-
nung zu ziehen; diese Korrektion erwies sich besonders bei dem
diamagnetischen Stoffe Wismut als vorteilhaft.

Die folgende Tabelle enthält die Ergebnisse der Beobachtun-
gen; \( x \) bedeutet darin den spezifischen Attraktionskoeffizienten
der Gravitationskonstante nach der Formel

\[
f_i = f(1 + x).
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{Untersuchte Stoffe} & \text{\( x - x' \)} \\
\hline
\text{Pt — Messing} & +0.05 \cdot 10^{-5} & \pm 0.05 \cdot 10^{-6} \\
\text{Glastränen — Messing} & -0.01 \cdot 10^{-5} & \pm 0.07 \cdot 10^{-6} \\
\text{Zwei teilige Glastränen — Messing} & +0.2 \cdot 10^{-6} & \pm 0.05 \cdot 10^{-6} \\
\text{Paraffin — Messing} & +0.41 \cdot 10^{-6} & \pm 0.44 \cdot 10^{-6} \\
\text{Ammoniumfluorid — Cu} & +0.13 \cdot 10^{-6} & \pm 0.57 \cdot 10^{-6} \\
\text{Manganfluorid — Cu} & +0.06 \cdot 10^{-6} & \pm 0.90 \cdot 10^{-6} \\
\text{Wismut — Messing} & -0.14 \cdot 10^{-6} & \pm 0.74 \cdot 10^{-6} \\
\hline
\end{array}
\]

Aus dieser Tabelle ist zu sehen, dass der Unterschied
der spezifischen Attraktionskoeffizienten in jedem Falle kleiner
ist, als der mittlere Fehler. Die mittleren Fehler sind alle von
derselben Größenordnung und ihr Mittelwert beträgt \( \pm 0.52 \cdot 10^{-6} \).

Die Gravitationskonstante ist für die untersuchten Stoffe all-
gemein bis zur Genauigkeit von \( 1 \cdot 2 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \), in einem
Falle sogar von \( 1 \cdot 5 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \) von der Beschaffenheit der
Körper unabhängig.

Einige untersuchten Stoffe, wie Paraffin und Ammonium-
fluorid geben Aufschluss darüber, dass der bei Vergleich von
Heliumkernen und Protonen auftretende sogenannte Massen-
defekt auf die Anziehungskräfte keinen Einfluss hat.

(Aus der Sitzung der III. Klasse der Ungarischen Akademie der Wissen-
schaften vom 18. März 1935.)
Dear Professor R. H. Dicks!

I read with great interest your valuable treatise about the Botvos experiment published in the "Scientific American" in the number of December 1961. I know your previous studies in this matter too. I am fully convinced of the great importance of your experiments carried out with up to date methods. The accuracy achieved by your experiment is much greater than the accuracy of the previous works done by R. Eotvos and his collaborators. The fundamental question of the independence of gravitational acceleration from the quality of material can be considered now as proved. I have to congratulate for your experimental results of high accuracy.

I was student and later on collaborator of Eotvos and I repeated his famous experiments in the years 1930-1935. I beg you to allow me some suggestions relating your remarks about the original experiments of Eotvos. I have to mention, that your interesting treatise was translated in Hungarian and published in the periodical "Fizikai Szemle." My suggestions were published in the same periodical. I enclose a copy of it in Hungarian.

As the Hungarian experiments were made by visual readings, the mass of the observer could have exert any influence to the position of equilibrium. But the observing time was always rather short and the observer did not stay for long time near to the instrument, as you already assumed it.

Concerning the influence of sudden changes of temperature, Eotvos took care to eliminate the unfavourable effects. The experiments were carried out in a dark room, where temperature was practically steady. In the room there was mounted a linen tent, the walls of it were filled with isolating material and the instrument was brought inside this tent. Besides the instrument had a triple shell of metal around the chamber containing the torsion balance, as Professor had already mentioned in the publication. Accordingly there was no reason of taking place any convection currents in the inside of the balance. Specifically prepared torsion wires were used, which had no any drift caused by elastic properties or by change of temperature. The positions of equilibrium appeared even in long observation sets very constant.

You have correctly suggested the disturbing effect of even quite small magnetic impurities in the moving parts of the balance. Eotvos and

*The original of this letter can be found in the Library of the Hungarian Academy of Sciences

his collaborators eliminated the assumable effect by compensating the geomagnetic field with permanent magnets and electromagnetic coils. The compensation was always controlled. The compensating magnets were so mounted, that they should not produce any translatory effect. Besides the materials used in the balance were controlled concerning their magnetic properties.

There is another question, that Botvos and his collaborators used a horizontal variometer, in which one of the masses was suspended lower, so that the horizontal gradient of the acceleration had influence too. This caused no disadvantage, as the method of Botvos counted on the effect of the gradient. When different materials were hanged on the balance, the centre of gravity was always in the same height and the balance took place in the same gravitational field. The optical diffraction in the telescope caused no erroneous readings. The 20-th part of a scale division could be precisely read by a skilled observer.

Concerning my own experiments carried out in the years 1930–1935, I have to mention, that I did not use the old Eotvos balance, but an improved new one, the thermical and magnetic effects were perfectly eliminated and very reliable torsion wires were used. The positions of equilibrium were kept in long observation sets constant. In this way it succeeded to extend the accuracy.

I would be very grateful, if you would be kind to publish a remark of my suggestions above concerning the original experiments of Botvos and his collaborators, if possible in the same periodical, in which your interesting study appeared.

With kindest regards

Yours faithfully

Budapest, 26. VII. 1963

/ Dr. J. Renner /
Budapest VII. Damjanich u 28/b

Six years of the fifth force

Ephraim Fischbach & Carrick Talmadge

The enunciation of the ‘fifth force’ hypothesis in 1986 spawned a generation of experiments searching for deviations from newtonian gravity. Although no compelling evidence for any new weak forces has emerged in the past six years, the searches for anomalous gravitational effects have produced a large number of important experimental and theoretical results.

The suggestion roughly six years ago of a possible gravity-like ‘fifth’ force of nature called attention to the fact that gravity, itself the ‘oldest’ of the known forces, was in some ways also the least well understood. As we discuss below, a fifth force coexisting with conventional gravity would lead to a net interaction between macroscopic objects which showed small deviations from the behaviour expected from the classical newtonian inverse-square law of gravity (see equations (2) and (3) below). Evidence for such non-newtonian behaviour in an appropriate system could thus be a sign of a new fundamental force, and this idea has provided part of the impetus for the current efforts to re-examine the experimental support for newtonian gravity over various distance scales. Although no compelling experimental evidence for any anomalous results has yet surfaced, there has nonetheless been a dividend from the intense effort of the past few years. This is the substantial improvement in the precision with which newtonian gravity can be checked to be experimentally supported. Moreover, the search for non-newtonian effects continues, spurred on both by remarkable advances in experimental techniques and by an improved understanding of the theoretical basis for such effects. Specifically, we now recognize that many models predict the existence of new intermediate-range forces, and relate the properties of these forces to the behaviour of physics at the Planck (energy) scale.

It is important to bear in mind that although we will focus here primarily on what has been learned in this field since the fifth force was proposed in 1986, this proposal was itself a natural outgrowth of much earlier experimental and theoretical studies of non-newtonian gravity. In the 1970s interest was stimulated by the work of Fujii and others who argued that various theoretical models led naturally to a new intermediate-range force, whose strength was comparable to (or perhaps slightly weaker than) gravity. Although Fujii’s theory was based on detailed dynamical considerations, its core is a few fundamental observations which have helped to motivate much of the subsequent work. We begin by noting that there are two major natural mass scales in physics, defined by $m_N = 1$ GeV/$c^2$ and $M_{Planck} = (hc/G)^{1/2} = 10^{19}$ GeV/$c^2$, where $m_N$ is the nucleon mass, $h$ is Planck’s constant, $c$ is the speed of light, and $G = (6.67259 \pm 0.00085) \times 10^{-11}$ N m$^2$/kg$^2$ is the newtonian gravitational constant. The ratio of these scales, $f = m_N/M_{Planck} = 10^{-19}$, introduces a new small dimensionless constant into physics.

This constant may have some dynamical significance if it determines the coupling strength of a new field to matter, just as the electric charge $e = e^2/hc = 1/137$ determines the strength of the coupling of photons to matter. If this were the case, then the analogue of $\alpha_{em}$ for this field would be $f^2/hc = 10^{-38}$, which corresponds to a force of gravitational strength $(f^2/M_{Planck}) = 1$. The second observation is that in various theories the product $\sqrt{(f^2/hc) m_N = 10^{-10}}$ eV/$c^2$ determines the mass of the new field. As the Compton wavelength (which characterizes the distance over which a force acts) associated with this field would be $\lambda = h/m_N \approx 2,000$ m, this combination of values for the parameters $f$ and $\mu$ could describe a new intermediate-range field of gravitational strength.

To understand how such a field could be detected experimentally, we note that the potential energy $V_\phi(r)$ describing the interaction of two point masses $m_1$ and $m_2$ through this force would be given by $V_\phi(r) = -G m_1 m_2 e^{-r/\lambda}$ (1), where by convention the sign has been chosen to make $V_\phi(r)$ correspond to an attractive force for $\alpha > 0$. Here $r = |r_1 - r_2|$ is the distance between the masses, and $\alpha$ is a dimensionless constant proportional to $f^2$ which determines the strength of the new force (relative to gravity). Because $\alpha$ can be either positive or negative in various theoretical models (corresponding to a force which is attractive or repulsive), the sign of $\alpha$ carries important physical information. For example, a repulsive force typically arises from the exchange of a vector (spin-1) meson between nucleons or electrons. Theories of such forces lead us to expect that the magnitudes of the effects they produce necessarily depend on the chemical compositions of the test masses. By contrast, an attractive force between like objects arises from the exchange of scalar (spin-0) and/or tensor (spin-2) fields, and these can lead to deviations from the $1/r^2$ law that are independent of the compositions of the test masses. It follows that an experimental determination of the sign of $\alpha$ can be used to discriminate among different theories of the fifth force. Returning to equation (1), we note that the sum of $V_\phi(r)$ and the usual newtonian potential $V_N(r)$ can be written as $V(r) = V_N(r) + V_\phi(r) = -G m_1 m_2 [(1 + \alpha e^{-r/\lambda})]/r$ (2).

In the form of equation (2), the presence of the new interaction described by $V_\phi(r)$ manifests itself as an apparent deviation from the usual $1/r$ newtonian potential. In practice nearly all experiments measure a force (or acceleration) rather than a potential, and from equation (2) we find for this force $F(r) = -\nabla V(r) = -G(r m_1 m_2 \hat{r})/(r^2)$ (3).

In equation (3) we have written the newtonian constant as $G_0$ to indicate that this is the constant that determines the strength of the interaction in the limit $r \to \infty$. This serves to distinguish $G_0$ from the constant $G_0 = G_0(1 + \alpha)$, which characterizes the interaction strength when $r/\lambda \ll 1$. (The latter regime is often referred to as the ‘laboratory scale’ on the assumption that $\lambda$ is larger than the typical separation of test masses in laboratory experiments.) It follows from equation (3) that any deviation of $G(r)$ from $G_0$ leads to a breakdown of the usual $1/r^2$ force law, and can be interpreted as evidence for a new force.

Elementary phenomenology

The work of Fujii and others, which led to the phenomenological framework given in equations (1)-(3), motivated further experimental and theoretical work, as discussed in ref. 7. We see from...
equation (3) that every experimental result implies some constraint on the parameters $\alpha$ and $\lambda$ which define the putative non-newtonian interaction. For technical reasons, the limits on $\alpha = \alpha(\lambda)$ implied by a given experiment are most stringent for values of $\lambda$ that are comparable to the characteristic separation $r$ of the test masses in that experiment. It follows that no single experiment can provide useful limits on $\alpha(\lambda)$ for all values of $\lambda$. Rather, a collection of experiments is needed, each optimized to measure $\alpha(\lambda)$ near a selected value of $\lambda$. By 1981 a number of such results were available, and these were collected together in an important paper by Gibbons and Whiting\(^6\) (hereafter referred to as GW).

The constraints on $\alpha$ and $\lambda$ implied by the data available to GW are shown by the dark shaded area in Fig. 1. In this and similar figures (see, for example, refs 9, 10), the shaded region in the $\alpha$-$\lambda$ plane is excluded by the data at the two-standard-deviation (2$\sigma$) level. The GW paper drew attention to the region ($10 \text{ m} < \lambda < 10^{10} \text{ m}$) where the limits on $\alpha$ (and hence on the validity of newtonian gravity) were very poor, as can be seen from Fig. 1. This gap in our knowledge simply reflects the difficulty in designing experiments in which the separation of the test masses is of order $10^{-3} \text{ m}$. Typically information on gravity for such mass separations comes from geophysical experiments, and hence this range of distances is often referred to as the 'geophysical window'.

Stacey and collaborators\(^{1-4,12}\) undertook to explore the 'geophysical window' by reviving the Airy method of determining the newtonian gravitational constant $G$. A schematic outline of the Airy method is shown in Fig. 2 for an idealized model of a spherical non-rotating Earth.\(^{13}\) Suppose we compare the acceleration $g(0)$ of a test mass at the surface of the Earth with its acceleration $g(z)$ at a depth $z$ below the surface. (The test mass in each case is simply the proof mass in a standard gravimeter, usually of the LaCoste-Romberg type.) It is straightforward to see that there are two different effects that contribute to $\Delta g(z) = g(z) - g(0)$, and that these have opposite signs. The first is the free-air gradient, which represents the increase in $g(z)$ arising from the circumstance that the test mass at $z$ is only a distance $(R - z)$ from the centre of the inner mass, rather than at $R$ as would be the case at the surface (see Fig. 2). The second effect, known as the double-Bouguer term, accounts for the fact that at a depth $z$, $g(z)$ will decrease because the test object is now attracted to the centre of the Earth by a smaller total mass (2$\sigma$ level, so that Newton's $1/r^2$ law of gravity holds, then the shell of matter of thickness $z$ does not contribute to $g(z)$.) From Fig. 2 and the preceding discussion we see that in this simplified model $\Delta g(z)$ is given by

$$\Delta g(z) = g(z) - g(0) = \frac{G(M - AM)}{(R - z)^2} - \frac{GM}{R^2}$$

(4)

where $\rho$ is the density of the material in the shell, and $g(0) = GM/R^2$. It follows from equation (4) that $G$ can be determined over geophysical scales by a series of local measurements to determine $g(z)$, $g(0)$ and $\rho$, and this constitutes the Airy method. In practice equation (4) must be modified to include the effects of the Earth's rotation and ellipticity\(^{11-13}\). The original analysis by Stacey and coworkers\(^{13}\) in the early 1980s in fact found an anomalously high value of $G$: their best value, obtained from a mine at Mount Hilton in Australia, was $G = (6.720 \pm 0.024) \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ which was larger than the laboratory value $G = (6.67259 \pm 0.00085) \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ by roughly twice their maximum admitted error.

We note for later purposes that as $G(r)$ always appears in combination with a mass such as $m_i$, $G(r)$ cannot be determined separately unless $m_i$ is independently known. In the Airy method the appropriate combination is $G(r)\Delta M$, and as $\Delta M$ can be determined from measurements of the local density $\rho$, $G(r)$ can be inferred for values of $r$ comparable to the depth of the measurement. But in some situations, such as tower experiments and tests involving planetary motion (see below), the appropriate masses cannot be determined independently. In these other cases $G$ cannot be obtained in absolute terms, and only the variation of $G(r)$ with $r$ can be measured.

During the same period of the early 1980s, Aronson et al.\(^{10}\) analysed high-energy Fermilab data on the $K^-K^0$ system to look for anomalous effects that might be associated with the existence of a new force. They used neutral kaons because the small mass difference $\Delta m = m_1 - m_2 = 4 \times 10^{-8} \text{ eV}/c^2$ of the weak interaction eigenstates $K_L$ and $K_S$ makes this system extremely sensitive to small external influences\(^{15,16}\). The presence of a new weak force would show up as an apparent dependence of $\Delta m$ on the velocity $v$ of the kaons (or their energy) in the laboratory, because the strength of an external field as seen in the rest frame of the kaons depends on $v$ through the Lorentz transformations. Aronson et al.\(^{10}\) also noted that similar velocity-dependent effects could show up in the other fundamental parameters of the $K^-K^0$ system, such as the lifetimes $\tau_L$ and $\tau_S$ of $K_L$ and $K_S$, and $CP$-violating parameter $\eta$ = amplitude ($K_L - \pi^+\pi^-)/amplitude (K_S - \pi^+\pi^-)$. Motivated by these theoretical

![FIG. 1 Constraints on the coupling constant α as a function of the range λ from composition-independent experiments. The dark shaded area indicates the status as of 1981, and the lighter region the current limits. For references to the earlier experiments which contribute to the curves, see refs 9, 10.]
considerations, they reanalysed data from a series of Fermilab experiments which determined the \( K^-\bar{K}^0 \) parameters for kaons in the energy range 30–130 GeV. They found evidence at a marginally interesting (\( \pm 3 \sigma \)) level for an energy variation of the kaon parameters, particularly for the phase \( \phi_\omega \) of \( \eta_\omega \), and this provided at that time another hint for a possible new force. We will return later to discuss the present status of kaon experiments.

The ‘fifth force’ hypothesis in 1986 arose from the recognition that the anomalies reported by both Stacey et al.\(^{1,2}\) and Aronson et al.\(^{13,14}\) could be explained in terms of the same new force, but only if it had properties that were somewhat different from those of \( V_\lambda (r) \) in equation (2). Additionally, the requisite force would be capable of producing anomalies in the classic experiment of Eötvös, Pékar and Fekete (EPF)\(^{17,18}\), which compared the accelerations of different pairs of materials to the Earth. This motivated a reanalysis\(^ {19} \) of the Eötvös experiment, from which two surprising observations emerged. First, the results of EPF indicated that for some pairs of materials, the acceleration differences for the two samples were relatively large unequal. The proposed fifth force departed from the earlier work of Fujiy by introducing a repulsive interaction proportional to the hypercharge of a test object \( Y = B + S \) (\( B \) is baryon number, \( S \) is strangeness), rather than to its mass as in equation (2). For ordinary bulk matter \( B = N + Z \) and \( S = 0 \), where \( N \) and \( Z \) respectively denote the number of neutrons and protons in a sample, and hence \( Y \) and \( B \) can be used interchangeably. But for certain elementary particles such as kaons (specifically \( K^0 \) and its antiparticle \( \bar{K}^0 \)), \( B = 0 \) and \( S \neq 0 \). Hence by selecting \( Y \) as the charge we allow interactions not only between samples of ordinary bulk matter, but also between bulk matter and kaons. Such an interaction could account for the anomalies reported by Aronson et al.\(^{13,14}\), and for this reason hypercharge emerged as the natural choice for the charge in the original fifth force theory. As our primary focus here is on macroscopic experiments, we can set \( Y = B \) and write \( V_\lambda (r) \) in the form

\[
V_\lambda (r) = \frac{f^2 B^2 g(z)}{r} e^{-r/R}
\]

(5)

where \( f \) is a new fundamental constant. It is straightforward to show that when \( V_\lambda (r) \) in equation (5) is combined with the newtonian potential \( V_\Sigma (r) \), the resulting expression for \( V(r) \) has the form given in equation (2), but with \( a \) replaced by \( a_\mu \)

\[
a_{\mu} = -\xi \left( \frac{B}{\mu} \right) \left( \frac{B}{\mu} \right)
\]

(6)

where \( \xi = f^2 / G_m \mu_\Sigma , \mu = m_\mu/m_\Sigma \) and \( m_\mu = m(\mu^+) \). As \( (B/\mu) \) and \( (B/\mu) \) vary from one material to another, \( a_\mu \) is a function of the compositions of the interacting materials. It then follows from equations (3) and (6) that the accelerations of two samples \( j \) and \( j' \) towards a common source \( i \) would depend on the constants \( a_{\mu} \) and \( a_{\mu'} \), respectively, and these are generally unequal. If \( i \) denotes the Earth then the accelerations of \( j \) and \( j' \) towards the Earth would depend on their compositions, and this could be tested by simultaneously dropping two dissimilar objects as Galileo is supposed to have done\(^ {21-23} \). Such an experiment, although relatively simple in principle, is extremely difficult in practice and has been carried out to high precision only recently\(^ {24,25} \), as we discuss below. Eötvös recognized, however, that an equivalent experiment could be done by suspending the two samples \( j \) and \( j' \) from a common bar, and searching for a torque on the bar arising from the differential force exerted on the samples by the Earth. Subsequent experiments\(^ {26-28} \) have generally been used the Sun as a source, and these set limits on possible new weak forces for which \( \lambda \) is of order \( \lambda A \).

It follows that in the model considered here, the acceleration difference \( \Delta a_{\mu} \) of \( j \) and \( j' \) towards the Earth is given by

\[
\Delta a_{\mu} = \xi \left( \frac{B}{\mu} \right) \left[ \frac{B}{\mu} - \frac{B}{\mu} \right] \mathcal{F} \]

(7)

where \( \mathcal{F} \) is the field strength (in units of acceleration) of the source, which in this case would be the Earth (denoted by \( \Theta \)). As in equation (12) below, \( B \) can be replaced by a mean general charge \( Q \), so that \( (B/\mu) \rightarrow (Q/\mu) \) and so on. It then follows that the experimentally determined sign of \( \Delta a_{\mu} \) (that is, whether \( j \) falls faster than \( j' \) or vice versa) depends on the signs of the factors \( (Q/\mu) \) and \( (Q/\mu) \), and on the direction of \( \mathcal{F} \). It should be emphasized that for a force of finite range the effective values of \( (Q/\mu) \) and \( \mathcal{F} \) are actually determined by the local chemical composition and the local matter distribution, respectively, and can vary from one location to another. As this information for the site of the original EPF experiment is not accurately known, the sign and magnitude of \( \lambda \) implied by this experiment cannot now be established unambiguously.

From equation (3) we see that with \( \alpha \rightarrow a_\mu \), an intermediate-range weak force can be detected through a modification of the inverse-square law \( (G(r) \neq constant) \) and/or through a composition-dependence of the net acceleration \( (a_\mu \neq constant) \). (The latter effect is often referred to as a violation of the weak equivalence principle or of the universality of free-fall.) It is evident from equation (3) that both effects are generally present in a given experiment. In practice, however, experiments are usually designed to isolate one or the other of these signals for a new force, and we will therefore focus on each of these classes of experiments separately.

**Composition-independent experiments**

We first consider the recent progress made in tests of the inverse-square law. Returning to Fig. 1, we see that the limits on the strength \( \xi = -\alpha \) of a possible non-newtonian component have improved substantially over several distance scales. Most notably the geophysical window at \( \lambda = 100 \text{ m} \) has been partially
closed by a series of experiments using geophysical sources, as we now discuss. In 1988, Eckhardt and collaborators\(^\text{26}\) introduced a new technique into the field, by measuring the gravitational acceleration \(g(z)\) as a function of height \(z\) using the 600-m tower of the television station WTVD in Garner, North Carolina. If newtonian gravity is correct then Laplace's equation can be used to predict \(g(z)\), given a knowledge of \(g\) on the surface of the Earth near the tower. Although this technique is in principle straightforward, it had never been attempted before 1988, apparently because of the widespread (and erroneous) belief that \(g(z)\) could not be measured with sufficient precision on towers. Although measuring \(g(z)\) on a tower is indeed more difficult than in the relatively protected environment of a mine-shaft, the tower method has an important compensating advantage. If there are pockets of unusually dense material below the surface of the Earth near the experiment, these will be located on average farther away from the gravimeter being used to determine \(g(z)\) in a tower experiment than for analogous measurements in a mine or borehole. The distortion by such local mass concentrations of the average gravitational field near the tower will have less effect than in a similar mine experiment. This 'smoothing' of the average gravitational field facilitates a comparison between the measured and predicted values of \(g(z)\). Eckhardt \textit{et al.} in fact demonstrated not only the feasibility of tower measurements, but also the ability of this technique to set useful limits on possible deviations from newtonian gravity.

In their original analysis\(^\text{26}\), Eckhardt \textit{et al.} found a discrepancy of \((-500 \pm 35) \, \mu\text{Gal} (1\,\mu\text{Gal} = 10^{-11} \, \text{m} \, \text{s}^{-2})\) between the measured and predicted values of \(g(z)\) at \(z = 562.24 \, \text{m}\). The negative sign corresponds to the effect expected from a new attractive non-newtonian force, which Eckhardt \textit{et al.} termed the 'sixth' force. But it was subsequently pointed out by Bartlett and Tew\(^\text{30}\) that the surface gravity database used in ref. 29, which was compiled by the Defense Mapping Agency, could have been biased by oversampling of the higher elevation terrain compared with that at lower elevation. Low-lying terrain is often relatively inaccessible, as would be the case if a stream were running through the region. As the gravimeters used in such experiments are expensive, it is not surprising that fewer measurements would be made, for instance, at the edge of a body of water, than at a nearby road running above the water. A difference of \(1 \, \text{m}\) between the average elevation of the actual topography and that of the sampled points could lead to a discrepancy of \(-309 \, \mu\text{Gal}\), which is comparable to the magnitude of the effect observed in ref. 29. Eckhardt \textit{et al.} have recently re-examined the effects of the terrain in their experiment, and now find a result consistent with newtonian gravity\(^\text{31}\). Similar null results have also been obtained recently in two other tower experiments\(^\text{32,33}\). Speake \textit{et al.}\(^\text{34}\) devoted considerable effort to understanding how tower motion affects the performance of the LaCoste-Romberg gravimeters used in these experiments. Their careful analysis supported the original claim by Eckhardt \textit{et al.} that \(g(z)\) could be measured on towers with sufficient precision to allow a meaningful comparison with the predictions of newtonian gravity. The limits implied by these experiments are partially responsible for filling in the 'geophysical window' in Fig. 1.

Another class of geophysical experiments which tests newtonian gravity over the same distance scales comprises the pumped lake experiments of Müller \textit{et al.}\(^\text{35,36}\) and Moore \textit{et al.}\(^\text{37}\). In these, the change in the local value of \(g\) is measured as a function of the height to which the reservoir is filled, and the experiments can thus be viewed as 'weighing' a slab of water whose lateral extent is hundreds of metres. The lake experiments are subject to a number of systematic uncertainties, and thus far have not achieved the sensitivity of the tower experiments.

Similarly, Zumberge \textit{et al.}\(^\text{38}\) very recently used the ocean as an attracting mass to determine newtonian constants \(G\) for matter interacting over a distance scale of \(-5,000 \, \text{m}\). This type of Airy experiment, although technically complex, offers a number of advantages over similar measurements in mines, lakes and boreholes. First, ocean experiments allow \(G\) to be studied over distance scales that are larger than those readily accessible in land-based experiments. Second, the density of the 5,000-m shell of matter (see Fig. 2) is not only much more uniform than in a mine, but varies in a known and reasonably smooth way with depth. (Mines, after all, are situated where they are precisely because of the inhomogeneities they contain in the form of relatively dense ores.) By combining various sea-surface measurements, Zumberge \textit{et al.} determined \(G\) from a generalized version of equation (4). Their result, \(G = (6.677 \pm 0.013) \times 10^{-11} \, \text{N m}^2 \, \text{kg}^{-2}\), agrees with the laboratory value.

This experiment helps fill in a large part of the geophysical window, and sets useful constraints down to values of \(\lambda = 0.1 \, \text{m}\). As noted previously, experiments typically give their most stringent limits for values of \(\lambda\) comparable to the characteristic separation of the test masses in that experiment. It might then seem surprising that an experiment carried out on a distance scale of \(-5,000 \, \text{m}\) can give a useful limit down to \(-0.1 \, \text{m}\). But the constraint in Fig. 1 is obtained by comparing the Zumberge value for \(G\) to the Cavendish value of \(G_0\), which is itself determined over a scale of a few centimetres. This discussion applies (with appropriate changes) to the mine/borehole and lake experiments.

The other regimes in which the limits on \(g\) have improved are at small \(\lambda\) (laboratory scales) and at larger \(\lambda\) (satellite and astronomical scales). At present the best laboratory limits come from the null experiments of Hoskins \textit{et al.}\(^\text{37}\) and Chen \textit{et al.}\(^\text{38}\). These experiments arose in part from work by Long\(^\text{39}\), who in 1976 claimed to see deviations from the inverse-square law. To illustrate the novel technology used in some of these experiments, we will briefly describe the null experiment of Hoskins \textit{et al.}, which tests Newton's law in the 2-5-cm range. These authors note that if the \(1/r^2\) law is exact then a test mass located inside an infinitely long hollow cylinder would experience no gravitational force, just as if it were inside a spherical shell. Although there are corrections for end effects in a finite cylinder,
these are small enough to allow detection of the force that would arise from a deviation from the 1/r^2 law. By monitoring the torque on a test mass inside the cylinder as the distance between the mass and the cylinder is varied, limits can be set on possible non-newtonian forces. Another novel technology is the laplacian detector of Paik,\textsuperscript{40,41}, which directly tests the implication of the 1/r^2 law that the gravitational potential \(\Phi(r)\) is a solution of the equation \(\nabla^2 \Phi(r) = 0\). Although the existing limits from this experiment are less stringent than those of refs 37 and 38, this technology offers the prospect of considerable improvements in sensitivity. Moreover, the laplacian detector can be used to study non-newtonian gravity in space, and efforts along this line are currently under way.

For values of \(\lambda\) in the range \(10^{-10} - 10^{-3}\) m there has been a considerable improvement in \(\xi\) which resulted from a comparison by Rapp\textsuperscript{42} of the values \(GM_0\) inferred from Earth-surface gravity measurements, and from a study of the orbits of the Lageos satellite and of the Moon about the Earth. The improved limits near \(\lambda \approx 10^{-11}\) m come from the analysis in ref. 9 of planetary data on \(G(r)\), as well as data on anomalous planetary precessions. To understand how these data can be used to set limits on \(\alpha\) and \(\lambda\), we recall from equation (3) that in the presence of a new force the newtonian constant \(G_0\) is replaced by the function \(G(r)\). It follows that under these circumstances Kepler's third law assumes the form

\[
a_p^2 = G(a_p)M_p(T_p/2\pi)^2
\]

where \(T_p\) is the period, and \(a_p\) is the physically measured semi-major axis, of planet \(p\). Given a set of values of \(a_p\) and \(T_p\) for the various planets, \(G(a_p)M_p\) can be determined and its constancy tested. The curves labelled 'Planetary' in Fig. 1 derive from an even more sensitive test, namely the anomalous precession of the perihelion of the orbit induced by a non-newtonian coupling. For the Yukawa interaction in equation (2) the anomalous precession \(\delta\theta_p\) is given by

\[
\delta\theta_p = \pi a (a/\lambda)^2 e^{-a/\lambda}
\]

where \(a\) is the mean value of the semi-major axis of the orbit. Because the precession of the perihelion is also predicted by general relativity (GR), one can extract a limit on \(\delta\theta_p\) (and thereby on \(\alpha\) and \(\lambda\)) only by assuming that GR is correct, for which ample independent evidence exists\textsuperscript{43}.

In summary, much progress has been made since 1981, but additional work needs to be done over the distance scales in the geophysical window. At present Eckhardt and coworkers are in the process of doing a new experiment using the 2,000-m WABG television tower near Indianola, Mississippi. This site was selected in part because the flatness of the surrounding terrain should help to minimize the problem of terrain bias encountered in their earlier work. Working in the other direction, the terrain problem noted by Bartlett and Tew\textsuperscript{30} apparently accounts for most of the anomalous results originally reported by Stacey et al. But results reported by Ander et al.\textsuperscript{44} from Greenland require further study to determine whether the apparent anomalous accelerations \(g(z)\) in that experiment could arise from subsurface concentrations of relatively dense material.

**Composition-dependent experiments**

We turn next to describe the recent progress in composition-dependent experiments. Before 1986, the only reliable data came from the original experiment of Eötvös, Pékér and Fekete (EPF)\textsuperscript{19,20}, which primarily compared the accelerations of various pairs of materials falling to the Earth, and from follow-up experiments\textsuperscript{26-28} which used the Sun as a source. The Eötvös apparatus (Fig. 3) was used to measure the differential torque on a torsion balance for 11 pairs of chemically different materials. As the experiment was originally intended to test the principle of universality of free fall, it was assumed that any acceleration difference between the test samples would arise from the force exerted on these materials from the Earth as a whole. But it was noted in ref. 1 that their results could also be used to set limits on interactions arising from a local source (such as a mountain, cliff, basement), if the chemical composition and matter distribution of the source were sufficiently well known. References 7, 19 and 20 describe the original EPF experiment in more detail.

The original Eötvös experiment can be used to set limits on intermediate-range composition-dependent forces, but because various details of this experiment remain unknown to us, these limits are necessarily somewhat model-dependent. Thus, as a practical matter, the only unambiguous limits on composition-dependent forces before 1986 applied to those for which \(\lambda \approx 1\) AU, which derive from the experiments of refs 26-28. We see from Fig. 4 that the whole distance scale from a few centimetres to 1 AU has now been filled in by high-precision experiments using both laboratory and geophysical sources. (As in Fig. 1, we show only the envelope of the limits derived from the various experiments, and not the individual results.) Most of the new experiments use torsion balances similar to those developed by Eötvös to compare the accelerations of their test masses. We
show in Fig. 5 the apparatus used Adelberger et al. (known as the Eot-Wash collaboration), who have carried out the most extensive searches for composition-dependent effects over a wide range of distance scales. As can be seen in Fig. 5, all of the test masses in the Eot-Wash balance are located in the same vertical position, in contrast to the case for the Eötvös balance. The Eötvös apparatus was originally designed to measure gravity gradients, which required a vertical separation of the masses. This separation is, however, undesirable in searches for composition-dependent effects, as it produces a substantial systematic bias which must be compensated for. The Eot-Wash balance introduced other innovations, such as compensating masses to cancel (vertical) gravity gradients, a high-precision turntable to rotate the apparatus smoothly and an optical read-out system. Adelberger and collaborators have compared the accelerations of different pairs of test masses towards various sources, including a large mass of lead bricks, the hillside adjacent to their apparatus, and the Earth. In all cases their results have been compatible with the weak equivalence principle, which is to say that they see no evidence for a composition-dependent fifth force. As an example, their most recent hillside limit on the acceleration difference $\Delta a_\alpha$ of aluminium and beryllium is

$$\Delta a_\alpha[\text{Be-Al}] = (2.0 \pm 2.2) \times 10^{-11} \text{ cm s}^{-2}$$

where $\mathbf{E}$ and $\mathbf{N}$ are unit vectors which point east and north respectively. For further details of the Eot-Wash experiments, see ref. 45.

A complementary design for a torsion balance has been introduced by Boynton et al. These authors measure the period $T(\theta)$ of a ring composed of two dissimilar materials oscillating in a horizontal plane, as a function of the initial orientation angle $\theta$ of the ring with respect to their source, a cliff located near Mount Index in Washington. It can be shown that a dependence of $T(\theta)$ on the orientation of their ring (which is a composition dipole) is a signal for a composition-dependent fifth force, just as the shift in the equilibrium position of the balance is for the EPF and Eot-Wash experiments. The first results published by Boynton et al. found a marginally significant signal that could be interpreted as evidence for a fifth force. Their signal was a nonzero value of the difference $\Delta T(\theta)/T(\theta)$ where $\Delta T(\theta) = T(\theta) - T(\theta + \pi)$, and where $\theta = 0$ corresponded to the aluminium half of their beryllium-aluminium ring being adjacent to the cliff. Boynton et al. found for $\Delta T(\theta)$

$$\Delta T(\theta)/T(\theta) = (-4.6 \pm 1.1) \times 10^{-6} \cos \theta$$

But a later experiment\(^7\) using a copper-polyethylene ring found no evidence for a fifth force. It is not yet clear how we should interpret the result in equation (11), and Boynton et al. are continuing their efforts with an improved apparatus. The result in equation (11) is not compatible with the more stringent limits of experiments that obtained null results (see Table 1 of ref. 3).

In addition to variants of torsion balances, a number of novel techniques have also been introduced, such as the floating-ball experiments of Thieberger\(^6\) and of Bizzeti et al.\(^4\), and the free-fall experiments of Niebauer et al.\(^4\) and Kuroda and Mio.\(^4\) Thieberger’s experiment is shown schematically in Fig. 6. The central feature is a hollow copper ball floating in water. A composition-dependent fifth-force $F_5$ (arising in his experiment from a cliff at the Palisades, New Jersey) would act differently on the test masses, which are the copper shell and the water it displaces. This unbalanced force causes the ball to move, and from its velocity we can infer the magnitude of $F_5$. The advantage of such an apparatus is that the high degree of symmetry of the test masses makes this experiment relatively insensitive to gravity gradients, which could simulate the presence of a fifth-force. In practice, the symmetry of Thieberger’s apparatus is broken by a small pin which protrudes from the sphere and which is needed to make the sphere buoyant. Bizzeti et al. have carried out a similar experiment (in a monastery near Vallambrosa, Italy) with an apparatus of even higher symmetry. They use a solid nylon sphere floating in a solution containing various salts, and the ball is kept buoyant by a gradient in the density of the solution.

Historically, Thieberger’s experiment was the first test of the fifth force hypothesis to search for a composition-dependent effect. In fact Thieberger detected such an effect, in the form of a steady drift of his sphere across the tank. The average velocity of his sphere, $v = (4.7 \pm 0.2) \text{ mm h}^{-1}$, could be accounted for by the model in equations (5)–(7) with $\alpha A = (1.2 \pm 0.4) \text{ m}$. By contrast, Bizzeti et al. have set an upper limit of $v < 0.010 \text{ mm h}^{-1}$ (1$\sigma$) on a drift of their sphere, and the corresponding limit for a coupling to $B$ is $\xi < 0.30 \text{ m} (1\sigma)$ for $\lambda \ll 1 \text{ km}$.

Another interesting route which is being explored is the use of gravity wave antennas as fifth force detectors.\(^5\) In simple conceptual terms, such an experiment consists of a spinning detector and a nearby rotating composition dipole whose axis passes through the centre-of-mass of the rotating masses. In the absence of a fifth force the rotor would drive the detector at twice the frequency $\omega$ of rotation (assuming that the axis of rotation is precisely aligned with the centre-of-mass of the rotor). In the presence of a composition-dependent fifth force, there would be an additional force at $\omega$, and if $\omega$ corresponds to the resonant frequency of the bar, then limits can be set on $\xi$. At present these limits are not yet competitive with those from other experiments, but as the technology of gravity-wave detectors improves, so will the sensitivity of such experiments.

In addition to composition-dependent experiments that use geophysical sources, other experiments combine $\xi$ for both smaller and larger values of $\lambda$. Using Fig. 4, we can summarize the present situation as follows. For small values of $\lambda$, experi-
materials have typically involved laboratory-size sources constructed using blocks of lead\(^{45,52-55}\). The obvious advantage of such experiments is that the source has known dimensions and composition, but their disadvantage is that the sensitivity of such experiments drops quickly as the range of the putative fifth force increases. As noted above, for larger \(\lambda\) the best limits on \(\xi\) come from experiments using geophysical sources, such as a canal lock\(^{56}\), a mountain or a cliff\(^{57,66}\). These have the advantage of using much larger sources than laboratory experiments, albeit ones whose compositions are less well known.

As \(\lambda\) increases, there is a region in which the best limits on \(\xi\) come from Galileo-type experiments\(^{61,62}\), which compare the accelerations of two test masses falling freely towards the Earth. These experiments are inherently less sensitive than those using torsion balances, but are in practice easier to analyse in the region \(10^9 \text{ m} < \lambda < 10^{10} \text{ m}\). This is a consequence of the fact that Galileo experiments are sensitive to the component of force parallel to \(g\), \(\langle F_\parallel\rangle\) whereas torsion balance experiments are necessarily sensitive only to the component of \(F\) perpendicular to \(g\). Over the intermediate range, \(F_\parallel\) is significantly less sensitive to fine details of the underlying structure of the Earth than is \(F_\perp\), and hence is much more precisely known. As the Galileo experiments can be interpreted with much higher certainty than the torsion balance experiments, they consequently give rise to the best limits in this region of \(\lambda\). Further improvements in the analysis of \(F_\parallel\) over these distance scales may eventually allow limits from torsion balance measurements to surpass those from Galileo measurements, which are indicated by the hatched (\(K^+\to\mu^+\nu\)) analysis of \(F_\parallel\) over these distance scales may eventually allow the best limits in this region of \(\lambda\). Further improvements in the best limits on this branching ratio are the torsion balance experiments, they consequently give rise to has been looked for in several recent experiments. The current necessary to take account of the possibility that a given pair of
We emphasize that this conclusion applies specifically to the case where $\gamma_\nu$ is a vector particle and couples to hypercharge; for a scalar field there are no meaningful limits from kaon decay. Similarly, if $\gamma_\nu$ couples to some more general linear combination of $B$ and $S$, the constraints arising from $K^-$ decay are not in direct conflict with other data (see ref. 60 for more details).

Another class of elementary-particle experiments can be used to explore models of the putative fifth force consists of those comparing the free-fall accelerations of particles and their antiparticles. In the 1960s Fairbank and coworkers attempted to measure the gravitational accelerations of $e^+$ and $e^-$, but these efforts were thwarted by the presence of residual electromagnetic fields produced by surface effects in their apparatus. Goldman, Hughes and Nieto again attempted to measure such effects but there was no evidence to suggest such effects would be relatively less important in an experiment comparing the accelerations of $p$ and $\bar{p}$ because of the larger masses of these particles. The impetus for carrying out such an experiment increased following the suggestion of a possible fifth force, as the proposed (vector) hypercharge field would be expected to produce a difference in the apparent gravitational accelerations of $p$ and $\bar{p}$. In most models this difference would be too small to be detected, but GHN also noted that some models containing both scalar and vector fields predicted an effect large enough to be measurable.

This claim has been called into question by several recent analyses.10-14. As was first noted by Schiff15, if matter and antimatter behave identically in a gravitational field, this could show up as an anomaly in the Eöt-Wash experiment. Using a similar argument, Adelberger et al.16 demonstrate that a fifth-force model that could produce an acceleration difference between $p$ and $\bar{p}$ would also produce an acceleration difference between the test masses in a typical Eöt-Wash experiment. Given the extra-ordinary sensitivity of the current Eöt-Wash experiments, Adelberger et al. argue that existing limits from the Eöt-Wash experiments preclude seeing any acceleration difference between $p$ and $\bar{p}$ at the sensitivity level expected in these experiments. See refs 71-74 and 76 for further discussion.

**Pushing the limits**

The experimental searches for both composition-independent and composition-dependent deviations from Newtonian gravity have made enormous advances since 1986, as shown in Figs 1 and 4. No compelling evidence has yet emerged that would indicate the presence of a fifth force, although the anomalies reported in the original Eöt-Wash experiment remain to be understood, as do those in the experiments of Thieberger16 and Boynton et al.17. On the experimental side, efforts continue to set even more stringent limits on possible deviations from Newtonian gravity, motivated in part by the recognition that such experiments may be our most powerful tool in exploring physics at the Planck scale. The connection, to which we have alluded earlier, between such experiments and physics at the Planck scale has been explored in several models. An excellent review of the wide class of theories that predict the existence of weak macroscopic forces has been given recently by Fujii18. Although it would be difficult to summarize the content of these models briefly, the main lesson is simply that there are many theories that predict such forces. It follows that if additional weak forces are not seen, as is the indication from present experiments, then the assumptions behind a wide class of theories will have been called into question.

The experiments we have discussed can be understood as extending the maximum energy scale that can be explored from $4 \times 10^{16}$ GeV (at the proposed Superconducting Super-Collider), to $\sim 10^{18}$ GeV, an energy beyond the capability of any accelerator. Viewed in this way, torsion balances, floating balls and tall towers may be the ultimate high-energy physics tools.

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THE SECOND COMING OF TOWER GRAVITY: AN UPDATE

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ABSTRACT

The Phillips Laboratory and Purdue University are conducting a tower gravity experiment near the town of Inverness, MS. Gravity is measured at six elevations on the 610 m WABG-TV tower as well as on the surface in an 8 km radius about the tower. These data are combined with archived data extending to 300 km. Using previously devised techniques, the surface data are analytically continued and compared with the observations. The current difference at the highest tower elevation surveyed so far, 493 m, is 34 μGal.
INTRODUCTION

In December 1991 the Phillips Laboratory (PL) Geophysics Directorate (formerly AFGL) published the final results of a search for non-Newtonian gravity. The experiment conducted on the WTVD tower in Clayton, NC, led to a null result. Many difficulties were encountered in the North Carolina experiment, not the least of which were systematic effects due to improperly modeled terrain. This led to a bias which resulted from the fact that the gravity survey elevations were not representative of the actual terrain. These problems were compounded by the sparsity of gravity data between 5 and 10 km from the tower, and by the inaccessibility of those areas due to dense regions of trees. Because there were some lingering uncertainties involving the final WTVD results, we embarked on a follow-on tower experiment in an area where all known uncertainties could be minimized.

After a lengthy search, PL selected the 610 m WABG-TV tower located northeast of Inverness, MS. Inverness is about 50 km east of the Mississippi River and about 350 km north of the Gulf of Mexico. The area surrounding the tower is extremely flat out to a distance of 40 km. Also, the area is free of any type of forests, permitting gravity measurements to be made at most desired locations. Furthermore, the existing gravity data in the area are much more extensive than that in Clayton, NC. We believe that, given these advantages, we shall be able to resolve many, if not all, of the previous difficulties.

SURFACE GRAVITY SURVEY AND ANALYTIC CONTINUATION

Based on existing data in the tower area, supplied by the Defense Mapping Agency (DMA), an inner zone survey out to 8 km was deemed sufficient. The survey plan called for a set of concentric rings with up to ten points in each ring. The ring spacing was such that each of the inner ten rings contributed nominally equal weight in the analytic continuation at the top of the tower. This led to ring radii of 150, 300, 450, 600, 800, 1050, 1400, 1900, 2600, and 3600 m from the tower. The remaining rings were placed at distances of 4900, 6400, and 8100 m. PL and Purdue began the near-tower gravity survey in the fall of 1991. The points were positioned using a combination of the Global Positioning System (GPS) and trigonometric leveling using an Electronic Distance Meter (EDM). Positioning all the points using GPS was not possible due to interference from the transmitter near the tower\(^1\). The EDM points were positioned to an accuracy (relative) of 1 m in the horizontal and 2 cm in the vertical; the GPS points were accurate (relatively) to 3 cm in the horizontal and 4 cm in the vertical. A total of
355 observations contained in 25 survey loops resulted in 123 gravity points surveyed in the region over a period of six months. A least-squares adjustment was performed on the data along with corrections for earth tide, gravimeter drift, and scale factor. The resultant rms error is 13 μGal with no individual errors greater than 30 μGal.

![Fig. 1](image.png)

Fig. 1 Survey determined elevations minus digitized elevations contoured at 0.5 m. Dots represent locations of survey points.

The next phase was the removal of the high wave number component of the gravity field from the surface measurements. Using U.S. Geological Survey topographic maps, with a reported accuracy of 76 cm, we digitized the elevations inside a 10 x 10 km region. We interpolated the digital terrain to the points of our survey, determined elevations, and compared the results. The USGS maps are good, but not perfect; the rms difference between our elevations and those of the maps is 85 cm (Figure 1). Especially striking is the effect of the catfish farms (large pools of raised earth where local farmers breed catfish) just south of the tower that do not appear on the USGS maps and thus show up as large elevation differences. Using the comparison results, we corrected the digitized elevations and computed terrain-corrected Bouguer anomalies for all points inside of 10 km. Due to the benign nature of the surrounding terrain the rms terrain correction to the Bouguer anomalies was a mere 0.4 μGal. The resultant gravity field is very smooth with an anomalous horizontal gradient of about 6.7 E in contrast to about 13 E in the North Carolina area (Figure 2).
Fig. 2 Terrain-corrected Bouguer anomalies. Dots show the location of survey points; contour interval is 1 mGal.

We merged our corrected data with archived data obtained from DMA which were accurate to 1-2 mGal and extended to 300 km from the tower. We selected 7781 points to be used in the analytic continuation from a total of 50292. We obtained digital data from DMA for use in terrain correcting all gravity points, but the voluminous amount of data made for large computational and storage requirements. So, given that the terrain corrections inside 10 km were very small, we computed simple Bouguer anomalies for the DMA points that fell outside of 10 km from the tower. The DMA points within 10 km of the tower were also terrain corrected using the digitized elevation data.

For points outside 10 km, we used a procedure similar to the one in North Carolina. There, we found that the DMA data were biased towards the higher elevations. Given that the bias extended to 20 km from the tower, we assumed a constant bias of 7 m from 20 km out to 200 km. This led to a constantly sloping residual of 42 μGal at the top of the tower and zero at the base. Presumably the DMA data around the WABG tower also contained some outer zone terrain bias despite the flat terrain.

Using the USGS and DMA digitized data together, we computed mean elevations out to 40 km from the tower. We also computed mean elevations of the
gravity data (≤40 km) and compared the results (Figure 3). The figure clearly shows a terrain bias beyond 10 km; the bias continues beyond the boundaries of the figure, growing to as large as 5 m. So, as in the WTVD experiment, the data are biased towards higher elevations even though in the inner survey area (< 10 km) we were very careful to insure unbiased data. We corrected for the bias out to 40 km based on the results of the comparison between digital elevations and gravity elevations shown in Figure 3. We then assumed that the 5 m bias at 40 km is constant out to 300 km (the full extent of the data).

![Figure 3: Azimuthally averaged elevations. Triangles are the elevations of the gravity points. Boxes are the digitized elevations of USGS (<10 km) and DMA (10 - 35 km).](image)

We analytically continued the surface Bouguer anomalies using a combination of a summation of Fourier-Bessel series as a reference field, and a numerical integration technique for the residuals from the reference field. We performed a four-step nested symmetric Bessel function fit. Residuals were computed after each fit which served as input to the succeeding fit at progressively smaller distances; this allowed for the resolution of higher wave numbers. Step 1 is a 35 parameter fit extending to 300 km; step 2 is a 6 parameter fit extending to 7 km; step 3 is a 3 parameter fit extending to 1.3 km; and the final step is a 2 parameter fit extending to 0.3 km. Weights were computed for residuals from this reference field which were then analytically continued. The terrain had been removed prior to analytic continuation, so its effect was then added to the predicted values at the various tower elevations.
The tower gravity experiment is currently incomplete. We have measured gravity at five elevations on the WABG tower using the LaCoste-Romberg gravimeter, G-152. The data were collected in four loops with a total of only nine observations. The tower elevations were determined to an accuracy of 2 cm using an EDM. All measurements have been made in less than ideal conditions with wind speeds exceeding 15 km/hr. We estimate the accuracy of the tower data to be in the range 20-25 μGal even though the rms errors are on the order of 10 μGal. An attempt to measure at a sixth elevation, 571 m above ground level, failed. At 571 m, both the galvanometer and the reading line on the gravimeter were disabled, for reasons that are as yet unclear. One possibility is the presence of a very strong magnetic field, although we cannot rule out other effects such as radio frequency interference (RFI). The WABG preliminary results (shown in Table 1) are plotted alongside those of the WTVD tower at commensurate elevations in Figure 4. The agreement is good and, with the exception of the 94 m level, the two results agree to within 16 μGal.

### Table 1. Preliminary WABG Analytic Continuation Results

<table>
<thead>
<tr>
<th>Elevation (m above ground)</th>
<th>Observed (mGal)</th>
<th>Predicted (mGal)</th>
<th>Observed-Predicted (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>9.445±.009</td>
<td>9.434±.054</td>
<td>0.011±.055</td>
</tr>
<tr>
<td>93.845</td>
<td>9.379±.022</td>
<td>9.363±.025</td>
<td>0.016±.033</td>
</tr>
<tr>
<td>194.363</td>
<td>9.300±.022</td>
<td>9.305±.019</td>
<td>-0.005±.029</td>
</tr>
<tr>
<td>292.564</td>
<td>9.233±.023</td>
<td>9.243±.020</td>
<td>-0.010±.030</td>
</tr>
<tr>
<td>388.511</td>
<td>9.148±.023</td>
<td>9.179±.023</td>
<td>-0.031±.033</td>
</tr>
<tr>
<td>493.589</td>
<td>9.078±.024</td>
<td>9.112±.026</td>
<td>-0.034±.035</td>
</tr>
</tbody>
</table>

### SUMMARY AND FUTURE PLANS

These results leave us with several tasks to perform: 1) obtain more tower data during better weather conditions (wind speeds <15 km/hr); 2) terrain correct the gravity data out to 40 km or beyond; 3) continue error analysis and obtain improved error estimates for analytically continued values; and 4) resolve the problem at the 571 m elevation so that gravity data can be collected. Previous tests have shown that RFI disables the galvanometer but has no effect on the reading line. In addition, mu-metal shielding around the gravimeter should protect it from
stray magnetic fields. So, either the previous tests are somehow incomplete, or there is some other yet unknown cause. Once all the above tasks have been completed to our satisfaction we should be able to present our final results for the WABG tower and for the topic of tower gravity in general.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Observed minus model results for the two tower experiments and their associated errors. The boxes are final WTVD results and the diamonds are preliminary WABG results.}
\end{figure}

ACKNOWLEDGEMENTS

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GLOBAL POSITIONING SYSTEM AND TELEVISION SIGNALS:
ARE THE TWO COMPATIBLE?

by

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ABSTRACT

In November 1991, the Phillips Laboratory (PL), Geophysics Directorate in cooperation with Purdue University, conducted a Global Positioning System (GPS), survey near Inverness, MS. The survey was carried out around the 610 m WABG-TV transmitting tower. The data were used in the ongoing program by PL to study possible departures from Newtonian gravity. Data were collected within an 8 km radius of the tower. Inside 2 km, half of the six GPS receivers were unable to lock onto incoming GPS signals. We believe that some form of tower transmissions, possibly microwave, interfered with the signal transmitted by the GPS satellites.
INTRODUCTION

In November 1991, the Geophysics Directorate of the Phillips Laboratory in collaboration with Purdue University performed a GPS survey near the town of Inverness, MS. The survey was designed to establish accurate geodetic positions where gravity measurements could subsequently be made. At the center of the survey area is the 610 m WABG-TV television transmitting tower which is the main focus of the ongoing non-Newtonian gravity program at PL (Jekeli et al., 1990, Romaides et al., 1991). The reason for using GPS is that it provides one of the most accurate methods of position determination for the surface gravity measurements. Gravity measurements are also being carried out on this tower as well as on the ground. These ground data, along with previously collected data (obtained from the Defense Mapping Agency Gravity Library), will be used to analytically predict what gravity should be at various elevations on the tower in the absence of non-Newtonian forces. The predicted values will then be compared to the tower measurements. Any error in the latitude, longitude, or elevation of the surface gravity also adds to the error of the predicted values. For the latitude and longitude the error is on the order of 2 μGal/m, and for the elevation 3 μGal/cm (1 μGal = 10⁻⁶ cm s⁻²). Given an allowed prediction error of 25 μGal, the latitudes and longitudes must be known to an accuracy of 20 - 50 cm, and the elevation 2 - 5 cm depending on the relative distance to the tower. GPS provides a convenient and rapid way for the survey to be completed given the unpredictability of the weather in that area. The survey consisted of 134 points extending to a radius of 8 km from the tower, but for the purposes of this paper we shall focus on the inner 2 km of the survey. It is in this area that we believe the television signal interfered with the reception of the GPS signal, severely hindering our survey efforts in that region, and leading to delays in the project completion.

THE INITIAL GPS SURVEY

The survey was divided into concentric rings where the inner 2 km radii were: 150, 300, 450, 600, 800, 1050, 1400, and 1900 m from the tower (rings A through H respectively). Each ring contained 10 points with every other ring offset by 18 degrees (Figure 1). We planned the survey employing a six GPS receiver configuration. The receivers were Ashtech Inc. dual-frequency GPS receivers, two M-XII models and four LD-XII models. The two M-XII models were our own and the four LD-XII were rented from Ashtech. We labeled GPS receivers 1, 2, 3, 1A, 2A, and 3A strictly for identification purposes. We had two survey teams each responsible for three receivers; one team had receivers 1, 2, and 3 and the other 1A, 2A, and 3A. At the time of the survey there were favorable constellations during the day that permitted five observing sessions of 1 hour and 45 minutes with 30 minutes allowed between setups. During the sessions there were
four satellites available most of the time with five (and possibly six) satellites available for about half of the time in each session. The survey was designed to form closed loops of six-sided polygons with each receiver placed at a vertex. The plan was to do two concentric rings a day working outward from the tower and repeating the outer of the two rings on each succeeding day. In each session three receivers would be set up on the inner ring and three on the outer ring. At the end of each session, four of the six receivers would be moved to new locations with the other two remaining fixed to form the next polygon. In this way five sessions would allow for the completion of two rings; also the first two points observed in session I would be the same as the last two points observed in session 5. (See Figure 2 for receiver configuration and session observations.) Because of some practical considerations involving accessibility, the placement of the points was not exactly as planned (see Figure 3).

On 13 November 1991 (Julian day 317) we began our GPS survey setting up on the A and B rings (150 and 300 m) with our six receivers. After about 10 minutes we noticed that only two of the six receivers had locked onto any satellites, the other four were in a search mode with none of them able to lock. (From this point on, we will refer to the receiver and antenna interchangeably even though it is the antenna that locks onto the GPS satellite signal and not the receiver itself.) The only two that locked were at the points designated B8 and B9 (See Figure 2), and even those two did not immediately lock onto the satellite signals as was usually the case. Even after an hour the situation did not change; the receivers on points B8 and B9 continued to collect data while the other four obtained no locks. Since there were no obstructions to hinder incoming satellite signals, the implication was that something associated with the television transmitter was creating interference. In an attempt to determine the extent of the problem, receiver 3 (selected at random) was placed at several selected points to ascertain whether a signal could be detected anywhere in the vicinity of the tower. When this receiver was located on the C and D rings located 450 m and 600 m respectively from the tower, no satellite locks were obtained, just as in the case of the A and B rings. However in the E ring (800 m from the tower), the receiver was able to lock onto four satellites in one position (located SSW from the tower) but not in another position located NW of the tower. Finally at the F ring, 1050 m from the tower, the receiver was able to lock onto all available satellites in all the azimuthal orientations that were tested. Due to the stringent time constraints we were working under, a thorough analysis of the interference problem was not possible at that time. So based on the preliminary analysis we deemed it best to begin the survey at the F and G rings, and to then try to obtain positions and elevations for the inner rings using some other method such as trigonometric leveling.

On 14 November 1991 (day 318) the receivers were placed on the F and G rings in the configuration called for in the survey plan (Figure 4). In the first session the six receivers were set on points G0, G1, G2, F9, F0, and F1. Problems again surfaced with only three of the six
receivers being able to lock onto the available satellites. After five complete observing sessions on
day 318, we had successfully collected data for seven out of ten points in the F ring and five out of
ten points in the G ring. On the next day (day 319) our receivers were set up on the G and H rings
beginning with points H0, H1, H2, G0, G1, and G2. This time four of the six receivers were able
to lock onto available satellites. As day 319 progressed the situation improved, and by the end of
the day we had obtained data for six out of ten points in the G ring (one more than the previous
day) and eight out of ten points in the H ring. Finally on day 320 we collected data for nine out of
ten points in the H ring and all ten points in the I ring (2400 m). Interestingly, for the H ring we
collected no data for H2 on day 319 but were successful on the following day. Similarly data were
collected for H4 on day 319 but not on day 320. The situation was different for H2 and H4. In
both of these cases the receivers were able to lock onto only one or two satellites even though in
each of the two observing sessions there were at least four and possibly five satellites available.
The H ring, situated 1.9 km from the tower, was the furthest ring from where any of our receivers
had trouble locking onto satellite signals. The remainder of the survey out to 8 km was completed
without problems. Also where possible we surveyed points in the inner rings using an Electronic
Distance Meter (EDM). We were able to position 33 points in this manner. The EDM
measurements were facilitated by the generally flat and featureless terrain. With the exception of a
few trees there were no significant obstructions that could account for the reception problems that
we encountered.

PRELIMINARY ANALYSIS

After the initial GPS survey, we decided to do a more rigorous analysis of this interference
problem. We computed WGS84 positions for the GPS surveyed points as well as for the points
on the inner rings that we had measured using trigonometric leveling. For the points where
obtaining GPS positions were unsuccessful, we used a U.S. Geological Survey topographic map
to obtain approximate positions. Figure 5 shows the result of this exercise. The unmarked points
are those with successfully obtained GPS positions. Those identified in italics are points where we
were unable to obtain satellite locks at any time, and the remainder are points where the receivers
were able to lock at one time but unable to lock at another time. As can be seen from Figure 5, the
problem areas do not appear to be random, rather there appear to be preferred directions. First,
there is only one point west of the tower (beyond the E ring), G7, where we were unable to obtain
a GPS measurement, most of the problems occurring to the east. But even in the east there
appeared to be preferred directions. The largest areas of difficulty seemed to be concentrated in
three azimuthal regions: NE, ESE, and SSE.
We examined the GPS satellite constellations for the points G1, G3, G5, and G7. These were the points that were observed twice but on both occasions the receivers were unable to obtain satellite locks. Even though the observing sessions were only separated by a day, all four points were observed at different times during the second day with a somewhat different satellite constellation. However there did not appear to be any differences in the satellite constellations that could give rise to any reception problem. Furthermore, stations where data were not collected were grouped with others in the same observing session where GPS signals were successfully obtained. This conflicting behavior of the different stations was unexpected since stations in any given observing session were seeing essentially the same satellite constellation even though they were separated by a small distance. A similar exercise was attempted with the points where data had been collected on one session but not on another session. Of particular interest were the points G4 and G0. At G4, data were successfully collected on day 319 but not on day 318. Examination of the satellite sky plots for G4 on days 318 and 319 again revealed no clues to the problem. Due to the similarity of the satellite configurations, the only other possibility was a variation of the television signal at different times of the day. Another example is G0, where data were successfully collected at the end of day 318, 1720 - 1928 hrs, but not at the start of the session, 0810 - 1005 hrs. Again, the two satellite sky plots for the sessions 1 and 5 respectively, revealed nothing significant. The only conclusive feature of this puzzle was that west of the tower there was only one station that caused a problem; the preponderance of the difficulties were to the east. It is interesting to note that in addition to the 610 m television transmitter, there is another smaller television tower (400 m) located about 500 m northwest. The region northwest of the tower is one of only two areas, the other being the southwest, where we had no problem receiving satellite signals. Also during the first attempts at observing on the A and B rings, the only two points where data were collected were the points B8 and B9, both located in the northwest quadrant. One final note, every day we observed the rings in a clockwise direction starting in the north where the first observations were made between 0730 and 0800 hrs, and ending in the north with final observations made between 1830 and 1930 hrs. This would mean that all afternoon observations were made in the west and all morning observations were made in the east. Once again, examination of the satellite sky plots for the morning and afternoon of day 318 did not reveal any good reason for the preferred directions, so again we suspected variations in the transmitting signal.

THE FOLLOW-UP SURVEY

As a result of the problems we encountered with GPS, we initially fell 26 points short of our objective of 134 points. Hence in March 1992 we returned to Inverness, MS both to finish our
survey and to more clearly define some of the problems we had been having. We also began our gravity survey at the known GPS points. This time we had only our own two M-XII receivers labeled 2A and 3. Without the luxury of six receivers the only plan was to set up one receiver on a previously surveyed point and to set up the other receiver on an unknown point. Then as a check the receiver on the known point would be moved to another known point. In this way coordinates for the unknown point could be computed from two known points thereby providing a check on the accuracy. The key to any experimental anomaly is repeatability, and so the question was whether we could repeat the observational problems we had encountered the previous November. We began our survey on 12 March 1992 (day 072) and at first data acquisition went smoothly for the most part. In the month of March the satellite constellation was favorable in the morning and from the middle of the afternoon in to the evening. We used the small window where there was a dearth of available satellites (1200 - 1500 hrs) to continue our testing. Using receiver 3 we tested several areas, including those that had previously proven problematical, but at no location were we unable to lock onto available satellites. At one point, we even placed receiver 3 at a distance of 3 m from the tower and it was able to lock onto all available satellites. It would appear that more questions were raised than were answered, but as the survey progressed to day 073 similar problems began to surface. This time with only two receivers and no rigid time constraints we finally began to unravel the mystery.

It turned out that only one of our two receivers, 2A, was experiencing the interference problems; the other receiver, 3, had no problem locking onto satellites in any direction or at any distance from the tower. It would appear the reason receiver 3 had problems on day 317 during the initial tests but experienced no such difficulties in March 1992, is simply that we did not allow the receiver enough time to acquire locks during the early tests. We interchanged the antennas and the receivers and the problem tracked the antenna. This was something we had not expected. The thought of a malfunctioning antenna (or receiver) had occurred to us but it was surprising that an antenna would function correctly at distances greater than 2 km from a television transmitter, yet experience difficulties at closer distances. The antennas for all six receivers were essentially identical; if one or more could receive GPS signals in the presence of a television transmitter it seemed unlikely that the others would behave differently. Yet this was exactly what was happening, although the solution was still not that simple. Even though receiver 2A was experiencing difficulties locking onto satellites, the problem was still directional to a certain extent. In the E ring for the points we tested, receiver 2A was able to lock onto satellites for points E5 and E6 but not for points E1, E7, E8, and E9. In the F ring we tested all but one point. Satellite locks were obtained for data points F1, F3, F5, F6, F7, and F9 but not for points F0, F2, and F4 (see Figure 5). So once again there appeared to be some directional variations in the television signal that were somehow interfering with the GPS signals.
Armed with this information we were able to complete most of the required survey. This was done simply by planning the survey so that receiver 3 was always closest to the tower or at the problem areas (e.g. F2 or G5), and 2A was further away from the tower and thus, from any interference. Unfortunately three of the GPS stations previously surveyed had been disturbed or destroyed before we had had the opportunity to obtain gravity measurements, so some of these needed to be resurveyed as well. Also with only two receivers, achieving the same level of redundancy and checking became more time consuming than had been anticipated. In addition to the GPS surveying, a great deal of time was spent completing the majority of the necessary gravity measurements. The heavy work schedule meant that by the end of this trip we were still about 12 points short of our final objective thus necessitating a third trip. Prior to returning to Inverness it occurred to us that perhaps RF interference was affecting one of our receivers. It was possible the shielding around the antenna cable for receiver 3 was superior to that of receiver 2A. We constructed some heavy-duty (RG-214) cables to be used in place of the existing antenna cables. We tested all cables before leaving to ensure they were functioning properly.

We returned for a third time to Mississippi on 5 April to complete the GPS survey. This time in addition to our two M-XII receivers we rented two LD-XII receivers from Ashtech Inc. As it turned out, one of the GPS receivers (receiver 2) was the same one we had rented in November 1991. The other receiver was totally new to us. This provided a unique opportunity to do some additional testing on the antennas in and around the tower. When we began the survey on 6 April (day 097) it became obvious that the only receiver having any problems locking onto satellite signals was 2A. We then decided to test for RF interference from the tower using the heavy-duty cables that we had constructed to replace the RG-58 cables we were previously using. We set up all four receivers at E8 employing the RG-214 cable on 2A. The results were the same, the 2A receiver was unable to lock onto any satellites while the remaining three receivers had no such difficulty. This was the last test we did with the GPS antennas. By this time plowing was well under way in the fields surrounding the tower, which destroyed several additional GPS points and brought the total number of points requiring surveying to 27. We proceeded to collect the final 23 GPS points (five less than we had planned) and completed the gravity survey without further problems.

**THE FINAL ANALYSIS**

In the end, 129 out of a proposed 134 GPS points were measured in and around the WABG-TV tower in Inverness, MS. The five remaining points were never surveyed due to the inaccessibility of the areas. The data were obtained on three separate trips: November 1991, and in March and April 1992. Given that fact, the resolution to the interference problem was not

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intuitively obvious at the time of the survey despite several pieces of evidence to that effect. Using all the data at our disposal some missing steps can be retraced, and some definitive conclusions can be drawn.

Looking back to November 1991 (day 318) we can make a few conclusive statements. We now know that receivers 1, 2, and 3 were placed on the F ring and receivers 1A, 2A, and 3A were placed on the G ring. Examination of the survey plan for the F ring indicates that only one of the three receivers was unable to lock onto the GPS signals. The survey plan for the F ring was as follows:

<table>
<thead>
<tr>
<th>Session</th>
<th>Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>F9, F0, F1</td>
</tr>
<tr>
<td>2</td>
<td>F1, F2, F3</td>
</tr>
<tr>
<td>3</td>
<td>F3, F4, F5</td>
</tr>
<tr>
<td>4</td>
<td>F5, F6, F7</td>
</tr>
<tr>
<td>5</td>
<td>F7, F8, F9</td>
</tr>
</tbody>
</table>

Given the clockwise direction we were working, it is easy to see how the one problem receiver could have been placed on F0, F2, and F4 during sessions 1, 2, and 3. Knowing that receiver 3 was operating correctly, and having tested receiver 2 in April, we concluded that receiver 1 was the one that was unable to lock onto the GPS signals at various azimuthal directions in the tower vicinity. There were no problems with any of the receivers in sessions 4 and 5. We also know that receivers 1, 2, and 3 were placed on B7, B8, and B9 on day 317 when these problems were first encountered. The above conclusions are also consistent with day 317 of the survey when two receivers in the B ring (undoubtedly 2 and 3) were able to lock onto signals and the third was not. For the G ring, the situation is less clear. The survey plan for the G ring on day 318 is given below:

<table>
<thead>
<tr>
<th>Session</th>
<th>Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>G0, G1, G2</td>
</tr>
<tr>
<td>2</td>
<td>G2, G3, G4</td>
</tr>
<tr>
<td>3</td>
<td>G4, G5, G6</td>
</tr>
<tr>
<td>4</td>
<td>G6, G7, G8</td>
</tr>
<tr>
<td>5</td>
<td>G8, G9, G0.</td>
</tr>
</tbody>
</table>

From past experience we knew receiver 2A was having problems. What we do not know for sure is which of the other two, 1A or 3A, experienced similar difficulties. On day 318 we failed to
collect data for points G0 (session 1), G1, G3, G4, G5, and G7. So in the first three sessions two out of three receivers were not collecting data, while in session 4 the problem was limited to one receiver. There are two puzzling aspects of this conclusion. Firstly, for station G0, data were successfully collected at the end of the day (session 5) but not at the beginning (session 1). Secondly, on the following day, day 319, only one receiver in each of the first three sessions failed to collect data whereas on day 318 it had been two receivers that had failed. The one problematic receiver on day 319 was obviously receiver 2A. Unfortunately we did not begin keeping track of where the receivers were placed until day 320. If one assumes some directional nature of interference, it is possible to construct a scenario where the receivers are placed on different stations on days 318 and 319 thus causing the inconsistency between the two days of observation. Once again we were faced with the conclusion that either receiver 1A or 3A was having difficulty locking onto GPS signals at a few azimuthal orientations. One final note on these receivers: on day 317 all three were placed on the A ring (A7, A8, and A9), and after an hour none of them had collected any data. So it would appear all three receivers had some degree of difficulty in receiving satellite signals.

On day 319 receivers 1, 2, and 3 were placed on the H ring beginning with H0, H1, and H2 and ending with H8, H9, and H0. The results of the data collection were: no data at H2 and at the second observation of H0. Once again we believed the problem was with receiver 1. Since we were solving for latitude, longitude and elevation, a minimum of four GPS satellites locks were required before the receiver would begin collecting data. As previously stated, for the H ring, and to a certain extent the G ring, the receiver was able to lock onto one or two of the four or five satellites available which was not enough to initiate data collection. This supported the theory that at greater distances from the tower the problems began to subside.

What could cause three out of six GPS receivers to be unable to lock onto signals from the GPS satellites in the vicinity of a television transmitter with no visible obstructions? The WABG-TV tower is 610 m tall and transmits at 100 kW with a frequency of 88 MHz. According to the engineers at the TV station, the transmitter is a circularly polarized antenna transmitting horizontally and vertically downward with no measurable decrease in the signal strength up to a distance of 80 km. This means that there is no variation of the television signal in any azimuthal orientation. The only component of the tower that could have any directional effect are the five 4 watt microwave transmitters located between 50 and 100 m above ground level. Three of these microwave dishes transmit to Greenville, and the other two to Greenwood. Greenville is WNW of the tower and Greenwood is NE of the tower. Interestingly, these are close to two of the four directions where we had difficulty in obtaining satellite locks. Perhaps this was all coincidence as the microwave transmitters still did not explain the problems to the east and the south. Further, incoming GPS signals arrive from all directions and not just from overhead so even though we were situated
directly under the microwave beam, there should not have been any reception problems. As
previously stated, there is another transmitting tower northwest of WABG-TV. This is the 400 m
tall WMAO-TV 5 MW transmitter, and is located between points C9 and C0. This tower contains
only one microwave transmitter and it transmits in the direction of Jackson (SSE of the tower). It
is again interesting to note that SSE was one of the four problem directions.

CONCLUSIONS AND RECOMMENDATIONS

What, if any, conclusions can we draw from our experience in Inverness, MS? The most
obvious conclusion is that GPS measurements cannot routinely be made in the direct vicinity (<2
km) of television and possibly radio transmitting towers. This, however, may be a luxury that a
surveyor cannot afford if a GPS survey needs to be conducted at an airport for instance, with
numerous transmitting towers. It does seem possible that some microwave signals emanating from
these towers create signal reception difficulties for some GPS receivers. In three out of the four
directions where we experienced signal reception difficulties there are microwave transmissions
present. The problems we encountered due east remain largely unexplained. We cannot devise a
mechanism to explain how the microwave transmission interferes with the GPS signal, but it is
possible that the signal to noise ratio is decreased making it difficult for less sensitive
GPS
antennas to obtain satellite locks. The GPS frequencies are at 1575.42 MHz for L1 and 1227.6
MHz for L2. It is possible that various harmonics of the microwave transmissions could fall near
the GPS frequencies thus causing some degree of receiver reception problems (Johannessen et al.,
1990). The problems undoubtedly will vary from one transmitter to another. Also it has been
shown that GPS signals are susceptible to jamming whether it is unintentional or deliberate
(Johannessen, 1992; Owen, 1992).

Given the fact that the GPS receiver manufacturers contend that television or radio
transmitters should pose no problems, how should one proceed? If a situation necessitates a GPS
survey near a transmitting tower, the only prudent course of action would be to test all receivers
thoroughly prior to commencement of the survey. The test should be conducted at various times of
the day and in all azimuthal orientations extending out to 4 km. Had we performed these tests and
been aware of all these problems in November 1991, it is quite possible we would have been able
to complete the entire GPS survey (134 points) without any additional trips. It is our belief that for
every type of transmitting tower some type of interference is present. Therefore any GPS surveys
that are performed in their vicinity will experience problems similar to the ones we encountered. It
is up to the surveyor to take precautionary steps in order to avoid potential problems, or using a
variation to the old salesperson adage: "let the surveyor beware."
ACKNOWLEDGEMENTS

Our sincerest thanks go to Sgt Joe Craig and A1C Mike Beaudet (both of the Phillips Laboratory, Hanscom AFB, MA) for their hard work during the bulk of the data collection in November 1991. Their tireless efforts were instrumental in the success of this experiment. The logistical requirements of this undertaking were enormous, and without their assistance we would have been unable to collect the amount data within the required time frame.

REFERENCES


FIGURE CAPTIONS

Figure 1. The original survey plan of the GPS points within a 2 km radius of the tower.

Figure 2. The original plan for the inner three rings (A, B, and C) with each six-sided polygon representing one session of observation. The figure depicts two days of observations consisting of five 1 hour and 45 minute sessions.

Figure 3. The final survey plan of the GPS points within 2 km of the tower.

Figure 4. The survey plan for the F, G, and H rings on Julian days 318 and 319 prior to the discovery of the interference problem.

Figure 5. The survey points of the F, G, and H rings. Points without designation are those where GPS data was successfully collected. The points labeled in italic are those where GPS data was not originally collected despite more than one attempt at obtaining data. The remaining points are ones where data was collected on one day but not on another day. The solid lines represent the approximate regions where reception difficulties were prevalent.
Figure 4