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DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORY

MELBOURNE, VICTORIA

Guided Weapons Technical Memorandum 021

THE THEORETICAL LAYERED AIR-DEFENCE CAPABILITY OF A SHIP ENGAGED AGAINST MULTIPLE ANTI-SHIP CAPABLE MISSILE ATTACKS

by

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SUMMARY

A probabilistic assessment of single-ship air-defence effectiveness against an attack by a stream or wave of anti-ship capable missiles (ASCMs) has been conducted for ships fitted with a mix of either 'hard kill or 'hard kill/soft kill' weapons systems.

Results are expressed in terms of the probability of a ship defeating all threats, or the expected number of hits achieved by the ASCMs.

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1. INTRODUCTION

In an ideal world, maritime air-defence is best realised by a strategy generally referred to as 'layered defence'; first, by air power denying a weapons platform a capacity to launch and conduct a strike, second, by an inner layer of ship-launched missile systems of progressively reducing range - missile systems are classed as either area defence missile systems (ADMS) or point defence missile systems (PDMS), and third, by a final layer of short-range, rapid rate-of-fire guns, generally classed as close-in weapon systems (CIWS).

The concept of layered defence had been well understood prior to the Falklands conflict of 1982, but save for the Super Powers, little impetus had been given to implementing such concepts in the air-defence of warships. The use of sea-skimming, Exocet anti-ship capable missiles (ASCMs) by the Argentinian air-force in the Falklands conflict of 1982 changed all that, and compelled many 'blue-water' navies to urgently re-examine the priorities of their anti-ship missile defence (ASMD) philosophies, particularly those related to layered defence.

In the Falklands conflict, two Royal Navy (RN) warship classes were assigned to provide the primary level of air-defence for the task force against both aircraft and ASCMs:

(1) The early '70s-designed, Type 42 destroyer, equipped with the area defence, medium range missile system *Seadart* - a missile system designed to provide protection for a group of ships, and

(2) The mid-'70s-designed, Type 22 frigate, equipped with the short-range PDMS, *Seawolf*, a missile system whose primary function was to provide quick-reaction defence for own ship.

There were occasions in the Falklands where one of each of the above ship types operated together, the tactics being, if both were subjected to air-attack, for their weapons systems to act in a layered manner. In one incident, involving the Type 42, HMS *Glasgow* and the Type 22, HMS *Brilliant*, the tactics were reputed to have worked well enough (except that *Glasgow* was hit by a 500 lb bomb which, fortunately for the crew, failed to explode) and a number of *Skyhawks* (A-4's) were successfully engaged and shot down by the air-defence combination. In a later incident involving the Type 42, HMS *Coventry* and HMS *Broadsword*, a failure in *Coventry*'s *Seadart* missile system, coupled with her cutting-across *Broadsword*'s *Seawolf* arc-of-fire at a critical moment in the engagement, resulted in the Type 42 destroyer being lost.

With this stop-gap form of air-defence seemingly discredited, there was considerable Defence Journal speculation following-on from the Falklands conflict, as to how the air-defence capability of RN units could be enhanced, and ultimately, improvements in air defence were realised by fitting a single Goalkeeper CIWS to each Batch 3-version of the Type 22s and two Phalanx CIWS to the Type 42s. 'Janes 1990/91' claims that the latest Batch 3-version of the Type 42 may also be fitted with the *Seawolf* PDMS, which would make it, potentially, a very effective air-defence platform.

As an interested and involved observer in these overseas developments, the Royal Australian Navy (RAN) sought similar means of upgrading the air-defence of its high value warships. The US-manufactured *Charles F. Adams* class guided missile destroyers (DDGs), (HMAS *Perth*, *Hobart* and *Brisbane*) were commissioned over the period 1962-5, their primary anti-air warfare (AAW) role being to protect a group of ships by the medium- to long-range deployment of their *Standard* surface-to-air missile (SAM) system. Only in very recent times was one of the DDGs, HMAS *Brisbane*, given a short-range, self-defence capability by the fitting of two Phalanx CIWS amidships.

From the late '70s onwards, force structure commitments required the RAN to progressively purchase or manufacture in-country, a further six *Oliver Hazard Perry* class Frigates (FFG-7), five of which are in-service (HMAS *Adelaide, Canberra, Sydney, Darwin, Melbourne*), and the sixth, *HMAS Newcastle*, which was launched in February 1992. This class of ship was also equipped with *Standard* missile, and later was also the first class of RAN ship to be fitted with Phalanx CIWS on a back-fit (in 1985/6) or installed-at-point-of-manufacture basis.

Consistent with this acceptance of the concept of layered AAW defence for ships, computer modelling techniques (generally event-stepping) have been developed which enable the parallel, sequential and interactive elements of a ship's combat system responses to ASCM attack to be simulated. The air-defence performance effectiveness of a ship is best expressed in terms of its probability of surviving a given ASCM attack density, and through computer simulations this effectiveness can be obtained over a wide range of engagement conditions. How well, or indeed how badly, the respective combatants perform is obviously ɜpendent on many engagement parameters, some of the more obvious being:

* Threat density - determined by the number of ASCMs in the attack, and their separation in both bearing and time when due to reach the ship.

* Ship surveillance radar - the function of which is to provide a ship with an initial long-range indication or warning of an impending attack.

* Electronic Support Measures (ESM) - a passive means of intercepting enemy radar transmissions.

* Combat system reaction times - defined as the time between a ship's ESM sensor confirming the presence of a threat and a missile being launched against that threat. This time also includes ship command, control and weapons allocation functions.

* Single-shot kill probability of the ship's missile systems and CIWS kill probability.

* Weapon system maximum and minimum engagement ranges.

limiting case of ship AAW effectiveness arises when the number of attackers exceeds the capacity of a ιip's weapons systems to engage them all; at this point a ship's air defences are said to be saturated. In this tuation the threat's ability to concentrate an attack over a given period of time has in effect outstripped the ιm of combat system reaction times governing various firepower channels, and attendant missile times-of-ght; ie ship survivability has become independent of the kill probability of its air-defence missiles and guns.

owever, if time-on-target characteristics of the total ASCM attack are compatible with those of combat 'stem reaction time and weapons systems time-of-flight, thereby making it possible for the ship to at least ıgage all threats (as distinct from successfully killing them), then ship survivability does become dependent ι the kill probability of its missile and gun systems.

ıere are many complex physical and combatant interactions operative at any one time in a single-ship ɜrsus a multiple-ASCM attack, and the diffuseness of the resultant engagement outcomes may make it fficult to differentiate between the contributions which various weapons systems make in defending the ιip. For the analyst, this problem of determining various weapon systems contributions to air-defence ıpability may be partially resolved by devising engagement scenarios where the ship's capacity to defeat l threats is not impeded by time constraints. This basic assumption makes it possible to conduct some 'obabilistic analysis for particular multiple-ASCM versus single-ship scenarios, and it is the purpose of this emorandum to describe the means by which some aspects of ship air-defence (or alternatively ASCM ɜnetration) effectiveness have been analytically determined.

is intended that the analysis in this Memorandum be regarded as more illustrative than exhaustive, thus hile the fundamental *raison d'etre* for the study was to stress the layered-defence effectiveness of ship air-ɜfence in terms of a capability of killing all threats, the analysis was further adapted and extended to cover e attacker's effectiveness in penetrating other forms of layered defence.

ıus scenarios were devised where ASCM attacks were assumed to approach the ship in:

(a) A stream attack configuration (a **stream** attack is one made by a number of ASCMs approaching sequentially from the same sector), and the ship was defended by 'hard kill' systems (**'hard kill'** being defined as the function of missile systems and/or CIWS to physically destroy their targets).

(b) A wave attack configuration (a **wave** attack involves multiple ASCMs approaching the ship from different angles at the same distance and speed), and the ship was defended by 'hard kill/soft kill' systems (**'soft kill'** in this context being defined as the act of seducing the ASCM away from a ship by various electronic counter measures (ECM) means, without necessarily destroying the threat).

special case of (b) above arise where, in terms of its affect on the timing of ship air-defence engagements, e intended wave attack reduces to a stream attack, ie rather than reaching the ship simultaneously, there e appreciable time intervals between successive ASCM arrivals. As a result the ship is able to maximise the deployment of its 'hard kill' and 'soft kill' systems. This level of defence continues until such time that an ASCM penetrates; whereupon any other ASCMs following are assumed to hit the ship.

For the wave attack (and its stream attack 'special case') analysis of Sections 4, 5 and 6, the typical 'probability of success': P/ 'probability of failure': Q definition/notation has been specifically chosen to relate more to the probability of the ASCMs successfully penetrating the ship's defences. These definitions hinge on the size and number of ASCM strikes that constitute the attack, and the likelihood, for each ASCM launched, of finally attacking the ship. Obviously, the above definitions highlight that whatever is done by the offence, impinges directly on the defence - and vice-versa.

2. GENERAL ASSUMPTIONS FOR MULTIPLE-ASCM VERSUS SINGLE-SHIP ENGAGEMENTS

2.1 ASCM stream attack versus 'hard kill' defended ship

For a ship defended by 'hard kill' weapons systems alone, the primary function of the analysis was to demonstrate the increase in ship survivability that could be obtained against a fixed level of ASCM attack through an increase in the number of channels of fire. The assumptions were:

(1) The stream attack was made by two or four ASCMs.

(2) Each ASCM was engaged by similar levels of missile firepower, thus if (p_K , q_K) are defined as missile single-shot kill/miss probabilities, and (p'_K , q'_K) are defined as the missile salvo kill/miss probabilities, then obviously for a single-missile salvo fired against each threat:

$$p'_K = p_K$$

and

$$q'_K = q_K$$

but, for a two-missile salvo

$$p'_K = 1 - (1 - p_K)^2 \quad (1)$$

and

$$q'_K = (1 - p_K)^2 \quad (2)$$

where, by definition:

$$p'_K + q'_K = 1,$$

and

$$p_K + q_K = 1,$$

(3) All ASCMs were capable of being intercepted prior to the time the first ASCM reached the ship.

(4) Each CIWS, subject to arc-of-fire limitations (see below), was able to engage only one ASCM. CIWS probability of kill was defined by p_C.

(5) All threats approached from a common direction, or sector. Given that the CIWS had arcs-of-fire, α_1 and α_2 (see figure 1), for a single CIWS-fitted ship, the probability of a CIWS being able to engage an ASCM in the attack, p_{A1} , was expressed by:

$$p_{A1} = \alpha_1/360. \quad (3)$$

while, for a ship fitted with two CIWS, the first ASCM penetrating through the PDMS was always engaged by a CIWS, but the probability of the second CIWS being able to engage a second ASCM penetrating through the PDMS, p_{A2}, was:

$$p_{A2} = \quad [(\alpha_1 + \alpha_2)/360 - 1]. \qquad (4)$$

(It should be stressed that in the context of this Memorandum, CIWS 'availability' refers to the ability of a CIWS to engage a threat, if so required.)

ASCM wave attack versus 'hard kill/soft kill'defended ship

: following assumptions applied for the wave attack scenario for a ship fitted with a 'hard kill/soft kill' ·defence mix:

(1) The plan of attack was for an initial strike of four ASCMs to saturate the known channels of 'hard kill' and 'soft kill' defences (see below). However, the analysis allowed for the possibility that not all ASCMs made the attack - for example, there might be operational procedure difficulties, or an ASCM might fail to acquire its target. The intended plan of attack was for two pairs of closely-separated ASCMs to attack from either side of the ship, figure 1.

(2) A PDMS channel, situated either side of the ship, was able to engage either the lead ASCM of each pair, or a single ASCM attacking from a particular direction.

(3) A ship deploying centroid chaff was assumed capable of seducing an attack from one direction, but there was not sufficient time for it to manoeuvre and deploy further centroid chaff to counter the ASCM attack from another direction; ie ECM was capable of seducing, at best, only one or other of the ASCM pairs.

(4) As described in the previous section, a limiting case of the wave attack scenario was defined to be where the ASCMs attack the ship one at a time; the ship is able to deploy both 'hard kill' and 'soft kill' defences against every threat until such time as one ASCM penetrates these defences.

(5) If insufficient damage - owing to such factors as ASCMs failing to attack, or all ASCMs being defeated by the ship's defences in the initial strike - was done to the ship by the initial ASCM strike, then additional, similar-size, ASCM strikes would be launched.

3. IMPROVEMENT IN SHIP AIR-DEFENCE CAPABILITY GAINED BY ADDITION OF EXTRA CIWS

: analysis of this section is concerned with determining the air-defence capability of a PDMS/CIWS fitted p based on the engagement scenario defined in Section 2.1. For a ship fitted with PDMS and a single VS engaged against the stream attack of two ASCMs, the decision tree of figure 2(a) indicates:

Prob.(killing both ASCMs) = Prob.(both ASCMs killed by PDMS) + Prob.(1 ASCM killed by PDMS, 1 ASCM by CIWS),

$$= \quad p'^2_K + 2p'_K q'_K p_{A1} p_C, \qquad (5)$$

ile for a ship fitted with a PDMS and two CIWS engaged against two closely-separated ASCMs, the ision tree of figure 2(b) likewise indicates:

Prob.(killing both ASCMs) = Prob.(both ASCMs killed by PDMS)
+
Prob.(1 ASCM killed by PDMS, 1 ASCM by CIWS)
+
Prob.(both ASCMs killed by CIWS),

$$= p'^2_K + 2p'_K q'_K p_C + q'^2_K p_{A2} p_C^2 . \qquad (6)$$

The difference between eq.(6) and eq.(5), Δp, yields the improvement in the probability of the ship's defences killing both ASCMs when two, rather than a single CIWS are fitted, and obviously,

$$\Delta p = 2p'_K q'_K (1 - p_{A1}) p_C + q'^2_K p_{A2} p_C^2 . \qquad (7)$$

For a ship fitted with PDMS and a single CIWS engaged against four closely-separated ASCMs, the equivalent function to eq.(5) may be derived using the decision tree of figure 2(c) as

Prob.(All 4 ASCMs are killed) = Prob.(All 4 ASCMs killed by PDMS)
+
Prob.(3 ASCMs killed by PDMS, 1 ASCM by CIWS).

$$= p'^4_K + 4p'^3_K q'_K p_{A1} p_C . \qquad (8)$$

For a ship fitted with PDMS and two CIWS engaged against four closely-separated ASCMs, the equivalent function to eq.(6) may be derived using the decision tree of figure 2(d) as

Prob.(All 4 ASCMs are killed) = Prob.(All 4 ASCMs killed by PDMS)
+
Prob.(3 ASCMs killed by PDMS, 1 ASCM by CIWS)
+
Prob.(2 ASCMs killed by PDMS, 2 ASCMs by CIWS).

$$= p'^4_K + 4p'^3_K q'_K p_C + 6p'^2_K q'^2_K p_{A2} p_C^2 . \qquad (9)$$

Once again the difference between eq.(9) and eq.(8) yields the improvement in the probability of the ship's defences killing all four ASCMs when an additional CIWS is fitted, and it is seen that this function equals

$$\Delta p = 4p'^3_K q'_K (1 - p_{A1}) p_C + 6p'^2_K q'^2_K p_{A2} p_C^2 . \qquad (10)$$

From eqs.(5), (6), (8) and (9) it is possible to generate functions of the probability of killing all ASCMs for all values of p'_K and p_C, figures 3(a), 3(b) and figures 4(a), 4(b). From eqs.(7) and (10) the magnitude of various kill probability improvement levels possible when an extra CIWS is added to a ship's air defence system can be calculated and these improvements are illustrated in figures 3(c) and 4(c). The p'_K and p_C axes define, of course, the baseline fits of ships fitted with a single layer of PDMS or CIWS.

4. EXPECTED NUMBER OF HITS BY WAVE ATTACKS OF FOUR ASCMs

The emphasis of the analysis of this section is to determine the air-defence penetration capability of a multiple-ASCM wave attack against a 'hard kill/soft kill' defended ship, see Section 2.2. For the engagement scenario where the launch platform's plan is to attack the ship with four closely-separated ASCMs, it is assumed that the ship's 'hard kill'/'soft kill' defensive response will ultimately be dependent on the magnitude and disposition of the ASCM attack. For example, if the intended lead missile of the first pair of ASCMs is unable (for whatever reason) to press home the attack, but the second ASCM does, then this latter missile will draw the ship's combined weapon and ECM defences. The probability of the second ASCM penetrating the ship's defences for this particular sequence of events occurring is simply:

Prob.(1st ASCM did not attack) x Prob.(2nd ASCM does attack) x $p_H p_S$, or $q_A p_A\, p_H p_S$, where

p_A is the probability that an individual ASCM attacks,

l conversely

q_A is the probability that an individual ASCM does not attack,

p_H is the probability of an ASCM surviving the ship's defences,

l conversely

q_H is the probability of a ship's 'hard kill' defences killing an ASCM.

ewise,

p_S is the probability of an ASCM surviving a ship's 'soft kill' defences,

conversely,

q_S is the probability of a ship's 'soft kill' defences seducing the ASCM.

ilarly, if the first ASCM pair and also the lead missile of the second ASCM-pair fail to attack, then if the l ASCM attacks, it will likewise have to penetrate the ship's combined defences; given that it does, the bability of this sequence of events occurring is $q_A^3 p_A\, p_H p_S$.

ummary, the level of defence encountered by a particular ASCM, and hence the expected number of hits the total attack, is dependent on the number and disposition of the preceding ASCMs in that attack.

ıs the expected number of hits for the first ASCM, E_1, is its probability of defeating the ship's defences,

$$E_1 = p_A p_H p_S \qquad (11)$$

le for the second ASCM, the expected number of hits, E_2, is the sum of the two probabilities:

Prob.(1st ASCM attacks) x [Prob.(2nd ASCM attacks) x p_S]

\+

Prob.(1st ASCM does not attack) x [Prob.(2nd ASCM attacks) x $p_H p_S$]

$$= p_A^2\, p_S + q_A p_A\, p_H p_S,$$

ıch simplifies to

$$E_2 = p_A p_S\, (1 - q_A q_H). \qquad (12)$$

ne ASCM from the first pair attacks, it will still be subjected to chaff decoy measures, but for the third CM - the lead ASCM of the second pair - the penetration probability of the third ASCM remains at p_H. wever, if neither of the lead pair of ASCMs attacks, then the first available (in this case third) ASCM, will e to survive both 'hard kill' and 'soft kill' measures (branch 4 of figure 5). For a similar argument to that lied for the second ASCM above, the expected number of hits, E_3, for the third ASCM will be:

$$E_3 = p_A(p_A^2 + 2q_A p_A)\, p_H + q_A^2 p_A\, p_H p_S.$$

$$E_3 = p_A p_H\, (1 - q_A^2\, q_S). \qquad (13)$$

If one of the lead pair of ASCMs is able to attack, but the third ASCM does not, then the fourth and final ASCM will have to survive the ship's air-defence missile/gun systems, see branches 2,4, and 6 of figure 6. However, if neither of the lead pair of ASCMs attacks, then whether the fourth ASCM is the sole ASCM to attack or not, determines whether the ship can respond with 'soft kill' alone, or combined 'hard kill/soft kill' measures, (branches 7 and 8 of figure 6 respectively).

Summing over the eight possible engagement outcomes, the expected number of hits, E_4, for the fourth ASCM yields:

$$p_A^4 + 2p_A^3 q_A \qquad \text{(branches 1, 3 and 5),}$$

$$(p_A^2 + 2p_A q_A)\, p_A q_A\, p_H \qquad \text{(branches 2, 4 and 6),}$$

$$q_A^2 p_A^2\, p_S \qquad \text{(branch 7),}$$

and

$$q_A^3 p_A\ p_H p_S \qquad \text{(branch 8),}$$

which, on summing, yields

$$E_4 \quad = \quad p_A\,(1 - q_A q_H)\,(1 - q_A^2\, q_S). \qquad (14)$$

Hence the total expected number of hits for the four-ASCM strike is obtained from eqs.(11), (12), (13) and (14) as

$$E(4) \quad = \quad p_A\left(1 + p_H - q_A q_H\right)\left(1 + p_S - q_A^2 q_S\right). \qquad (15)$$

5. ASCM SALVO-SIZE REQUIRED TO ACHIEVE SPECIFIED LEVELS OF SHIP DAMAGE

The analysis of this section determines M_L, the expected number of ASCMs launched to inflict the required level of ship damage, given that a hit (or hits) achieved by any strike will render the ship defenceless against further ASCMs; however, if there are no hits obtained by the first strike, then a further strike by four ASCMs will be made, etc.

Thus, if i hits are made by the first ASCM strike, the number of ASCMs ultimately required to be launched to achieve M_D hits can be determined from the decision tree representation of figure 7 as:

$$M_{L1} \quad = \quad \sum_{i=1}^{i=4} \left[\, 4 + (M_D - i)/p_A \right] p_i \qquad (16)$$

where

p_i is the probability of the ASCM strike achieving i hits on the ship, and

M_{L1} is the expected number of ASCMs launched, given that the first ASCM strike achieved at least one hit.

The summation of eq.(16) is only valid for $M_D \geq i$, and by definition $M_L \geq 4$. For example, if only two hits are required to cripple the ship, but three hits are obtained, then $M_L = 4$ and there is no need to launch further ASCM strikes.

With reference to eq.(16), since the term

$$\sum_{i=1}^{i=4} i p_i$$

is, by definition, the expected number of hits, E(4), for the four-ASCM strike, and

$$\sum_{i=1}^{i=4} p_i = 1 - p_0 = q_0$$

where, from the abbreviated decision tree of figure 8, it is seen that the probability, p_0, of no hits being recorded by the initial four-ASCM strike is given by:

$$p_0 = q_A^4 + 4p_A q_A^3 (1 - p_H p_S) + 2p_A^2 q_A^2 (1 - p_H p_S)(q_S + 2q_H) + 2p_A^3 q_A (1 - p_H p_S) q_H q_S \tag{17}$$

and hence, eq.(16) may be written,

$$M_{L1} = [4 + M_D/p_A] q_0 - E(4)/p_A. \tag{18}$$

However, if no hits are recorded, a similar four-ASCM strike is repeated, the number of ASCMs required to achieve the required level of ship damage, M_{L2}, given that at least one of the ASCMs in the second strike does hit, will be

$$M_{L2} = M_{L1} + p_0([8 + M_D/p_A] q_0 - E(4)/p_A) . \tag{19}$$

Similarly, if this second ASCM strike fails, a third strike is called, and

$$M_{L3} = M_{L2} + p_0^2([12 + M_D/p_A] q_0 - E(4)/p_A) . \tag{20}$$

Hence with N four-ASCM strike failures, followed-on by the (N+1) th attack recording a hit, the total number of ASCMs launched to obtain M_D hits, can be expressed as:

$$M_{LN} = \left[M_D q_0 - E(4)\right] \left(1 + p_0 + p_0^2 + ... + p_0^{N-1} \right)/p_A + 4q_0 (1 + 2p_0 + 3p_0^2 + ... + Np_0^{N-1}) . \tag{21}$$

On summing the geometric and arithmetic/geometric series of eq.(21)

$$M_{LN} = \left[M_D q_0 - E(4)\right] \left(1 - p_0^N \right) /q_0 p_A + 4q_0 [(1 - p_0^N)/ q_0^2 - Np_0^N/q_0] \tag{22}$$

Finally, for large N, eq.(22) equals:

$$M_L = \frac{M_D}{p_A} - \frac{E(4)}{p_A q_0} + \frac{4}{q_0}. \tag{23}$$

6. EXPECTED NUMBER OF HITS FOR ASCM STREAM ATTACKS

For a multiple-ASCM strike to achieve the desired level of saturation of a ship's air defences requires a considerable amount of co-ordination skill (or perhaps luck) on the part of the attacking forces, and conceivably all ASCMs may not achieve what is termed a satisfactory simultaneous time-on-target (STOT).

An alternative methodology was therefore developed to evaluate the reduction in expected number of ASCM hits for a scenario where STOT was not achieved, but to the extent that the ship is able to deploy both 'hard kill' and 'soft kill' defences against every threat, the wave attack reduces to a stream attack. This maximum level of air-defence capability continues up till the time a particular ASCM penetrates, whereupon any subsequent ASCMs that attack are assumed to also hit the ship.

Based on these assumptions, a decision tree for this engagement scenario can be constructed to cover all possible outcomes for this engagement scenario, figure 9. If the first ASCM defeats the ship's defences, the expected number of hits resulting from this, and subsequent ASCMs would be:

$$p_A p_H p_S \, [1 + (N\text{-}1) p_A].$$

Where the first ASCM in the stream either did not attack or fails to penetrate the ship's defences, but the second ASCM attacks and penetrates, then the expected number of hits arising from this, and subsequent ASCMs equals:

$$(1 - p_A p_H p_S) \, p_A p_H p_S \, [1 + (N\text{-}2) p_A].$$

Similarly where the first penetration of the ship's defences is achieved by the third ASCM, the equivalent expected number of hits would be:

$$(1 - p_A p_H p_S)^2 \, p_A p_H p_S \, [1 + (N\text{-}3) p_A],$$

and finally, for the N th ASCM, the expected number of hits would be:

$$(1 - p_A p_H p_S)^{N-1} \, p_A p_H p_S.$$

The term $(1 - p_A p_H p_S)$, or q_F, is the probability of an ASCM mission failure, either through incorrect functioning or inability to penetrate the ship's defences. Thus after some arrangement of terms, and summing for the individual hit probabilities, the expected number of hits for the planned attack by N ASCMs is given by:

$$p_A p_H p_S [\, (1 + N p_A)\,(1 + q_F + q_F^2 + ... + q_F^{N-1})$$

$$- \, p_A \,(\, 1 + 2\, q_F + 3\, q_F^2 + ... + N\, q_F^{\,N-1})] \qquad (24)$$

Summing the geometric and arithmetic-geometric series of eq.(24) in the usual manner, the expected number of hits for the total attack, E(N), is given by:

$$E(N) \quad = \quad N p_A + (\,1 - q_F^N)\,[1 - p_A/(1 - q_F)]. \qquad (25)$$

For large N, eq.(25) asymptotes to:

$$E(N) \quad = \quad N p_A - (1 - p_H p_S)/p_H p_S \,.$$

and for $p_A = 1$,

$$E(N) \quad = \quad N{+}1 - \left[1 - (1 - p_H p_S)^{N+1}\right]/p_H p_S. \qquad (26)$$

The dependence of E(4) and M_L on p_A, p_H and p_S is illustrated in figures 10, 11, 12 and 13, for both wave and stream modes of ASCM attack.

7. DISCUSSION

The two examples of layered air-defence systems analysis developed for this Memorandum indicate there are numerous options available for refining and extending the level of complexity for various single-ship versus multiple-ASCM engagement scenarios.

For example, for the scenario where a wave attack of four ASCMs attacked the 'hard kill/soft kill' defended ship, the form of defence meant there was no defence possible against the fourth, and final, ASCM; in this instance, the ship's air defences would be classed as saturated. On the other hand, if the ship were fitted with two CIWS, and the time interval between threats permitted more than a single engagement, or with centroid chaff whose effectiveness was not assumed to be directionally dependent, then the four ASCMs would not necessarily constitute a saturation level attack; however, analysis of such a scenario would involve a considerable expansion of the current methodology. Alternatively, if it were required to determine the effect of p_A for the ASCM stream attack versus PDMS/CIWS defended ship, extra decision trees based on p_A could be readily generated to yield probabilities of ship survival against attacks by one, two, three or four ASCMs.

The following numerical examples, and figures 3,4, 10-13 provide some indication of the wide divergence of ship air-defence capability results that are possible for the engagement scenarios defined for this study.

Where two ASCMs attack a ship defended by PDMS and a single CIWS, whose arc-of-fire is 270 deg. (ie from eq.(3), $p_{A1} = 0.75$), eq.(5) and figure 3(a) can be used to show:

1. P(both ASCMs killed) = 0.05 when $p_C = 0.$ and $p'_K = 0.05^{1/2} = 0.22$
2. P(both ASCMs killed) = 0.05 when $p_C = 1.$ and $p'_K = 0.03$
3. P(both ASCMs killed) = 0.40 when $p_C = 0.4$ and $p'_K = 0.50.$

Where two CIWS (each having an arc-of-fire of 270 deg. ie from eq.(4), $p_{A2} = 0.50$) provide extra 'back-up' support for the PDMS for a similar engagement scenario to that described above, eq.(6) and figure 3(b) show for:

4. P(both ASCMs killed) = 0.05 when $p'_K = 0.$ and $p_C = 0.10^{1/2} = 0.32.$
5. P(both ASCMs killed) = 0.50 when $p_C = 0.70$ and $p'_K = 0.30$
6. P(both ASCMs killed) = 0.85 when $p_C = 0.80$ and $p'_K = 0.70.$

The top left-hand corner of figure 3(c) illustrates that as p'_K tends to zero, the probability of both ASCMs being killed (for a guaranteed CIWS kill, $p_C = 1$) equals the probability of both CIWS being available for their respective engagements, p_{A2}.

For an attack by four closely-separated ASCMs on a ship defended by a PDMS and a single CIWS, eq.(8) and figure 4(a) indicate, for $p_{A1} = 0.75$:

7. P(all four ASCMs killed) = 0.05 when $p_C = 0$ and $p'_K = 0.05^{1/4} = 0.47$
8. P(all four ASCMs killed) = 0.05 when $p_C = 0.74$ and $p'_K = 0.30$
9. P(all four ASCMs killed) = 0.49 when $p_C = 0.80$ and $p'_K = 0.70.$

Where the ship is fitted with two CIWS (again, $p_{A2} = 0.5$) in addition to a PDMS, eq.(9) and figure 4(b) show:

10. P(all four ASCMs killed) = 0.46 when $p_C = 0.70$ and $p'_K = 0.60$

11. P(all four ASCMs killed) $= 0.79$ when $p_C = 0.70$ and $p'_K = 0.84$

12. P(all four ASCMs killed) $= 0.65$ when $p_C = 0.80$ and $p'_K = 0.70$.

In the case of 10. and 11. (above) p'_K values of 0.60 and 0.84 have been selected to illustrate the improvement in P(all 4 ASCMs killed) if two-missile, rather than single-missile salvos, are launched at each ASCM. (Note: for a two-missile salvo $p'_K = 1 - (1 - 0.6)^2 = 0.84$, see eq(1).)

Finally, use of elementary calculus shows that after differentiation of eq.(10), the maximum improvement ($\Delta p = 0.260$) of fitting two, rather than one, CIWS occurs (again for $p_{A2} = 0.5$) when:

$$p_C = 1 \text{ and } p'_K = (15 - \sqrt{33})/16.$$

The analysis derived for the 'hard kill/soft kill' fitted ship when attacked by an intended wave strike of four ASCMs determines the expected number of hits as an individual ASCM's probability of attack, p_A, increases from 0.3 to 1.0 in 0.1 steps, figure 10. For the following discussion p_A is set at a nominal value of 0.8.

If decoy measures are totally effective (ie $p_S = 0$), an ASCM probability of defeating a ship's 'hard kill' defences of $p_H = 0.42$ is required if the expected number of hits by the ASCM strike is to be close to unity. When $p_S = 0.5$ for this value of p_H, the expected number of hits for an intended four-ASCM strike rises to 1.54.

A stream attack by ASCMs, where time intervals between successive ASCMs are sufficiently large, gives the ship's defences extra time to re-group and defeat each incoming threat. Again, where 'soft kill' measures perform perfectly (ie $p_S = 0$), ship survivability is independent of its 'hard kill' defences, since all ASCMs are seduced away from the ship. For $p_H = 0.42$ and $p_S = 0.5$, an expected 1.24 hits would be obtained from the first four ASCMs launched, see figure 11.

Figures 12 and 13 show plots of the expected number of ASCMs required to be launched in order to attain a given ship-damage criterion, which in this case is assumed to be four hits. For the values of p_A, p_H discussed above and $p_S = 0.5$, it can be shown from eq.(23) that for an attack conducted by planned, closely-separated waves of four ASCMs, 7.40 ASCMs will need to be launched to obtain the expected four hits. For the special case of the stream attack by successive, single ASCMs, then again from eq.(25), 8.75 ASCMs are required.

8. CONCLUSIONS

A probabilistic decision tree approach has been used to provide some estimates of a single ship's layered air-defence capability for scenarios where weapon system deployment in response to an multiple-ASCM attack was assessed as feasible for given combat system reaction times. The prime intention of this study was to express this capability in generalised terms of the more critical engagement parameters (in particular, threat density and weapons systems probabilities of kill), and in as basic a form as practicable.

A limited number of single-ship versus multiple-ASCM scenarios were considered, and assumptions made concerning ship awareness and command and control responses, and ASCM attack density and engagement geometry. The analytic techniques developed for determining single-ship air-defence effectiveness, or alternatively the number of hits that an ASCM strike might obtain against a given level of ship air-defence, were not exhaustive in their application, which suggests there is further scope for modifying the separate methodologies, or indeed unifying them into a single methodology.

The development of these probabilistic methodologies provides a measure of the relative levels of improvement that are possible through the implementation of layered air-defence systems in ships.

NOTATION

E_i (i=1,4)	expected number of hits for ith missile in stream attack
E(N)	expected number of hits for stream attack by N missiles
M_D	number of ASCMs hitting ship to achieve ship damage criteria
M_L	expected number of ASCMs launched to achieve ship damage criteria
M_{Li} (i=1,N)	number of ASCMs launched, given at least one hit by i th ASCM strike
p_A	probability of ASCM attacking ship
q_A	probability of ASCM not attacking ship
p_{A1}	probability of CIWS 1 engaging threat
p_{A2}	probability of CIWS 2 engaging threat
p_C	CIWS kill probability
p_H	probability of ASCM penetrating ship's 'hard kill' defences
q_H	probability of ASCM not penetrating ship's 'hard kill' defences
p_i (i=1,4)	probability of ASCM strike achieving i hits
p_K	missile single-shot kill probability
q_K	missile single-shot miss probability
p'_K	missile salvo kill probability
q'_K	missile salvo miss probability
p_S	probability of ASCM not being seduced by 'soft kill'' systems
q_S	probability of ASCM being seduced by 'soft kill' systems
q_F	probability of mission failure for ASCM
Δp	kill probability differential for two- and single-CIWS fitted ships
p_0	probability of no ASCM penetrating ship's air defences
q_0	probability of at least one ASCM penetrating ship's air defences
α_1	CIWS 1 arc-of-fire
α_2	CIWS 2 arc-of-fire

α_1 CIWS 1 arc-of-fire

α_2 CIWS 2 arc-of-fire

$$\Phi = 90 - (360 - \alpha_1)/2 + 90 - (360 - \alpha_2)/2$$
$$= -180 + (\alpha_1 + \alpha_2)/2$$
$$p_{A2} = 2\Phi/360 = [(\alpha_1 + \alpha_2)/360 - 1]$$

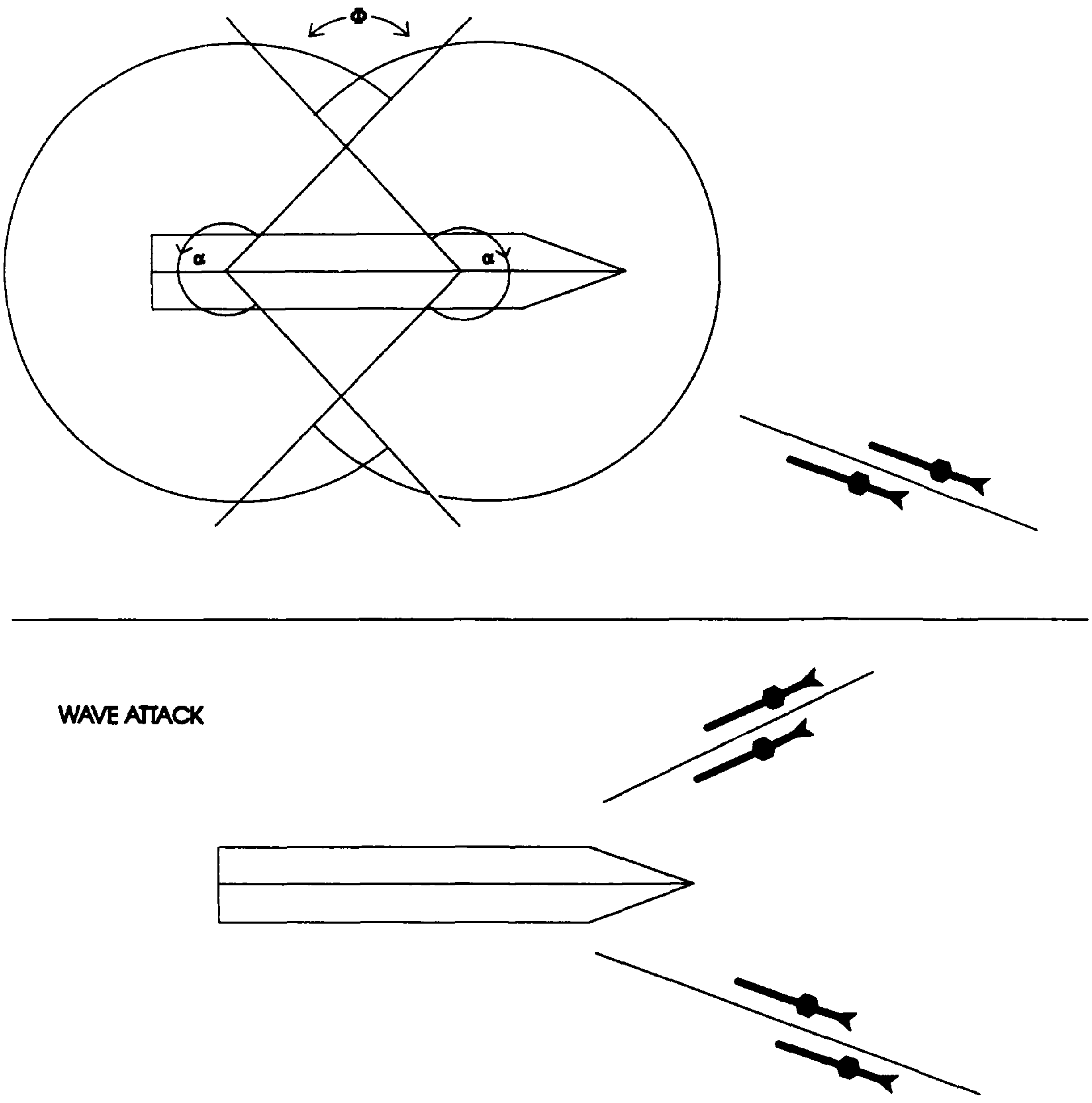


Figure 1 **Engagement geometry and CIWS arcs-of-fire**

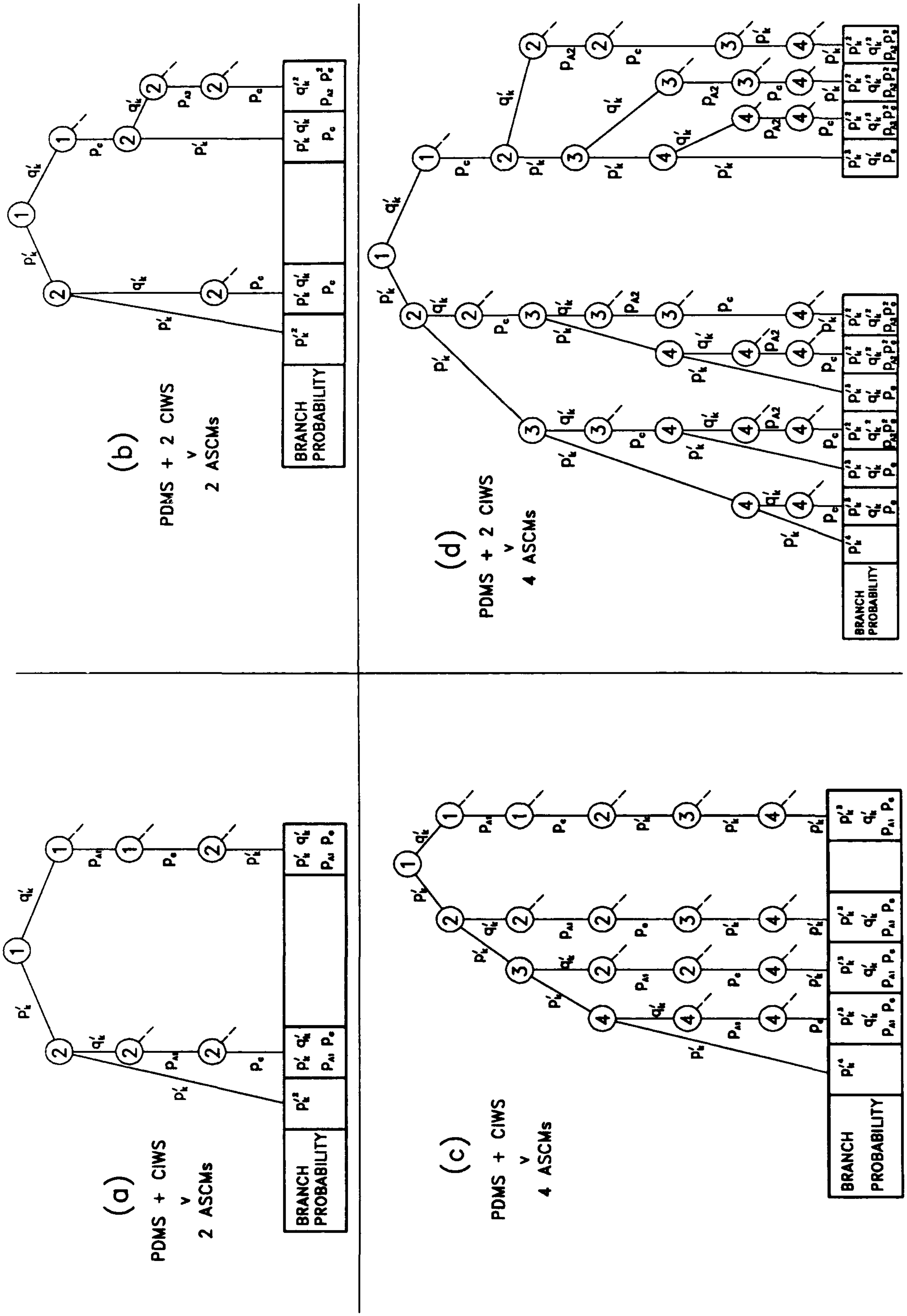


Figure 2 Decision tree branches indicating successful engagements of stream attacks of two and four ASCMs by PDMS and CIWS-fitted ships

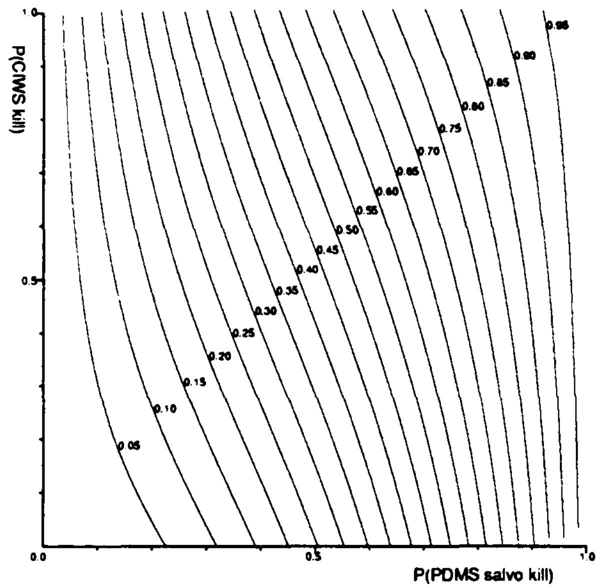


FIGURE 3(a)
PROBABILITY CONTOURS OF BOTH ASCMs BEING KILLED FOR SHIP DEFENDED BY PDMS AND SINGLE CIWS

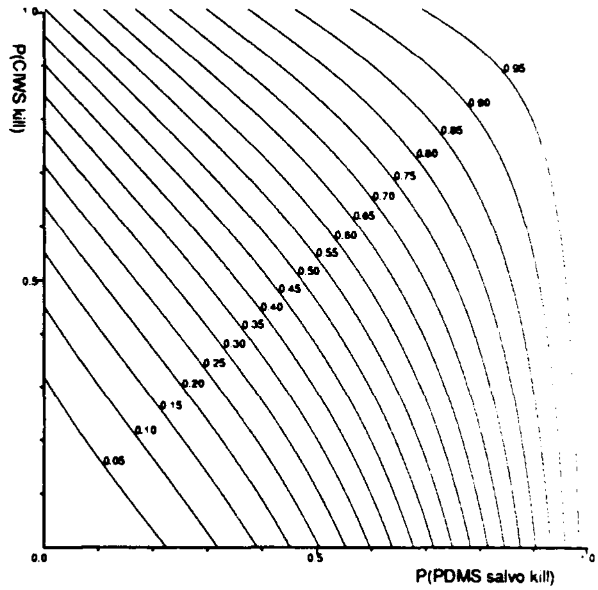


FIGURE 3(b)
PROBABILITY CONTOURS OF BOTH ASCMs BEING KILLED FOR SHIP DEFENDED BY PDMS AND TWO CIWS

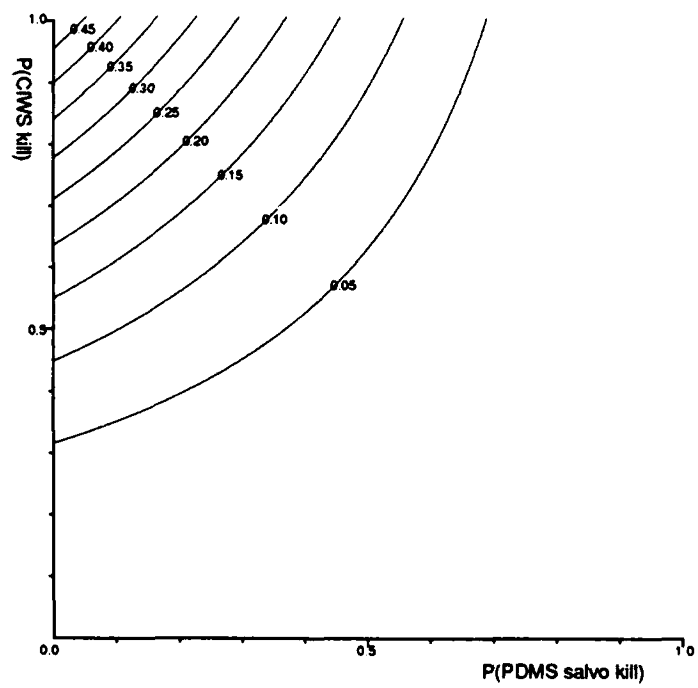
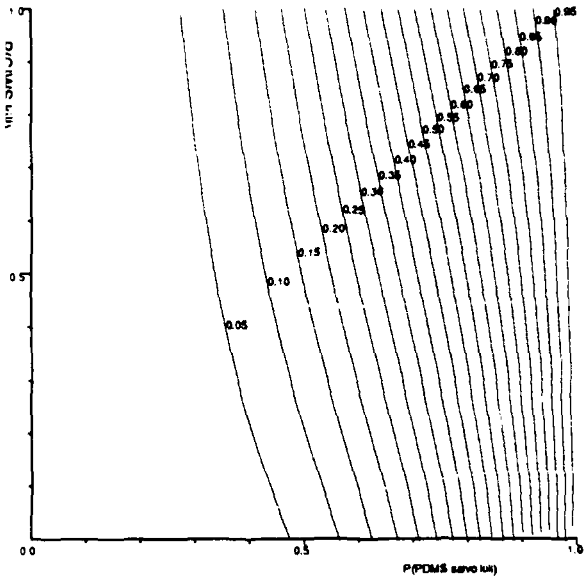


FIGURE 3(c)
CONTOURS OF IMPROVEMENT IN PROBABILITY OF KILLING BOTH ASCMs WHEN SECOND CIWS IS ADDED TO SHIP WEAPONS SYSTEMS FIT

Figure 3 Probability of stream attack of two ASCMs being killed by PDMS/CIWS layered defence



GURE 4(a)
OBABILITY CONTOURS OF ALL FOUR ASCMs BEING
LLED FOR SHIP DEFENDED BY PDMS AND SINGLE
WS

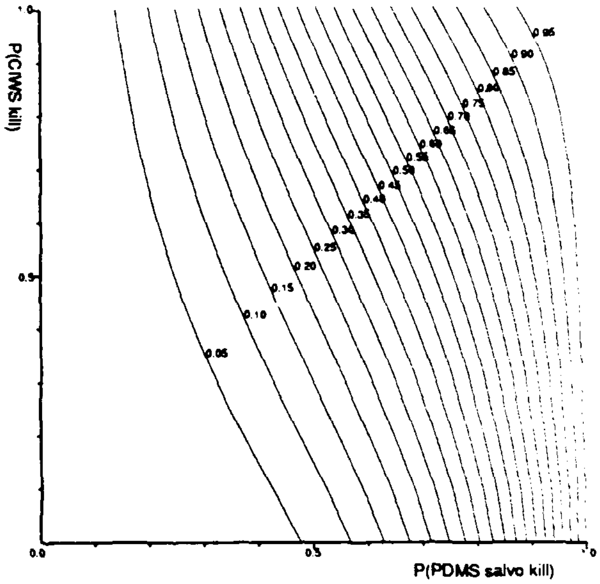


FIGURE 4(b)
PROBABILITY CONTOURS OF ALL FOUR ASCMs BEING KILLED FOR SHIP DEFENDED BY PDMS AND TWO CIWS

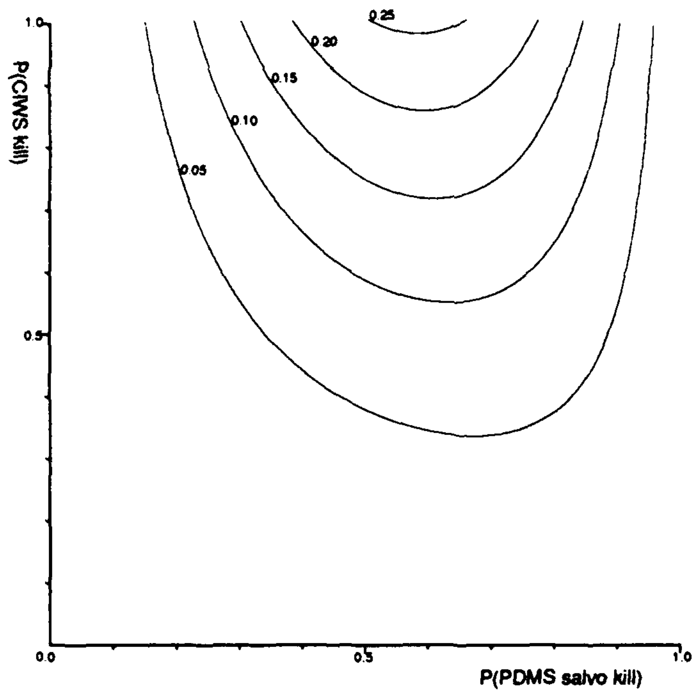
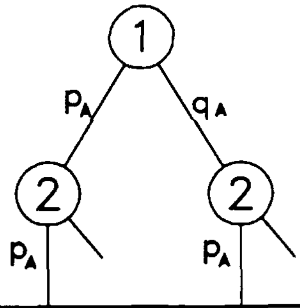


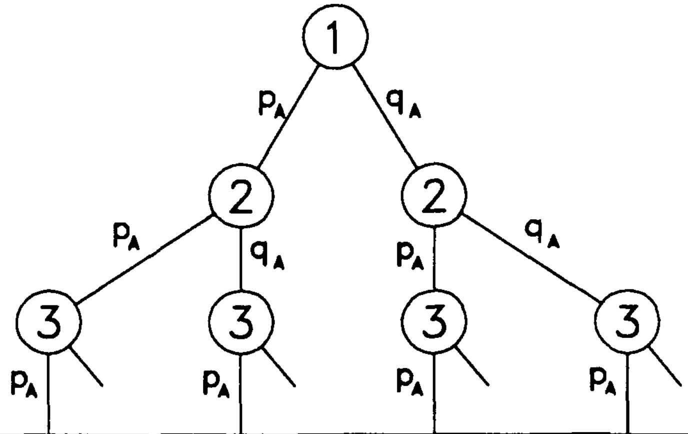
FIGURE 4(c)
CONTOURS OF IMPROVEMENT IN PROBABILITY OF KILLING ALL FOUR ASCMs WHEN SECOND CIWS IS ADDED TO SHIP WEAPONS SYSTEMS FIT

gure 4 Probability of stream attack of four ASCMs being killed by PDMS/CIWS layered defence

p_A	:	Prob (ASCM attacks)
p_H	:	Prob (ASCM penetrates hard kill defence)
p_S	:	Prob (ASCM penetrates soft kill defence)



BRANCH NUMBER	1	2
P(2nd ASCM PENETRATES)	p_A^2 p_S	$p_A q_A$ $p_H p_S$



BRANCH NUMBER	1	2	3	4
P(3rd ASCM PENETRATES)	p_A^3 p_H	$p_A^2 q_A$ p_H	$p_A^2 q_A$ p_H	$p_A q_A^2$ $p_H p_S$

Figure 5 **Decision trees for expected number of hits for second and third ASCMs in intended wave attack of four ASCMs**

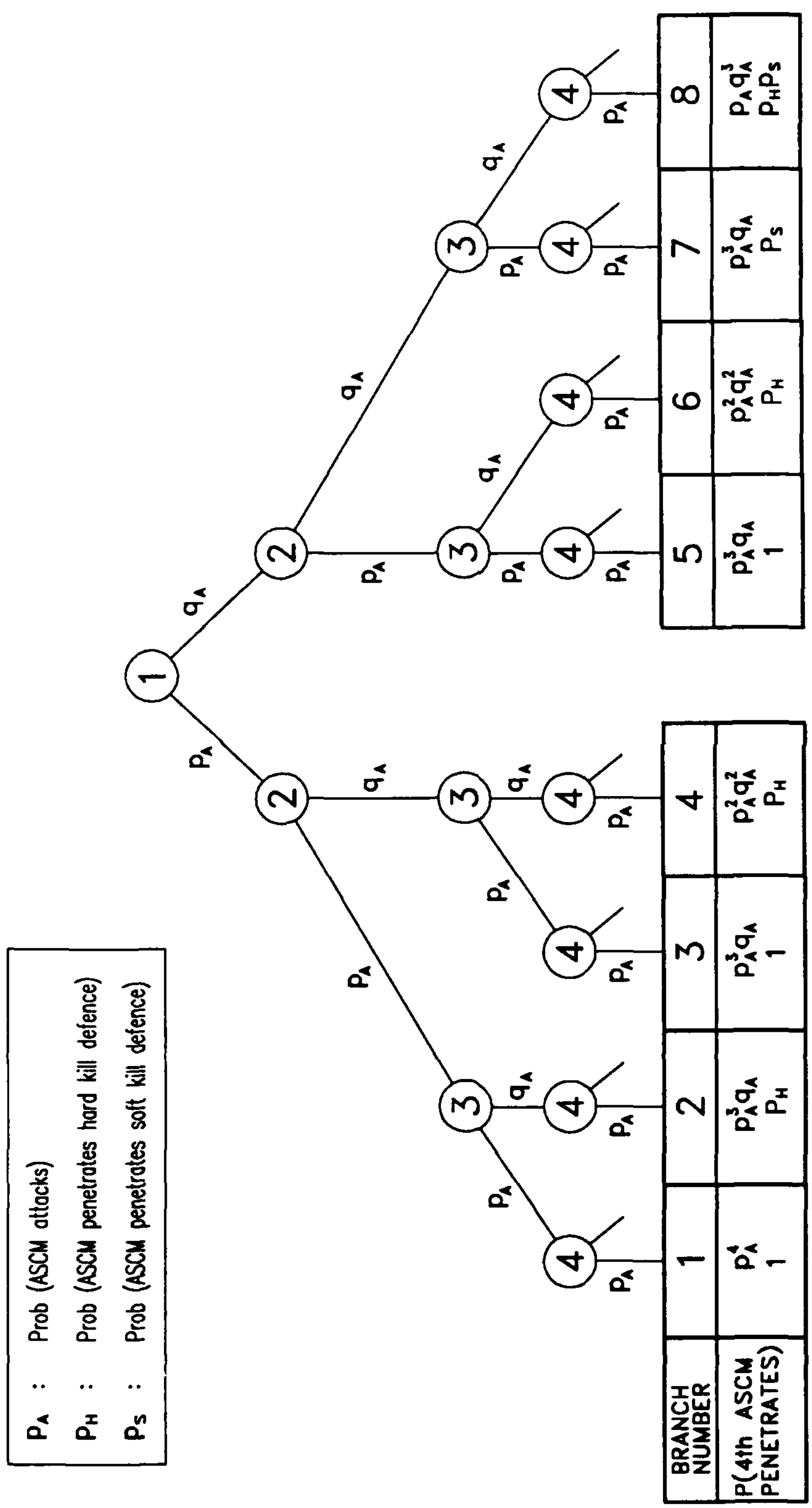


Figure 6 **Decision tree for expected number of hits for fourth ASCM in intended wave attack of four ASCMs**

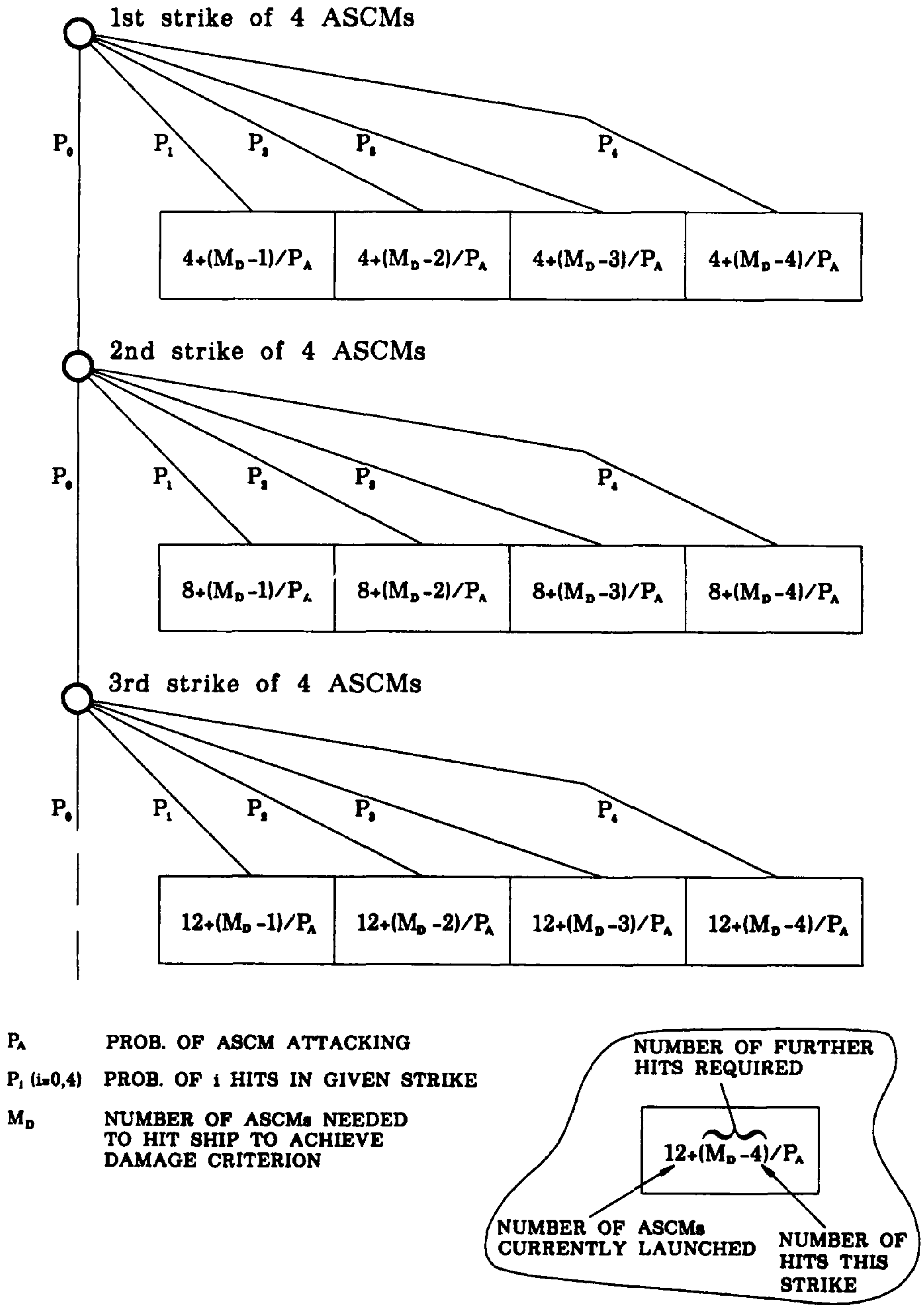
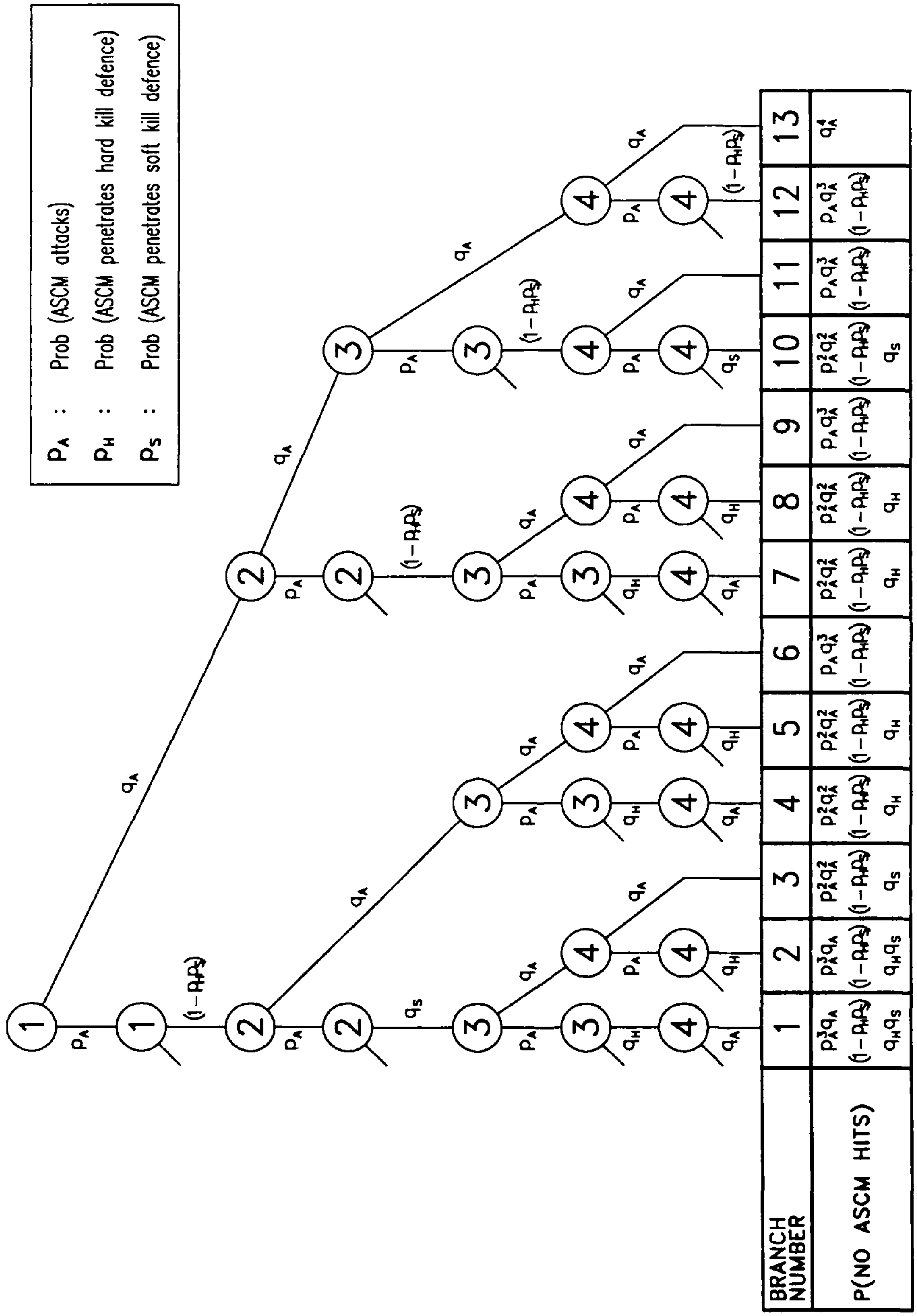


Figure 7 Decision tree for expected number of ASCMs launched in wave attack to achieve ship damage criterion



BRANCH NUMBER	P(NO ASCM HITS)
1	$p_A^3 q_A (1-P_H P_S) q_H q_S$
2	$p_A^3 q_A (1-P_H P_S) q_H q_S$
3	$p_A^2 q_A^2 (1-P_H P_S) q_S$
4	$p_A^2 q_A^2 (1-P_H P_S) q_H$
5	$p_A^2 q_A^2 (1-P_H P_S) q_H$
6	$p_A q_A^3 (1-P_H P_S)$
7	$p_A^2 q_A^2 (1-P_H P_S) q_H$
8	$p_A^2 q_A^2 (1-P_H P_S) q_H$
9	$p_A q_A^3 (1-P_H P_S)$
10	$p_A^2 q_A^2 (1-P_H P_S) q_S$
11	$p_A q_A^3 (1-P_H P_S)$
12	$p_A q_A^3 (1-P_H P_S)$
13	q_A^4

Figure 8 **Partial decision tree indicating those branches which result in no hit on ship by intended wave attack of four ASCMs**

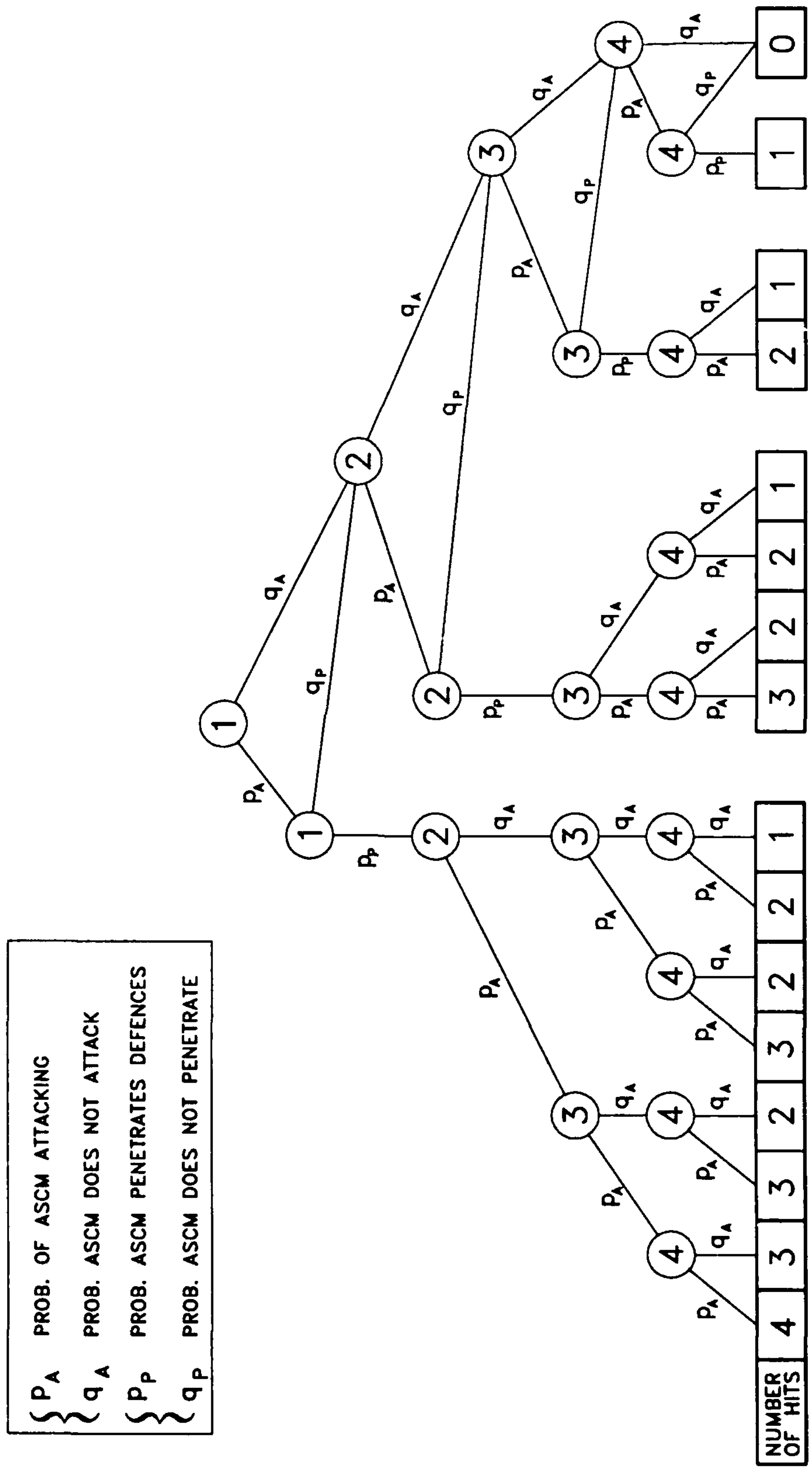


Figure 9 **Decision tree for expected number of hits resulting from multiple-ASCM stream attack**

P_A : PROB(Individual ASCM available for attack)
P_H : PROB(Individual ASCM penetrates hard kill defences)
P_S : PROB(Individual ASCM penetrates soft kill defences)

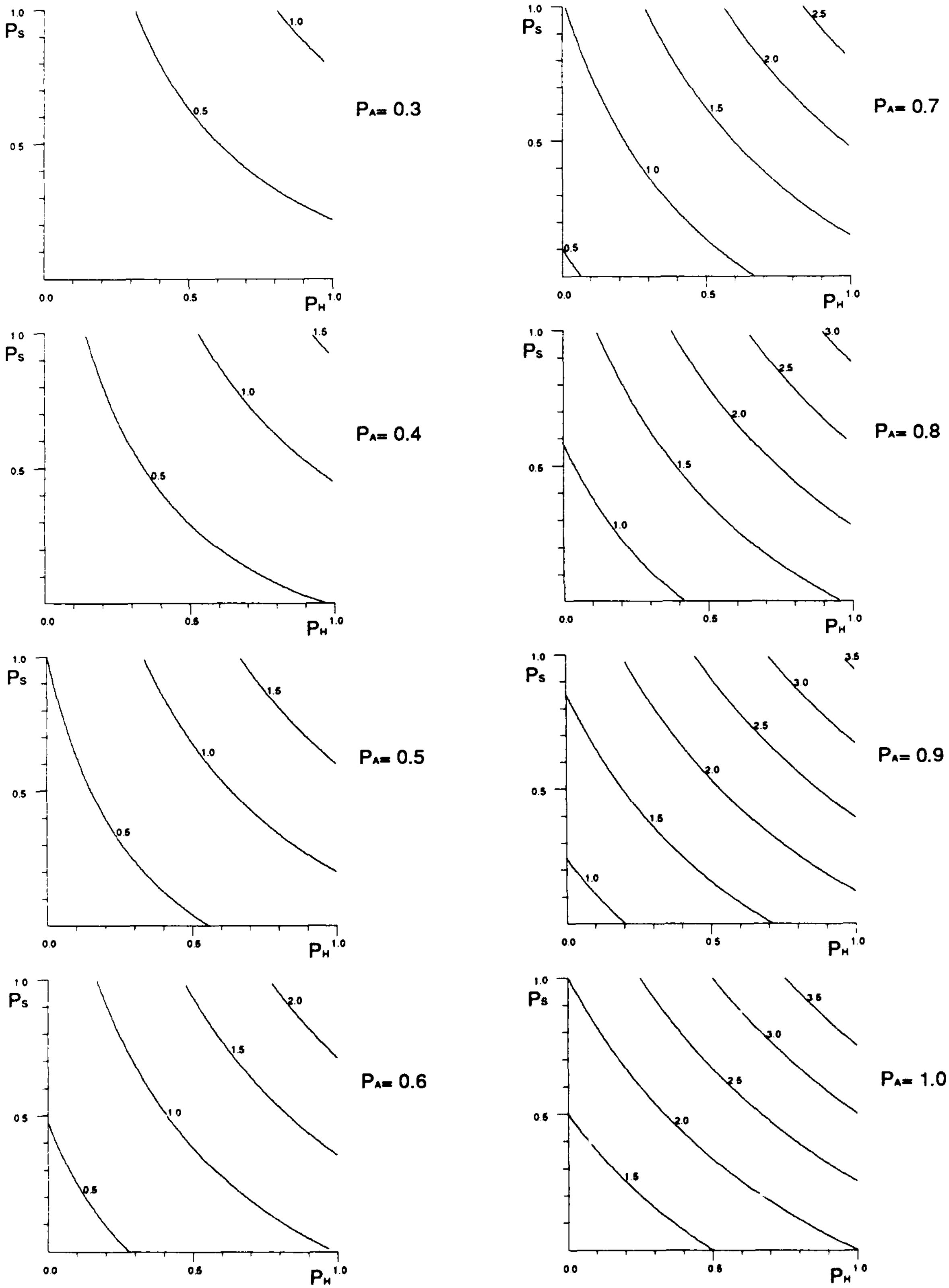


Figure 10 Expected number of hits by wave attack of four ASCMs on a ship as a function of attack probability and 'hard kill/soft kill' penetration probability

P_A : PROB(Individual ASCM available for attack)
P_H : PROB(Individual ASCM penetrates hard kill defences)
P_S : PROB(Individual ASCM penetrates soft kill defences)

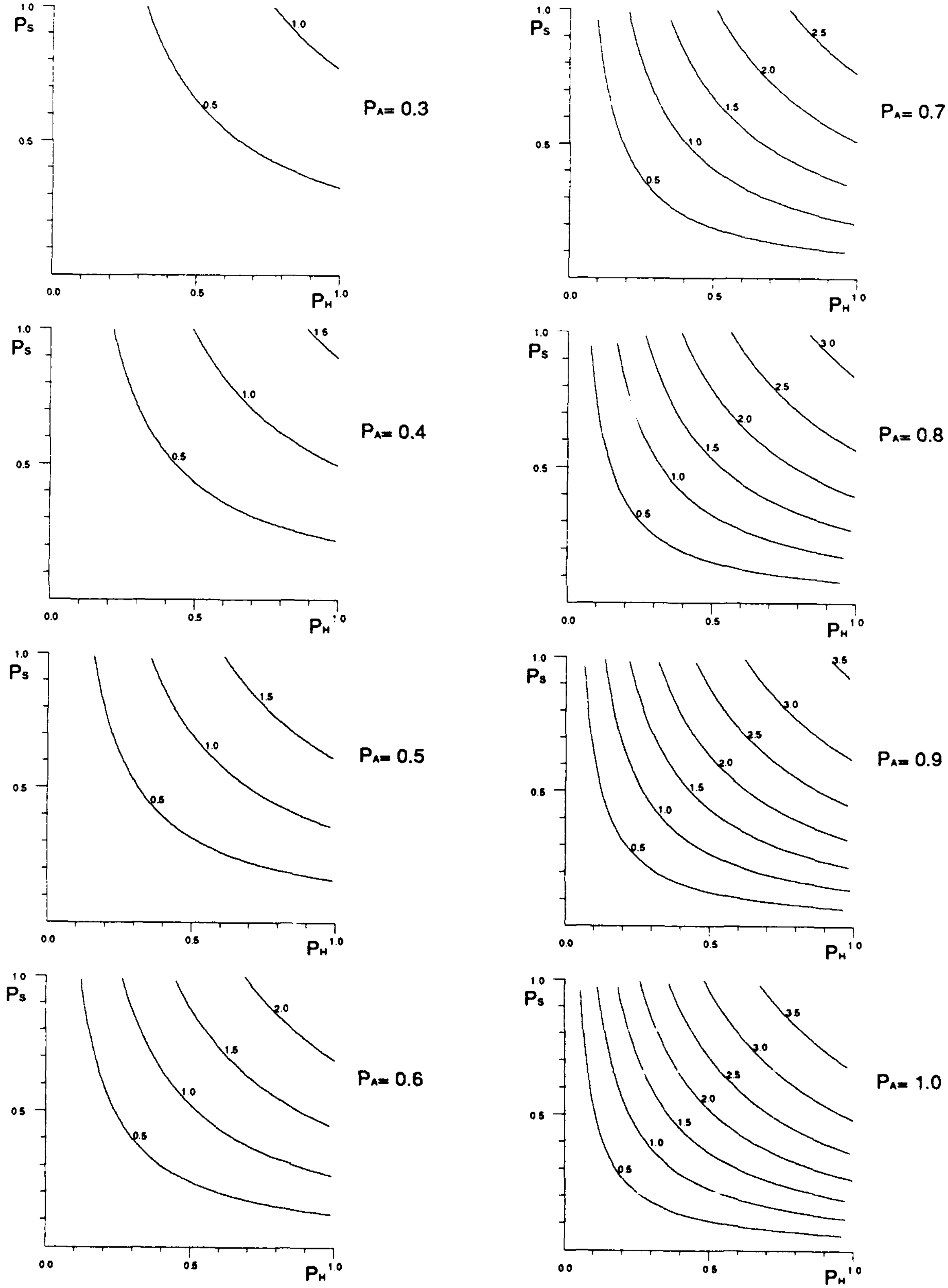


Figure 11 Expected number of hits by stream attack of four ASCMs on a ship as a function of attack probability and 'hard kill/soft kill' penetration probability

P_A : PROB(Individual ASCM available for attack)
P_H : PROB(Individual ASCM penetrates hard kill defences)
P_S : PROB(Individual ASCM penetrates soft kill defences)

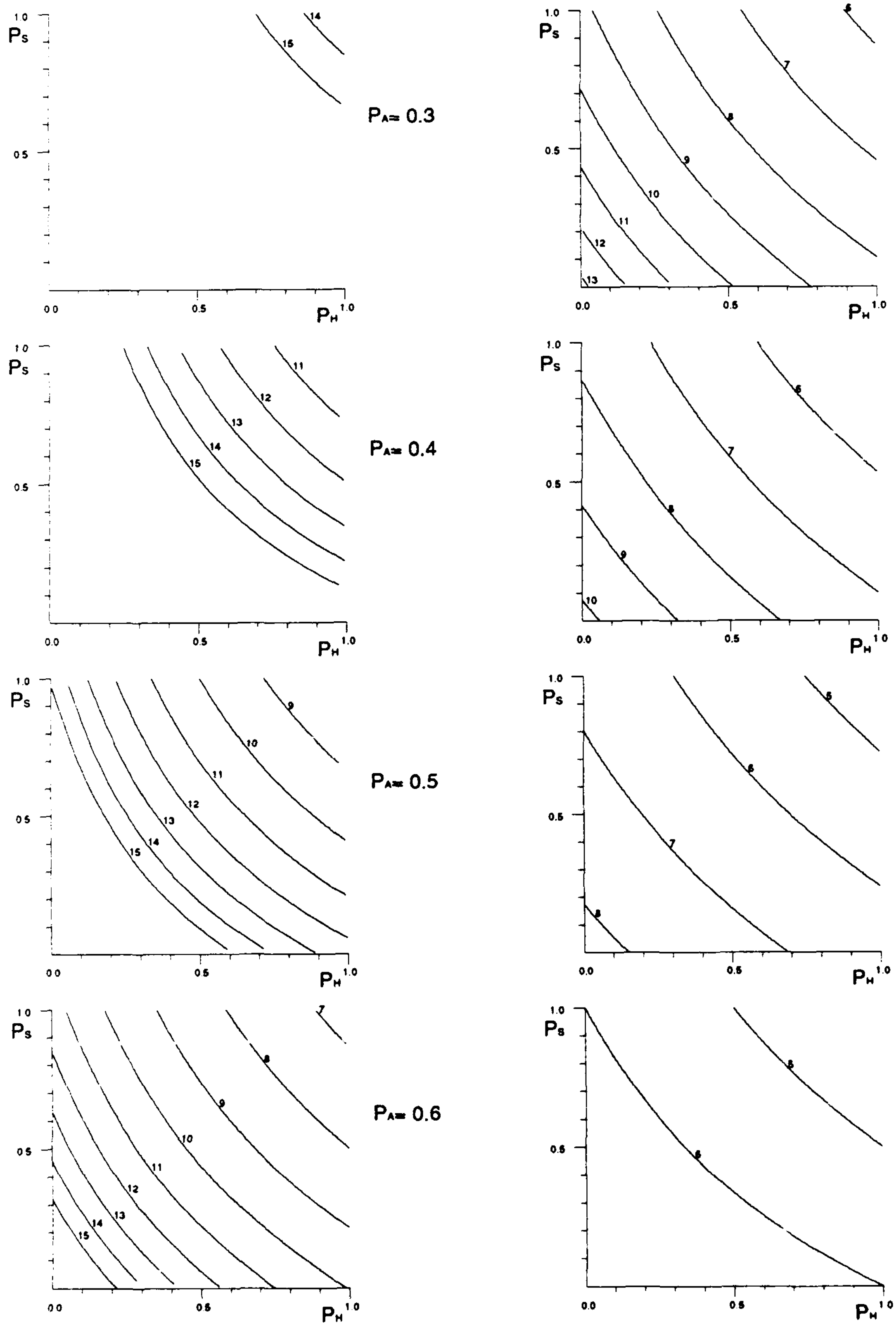


Figure 12 Average number of ASCMs launched in wave attack to achieve four hits on a ship as a function of attack probability and 'hard kill/soft kill' penetration probability

P_A : PROB(Individual ASCM available for attack)
P_H : PROB(Individual ASCM penetrates hard kill defences)
P_S : PROB(Individual ASCM penetrates soft kill defences)

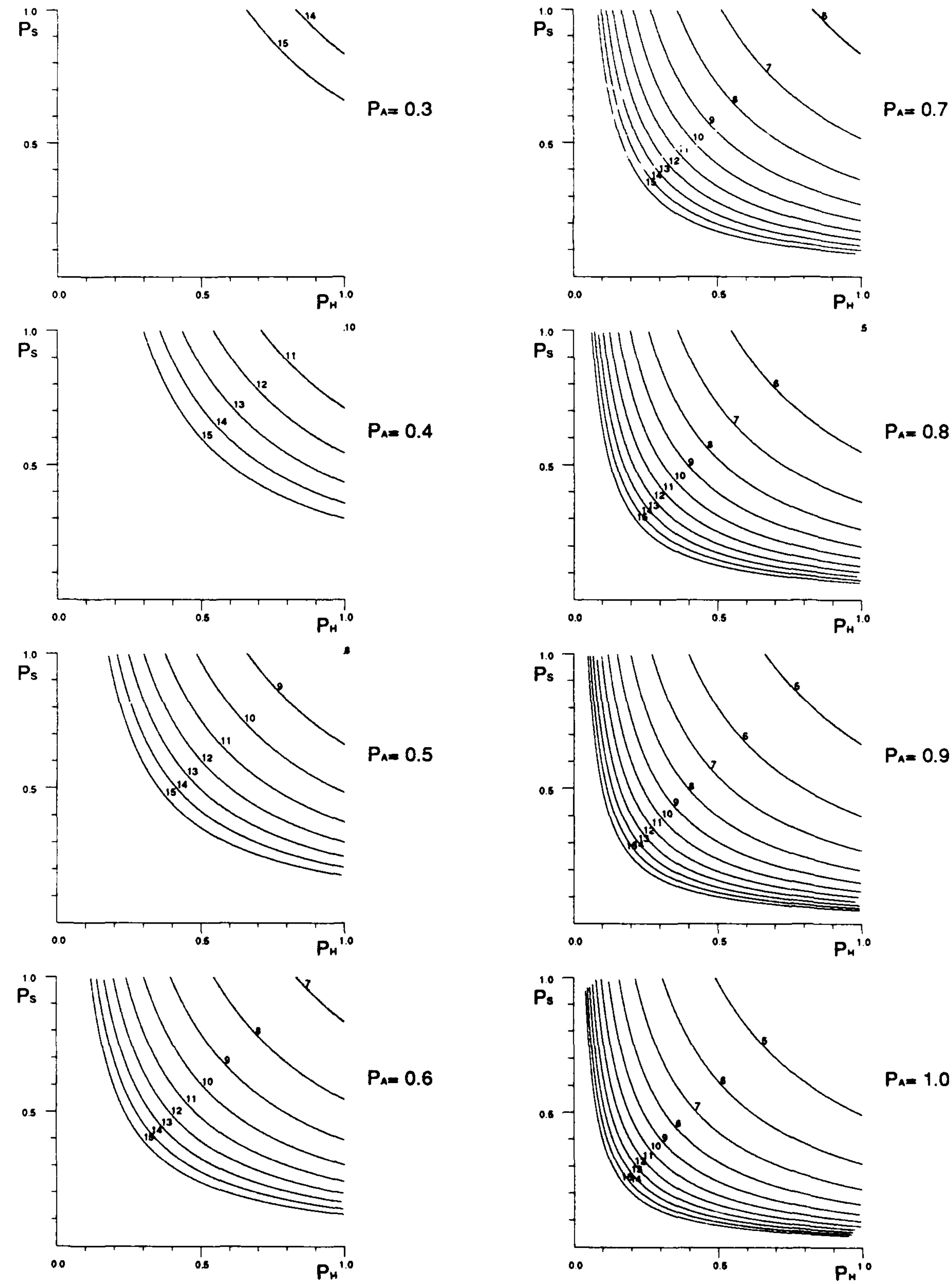


Figure 13 Average number of ASCMs launched in stream attack to achieve four hits on a ship as a function of attack probability and 'hard kill/soft kill' penetration probability

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16. ABSTRACT
A probabilistic assessment of single-ship air-defence effectiveness against an attack by a stream or wave of anti-ship capable missiles (ASCMs) has been conducted for ships fitted with a mix of either 'hard kill' or 'hard kill//soft kill' weapons systems.

Results are expressed in terms of the probability of a ship defeating all threats, or the expected number of hits achieved by the ASCMs.

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