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Derivation of the Generalized, Average Euclidean Distance Function for the PDI Model

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PREFACE

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A handwritten signature in dark ink, appearing to read 'P. A. La Brecque', is positioned above the printed name.

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13. ABSTRACT (Maximum 200 words) This report derives distance functions that form the basis for the Population Density Index (PDI) model, which is a three-parameter square-root model for measuring discrete spatial density in finite populations. The PDI and its methods have been applied to facilities layout methodologies in submarine environments at the Naval Undersea Warfare Center Division, Newport, RI, resulting in several U.S. patent applications. The emphasis here is on the "micro-population" model in which the linear units are "feet." The derivations relate Cartesian rectangular coordinate systems to uniform unit and nonunit lattices, as well as to the nonlattice distribution. Other proofs relate to the bounds of the calculated density measure and the density rate index called "effective distance." Alternative distance functions are discussed, and examples of the numerical calculations are provided. Also derived is the algorithm for selecting a rectangular lattice conformal to a quadrilateral area and for calculating interpoint distance in a PDI lattice. A table of computer-generated unit lattice average Euclidean distances for up to 10,000 density points is included.				
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DERIVATION OF THE GENERALIZED, AVERAGE EUCLIDEAN DISTANCE FUNCTION FOR THE PDI MODEL

INTRODUCTION

Research has demonstrated spatial density (or crowding) to be a significant stressor in animal and human populations (Galle, Grove, and McPherson, 1972; Baum and Epstein, 1975). In previous papers, the author formulated and tested a mathematical model and methodology for measuring discrete spatial density in human populations (O'Brien (1989, 1990a, 1990b)). The model, called the population density index (PDI) model, was demonstrated to provide a more accurate and flexible approach for discrete spatial density measurement than the conventional formulation. The traditional approach to measuring human physical density involves two parameters: the number of persons (n) and the geometric area (A) in which the persons dwell. The equation $D = n/A$ serves as the conceptual and computational definition for "density," "congestion," "population density," or "physical crowding," each term used interchangeably. In contrast, the PDI model is based on three parameters: n , A , and inter-object distance. The derivation of the PDI model metrics is patterned on the "square-root law" of average distances used in the physical sciences. The capability to model inter-object distance within a defined geometric plane is a significant enhancement to discrete spatial density measurement. In O'Brien (1991a), the PDI model was generalized to any finite number of density points (i.e., people).

The motivation for developing the PDI formula and model was the need to be able to measure crowding among people from variable spatial configurations such as in a typical dynamic workplace environment. The conventional density model assumes that a static description is adequate without taking into account the way in which people use an environment over time.

The PDI model has been used at the Naval Undersea Warfare Center (NUWC) Division, Newport, for density measurement (O'Brien and Kanter, 1988; Kanter and O'Brien, 1989a; 1989b) in submarine attack center concept of operations experiments (Wallin, 1987). Practical applications of the PDI model resulting from research at NUWC have been documented for a variety of disciplines in several U.S. patent applications (O'Brien, 1991c, 1991d, 1991e, 1991f).

The purpose of this report is to provide a more rigorous derivation of the PDI model than currently exists. The basis of the PDI model is the distance function in Euclidean space. All of

the measures in the model are related to distance. Thus, an attempt is made to characterize the PDI distance function in R^2 (two-dimensional Euclidean space).

DERIVATION OF THE DISTANCE FUNCTION

GENERAL CASE LATTICES

The notation and structure of this section is patterned on Morrey (1962, Chapter 8, "The Definite Integral"), where the theory of area and concept of functional uniform continuity are developed in detail. Also, the ideas of inner and outer areas of bounded sets and the idea of a planar figure developed in Morrey are germane to the present development.

In the X-Y Euclidean plane (quadrant I) of figure 1, any two consecutive abscissa (horizontal) or ordinate (vertical) points (denoted by a large dot $\bullet$) are assumed to be equidistant with interpoint spacing parameter δ . That is, the directed distances of the collinear point pairs $(P_1P_2) = [(x_k, y_l), (x_{k+1}, y_l)]$ and $(P_3P_4) = [(x_m, y_j), (x_m, y_{j+1})]$ are

$$\begin{aligned}\overline{P_1P_2} &= |x_{k+1} - x_k| = \delta, \\ \overline{P_3P_4} &= |y_{j+1} - y_j| = \delta,\end{aligned}\tag{1}$$

where x_k is a representative abscissa and y_j is a representative ordinate; $(x_k, y_j) > 0$. Generally, x_k, y_j will not be lattice (integer) points. In this report the units of the interpoint distance parameter δ for human populations are assumed to be feet ($\delta \geq 1$).

The interior rectangular lattice shown in figure 1 consists of n (a nonprime number) finite points arranged uniformly with R row (horizontal) and C column (vertical) points such that $n = RC$ ($n \geq 2$). The selection of an RC configuration and the computation of δ are explained in appendix A.

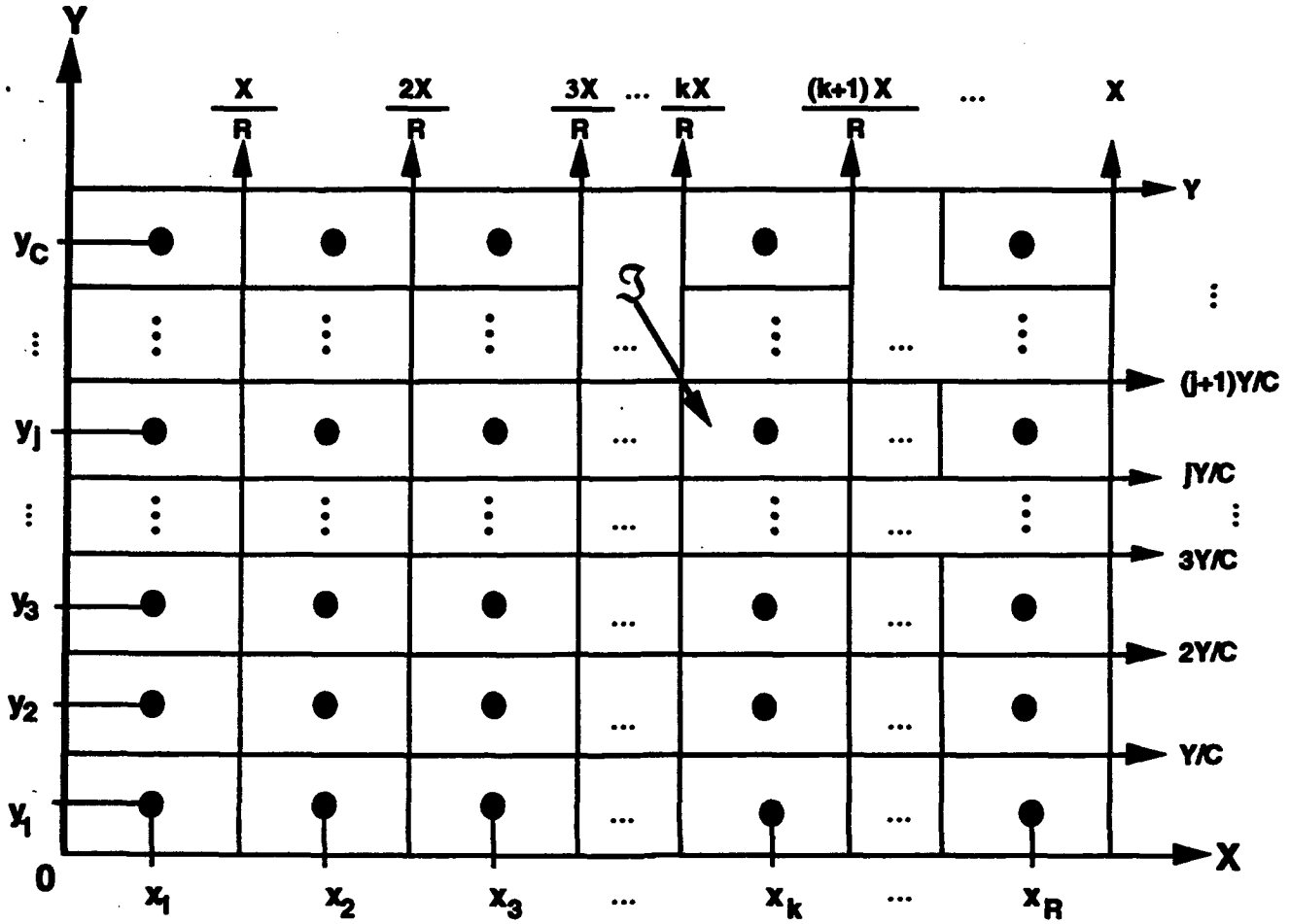


Figure 1. General Case Lattice

For a representative region 3 bounded by the nonnegative curves (see figure 1)

$$f_A(x) = (j+1)Y/C \quad (2)$$

$$g_A(x) = jY/C, \quad g_A(x) < f_A(x),$$

$$kX/R \leq x \leq (k+1)X/R, \quad 0 \leq k \leq R-1, 0 \leq j \leq C-1,$$

the area $A(3)$ is defined as

$$\begin{aligned} A(3) &= \int_{kX/R}^{(k+1)X/R} [f_A(x) - g_A(x)] dx, \\ &= XY/RC = A/n, \end{aligned} \quad (3)$$

which is seen to be a rectangle. For human populations with feet as the linear units, the restriction will be placed on the value of A/n ; viz., $A \geq n$. When $A = n$, the RC rectangular or square uniform discrete distribution is referred to as a "unit lattice"; otherwise, the homogeneous distribution of points is called a "nonunit lattice." The distinction will be understood in context. Each such rectangle will be obtained by dividing the total study area $n[A(3)]$ into $n = RC$ partitioned rectangles each, with area given by equation (3).

The connected density points in each of the horizontal and vertical intervals are defined by relations (or multiple-valued discrete constant functions):

$$\begin{aligned} f(R) = X &= (R - 1)\delta + p, \quad p > 0, \\ f(C) = Y &= (C - 1)\delta + q, \quad q > 0. \end{aligned} \tag{4}$$

Equations (4) indicate that each X or Y interval consists of two components: the length of the density points segment $[(R - 1)\delta$ or $(C - 1)\delta]$ and an excess factor (p or q). The region outside the perimeter of the uniform point arrangement [equal to $A - (R - 1)(C - 1)\delta^2$] is required to accommodate environmental objects (furniture, equipment, displays, etc.). Each of the CX intervals and RY intervals is defined by the constant functions in equations (4). The interval X will be partitioned into R subintervals, each subpartition of which will have the length shown in figure 1, and Y will be similarly divided and have the length shown in figure 1.

The derivation of the coordinate system for the general-case lattice will allow a precise graph to be drawn of any uniform rectangular distribution on a rectangular Cartesian X-Y coordinate system such that the interior RC lattice is contained within the XY exterior region. The coordinates of the density points derived from equation (4) will be generated by

$$\begin{aligned} x_k &= p/2 + (k - 1)\delta, \quad 1 \leq k \leq R, \\ y_j &= q/2 + (j - 1)\delta, \quad 1 \leq j \leq C. \end{aligned} \tag{5}$$

Then, the coordinate system for the general case will be defined as

$$\begin{aligned} (x_k, y_j) &= [(x_1, y_1), (x_2, y_1), \dots, (x_k, y_j), \dots, (x_R, y_C)] \\ &= \left[\left(\frac{p}{2}, \frac{q}{2} \right), \left(\frac{p}{2} + \delta, \frac{q}{2} \right), \dots, \left(\frac{p}{2} + (k - 1)\delta, \frac{q}{2} + (j - 1)\delta \right), \dots, \right. \\ &\quad \left. \left(\frac{p}{2} + (R - 1)\delta, \frac{q}{2} + (C - 1)\delta \right) \right]. \end{aligned} \tag{6}$$

The coordinate system of equation (6) applies to either a unit or nonunit lattice because it is derived from the general case. An example of the use of equation (6) is depicted in figure 2. The coordinates were generated from the following assumptions: $n = 6$; $R = 3$, $C = 2$ (from equation (A-4) in appendix A); $A = X \times Y = 16 \times 6$; $p/2 = 4$, $q/2 = 1$ (from (4)); $\delta = 4$ (from equation (A-5) in appendix A). Then the coordinate points are generated by $x_k = 4 + 4(k - 1)$; $y_j = 1 + 4(j - 1)$. The plot points are obtained from all $k \times j$ combinations ($k = 1, 2, 3$; $j = 1, 2$).

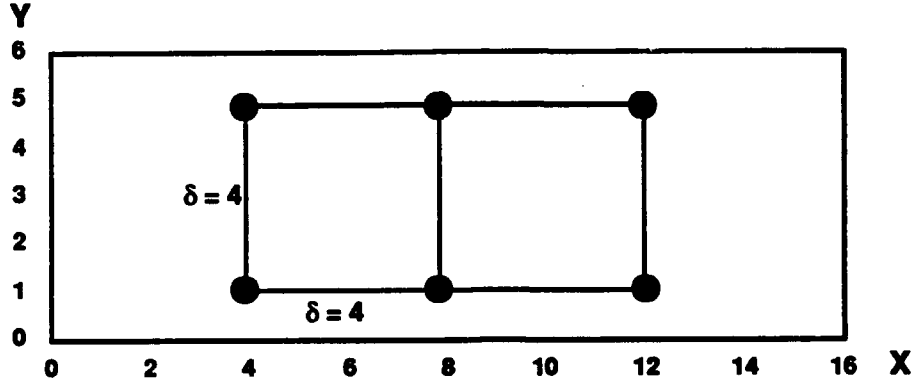


Figure 2. Example of General Case PDI Graph

SPECIAL CASE LATTICES

Each density point ($\bullet$) is now assumed to be the centroid (center of mass) bounded by its respective planar region (see figure 3). Let a representative region be called S . The area of S can be determined by first defining the nonnegative curves as the boundaries of S :

$$f_B(x) = (j + 1)\delta, \quad (7)$$

$$g_B(x) = j\delta, \quad g_B(x) < f_B(x),$$

$$k\delta \leq x \leq (k + 1)\delta, \quad 0 \leq k \leq R - 1, \quad 0 \leq j \leq C - 1.$$

The special case of equation (7) can be derived from equation (2) by assuming that $\delta = p = q = X/R = Y/C$ in equation (4) of the general case (i.e., proportionate commensurability between the dimensions of the outer and inner rectangular areas).

The area of S is then found by integrating between the curves $f_B(x)$ and $g_B(x)$ in the x interval, and applying the Fundamental Theorem of Calculus:

$$A(S) = \int_{k\delta}^{(k+1)\delta} [f_B(x) - g_B(x)] dx = \delta^2, \quad \delta^2 \geq 1. \quad (8)$$

This is intuitively the area of a square figure. The figure will be obtained by dividing the total area $n[A(S)]$ into $n = RC$ partitions after determining which lattice configuration will accommodate best the n points into a rectangular configuration with associated interpoint spacing parameter δ (see appendix A). Note that for commensurate (unit or nonunit) lattices, the interpoint spacing parameter is related to the region in equation (8).

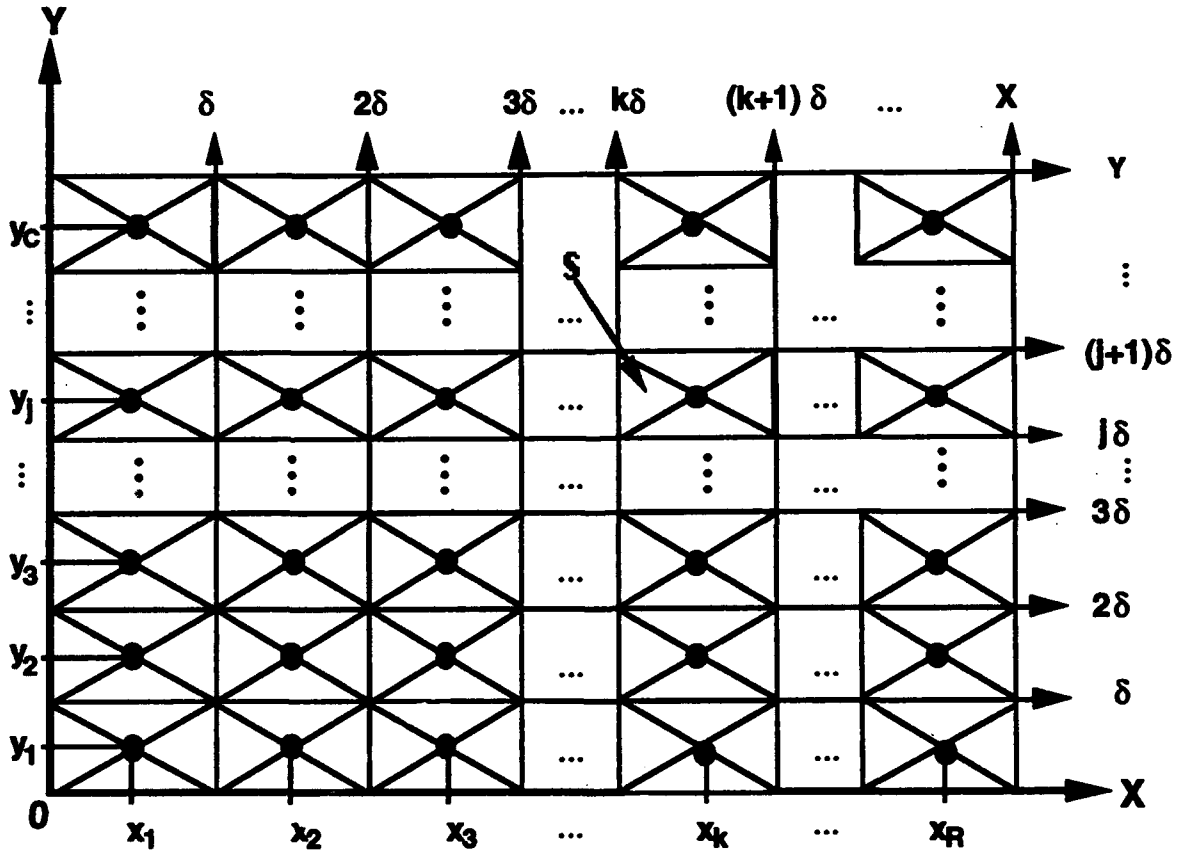


Figure 3. Special Case Lattice

Bers (1969, Vol. II, chapter 8, section 8, "Centroids of Plane Regions and Curves") gives the following definitional formulas for determining the coordinate points (x_k, y_j) of the centroid in region S:

$$(x_k, y_j): x_k = \frac{\int_{k\delta}^{(k+1)\delta} x [f_B(x) - g_B(x)] dx}{\int_{k\delta}^{(k+1)\delta} [f_B(x) - g_B(x)] dx} = \frac{\delta(2k+1)}{2},$$

$$(x_k, y_j): y_j = \frac{\int_{k\delta}^{(k+1)\delta} 1/2 [f_B(x)^2 - g_B(x)^2] dx}{\int_{k\delta}^{(k+1)\delta} [f_B(x) - g_B(x)] dx} = \frac{\delta(2j+1)}{2}.$$
(9)

Here, (x_k, y_j) represents the rule for locating all and every density point (centroid) in the entire XY area, given concisely as

$$(x_k, y_j) = \left[\left(\frac{\delta}{2} + (k-1)\delta \right), \left(\frac{\delta}{2} + (j-1)\delta \right) \right]$$

$$= [(x_1, y_1), (x_2, y_1), (x_3, y_1), \dots, (x_k, y_1),$$

$$(x_1, y_2), \dots, (x_k, y_2), \dots, (x_k, y_j), \dots, (x_R, y_C)]$$

$$= \left[\left[\frac{\delta}{2}, \frac{\delta}{2} \right], \left[\frac{3\delta}{2}, \frac{\delta}{2} \right], \left[\frac{5\delta}{2}, \frac{\delta}{2} \right], \dots, \left[\frac{\delta(2R-1)}{2}, \frac{\delta}{2} \right], \right.$$
(10)

$$\left. \left[\frac{\delta}{2}, \frac{3\delta}{2} \right], \dots, \left[\frac{\delta(2R-1)}{2}, \frac{3\delta}{2} \right], \dots, \right.$$

$$\left. \left[\frac{\delta}{2} + (k-1)\delta, \frac{\delta}{2} + (j-1)\delta \right], \dots, \left[\frac{\delta(2R-1)}{2}, \frac{\delta(2C-1)}{2} \right] \right\},$$

$$1 \leq k \leq R, \quad 1 \leq j \leq C.$$

The coordinate system of equation (10) applies to unit lattices and commensurate nonunit lattices. Figure 4 is an example of equation (10) applied to a 3 x 2 unit lattice ($\delta = 1$ from equation (A-5) in appendix A). Note that $X/R = Y/C = p = q = \delta = \sqrt{XY/RC} = 1$ because all unit lattices are commensurate. The graph is plotted from equation (10) by $x_k = k - 0.5$; $y_j = j - 0.5$ ($k = 1, 2, 3$; $j = 1, 2$).

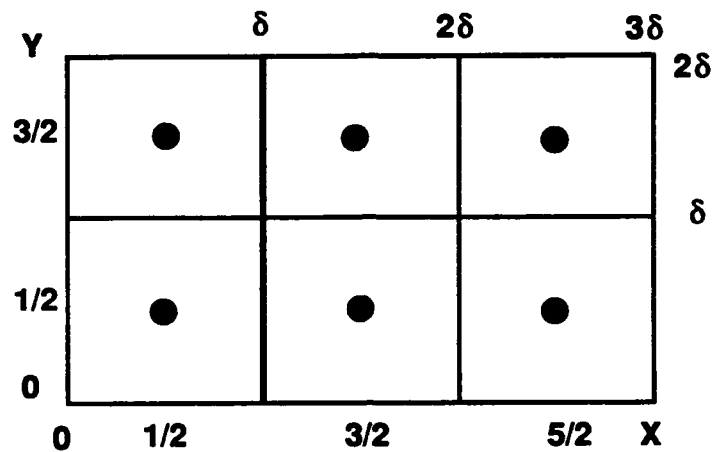


Figure 4. Example of Special Case PDI Graph (Unit Lattice)

Figure 5 is an example of a graph for a nonunit commensurate lattice with $n = 15$ points within area of 40 ft x 24 ft. Note that $X/R = Y/C = p = q = \delta = \sqrt{XY/RC} = 8$. Plot points are generated from equation (10): $x_k = 4 + 8(k - 1)$; $y_j = 4 + 8(j - 1)$.

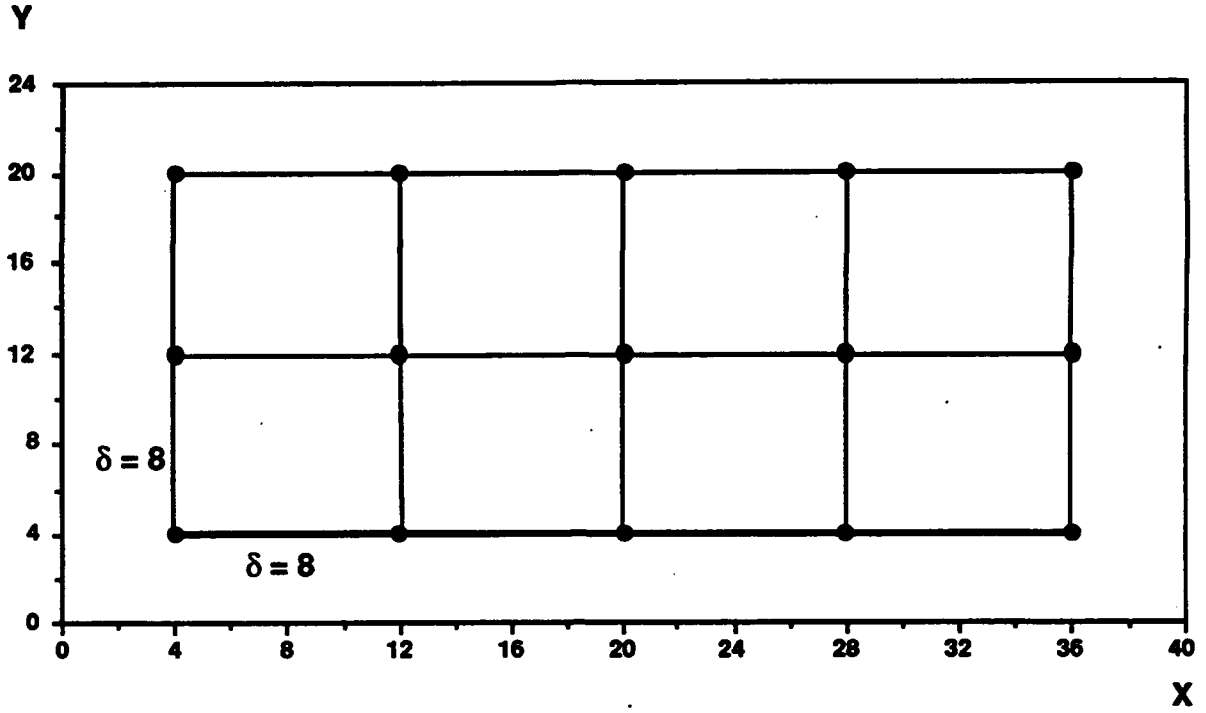


Figure 5. Example of Special Case PDI Graph (Commensurate Nonunit Lattice)

GENERALIZED DISTANCE FUNCTION IN A LATTICE

Since the coordinate points in the PDI lattice can now be specified completely, the PDI “exact” and “approximate” distance formulas can be derived (O’Brien, 1990b, 1991b). Here is derived the generalized, Euclidean distance formula for any PDI lattice (nonunit lattice and thereby the unit lattice as a special case) and any nonuniform distribution. First shown is the derivation for a lattice using the general case notation system. The derivation applies equally to the special case by assuming commensurability.

Let any density point in the X-Y plane be called (x_k, y_j) and let a second distinct point be called (x_{k+i}, y_{j+l}) . Then, from equation (6),

$$(x_k, y_j) = (p/2 + (k - 1)\delta, q/2 + (j - 1)\delta), \quad (11)$$

$$(x_{k+i}, y_{j+l}) = (p/2 + (k + i - 1)\delta, q/2 + (j + l - 1)\delta),$$

$$1 \leq k \leq R - 1, \quad 1 \leq j \leq C - 1,$$

$$2 \leq k+i \leq R, \quad 2 \leq j+l \leq C.$$

Bers (1969, Vol. I) shows that the classical Pythagorean distance formula for any two points in a Cartesian plane can generally be derived from the integral calculus arc length formula, given for our notation as

$$\mathcal{L} = \int_{x_k}^{x_{k+i}} \sqrt{1 + [f'(x)]^2} dx. \quad (12)$$

The quantity $f'(x)$ is the first derivative of the function $f(x)$, taken to be a generalized single-valued relation for two points in a Euclidean X-Y plane specified by the first degree equation $f(x) = a + bx$, for the slope intercept a and linear slope b .

Since $f'(x) = \frac{d[f(x)]}{dx} = D_x(a + bx) = b$, the constant slope of the points in equation (11) can be defined as $b = \frac{y_{j+l} - y_j}{x_{k+i} - x_k} = \frac{\delta l}{\delta i}$. Then,

$$\begin{aligned} \mathcal{L} &= \int_{x_k}^{x_{k+i}} \sqrt{1 + \left(\frac{l}{i}\right)^2} dx, \\ &= \delta \sqrt{i^2 + l^2}, \end{aligned} \quad (13)$$

which is seen to be of the form for the standard bivariate Pythagorean theorem scaled by a constant:

$$\mathcal{L} = \delta \sqrt{(x_{k+i} - x_k)^2 + (y_{j+l} - y_j)^2}. \quad (14)$$

Bers (1969, Vol. I, p. 279) terms equation (12) the "length formula." It may also be viewed as an average -- the average length of one pair of points. The length (distance) between any one pair of points in the uniform RC lattice can be generalized to an average among all possible pairs of RC points since each point pair defines a simple linear function each of which possesses a piecewise continuous first derivative. The average pair-to-pair distance, summed over all pairs of points, will be the average of all the line-to-line curves (total length), since the connected graph defines a multiple-valued relation (Bers, 1969, Vol. I, page 279). That is, the uniform average distance in the total lattice is

$$\bar{d} = \frac{\sum_{k=j=1}^n L_{kj}}{C(n,2)}, \quad (15)$$

where

$$C(n,2) = \frac{(RC)!}{2!(RC-2)!} = \frac{n(n-1)}{2}, \quad (n \geq 2), \quad (16)$$

is the combinatorial expression specifying the total number of nonredundant pairwise-connected lines from n nodes and the exact summation index limits are given in equation (11). The uniform lattice distance equation (15) can be further expressed in a more computational convenient form as

$$\bar{d} = \delta \bar{\Delta}, \quad (17)$$

where δ is given in appendix A and $\bar{\Delta}$ is the unit lattice average distance, which has been derived in O'Brien (1991a) as

$$\bar{\Delta} = \frac{12 \sum_{i=1}^{R-1} \sum_{j=1}^{C-1} (R-i)(C-j) \sqrt{i^2 + j^2} + RC(R^2 + C^2 - 2)}{3(RC)(RC-1)}, \quad (18)$$

where R is the number of horizontal points in each row of the unit lattice, C is the number of vertical points in each column of the unit lattice, and RC is the total number of density points in the unit lattice.

An accurate approximation to equation (18) exists when n is not small. This relation is derived under the assumption that there is a continuous uniform distribution within a rectangular plane. The objective is to find the average distance between any two randomly selected points of a convex set. The approximation formula* (Santalo, 1976, formula 4.18, page 49) is as follows:

$$\bar{\Delta}' = \frac{1}{15} \left[\frac{R^3}{C^2} + \frac{C^3}{R^2} + d \left(3 - \frac{R^2}{C^2} - \frac{C^2}{R^2} \right) + \frac{5}{2} \left[\frac{C^2}{R} \ln \left(\frac{R+d}{C} \right) + \frac{R^2}{C} \ln \left(\frac{C+d}{R} \right) \right] \right], \quad (19)$$

* The author gratefully acknowledges an anonymous referee of *The American Mathematical Monthly* for suggesting equation (19) (in correspondence related to O'Brien, 1990c).

where $d = \sqrt{R^2 + C^2}$ and $\ln$ is the natural logarithm operator.

Calculations have shown equation (19) to be a good approximation to equation (18). For example, for n under 100, the maximum discrepancy is less than 10 percent. Equation (19) is an interesting example where a continuous distribution relation is applied to a discrete distribution to obtain an approximation to the latter. In the limiting case, as RC approaches infinity, the difference between equations (18) and (19) approaches zero.

In conclusion, for any finite, discrete, uniform distribution with distance between any two points δ , the generalized average Euclidean distance in any PDI lattice among all possible pairs of RC points will be $\bar{d} = \delta \bar{\Delta}$ or $\delta \bar{\Delta}' \approx \bar{d}$. If a unit lattice ($\delta = 1$), then $\bar{d} = \bar{\Delta}$ or $\bar{\Delta}' \approx \bar{d}$. Selected values of $\bar{\Delta}$ calculated from equation (18) are given in appendix B for all RC configurations from $R \times C = 2 \times 2$ to $R \times C = 100 \times 100$ ($n = 10,000$ density points).

GENERALIZED DISTANCE FUNCTION IN A NONLATTICE

Here, density points can fall anywhere within the X - Y geometric area, subject to restrictions specified earlier. The average Euclidean distance is calculated by equation (15) from known coordinate points as

$$\bar{d} = \frac{\sum_{i < j} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{C(n,2)}, \quad (20)$$

where (x_i, y_i) , (x_j, y_j) ($i = 1, 2, \dots, n$; $j = 1, 2, \dots, n$) denote the coordinate locations for the density points ($n > 1$) within the rectangular study area $X \times Y$ with arbitrary origin O . Equation (20) is the form used for calculating the population density measure of observed density points (i.e., PDI_{act} , defined below in equation (29)).

An approximation to equation (20) is useful because exact (x_i, y_i) , (x_j, y_j) coordinates cannot always be obtained. Recently, O'Brien (1991b) derived an approximation PDI method by assuming knowledge of the relative location of the density objects when (x_i, y_i) , (x_j, y_j) data were unavailable.

If one assumes that the study area $A = X \times Y$ has been partitioned into $n = RC$ rectangles, each with subarea given by equation (3), then the following abbreviated calculation routines can be derived.

Define a cell density measure,

$$D_{jk} = n_{jk} / A_{jk}, \quad (21)$$

where n_{jk} is the number of objects observed to be within each of the subareas $A_{jk} = A/n$ ($j = 1, 2, \dots, R; k = 1, 2, \dots, C$), $0 \leq n_{jk} \leq A_{jk}$, $0 \leq D_{jk} \leq 1$. Then, define a cell indicator variable I :

$$I_{jk} = \begin{cases} 1 & \text{if } D_{jk} \neq 0, \\ 0 & \text{if } D_{jk} = 0. \end{cases} \quad (22)$$

Let

$$m = \sum_{k=1}^R \sum_{j=1}^C I_{jk}, \quad nD \leq m \leq n, \quad (23)$$

where $D = n/A$ is obtained from equation (21) as an average cell density with weights spread over all cells; i.e., $D = \sum_{k=1}^R \sum_{j=1}^C D_{jk} n^{-1}$. The measure m represents the total number of RC partitions occupied by at least one object. In practice, n_{jk} is taken as the smallest integer value. Likewise, m is taken to be the largest integer value.

Hence, equation (17) can be redefined to give the following approximation to equation (20):

$$\bar{d}' = \delta'_{\text{eff}} \bar{\Delta}, \quad (24)$$

where

$$\delta'_{\text{eff}} = \left(\frac{\sum_{k=1}^R \sum_{j=1}^C D_{jk}}{m} \right)^{-1/2} = \left(\frac{mA_{jk}}{n} \right)^{1/2}, \quad 1 \leq \delta'_{\text{eff}} \leq D^{-1/2}. \quad (25)$$

δ'_{eff} is obtained from equation (21) as an average cell density with weights spread over only the m occupied cells. The limits of equation (24) follow immediately by substituting the lower and upper limits of m given in equation (23); viz., $\bar{\Delta} \leq \bar{d}' \leq \bar{\Delta} \sqrt{A/n}$. Noting that $1 \leq \delta \leq \sqrt{A/n}$ (see appendix A) and assuming, in practice, that $1 \leq \delta'_{\text{eff}} \leq \delta$, it then follows that

$$\bar{d}_{\text{max}} \leq \bar{d}' \leq \bar{d}_{\text{min}}, \quad (26)$$

where $\bar{d}_{\min}$ and $\bar{d}_{\max}$ are, respectively, the lower and upper distance measures in the exact PDI model (O'Brien, 1990b). The relationship of (26) translates directly into a proof of the bounds of the approximate PDI measure ($PDI'_{act} = \sqrt{D/\bar{d}} = [D\sqrt{n/m}]/\bar{A}$; i.e., PDI'_{act}) is bounded by the $PDI_{\min}$ and $PDI_{\max}$ relations defined in O'Brien (1990b) and in equations (27) and (28) below.

ALTERNATIVE DISTANCE MODELS

Thus far, the distance function has been derived for a rectangular configuration of points by assuming a rectangular exterior region. Mathematically, there is good reason for doing this because a square or rectangle can be drawn around any closed curve (Steinhaus, 1969).

Occasionally, the environment of interest may be modeled by curved configurations such as ellipses or circles, the latter being the easier to work with. Circular distributions have two advantages. First, for regions nearly square, a circle offers a more compact concentration of points, which may provide more realistic bounds on the density measure for highly cluttered environments. Second, any number n of points (including prime numbers) can be placed uniformly on a circle of radius r with linear point-to-point distance $d = 2r \sin(180/n)$. Based on this chord length measure, the author recently constructed a PDI model for discrete spatial density for circular distributions (O'Brien, 1992).

SELECTED PROOFS

PROOF THAT $PDI_{\min} \leq PDI_{\text{act}} \leq PDI_{\max}$

First, a statement of the relationships involved in this proof is given as follows:

$$\text{Lower bound: } PDI_{\min} = \frac{1}{\delta \bar{\Delta}} \sqrt{\frac{n}{A}}, \quad (27)$$

$$\text{Upper bound: } PDI_{\max} = \frac{1}{\bar{\Delta}} \sqrt{\frac{n}{A}}, \quad (28)$$

$$\text{Actual PDI: } PDI_{\text{act}} = \frac{1}{\bar{d}_{\text{act}}} \sqrt{\frac{n}{A}}. \quad (29)$$

The terms n , A , $\bar{\Delta}$, and $\bar{d}_{\text{act}}$ (equivalent to equation (20)) are used here as defined in this report; δ is defined in appendix A.

Now, to the proof. From the relationships of equations (27), (28), and (29), a formal statement of the relationship to be proven is as follows:

$$\frac{1}{\delta \bar{\Delta}} \sqrt{\frac{n}{A}} \leq \frac{1}{\bar{d}_{\text{act}}} \sqrt{\frac{n}{A}} \leq \frac{1}{\bar{\Delta}} \sqrt{\frac{n}{A}}. \quad (30)$$

To prove that equation (30) is a true statement, three assumptions are required:

$$\bar{d}_{\text{act}}, \delta, \text{ and } \bar{\Delta} \text{ are measured in linear units of feet,} \quad (31)$$

$$\delta \geq 1, \quad (32)$$

$$\bar{d}_{\text{act}} \leq \delta \bar{\Delta}. \quad (33)$$

The first assumption (31) is self-explanatory. The second assumption (32) is deemed reasonable because it amounts to saying that if persons are positioned uniformly the head-to-head distance (δ) is about 1 foot. Although (32) would not be a reasonable assumption for areal units of, say, square miles, (32) is reasonable when the areal units are square feet. (See O'Brien, 1991f, for the finite "macro" PDI model when areal units are square miles.)

The third assumption (33) states that, for a given geometric area to be studied in a density analysis, the actual clustering of the density points (i.e., people) in that area (with associated density $\bar{d}_{\text{act}}$) will not be greater than the maximum theoretical dispersion provided by the

relation $\delta\bar{\Delta}$ (equation (17)). The region outside $\delta\bar{\Delta}$ is assumed to contain physical objects such as furniture, equipment, etc., making it unlikely that density points will be observed in that region. Empirical evidence from Monte Carlo simulations in O'Brien (1989) is cited in support of (33). In effect, (33) assumes that the persons are maximally dispersed in accord with the relation $\delta\bar{\Delta}$. Figure 6 describes the essential meaning of (33).

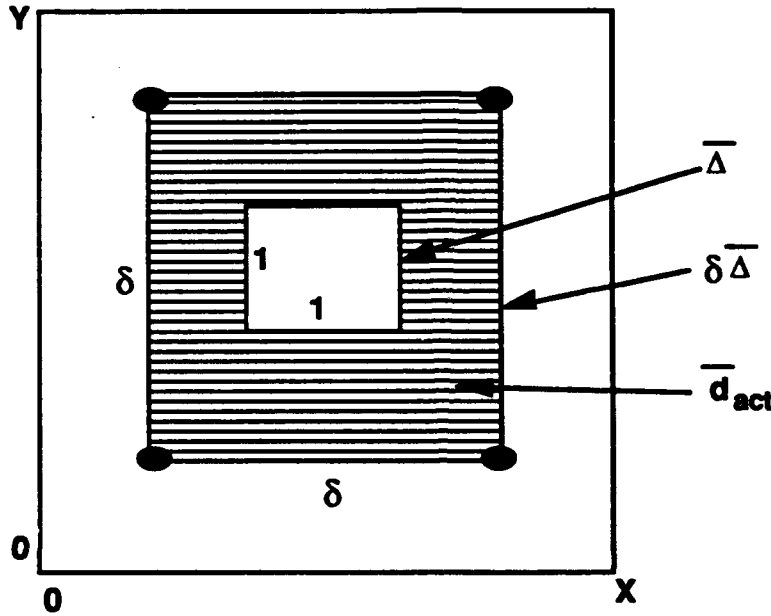


Figure 6. Intuitive Justification for Third Assumption

The formal proof of equation (30) can now be given in detail. The proof is presented in three parts. The first part states

$$\frac{1}{\delta\bar{\Delta}} \sqrt{\frac{n}{A}} \leq \frac{1}{\bar{d}_{act}} \sqrt{\frac{n}{A}}. \quad (34)$$

Simplifying and rearranging the terms of equation (34) gives the following relationship:

$$\bar{d}_{act} \leq \delta\bar{\Delta}, \quad (35)$$

which follows directly from (33).

The second part of the proof states that

$$\frac{1}{\bar{d}_{act}} \sqrt{\frac{n}{A}} \leq \frac{1}{\bar{\Delta}} \sqrt{\frac{n}{A}}. \quad (36)$$

Simplifying and rearranging the terms of equation (36) gives the following relationship:

$$\bar{\Delta} \leq \bar{d}_{act}. \quad (37)$$

From (33) the following relationship can be established:

$$\frac{\bar{d}_{act}}{\bar{\Delta}} \leq \delta, \quad (38)$$

from which it can be deduced that

$$\frac{\bar{\Delta}}{\bar{d}_{act}} \leq \frac{1}{\delta}. \quad (39)$$

Since, by (32), it follows that $1/\delta \leq 1$, then it can be deduced that $\bar{\Delta}/\bar{d}_{act} \leq 1$, from which it follows that $\bar{\Delta} \leq \bar{d}_{act}$.

The third part of the proof asserts that

$$\frac{1}{\delta \bar{\Delta}} \sqrt{\frac{n}{A}} \leq \frac{1}{\bar{\Delta}} \sqrt{\frac{n}{A}}. \quad (40)$$

The relationship between the lower and upper limits of equation (40) follows necessarily from the proofs given for equations (34) and (36) by the transitivity property of relations. It can be readily seen that equation (40) reduces algebraically to $\delta \geq 1$, which follows directly from (32). Thus, the statement of equation (30) has been shown to be true as derived from the stated definitions and assumptions.

The proof that the approximate PDI formula is bounded by the minimum and maximum bounds given in equations (27) and (28) follows from equation (26) and from the definition of the approximate PDI measure.

PROOFS FOR δ_{eff}

From O'Brien (1990b, equation(9)), δ_{eff} is defined as

$$\delta_{\text{eff}} = \frac{\bar{d}_{\text{act}}}{\bar{\Delta}}. \quad (41)$$

The objective is to show that $\delta_{\text{eff}} \geq 1$. Since $\frac{\bar{d}_{\text{act}}}{\bar{\Delta}} \geq 1$, as proven from equation (36), equation (41) follows.

The proof that $\delta/\delta_{\text{eff}} \geq 1$ is as follows:

By definition, $\delta_{\text{eff}} = \frac{\bar{d}_{\text{act}}}{\bar{\Delta}}$; then, $\delta/\delta_{\text{eff}} = \frac{\delta\bar{\Delta}}{\bar{d}_{\text{act}}} \geq 1$, which follows because it reduces to $\delta\bar{\Delta} \geq \bar{d}_{\text{act}}$, which was established previously in (33).

Because the quantity δ_{eff} is a "pure number" (i.e., it has no dimensions because they cancel out as in the above definition), it provides a pure measure of relative change in population density.

The reader may also note that in the approximation model $\delta'_{\text{eff}} \geq 1$ and $\delta/\delta'_{\text{eff}} \geq 1$ follows from the derived limits given in equation (25) and the relationship given in equation (26).

SUMMARY

This report has presented derivations of various distance functions that relate to the author's three-parameter square-root model for measuring discrete spatial density in finite populations. The model, called the Population Density Index (PDI) model, was developed to capture dynamic density relations among persons within a naturalistic environment. An "exact" model and an "approximate" model were presented.

The derivations related a generalized Euclidean distance function to the fundamental measures in the model (PDI_{act} , the approximation measure PDI'_{act} , their lower and upper bounds, and the density rate indices δ_{eff} and δ'_{eff}). Coordinate systems were derived for plotting graphs of the PDI lattices and calculating the distance measures.

Also derived was the algorithm required to select a conformal lattice and the average uniform distance among the lattice points based on the number of density points to be analyzed within the reference quadrilateral area.

Average Euclidean distance values ($\bar{\Delta}$) were presented for unit lattices up to a 100 x 100 matrix. Using these values, researchers will be able to compute lower and upper bounds of the PDI measures for up to 10,000 density objects.

APPENDIX A SELECTING A UNIT LATTICE AND INTERPOINT DISTANCE PARAMETER

DERIVATION OF THE ALGORITHM

In this appendix, the algorithm is presented for (1) determining a unique finite, discrete, conformal RC lattice and (2) computing the average interpoint distance among the RC points.

To begin, it is assumed that n (sample size) and $A = X \times Y$ (the outer rectangular geometric area) are known. If n is a prime number (like 5 or 13 or 29), augment n by 1 before determining the rectangular/square dimensions of the unit lattice. The derivation of the algorithm for selecting an RC lattice is developed from concepts of number theory (Ore, 1967). In particular, interest is centered on sets and subsets of composite numbers that can be expressed as rectangular or square integers; i.e., positive (nonprime) integers that are two-integer products.

The value of n can be expressed in terms of the prime factors of the whole number:

$$n = \prod_{j=1}^r P_j^{\alpha_j}, \quad (A-1)$$

where P_j represents the j th prime number and α_j is the number of occurrences of the j th prime number of n . For example, composite 60 can be decomposed into $P_1^{\alpha_1} P_2^{\alpha_2} P_3^{\alpha_3} = 2^2 \times 3 \times 5$. Next, it is desired to derive the total number of possible RC ($n = R \times C$) product configurations of n in order to create the set of RC configurations; the latter will be a subset of the former. This number can be derived as follows.

Let $\tau(n)$ represent the number of all possible configurations of a composite integer n . Then it can be shown that this quantity is obtained from equation (A-1) by

$$\tau(n) = \prod_{j=1}^r (\alpha_j + 1). \quad (A-2)$$

For example, 60 can be partitioned into $(2+1)(1+1)^2 = 12$ two-integer products.

Next, the set of the $\tau(n)$ configurations is examined to select only those nontrivial and/or nonredundant configurations. Let $\Phi(RC)$ represent the total number of nonredundant and nontrivial $R \times C$ configurations for composite n , $\tau(n) \supset \Phi(RC)$. The trivial configurations are those for which $n = n \times 1$ or $1 \times n$, and the redundant configurations are the multiplicative, commutative equivalents of $R \times C$; i.e., $R \times C = C \times R$ ($R \geq C$) (e.g., $10 \times 4 = 4 \times 10$). Then,

$$\Phi(RC) = \frac{\tau(n) - 2 + S}{2}, \quad (A-3)$$

where $S = 0$ when n is a rectangular number, and $S = 1$ when n is a square number.* The set of all such specified configurations is denoted P of size $\Phi(RC) = m$; $P = \{R_1C_1, R_2C_2, \dots, R_iC_i, \dots, R_mC_m\}$, $(R_i \geq C_i)$. For example, if $n = 60$, then $\Phi(RC) = [(3 \times 2 \times 2) - 2 + 0]/2 = 5$; $P = \{30 \times 2, 20 \times 3, 15 \times 4, 12 \times 5, 10 \times 6\}$. Note that the trivial $(60 \times 1, 1 \times 60)$ and redundant commutative equivalent configurations $(2 \times 30, 3 \times 20, 4 \times 15, 5 \times 12, 6 \times 10)$ have been eliminated from P . Likewise, for $n = 100$, $\Phi(100) = \Phi(2^2 \times 5^2) = [(3 \times 3) - 2 + 1]/2 = 4$; $P = \{50 \times 2, 25 \times 4, 20 \times 5, 10 \times 10\}$.

Selection of a unique RC lattice with interpoint distance parameter δ is accomplished by the following guidelines.

Select the $R \times C$ lattice configuration (usually one) with dimensions most commensurate with the exterior $X \times Y$ dimensions; i.e., the one for which $X/Y - R/C$ is a minimum absolute difference ($X \geq Y, R \geq C$). Determine the uniform interpoint spacing parameter $\delta = \sqrt{A/n} = \sqrt{XY/RC}$ as defined in O'Brien (1990b, equation (3)). Next, test for conformity of the dimensions of the selected lattice to the study area dimensions by the quantities $(R - 1)\delta$ and $(C - 1)\delta$. If either of the R, C dimensions is nonconformal (i.e., $(R - 1)\delta \geq X$ or $(C - 1)\delta \geq Y$), then conform the lattice dimensions by adjusting δ by the relation $\delta = \min[X/(R - 1), Y/(C - 1)] - 0.1$. Finally, in the rarest of instances, when commensurability is achieved simultaneously by more than one lattice configuration, the researcher should approximate δ as above for each configuration, and then the $R \times C$ configuration will be that associated with the maximum δ value. If plural maxima δ occur, select the $R \times C$ configuration associated with the smallest value of $\bar{\Delta}$, given in appendix B.

The symbolic specification of the above guidelines can be stated as follows. Because the desired discrete $R \times C$ lattice must be unique, the selection mechanism requires a complex

* Equation (A-3) is not proven nor could a proof be found in the mathematical literature. Its correctness seems intuitively obvious. For example, for a number to be square, it is necessary and sufficient that all exponents in the prime factorization (equation (A-1)) be even (Ore, p. 42), which implies that $\tau(n)$ is odd, as is $\tau(n) - 2$, but adding 1 (S) makes $\Phi(RC)$ even. Finally, dividing by 2 eliminates the rectangular duplicates in $\tau(n) + S - 2$. The same logic applies to rectangular numbers, thus completing the proof outline.

two-step procedure. First, the following commensurability relation is determined from the dimensions of A and each element of the set P:

$$R_k C_k = \min_{1 \leq i \leq m} \left| \left[\frac{\max(X, Y)}{\min(X, Y)} - \frac{\max(R_i, C_i)}{\min(R_i, C_i)} \right] \right| \quad (1 \leq k \leq m). \quad (\text{A-4})$$

Then, based on equation (A-4) above and equation (4) in the main body of the text, δ is determined from one of the following four mutually exclusive and exhaustive conditions:

$$\delta = \begin{cases} \sqrt{\frac{\bar{\Delta}}{n}} & \text{if } k = 1 \text{ and } p > 0 \text{ and } q > 0 \quad (\text{A-5}) \\ \min \left[\left(\frac{X}{R-1}, \frac{Y}{C-1} \right) - 0.1 \right] & \text{if } k = 1 \text{ and } p \leq 0 \text{ or } q \leq 0 \quad (\text{A-6}) \\ \max_{1 \leq l \leq k} \left\{ \min_{2 \leq k \leq m} \left[\left(\frac{X}{R_k-1}, \frac{Y}{C_k-1} \right) - 0.1 \right] \right\} & \text{if } k > 1 \text{ and } l = 1 \quad (\text{A-7}) \\ \min_{2 \leq l \leq k} [\bar{\Delta}(R_l, C_l)] & \text{if } k > 1 \text{ and } l > 1 \quad (\text{A-8}) \end{cases}$$

where p, q are defined in equation (4). In (A-5) through (A-8), $\delta \geq 1$ by definition. Also, it may be proven that $\delta \leq \sqrt{\bar{\Delta}/n}$ based on equation (4) where it can be deduced that $(R-1)\delta < X$, $(C-1)\delta < Y$, and for commensurate lattices ($\delta \leq \sqrt{\bar{\Delta}/n}$), $\delta = X/R = Y/C$. This relationship places an upper bound on δ that is important in the proofs and derivations of the text.

Figure A-1 summarizes the algorithm for the RC lattice selection and computation of δ . In summary, if $k = 1$, $R_k C_k$ is the lattice selected from equation (A-4) and δ is selected from equation (A-5) or equation (A-6). If $k > 1$, δ is selected from equation (A-7) and R & C is selected as the lattice associated with the maximum δ in equation (A-7). Finally, if (A-7) provides a plurality of δ values, then (A-8) is used, which selects the R_l, C_l ($2 \leq l \leq k$) lattice associated with the smallest $\bar{\Delta}$ value. Appendix B contains the required $\bar{\Delta}$ values computed to five decimal places. Note that for a unit lattice, or commensurate nonunit lattice, $k = 1$ and equation (A-5) computes the correct δ . Hansen et al. (1953, Vol. I) provides an interesting discussion of commensurate nonunit lattices related to a square-root law for distances in the field of discrete finite-population sampling theory when equation (A-5) applies.

Thus, equations (A-4) through (A-8) provide a unique, conforming lattice with associated interpoint distance parameter δ . A table of prime numbers and factorizations of composite numbers is an indispensable tool for implementing equation (A-4). See Lehmer (1941, 1961) for extensive tables and Abramowitz and Stegun (1964) for abbreviated tables.

These calculations assure that the lengths of the R and C line segments of the nonunit lattice, $(R - 1)\delta$ and $(C - 1)\delta$, containing human density points do not exceed the dimensions of the study area. The utility of adjusting δ (when so required) as recommended resides in plotting minimum/maximum dispersions of the RC density points in the study area as given in equations (6) and (10).

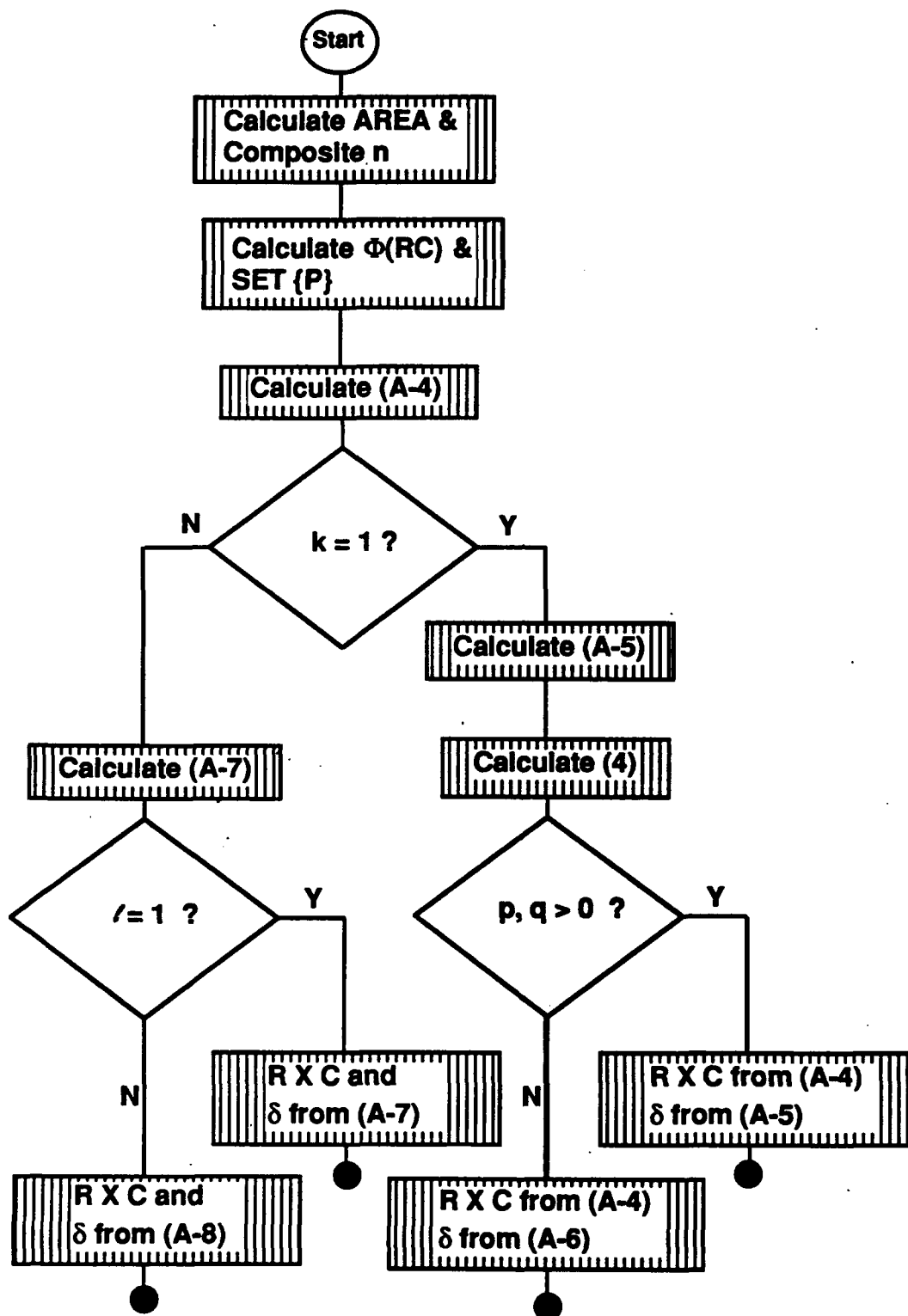


Figure A-1. Flowchart for Determining R H C Unit Lattice and Interpoint Distance Parameter δ

NUMERICAL EXAMPLES

Three artificial examples are selected to demonstrate the procedures. A complete setup is provided. The flowchart in figure A-1 is useful in tracing the decision logic.

In the first example, the data are as follows: $n = 12$, $A = 25 \times 25 \text{ ft}^2$. It is obvious that $n = 12$ provides two nontrivial, nonredundant choices ($\Phi(12) = 2$); viz., $R_1C_1 = 6 \times 2$ or $R_2C_2 = 4 \times 3$. Because $X/Y = 1$, $R_2C_2 = 4 \times 3$ comes closest to satisfying equation (A-4). Because $k = 1$, first compute $\delta = 7.22$ (from equation (A-5)); R, C is found to be conformal (each row/column "fits" inside the outside 25 ft^2 area in accord with equation (4)). Thus, $R = 4$, $C = 3$, and $\delta = 7.22$.

In the second example, $n = 64$ and $A = 50 \times 5 \text{ ft}^2$. This example is one of those rare possibilities. For $n = 64$, $\tau(64) = 7$; $\Phi(RC) = [7 + 1 - 2]/2 = 3$, and $P = \{32 \times 2, 16 \times 4, 8 \times 8\}$. Applying equation (A-4) shows that 32×2 and 16×4 are equally commensurate ($k > 1$); i.e., $|10 - 16| = |10 - 4|$. Thus, because $k = 2$ and l is undetermined, apply equation (A-7), giving $\delta = \max \{1.57, 1.51\} = 1.57 (l = 1)$. The configuration associated with the largest δ value is 16×4 . Thus, $R = 16$, $C = 4$, and $\delta = 1.57$ for this data distribution.

As an example requiring equation (A-8) for determining $R \times C$ and δ , consider the data: $A = 80 \times 16 \text{ ft}^2$, $n = 32$, $P = \{16 \times 2, 8 \times 4\}$. Here, applying (A-4) to the above data distributions produces $|5 - 8| = |5 - 2|$ (i.e., $k = 2$), and (A-7) produces $\delta = \max \{(5.23, 5.23)\} (l > 1)$, which is clearly ambiguous. But $\min [\bar{\Delta}(16 \times 2), \bar{\Delta}(8 \times 4)] = \min (5.59, 3.27) = \bar{\Delta}(8 \times 4)$. Thus, $R \times C = 8 \times 4$, and $\delta = 5.23$.

In general, the reader will note that (A-7) or (A-8) will be required for determining δ whenever the study area ratio X/Y is equal to the average of the ratios of two equally commensurate lattices. The above examples bear out this relationship.

APPENDIX B **UNIT LATTICE AVERAGE EUCLIDEAN DISTANCE VALUES**

Dimensions of Sublattice 100 by 100			Average Distance		
A	C				
2	2	1.12500	2	66	21.06670
2	3	1.40000	2	67	22.10707
2	4	1.70000	2	68	22.85000
2	5	2.02500	2	69	23.00000
2	6	2.37500	2	70	23.50000
2	7	2.75000	2	71	23.90000
2	8	3.15000	2	72	24.10000
2	9	3.57500	2	73	24.20000
2	10	4.02500	2	74	24.20000
2	11	4.50000	2	75	24.10000
2	12	4.97500	2	76	23.90000
2	13	5.45000	2	77	23.60000
2	14	5.92500	2	78	23.20000
2	15	6.40000	2	79	22.70000
2	16	6.87500	2	80	22.10000
2	17	7.35000	2	81	21.40000
2	18	7.82500	2	82	20.60000
2	19	8.30000	2	83	19.70000
2	20	8.77500	2	84	18.70000
2	21	9.25000	2	85	17.60000
2	22	9.72500	2	86	16.40000
2	23	10.20000	2	87	15.10000
2	24	10.67500	2	88	13.70000
2	25	11.15000	2	89	12.20000
2	26	11.62500	2	90	10.60000
2	27	12.10000	2	91	8.90000
2	28	12.57500	2	92	7.10000
2	29	13.05000	2	93	5.20000
2	30	13.52500	2	94	3.20000
2	31	14.00000	2	95	1.10000
2	32	14.47500	2	96	0.00000
2	33	14.95000	2	97	0.00000
2	34	15.42500	2	98	0.00000
2	35	15.90000	2	99	0.00000
2	36	16.37500	2	100	0.00000
2	37	16.85000	2	101	0.00000
2	38	17.32500	2	102	0.00000
2	39	17.80000	2	103	0.00000
2	40	18.27500	2	104	0.00000
2	41	18.75000	2	105	0.00000
2	42	19.22500	2	106	0.00000
2	43	19.70000	2	107	0.00000
2	44	20.17500	2	108	0.00000
2	45	20.65000	2	109	0.00000
2	46	21.12500	2	110	0.00000
2	47	21.60000	2	111	0.00000
2	48	22.07500	2	112	0.00000
2	49	22.55000	2	113	0.00000
2	50	23.02500	2	114	0.00000
2	51	23.50000	2	115	0.00000
2	52	23.97500	2	116	0.00000
2	53	24.45000	2	117	0.00000
2	54	24.92500	2	118	0.00000
2	55	25.40000	2	119	0.00000
2	56	25.87500	2	120	0.00000
2	57	26.35000	2	121	0.00000
2	58	26.82500	2	122	0.00000
2	59	27.30000	2	123	0.00000
2	60	27.77500	2	124	0.00000
2	61	28.25000	2	125	0.00000
2	62	28.72500	2	126	0.00000
2	63	29.20000	2	127	0.00000
2	64	29.67500	2	128	0.00000
2	65	30.15000	2	129	0.00000
2	66	30.62500	2	130	0.00000
2	67	31.10000	2	131	0.00000
2	68	31.57500	2	132	0.00000
2	69	32.05000	2	133	0.00000
2	70	32.52500	2	134	0.00000
2	71	33.00000	2	135	0.00000
2	72	33.47500	2	136	0.00000
2	73	33.95000	2	137	0.00000
2	74	34.42500	2	138	0.00000
2	75	34.90000	2	139	0.00000
2	76	35.37500	2	140	0.00000
2	77	35.85000	2	141	0.00000
2	78	36.32500	2	142	0.00000
2	79	36.80000	2	143	0.00000
2	80	37.27500	2	144	0.00000
2	81	37.75000	2	145	0.00000
2	82	38.22500	2	146	0.00000
2	83	38.70000	2	147	0.00000
2	84	39.17500	2	148	0.00000
2	85	39.65000	2	149	0.00000
2	86	40.12500	2	150	0.00000
2	87	40.60000	2	151	0.00000
2	88	41.07500	2	152	0.00000
2	89	41.55000	2	153	0.00000
2	90	42.02500	2	154	0.00000
2	91	42.50000	2	155	0.00000
2	92	42.97500	2	156	0.00000
2	93	43.45000	2	157	0.00000
2	94	43.92500	2	158	0.00000
2	95	44.40000	2	159	0.00000
2	96	44.87500	2	160	0.00000
2	97	45.35000	2	161	0.00000
2	98	45.82500	2	162	0.00000
2	99	46.30000	2	163	0.00000
2	100	46.77500	2	164	0.00000
2	101	47.25000	2	165	0.00000
2	102	47.72500	2	166	0.00000
2	103	48.20000	2	167	0.00000
2	104	48.67500	2	168	0.00000
2	105	49.15000	2	169	0.00000
2	106	49.62500	2	170	0.00000
2	107	50.10000	2	171	0.00000
2	108	50.57500	2	172	0.00000
2	109	51.05000	2	173	0.00000
2	110	51.52500	2	174	0.00000
2	111	52.00000	2	175	0.00000
2	112	52.47500	2	176	0.00000
2	113	52.95000	2	177	0.00000
2	114	53.42500	2	178	0.00000
2	115	53.90000	2	179	0.00000
2	116	54.37500	2	180	0.00000
2	117	54.85000	2	181	0.00000
2	118	55.32500	2	182	0.00000
2	119	55.80000	2	183	0.00000
2	120	56.27500	2	184	0.00000
2	121	56.75000	2	185	0.00000
2	122	57.22500	2	186	0.00000
2	123	57.70000	2	187	0.00000
2	124	58.17500	2	188	0.00000
2	125	58.65000	2	189	0.00000
2	126	59.12500	2	190	0.00000
2	127	59.60000	2	191	0.00000
2	128	60.07500	2	192	0.00000
2	129	60.55000	2	193	0.00000
2	130	61.02500	2	194	0.00000
2	131	61.50000	2	195	0.00000
2	132	61.97500	2	196	0.00000
2	133	62.45000	2	197	0.00000
2	134	62.92500	2	198	0.00000
2	135	63.40000	2	199	0.00000
2	136	63.87500	2	200	0.00000
2	137	64.35000	2	201	0.00000
2	138	64.82500	2	202	0.00000
2	139	65.30000	2	203	0.00000
2	140	65.77500	2	204	0.00000
2	141	66.25000	2	205	0.00000
2	142	66.72500	2	206	0.00000
2	143	67.20000	2	207	0.00000
2	144	67.67500	2	208	0.00000
2	145	68.15000	2	209	0.00000
2	146	68.62500	2	210	0.00000
2	147	69.10000	2	211	0.00000
2	148	69.57500	2	212	0.00000
2	149	70.05000	2	213	0.00000
2	150	70.52500	2	214	0.00000
2	151	71.00000	2	215	0.00000
2	152	71.47500	2	216	0.00000
2	153	71.95000	2	217	0.00000
2	154	72.42500	2	218	0.00000
2	155	72.90000	2	219	0.00000
2	156	73.37500	2	220	0.00000
2	157	73.85000	2	221	0.00000
2	158	74.32500	2	222	0.00000
2	159	74.80000	2	223	0.00000
2	160	75.27500	2	224	0.00000
2	161	75.75000	2	225	0.00000
2	162	76.22500	2	226	0.00000
2	163	76.70000	2	227	0.00000
2	164	77.17500	2	228	0.00000
2	165	77.65000	2	229	0.00000
2	166	78.12500	2	230	0.00000
2	167	78.60000	2	231	0.00000
2	168	79.07500	2	232	0.00000
2	169	79.55000	2	233	0.00000
2	170	80.02500	2	234	0.00000
2	171	80.50000	2	235	0.00000
2	172	80.97500	2	236	0.00000
2	173	81.45000	2	237	0.00000
2	174	81.92500	2	238	0.00000
2	175	82.40000	2	239	0.00000
2	176	82.87500	2	240	0.00000
2	177	83.35000	2	241	0.00000
2	178	83.82500	2	242	0.00000
2	179	84.30000	2	243	0.00000
2	180	84.77500	2	244	0.00000
2	181	85.25000	2	245	0.00000
2	182	85.72500	2	246	0.00000
2	183	86.20000	2	247	0.00000
2	184	86.67500	2	248	0.00000
2	185	87.15000	2	249	0.00000
2	186	87.62500	2	250	0.00000
2	187	88.10000	2	251	0.00000
2	188	88.57500	2	252	0.00000
2	189	89.05000	2	253	0.00000
2	190	89.52500	2	254	0.00000
2	191	90.00000	2	255	0.00000
2	192	90.47500	2	256	0.00000
2	193	90.95000	2	257	0.00000
2	194	91.42500	2	258	0.00000
2	195	91.90000	2	259	0.00000
2	196	92.37500	2	260	0.00000
2	197	92.85000	2	261	0.00000
2	198	93.32500	2	262	0.00000
2	199	93.80000	2	263	0.00000
2	200	94.27500	2	264	0.00000
2	201	94.75000	2	265	0.00000
2	202	95.22500	2	266	0.00000
2	203	95.70000	2	267	0.00000
2	204	96.17500	2	268	0.00000
2	205	96.65000	2	269	0.00000
2	206	97.12500	2	270	0.00000
2	207	97.60000	2	271	0.00000
2	208	98.07500	2	272	0.00000
2	209	98.55000	2	273	0.00000
2	210	99.02500	2	274	0.00000
2	211	99.50000	2	275	0.00000
2	212	99.97500	2	276	0.00000
2	213	100.45000	2	277	0.00000

4	60	22.80240	5	30	13.62027
4	69	23.21420	5	39	13.26103
4	70	23.54634	5	40	13.60230
4	71	23.87823	5	41	14.01733
4	72	24.21135	5	42	14.34045
4	73	24.54240	5	43	14.67054
4	74	24.87440	5	44	15.00450
4	75	25.20630	5	45	15.33270
4	76	25.53672	5	46	15.66312
4	77	25.87086	5	47	15.99550
4	78	26.20297	5	48	16.32215
4	79	26.53330	5	49	16.65191
4	80	26.86731	5	50	16.98174
4	81	27.19976	5	51	17.31160
4	82	27.53204	5	52	17.64172
4	83	27.86434	5	53	17.97200
4	84	28.19666	5	54	18.30211
4	85	28.52900	5	55	18.63244
4	86	28.86130	5	56	18.96280
4	87	29.19375	5	57	19.29330
4	88	29.52613	5	58	19.62393
4	89	29.85854	5	59	19.95457
4	90	30.19097	5	60	20.28520
4	91	30.52341	5	61	20.61580
4	92	30.85586	5	62	20.94654
4	93	31.18830	5	63	21.27704
4	94	31.52080	5	64	21.60804
4	95	31.85334	5	65	21.93900
4	96	32.18580	5	66	22.27097
4	97	32.51842	5	67	22.60212
4	98	32.85097	5	68	22.93332
4	99	33.18353	5	69	23.26457
4	100	33.51611	5	70	23.59587
5	3	2.63371	5	71	23.92722
5	4	2.81777	5	72	24.25861
5	5	3.13364	5	73	24.59001
5	6	3.47825	5	74	24.92153
5	7	3.76989	5	75	25.25303
5	8	4.06625	5	76	25.58459
5	9	4.36721	5	77	25.91610
5	10	4.67171	5	78	26.24761
5	11	4.97915	5	79	26.57940
5	12	5.28909	5	80	26.91110
5	13	5.59115	5	81	27.24291
5	14	5.89504	5	82	27.57467
5	15	6.19832	5	83	27.90647
5	16	6.50470	5	84	28.23823
5	17	6.80350	5	85	28.57015
5	18	7.10460	5	86	28.90203
5	19	7.40803	5	87	29.23394
5	20	7.71380	5	88	29.56580
5	21	8.02164	5	89	29.89764
5	22	8.33114	5	90	30.22943
5	23	8.64230	5	91	30.56134
5	24	8.95500	5	92	30.89300
5	25	9.26930	5	93	31.22504
5	26	9.58523	5	94	31.55682
5	27	9.90280	5	95	31.88812
5	28	10.22110	5	96	32.21920
5	29	10.54171	5	97	32.55030
5	30	10.86267	5	98	32.88164
5	31	11.18407	5	99	33.21274
5	32	11.50600	5	100	33.54380
5	33	11.82853	6	7	3.16413
5	34	12.15170	6	8	3.43206
5	35	12.47521	6	9	3.70072
5	36	12.79721			
5	37	13.10433			
6	9	3.90714	6	75	23.21411
6	10	4.27479	6	76	23.64502
6	11	4.64731	6	77	24.07530
6	12	5.01480	6	78	24.50590
6	13	5.38634	6	79	24.93600
6	14	5.75400	6	80	25.36610
6	15	6.11774	6	81	25.79630
6	16	6.48397	6	82	26.22640
6	17	6.85080	6	83	26.65650
6	18	7.21830	6	84	27.08660
6	19	7.58640	6	85	27.51670
6	20	7.95490	6	86	27.94680
6	21	8.32300	6	87	28.37690
6	22	8.69170	6	88	28.80700
6	23	9.06000	6	89	29.23710
6	24	9.42890	6	90	29.66720
6	25	9.79740	6	91	30.09730
6	26	10.16650	6	92	30.52740
6	27	10.53520	6	93	30.95750
6	28	10.90400	6	94	31.38760
6	29	11.27290	6	95	31.81770
6	30	11.64190	6	96	32.24780
6	31	12.01100	6	97	32.67790
6	32	12.38020	6	98	33.10800
6	33	12.74940	6	99	33.53810
6	34	13.11870	6	100	33.96820
6	35	13.48800	7	7	3.60444
6	36	13.85730	7	8	3.94014
6	37	14.22660	7	9	4.27584
6	38	14.59590	7	10	4.61154
6	39	14.96520	7	11	4.94724
6	40	15.33450	7	12	5.28294
6	41	15.70380	7	13	5.61864
6	42	16.07310	7	14	5.95434
6	43	16.44240	7	15	6.29004
6	44	16.81170	7	16	6.62574
6	45	17.18100	7	17	6.96144
6	46	17.55030	7	18	7.29714
6	47	17.91960	7	19	7.63284
6	48	18.28890	7	20	7.96854
6	49	18.65820	7	21	8.30424
6	50	19.02750	7	22	8.63994
6	51	19.39680	7	23	8.97564
6	52	19.76610	7	24	9.31134
6	53	20.13540	7	25	9.64704
6	54	20.50470	7	26	9.98274
6	55	20.87400	7	27	10.31844
6	56	21.24330	7	28	10.65414
6	57	21.61260	7	29	10.98984
6	58	21.98190	7	30	11.32554
6	59	22.35120	7	31	11.66124
6	60	22.72050	7	32	11.99694
6	61	23.08980	7	33	12.33264
6	62	23.45910	7	34	12.66834
6	63	23.82840	7	35	13.00404
6	64	24.19770	7	36	13.33974
6	65	24.56700	7	37	13.67544
6	66	24.93630	7	38	14.01114
6	67	25.30560	7	39	14.34684
6	68	25.67490	7	40	14.68254
6	69	26.04420	7	41	15.01824
6	70	26.41350	7	42	15.35394
6	71	26.78280	7	43	15.68964
6	72	27.15210	7	44	16.02534
6	73	27.52140	7	45	16.36104
6	74	27.89070	7	46	16.69674
6	75	28.26000	7	47	17.03244
6	76	28.62930	7	48	17.36814
6	77	29.00000	7	49	17.70384
6	78	29.37070	7	50	18.03954
6	79	29.74140	7	51	18.37524
6	80	30.11210	7	52	18.71094
6	81	30.48280	7	53	19.04664
6	82	30.85350	7	54	19.38234
6	83	31.22420	7	55	19.71804
6	84	31.59490	7	56	20.05374
6	85	31.96560	7	57	20.38944
6	86	32.33630	7	58	20.72514
6	87	32.70700	7	59	21.06084
6	88	33.07770	7	60	21.39654
6	89	33.44840	7	61	21.73224
6	90	33.81910	7	62	22.06794
6	91	34.18980	7	63	22.40364
6	92	34.56050	7	64	22.73934
6	93	34.93120	7	65	23.07504
6	94	35.30190	7	66	23.41074
6	95	35.67260	7	67	23.74644
6	96	36.04330	7	68	24.08214
6	97	36.41400	7	69	24.41784
6	98	36.78470	7	70	24.75354
6	99	37.15540	7	71	25.08924
6	100	37.52610	7	72	25.42494

7	47	14.17982	8	20	7.47843
7	48	14.36584	8	21	7.38636
7	49	14.33394	8	22	6.23861
7	50	17.16126	8	23	6.08644
7	51	17.46677	8	24	6.31673
7	52	17.81643	8	25	9.33194
7	53	18.14426	8	26	9.54634
7	54	18.47225	8	27	9.86149
7	55	18.80036	8	28	10.17763
7	56	19.12846	8	29	10.49439
7	57	19.45707	8	30	10.81231
7	58	19.78622	8	31	11.13074
7	59	20.11439	8	32	11.44922
7	60	20.44260	8	33	11.76822
7	61	20.77080	8	34	12.08779
7	62	21.09902	8	35	12.40789
7	63	21.42815	8	36	12.72819
7	64	21.75834	8	37	13.04864
7	65	22.08871	8	38	13.37877
7	66	22.41915	8	39	13.70869
7	67	22.74978	8	40	14.03856
7	68	23.07726	8	41	14.36848
7	69	23.40636	8	42	14.69841
7	70	23.73672	8	43	14.99877
7	71	24.06687	8	44	15.31764
7	72	24.39686	8	45	15.64171
7	73	24.72659	8	46	15.96468
7	74	25.05637	8	47	16.29186
7	75	25.38672	8	48	16.61723
7	76	25.71693	8	49	16.94889
7	77	26.04720	8	50	17.28084
7	78	26.37754	8	51	17.60611
7	79	26.70783	8	52	17.93147
7	80	27.03836	8	53	18.26063
7	81	27.36896	8	54	18.59479
7	82	27.69947	8	55	18.93171
7	83	28.03009	8	56	19.26842
7	84	28.36076	8	57	19.60489
7	85	28.69146	8	58	19.94332
7	86	29.02225	8	59	20.28119
7	87	29.35307	8	60	20.61943
7	88	29.68394	8	61	20.95679
7	89	30.01488	8	62	21.29471
7	90	30.34580	8	63	21.63264
7	91	30.67679	8	64	21.97111
7	92	31.00783	8	65	22.31049
7	93	31.33891	8	66	22.64979
7	94	31.67002	8	67	22.98960
7	95	32.00117	8	68	23.32932
7	96	32.33234	8	69	23.66914
7	97	32.66359	8	70	23.99896
7	98	32.99485	8	71	24.32890
7	99	33.32614	8	72	24.65812
7	100	33.65747	8	73	24.98660
8	9	4.28814	8	74	25.31539
8	10	4.46341	8	75	25.64407
8	11	4.73626	8	76	25.97263
8	12	5.01341	8	77	26.30104
8	13	5.29608	8	78	26.62954
8	14	5.58281	8	79	26.95779
8	15	5.87366	8	80	27.28572
8	16	6.16829	8	81	27.61466
8	17	6.46683	8	82	27.94388
8	18	6.76934	8	83	28.27341
8	19	7.07580	8	84	28.60344
8	20	7.37721	8	85	28.93382
8	86	29.06787	9	86	29.04386
8	87	29.42729	9	87	29.37606
8	88	29.78754	9	88	21.29789
8	89	30.06788	9	89	21.62649
8	90	30.41822	9	90	21.95183
8	91	30.74603	9	91	22.27791
8	92	31.07913	9	92	22.60404
8	93	31.40944	9	93	22.93449
8	94	31.73621	9	94	23.26326
8	95	32.07063	9	95	23.59016
8	96	32.40149	9	96	23.91621
8	97	32.72819	9	97	24.24436
8	98	33.06234	9	98	24.57409
8	99	33.39372	9	99	24.90296
8	100	33.72454	9	100	25.23142
9	9	4.72942	9	74	25.56090
9	10	4.94351	9	75	25.89043
9	11	5.25336	9	76	26.21741
9	12	5.55902	9	77	26.54426
9	13	5.86042	9	78	26.87819
9	14	6.09450	9	79	27.20421
9	15	6.36214	9	80	27.53331
9	16	6.67806	9	81	27.86256
9	17	6.98986	9	82	28.19176
9	18	7.29742	9	83	28.52119
9	19	7.54722	9	84	28.85081
9	20	7.84912	9	85	29.18060
9	21	8.17232	9	86	29.50955
9	22	8.47643	9	87	29.83917
9	23	8.78050	9	88	30.16804
9	24	9.09490	9	89	30.49641
9	25	9.40941	9	90	30.82342
9	26	9.71402	9	91	31.15036
9	27	10.02893	9	92	31.48022
9	28	10.34406	9	93	31.81122
9	29	10.65413	9	94	32.14426
9	30	10.96904	9	95	32.47836
9	31	11.28484	9	96	32.80851
9	32	11.60134	9	97	33.13872
9	33	11.91856	9	98	33.46897
9	34	12.23447	9	99	33.79926
9	35	12.54997	10	10	3.23929
9	36	12.87006	10	11	3.56829
9	37	13.19089	10	12	3.77129
9	38	13.51352	10	13	6.04046
9	39	13.83445	10	14	6.32469
9	40	14.15582	10	15	6.99777
9	41	14.47783	10	16	6.88443
9	42	14.79949	10	17	7.18429
9	43	15.12123	10	18	7.47706
9	44	15.44389	10	19	7.77227
9	45	15.76690	10	20	8.06905
9	46	16.09023	10	21	8.36953
9	47	16.41386	10	22	8.67111
9	48	16.73782	10	23	8.97443
9	49	17.06294	10	24	9.27924
9	50	17.38834	10	25	9.58579
9	51	17.71120	10	26	9.89341
9	52	18.03329	10	27	10.20235
9	53	18.35151	10	28	10.51344
9	54	18.66494	10	29	10.82599
9	55	19.01243	10	30	11.13872
9	56	19.33050	10	31	11.44976
9	57	19.64837	10	32	11.76299
9	58	19.96603	10	33	12.07713
9	59	20.28176	10	34	12.39265

10	26	12.70001	11	11	5.70005
10	27	12.00002	11	12	6.00010
10	28	12.30003	11	13	6.30015
10	29	12.60004	11	14	6.60020
10	30	12.90005	11	15	6.90025
10	31	13.20006	11	16	7.20030
10	32	13.50007	11	17	7.50035
10	33	13.80008	11	18	7.80040
10	34	14.10009	11	19	8.10045
10	35	14.40010	11	20	8.40050
10	36	14.70011	11	21	8.70055
10	37	15.00012	11	22	9.00060
10	38	15.30013	11	23	9.30065
10	39	15.60014	11	24	9.60070
10	40	15.90015	11	25	9.90075
10	41	16.20016	11	26	10.20080
10	42	16.50017	11	27	10.50085
10	43	16.80018	11	28	10.80090
10	44	17.10019	11	29	11.10095
10	45	17.40020	11	30	11.40100
10	46	17.70021	11	31	11.70105
10	47	18.00022	11	32	12.00110
10	48	18.30023	11	33	12.30115
10	49	18.60024	11	34	12.60120
10	50	18.90025	11	35	12.90125
10	51	19.20026	11	36	13.20130
10	52	19.50027	11	37	13.50135
10	53	19.80028	11	38	13.80140
10	54	20.10029	11	39	14.10145
10	55	20.40030	11	40	14.40150
10	56	20.70031	11	41	14.70155
10	57	21.00032	11	42	15.00160
10	58	21.30033	11	43	15.30165
10	59	21.60034	11	44	15.60170
10	60	21.90035	11	45	15.90175
10	61	22.20036	11	46	16.20180
10	62	22.50037	11	47	16.50185
10	63	22.80038	11	48	16.80190
10	64	23.10039	11	49	17.10195
10	65	23.40040	11	50	17.40200
10	66	23.70041	11	51	17.70205
10	67	24.00042	11	52	18.00210
10	68	24.30043	11	53	18.30215
10	69	24.60044	11	54	18.60220
10	70	24.90045	11	55	18.90225
10	71	25.20046	11	56	19.20230
10	72	25.50047	11	57	19.50235
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10	74	26.10049	11	59	20.10245
10	75	26.40050	11	60	20.40250
10	76	26.70051	11	61	20.70255
10	77	27.00052	11	62	21.00260
10	78	27.30053	11	63	21.30265
10	79	27.60054	11	64	21.60270
10	80	27.90055	11	65	21.90275
10	81	28.20056	11	66	22.20280
10	82	28.50057	11	67	22.50285
10	83	28.80058	11	68	22.80290
10	84	29.10059	11	69	23.10295
10	85	29.40060	11	70	23.40300
10	86	29.70061	11	71	23.70305
10	87	30.00062	11	72	24.00310
10	88	30.30063	11	73	24.30315
10	89	30.60064	11	74	24.60320
10	90	30.90065	11	75	24.90325
10	91	31.20066	11	76	25.20330
10	92	31.50067	11	77	25.50335
10	93	31.80068	11	78	25.80340
10	94	32.10069	11	79	26.10345
10	95	32.40070	11	80	26.40350
10	96	32.70071	11	81	26.70355
10	97	33.00072	11	82	27.00360
10	98	33.30073	11	83	27.30365
10	99	33.60074	11	84	27.60370
10	100	33.90075	11	85	27.90375
10	101	34.20076	11	86	28.20380
10	102	34.50077	11	87	28.50385
10	103	34.80078	11	88	28.80390
10	104	35.10079	11	89	29.10395
10	105	35.40080	11	90	29.40400
10	106	35.70081	11	91	29.70405
10	107	36.00082	11	92	30.00410
10	108	36.30083	11	93	30.30415
10	109	36.60084	11	94	30.60420
10	110	36.90085	11	95	30.90425
10	111	37.20086	11	96	31.20430
10	112	37.50087	11	97	31.50435
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10	116	38.70091	11	101	32.70455
10	117	39.00092	11	102	33.00460
10	118	39.30093	11	103	33.30465
10	119	39.60094	11	104	33.60470
10	120	39.90095	11	105	33.90475
10	121	40.20096	11	106	34.20480
10	122	40.50097	11	107	34.50485
10	123	40.80098	11	108	34.80490
10	124	41.10099	11	109	35.10495
10	125	41.40100	11	110	35.40500
10	126	41.70101	11	111	35.70505
10	127	42.00102	11	112	36.00510
10	128	42.30103	11	113	36.30515
10	129	42.60104	11	114	36.60520
10	130	42.90105	11	115	36.90525
10	131	43.20106	11	116	37.20530
10	132	43.50107	11	117	37.50535
10	133	43.80108	11	118	37.80540
10	134	44.10109	11	119	38.10545
10	135	44.40110	11	120	38.40550
10	136	44.70111	11	121	38.70555
10	137	45.00112	11	122	39.00560
10	138	45.30113	11	123	39.30565
10	139	45.60114	11	124	39.60570
10	140	45.90115	11	125	39.90575
10	141	46.20116	11	126	40.20580
10	142	46.50117	11	127	40.50585
10	143	46.80118	11	128	40.80590
10	144	47.10119	11	129	41.10595
10	145	47.40120	11	130	41.40600
10	146	47.70121	11	131	41.70605
10	147	48.00122	11	132	42.00610
10	148	48.30123	11	133	42.30615
10	149	48.60124	11	134	42.60620
10	150	48.90125	11	135	42.90625

13	32	13.29028	13	96	23.20000
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13	34	13.29139	13	100	24.14175
13	35	13.29021	14	14	7.21010
13	36	13.29004	14	15	7.20000
13	37	13.29070	14	16	7.20754
13	38	14.13515	14	17	6.11000
13	39	14.04435	14	18	6.20015
13	40	14.77726	14	19	6.27100
13	41	15.00000	14	20	6.20000
13	42	15.00000	14	21	6.20077
13	43	15.71904	14	22	6.20000
13	44	16.00000	14	23	6.21002
13	45	16.20115	14	24	10.10136
13	46	16.00700	14	25	10.20000
13	47	16.00454	14	26	10.00000
13	48	17.20100	14	27	10.00000
13	49	17.01000	14	28	11.20000
13	50	17.20011	14	29	11.20000
13	51	18.20000	14	30	11.00000
13	52	18.27500	14	31	12.10001
13	53	18.00000	14	32	12.00000
13	54	19.21004	14	33	13.70072
13	55	19.20045	14	34	13.00000
13	56	19.00000	14	35	13.00070
13	57	20.17070	14	36	13.71200
13	58	20.00704	14	37	14.00000
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13	61	21.00011	14	40	14.00004
13	62	21.70000	14	41	15.20010
13	63	22.10700	14	42	15.07430
13	64	22.00000	14	43	15.00725
13	65	22.70000	14	44	16.20074
13	66	23.07000	14	45	16.21000
13	67	23.00000	14	46	16.20000
13	68	23.70000	14	47	17.10000
13	69	24.00707	14	48	17.00000
13	70	24.27100	14	49	17.77000
13	71	24.00010	14	50	18.00000
13	72	25.00000	14	51	18.01000
13	73	25.20000	14	52	18.70000
13	74	25.07000	14	53	19.00000
13	75	25.00001	14	54	19.20000
13	76	26.20014	14	55	19.00000
13	77	26.00000	14	56	20.00010
13	78	26.07007	14	57	20.20000
13	79	27.20000	14	58	20.00000
13	80	27.00010	14	59	20.00000
13	81	27.00000	14	60	21.20000
13	82	28.27007	14	61	21.00000
13	83	28.00000	14	62	21.00000
13	84	28.00000	14	63	22.20000
13	85	29.20000	14	64	22.00000
13	86	29.07000	14	65	22.00000
13	87	29.00000	14	66	23.21000
13	88	30.20000	14	67	23.00000
13	89	30.00000	14	68	23.00000
13	90	31.21070	14	69	24.17010
13	91	31.21070	14	70	24.00000
13	92	31.00000	14	71	24.00000
13	93	31.00000	14	72	25.10000
13	94	32.10000	14	73	25.07000
13	95	33.20000	14	74	25.70015
13	96	33.00000	14	75	26.11015
13	97	33.17000	14	76	26.00000

14	77	26.70007	15	57	28.07010
14	78	27.00000	15	58	28.70000
14	79	27.01000	15	59	21.10000
14	80	27.74172	15	60	21.00000
14	81	28.00000	15	61	21.70000
14	82	28.20000	15	62	22.00000
14	83	28.70000	15	63	22.00000
14	84	29.00000	15	64	22.70000
14	85	29.20010	15	65	23.00000
14	86	29.00000	15	66	23.00000
14	87	29.01070	15	67	23.00000
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14	89	30.07000	15	69	24.21000
14	90	30.00000	15	70	24.00000
14	91	31.20000	15	71	24.00000
14	92	31.00000	15	72	25.20000
14	93	31.07007	15	73	25.00000
14	94	32.00000	15	74	25.00000
14	95	32.00000	15	75	26.20071
14	96	32.00000	15	76	26.07000
14	97	33.20000	15	77	26.00000
14	98	33.01170	15	78	26.00000
14	99	33.00000	15	79	27.00000
14	100	34.20010	15	80	27.00000
15	15	7.00000	15	81	28.10000
15	16	8.00000	15	82	28.00000
15	17	8.00000	15	83	28.00000
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15	23	10.00000	15	89	30.70011
15	24	10.20000	15	90	31.10000
15	25	10.01070	15	91	31.00000
15	26	10.00000	15	92	31.00000
15	27	11.10000	15	93	32.00000
15	28	11.00000	15	94	32.00000
15	29	11.70000	15	95	32.70000
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16	41	15.62200	17	23	10.50205
16	42	15.93090	17	24	10.70344
16	43	16.23990	17	25	11.00454
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16	47	17.48500	17	29	12.37304
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16	49	18.10771	17	31	13.20431
16	50	18.42107	17	32	13.60990
16	51	18.73497	17	33	13.99430
16	52	19.04830	17	34	14.36430
16	53	19.36104	17	35	14.70903
16	54	19.67300	17	36	15.03402
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16	56	20.29113	17	38	15.62404
16	57	20.59222	17	39	15.89150
16	58	20.88934	17	40	16.14000
16	59	21.18200	17	41	16.36132
16	60	21.47042	17	42	16.55780
16	61	21.75483	17	43	16.73040
16	62	22.03522	17	44	16.78224
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16	64	22.58403	17	46	17.24891
16	65	22.85250	17	47	17.48977
16	66	23.11700	17	48	17.77020
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16	68	23.63697	17	50	18.50097
16	69	23.89242	17	51	18.90032
16	70	24.14574	17	52	19.31013
16	71	24.39690	17	53	19.53143
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16	73	24.89033	17	55	20.10053
16	74	25.13242	17	56	20.47430
16	75	25.37241	17	57	20.78947
16	76	25.61032	17	58	21.10809
16	77	25.84604	17	59	21.42111
16	78	26.07953	17	60	21.72713
16	79	26.31097	17	61	22.02433
16	80	26.54047	17	62	22.37130
16	81	26.76804	17	63	22.68902
16	82	26.99363	17	64	22.98800
16	83	27.21720	17	65	23.25000
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16	85	27.65841	17	67	23.94242
16	86	27.87602	17	68	24.20134
16	87	28.09163	17	69	24.60090
16	88	28.30527	17	70	24.83064
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16	92	29.14043	17	74	26.20000
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17	23	10.50205	18	72	25.70004
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17	25	11.00454	18	74	26.20721
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21	64	23.97279	22	51	21.25000
21	65	24.28729	22	52	21.60000
21	66	24.60129	22	53	21.95000
21	67	24.91500	22	54	22.30000
21	68	25.22931	22	55	22.65000
21	69	25.54439	22	56	23.00000
21	70	25.86001	22	57	23.35000
21	71	26.17637	22	58	23.70000
21	72	26.49304	22	59	24.05000
21	73	26.81000	22	60	24.40000
21	74	27.12729	22	61	24.75000
21	75	27.44500	22	62	25.10000
21	76	27.76311	22	63	25.45000
21	77	28.08166	22	64	25.80000
21	78	28.40067	22	65	26.15000
21	79	28.71971	22	66	26.50000
21	80	29.03910	22	67	26.85000
21	81	29.35874	22	68	27.20000
21	82	29.67889	22	69	27.55000
21	83	29.99904	22	70	27.90000
21	84	30.31949	22	71	28.25000
21	85	30.64000	22	72	28.60000
21	86	30.96074	22	73	28.95000
21	87	31.28166	22	74	29.30000
21	88	31.60271	22	75	29.65000
21	89	31.92389	22	76	30.00000

22	77	27.81604	23	65	24.32279
22	78	28.22641	23	66	24.63374
22	79	28.53226	23	67	24.94522
22	80	28.84939	23	68	25.25712
22	81	29.16770	23	69	25.56943
22	82	29.48447	23	70	25.88214
22	83	29.82240	23	71	26.19524
22	84	30.14089	23	72	26.50871
22	85	30.45928	23	73	26.82255
22	86	30.77770	23	74	27.13674
22	87	31.09641	23	75	27.45127
22	88	31.41576	23	76	27.76614
22	89	31.73512	23	77	28.08133
22	90	32.05470	23	78	28.39684
22	91	32.37460	23	79	28.71264
22	92	32.69480	23	80	29.02877
22	93	33.01471	23	81	29.34518
22	94	33.33511	23	82	29.66184
22	95	33.65570	23	83	29.97882
22	96	33.97640	23	84	30.29600
22	97	34.29746	23	85	30.61356
22	98	34.61890	23	86	30.93129
22	99	34.93993	23	87	31.24929
22	100	35.26142	23	88	31.56752
23	23	12.00000	23	89	31.88600
23	24	12.26572	23	90	32.20469
23	25	12.53042	23	91	32.52362
23	26	12.79773	23	92	32.84276
23	27	13.06733	23	93	33.16212
23	28	13.33932	23	94	33.48169
23	29	13.61390	23	95	33.80146
23	30	13.89023	23	96	34.12143
23	31	14.16832	23	97	34.44160
23	32	14.44854	23	98	34.76185
23	33	14.73000	23	99	35.08229
23	34	15.01282	23	100	35.40321
23	35	15.29691	24	24	12.52474
23	36	15.58339	24	25	12.78462
23	37	15.87239	24	26	13.04517
23	38	16.16394	24	27	13.31224
23	39	16.45807	24	28	13.57769
23	40	16.74584	24	29	13.85037
23	41	17.03635	24	30	14.13317
23	42	17.32974	24	31	14.41697
23	43	17.62597	24	32	14.70464
23	44	17.92422	24	33	14.99614
23	45	18.22446	24	34	15.28734
23	46	18.52663	24	35	15.58013
23	47	18.83071	24	36	15.87450
23	48	19.13684	24	37	16.16951
23	49	19.44493	24	38	16.46782
23	50	19.75401	24	39	16.76807
23	51	20.06526	24	40	17.06988
23	52	20.36863	24	41	17.37461
23	53	20.67401	24	42	17.68190
23	54	20.98149	24	43	17.99240
23	55	21.29002	24	44	18.30677
23	56	21.59959	24	45	18.62423
23	57	21.91017	24	46	18.94481
23	58	22.22184	24	47	19.26861
23	59	22.53460	24	48	19.59490
23	60	22.77450	24	49	19.92304
23	61	23.06321	24	50	20.25359
23	62	23.35283	24	51	20.58664
23	63	23.70199	24	52	20.92210
23	64	24.01211	24	53	20.22790
24	54	21.13021	25	44	19.38277
24	55	21.43301	25	45	19.69681
24	56	21.73807	25	46	19.99904
24	57	22.04537	25	47	20.30484
24	58	22.35489	25	48	20.61364
24	59	22.66660	25	49	20.92613
24	60	22.98129	25	50	21.24040
24	61	23.29854	25	51	21.55745
24	62	23.61833	25	52	21.87729
24	63	23.94043	25	53	22.19790
24	64	24.26484	25	54	22.52025
24	65	24.59274	25	55	22.84432
24	66	24.92351	25	56	23.17007
24	67	25.25774	25	57	23.49749
24	68	25.59541	25	58	23.82656
24	69	25.93652	25	59	24.15807
24	70	26.28104	25	60	24.49193
24	71	26.62796	25	61	24.82829
24	72	26.97828	25	62	25.16707
24	73	27.33200	25	63	25.50829
24	74	27.68914	25	64	25.85193
24	75	28.04969	25	65	26.19807
24	76	28.41365	25	66	26.54671
24	77	28.78102	25	67	26.89784
24	78	29.15172	25	68	27.25146
24	79	29.52573	25	69	27.60757
24	80	29.90301	25	70	27.96617
24	81	30.28352	25	71	28.32726
24	82	30.66721	25	72	28.69084
24	83	31.05406	25	73	29.05690
24	84	31.44406	25	74	29.42544
24	85	31.83729	25	75	29.79647
24	86	32.23374	25	76	30.16999
24	87	32.63351	25	77	30.54601
24	88	33.03660	25	78	30.92454
24	89	33.44301	25	79	31.30567
24	90	33.85274	25	80	31.68939
24	91	34.26589	25	81	32.07571
24	92	34.68246	25	82	32.46464
24	93	35.10246	25	83	32.85617
24	94	35.52589	25	84	33.25030
24	95	35.95274	25	85	33.64703
24	96	36.38301	25	86	34.04636
24	97	36.81671	25	87	34.44829
24	98	37.25384	25	88	34.85282
24	99	37.69440	25	89	35.26005
24	100	38.13841	25	90	35.66998
25	23	13.04572	25	91	36.08261
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25	26	14.01924	25	94	37.33670
25	27	14.34951	25	95	37.76013
25	28	14.68284	25	96	38.18626
25	29	15.01921	25	97	38.61509
25	30	15.35864	25	98	39.04662
25	31	15.70117	25	99	39.48085
25	32	16.04680	25	100	39.91778
25	33	16.39553	26	26	13.94673
25	34	16.74736	26	27	14.29856
25	35	17.10229	26	28	14.65239
25	36	17.46032	26	29	15.00822
25	37	17.82145	26	30	15.36605
25	38	18.18568	26	31	15.72588
25	39	18.55301	26	32	16.08771
25	40	18.92344	26	33	16.45154
25	41	19.29697	26	34	16.81737
25	42	19.67360	26	35	17.18520
25	43	20.05333	26	36	17.55503
25	44	20.43616	26	37	17.92686
25	45	20.82209	26	38	18.29969
25	46	21.21112	26	39	18.67452
25	47	21.60325	26	40	19.05135
25	48	21.99848	26	41	19.43018
25	49	22.39681	26	42	19.81101
25	50	22.79824	26	43	20.19384
25	51	23.20277	26	44	20.57867
25	52	23.61040	26	45	20.96550
25	53	24.02113	26	46	21.35433
25	54	24.43496	26	47	21.74616
25	55	24.85179	26	48	22.14099
25	56	25.27162	26	49	22.53882
25	57	25.69445	26	50	22.93965
25	58	26.12028	26	51	23.34348
25	59	26.54911	26	52	23.75031
25	60	26.98094	26	53	24.16014
25	61	27.41577	26	54	24.57297
25	62	27.85360	26	55	24.98880
25	63	28.29443	26	56	25.40763
25	64	28.73826	26	57	25.82946
25	65	29.18509	26	58	26.25429
25	66	29.63492	26	59	26.68212
25	67	30.08775	26	60	27.11295
25	68	30.54358	26	61	27.54678
25	69	31.00241	26	62	27.98361
25	70	31.46424	26	63	28.42344
25	71	31.92907	26	64	28.86627
25	72	32.39690	26	65	29.31210
25	73	32.86773	26	66	29.76093
25	74	33.34156	26	67	30.21276
25	75	33.81839	26	68	30.66759
25	76	34.29822	26	69	31.12542
25	77	34.78105	26	70	31.58625
25	78	35.26688	26	71	32.05008
25	79	35.75571	26	72	32.51691
25	80	36.24754	26	73	32.98674
25	81	36.74237	26	74	33.45957
25	82	37.24020	26	75	33.93540
25	83	37.74103	26	76	34.41423
25	84	38.24486	26	77	34.89606
25	85	38.75169	26	78	35.38089
25	86	39.26152	26	79	35.86872
25	87	39.77435	26	80	36.35955
25	88	40.29018	26	81	36.85338
25	89	40.80901	26	82	37.35021
25	90	41.33084	26	83	37.85004
25	91	41.85567	26	84	38.35287
25	92	42.38350	26	85	38.85870
25	93	42.91433	26	86	39.36753
25	94	43.44816	26	87	39.87936
25	95	43.98499	26	88	40.39419
25	96	44.52482	26	89	40.91202
25	97	45.06765	26	90	41.43285
25	98	45.61348	26	91	41.95668
25	99	46.16231	26	92	42.48351
25	100	46.71414	26	93	43.01334
26	26	14.38944	26	94	43.54617
26	27	14.82227	26	95	44.08200
26	28	15.25810	26	96	44.62083
26	29	15.69693	26	97	45.16266
26	30	16.13876	26	98	45.70749
26	31	16.58359	26	99	46.25532
26	32	17.03142	26	100	46.80615
26	33	17.48225	27	27	15.72300
26	34	17.93608	27	28	16.17283
26	35	18.39291	27	29	16.62566
26	36	18.85274	27	30	17.08149
26	37	19.31557	27	31	17.53932
26	38	19.78140	27	32	18.00015
26	39	20.25023	27	33	18.46398
26	40	20.72206	27	34	18.93081
26	41	21.19689	27	35	19.39964
26	42	21.67472	27	36	19.87147
26	43	22.15555	27	37	20.34630
26	44				

26	26	16.00194	27	27	14.00777
26	26	16.26190	27	28	14.34054
26	27	16.56342	27	29	14.61270
26	28	16.84423	27	30	14.88012
26	29	17.13004	27	31	15.14049
26	30	17.41660	27	32	15.41230
26	31	17.70366	27	33	15.68184
26	32	17.99196	27	34	15.94622
26	33	18.28144	27	35	16.20236
26	34	18.57204	27	36	16.45918
26	35	18.86377	27	37	16.71686
26	36	19.15652	27	38	16.97551
26	37	19.45028	27	39	17.23520
26	38	19.74500	27	40	17.49586
26	39	20.04064	27	41	17.75746
26	40	20.33720	27	42	18.01999
26	41	20.63460	27	43	18.28353
26	42	20.93283	27	44	18.54800
26	43	21.23196	27	45	18.81339
26	44	21.53193	27	46	19.07972
26	45	21.83271	27	47	19.34699
26	46	22.13340	27	48	19.61520
26	47	22.43489	27	49	19.88432
26	48	22.73690	27	50	20.15430
26	49	23.03926	27	51	20.42517
26	50	23.34214	27	52	20.69690
26	51	23.64542	27	53	20.96949
26	52	23.94919	27	54	21.24292
26	53	24.25342	27	55	21.51719
26	54	24.55810	27	56	21.79230
26	55	24.86321	27	57	22.06826
26	56	25.16874	27	58	22.34506
26	57	25.47469	27	59	22.62270
26	58	25.78106	27	60	22.90119
26	59	26.08784	27	61	23.18052
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26	61	26.70260	27	63	23.74170
26	62	27.01057	27	64	24.02354
26	63	27.31894	27	65	24.30621
26	64	27.62771	27	66	24.58970
26	65	27.93688	27	67	24.87400
26	66	28.24644	27	68	25.15911
26	67	28.55639	27	69	25.44502
26	68	28.86674	27	70	25.73174
26	69	29.17749	27	71	26.01926
26	70	29.48864	27	72	26.30759
26	71	29.79990	27	73	26.59674
26	72	30.11146	27	74	26.88670
26	73	30.42332	27	75	27.17746
26	74	30.73547	27	76	27.46902
26	75	31.04792	27	77	27.76139
26	76	31.36067	27	78	28.05456
26	77	31.67372	27	79	28.34854
26	78	31.98707	27	80	28.64332
26	79	32.30072	27	81	28.93890
26	80	32.61467	27	82	29.23528
26	81	32.92892	27	83	29.53246
26	82	33.24347	27	84	29.83044
26	83	33.55832	27	85	30.12922
26	84	33.87347	27	86	30.42880
26	85	34.18892	27	87	30.72918
26	86	34.50467	27	88	31.03036
26	87	34.82072	27	89	31.33234
26	88	35.13707	27	90	31.63512
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26	90	35.77067	27	92	32.24308
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27	31	35.05834	28	32	32.96050
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27	41	38.45529	28	42	36.15130
27	42	38.81290	28	43	36.48138
27	43	39.17371	28	44	36.81346
27	44	39.53772	28	45	37.14754
27	45	39.90493	28	46	37.48362
27	46	40.27534	28	47	37.82170
27	47	40.64895	28	48	38.16178
27	48	41.02576	28	49	38.50386
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27	50	41.78898	28	51	39.19402
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27	61	46.20999	28	62	43.13290
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27	63	47.05411	28	64	43.87506
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27	68	49.21926	28	69	45.76546
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27	70	50.10768	28	71	46.53562
27	71	50.55669	28	72	46.92370
27	72	51.00890	28	73	47.31378
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27	74	51.92292	28	75	48.09994
27	75	52.38473	28	76	48.49602
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27	78	53.78936	28	79	49.69626
27	79	54.26397	28	80	50.10034
27	80	54.74178	28	81	50.50642
27	81	55.22279	28	82	50.91450
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27	85	57.17883	28	86	52.56682
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27	87	58.17605	28	88	53.40498
27	88	58.67946	28	89	53.82706
27	89	59.18607	28	90	54.25114
27	90	59.69588	28	91	54.67722
27	91	60.20889	28	92	55.10530
27	92	60.72510			

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20	86	21.82293	30	81	20.84674
20	87	22.23494	30	82	20.85436
20	88	22.84794	30	83	21.16536
20	89	23.06032	30	84	21.47872
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30	33	16.44844	30	99	26.17010
30	34	16.71829	30	100	26.48522
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30	49	20.91314	31	45	19.99572
30	50	21.20309	31	46	20.27968
30	51	21.49401	31	47	20.56479
30	52	21.78596	31	48	20.85101
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30	67	26.25698	31	63	25.25426
30	68	26.56021	31	64	25.55388
30	69	26.86401	31	65	25.85412
30	70	27.16833	31	66	26.15497
30	71	27.47316	31	67	26.45641
30	72	27.77849	31	68	26.75843
30	73	28.08431	31	69	27.06101
30	74	28.39059	31	70	27.36413
31	71	27.68779	32	60	26.96336
31	72	27.97194	32	61	27.26072
31	73	28.27600	32	62	27.56264
31	74	28.58173	32	63	27.86811
31	75	28.88737	32	64	28.16810
31	76	29.19345	32	65	28.47141
31	77	29.49994	32	66	28.77542
31	78	29.80688	32	67	29.08012
31	79	30.11424	32	68	29.38610
31	80	30.42215	32	69	29.69453
31	81	30.73053	32	70	29.99642
31	82	31.03934	32	71	30.30275
31	83	31.34764	32	72	30.60950
31	84	31.65751	32	73	30.91668
31	85	31.96783	32	74	31.22426
31	86	32.27712	32	75	31.53223
31	87	32.58735	32	76	31.84060
31	88	32.89822	32	77	32.14934
31	89	33.20942	32	78	32.45843
31	90	33.52088	32	79	32.76782
31	91	33.83260	32	80	33.07774
31	92	34.14465	32	81	33.38791
31	93	34.45701	32	82	33.69841
31	94	34.76964	32	83	34.00924
31	95	35.08260	32	84	34.32039
31	96	35.39583	32	85	34.63185
31	97	35.70933	32	86	34.94362
31	98	36.02310	32	87	35.25569
31	99	36.33713	32	88	35.56806
31	100	36.65143	32	89	35.88069
32	32	16.63330	32	90	36.19362
32	33	16.95493	32	91	36.50682
32	34	17.21658	32	100	36.82029
32	35	17.48417	33	33	17.21446
32	36	17.75162	33	34	17.47604
32	37	18.02004	33	35	17.73963
32	38	18.29173	33	36	18.00699
32	39	18.56429	33	37	18.27234
32	40	18.83639	33	38	18.54133
32	41	19.11399	33	39	18.81197
32	42	19.39182	33	40	19.08420
32	43	19.66943	33	41	19.35794
32	44	19.94617	33	42	19.63318
32	45	20.22018	33	43	19.90900
32	46	20.51261	33	44	20.18770
32	47	20.79863	33	45	20.46787
32	48	21.08939	33	46	20.74760
32	49	21.38405	33	47	21.02925
32	50	21.68274	33	48	21.31226
32	51	21.98449	33	49	21.59629
32	52	22.28921	33	50	21.88140
32	53	22.51088	33	51	22.16784
32	54	22.80947	33	52	22.45472
32	55	23.10695	33	53	22.74297
32	56	23.39329	33	54	23.03195
32	57	23.68646	33	55	23.32195
32	58	23.98044	33	56	23.61282
32	59	24.27519	33	57	23.90458
32	60	24.57071	33	58	24.19711
32	61	24.86698	33	59	24.49047
32	62	25.16390	33	60	24.78460
32	63	25.46134	33	61	25.07940
32	64	25.75904	33	62	25.37599
32	65	26.05698	33	63	25.67141
32	66	26.35630	33	64	25.96842
32	67	26.65887	33	65	26.26699

23	66	26.54444	34	63	26.47869
23	67	26.54335	34	64	26.77295
23	68	27.18290	34	65	27.67043
23	69	27.64306	34	66	27.34897
23	70	27.76377	34	67	27.66781
23	71	26.96385	34	68	27.96744
23	72	26.54404	34	69	28.26754
23	73	26.54824	34	70	28.54420
23	74	26.67212	34	71	28.86941
23	75	29.27530	34	72	29.17116
23	76	29.57937	34	73	29.47341
23	77	29.64371	34	74	29.77518
23	78	26.18482	34	75	29.67943
23	79	26.48376	34	76	29.38316
23	80	26.79949	34	77	29.68736
23	81	21.18682	34	78	29.98862
23	82	21.43219	34	79	21.29712
23	83	21.71814	34	80	21.68865
23	84	22.62681	34	81	21.98669
23	85	22.33426	34	82	22.21496
23	86	22.64246	34	83	22.52173
23	87	22.95091	34	84	22.82869
23	88	23.25875	34	85	23.13643
23	89	23.56590	34	86	23.44425
23	90	23.87357	34	87	23.75263
23	91	24.18049	34	88	24.06129
23	92	24.48673	34	89	24.37027
23	93	24.79329	34	90	24.67940
23	94	25.12610	34	91	24.98927
23	95	25.43137	34	92	25.29926
23	96	25.74284	34	93	25.60937
23	97	26.06463	34	94	25.92029
23	98	26.36673	34	95	26.23113
23	99	26.67910	34	96	26.54236
23	100	26.99174	34	97	26.85369
24	34	17.73564	34	100	27.16370
24	35	17.99721	35	35	18.26462
24	36	16.26670	35	36	18.51830
24	37	16.52663	35	37	18.76179
24	38	16.78311	35	38	19.04699
24	39	19.06187	35	39	19.31391
24	40	19.33235	35	40	19.58246
24	41	19.60419	35	41	19.85260
24	42	19.87762	35	42	20.12426
24	43	20.15244	35	43	20.39737
24	44	20.42872	35	44	20.67189
24	45	20.70630	35	45	20.94777
24	46	20.98515	35	46	21.22495
24	47	21.26323	35	47	21.50339
24	48	21.54630	35	48	21.78304
24	49	21.82882	35	49	22.06386
24	50	22.11244	35	50	22.34511
24	51	22.39784	35	51	22.62685
24	52	22.68266	35	52	22.91285
24	53	22.96929	35	53	23.19896
24	54	23.25686	35	54	23.48416
24	55	23.54540	35	55	23.77122
24	56	23.83402	35	56	24.05920
24	57	24.12512	35	57	24.34807
24	58	24.41626	35	58	24.63781
24	59	24.70623	35	59	24.92839
24	60	25.00090	35	60	25.21979
24	61	25.29452	35	61	25.51197
24	62	25.58679	35	62	25.80482
24	63	25.88380	35	63	26.09841
24	64	26.17956	35	64	26.39362

26	65	26.66614	36	65	27.19729
26	66	26.96394	36	66	27.48251
26	67	27.26639	36	67	27.76639
26	68	27.57749	36	68	28.06491
26	69	27.87822	36	69	28.36286
26	70	26.17584	36	70	28.67361
26	71	26.47246	36	71	28.97815
26	72	26.77196	36	72	29.27767
26	73	29.07286	36	73	29.57933
26	74	29.37264	36	74	29.87637
26	75	29.67260	36	75	30.17713
26	76	29.97246	36	76	30.47713
26	77	30.27762	36	77	30.77378
26	78	30.58029	36	78	31.06186
26	79	30.88341	36	79	31.35441
26	80	31.18782	36	80	31.64743
26	81	31.49184	36	81	31.94091
26	82	31.79634	36	82	32.23484
26	83	32.10032	36	83	32.52921
26	84	32.40589	36	84	32.82400
26	85	32.71156	36	85	33.11929
26	86	33.01785	36	86	33.41462
26	87	33.32443	36	87	33.72082
26	88	33.63139	36	88	34.02722
26	89	33.93873	36	89	34.33400
26	90	34.24644	36	90	34.64115
26	91	34.55451	36	91	34.94864
26	92	34.86233	36	92	35.25452
26	93	35.17170	36	93	35.56474
26	94	35.48080	36	94	35.87323
26	95	35.79024	36	95	36.18217
26	96	36.09999	36	96	36.49134
26	97	36.41067	36	97	36.80051
26	98	36.72045	36	98	37.11073
26	99	37.03113	36	99	37.42090
26	100	37.34211	36	100	37.73023
26	34	16.77882	37	34	19.54674
26	35	19.63934	37	35	19.82462
26	36	19.90290	37	36	20.09899
26	37	19.56796	37	37	20.38660
26	38	19.63474	37	38	20.62377
26	39	20.18210	37	39	20.86345
26	40	20.37390	37	40	21.14459
26	41	20.64439	37	41	21.42712
26	42	20.91721	37	42	21.71201
26	43	21.19141	37	43	22.00050
26	44	21.46489	37	44	22.29283
26	45	21.74374	37	45	22.54032
26	46	22.02170	37	46	22.81916
26	47	22.30182	37	47	23.09913
26	48	22.58141	37	48	23.38020
26	49	22.86191	37	49	23.66233
26	50	23.14540	37	50	23.94549
26	51	23.42910	37	51	24.22963
26	52	23.71373	37	52	24.51477
26	53	23.99833	37	53	24.80002
26	54	24.28587	37	54	25.08777
26	55	24.57333	37	55	25.37541
26	56	24.86167	37	56	25.66429
26	57	25.15084	37	57	25.95306
26	58	25.44091	37	58	26.24411
26	59	25.73176	37	59	26.53529
26	60	26.02330	37	60	26.82706
26	61	26.31577	37	61	27.11943
26	62	26.60890	37	62	27.41292
26	63	26.90274	37	63	27.70661

37	60	20.00130	30	71	29.10131
37	69	20.22090	30	72	29.39734
37	70	20.80204	30	73	29.60394
37	71	20.60504	30	74	29.99120
37	72	29.10062	30	75	30.22090
37	73	29.40440	30	76	30.50736
37	74	29.70275	30	77	30.80626
37	75	30.00166	30	78	31.10570
37	76	30.30112	30	79	31.40505
37	77	30.60111	30	80	31.70411
37	78	30.90161	30	81	32.00707
37	79	31.20202	30	82	32.30630
37	80	31.50412	30	83	32.60941
37	81	31.80412	30	84	32.90277
37	82	32.10030	30	85	33.20639
37	83	32.40149	30	86	33.50004
37	84	32.70406	30	87	33.80332
37	85	33.00006	30	88	34.10642
37	86	33.30209	30	89	34.41112
37	87	33.70754	30	90	34.61605
37	88	34.01259	30	91	35.12135
37	89	34.31005	30	92	35.42703
37	90	34.62509	30	93	35.73310
37	91	34.93011	30	94	36.03052
37	92	35.23671	30	95	36.34031
37	93	35.54360	30	96	36.63245
37	94	35.85100	30	97	36.90092
37	95	36.15807	30	98	37.26070
37	96	36.46605	30	99	37.57090
37	97	36.77502	30	100	37.60837
37	98	37.08370	39	39	20.34160
37	99	37.39269	39	40	20.60316
37	100	37.70200	39	41	20.90421
38	39	19.82045	39	42	21.13100
38	40	20.00195	39	43	21.39740
38	41	20.24516	39	44	21.64522
38	42	20.41003	39	45	21.83409
38	43	20.67649	39	46	22.00813
38	44	21.10460	39	47	22.47716
38	45	21.41395	39	48	22.73047
38	46	21.60404	39	49	23.02503
38	47	21.83711	39	50	23.20062
38	48	22.23070	39	51	23.57777
38	49	22.50257	39	52	23.63647
38	50	22.78160	39	53	24.13506
38	51	23.05000	39	54	24.41532
38	52	23.31744	39	55	24.69602
38	53	23.61702	39	56	24.97022
38	54	23.89766	39	57	25.26210
38	55	24.17930	39	58	25.50039
38	56	24.46210	39	59	25.83131
38	57	24.74501	39	60	26.11732
38	58	25.03006	39	61	26.40430
38	59	25.31604	39	62	26.68200
38	60	25.60231	39	63	26.90060
38	61	25.88805	39	64	27.20990
38	62	26.17603	39	65	27.48597
38	63	26.46704	39	66	27.63676
38	64	26.75403	39	67	28.14232
38	65	27.04741	39	68	28.43457
38	66	27.33070	39	69	28.72790
38	67	27.62354	39	70	29.02110
38	68	28.21699	39	71	29.31535
38	69	28.51111	39	72	29.61023
38	70	28.80809	39	73	29.90573
			39	74	30.20104

39	75	30.49083	40	80	32.10061
39	76	30.78079	40	81	32.40030
39	77	31.08061	40	82	32.70400
39	78	31.38130	40	83	33.00470
39	79	31.68080	40	84	33.30510
39	80	31.98020	40	85	33.60000
39	81	32.28021	40	86	33.90730
39	82	32.58063	40	87	34.20905
39	83	32.88132	40	88	34.60125
39	84	33.18209	40	89	34.90007
39	85	33.48472	40	90	35.20000
39	86	33.78700	40	91	35.51030
39	87	34.08972	40	92	35.81424
39	88	34.40206	40	93	36.11049
39	89	34.70645	40	94	36.42514
39	90	35.01041	40	95	36.72010
39	91	35.31479	40	96	37.03354
39	92	35.61954	40	97	37.33530
39	93	35.92472	40	98	37.64040
39	94	36.23025	40	99	37.95100
39	95	36.53610	40	100	38.25000
39	96	36.84242	41	41	21.30410
39	97	37.14904	41	42	21.60560
39	98	37.45600	41	43	21.90064
39	99	37.76330	41	44	22.17322
39	100	38.07004	41	45	22.43020
40	40	20.06292	41	46	22.70679
40	41	21.12430	41	47	22.97860
40	42	21.30747	41	48	23.24591
40	43	21.63214	41	49	23.51744
40	44	21.91635	41	50	23.79022
40	45	22.18099	41	51	24.06421
40	46	22.45507	41	52	24.33930
40	47	22.72851	41	53	24.61569
40	48	23.00727	41	54	24.89310
40	49	23.27931	41	55	25.17150
40	50	23.54480	41	56	25.45100
40	51	23.82000	41	57	25.73161
40	52	24.09600	41	58	26.01311
40	53	24.37402	41	59	26.29035
40	54	24.65325	41	60	26.57001
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40	56	25.21403	41	62	27.10020
40	57	25.49632	41	63	27.43425
40	58	25.77077	41	64	27.72103
40	59	26.04254	41	65	28.00061
40	60	26.34722	41	66	28.29097
40	61	26.63276	41	67	28.50000
40	62	26.91919	41	68	28.77533
40	63	27.20642	41	69	29.10090
40	64	27.49446	41	70	29.45776
40	65	27.78330	41	71	29.74060
40	66	28.07207	41	72	30.04229
40	67	28.36310	41	73	30.33504
40	68	28.65423	41	74	30.62941
40	69	28.94500	41	75	30.92500
40	70	29.23040	41	76	31.21090
40	71	29.52150	41	77	31.51404
40	72	29.80824	41	78	31.81000
40	73	30.10001	41	79	32.10707
40	74	30.41000	41	80	32.40000
40	75	30.71010	41	81	32.70200
40	76	31.00020	41	82	33.00124
40	77	31.29000	41	83	33.29012
40	78	31.58000	41	84	33.58000
40	79	31.89020	41	85	33.89030

41	86	34.19949	42	83	34.51227
41	87	34.50044	42	84	34.51313
41	88	34.50179	42	85	37.13430
41	89	35.18341	42	86	37.22282
41	90	35.40852	42	87	37.72608
41	91	35.70004	42	88	38.03044
41	92	36.01101	42	89	38.23520
41	93	36.31436	42	100	38.44031
41	94	36.61811	43	43	22.42667
41	95	36.92225	43	44	22.68009
41	96	37.22676	43	45	22.93102
41	97	37.53103	43	46	23.21543
41	98	37.83690	43	47	23.48127
41	99	38.14281	43	48	23.74840
41	100	38.44844	43	49	24.01782
42	42	21.90845	43	50	24.28604
42	43	22.16684	43	51	24.55794
42	44	22.42943	43	52	24.83023
42	45	22.69432	43	53	25.10370
42	46	22.96027	43	54	25.37830
42	47	23.22742	43	55	25.65401
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42	61	27.09847	43	69	29.61341
42	62	27.37931	43	70	29.90233
42	63	27.66401	43	71	30.18700
42	64	27.94955	43	72	30.46233
42	65	28.23590	43	73	30.77233
42	66	28.52304	43	74	31.04499
42	67	28.81089	43	75	31.32726
42	68	29.09941	43	76	31.60620
42	69	29.38899	43	77	31.94373
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42	71	29.96988	43	79	32.53254
42	72	30.26134	43	80	32.82740
42	73	30.55446	43	81	33.12341
42	74	30.84823	43	82	33.41994
42	75	31.13961	43	83	33.71684
42	76	31.43361	43	84	34.01424
42	77	31.72820	43	85	34.31214
42	78	32.02330	43	86	34.61053
42	79	32.31912	43	87	34.90940
42	80	32.61582	43	88	35.20874
42	81	32.91323	43	89	35.50858
42	82	33.20932	43	90	35.80800
42	83	33.50750	43	91	36.10890
42	84	33.80684	43	92	36.41043
42	85	34.10476	43	93	36.71216
42	86	34.40432	43	94	37.01414
42	87	34.70395	43	95	37.31651
42	88	35.00424	43	96	37.61920
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42	90	35.60617	43	98	38.22597
42	91	35.90776	43	99	38.52949
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44	44	23.94794	44	54	25.87947
44	45	23.20934	44	55	26.14345
44	46	23.47223	44	56	26.41783
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44	49	24.26934	44	59	27.24407
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44	54	25.62354	44	64	28.64633
44	55	25.89780	44	65	28.92892
44	56	26.17330	44	66	29.21245
44	57	26.44976	44	67	29.49678
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44	61	27.56863	44	71	30.64182
44	62	27.84694	44	72	30.92990
44	63	28.12914	44	73	31.21849
44	64	28.41320	44	74	31.50815
44	65	28.69611	44	75	31.79837
44	66	28.98083	44	76	32.08963
44	67	29.26630	44	77	32.38063
44	68	29.55263	44	78	32.67245
44	69	29.83970	44	79	32.96396
44	70	30.12749	44	80	33.25836
44	71	30.41599	44	81	33.55204
44	72	30.70520	44	82	33.84636
44	73	30.99509	44	83	34.14126
44	74	31.28643	44	84	34.43688
44	75	31.57945	44	85	34.73243
44	76	31.86689	44	86	35.02909
44	77	32.16113	44	87	35.32606
44	78	32.45421	44	88	35.62350
44	79	32.74787	44	89	35.92143
44	80	33.04210	44	90	36.21984
44	81	33.33689	44	91	36.51870
44	82	33.63223	44	92	36.81801
44	83	33.92812	44	93	37.11776
44	84	34.22452	44	94	37.41790
44	85	34.52144	44	95	37.71834
44	86	34.81897	44	96	38.01908
44	87	35.11676	44	97	38.32101
44	88	35.41518	44	98	38.62304
44	89	35.71406	44	99	38.92540
44	90	36.01337	44	100	39.22764
44	91	36.31313	44	44	23.99049
44	92	36.61337	44	47	24.25104
44	93	36.91400	44	48	24.51144
44	94	37.21510	44	49	24.77044
44	95	37.51689	44	50	25.04434
44	96	37.81840	44	51	25.31110
44	97	38.12077	44	52	25.57920
44	98	38.42340	44	53	25.84680
44	99	38.72632	44	54	26.11304
44	100	39.02994	44	55	26.38047
45	45	23.46921	45	56	26.64342
45	46	23.73060	45	57	26.90725
45	47	24.00344	45	58	27.17114
45	48	24.28799	45	59	27.43608
45	49	24.57331	45	60	27.70490
45	50	24.79936	45	61	28.04283
45	51	25.08040	45	62	28.32164
45	52	25.32795	45	63	28.60137
45	53	25.59043	45	64	28.88190

66	65	29.14344	47	77	32.82437
66	66	29.44881	47	78	33.11430
66	67	29.72897	47	79	33.66485
66	68	30.01283	47	80	33.89481
66	69	30.29767	47	81	33.98778
66	70	30.58317	47	82	34.28000
66	71	30.86942	47	83	34.57300
66	72	31.15639	47	84	34.86644
66	73	31.44407	47	85	35.16047
66	74	31.73244	47	86	35.45500
66	75	32.02140	47	87	35.75000
66	76	32.31110	47	88	36.04543
66	77	32.60133	47	89	36.34170
66	78	32.89248	47	90	36.63825
66	79	33.18467	47	91	36.93520
66	80	33.47620	47	92	37.23279
66	81	33.76801	47	93	37.53075
66	82	34.06235	47	94	37.82916
66	83	34.35824	47	95	38.12802
66	84	34.65568	47	96	38.42730
66	85	34.95463	47	97	38.72701
66	86	35.25413	47	98	39.02713
66	87	35.55716	47	99	39.32764
66	88	35.86367	47	100	39.62854
66	89	36.17360	48	00	39.92984
66	90	36.48614	48	01	40.23141
66	91	36.79929	48	02	40.53325
66	92	37.11300	48	03	40.83536
66	93	37.42828	48	04	41.13774
66	94	37.74513	48	05	41.44039
66	95	38.06356	48	06	41.74332
66	96	38.38358	48	07	42.04654
66	97	38.70519	48	08	42.35005
66	98	39.02840	48	09	42.65386
66	99	39.35321	48	10	42.95797
66	100	39.67962	48	11	43.26238
47	00	40.00763	48	12	43.56709
47	01	40.33724	48	13	43.87210
47	02	40.66845	48	14	44.17741
47	03	40.99926	48	15	44.48302
47	04	41.33067	48	16	44.78893
47	05	41.66268	48	17	45.09514
47	06	41.99529	48	18	45.40165
47	07	42.32850	48	19	45.70846
47	08	42.66231	48	20	46.01557
47	09	42.99662	48	21	46.32298
47	10	43.33253	48	22	46.63069
47	11	43.66904	48	23	46.93870
47	12	44.00615	48	24	47.24701
47	13	44.34386	48	25	47.55562
47	14	44.68217	48	26	47.86453
47	15	45.02108	48	27	48.17374
47	16	45.36059	48	28	48.48325
47	17	45.69970	48	29	48.79306
47	18	46.03941	48	30	49.10317
47	19	46.37972	48	31	49.41358
47	20	46.72073	48	32	49.72429
47	21	47.06144	48	33	50.03530
47	22	47.40285	48	34	50.34661
47	23	47.74496	48	35	50.65822
47	24	48.08677	48	36	50.97013
47	25	48.42828	48	37	51.28234
47	26	48.76949	48	38	51.59485
47	27	49.11030	48	39	51.90766
47	28	49.45081	48	40	52.22077
47	29	49.79102	48	41	52.53418
47	30	50.13103	48	42	52.84789
47	31	50.47084	48	43	53.16190
47	32	50.81045	48	44	53.47621
47	33	51.14986	48	45	53.79082
47	34	51.48907	48	46	54.10573
47	35	51.82808	48	47	54.42094
47	36	52.16689	48	48	54.73645
47	37	52.50550	48	49	55.05226
47	38	52.84391	48	50	55.36837
47	39	53.18212	48	51	55.68478
47	40	53.52013	48	52	55.99949
47	41	53.85794	48	53	56.31450
47	42	54.19555	48	54	56.62981
47	43	54.53296	48	55	56.94542
47	44	54.87017	48	56	57.26133
47	45	55.20718	48	57	57.57754
47	46	55.54399	48	58	57.89405
47	47	55.88060	48	59	58.21086
47	48	56.21691	48	60	58.52797
47	49	56.55302	48	61	58.84538
47	50	56.88893	48	62	59.16309
47	51	57.22464	48	63	59.48110
47	52	57.56015	48	64	59.79941
47	53	57.89546	48	65	60.11802
47	54	58.23057	48	66	60.43693
47	55	58.56548	48	67	60.75614
47	56	58.89919	48	68	61.07565
47	57	59.23270	48	69	61.39546
47	58	59.56601	48	70	61.71557
47	59	59.89912	48	71	62.03598
47	60	60.23203	48	72	62.35669
47	61	60.56474	48	73	62.67770
47	62	60.89725	48	74	62.99891
47	63	61.22956	48	75	63.32032
47	64	61.56167	48	76	63.64193
47	65	61.89358	48	77	63.96374
47	66	62.22529	48	78	64.28575
47	67	62.55680	48	79	64.60796
47	68	62.88811	48	80	64.93037
47	69	63.21922	48	81	65.25298
47	70	63.55013	48	82	65.57579
47	71	63.88084	48	83	65.89880
47	72	64.21135	48	84	66.22201
47	73	64.54166	48	85	66.54542
47	74	64.87177	48	86	66.86903
47	75	65.20168	48	87	67.19284
47	76	65.53139	48	88	67.51685
47	77	65.86090	48	89	67.84106
47	78	66.19021	48	90	68.16547
47	79	66.51932	48	91	68.49008
47	80	66.84823	48	92	68.81489
47	81	67.17694	48	93	69.13990
47	82	67.50545	48	94	69.46511
47	83	67.83376	48	95	69.79052
47	84	68.16187	48	96	70.11613
47	85	68.48978	48	97	70.44194
47	86	68.81749	48	98	70.76795
47	87	69.14490	48	99	71.09416
47	88	69.47211	48	100	71.42057
47	89	69.79912	48	01	71.74718
47	90	70.12593	48	02	72.07399
47	91	70.45254	48	03	72.40090
47	92	70.77895	48	04	72.72791
47	93	71.10516	48	05	73.05452
47	94	71.43117	48	06	73.38133
47	95	71.75698	48	07	73.70834
47	96	72.08259	48	08	74.03455
47	97	72.40800	48	09	74.36036
47	98	72.73321	48	10	74.68577
47	99	73.05822	48	11	75.01078
47	100	73.38293	48	12	75.33539
48	00	73.70734	48	13	75.65970
48	01	74.03145	48	14	75.98371
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48	03	74.67877	48	16	76.63083
48	04	75.00198	48	17	76.95394
48	05	75.32489	48	18	77.27675
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48	07	75.96951	48	20	77.92177
48	08	76.29122	48	21	78.24398
48	09	76.61263	48	22	78.56599
48	10	76.93374	48	23	78.88770
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48	13	77.89467	48	26	79.85103
48	14	78.21418	48	27	80.17154
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48	16	78.85200	48	29	80.81166
48	17	79.17031	48	30	81.13127
48	18	79.48822	48	31	81.45058
48	19	79.80583	48	32	81.76959
48	20	80.12314	48	33	82.08830
48	21	80.44015	48	34	82.40671
48	22	80.75686	48	35	82.72482
48	23	81.07327	48	36	83.04263
48	24	81.38938	48	37	83.36014
48	25	81.70519	48	38	83.67735
48	26	82.02070	48	39	83.99426
48	27	82.33591	48	40	84.31087
48	28	82.65082	48	41	84.62718
48	29	82.96543	48	42	84.94319
48	30	83.27974	48	43	85.25890
48	31	83.59375	48	44	85.57431
48	32	83.90746	48	45	85.88942
48	33	84.22087	48	46	86.20423
48	34	84.53398	48	47	86.51874
48	35	84.84679	48	48	86.83295
48	36	85.15930	48	49	87.14686
48	37	85.47151	48	50	87.46047
48	38	85.78342	48	51	87.77378
48	39	86.09503	48	52	88.08679
48	40	86.40634	48	53	88.39950
48	41	86.71735	48	54	88.71191
48	42	87.02806	48	55	89.02402
48	43	87.33847	48	56	89.33583
48	44	87.64858	48	57	89.64734
48	45	87.95839	48	58	89.95855
48	46	88.26780	48	59	90.26946
48	47	88.57691	48	60	90.58007
48	48	88.88572	48	61	90.89038
48	49	89.19423	48	62	91.20039
48	50	89.50244	48	63	91.51010
48	51	89.81035	48	64	91.81951
48	52	90.11796	48	65	92.12862
48	53	90.42527	48	66	92.43743
48	54	90.73228	48	67	92.74594
48	55	91.03899	48	68	93.05415
48	56	91.34540	48	69	93.36206
48	57	91.65151	48	70	93.66967
48	58	91.95732	48	71	93.97698
48	59	92.26283	48	72	94.28399
48	60	92.56814	48	73	94.59070
48	61	92.87325	48	74	94.89711
48	62	93.17816	48	75	95.20322
48	63	93.48287	48	76	95.50903
48	64	93.78738	48	77	95.81454
48	65	94.09169	48	78	96.11975
48	66	94.39580	48	79	96.42466
48	67	94.69971	48	80	96.72927
48	68	95.00342	48	81	97.03358
48	69	95.30693	48	82	97.33759
48	70	95.61024	48	83	97.64130
48	71	95.91335	48	84	97.94481
48	72	96			

81	69	31.67188	82	66	36.54947
81	70	31.75178	82	67	36.64682
81	71	32.63249	82	68	37.13693
81	72	32.91897	82	69	37.42239
81	73	32.89422	82	70	37.71437
81	74	32.67921	82	71	38.00600
81	75	33.10283	82	72	38.29890
81	76	33.44734	82	73	38.59342
81	77	33.73249	82	74	38.88743
81	78	34.01839	82	75	39.18182
81	79	34.30477	82	76	39.47689
81	80	34.59109	82	77	39.77232
81	81	34.87948	82	78	40.06839
81	82	35.16882	82	79	40.36483
81	83	35.45781	82	80	40.66129
81	84	35.74639	100	81	40.95827
81	85	36.03478	83	82	41.25564
81	86	36.32340	83	83	41.55339
81	87	36.61277	83	84	41.85113
81	88	36.90181	83	85	42.14985
81	89	37.19094	83	86	42.44855
81	90	37.48027	83	87	42.74722
81	91	37.76920	83	88	43.04589
81	92	38.05819	83	89	43.34454
81	93	38.34729	83	90	43.64319
81	94	38.63640	83	91	43.94184
81	95	38.92549	83	92	44.24049
81	96	39.21459	83	93	44.53914
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81	99	40.08189	83	96	45.43509
82	82	40.37099	83	97	45.73374
82	83	40.66009	83	98	46.03239
82	84	40.94919	83	99	46.33104
82	85	41.23829	83	100	46.62969
82	86	41.52739	84	84	46.92834
82	87	41.81649	84	85	47.22699
82	88	42.10559	84	86	47.52564
82	89	42.39469	84	87	47.82429
82	90	42.68379	84	88	48.12294
82	91	42.97289	84	89	48.42159
82	92	43.26199	84	90	48.72024
82	93	43.55109	84	91	49.01889
82	94	43.84019	84	92	49.31754
82	95	44.12929	84	93	49.61619
82	96	44.41839	84	94	49.91484
82	97	44.70749	84	95	50.21349
82	98	45.00000	84	96	50.51214
82	99	45.29250	84	97	50.81079
82	100	45.58500	84	98	51.10944
82	101	45.87750	84	99	51.40809
82	102	46.17000	84	100	51.70674
82	103	46.46250	84	101	52.00539
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82	105	47.04750	84	103	52.60269
82	106	47.34000	84	104	52.90134
82	107	47.63250	84	105	53.20000
82	108	47.92500	84	106	53.49864
82	109	48.21750	84	107	53.79729
82	110	48.51000	84	108	54.09594
82	111	48.80250	84	109	54.39459
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82	115	49.97250	84	113	55.58919
82	116	50.26500	84	114	55.88784
82	117	50.55750	84	115	56.18649
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82	119	51.14250	84	117	56.78379
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82	121	51.72750	84	119	57.38109
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82	123	52.31250	84	121	57.97839
82	124	52.60500	84	122	58.27704
82	125	52.89750	84	123	58.57569
82	126	53.19000	84	124	58.87434
82	127	53.48250	84	125	59.17299
82	128	53.77500	84	126	59.47164
82	129	54.06750	84	127	59.77029
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82	131	54.65250	84	129	60.36759
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82	134	55.53000	84	132	61.26354
82	135	55.82250	84	133	61.56219
82	136	56.11500	84	134	61.86084
82	137	56.40750	84	135	62.15949
82	138	56.70000	84	136	62.45814
82	139	56.99250	84	137	62.75679
82	140	57.28500	84	138	63.05544
82	141	57.57750	84	139	63.35409
82	142	57.87000	84	140	63.65274
82	143	58.16250	84	141	63.95139
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82	153	61.08750	84	151	66.93789
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82	157	62.25750	84	155	68.13249
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82	160	63.13500	84	158	69.02844
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82	164	64.30500	84	162	70.22304
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82	166	64.89000	84	164	70.82034
82	167	65.18250	84	165	71.11899
82	168	65.47500	84	166	71.41764
82	169	65.76750	84	167	71.71629
82	170	66.06000	84	168	72.01494
82	171	66.35250	84	169	72.31359
82	172	66.64500	84	170	72.61224
82	173	66.93750	84	171	72.91089
82	174	67.23000	84	172	73.20954
82	175	67.52250	84	173	73.50819
82	176	67.81500	84	174	73.80684
82	177	68.10750	84	175	74.10549
82	178	68.40000	84	176	74.40414
82	179	68.69250	84	177	74.70279
82	180	68.98500	84	178	75.00144
82	181	69.27750	84	179	75.30009
82	182	69.57000	84	180	75.59874
82	183	69.86250	84	181	75.89739
82	184	70.15500	84	182	76.19604
82	185	70.44750	84	183	76.49469
82	186	70.74000	84	184	76.79334
82	187	71.03250	84	185	77.09199
82	188	71.32500	84	186	77.39064
82	189	71.61750	84	187	77.68929
82	190	71.91000	84	188	77.98794
82	191	72.20250	84	189	78.28659
82	192	72.49500	84	190	78.58524
82	193	72.78750	84	191	78.88389
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82	195	73.37250	84	193	79.48119
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82	197	73.95750	84	195	80.07849
82	198	74.25000	84	196	80.37714
82	199	74.54250	84	197	80.67579
82	200	74.83500	84	198	80.97444
82	201	75.12750	84	199	81.27309
82	202	75.42000	84	200	81.57174
82	203	75.71250	84	201	81.87039
82	204	76.00500	84	202	82.16904
82	205	76.29750	84	203	82.46769
82	206	76.59000	84	204	82.76634
82	207	76.88250	84	205	83.06499
82	208	77.17500	84	206	83.36364
82	209	77.46750	84	207	83.66229
82	210	77.76000	84	208	83.96094
82	211	78.05250	84	209	84.25959
82	212	78.34500	84	210	84.55824
82	213	78.63750	84	211	84.85689
82	214	78.93000	84	212	85.15554
82	215	79.22250	84	213	85.45419
82	216	79.51500	84	214	85.75284
82	217	79.80750	84	215	86.05149
82	218	80.10000	84	216	86.35014
82	219	80.39250	84	217	86.64879
82	220	80.68500	84	218	86.94744
82	221	80.97750	84	219	87.24609
82	222	81.27000	84	220	87.54474
82	223	81.56250	84	221	87.84339
82	224	81.85500	84	222	88.14204
82	225	82.14750	84	223	88.44069
82	226	82.44000	84	224	88.73934
82	227	82.73250	84	225	89.03799
82	228	83.02500	84	226	89.33664
82	229	83.31750	84	227	89.63529
82	230	83.61000	84	228	89.93394
82	231	83.90250	84	229	90.23259
82	232	84.19500	84	230	90.53124
82	233	84.48750	84	231	90.82989
82	234	84.78000	84	232	91.12854
82	235	85.07250	84	233	91.42719
82	236	85.36500	84	234	91.72584
82	237	85.65750	84	235	92.02449
82	238	85.95000	84	236	92.32314
82	239	86.24250	84	237	92.62179
82	240	86.53500	84	238	92.92044
82	241	86.82750	84	239	93.21909
82	242	87.12000	84	240	93.51774
82	243	87.41250	84	241	93.81639
82	244	87.70500	84	242	94.11504
82	245	88.00000	84	243	94.41369
82	246	88.29250	84	244	94.71234
82	247	88.58500	84	245	95.01099
82	248	88.87750	84	246	95.30964
82	249	89.17000	84	247	95.60829
82	250	89.46250	84	248	95.90694
82	251	89.75500	84	249	96.20559
82	252	90.04750	84	250	96.50424
82	253	90.34000	84	251	96.80289
82	254	90.63250	84	252	97.10154
82					

56	56	40.02983	58	77	35.38811
56	59	41.22283	58	78	35.43889
56	100	41.51630	58	79	35.43641
57	57	39.72484	58	80	35.21845
57	58	39.98699	58	81	35.49884
57	59	39.24830	58	82	35.78113
57	60	39.51284	58	83	37.04847
57	61	39.77604	58	84	37.30641
57	62	31.04239	58	85	37.62999
57	63	31.30913	58	86	37.91429
57	64	31.57682	58	87	38.19983
57	65	31.84569	58	88	38.48446
57	66	32.11544	58	89	38.77040
57	67	32.38613	58	90	39.05700
57	68	32.65774	58	91	39.34426
57	69	32.93026	58	92	39.63199
57	70	33.20366	58	93	39.92026
57	71	33.47782	58	94	40.20900
57	72	33.75363	58	95	40.49842
57	73	34.03096	58	96	40.78828
57	74	34.30869	58	97	41.07864
57	75	34.58621	58	98	41.36951
57	76	34.86149	58	99	41.66096
57	77	35.14884	58	100	41.95269
57	78	35.43831	59	59	36.76749
57	79	35.73081	59	60	31.02873
57	80	36.02601	59	61	31.29100
57	81	36.32390	59	62	31.55413
57	82	36.62467	59	63	31.81904
57	83	36.92869	59	64	32.08489
57	84	37.23587	59	65	32.35116
57	85	37.54606	59	66	32.61871
57	86	37.85921	59	67	32.88723
57	87	37.94894	59	68	33.15670
57	88	38.25328	59	69	33.42709
57	89	38.52020	59	70	33.69838
57	90	38.78969	59	71	33.97055
57	91	39.11774	59	72	34.24384
57	92	39.40635	59	73	34.51745
57	93	39.69849	59	74	34.79215
57	94	39.99316	59	75	35.06783
57	95	40.27834	59	76	35.34383
57	96	40.56606	59	77	35.62099
57	97	40.85726	59	78	35.89860
57	98	41.14894	59	79	36.17794
57	99	41.44113	59	80	36.45861
57	100	41.73378	59	81	36.73919
58	56	39.34617	59	82	37.01725
58	59	39.50741	59	83	37.29459
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58	61	31.03320	59	85	37.84638
58	62	31.29788	59	86	38.12185
58	63	31.56348	59	87	38.39807
58	64	31.83014	59	88	38.67500
58	65	32.09781	59	89	38.95263
58	66	32.36648	59	90	39.23085
58	67	32.63606	59	91	39.50921
58	68	32.90659	59	92	39.78841
58	69	33.17804	59	93	40.06842
58	70	33.45038	59	94	40.34938
58	71	33.72369	59	95	40.63027
58	72	33.99786	59	96	41.01109
58	73	34.27284	59	97	41.39182
58	74	34.54827	59	98	41.59146
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60	62	31.01238	61	68	39.18007
60	63	32.07374	61	69	39.46343
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60	66	32.89210	61	72	40.31707
60	67	33.16843	61	73	40.60277
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60	71	34.21878	61	77	41.75104
60	72	34.48678	61	78	42.03943
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60	75	35.31179	62	100	42.90769
60	76	35.58706	62	62	32.38149
60	77	35.86316	62	63	32.58270
60	78	36.13999	62	64	32.88490
60	79	36.41757	62	65	33.11420
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60	82	37.25462	62	68	33.91426
60	83	37.53503	62	69	34.18151
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60	87	38.66325	62	73	35.25950
60	88	38.94688	62	74	35.53127
60	89	39.23112	62	75	35.80379
60	90	39.51894	62	76	36.07711
60	91	39.80130	62	77	36.35123
60	92	40.08727	62	78	36.62613
60	93	40.37593	62	79	36.90179
60	94	40.66194	62	80	37.17819
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60	96	41.23066	62	82	37.73318
60	97	41.52584	62	83	38.01174
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61	67	33.39822	62	93	40.83290
61	68	33.66994	62	94	41.11831
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61	71	34.44017	62	97	41.97764
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61	78	36.36264	63	66	33.63994
61	79	36.63906	63	67	33.90364
61	80	36.90941	63	68	34.16910
61	81	37.21449	63	69	34.43532
61	82	37.48036	63	70	34.70240
61	83	37.74776	63	71	34.97054
61	84	38.01621	63	72	35.23951
61	85	38.28374	63	73	35.50928
61	86		63	74	35.78004

63	75	34.08157	65	88	34.64811
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63	77	34.59766	65	90	35.21141
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63	85	38.80829	65	98	37.36432
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63	94	41.32408	66	07	39.84615
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63	97	42.20891	66	10	40.68571
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64	28	40.73125	66	41	39.80867
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64	47	35.98874	66	60	45.20382
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64	51	37.05412	66	64	46.38186
64	52	37.32263	66	65	46.67887
64	53	37.59194	66	66	46.97688
64	54	37.86214	66	67	47.27489
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71	72	37.26474	73	81	40.17987
71	73	37.54653	73	82	40.44700
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71	75	38.07373	73	84	40.98890
71	76	38.35850	73	85	41.25617
71	77	38.64113	73	86	41.52711
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71	80	39.40483	73	89	42.34420
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72	86	41.27846	74	97	44.79290
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72	92	42.92276	75	77	39.63216
72	93	43.20023	75	78	39.89583
72	94	43.47744	75	79	40.15877
72	95	43.75420	75	80	40.42333
72	96	44.03350	75	81	40.68872
72	97	44.31286	75	82	40.95490
72	98	44.59219	75	83	41.22180
72	99	44.87223	75	84	41.48943
72	100	45.15282	75	85	41.75613
73	73	38.06430	75	86	42.02730
73	74	38.32744	75	87	42.29737
73	75	38.59040	75	88	42.56847
73	76	38.85243	75	89	42.83947
73	77	39.11624	75	90	43.11157
73	78	39.38090	75	91	43.38435
73	79	39.64641	75	92	43.65780
73	80	43.82189	75	87	43.08247
73	81	44.20644	75	88	43.25070
73	82	44.48262	75	89	43.50632
73	83	44.75083	75	90	43.76072
73	84	45.01443	75	91	44.12097
73	85	45.21104	75	92	44.40672
73	86	45.50043	75	93	44.67843
73	87	45.84064	75	94	44.94890
73	88	39.63037	75	95	45.21783
73	89	39.89280	75	96	45.48140
73	90	40.15040	75	97	45.74070
73	91	40.41384	75	98	46.00040
73	92	40.68064	75	99	46.26119
73	93	40.94456	75	100	46.50230
73	94	41.20060	75	79	41.18446
73	95	41.47060	75	80	41.45854
73	96	41.74290	75	81	41.71781
73	97	42.01854	75	82	41.98283
73	98	42.27890	75	83	42.24367
73	99	42.54413	75	84	42.50025
73	100	42.81000	75	85	42.77341
74	74	43.00040	75	86	43.04033
74	75	43.28967	75	87	43.30682
74	76	43.53183	75	88	43.57343
74	77	43.80447	75	89	43.84180
74	78	44.17777	75	90	44.11044
74	79	44.45172	75	91	44.38000
74	80	44.72431	75	92	44.65025
74	81	45.00183	75	93	44.92117
74	82	45.27734	75	94	45.18877
74	83	45.55381	75	95	45.45581
74	84	45.83064	75	96	45.72790
74	85	46.10047	75	97	46.01142
74	86	46.35173	75	98	46.29034
74	87	46.61885	75	99	46.56932
74	88	46.87463	75	100	46.83864
74	89	46.93764	75	80	41.71802
74	90	41.29131	80	81	41.97832
74	91	41.46370	80	82	42.23890
74	92	41.73186	80	83	42.50160
74	93	42.00711	80	84	42.76515
74	94	42.28183	80	85	43.02840
74	95	42.55151	80	86	43.29460
74	96	42.79043	80	87	43.56046
74	97	43.04068	80	88	43.82707
74	98	43.29065	80	89	44.09443
74	99	43.54012	80	90	44.36347
74	100	43.80020	80	91	44.63124
75	75	44.15312	80	92	44.89870
75	76	44.42462	80	93	45.17084
75	77	44.69770	80	94	45.44104
75	78	44.97150	80	95	45.71213
75	79	45.24460	80	96	45.98125
75	80	45.52100	80	97	46.25061
75	81	45.79073	80	98	46.52130
75	82	46.07060	80	99	46.79230
75	83	46.34960	80	100	47.06060
75	84	46.67360	81	81	42.23710
75	85	46.94290	81	82	42.49630
75	86	47.19617	81	83	42.76020
75	87	47.45097	81	84	43.02880
75	88	47.70290	81	85	43.29544
75	89	47.97761	81	86	43.56073
75	90	48.25823	81	87	43.81570
75	91	48.51622	81	88	44.08180
75	92	48.78497	81	89	44.34614

81	99	44.61840	85	99	44.86373
81	91	44.64334	85	87	44.86880
81	88	45.15306	85	88	45.19422
81	83	45.42142	85	89	45.37141
81	84	45.45145	85	90	45.62874
81	85	45.84214	85	91	45.80040
81	86	46.23381	85	92	46.14618
81	87	46.50831	85	93	46.45247
81	89	46.77614	85	94	46.69040
81	90	47.06139	85	95	46.94711
81	100	47.30825	85	96	47.23844
82	82	42.78453	85	97	47.50443
82	83	42.81944	85	98	47.77490
82	84	42.86188	85	99	48.04437
82	85	42.94404	85	100	48.31527
82	86	43.02772	86	86	44.84482
82	87	44.07130	86	87	45.10840
82	88	44.33684	86	88	45.34684
82	89	44.60872	86	89	45.62040
82	90	44.84621	86	90	45.85291
82	91	45.13440	86	91	46.13700
82	92	45.46430	86	92	46.42182
82	93	45.67284	86	93	46.68724
82	94	45.94215	86	94	46.96337
82	95	46.21300	86	95	47.22044
82	96	46.48546	86	96	47.46847
82	97	46.78382	86	97	47.70432
82	98	47.02844	86	98	48.02322
82	99	47.29830	86	99	48.32470
82	100	47.57142	86	100	48.54484
83	83	43.27891	87	87	45.34830
83	84	43.54100	87	88	45.62844
83	85	43.80289	87	89	45.90830
83	86	44.06187	87	90	46.18444
83	87	44.32800	87	91	46.41421
83	88	44.58323	87	92	46.67827
83	89	44.85814	87	93	46.94304
83	90	45.12347	87	94	47.20651
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83	92	45.45741	87	96	47.74152
83	93	45.62822	87	97	48.00904
83	94	46.18372	87	98	48.27721
83	95	46.46230	87	99	48.54663
83	96	46.73274	87	100	48.81549
83	97	47.00322	88	88	45.84674
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84	84	45.80135	88	92	46.93532
84	85	46.06337	88	93	47.19954
84	86	46.23424	88	94	47.46426
84	87	46.50490	88	95	47.72944
84	88	46.80432	88	96	47.98879
84	89	45.11444	88	97	48.25237
84	90	45.27820	88	98	48.51002
84	91	45.44643	88	99	48.79011
84	92	45.61137	88	100	49.06403
84	93	46.17042	89	89	46.06013
84	94	46.44014	89	90	46.30019
84	95	46.71430	89	91	46.53100
84	96	46.84344	89	92	47.13335
84	97	47.25340	89	93	47.48482
84	98	47.52378	89	94	47.72041
84	99	47.79400	89	95	47.95045
84	100	48.06445	89	96	48.22004
85	85	44.22845	89	97	48.51691
89	89	46.78343	90	90	51.02100
89	90	49.05100	90	100	51.84291
89	100	49.21902	100	100	52.14334
90	80	46.82904			
90	91	47.18094			
90	92	47.45234			
90	93	47.71409			
90	94	47.97612			
90	95	48.24804			
90	96	48.50472			
90	97	48.77205			
90	98	49.03004			
90	99	49.28409			
90	100	49.57890			
91	91	47.48467			
91	92	47.71193			
91	93	47.97372			
91	94	48.23623			
91	95	48.49944			
91	96	48.76334			
91	97	49.02794			
91	98	49.29323			
91	99	49.55817			
91	100	49.82874			
92	92	47.97235			
92	93	48.23330			
92	94	48.49504			
92	95	48.75786			
92	96	49.02071			
92	97	49.28463			
92	98	49.54920			
92	99	49.81442			
92	100	50.08031			
93	93	48.45042			
93	94	48.70467			
93	95	49.01643			
93	96	49.27891			
93	97	49.54287			
93	98	49.80822			
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95	100	50.84440			
96	96	50.08775			
96	97	50.31879			
96	98	50.56082			
96	99	50.84284			
96	100	51.10463			
97	97	50.87912			
97	98	50.84016			
97	99	51.10100			
97	100	51.36438			
98	98	51.10050			
98	99	51.36153			
98	100	51.62325			

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