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A DATA ANALYSIS OF SUCCESS IN OCS, THE USE OF ASVAB WAIVERS, AND RACE

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A DATA ANALYSIS OF SUCCESS IN OCS, THE USE OF ASVAB WAIVERS, AND RACE

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L. R. Whitaker

Abstract

Success in Officers Candidate School (OCS) occurs at the same rate regardless of whether the candidates received a mental aptitude qualification waiver based upon their score on the electronics portion of the Armed Services Vocational Aptitude Battery (ASVAB). However, these rates do change with race and time; and the result is an apparent contradiction because the macro rates (those rates computed overall without discriminating race and time) exhibit different success rates depending upon the presence of a waiver or not. The data are studied to expose the contradiction and develop sharper models.

I. Introduction

The accession of officers into the Marine Corps includes using one of three mental aptitude test scores: Armed Services Vocational Aptitude Battery Electronics Repair Composite (called ASVAB herein), the Scholastic Aptitude Test (SAT), and the American College Test (ACT). Historically, 55% of the officers entering the Corps use the first of these three, and the qualification threshold is a score of 120. But a candidate can receive a waiver of this minimum provided his score is 115 or better. The paper treats only those using the ASVAB test.

Based on data collected over the fiscal years 1988 through 1992 and broken out by race, personnel at the Manpower Analysis (MA) Branch at Marine Corps Headquarters noticed that success at the Officer Candidate School (OCS) appears to be independent of whether an officer has received an ASVAB waiver. Specifically, there are four racial groups, Caucasian, Black, Hispanic, and Other. The Other group consists of American Indian, Alaskan Native, Asian, and Pacific Islander in the large. When collapsed over time, the four 2 x 2 contingency table tests for independence yield the chi square test statistics .6678, 2.841, .7983, .5767 for the respective races, each with one degree of freedom. None of these are significant. However, when the data are further collapsed over race and a single test for independence is performed, then the relationship is highly significant. This latter 2 x 2 table appears in Table 1. The chi square statistic is 11.87 and the p-value is 0.00057.

On the surface, it appears that we have contradictory results. On the one hand, OCS candidate success and the presence of a waiver are independent when Caucasians, Blacks, Hispanics and Others are considered separately. On the other hand, there is dependence in the collapsed table when race is not accounted for, with strong evidence that the chance of success without a waiver is greater than that with a waiver.

Table 1. Macro Analysis of Success and Waiver

	Waiver	No Waiver	Total
Success	754	7449	8203
Failure	299	2303	2602
Total	1053	9752	10805

A short answer to the contradiction can be obtained through an interpretation of the two success rates. They are not significantly different for waiver and non-waiver within racial groups. But the rates change sharply from group to group. Indeed, the use of the waiver varies markedly from group to group and, to a lesser extent, from year to year. This is surely related to the implementation of the Marine Corps Affirmative Action Plan.

This paper contains an explanation of the contradiction and attention is drawn to other interesting facets as well. In Section II the raw data are presented and all 2 x 2 tables of success/failure by waiver/non-waiver are studied for each year/racial group pair. Generally, independence is tenable. To explain the non-independence, the full data, aggregated over years and with race as a factor, are then subjected to a log-linear analysis in Section III. In Section IV, we fit models with time as a factor including the use of the waiver by year and race. These models could be valuable because an ill-advised long-term overuse of the waiver could lead to inequities in the future advancement to higher rank [3].

Categorical data is prevalent in military OR. Thus, we take a careful look at the data and provide details that would normally be omitted so that certain usage may be illustrated. In particular, in the next section, attention is drawn to the rather interesting effects when conditional tests are used, and in Section III the steps for fitting a loglinear model are presented.

The factors of interest are success or failure of OCS candidates to qualify for the OCS program, whether the candidate used an ASVAB (lower mental category) waiver, fiscal year, and race. The data (see Table 2) consists of counts

$$D_{ijkl}$$

where $i = 1, 2$ indicates success or failure, $j = 1, 2$ indicates presence or absence of waivers, $k = 1, \ldots, 5$ indicates the fiscal year FY88 to FY92 and $l = 1, \ldots, 4$ indicates race, in the order given earlier.

Table 2. Frequency Counts by Category

Candidates Qualifying with ASVAB Waiver

	FY	White	Black	Hispanic	Other	Total
Success in OCS	FY88	100	11	10	12	133
	FY89	142	37	12	20	211
	FY90	102	30	20	11	163
	FY91	77	22	14	2	115
	FY92	70	36	22	4	132
	Total	491	136	78	49	754

	FY	White	Black	Hispanic	Other	Total
Failure in OCS	FY88	22	8	5	1	36
	FY89	30	15	11	7	63
	FY90	35	16	10	3	64
	FY91	21	22	6	3	52
	FY92	45	31	8	0	84
	Total	153	92	40	14	299

Candidates Qualifying without ASVAB Waiver

	FY	White	Black	Hispanic	Other	Total
Success in OCS	FY88	1113	48	48	95	1304
	FY89	1533	56	80	111	1780
	FY90	1263	77	76	109	1525
	FY91	1013	58	78	39	1188
	FY92	1390	87	108	67	1652
	Total	6312	326	390	421	7449

	FY	White	Black	Hispanic	Other	Total
Failure in OCS	FY88	234	14	16	31	295
	FY89	323	18	22	35	398
	FY90	350	50	41	38	479
	FY91	430	35	38	24	527
	FY92	481	50	48	25	604
	Total	1818	167	165	153	2303

II. Individual Contingency Tables

Suppose the full data are broken into twenty (5 years, 4 races) 2 x 2 contingency tables and subjected to individual analyses. It is instructive to apply the most often used procedures to each and gain experience in their use and effect.

Let us simplify the notation and let $n_{ij} = D_{ijkl}$ be the counts with year and race held fixed, $i = 1, 2$ indicates success or failure in OCS, and $j = 1, 2$ indicates presence or absence of waiver, respectively. Under independence the expected frequencies are estimated by

$$\hat{m}_{ij} = n_{i+}n_{+j} / N \qquad \text{with } N = \sum\sum n_{ij},$$

and the plus indicates summation over the replaced subscript. The familiar Pearson Chi Square and Log Likelihood statistics are given by

$$\chi^2 = \sum_{i=1}^{2}\sum_{j=1}^{2}\left(n_{ij} - \hat{m}_{ij}\right)^2 / \hat{m}_{ij}$$

$$G^2 = 2\sum_{i=1}^{2}\sum_{j=1}^{2} n_{ij} \ln\left(n_{ij} / \hat{m}_{ij}\right).$$

Each is asymptotically distributed as chi square with one degree of freedom.

The use of the odds ratio is also popular especially in 2 x 2 tables. It summarizes the strength and type of dependence between the two categories. Letting $\{\Pi_{ij}\}$ be the cell probabilities, the odds ratio is defined by

$$\theta = \Pi_{11}\Pi_{22} / \Pi_{12}\Pi_{21}$$

and, in our context, represents the odds of OCS success using waivers divided by the odds of success without the use of waivers. The null value $\theta = 1$ represents "no effect" of waivers, or independence. The maximum likelihood estimator of θ is

$$\hat{\theta} = n_{11}n_{22} / n_{12}n_{21}.$$

The null distribution of $\ln(\hat{\theta})$ is well approximated by the normal distribution [1] with the variance estimated by $\left[\hat{\sigma}(\ln \hat{\theta})\right]^2 = \sum_{1}^{2}\sum_{1}^{2} 1/n_{ij}$.

Thus, a third test statistic is

$$Z = \ln(\hat{\theta}) / \left[\sum\sum 1/n_{ij}\right]^{1/2}.$$

Concern for the use of asymptotics has led the authors to consider Fisher's Exact Test as well, [1, p60ff]. Under the null hypothesis of independence, an exact distribution that is free of any unknown parameters results from conditioning on the totals in both margins. The result is a hypergeometric distribution

$$\binom{n_{+1}}{n_{11}}\binom{n_{+2}}{n_{12}} \Big/ \binom{N}{n_{1+}}.$$

Since the totals in the margins are given, only n_{11} need be considered as variable. Its range is

$$\max(0, n_{+1} + n_{1+} - N) \le n_{11} \le \min(n_{+1}, n_{1+}).$$

Exact two-sided p-values are obtained by summing probabilities of tables that are at least as rare under the null hypothesis as the observed table. Only those tables that have hypergeometric probabilities at least as small as the observed configuration are used [2].

The results of the four procedures are given in Table 3, which contains the values of total populations, N; the odds ratios, $\hat{\theta}$; $\ln(\hat{\theta})$; the standard deviation of $\ln(\hat{\theta})$; and the four p-values. Within cells the racial levels are Caucasian, Black, Hispanic, Other, respectively. There are some blank entries for the last case because $n_{21} = 0$.

Perhaps the first thing to notice is the agreement of p-values for the three asymptotic procedures. Only for the smaller values of N do they show much separation. On the other hand, the p-values for Fisher's Exact Test generally tend to be higher. The main reason for this is the conditioning on both margin totals. Such is not the case in the other procedures. In the former case, the nuisance parameters are eliminated while in the latter three procedures they are estimated.

The differences in p-values do not lead to conflicting conclusions, however. Two cases of the twenty are significant: Hispanics '89 and Caucasians '92. In both of these cases the odds for success are smaller if waivers are used. The opposite is true for Caucasians '91, a case that might be controversial as $p \sim .08$.

Table 3. Two-Sided p-values

		N	$\hat{\theta}$	$\ln\hat{\theta}$	$\hat{\sigma}(\ln\hat{\theta})$	Fisher	Z	X^2	G^2
FY88	Cauc.	1469	.956	-.045	.246	.804	.854	.854	.854
	Black	81	.401	-.914	.555	.139	.100	.094	.104
	Hisp.	79	.667	-.405	.619	.527	.513	.511	.518
	Other	139	3.916	1.365	1.061	.298	.198	.168	.126
FY89	Cauc.	2028	.997	-.003	.210	1.000	.990	.990	.990
	Black	126	.793	-.232	.409	.681	.570	.570	.571
	Hisp.	125	.300	-1.204	.482	.017	.012	.010	.014
	Other	173	.901	-.104	.480	.810	.828	.828	.829
FY90	Cauc.	1750	.808	-.213	.205	.285	.297	.296	.304
	Black	173	1.218	.197	.359	.723	.583	.583	.582
	Hisp.	147	1.079	.076	.433	1.000	.861	.861	.860
	Other	161	1.278	.245	.678	1.000	.717	.717	.712
FY91	Cauc.	1541	1.556	.442	.253	.085	.080	.078	.070
	Black	137	.603	-.506	.370	.196	.172	.170	.172
	Hisp.	136	1.137	.128	.527	1.000	.808	.808	.807
	Other	68	.410	-.892	.949	.379	.348	.335	.342
FY92	Cauc.	1986	.538	-.620	.198	.002	.002	.002	.002
	Black	204	.667	-.405	.303	.223	.181	.180	.182
	Hisp.	186	1.222	.200	.448	.828	.654	.654	.651
	Other	96				.570		.225	.116

III. General Models

The four factors; success/failure, waiver/no waiver, year (1, ..., 5), and race (1, ..., 4); are denoted as A, B, C, D, respectively. Since the total number of OCS candidates is not fixed, the data D_{ijkl} will be assumed to be generated from an independent Poisson sampling scheme, i.e., D_{ijkl} are independent Poisson random variables with respective parameters (m_{ijkl}) where $m_{ijkl} = E[D_{ijkl}]$. To interpret the results given in the introduction we first fit a loglinear model to the counts collapsed over years, i.e., to

$$D_{ij+l} = \sum_{k=1}^{5} D_{ijkl}.$$

The saturated loglinear model parameterizes $m_{ij+l} = E\left[D_{ij+l}\right]$ as

$$\ln m_{ij+l} = \mu + \lambda_i^A + \lambda_j^B + \lambda_l^D + \lambda_{ij}^{AB} + \lambda_{il}^{AD} + \lambda_{jl}^{BD} + \lambda_{ijl}^{ABD},$$

$$i = 1,2 \qquad j = 1,2 \qquad l = 1,\ldots,4,$$

where the λ's are the effects and interaction terms corresponding to the variables A, B, D. Using standard notation [1], this saturated model can be represented as [ABD], i.e., the third order interaction term ABD and all lower order terms made up of subsets of the variables A, B, and D are included in the model. We begin by fitting the model with all two-way interaction terms along with all main effects, i.e., the model [AB] [AD] [BD]. This gives a likelihood ratio test statistic of 2.55 with 3 degrees of freedom and a p-value of .466. This model does fit the data. To see whether a more parsimonious model can be fit we remove two-way interaction terms one at a time. This yields the model [AD] [BD]. The overall likelihood ratio test statistic is 4.84 with 4 degrees of freedom giving an acceptable p-value of .31. To see whether anything has been lost by removing the AB interaction term, we test the null hypothesis [AD] [BD] versus the alternative [AB] [AD] [BD]. The test statistic 1.99 with 1 degree of freedom has a p-value of

.256. There is not enough evidence to indicate that the AB term should be included. Further, deleting terms from the [AD] [BD] model yields models with unacceptable fits, i.e., those with likelihood ratio test statistics having p-values less than .05. Finally, the standardized residuals for the [AD] [BD] model range from -.843 to 1.090. Thus, the model [AD] [BD] is selected and fits the data (collapsed over years) reasonably well.

The question now becomes, can this model account for the results that motivated the study. The probabilistic interpretation of the model [AD] [BD] is that conditional on the levels of factor D (race), the variables A and B are independent. To see this note that the joint probability mass function (pmf) of the variables A, B, C, D is

$$P_{ijkl} = \frac{m_{ijkl}}{m_{++++}},$$

for $i = 1, 2$; $j = 1, 2$; $k = 1, \ldots, 5$; and $l = 1, \ldots, 4$. The model [AB] [BD] fitted to the data collapsed over years corresponds to

$$\ln m_{ij+l} = \mu + \lambda_i^A + \lambda_j^B + \lambda_l^D + \lambda_{il}^{AD} + \lambda_{jl}^{BD}. \qquad (2.1)$$

Thus the conditional pmf of A given that B is at level j and D is at level l can be found from this model to be

$$P_{i|jl} = \frac{P_{ij+l}}{P_{+j+l}},$$

$$= \frac{\exp\left\{\mu + \lambda_i^A + \lambda_l^D + \lambda_{il}^{AD}\right\}}{\sum_i \exp\left\{\mu + \lambda_i^A + \lambda_l^D + \lambda_{il}^{AD}\right\}}. \qquad (2.2)$$

Since the right hand side of (2.2) is not a function of j, we see that the conditional pmf of A given B, D is the same as the conditional pmf of A given D. Thus given D, the factors A and B are independent.

However, A and B are not independent by themselves alone. The marginal probabilities of these two factors can be developed from the model (2.1) by summing

$$\exp\left\{\mu+\lambda_i^A\right\}\sum_\ell\sum_j\exp\left\{\lambda_j^B+\lambda_\ell^D+\lambda_{i\ell}^{AD}+\lambda_{j\ell}^{BD}\right\}$$

and

$$\exp\left\{\mu+\lambda_j^B\right\}\sum_\ell\sum_i\exp\left\{\lambda_i^A+\lambda_\ell^D+\lambda_{i\ell}^{AD}+\lambda_{j\ell}^{BD}\right\}$$

and forming the appropriate normalizations. The joint probability is not the product of these probabilities. Thus the model supports the observation made earlier that success of the OCS candidate is not independent of whether the ASVAB waiver has been used for entry. These two variables are independent, however, when broken out by race.

The following probabilities help interpret the dependence between A and B. The probabilities of success given race are estimated to be .78, .64, .70, .74 for Caucasians, Blacks, Hispanics and Others, respectively. (The empirical rates and the modeled rates are the same to two decimal places.) The proportions of candidates in each race which possess a waiver are .07, .32, .18, .10, and the proportions of candidates who don't possess a waiver in each race are the complementary values, .93, .68, .82, .90. The greatest proportion of candidates who don't possess a waiver are Caucasians (93%), with a good chance of success (78%). However, candidates that do utilize the waiver are divided primarily between Blacks (32%) and Hispanics (18%). Because the probability of success for these two races differ (67%) and (70%) respectively, we see that the overall probability of success with a waiver is lower than without a waiver. Also, the four success rates decrease monotonically as the four waiver use rates increase.

IV. Temporal Analysis

The above analysis responds to the question posed in the introduction. But it is also of interest to consider the other factor, C, the fiscal year. If including the variable race sheds light on the dependence between having a waiver and success of the OCS candidate, perhaps considering this fourth variable will add to an understanding of this data set.

Perhaps the most direct way to proceed is to consider the most general four factor model that reflects independence of factors A and B. In the notation established this would be [ACD] [BCD]. All interactions involving A and B are zero. Doing so produces a likelihood ratio p-value of .049. This is rather small for our tastes. Study of the residuals reveals two outlier cells: unsuccessful Hispanics with a waiver in FY89 and unsuccessful Caucasians with a waiver in FY92. These two cells belong to the same cases that exhibited low p-values in Table 3.

It appears that the loglinear modeling system must provide for some AB interactive terms. Accordingly we apply the strategy which fits the models with all three way and lower order terms; all two way and lower order terms; and all one way terms. Then the overall model with the fewest terms and an acceptable overall fit is used as a starting point for further deletion of terms within the chosen set. The first model fit was the one with all three way interactions. This gives an overall fit with a p-value of .0387. However, as terms are deleted the p-value increases and the model [ABC] [BCD] [ACD] gives a slightly higher p-value for overall fit of .0657. Further deletion of terms leads to the model [ABC] [BCD] [AD] with p-value .22.

The fact that the deletion of additional terms appears to improve the fit can be explained by noting the increase in the degrees of freedom. For the model with all three way interaction terms, the likelihood ratio test statistic is 21.95 with 12 degrees of freedom, deleting the ABD term increases degrees of freedom to 15 and the test statistic to 24.01 and the deletion of the ABD term increases the degrees of freedom to 19 and the test statistic to 29.548. Therefore deleting terms does not increase the test statistic very much compared to the gain in degrees of freedom.

Deleting either the ABC or BCD terms from the [AD] [ABC] [BCD] model results in models with much lower p-values for overall goodness of fit and standardized residuals that are of much larger magnitude than those of the [AD] [ABC] [BCD] model. Since the standardized residuals for this model range between -1.78 to 1.81, this model appears to give an adequate fit. In passing, we note that all AB interactive terms are modest in size.

The estimated probabilities of success given race, waiver status and fiscal year $\left(\hat{p}_{i|jkl}\right)$ are plotted against year (k) in Figures 1 and 2. There is a general decrease in the probability of success over time in all four racial groups regardless of waiver status. In fact, when the model [AD] [BD] is fit to years separately, only 1992 fails to fit with a p-value = .01. It appears that for the first four years this trend is reasonably well modeled as independent of waiver status. The presence of the ABC interaction term in the temporal model is a consequence of changes in 1992, specifically the outlier cell cited earlier.

The presence of the BCD interaction term can be explained by changes in the number of waivers utilized over time. To examine this, we fit a logistic regression model where the response variable is one or zero according to whether an individual received a waiver or not, and the explanatory variables are years and race. Since years is in fact an ordinal variable, it was scored as the integers 1 to 5 for the years 1988 to 1992. This saves degrees of freedom and helps detect monotonic trends.

Figure 1

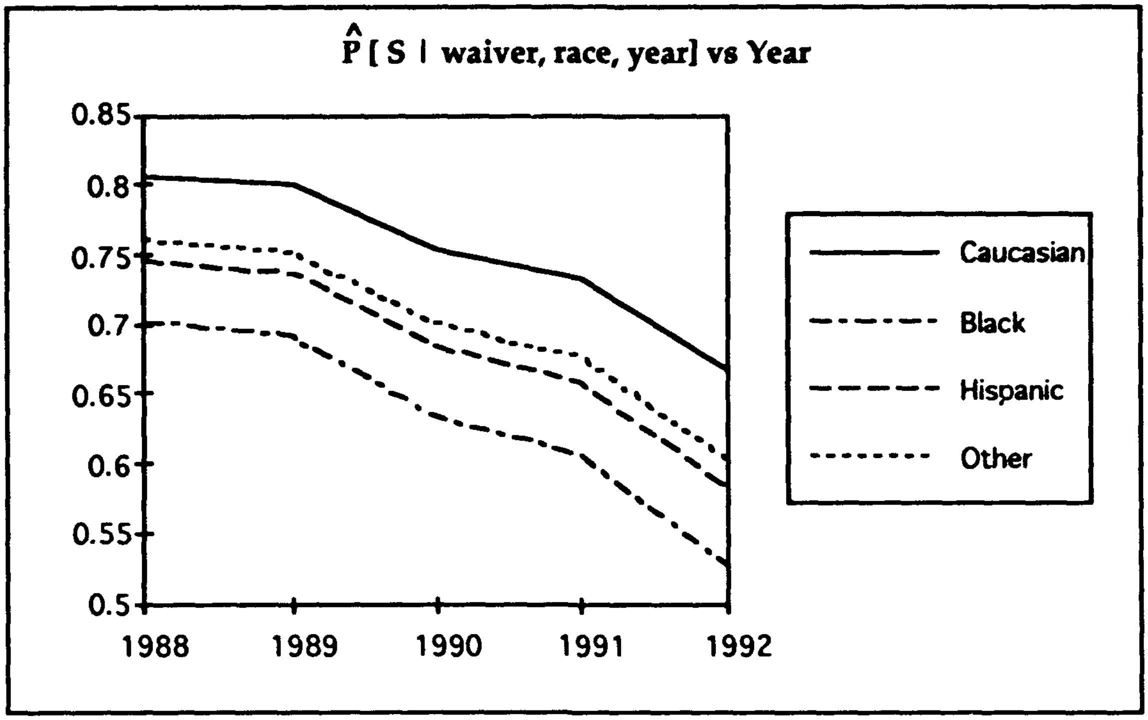
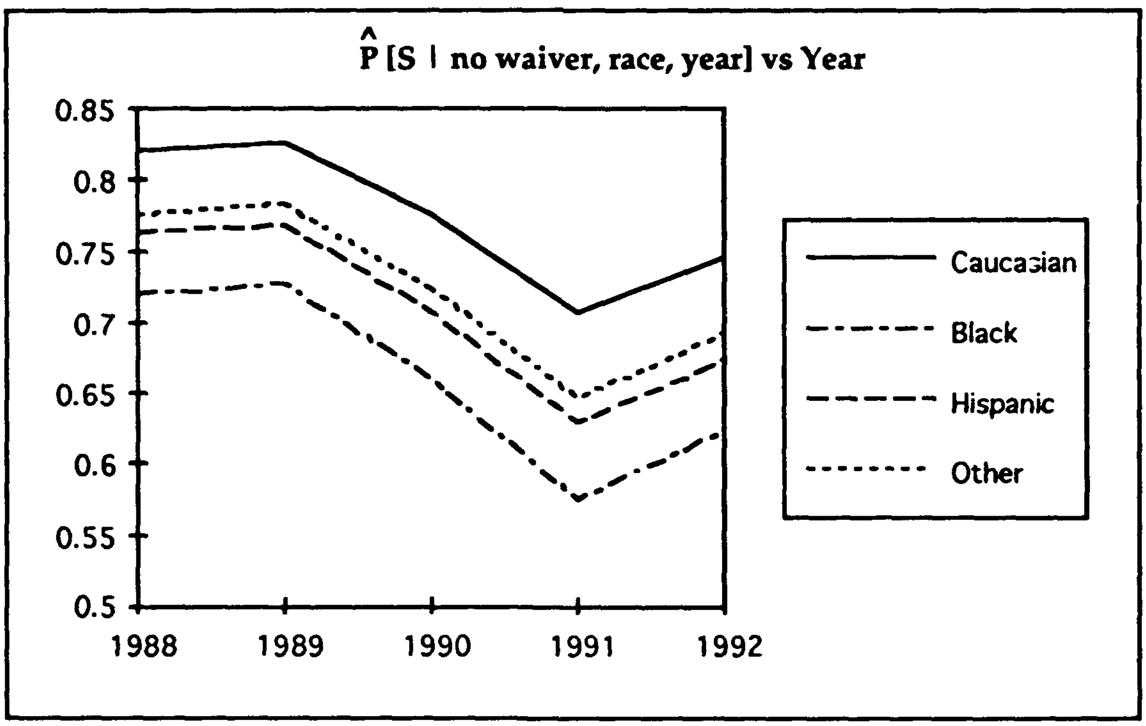


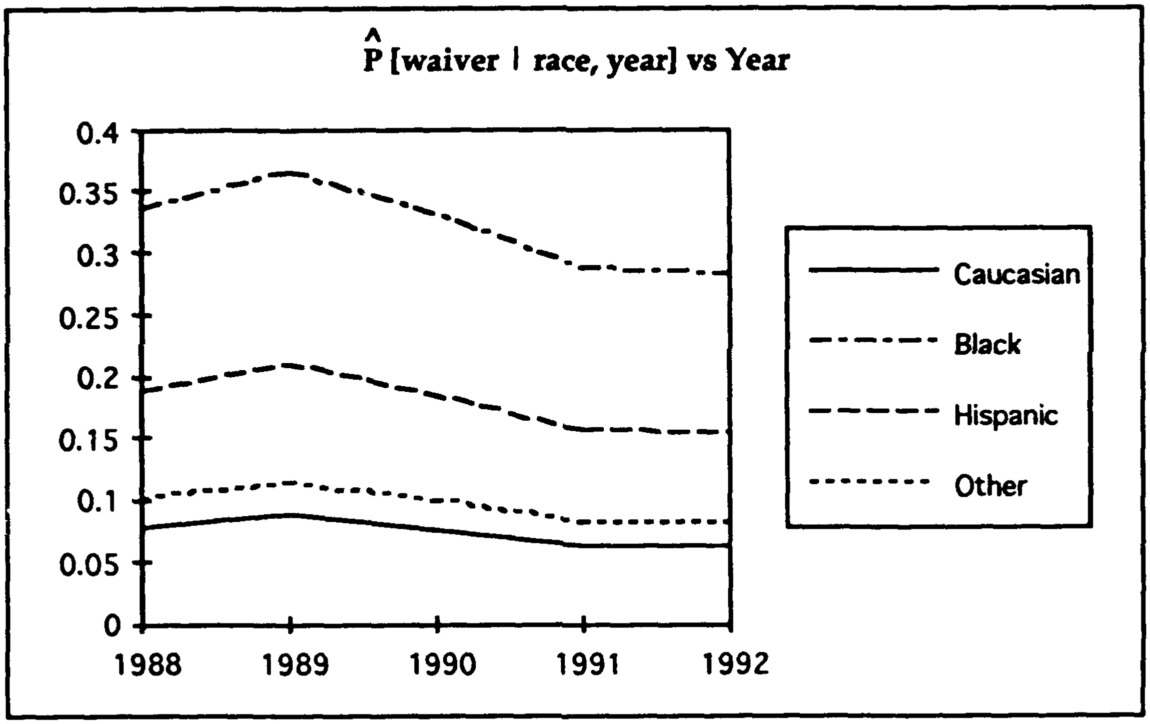
Figure 2



The model with a cubic term in years gives an adequate fit to the data (p-value = .112). This model fits the data somewhat better than the model that fits the year as a categorical variable.

The fitted values are the estimates of the conditional probabilities that an officer receives a waiver given year and race. These are plotted by race in Figure 3. From this plot it can be seen that except for 1989 there has been a general decline in the proportion of waivers awarded for each race.

Figure 3



In conclusion, we have accounted for the nature of the paradox stated in the introduction by the use of loglinear analysis after collapsing the data over time. The odds ratio analysis serves to support the independence vs. waiver hypothesis at a micro-level, and deeper loglinear modeling can be used to quantify the changes in probabilities as functions of race and time. The final analysis collapses the data over OCS success or failure and treats the use of the waiver. It appears to be diminishing in time but there are some rather prominent separations by race. Some additional study in these areas can be found in [3].

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APPENDIX A

Algorithm to produce p-values for the hypergeometric distribution.

Let us view our basic 2 x 2 table as

a	b	S
c	d	F
n_1	n_2	N

In the context of the report, a is the number of successful candidates among the n_1 that used waivers; b is the number of successful candidates among the n_2 that did not use waivers, etc. The probabilistic structure used is a conditional one,

$$P(a|a+b=S)=\frac{\binom{n_1}{a}\binom{n_2}{b}}{\binom{N}{S}} \tag{A.1}$$

which is a hypergeometric probability function. For the present purposes it is useful to describe the variable range constraints rather elaborately:

$$\max(0, S-n_2) \le a \le \min(S, n_1)$$

$$\max(0, S-n_1) \le b \le \min(S, n_2)$$

$$\max(0, F-n_2) \le c \le \min(F, n_1)$$

$$\max(0, F-n_1) \le d \le \min(F, n_2)$$

Let us analyze the computations. Let P_0 be the value of (A.1) for the observed table. The p-value is the sum of all probabilities (A.1) which are less than or equal to P_0. Let

$$C = n_1!\, n_2!\, S!\, F!/N!$$

Then (A.1) can be expressed as

$$P = C\frac{1}{a!b!c!d!} \tag{A.2}$$

In the p-value computation the value of C is fixed and only the other factor in (A.2) changes as the summation takes place. It is often wise to use logarithms in the computation because the factorials can get quite large. Also the two-sided p-value computation is managed by identifying the two tails of the distribution and summing their contributions.

Our approach is to first identify the variable (a, b, c, d) that has the shortest range in the specific situation. To do this we compute the empirical odds ratio

$$\hat{\theta} = \frac{ad}{bc}$$

and determine the case $\hat{\theta} \le 1$ or $\hat{\theta} > 1$. This identifies the tail that contains the experimental result. That is, we view the testing problem as H_0: $p_1 = p_2$ vs H_1: $p_1 \ne p_2$. The two estimators are

$$\hat{p}_1 = a / n_1 \quad \text{and} \quad \hat{p}_2 = b / n_2.$$

It is easily seen that $\hat{p}_1 \le \hat{p}_2$ is equivalent to $\hat{\theta} \le 1$; and the opposite case with $\hat{\theta} > 1$. Thus if $\hat{\theta} \le 1$ we choose $M = \min(a, d)$ and sum the hypergeometric terms for that tail of the distribution. Of course, if $\hat{\theta} > 1$ we choose $M = \min(b, c)$ for the single tail sum. If $M = 0$ in either case then P_0 is the total probability for that tail.

To illustrate, we have

$$P_0 = C / a!\, d!\, b!\, c!$$

and for $\hat{\theta} \le 1$ we form the successive terms

$$R_0 = 1, R_1 = R_0 \frac{ad}{(b+1)(c+1)}, R_2 = R_1 \frac{(a-1)(d-1)}{(b+2)(c+2)}, \ldots, R_M = R_{M-1} \frac{(a+1-M)(d+1-M)}{(b+M)(c+M)} \tag{A.3}$$

and the single tail probability is $P_0 \sum_{i=0}^{M} R_i$.

On the other hand, if $\hat{\theta} > 1$ the R's are formed differently. That is

$$R_0 = 1, R_1 = R_0 \frac{bc}{(a+1)(d+1)}, R_2 = R_1 \frac{(b-1)(c-1)}{(a+2)(d+2)}, \ldots, R_M = R_{M-1} \frac{(b+1-M)(c+1-M)}{(a+M)(d+M)} \tag{A.4}$$

To manage the opposite tail let us redefine the R's in the following way. For the case $\hat{\theta} \leq 1$ we change to $M = \min(b, c)$ and choose

$$R_1 = \frac{bc}{(a+1)(d+1)}, R_2 = R_1 \frac{(b-1)(c-1)}{(a+2)(d+2)}, \ldots, R_M = R_{M-1} \frac{(b+1-M)(c+1-M)}{(a+M)(d+M)} \qquad \text{(A.5)}$$

which matches (A.4) except that $R_0 = 1$ is not in the set. The opposite tail probability is obtained by summing

$$P_0 \sum R_i \quad \text{for all } R_i \leq 1. \qquad \text{(A.6)}$$

The opposite tail for the case $\hat{\theta} > 1$ is managed similarly. This time $M = \min(a, d)$ and define a new set of R's according to the form of (A.3), but omitting $R_0 = 1$. Then apply the formula (A.6).

APPENDIX B

The estimated coefficients, their standard errors and p-values for the model [AD] [ABC] [BCD] are given in Table B1. The coefficients are constrained so that one level of each factor has a coefficient that is set to zero. For example, for factor A there is only one estimated coefficient $\hat{\lambda}_1^A$ corresponding to success at OCS; the coefficient corresponding to failure in OCS $\hat{\lambda}_2^A$ is set to zero. Thus, the estimated value .9438 is a contrast and the t-value 19.45 tests the null hypothesis that the main effects for levels 1 and 2 of factor A are the same. Since A has only 2 levels this is equivalent to $H_0: \lambda_1^A = \lambda_2^A = 0$. The main effects in Table B1 are labeled as follows:

A	λ_1^A	(Success in OCS)
B	λ_1^B	(Waiver)
C1	λ_2^C	(FY89)
C2	λ_3^C	(FY90)
C3	λ_4^C	(FY91)
C4	λ_5^C	(FY92)
D1	λ_3^D	(Hispanic)
D2	λ_4^D	(Other)
D3	λ_1^D	(Caucasian)

All other main effects are set to zero. Interaction terms are similarly treated.

Table B1

	Value	Std. Error	t-value
(Intercept)	2.855	0.147	19.45
A	0.944	0.102	9.24
D1	-0.135	0.198	-0.68
D2	0.481	0.180	2.67
D3	2.640	0.144	18.35
B	-1.122	0.297	-3.77
C1	0.151	0.183	0.83
C2	0.910	0.165	5.50
C3	0.820	0.173	4.73
C4	1.087	0.163	6.68

A:D1	0.225	0.116	1.94
A:D2	0.304	0.121	2.51
A:D3	0.576	0.085	6.80
A:B	-0.085	0.199	-0.43
A:C1	0.036	0.085	0.42
A:C2	-0.278	0.083	-3.36
A:C3	-0.638	0.083	-7.70
A:C4	-0.438	0.080	-5.48
B:C1	0.888	0.364	2.44
B:C2	0.182	0.358	0.51
B:C3	0.300	0.365	0.82
B:C4	0.633	0.342	1.85
B:D1	-0.264	0.389	-0.68
B:D2	-1.084	0.392	-2.76
B:D3	-1.217	0.280	-4.35
C:D1	0.288	0.235	1.23
C:D2	-0.025	0.211	-0.12
C:D3	0.133	0.176	0.76
C:D4	-0.101	0.220	-0.46
C:D5	-0.546	0.197	-2.77
C:D6	-0.513	0.160	-3.22
C:D7	0.220	0.227	0.97
C:D8	-1.057	0.226	-4.68
C:D9	-0.269	0.169	-1.59
C:D10	0.119	0.214	0.56
C:D11	-1.080	0.206	-5.25
C:D12	-0.422	0.158	-2.68
A:B:C1	-0.083	0.252	-0.33
A:B:C2	-0.031	0.254	-0.12
A:B:C3	0.210	0.266	0.79
A:B:C4	-0.306	0.249	-1.23
B:C:D1	-0.865	0.487	-1.78
B:C:D2	-0.075	0.472	-0.16
B:C:D3	-0.752	0.493	-1.52
B:C:D4	-0.648	0.462	-1.40
B:C:D5	-0.248	0.480	-0.52
B:C:D6	-0.245	0.512	-0.48
B:C:D7	-0.710	0.635	-1.12
B:C:D8	-1.308	0.661	-1.98
B:C:D9	-0.792	0.343	-2.31
B:C:D10	-0.220	0.341	-0.65
B:C:D11	-0.740	0.351	-2.11
B:C:D12	-0.806	0.332	-2.43

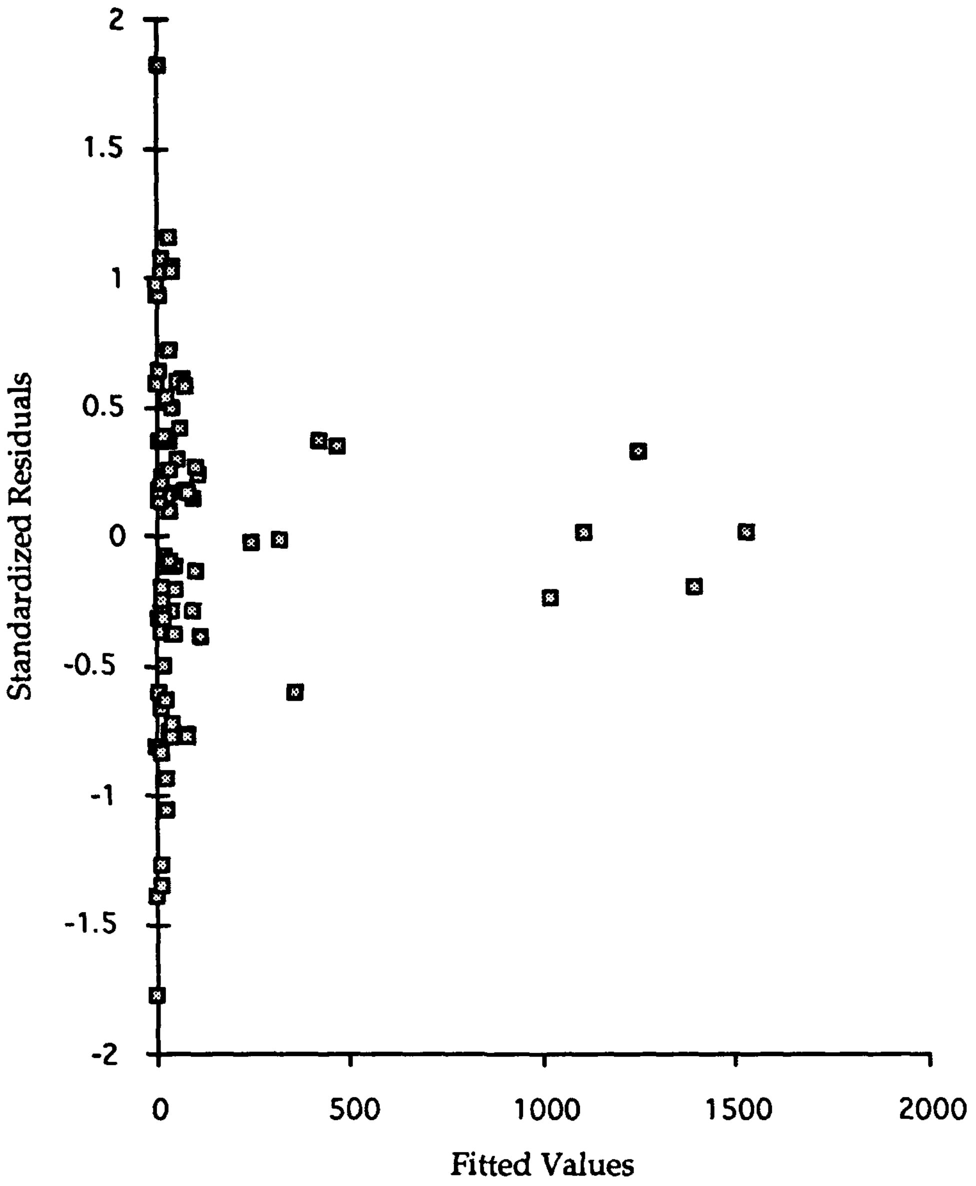
Table B2 contains the fitted cell means along with the standardized residuals. The standardized residuals are plotted against the fitted values in Figure B1.

Table B2

	count			Fitted Values	Std. Residuals
1	100	FY88	Cauc.	98.540	0.147
2	11		Black	13.347	-0.663
3	10		Hisp.	11.208	-0.368
4	12		Other	9.905	0.644
5	142	FY89	Cauc.	37.662	0.368
6	37		Black	36.016	0.163
7	12		Hisp.	16.980	-1.276
8	20		Other	20.342	-0.076
9	102	FY90	Cauc.	103.464	-0.144
10	30		Black	29.175	0.152
11	20		Hisp.	20.539	-0.119
12	11		Other	9.822	0.369
13	77	FY91	Cauc.	71.784	0.608
14	22		Black	26.670	-0.933
15	14		Hisp.	13.166	0.227
16	2		Other	3.380	-0.813
17	70	FY92	Cauc.	76.628	-0.768
18	36		Black	35.432	0.095
19	22		Hisp.	17.527	1.027
20	4		Other	2.414	0.932
21	1113	FY88	Cauc.	1112.644	0.011
22	48		Black	44.632	0.498
23	48		Hisp.	48.823	-0.118
24	95		Other	97.901	-0.295
25	1533	FY89	Cauc.	1532.608	0.010
26	56		Black	53.801	0.298
27	80		Hisp.	78.468	0.172
28	111		Other	115.123	-0.387
29	1263	FY90	Cauc.	1251.554	0.323
30	77		Black	83.893	-0.763
31	76		Hisp.	82.953	-0.774
32	109		Other	106.601	0.232
33	1013	FY91	Cauc.	1020.580	-0.238
34	58		Black	53.558	0.599
35	78		Hisp.	73.037	0.574
36	39		Other	40.826	-0.288
37	1390	FY92	Cauc.	1397.537	-0.202

38	87		Black	85.477	0.164
39	108		Hisp.	105.300	0.262
40	67		Other	63.687	0.412
41	22	FY88	Cauc.	23.460	-0.305
42	8		Black	5.653	0.928
43	5		Hisp.	3.792	0.591
44	1		Other	3.095	-1.389
45	30	FY89	Cauc.	34.338	-0.757
46	15		Black	15.984	-0.249
47	11		Hisp.	6.020	1.817
48	7		Other	6.658	0.131
49	35	FY90	Cauc.	33.536	0.251
50	16		Black	16.825	-0.203
51	10		Hisp.	9.461	0.174
52	3		Other	4.178	-0.607
53	21	FY91	Cauc.	26.216	-1.056
54	22		Black	17.330	1.076
55	6		Hisp.	6.834	-0.326
56	3		Other	1.620	0.968
57	45	FY92	Cauc.	38.372	1.041
58	31		Black	31.568	-0.101
59	8		Hisp.	12.473	-1.357
60	0		Other	1.586	-1.781
61	243	FY88	Cauc.	243.356	-0.023
62	14		Black	17.368	-0.837
63	16		Hisp.	15.177	0.209
64	31		Other	28.099	0.538
65	323	FY89	Cauc.	323.392	-0.022
66	18		Black	20.199	-0.499
67	22		Hisp.	23.532	-0.319
68	36		Other	31.877	0.715
69	350	FY90	Cauc.	361.446	-0.605
70	50		Black	43.107	1.024
71	41		Hisp.	34.047	1.154
72	38		Other	40.399	-0.381
73	430	FY91	Cauc.	422.420	0.368
74	35		Black	39.442	-0.721
75	38		Hisp.	42.963	-0.773
76	24		Other	22.174	0.383
77	481	FY92	Cauc.	473.463	0.345
78	50		Black	51.523	-0.213
79	48		Hisp.	50.700	-0.383
80	25		Other	28.313	-0.635

Figure B1


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Standardized Residuals
0
500
1000
1500
2000
Fitted Values

APPENDIX C

Short analysis of the model [ACD] [BCD].

This model features the conditional independence of factors A and B given the levels of C and D, coupled with a fully saturated modeling of the joint distribution of C and D. Thus the loglinear representation can be made more succinct than the direct representation. Since

$$p_{ij|k\ell} = p_{i|k\ell}\, p_{j|k\ell},$$

the maximum likelihood estimates of the two factors on the right hand side are

$$\frac{n_{i+k\ell}}{n_{++k\ell}} \quad \textit{and} \quad \frac{n_{+jk\ell}}{n_{++k\ell}}$$

respectively. It follows that for each k, ℓ pair, the loglinear model of the left hand side may be expressed as

$$\ln\left(m_{ij|k\ell}\right) = \text{const} + \lambda^{A}_{i|k\ell} + \lambda^{B}_{j|k\ell}$$

and estimates of these parameters can be obtained rather easily from the twenty 2 x 2 tables that lie behind Table 3. The maximum likelihood estimators of $m_{ij\,|\,k\ell}$ are

$$n_{i+k\ell}\, n_{+jk\ell} \,/\, n_{++k\ell}$$

and match the expected frequencies in the 2 x 2 contingency table computations.

Next, the model calls for the saturated version of $p_{k\ell}$, so that

$$\ln m_{++k\ell} = \mu + \lambda^{C}_{k} + \lambda^{D}_{\ell} + \lambda^{CD}_{k\ell}$$

with the customary constraints. The maximum likelihood estimators are

$$\hat{m}_{++k\ell} = n_{++k\ell}$$

and it follows from the rules of conditional and marginal expectation

$$m_{ijk\ell} = m_{k\ell}\, p_{ij|k\ell}$$

lead to the estimates

$$\hat{m}_{ijk\ell} = m_{i+k\ell}\, m_{+jk\ell} \,/\, m_{++k\ell}.$$

This point is especially convenient in that it allows chi squared test statistics for the model [ACD] [BCD] to be constructed merely by summing the individual chi squared statistics computed from the original twenty contingency tables. The degrees of freedom for this sum are the total of the individual table degrees of freedom.

APPENDIX D

This appendix contains the details of a logistic regression model that treats the response variable of whether an individual possesses a waiver or not and using explanatory variables of years and race. Following the notation established in the paper, let

$$P_{k\ell} = \frac{m_{+1k\ell}}{m_{++k\ell}} \qquad \begin{array}{l} \text{for } k = 1,\ldots,5 \\ \text{for } \ell = 1,\ldots,4, \end{array}$$

be the probability that an individual of race ℓ in year k possesses a waiver. Because years is an ordinal variable, it is treated as numeric with FY88, ..., FY92 scored as 1, ..., 5 respectively. The logistic regression models fit to $\ln(P_{k\ell}/(1-P_{k\ell}))$ along with the likelihood ratio test statistic G^2 and the corresponding p-values are as follows:

Model	G^2	degrees of freedom	p-value
1. $\mu + \lambda^C k + \lambda^D_\ell$	25.11	15	.048
2. $\mu + \lambda^C_1 k + \lambda^C_1 k^2 + \lambda^D_\ell$	23.43	14	.054
3. $\mu + \lambda^C_1 k + \lambda^C_2 k^2 + \lambda^C_3 k^3 + \lambda^D_\ell$	19.36	13	.112
4. $\mu + \lambda^C_1 k + \lambda^C_2 k^2 + \lambda^C_3 k^3 + \lambda^C_4 k^4 + \lambda^D_\ell$	18.95	12	.090

The fits of models number 1 and 2 are inadequate. This is confirmed in Figures D1 and D2 where the standardized residuals are plotted against years. The pattern of the residuals in both figures suggests that higher order polynomials in k need to be fit to the data. The model 3 fit is acceptable and the residuals (Figure D3) appear to be evenly scattered when plotted against years. The hypothesis test between models 3 and 4 has likelihood ratio test statistic 19.36–18.95 with 1 degree of freedom and p-value .52. Note that model 4 is equivalent to fitting the logistic regression model where both race and years are treated as categorical variables.

The standardized residuals and fitted values given in Table D1 are plotted in Figure D4.

Table D1

Race	Year	Fitted $P_{k\ell}$	Standardized Residuals
Cauc.	FY88	0.0775	0.7799
Cauc.	FY89	0.0876	-0.4536
Cauc.	FY90	0.0760	0.3658
Cauc.	FY91	0.0627	0.1402
Cauc.	FY92	0.0618	-0.7288
Black	FY88	0.3361	-1.9954
Black	FY89	0.3665	1.0672
Black	FY90	0.3311	-1.8583
Black	FY91	0.2873	0.8670
Black	FY92	0.2841	1.3853
Hisp.	FY88	0.1882	0.0378
Hisp.	FY89	0.2094	-0.7101
Hisp.	FY90	0.1848	0.5946
Hisp.	FY91	0.1558	-0.2835
Hisp.	FY92	0.1537	0.2836
Other	FY88	0.1010	-0.2953
Other	FY89	0.1138	1.6707
Other	FY90	0.0990	-0.5201
Other	FY91	0.0821	-0.2613
Other	FY92	0.0809	-1.5438

The coefficients λ_ℓ^D $\ell = 1, \ldots, 5$ corresponding to the factor race are over-parameterized without an additional constraint. Familiar constraints are the "sum to zero" and the "set to zero" constraints. Statistical packages usually use either one of these constraints. S-PLUS, the package used for the analysis presented in this paper uses neither of these constraints. Instead, let

$$x_1^T = (1, k, k^2, k^3, 0, 0, 3)$$

$$x_2^T = (1, k, k^2, k^3, -1, -1, -1)$$

$$x_3^T = (1, k, k^2, k^3, 1, -1, -1)$$

$$x_4^T = (1, k, k^2, k^3, 0, 2, -1).$$

Then for example in model 3, the fitted values $\hat{P}_{k\ell}$ can be found by

$$\ln\left(\frac{\hat{P}_{k\ell}}{1-\hat{P}_{k\ell}}\right) = x_\ell^T \hat{\beta}$$

where $\hat{\beta}^T = (\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_T)$ (along with estimated standard deviations and t-values) are given in Table D2.

Table D2

i		$\hat{\beta}_i$	std error	t-values
1	(Intercept)	-2.3634	.3800	-6.219
2	(λ_1^c)	1.0065	.4822	2.0687
3	(λ_2^c)	-.3842	.1791	-2.145
4	(λ_3^c)	.0399	.0198	2.012
5		-.3906	.0647	-6.034
6		-.3717	.0496	-7.501
7		-.2583	.0186	-13.9202

S-PLUS uses helmert polynomials to generate the linear combinations of the parameters used for each level of a categorical factor.

Figure D1

Standardized Residuals vs. Years

for the model $\ln\left(\frac{P_{k\ell}}{1-P_{k\ell}}\right)=\mu+\lambda_1^C k+\lambda_\ell^D$

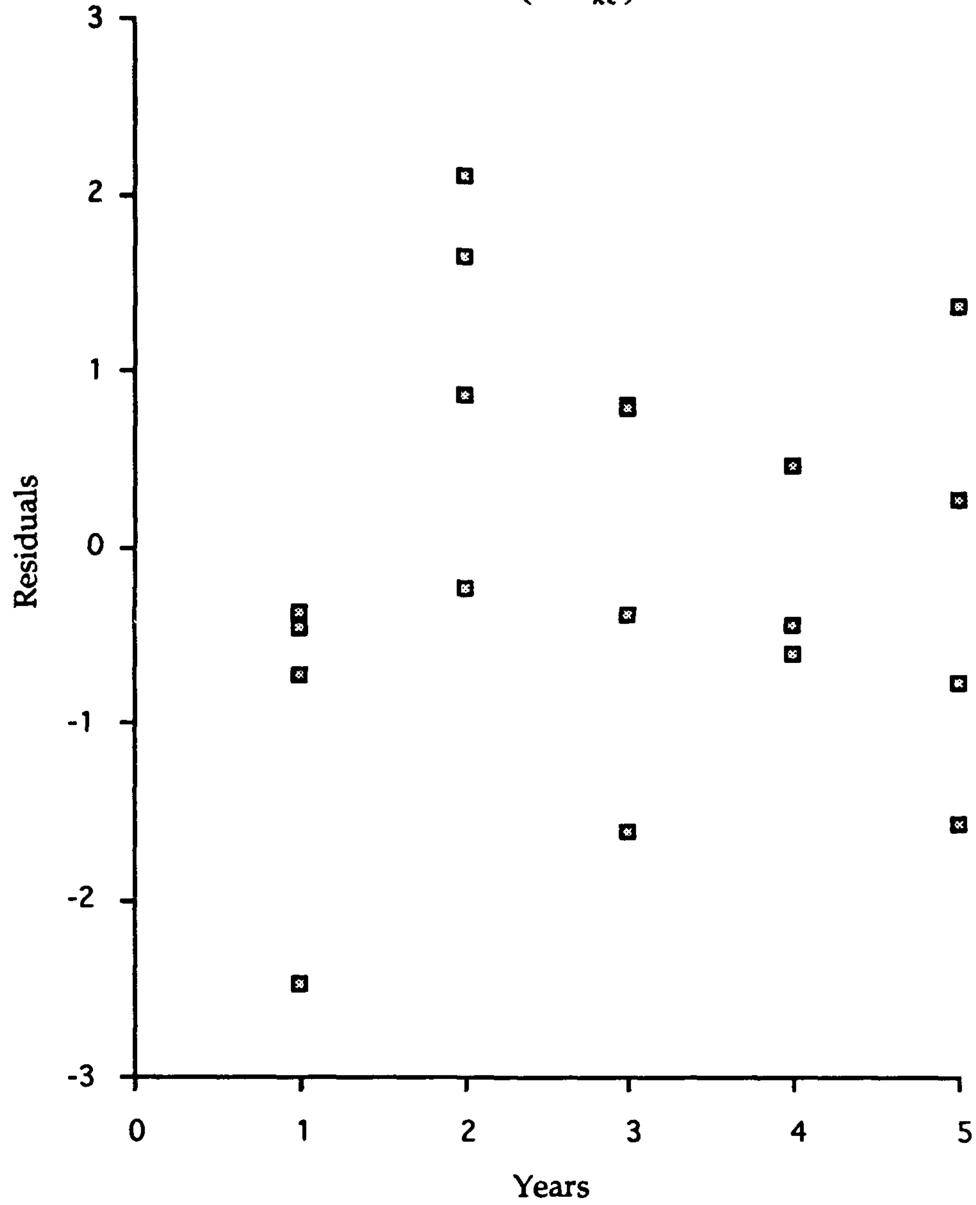


Figure D2

Standardized Residuals vs. Years

for the model $\ln\left(\frac{P_{k\ell}}{1-P_{k\ell}}\right) = \mu + \lambda_1^C k + \lambda_2^C k^2 + \lambda_\ell^D$

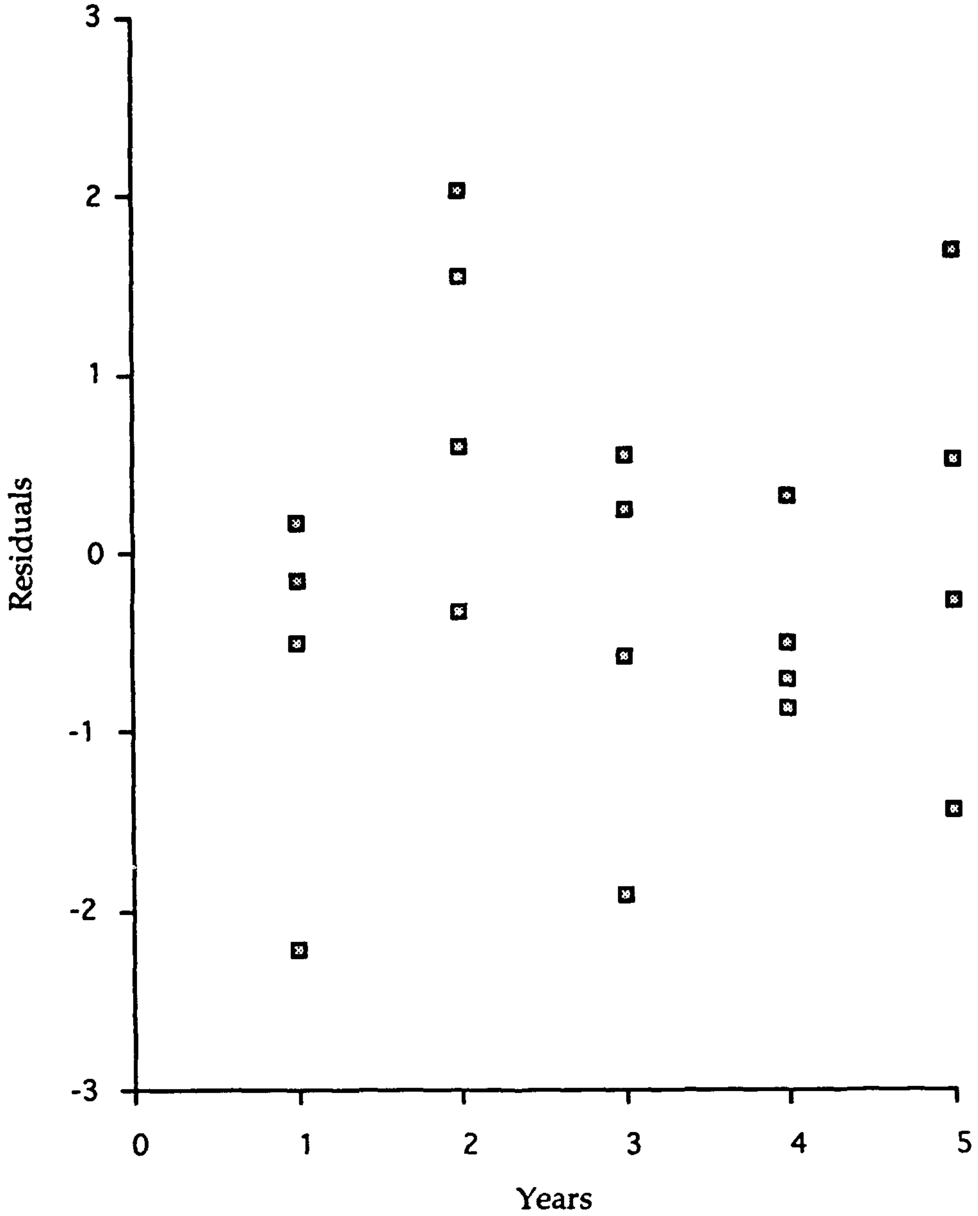


Figure D3

Standardized Residuals vs. Years

for the model $\ln\left(\frac{P_{k\ell}}{1-P_{k\ell}}\right) = \mu + \lambda_1^C k + \lambda_2^C k^2 + \lambda_3^C k^3 + \lambda_\ell^D$

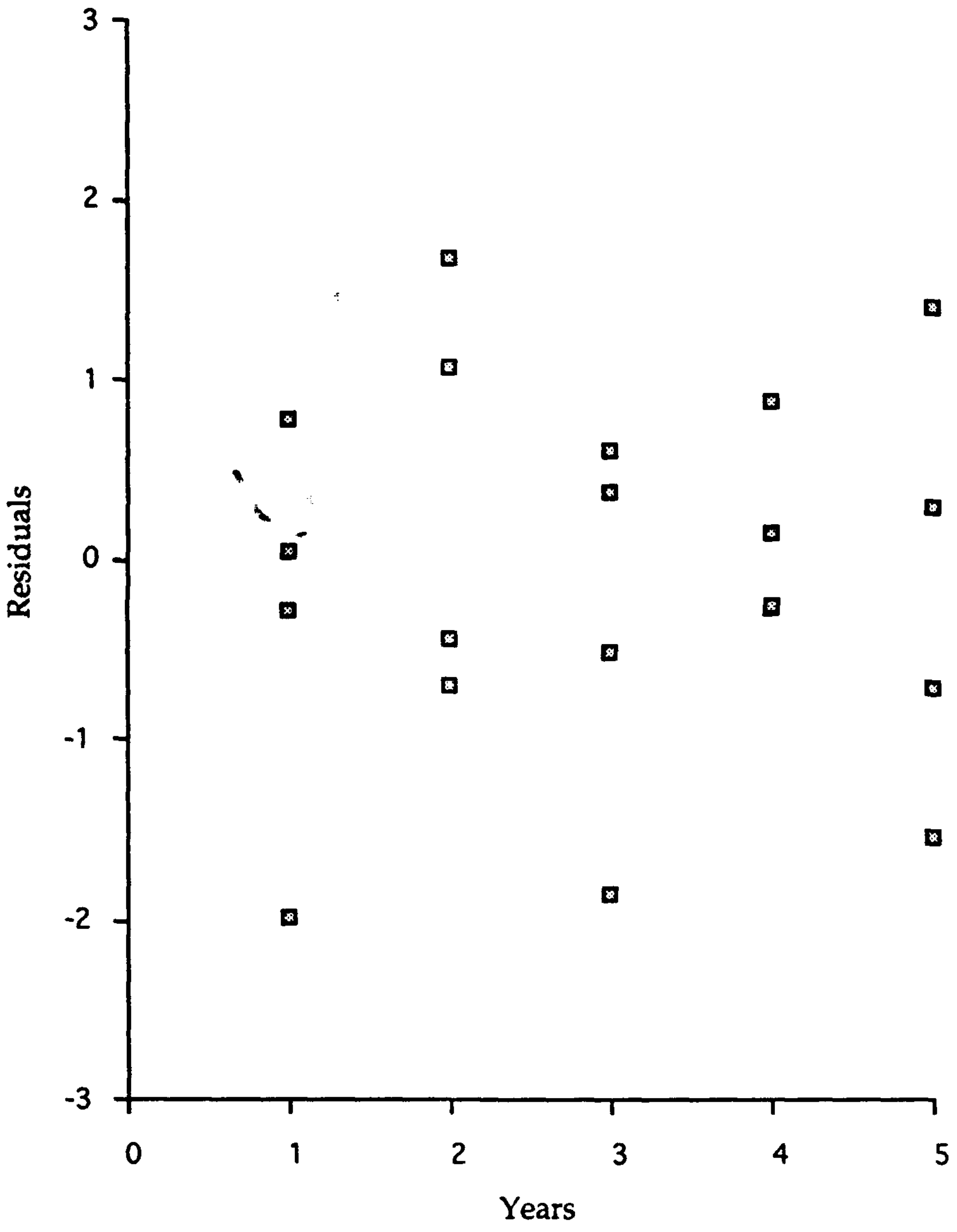
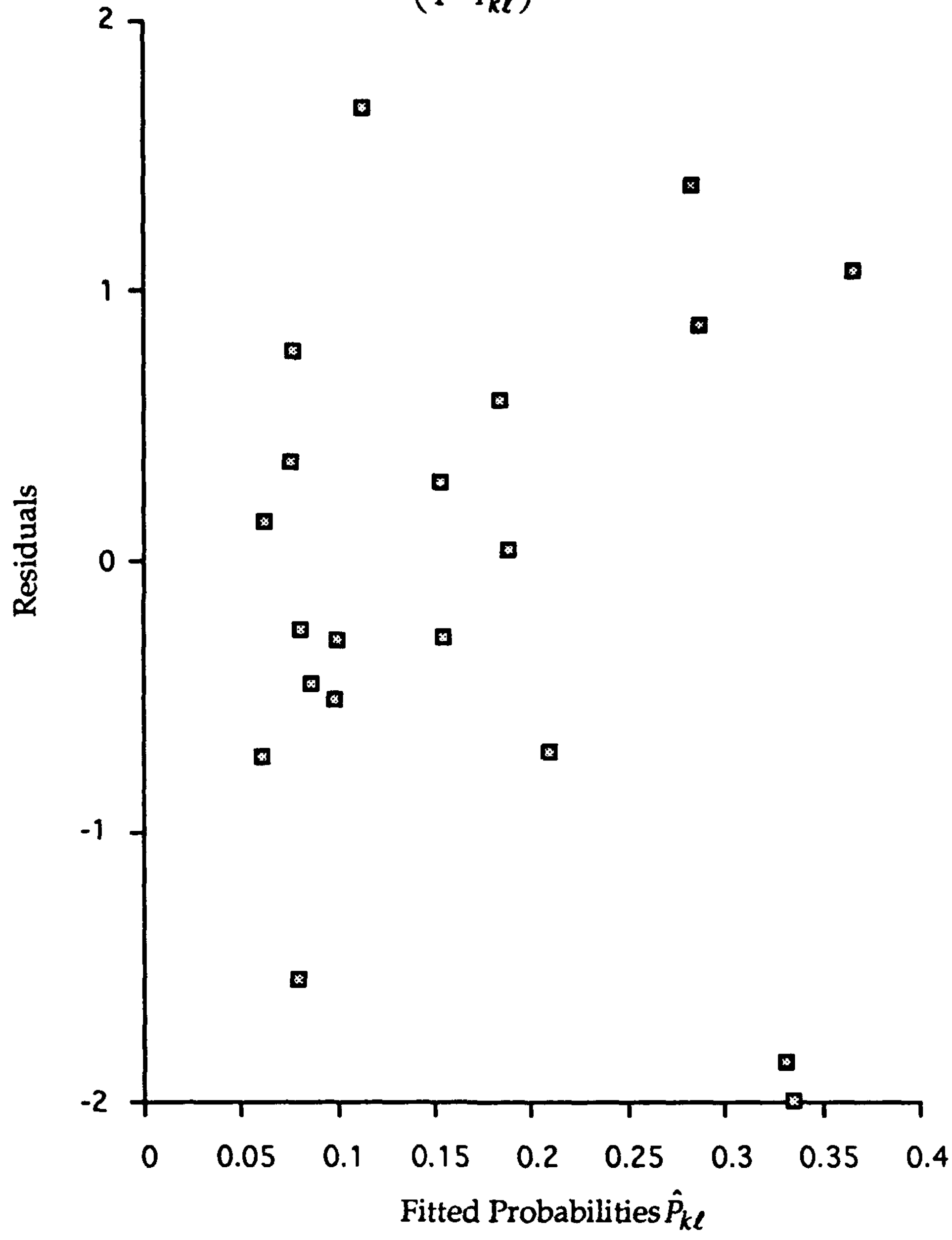


Figure D4

Standardized Residuals vs. Fitted Probabilities

for the model $\ln\left(\frac{P_{k\ell}}{1-P_{k\ell}}\right) = \mu + \lambda_1^C k + \lambda_2^C k^2 + \lambda_3^C k^3 + \lambda_\ell^D$



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