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This is the final report for a project studying the perturbed sine-Gordon pde and the nonlinear Schrodinger pde. We studied low-dimensional chaos in these dissipative pdes, in order to understand the onset of chaos, the underlying geometry of the pde, and how the chaos is low-dimensional.

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Final Report
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We have continued our study of chaos and coherence in dissipative partial differential equations that are "close" to being completely integrable. A review article has been completed which discusses this subject in great detail. The title is "Whiskered Tori for Integrable Pde's: Chaotic Behavior in Near Integrable Pde's" by David W. McLaughlin and Edward A. Overman II. This discusses in detail the analytical theory and perturbation theory for our study of chaos and coherence. It relates this theoretical work to our computational experiments.

This paper is mainly concerned with studying the theory and application of the nonlinear Schrödinger equation,

$$2iq_t(x,t) + q_{xx}(x,t) + 2|q(x,t)|^2 q(x,t) = 0\,.$$

This partial differential equation describes the evolution of the envelope, $q(x,t)$, of a rapidly oscillating wavetrain. It is applicable to many physical models, such as deep-water wave theory and energy transport in many systems. One very practical application is in laser optics. In fact, AT&T will be laying a fiber based on this nonlinear equation between the United States and Asia in the late 1990's. The advantage of using a *nonlinear* fiber rather than a linear fiber is that in a linear fiber a pulse will continue spreading out as it moves down the fiber, and so many "repeaters" are needed to condense and amplify the pulse. In a nonlinear fiber the nonlinearity can focus the pulse itself and prevent it from spreading out.

This paper begins by discussing the possible behavior of the solutions of the nonlinear Schrödinger equation under weak perturbations. In the real world the mathematical models of physical systems always involve some perturbations of the underlying NLS equation. Usually, the perturbation is treated as a small change in the unperturbed equation. However, it is shown numerically that there is a wide range of possible responses under weak perturbations. These range up to chaotic solutions, although the chaos itself is rather low-dimensional. This chaotic response is quite interesting because large-scale structures are still present. The chaotic motion can be thought of as a localized structure which is undergoing oscillations due to an underlying chaotic "bath".

Next, the analytical structure, that is, the inverse spectral transform, is developed in order to be able to derive the solutions of the unperturbed NLS equation. In addition, this theory is used to explain the regular and the chaotic motion of the perturbed NLS equation. This theory includes the introduction of a Morse function which shows the underlying geometric structure of the NLS equation. It shows the saddle point structure of the constants of the motion and the Bäcklund transformations which are used to derive the homoclinic orbits. With this geometric picture it is easy to understand the chaotic motion in the perturbed NLS equation. In addition, this analytic and geometric structure is used to show that the numerical calculations are behaving exactly as the theory predicts. It is rare that dynamical systems theory can be used to help explain the chaotic response of a high-dimension system, such as a partial differential equation. Because of the combination of

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analytical and geometrical understanding which has been developed for the NLS equation, it is possible to do so here. Also, this combination of theory and numerics allows us to understand the role of the homoclinic orbits in the onset of chaos.

Finally, a Melnikov analysis of a perturbed model system, which is a low-mode truncation of the perturbed NLS equation, is described. At present, dynamical systems theory can only be applied in low-dimensional models. This model system is four-dimensional and so we use the known homoclinic structure of the NLS equation, which can be continued down to this model system, to study the onset of chaos in the perturbed system. Much of the analysis which is developed here is based on numerical work. In particular, the onset of chaos is a singular perturbation problem and so the numerics has proved invaluable in determining the "slow" and the "fast" variables. This is a novel application of Melnikov and Shilnokov theory in order to understand the effect of the underlying homoclinic orbit on the onset of chaos. This is the type of analysis that is necessary to extend dynamical systems theory so that it is applicable to the high-dimensional systems that can be derived from physical models.

A second paper has been completed which continues our attempts to use singular perturbation theory to understand the onset of chaos. The title is "Homoclinic Orbits in a Four Dimensional Model of a Perturbed NLS Equation: A Geometric Singular Perturbation Study" by David W. McLaughlin, Edward A. Overman II, Stephen Wiggins, and Chuyu Xiong. Our main advance is to study how homoclinic orbits, which exist in the completely integrable model, can continue to exist in the dissipative system. Since this homoclinic orbit seems to be the source of the chaos, this use of Melnikov and Shilnikov methods to study the persistence of homoclinic orbits may prove invaluable in other systems.

This paper studies a mechanism for the onset of chaos in a two-mode ode which is obtained from the driven, damped nonlinear Schrödinger pde by mode truncation. We study, both analytically and numerically, homoclinic orbits in this ode. We show how the homoclinic orbit in the perturbed ode arises from a specific homoclinic orbit both numerically and analytically. We begin with a representation of certain invariant manifolds by fibers. The existence of the homoclinic orbit then follows from a Melnikov argument combined with methods from geometric singular perturbation theory. Next, these homoclinic orbits are constructed numerically with a bifurcation code. These numerical studies find some members of the family of homoclinic orbits where were predicted by the theory. Finally, the existence of a chaotic symbol dynamics is established using a "Smale horseshoe".

A short paper which studies some analytical solutions which give rise to homoclinic orbits is nearly complete. the title is "Novel Periodic Solutions of the Sine-Gordon Equation and their Stability" by Gregory Forest, Edward A. Overman II, Peter Christiansen, Mads Peter Sørensen, Randy Flesch, David W. McLaughlin, and Robert D. Parmentier.

A 50 minute talk was given on the subject of chaos, coherence, and homoclinic orbits at the NATO conference, ***Future Directions of Nonlinear Dynamics in Physical and Biological Systems*** at the Technical University of Denmark in July 1992.

Chuyu Xiong, who was supported by this and the previous AFOSR grant both as a graduate student and an instructor, is now a post-doctoral student at the University of Indiana. He is supported by Roger Temam and is also working with Michael Jolley.