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MICROWAVE SCATTERING FROM A RANDOM MEDIUM LAYER WITH A RANDOM INTERFACE

Saba Mudaliar

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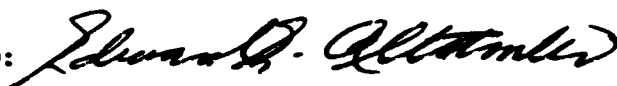


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Microwave Scattering From A Random Medium Layer With A Random Interface

1. INTRODUCTION

In the field of wave propagation and scattering from layered media a problem of great practical interest and importance is the one where both medium parameters and interfaces have random fluctuations. Most natural objects are best represented and studied by this kind of model. Indeed the topic of wave scattering from random surfaces and that of propagation and scattering in random media have been extensively studied by Beckmann and Spizzichino¹, and by Ishimaru². However, little has been reported about the problem that involves both random media and random boundaries. Within the framework of radiative transfer theory, Fung and Chen³ and Fung and Eom⁴ have solved this problem and illustrated its usefulness by applying it to several practical situations. Unfortunately the numerical procedure that they have used has obscured much physical insight. Furutsu⁵ has also considered a similar problem and has given a comprehensive analysis. However his work does not seem to be readily amenable to numerical computations. We provide in this report a first-order perturbation solution that is at once physically transparent and computationally elementary.

In Section 2 the geometry of the problem is described. The problem is then mathematically formulated in the next section. Section 4 is devoted to the derivation of the scattered field. In the following section the bistatic scattering coefficients are calculated. Section 6 contains a brief discussion of some of the properties of the results. Section 7 gives the conclusions.

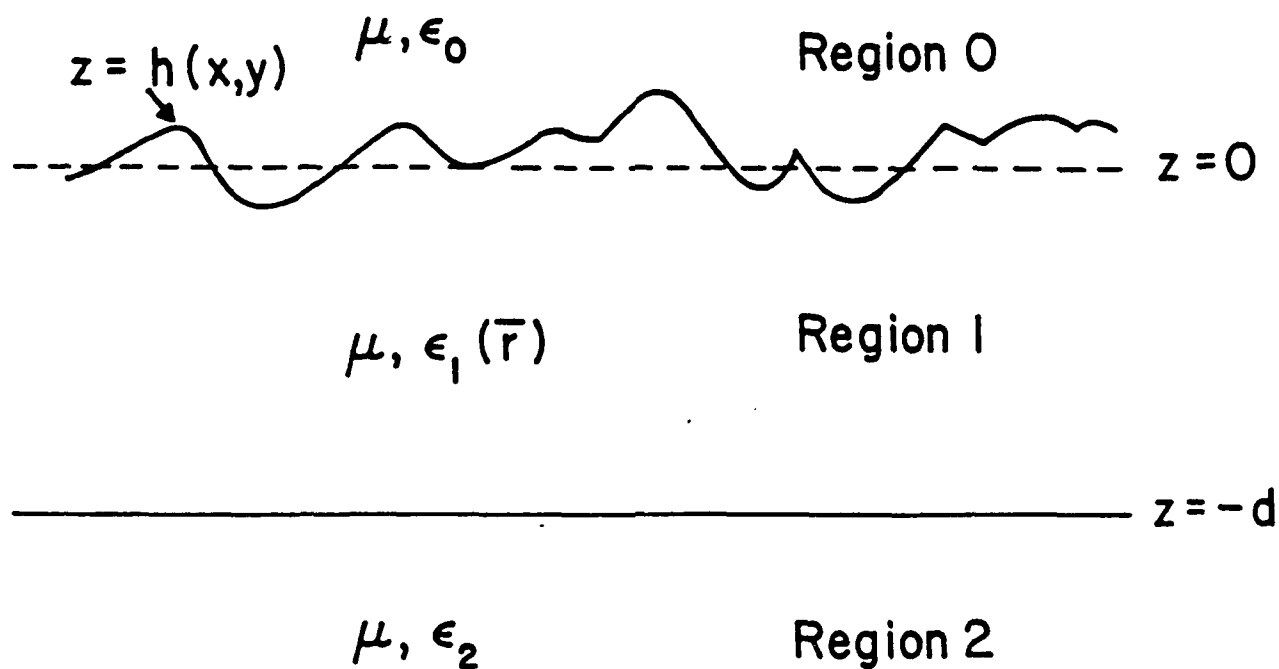


Figure 1. GEOMETRY OF THE PROBLEM

2. GEOMETRY OF THE PROBLEM

Fig. 1 shows the geometry of the problem. In this two-layer problem the bottom interface is planar while the top rough interface is described by the random function $z = h(x,y)$. Thus we have three regions: Region 0, $z > h(x,y)$, is free space with permittivity ϵ_0 ; Region 1, $-d < z < h(x,y)$ is the layer with randomly inhomogeneous permittivity $\epsilon_1(\bar{r})$; Region 2, $z < -d$, constitutes a homogeneous medium of permittivity ϵ_2 . All three regions have the same permeability μ . The permittivity of the layer $\epsilon_1(\bar{r})$ may be written as

$$\epsilon_1(\bar{r}) = \epsilon_{1m} + \epsilon_{1f}(\bar{r}) \quad (1)$$

where $\epsilon_{1m} \equiv \langle \epsilon_1(\bar{r}) \rangle$ is the mean part and $\epsilon_{1f}(\bar{r})$ is the fluctuating part. Both $h(x,y)$ and $\epsilon_{1f}(\bar{r})$ have zero means and small variances. The variances of $h(x,y)$ and $\epsilon_{1f}(\bar{r})$ are denoted as σ_h^2 and σ_v^2 respectively. Further, three correlation functions are defined as follows.

$$C_v(\bar{r}_1 - \bar{r}_2) = \langle \epsilon_{1f}(\bar{r}_1) \epsilon_{1f}^*(\bar{r}_2) \rangle \quad (2a)$$

$$C_b(\bar{r}_{11} - \bar{r}_{21}) = \langle h(\bar{r}_{11}) h(\bar{r}_{21}) \rangle \quad (2b)$$

$$C_c(\bar{r}_1 - \bar{r}_{21}) = \langle \epsilon_{1f}(\bar{r}_1) h(\bar{r}_{21}) \rangle \quad (2c)$$

Here and henceforth $\hat{x}\hat{x} + \hat{y}\hat{y}$ is denoted as $\bar{r}_1$.

3. FORMULATION

For a plane wave incident on the random layer from above (Region 0) we are interested in the far-zone scattered field in Region 0.

Mathematically we may formulate this problem as follows.

Let $\bar{E}_0(\bar{r})$, $\bar{E}_1(\bar{r})$ and $\bar{E}_2(\bar{r})$ denote the electric fields in Region 0, Region 1 and Region 2 respectively. These electric fields satisfy the wave equations:

$$\nabla \times \nabla \times \bar{E}_0(\bar{r}) - k_0^2 \bar{E}_0(\bar{r}) = 0 \quad , \quad z > h \quad (3a)$$

$$\nabla \times \nabla \times \bar{E}_1(\bar{r}) - k_{1m}^2 \bar{E}_1(\bar{r}) = q(\bar{r}) \bar{E}_1(\bar{r}) \quad , \quad -d < z < h \quad (3b)$$

$$\nabla \times \nabla \times \bar{E}_2(\bar{r}) - k_2^2 \bar{E}_2(\bar{r}) = 0 \quad , \quad z < -d \quad (3c)$$

where

$$k_\ell^2 = \omega^2 \mu \epsilon_\ell \quad , \quad \ell = 0, 2 \quad (4a)$$

$$k_{1m}^2 = \omega^2 \mu \epsilon_{1m} \quad (4b)$$

$$q(\bar{r}) = \omega^2 \mu \epsilon_{1f}(\bar{r}) \quad (5)$$

Further, the electric fields must also satisfy the boundary conditions

$$\hat{n} \times \bar{E}_0(\bar{r}_1, h) = \hat{n} \times \bar{E}_1(\bar{r}_1, h) \quad (6a)$$

$$\hat{n} \times [\nabla \times \bar{E}_0](\bar{r}_1, h) - \hat{n} \times [\nabla \times \bar{E}_1](\bar{r}_1, h) \quad (6b)$$

$$\hat{z} \times \bar{E}_1(\bar{r}_1, -d) - \hat{z} \times \bar{E}_2(\bar{r}_1, -d) \quad (7a)$$

and

$$\hat{z} \times [\nabla \times \bar{E}_1](\bar{r}_1, -d) - \hat{z} \times [\nabla \times \bar{E}_2](\bar{r}_1, -d) \quad (7b)$$

where $\hat{n}$ is the unit vector normal to the rough interface pointing into Region 0. The task now is to solve this system of equations. In particular we need to find $\lim_{r \rightarrow \infty} \bar{E}_0^s(\bar{r})$, the far zone incoherent part of the field in Region 0.

4. ANALYSIS

We first consider the situation where the boundary is unperturbed, that is when $h(\bar{r}_1) = 0$. The electric fields in this situation are labelled by the superscript (0). These fields satisfy the following wave equations and the boundary conditions.

$$\nabla \times \nabla \times \bar{E}_0^{(0)}(\bar{r}) - k_0^2 \bar{E}_0^{(0)}(\bar{r}) = 0, \quad z > 0 \quad (8a)$$

$$\begin{aligned} \nabla \times \nabla \times \bar{E}_1^{(0)}(\bar{r}) - k_{1m}^2 \bar{E}_1^{(0)}(\bar{r}) &= q(\bar{r}) \bar{E}_1^{(0)}(\bar{r}), \\ &-d < z < 0 \end{aligned} \quad (8b)$$

$$\hat{z} \times \bar{E}_0^{(0)}(\bar{r}_1, 0) = \hat{z} \times \bar{E}_1^{(0)}(\bar{r}_1, 0) \quad (9a)$$

$$\hat{z} \times [\nabla \times \bar{E}_0^{(0)}](\bar{r}_1, 0) = \hat{z} \times [\nabla \times \bar{E}_1^{(0)}](\bar{r}_1, 0) \quad (9b)$$

The solutions to Eqs.(8) and (9) may be written as

$$\bar{E}_0^{(0)}(\bar{r}) = \bar{E}_0^{(00)}(\bar{r}) + \int_{-d}^0 dz_1 \int_{-\infty}^{\infty} d^2\bar{r}_{1\perp} \bar{G}_{01}^{(00)}(\bar{r}, \bar{r}_1) q(\bar{r}_1) \cdot \bar{E}_1^{(0)}(\bar{r}_1) \quad (10)$$

$$\bar{E}_1^{(0)}(\bar{r}) = \bar{E}_1^{(00)}(\bar{r}) + \int_{-d}^0 dz_1 \int_{-\infty}^{\infty} d^2\bar{r}_{1\perp} \bar{G}_{11}^{(00)}(\bar{r}, \bar{r}_1) q(\bar{r}_1) \cdot \bar{E}_1^{(0)}(\bar{r}_1) \quad (11)$$

where $\bar{G}_{01}^{(00)}(\bar{r}, \bar{r}_1)$ and $\bar{G}_{11}^{(00)}(\bar{r}, \bar{r}_1)$ are the dyadic Green's functions: the first subscript stands for the region where the point of observation is located while the second subscript indicates the region enclosing the source. Also, the arguments $\bar{r}$ and $\bar{r}_1$ denote respectively the points of observation and source. The superscript (00) indicates the situation when both the medium and the boundary are unperturbed, that is, when $h(\bar{r}_1) = 0$ and $\epsilon_{1f}(\bar{r}) = 0$.

For small σ_v we may approximate Eq.(10) as

$$\bar{E}_0^{(0)}(\bar{r}) = \bar{E}_0^{(00)}(\bar{r}) + \int_{-d}^0 dz_1 \int_{-\infty}^{\infty} d^2\bar{r}_{1\perp} \bar{G}_{01}^{(00)}(\bar{r}, \bar{r}_1) q(\bar{r}_1) \cdot \bar{E}_1^{(00)}(\bar{r}_1) \quad (12)$$

Also when σ_b and $|\nabla h|$ are small we can approximate the field on the rough interface as

$$\bar{E}_0(\bar{r}_1, h) = \bar{E}_0(\bar{r}_1, 0) + h \partial_z \bar{E}_0(\bar{r}_1, 0) \quad (13a)$$

$$\bar{E}_1(\bar{r}_1, h) = \bar{E}_1(\bar{r}_1, 0) + h \partial_z \bar{E}_1(\bar{r}_1, 0) \quad (13b)$$

Noting that

$$\bar{n} = \hat{z} - \nabla h \quad (14)$$

we may write

$$\begin{aligned} \bar{n} \times \bar{E}_\ell(\bar{r}_1, h) &= (\hat{z} \times \nabla h) E_{\ell z}(\bar{r}_1, 0) + \left[\bar{I} + \hat{z} \nabla h \right] \cdot \hat{z} \times E_\ell(\bar{r}_1, 0) \\ &\quad + \hat{z} h \left[\partial_z \times \bar{E}_\ell \right] (\bar{r}_1, 0) ; \quad \ell = 0, 1 \end{aligned} \quad (15)$$

We now express $\bar{E}_\ell(\bar{r})$ as a perturbation series

$$\bar{E}_\ell(\bar{r}) = \sum_{m=0}^{\infty} \delta^m \bar{E}_\ell^{(m)}(\bar{r}) \quad (16)$$

where δ is the small parameter of the problem. From Eqs. (16) and (15)

we have

$$\begin{aligned}
\bar{n} \times \bar{E}_\ell(\bar{r}_\perp, h) &\approx \left[\bar{I} + \hat{z} \nabla h \right] \cdot \hat{z} \times \bar{E}_\ell^{(0)}(\bar{r}_\perp, 0) \\
&+ \delta \hat{z} \times \bar{E}_\ell^{(1)}(\bar{r}_\perp, 0) + (\hat{z} \times \nabla h) \cdot \bar{E}_{\ell z}^{(0)}(\bar{r}_\perp, 0) \\
&+ \hat{z} h \left[\partial_z \times \bar{E}_\ell^{(0)} \right] (\bar{r}_\perp, 0) ; \quad \ell = 0, 1
\end{aligned} \tag{17}$$

On substituting Eq.(17) in Eq.(6a) we obtain the following relations.

Zeroth-order relation:

$$\hat{z} \times \bar{E}_0^{(0)}(\bar{r}_\perp, 0) = \hat{z} \times \bar{E}_1^{(0)}(\bar{r}_\perp, 0) \tag{18}$$

first-order relation:

$$\hat{z} \times \bar{E}_0^{(1)}(\bar{r}_\perp, 0) = \hat{z} \times \bar{E}_1^{(1)}(\bar{r}_\perp, 0) + \bar{J}(\bar{r}) \tag{19}$$

where

$$\bar{J}(\bar{r}) = \left[(\hat{z} \times \nabla h) \cdot \hat{z} + h \hat{z} \cdot \partial_z \times \bar{I} \right] \cdot \left[\bar{E}_1^{(0)} - \bar{E}_0^{(0)} \right] (\bar{r}_\perp, 0) \tag{20}$$

Also from Eqs.(13) and (14)

$$\begin{aligned}
\bar{n} \times \left[\nabla \times \bar{E}_\ell \right] (\bar{r}_\perp, h) &= (\hat{z} \times \nabla h) \left[\hat{z} \cdot \nabla \times \bar{E}_\ell \right] (\bar{r}_\perp, 0) \\
&+ \left[\bar{I} + \hat{z} \nabla h \right] \cdot \left[\hat{z} \times \nabla \times \bar{E}_\ell \right] (\bar{r}_\perp, 0) \\
&+ \left[\hat{z} h \partial_z \times \nabla \times \bar{E}_\ell \right] (\bar{r}_\perp, 0)
\end{aligned} \tag{21}$$

On substituting Eq.(16) into Eq.(21) we have

$$\begin{aligned}
\bar{n} \times \left[\nabla \times \bar{E}_\ell \right] (\bar{r}_\perp, h) &= (\hat{z} \times \nabla h) \left[\hat{z} \cdot \nabla \times \bar{E}_\ell^{(0)} \right] (\bar{r}_\perp, 0) \\
&+ \left[\bar{I} + \hat{z} \nabla h \right] \cdot \left[\hat{z} \times \nabla \times \bar{E}_\ell^{(0)} \right] (\bar{r}_\perp, 0) \\
&+ \delta \hat{z} \times \left[\nabla \times \bar{E}_\ell^{(1)} \right] (\bar{r}_\perp, 0) \\
&+ \left[\hat{z} h \partial_z \times \nabla \times \bar{E}_\ell^{(0)} \right] (\bar{r}_\perp, 0)
\end{aligned} \tag{22}$$

Equations (22) and (6b) yield the following relations.

Zeroth-order relation:

$$\hat{z} \times \left[\nabla \times \bar{E}_0^{(0)} \right] (\bar{r}_\perp, 0) = \hat{z} \times \left[\nabla \times \bar{E}_1^{(0)} \right] (\bar{r}_\perp, 0) \tag{23}$$

first-order relation:

$$\hat{z} \times \left[\nabla \times \bar{E}_0^{(1)} \right] (\bar{r}_\perp, 0) = \hat{z} \times \left[\nabla \times \bar{E}_1^{(1)} \right] (\bar{r}_\perp, 0) + \bar{J}(\bar{r}_\perp) \tag{24}$$

where

$$\bar{J}(\bar{r}_1) = \left[(\hat{z} \times \nabla h)(\hat{z} \cdot \nabla \times \bar{I}) + h \hat{z} \cdot \partial_z \times \nabla \times \bar{I} \right] \cdot \left[\bar{E}_1^{(0)} - \bar{E}_0^{(0)} \right](\bar{r}_1, 0) \quad (25)$$

The zeroth-order relations of Eqs.(18) and (25) are indeed a replica of Eq.(9). Moreover we do have an explicit zeroth-order solution by means of Eq.(12). So our task now is to find the first-order solution. In this context we note that the first-order electric fields, $\bar{E}_0^{(1)}(\bar{r})$ and $\bar{E}_1^{(1)}(\bar{r})$ satisfy the wave equations

$$\nabla \times \nabla \times \bar{E}_0^{(1)}(\bar{r}) - k_0^2 \bar{E}_0^{(1)}(\bar{r}) = 0, \quad z > 0 \quad (26a)$$

$$\nabla \times \nabla \times \bar{E}_1^{(1)}(\bar{r}) - k_{1m}^2 \bar{E}_1^{(1)}(\bar{r}) = 0, \quad -d < z < h \quad (26b)$$

The above equations, in conjunction with Eqs. (19) and (24), lead to the following solution for $\bar{E}_0^{(1)}(\bar{r})$.

$$\bar{E}_0^{(1)}(\bar{r}) = \int_{-\infty}^{\infty} d^2 \bar{r}'_1 \left\{ \bar{G}_{01}^{(0)}(\bar{r}, \bar{r}'_1) \cdot \bar{J}(\bar{r}'_1) + \left[\nabla' \times \bar{G}_{01}^{(0)}(\bar{r}, \bar{r}'_1) \right] \cdot \bar{\mathcal{J}}(\bar{r}'_1) \right\} \quad (27)$$

Thus our first-order solution in Region 0 is expressed as

$$\bar{E}_0(\bar{r}) \approx \bar{E}_0^{(0)}(\bar{r}) + \bar{E}_0^{(1)}(\bar{r}) \quad (28)$$

where $\bar{E}_0^{(0)}(\bar{r})$ and $\bar{E}_0^{(1)}(\bar{r})$ are given by Eqs. (11) and (27). The scattered field

$\bar{E}_0^s(\bar{r}) = \bar{E}_0(\bar{r}) - \bar{E}_0^{(00)}(\bar{r})$ is then

$$\begin{aligned} \bar{E}_0^s(\bar{r}) = & \int_{-d}^0 dz_1 \int_{-\infty}^{\infty} d^2\bar{r}_{1\perp} \bar{G}_{01}^{(00)}(\bar{r}, \bar{r}_1) q(\bar{r}_1) \cdot \bar{E}_1^{(00)}(\bar{r}_1) \\ & + \int_{-\infty}^{\infty} d^2\bar{r}'_1 \left\{ \bar{G}_{01}^{(0)}(\bar{r}, \bar{r}'_1) \cdot \bar{J}(\bar{r}'_1) + \left[\nabla' \times \bar{G}_{01}^{(0)}(\bar{r}, \bar{r}'_1) \right] \cdot \bar{J}(\bar{r}'_1) \right\} \end{aligned} \quad (29)$$

but we know that (see Eq. (12))

$$\bar{G}_{01}^{(0)}(\bar{r}, \bar{r}') = \bar{G}_{01}^{(00)}(\bar{r}, \bar{r}') + \int_{-d}^0 dz_1 \int_{-\infty}^{\infty} d^2\bar{r}_{1\perp} \bar{G}_{01}^{(00)}(\bar{r}, \bar{r}_1) q(\bar{r}_1) \cdot \bar{G}_{11}^{(00)}(\bar{r}_1, \bar{r}') \quad (30)$$

Further, we are interested in the far-zone solution for the scattered field $\bar{E}_0^s(\bar{r}) = \lim_{r \rightarrow \infty} \bar{E}_0^s(\bar{r})$. Therefore we need to use the asymptotic

form of the dyadic Green's function $\bar{G}_{01}^{(00)}(\bar{r}, \bar{r}') = \lim_{r \rightarrow \infty} \bar{G}_{01}^{(00)}(\bar{r}, \bar{r}')$.

Noting these comments we obtain from Eqs. (29), (30), (20), (25), (10)

and (11) the following first-order solution for $\bar{E}_0^s(\bar{r})$.

$$\begin{aligned} \bar{E}_0^s(\bar{r}) = & \int_{-d}^0 dz' \int_{-\infty}^{\infty} d^2\bar{r}'_1 \bar{G}_{01}^{(00)}(\bar{r}, \bar{r}') q(\bar{r}') \cdot \bar{E}_1^{(00)}(\bar{r}') \\ & + \int_{-\infty}^{\infty} d^2\bar{r}'_1 \left\{ \bar{G}_{01}^{(00)}(\bar{r}, \bar{r}') \cdot \dot{\bar{J}}(\bar{r}') + \left[\nabla' \times \bar{G}_{01}^{(00)}(\bar{r}, \bar{r}') \right] \cdot \dot{\bar{J}}(\bar{r}') \right\} \end{aligned} \quad (31)$$

where

$$\dot{\bar{J}}(\bar{r}) = \left[(\hat{z} \times \nabla h) \hat{z} + h \hat{z} \partial_z \times \bar{I} \right] \cdot \left[\bar{E}_1^{(00)} - \bar{E}_0^{(00)} \right](\bar{r}_1, 0) \quad (32)$$

$$\dot{\bar{J}}(\bar{r}_1) = \left[(\hat{z} \times \nabla h)(\hat{z} \cdot \nabla \times \bar{I}) + h \hat{z} \partial_z \times \nabla \times \bar{I} \right] \cdot \left[\bar{E}_1^{(00)} - \bar{E}_0^{(00)} \right](\bar{r}_1, 0) \quad (33)$$

Explicit expressions for $\bar{G}_{01}^{(00)}(\bar{r}, \bar{r}')$ and $\bar{E}_1^{(00)}(\bar{r}_1)$ are given in Appendixes A and B.

5. SCATTERING COEFFICIENTS

Suppose a plane wave $\bar{E}_1(\bar{r})$, given as

$$\bar{E}^i(\bar{r}) = \hat{\alpha}_{0i}^- E^i \exp \left[i \bar{\kappa}_{0i} \cdot \bar{r} \right]. \quad (34)$$

is incident on the random medium layer. The bistatic scattering coefficients $\gamma_{\alpha\beta}$ can then be readily calculated using the following definition [Peake, 1959]⁶

$$\gamma_{\alpha\beta} = \lim_{\substack{r \rightarrow \infty \\ a \rightarrow \infty}} \frac{4\pi r^2 \langle |\bar{E}_0^s(r)|^2 \rangle_\beta}{a \cos \theta_{0i} |E^i|^2} \quad (35)$$

where the subscripts α and β stand for the polarization of the incident wave and scattered wave respectively; a denotes the illuminated area; θ_{0i} is the angle of incidence. On using Eq.(31) in Eq.(35) $\gamma_{\alpha\beta}$ may be evaluated. Omitting the rather tedious intermediate steps the final result is presented in the following compact form.

$$\gamma_{\alpha\beta} = \frac{1}{4\pi \cos \theta_{0i}} \left[\Gamma_{\alpha\beta}^{(v)} + \Gamma_{\alpha\beta}^{(b)} + \Gamma_{\alpha\beta}^{(c)} \right] ; \quad \{ \alpha, \beta \} = \{ h, v \} \quad (36)$$

where

$$\Gamma_{\alpha\beta}^{(v)} = \sum_{j=1}^5 \Delta_j \quad (37)$$

$$\Delta_1 = \frac{|A_{\alpha i} A_{\beta s}|^2}{2(k_{1mzs}'' + k_{1mzi}'')} \left\{ \exp \left[-2(k_{1mzs}'' + k_{1mzi}'')d \right] - 1 \right\} \\ \cdot (\hat{\beta}_{1s}^+ \cdot \hat{\alpha}_{1i}^-)^2 \Phi_v(\bar{k}_{1i} - \bar{k}_{1s}, -k_{1mzs}' - k_{1mzi}') \quad (38a)$$

$$\Delta_2 = \frac{|B_{\alpha i} A_{\beta s}|^2}{2|k_{1mzi}'' - k_{1mzs}''|} \left\{ \exp \left[2(-k_{1mzs}'' + k_{1mzi}'')d \right] - 1 \right\} \\ \cdot (\hat{\beta}_{1s}^+ \cdot \hat{\alpha}_{1i}^-)^2 \Phi_v(\bar{k}_{1i} - \bar{k}_{1s}, -k_{1mzs}' + k_{1mzi}') \quad (38b)$$

$$\Delta_3 = \frac{|A_{\alpha i} B_{\beta s}|^2}{2|k_{1mzs}'' - k_{1mzi}''|} \left\{ \exp \left[2(k_{1mzs}'' - k_{1mzi}'')d \right] - 1 \right\}$$

$$(\hat{\beta}_{1s}^- \cdot \hat{\alpha}_{1i}^-)^2 \Phi_v(\bar{k}_{1i} - \bar{k}_{1s}, k_{1mzs}' - k_{1mzi}') \quad (38c)$$

$$\Delta_4 = \frac{|B_{\alpha i} B_{\beta s}|^2}{2(k_{1mzs}'' + k_{1mzi}'')} \left\{ \exp \left[2(k_{1mzs}'' + k_{1mzi}'')d \right] - 1 \right\}$$

$$(\hat{\beta}_{1s}^- \cdot \hat{\alpha}_{1i}^+)^2 \Phi_v(\bar{k}_{1i} - \bar{k}_{1s}, k_{1mzs}' + k_{1mzi}') \quad (38d)$$

$$\Delta_5 = \Delta \delta_{\alpha\beta} 2d |A_{\alpha i} B_{\beta i}|^2 \Phi_v(2\bar{k}_{1i}, 0) \quad (38e)$$

$$\Gamma_{\alpha\beta}^{(b)} = \omega^2 \mu^2 \Phi_b(\bar{k}_{1i} - \bar{k}_{1s})$$

$$\left| \hat{z} \cdot \bar{U}_s^\beta \times \bar{I} \cdot \left[(P_z^\alpha - Q_z^\alpha) (\bar{k}_{1i} - \bar{k}_{1s}) + i(\bar{P}'^\alpha - \bar{Q}'^\alpha) \right] \right. \\ \left. + \hat{z} \cdot \bar{V}_s^\beta \times \bar{I} \cdot \left[(M_z^\alpha - N_z^\alpha) (\bar{k}_{1i} - \bar{k}_{1s}) + i(\bar{M}'^\alpha - \bar{N}'^\alpha) \right] \right|^2$$

$$(39)$$

and

$$\Gamma_{\alpha\beta}^{(c)} \geq -2\omega\mu \operatorname{Re} \left[\left\{ \begin{aligned} & B_{\alpha i} A_{\beta s} (\hat{\beta}_{1s}^+ \cdot \hat{\alpha}_{1i}^+)^2 \Phi_c(\bar{k}_{1i} - \bar{k}_{1s}, -k'_{1mzs} + k'_{1mzi}) \\ & A_{\alpha i} B_{\beta s} (\hat{\beta}_{1s}^- \cdot \hat{\alpha}_{1i}^-)^2 \Phi_c(\bar{k}_{1i} - \bar{k}_{1s}, k'_{1mzs} - k'_{1mzi}) \\ & + A_{\alpha i} A_{\beta s} (\hat{\beta}_{1s}^+ \cdot \hat{\alpha}_{1i}^-)^2 \Phi_c(\bar{k}_{1i} - \bar{k}_{1s}, -k'_{1mzs} - k'_{1mzi}) \end{aligned} \right\} \right]$$

$$\cdot \left\{ \hat{z} \cdot \bar{U}_s^\beta \times \bar{I} \cdot \left[(P_z^\alpha - Q_z^\alpha) (\bar{k}_{1i} - \bar{k}_{1s}) + i (\bar{P}'^\alpha - \bar{Q}'^\alpha) \right] \right.$$

$$\left. + \hat{z} \cdot \bar{V}_s^\beta \times \bar{I} \cdot \left[(M_z^\alpha - N_z^\alpha) (\bar{k}_{1i} - \bar{k}_{1s}) + i (\bar{M}'^\alpha - \bar{N}'^\alpha) \right] \right\}^*$$

$$; k''_{1mzs} \geq k''_{1mzi} \quad (40)$$

where

$$\bar{U}_s^p = A_{ps} \hat{p}_{1s}^+ + B_{ps} \hat{p}_{1s}^- \quad ; p = h, v \quad (41)$$

$$\bar{V}_s^h = -A_{hs} \hat{v}_{1s}^+ - B_{ps} \hat{v}_{1s}^- \quad (42a)$$

$$\bar{V}_s^v = A_{hs} \hat{h}_{1s}^+ + B_{vs} \hat{h}_{1s}^- \quad (42b)$$

$$\bar{P}^h = - \left[B_{hi} \hat{v}_{1i}^+ + A_{hi} \hat{v}_{1i}^- \right] / \eta_1 \quad (43a)$$

$$\bar{Q}^h = - \left[R_{hi} \hat{v}_{0i}^+ + \hat{v}_{0i}^- \right] / \eta_0 \quad (43b)$$

$$\bar{P}'^h = - ik_{lmzi} \left[B_{hi} \hat{v}_{1i}^+ - A_{hi} \hat{v}_{1i}^- \right] / \eta_1 \quad (44a)$$

$$\bar{Q}'^h = - ik_{0zi} \left[R_{hi} \hat{v}_{0i}^+ - \hat{v}_{0i}^- \right] / \eta_0 \quad (44b)$$

$$\bar{P}'^v = - \bar{P}'^h \left[\text{Replacements} \left\{ \begin{array}{l} h \rightarrow v \\ v \rightarrow h \end{array} \right\} \right] \quad (45a)$$

$$\bar{Q}'^v = - \bar{Q}'^h \left[\text{Replacements} \left\{ \begin{array}{l} h \rightarrow v \\ v \rightarrow h \end{array} \right\} \right] \quad (45b)$$

$$\bar{P}'^{v,h} = - \bar{P}'^h \left[\text{Replacements} \left\{ \begin{array}{l} h \rightarrow v \\ v \rightarrow h \end{array} \right\} \right] \quad (46a)$$

$$\bar{Q}'^{v,h} = - \bar{Q}'^h \left[\text{Replacements} \left\{ \begin{array}{l} h \rightarrow v \\ v \rightarrow h \end{array} \right\} \right] \quad (46b)$$

$$\bar{M}^p = B_{pi} \hat{p}_{1i}^+ + A_{pi} \hat{p}_{1i}^- ; p = h, v \quad (47a)$$

$$\bar{N}^p = R_{pi} \hat{p}_{0i}^+ + \hat{p}_{0i}^- ; p = h, v \quad (47b)$$

$$\bar{M}'^p = ik_{lmzi} \left[B_{pi} \hat{p}_{1i}^+ - A_{pi} \hat{p}_{1i}^- \right] ; p = h, v \quad (48a)$$

$$\bar{N}'^p = ik_{0zi} \left[R_{hi} \hat{p}_{0i}^+ - \hat{p}_{0i}^- \right] ; p = h, v \quad (48b)$$

$$R_P = \frac{R_{01}^P + R_{12}^P \exp[i 2 k_{1mz} d]}{1 + R_{01}^P R_{12}^P \exp[i 2 k_{1mz} d]} \quad (49)$$

$$\eta_0 = \left[\mu / \epsilon_0 \right]^{.5} \quad (50a)$$

$$\eta_1 = \left[\mu / \epsilon_{1m} \right]^{.5} \quad (50b)$$

$$\Delta = \begin{cases} 1 & \text{if } \bar{k}_{1s} = -\bar{k}_{1i} \\ 0 & \text{otherwise} \end{cases} \quad (51)$$

$$\delta_{\alpha\beta} = \begin{cases} 1 & \text{if } \alpha = \beta \\ 0 & \text{otherwise} \end{cases} \quad (52)$$

The other quantities are defined in Appendix A. We have denoted the spectral densities of the correlation functions C_t , $t = v, b, c$ by Φ_t . The superscripts ' and " over the various propagation constants indicate their real and imaginary parts respectively. The subscript i indicates that the angle associated with that quantity is the angle of incidence. Similarly the subscript s corresponds to the angle of scattering. $\hat{h}_1$ and $\hat{v}_1$ are the unit polarization vectors of the field in Region 1 (unperturbed situation) and they stand for horizontal and vertical polarizations respectively. These vectors are defined in Appendix A.

6. DISCUSSION

The representation in which I have presented the scattering coefficients is very suitable for physical interpretation. There are three terms $\Gamma_{\alpha\beta}^{(v)}$, $\Gamma_{\alpha\beta}^{(b)}$ and $\Gamma_{\alpha\beta}^{(c)}$ which constitute the scattering coefficient $\gamma_{\alpha\beta}$. They represent volume scattering, boundary scattering and cross scattering (between volume and boundary inhomogeneities). The five terms that make up $\Gamma_{\alpha\beta}^{(v)}$ are schematically described in Fig.2 by corresponding scattering diagrams. We have represented the wave path by a solid line and corresponding complex conjugate by a dotted line.

Although it is just a particular case of our general result, backscattering deserves special attention because of its importance in several practical applications. To obtain expressions for backscattering coefficients $\sigma_{\alpha\beta}^0$ we let $\bar{k}_{1s} = -\bar{k}_{1i}$ in Eqs.(36)-(40). This result is:

$$\sigma_{\alpha\beta}^0 = \frac{1}{4\pi} \left[\sigma_{\alpha\beta}^{(v)} + \sigma_{\alpha\beta}^{(b)} + \sigma_{\alpha\beta}^{(c)} \right] ; \{ \alpha, \beta \} = \{ h, v \} \quad (53)$$

where

$$\sigma_{\alpha\beta}^{(v)} = \sum_{j=-1}^3 \Delta_j \delta_{\alpha\beta} \quad (53)$$

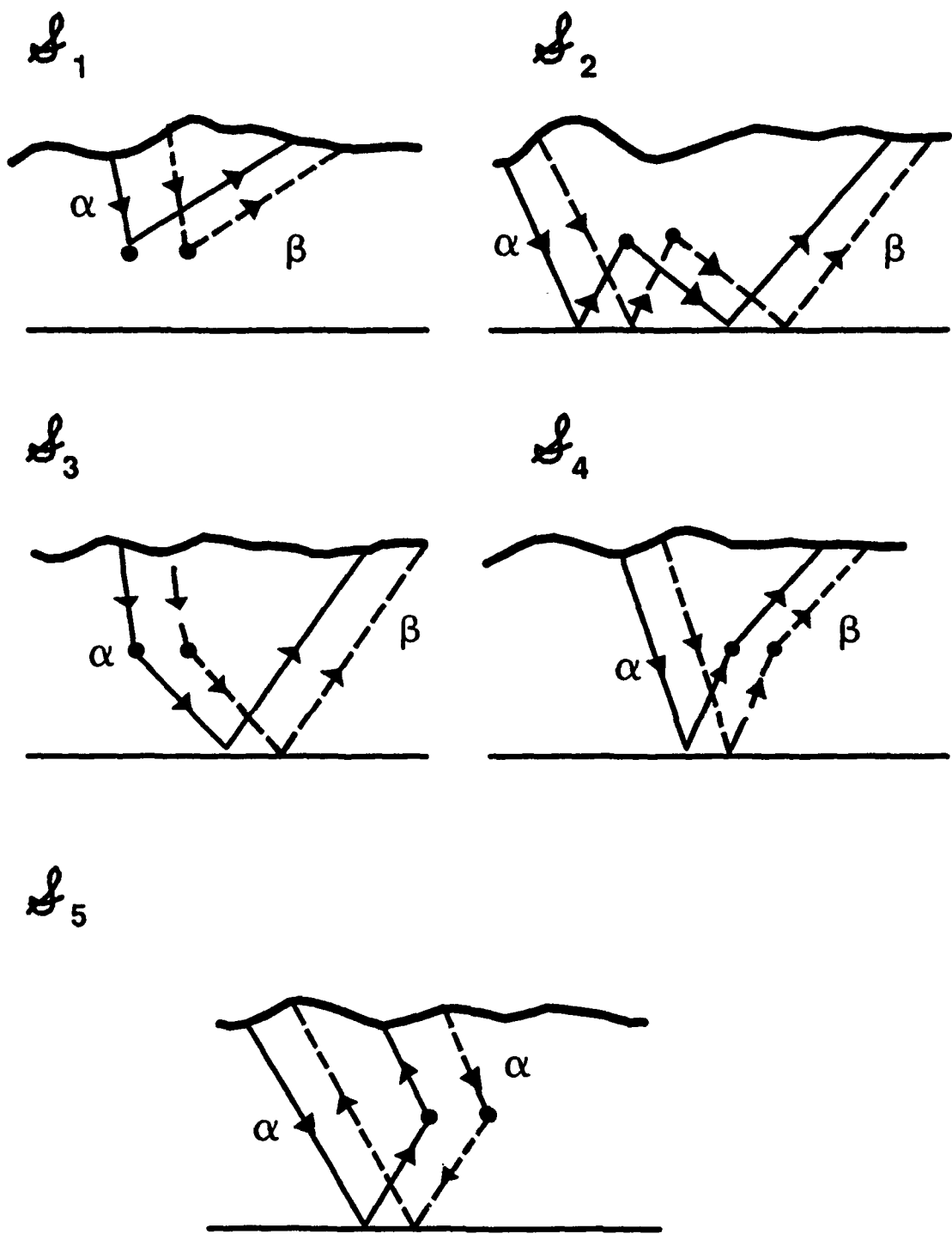


Figure 2 VOLUME SCATTERING

$$\Delta_1 = \frac{|A_{\alpha i} A_{\beta i}|^2}{4k_{lmzi}''} \left\{ \exp \left[-4k_{lmzi}'' d \right] - 1 \right\} \Phi_v(2\bar{k}_{li}, -2k_{lmzi}') \quad (55a)$$

$$\Delta_2 = 4 |A_{\alpha i} B_{\beta i}|^2 d \Phi_v(2\bar{k}_{li}, 0) \quad (55b)$$

$$\Delta_3 = \frac{|B_{\alpha i} B_{\beta i}|^2}{4k_{lmzi}''} \left\{ \exp \left[4k_{lmzi}'' d \right] - 1 \right\} \Phi_v(2\bar{k}_{li}, 2k_{lmzi}') \quad (55c)$$

$$\sigma_{\alpha\beta}^{(b)} = \omega^2 \mu^2 \Phi_b(2\bar{k}_{li}) \delta_{\alpha\beta}$$

$$\left| \begin{aligned} & \hat{z} \cdot \bar{U}_i^\beta \times \bar{I} \cdot \left[2 (P_z^\alpha - Q_z^\alpha) \bar{k}_{li} + i (\bar{P}'^\alpha - \bar{Q}'^\alpha) \right] \\ & + \hat{z} \cdot \bar{V}_i^\beta \times \bar{I} \cdot \left[2 (M_z^\alpha - N_z^\alpha) \bar{k}_{li} + i (\bar{M}'^\alpha - \bar{N}'^\alpha) \right] \end{aligned} \right|^2 \quad (56)$$

$$\begin{aligned} \sigma_{\alpha\beta}^{(c)} = 2\omega\mu \delta_{\alpha\beta} \text{Re} \left[\right. & \left\{ A_{\alpha i} B_{\beta i} \chi_1 \Phi_c(2\bar{k}_{li}, 0) \right. \\ & \left. + A_{\alpha i} A_{\beta i} \chi_2 \Phi_c(2\bar{k}_{li}, -2k_{lmzi}') \right\} \\ & \left. \left\{ \hat{z} \cdot \bar{U}_i^\beta \times \bar{I} \cdot \left[2 (P_z^\alpha - Q_z^\alpha) \bar{k}_{li} + i (\bar{P}'^\alpha - \bar{Q}'^\alpha) \right] \right. \right. \\ & \left. \left. + \hat{z} \cdot \bar{V}_i^\beta \times \bar{I} \cdot \left[2 (M_z^\alpha - N_z^\alpha) \bar{k}_{li} + i (\bar{M}'^\alpha - \bar{N}'^\alpha) \right] \right\}^* \right] \quad (57) \end{aligned}$$

and where

$$x_1 = \begin{cases} -1 & \text{if } \alpha = h \\ -\cos 2\theta_{1i} & \text{if } \alpha = v \end{cases} \quad (58a)$$

$$x_2 = \begin{cases} -1 & \text{if } \alpha = h \\ +1 & \text{if } \alpha = v \end{cases} \quad (58b)$$

The most striking feature of the above result is the disappearance of depolarization in the case of backscattering. This is not too surprising when we consider the fact that our result is a first-order perturbation solution to a scattering problem which has isotropic characteristics. The other feature is that of enhancement. As shown in Fig. 2 there are certain scattering processes that constructively interfere and hence contribute only in the backscattering direction.

Next we try to relate our results to those of others. We mentioned earlier that $\Gamma_{\alpha\beta}^{(v)}$ is the contribution due to volume scattering. Indeed it is easy to verify that our $(4\pi \cos\theta_{0i})^{-1} \Gamma_{\alpha\beta}^{(v)}$ is equivalent to that of Zuniga and Kong [1980]⁷. However, physical processes are more clearly identifiable in ours. We next consider the boundary scattering term, $\Gamma_{\alpha\beta}^{(b)}$. Barrick and Peake [1967]⁸, have used the Rayleigh-Rice method to calculate the bistatic scattering coefficients of a slightly rough surface. To compare our results with theirs we need to let $d \rightarrow \infty$ in $(4\pi \cos\theta_{0i})^{-1} \Gamma_{\alpha\beta}^{(b)}$. On doing this and after some algebraic simplifications we notice that our results are in agreement with theirs.

It is pleasing to observe the structure of the results for $\gamma_{\alpha\beta}$ and $\sigma_{\alpha\beta}^0$. Indeed several conclusions may be drawn on closer examination of those expressions. Such a task will be undertaken in a future report which will also present numerical examples.

We have thus solved the problem of scattering from a random medium layer with a random interface using a first-order approximation. Our solution is therefore a single scattering solution. Quite obviously our result is meaningful only when the random inhomogeneities are small. If not, the phenomenon of multiple scattering will play a dominant role and it must be properly taken into consideration. We notice that in our first-order perturbation method the various scattering contributions, namely, volume, boundary, and volume-boundary, are merely additive. This is certainly not so in multiple scattering where various interactions take place and the physical picture is fairly complex [Mudaliar and Lee, 1991]⁹. Thus there are limitations to the situations where these results can be applied. Nevertheless this work has given a frame-work to analyze the more general case of multiple scattering.

7. CONCLUSION

We have considered the problem of microwave scattering from a random medium layer with a random interface. Assuming that the random fluctuations are small, a simple perturbation solution for the scattered field is obtained. Using this, the bistatic scattering coefficients are calculated and expressed in a compact meaningful form. With the help of schematic scattering diagrams the various terms

that constitute the scattering coefficients are explained. The special case of backscattering is considered in some detail. Since this result is essentially a single scattering approximation, one natural extension of this work will be to study the case of multiple scattering. This is left for future work.

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Appendix A

Asymptotic Dyadic Green's Function

$$\bar{G}_0^{(00)}(\bar{r}, \bar{r}') = \frac{e^{ik_0 r}}{4\pi r} e^{-i\bar{k}_1 \cdot \bar{r}'} \bar{g}(\bar{k}_{1m}, z') \quad (A1)$$

where

$$\begin{aligned} \bar{g}(\bar{k}_{1m}, z') = & \hat{h}_0^+ \hat{h}_1^+ A_h e^{-ik_{1m} z'} + \hat{h}_0^+ \hat{h}_1^- B_h e^{ik_{1m} z'} \\ & + \hat{v}_0^+ \hat{v}_1^+ A_v e^{-ik_{1m} z'} + \hat{v}_0^+ \hat{v}_1^- B_v e^{ik_{1m} z'} \end{aligned} \quad (A2)$$

$$A_p = \frac{X_{01}^p}{D_p} \quad ; \quad p = h, v \quad (A3)$$

$$B_p = A_p R_{12}^p e^{i2k_{1m} d} \quad ; \quad p = h, v \quad (A4)$$

$$D_p = 1 + R_{01}^p R_{12}^p e^{i2k_{1m} d} \quad ; \quad p = h, v \quad (A5)$$

$$R_{ij}^h = \frac{k_{iz} - k_{jz}}{k_{iz} + k_{jz}} \quad ; \quad \{i, j\} = \{0, 1, 2\} \quad (A6a)$$

$$R_{ij}^v = \frac{\epsilon_j k_{iz} - \epsilon_i k_{jz}}{\epsilon_j k_{iz} + \epsilon_i k_{jz}} ; \quad \{ i, j \} = \{ 0, 1, 2 \} \quad (\text{A6b})$$

$$X_{ij}^h = \frac{2k_{iz}}{k_{iz} + k_{jz}} ; \quad \{ i, j \} = \{ 0, 1, 2 \} \quad (\text{A7a})$$

$$X_{ij}^v = \frac{k_i}{k_j} \frac{2\epsilon_j k_{iz}}{\epsilon_j k_{iz} + \epsilon_i k_{jz}} ; \quad \{ i, j \} = \{ 0, 1, 2 \} \quad (\text{A7b})$$

$$\hat{h}_\ell^+ = \hat{h}_\ell^- = \frac{1}{k_\perp} \left(\hat{x}k_y - \hat{y}k_x \right) ; \quad \ell = 1, 2, 3 \quad (\text{A8})$$

$$\hat{v}_\ell^+ = \hat{h}_\ell^+ \times \hat{k}_\ell ; \quad \ell = 1, 2, 3 \quad (\text{A9a})$$

$$\hat{v}_\ell^- = \hat{h}_\ell^- \times \hat{k}_\ell ; \quad \ell = 1, 2, 3 \quad (\text{A9b})$$

$$\bar{k}_\ell = \hat{x}k_x + \hat{y}k_y + \hat{z}k_{\ell z} \quad (\text{A10a})$$

$$\bar{\kappa}_\ell = \hat{x}k_x + \hat{y}k_y - \hat{z}k_{\ell z} \quad (\text{A10b})$$

$$k_{\ell z} = \left(k_\ell^2 - k_\perp^2 \right)^{.5} \quad (\text{A11})$$

$$k_\ell^2 = \omega^2 \mu \epsilon_\ell \quad (\text{A12})$$

In the above $k_\perp$ stands for k_{1m} . This convention is adopted here for notational conciseness.

Appendix B

Unperturbed Electric Fields in Region 1

TE case

$$E_1^{(oo)}(r) = E_{oi} \left[B_{hi} \hat{h}_{li}^+ e^{i\bar{k}_{li} \cdot \bar{r}} + A_{hi} \hat{h}_{li}^- e^{i\bar{\kappa}_{li} \cdot \bar{r}} \right] \quad (B1)$$

TM case

$$E_1^{(oo)}(r) = E_{oi} \left[B_{vi} \hat{v}_{li}^+ e^{i\bar{k}_{li} \cdot \bar{r}} + A_{vi} \hat{v}_{li}^- e^{i\bar{\kappa}_{li} \cdot \bar{r}} \right] \quad (B2)$$

All quantities in the above equations are defined in Appendix A.

Nomenclature

h : zero-mean random function describing surface height

ϵ_j : permittivity of the medium in Region j

ϵ_{1m} : mean part of ϵ_1

ϵ_{1f} : fluctuating part of ϵ_1

C_t : correlation function; $t = \{v, b, c\}$

σ_t^2 : variance associated with $t = \{v, b, c\}$

Φ_t : spectral density

$\bar{r}_1$: projection of $\bar{r}$ on the xy plane

$\bar{E}_j$: electric field in Region j

$\bar{E}_j^{(0)}$: electric field in Region j when the boundary is unperturbed

$\bar{E}_j^{(00)}$: electric field in Region j when both
boundary and medium are unperturbed

$\bar{\bar{G}}_{01}$: dyadic Green's function for source in Region 1
and observation point in Region 0

$\overline{\overline{G}}_{01}^{(0)}$, $\overline{\overline{G}}_{01}^{(00)}$: dyadic Green's functions; superscript conventions
 same as in E_j

$\overline{E}_0^s$: incoherent scattered field in Region 0

$\overline{E}_0^i$: incident field in Region 0

θ_{0i} : angle of incidence

$\gamma_{\alpha\beta}$: bistatic scattering coefficient;

α :- polarization of the incident field

β :- polarization of the scattered field

$\sigma_{\alpha\beta}^0$: back scattering coefficient

$\overline{k}_j$: propagation vector in Region j

k_{1m} : mean part of k

k_{1mz} : z-component of the $\overline{k}_{1m}$

k_{1mzs} : k_{1mz} evaluated in the scattered (observation) direction

k_{1mzi} : k_{1mz} evaluated in the direction of the incident wave

$\hat{h}_0$: unit vector for horizontal polarization in Region 0

$\hat{v}_0$: unit vector for vertical polarization in Region 1

$\hat{\alpha}^+$: unit polarization vector ($\alpha = h$ or v) for upward travelling wave

$\hat{\alpha}^-$: unit polarization vector for downward travelling wave