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Angular Momentum — Can Be Used in Ballistic Tracking

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The correlation and tracking problem for ballistic objects is a major concern for the Strategic Defense Initiative (SDI). In this report, the use of angular momentum for gating and assignment of reports and tracks of ballistic objects is proposed. The various errors and appropriate statistical tests that could be used on angular momentum to potentially improve the tracking and correlation, multi-track hypothesis and Kalman filters are studied. The investigation is performed in earch centered inertial Cartesian coordinates for position, velocity and acceleration in tracking, gating and assignment. A discussion is given of a brief investigation, the theory developed, the results of numerical test and the conclusions.

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Angular Momentum – Can Be Used in Ballistic Tracking

Introduction

For any ballistic flight without drag and maneuvering, conservation of angular momentum applies. Conservation of angular momentum is part of the basis for orbital mechanics and Keppler orbits. The question can be raised, "Why not use this conservation principle in tracking for gating and assignment?" Interest in this matter first arose in studying misassociations in our own tracking algorithm. The BEAST program [1] [2] uses earth centered inertial Cartesian coordinates for position, velocity and acceleration in tracking, gating and assignment. Therefore, the proposition is not to use primitive orbital variables from the Keppler orbits, but instead combine state variables, position and velocity, to form a new state variable, angular momentum. This new variable will have errors associated with it, errors based on the position - velocity errors. Since angular momentum is a conserved quantity, the errors in the angular momentum will be conserved if the prediction methods used to update position and velocity or to project the trajectories forward in time conserve angular moment. The various errors and appropriate statistical tests could be applied to multi-track hypothesis, Kalman filters and the assignment problem based on angular momentum to potentially improve tracking and correlation. This paper is a discussion of a brief investigation of this possibility, the theory developed, the results of numerical test and the conclusions.

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Investigation

In the BEAST program the track - report assignment is performed by a greedy nearest neighbor assignment method having each report pass two gates using distance and velocity. Misassociations between tracks and reports, i.e. assigning the wrong report to a track, occur as with any tracking algorithm. For BEAST, less than a percent of the 12,000 tracks are misassociated under relatively ideal circumstances. Since all associations cannot be made between the tracks and reports because of the gate sizes used, these lost associations resulted in additional tracks being generated. This is deemed preferable to forcing every association which would increase significantly the number of misassociations. A relationship exists between the gate size, the number of misassociations and the number of lost associations. If the gate size is increased to reduce lost associations, there is an increase in misassociations as with most tracking algorithms. If the gates size is decreased to reduce misassociations, there is an increase in lost associations. Therefore, the misassociations were investigated to discover their source and to find possible methods to reduce misassociations that would not result in increase lost associations, (and more new tracks), and in a major increase in computational complexity.

The investigation began by studying the misassociations and lost associations which occurred in the BEAST program. A parametric study was performed on the position and velocity gate sizes to minimize misassociations and loss associations in general. During this study, one track continuously had a misassociation. This track had not been updated for five scan periods (150 seconds) and would have been eliminated, if the tracker had not updated it improperly during the scan appearing in Table 1. The position and velocity data obtained from the program for the track, the misassociated report and original track object data are presented in Table 1.

The gates and assignment algorithm use the sum of the errors squared of the position and velocity relative to the track. The sum of the errors squared was used to reduce computational work within the program. The position and velocity component errors, the basis for the sum, are given in Table 2. The square root of the sum of the errors squared for the position and velocity errors of each object is given in Table 3. The gates used for the association were 5.5 km for position and 0.5 km/sec for velocity. The misassociated object passed the gates and was assigned to the track based on its closeness to the track. For this case, even opening the gating windows wider would not have helped. The greedy nearest assignment method would have made this misassignment anyway, since considering the errors, the assigned report was closer than the correct report.

The question that arose was "Is there some way to combine position and velocity and their errors into a single parameter allowing the opening of the individual position and velocity gates?" This would be similar to combining the individual position component errors, **x**, **y** , **z**, into a distance error A single parameter can be formed by a weighted sum of the absolute errors in position and velocity or distance and speed. This method is artificial and the weights and parameter may not have physical meaning. Linear momentum, **P**, would not be any better because it is constantly changing. But, angular momentum, **L**, is conserved for the ballistic case and is a combination of position and velocity. Angular momentum about the earth's center is defined in this paper as the cross product of position and velocity measured in earth center inertial coordinates. In terms of the position and velocity components, angular momentum, in vector form, is

$$\mathbf{L} = (yw - zv)\mathbf{i} + (zu - xw)\mathbf{j} + (xv - yu)\mathbf{k}$$

The angular momentum for the track and both objects is given in Table 4. From the table, differences can be observed in the angular momentum of the track and of the two objects. The errors for the original object and the misassociated object with respect to the track are listed along with square root of the sum of the errors squared in Table 5. It is very obvious that the angular momentum of the track, which had not been updated in 5 scans, matched the angular momentum of the original object very well, but it did not match the angular momentum of the misassociated object, even though position and velocity were apparently closer in the latter case. If the angular momentum had been used as the gating variable, the misassociation would never have been made.

Why is the angular momentum this close? Standard error analysis will provide some insight into the reason why the angular momentum is a good gating variable. In any statistics book, [3], the formulas for error propagation can be found. The error propagation for addition and subtraction of items is given by

$$E_{sum}^2 = \sum_{i=1}^{n} e_i^2$$

POSITION AND VELOCITY DATA FOR TRACK AND OBJECTS

Item	Position Components (km)			Velocity Components (km/sec)		
	x	y	z	u	v	w
track	867.22	2707.24	6549.80	0.8603	−4.6299	4.0719
original object	867.61	2708.78	6552.93	0.8627	−4.6187	4.0908
misassociated object	867.47	2708.54	6549.37	0.8608	−4.6533	4.1045

Table 1: Table of positions and velocities

POSITION AND VELOCITY COMPONENTS ERRORS

Item	Position Errors (km)			Velocity Errors (km/sec)		
	δx	δy	δz	δu	δv	δw
original object	0.390	1.537	3.130	0.0024	0.0112	0.0189
misassociated object	0.247	1.299	−0.427	0.0005	−0.0234	0.0326

Table 2: Table of position and velocity errors of objects with respect to track

SQUARE ROOT OF THE SUM OF THE ERRORS SQUARED

Object	Positional Error (km)	Velocity Error (km/sec)
original object	3.509	0.0221
misassociated object	1.3895	0.0101

Table 3: Table of square root of the sum of the errors squared of objects with respect to track

ANGULAR MOMENTUM FOR TRACK AND OBJECTS

Item	Angular Momentum Components (km^2/sec)		
	i	j	k
track	41348.52	2103.54	−6311.20
original object	41347.39	2103.84	−6311.08
misassociated object	41593.39	1749.63	−6232.66

Table 4: Table of angular momentum of track, original and misassociated objects

where

E_{sum} is the error for either addition or subtraction,

ϵ_i is the error for each individual term.

The error propagation for multiplication of two number A and B is given by

$$E^2_{prod} = \mathrm{A}^2 E(\mathrm{B})^2 + \mathrm{B}^2 E(\mathrm{A})^2$$

where

$E(\mathrm{A})$ and $E(\mathrm{B})$ are the associated errors

E_{prod} is the error for the product of two numbers

These are the two formulas required to calculate the error propagation in a cross product. Each component of the angular momentum is the difference of two multiplication products. For the **i** component of the angular momentum, the estimated error is

$$E(\mathbf{L_x})^2 = [(\mathrm{y}E(\mathrm{w}))^2 + (\mathrm{w}E(\mathrm{y}))^2 + (\mathrm{z}E(\mathrm{v}))^2 + (\mathrm{v}E(\mathrm{z}))^2],$$

where

y is the y component of position,

$E(\mathrm{y})$ is the error in the y component of position,

z is the z component of position,

$E(\mathrm{z})$ is the error in the z component of position,

v is the y component of velocity,

$E(\mathrm{v})$ is the error in the v component of velocity,

w is the z component of velocity,

$E(\mathrm{w})$ is the error in the w component of velocity.

This formula can be recursively used for the other j and k components of the angular momentum. The estimated errors for each component on the angular momentum based on error analysis and the square root of the sum of the errors squared are given in Table 6. From the table, it is observed that the original object's estimated error for angular momentum is smaller than the misassociated object's estimated error for angular momentum.

Comparing the actual and the estimated angular momentum errors is very interesting. The estimated errors for the misassociated object are, in general, smaller than the actual error.

On the other hand, the estimated errors for the original object are larger than the actual errors. This may be explained by the following. For the misassociated object, there exists a real difference in position and velocity components with respect to the track. For the original object, there exists a gaussian error in position and velocity components with respect to the track. Since angular momentum is conserved, the actual position and velocity errors show significant cancellation relative to the estimates.

To determine a formula for the difference in angular momentum for real difference in position and velocity, the following position and velocity vectors can be formed for the misassociated object using the track's position and velocity vectors.

i	j	k
$x_{track} + \delta x$	$y_{track} + \delta y$	$z_{track} + \delta z$
$u_{track} + \delta u$	$v_{track} + \delta v$	$w_{track} + \delta w$

Now, the **i** component of the angular momentum for the misassociated object is

$$(y_{track} + \delta y)\,(w_{track} + \delta w) - (z_{track} + \delta z)\,(v_{track} + \delta v).$$

Expanding the terms and grouping for the track's angular momentum and the difference, the following is obtained:

$$y_{track}w_{track} - z_{track}v_{track} + \left[y_{track}\delta w + w_{track}\delta y - z_{track}\delta v - v_{track}\delta z + \delta y\delta w - \delta z\delta v\right]$$

The first two terms are the angular momentum of the track, and the terms in the brackets in this expansion are the actual differences in the angular momentum for the misassociated object with respect to the track. These terms are not Gaussian in nature, since the difference in position and velocity are systematic. But even this does not explain the closeness of the track's and original object's angular momentum. The error analysis shows that the angular momentum errors could be much larger. Why are these two angular momenta so close when the position and velocity errors are so large?

The answer to the last question comes from two places. First, the position and velocity errors of the original object with respect to the track are the results of the track and object moving for 5 scan periods or 150 seconds. The original position and velocity errors that the track started with have been increased by a large factor. This effect can be seen from the equations of motion.

$$x = x_o + vt + \frac{1}{2}a(x)t^2$$

$$v = v_o + a(x)t$$

The position components would show the largest effects since the velocity error is multiplied by time. The velocity, in addition to the velocity errors, would be affected by the position errors through the acceleration which is a function of position. Therefore, the angular momentum error analysis based on the position and velocity errors result in large error bands. Actually, the angular momentum is a conserved quantity. If the method used to update the position and velocity for the track and object conserves angular momentum, then **the original errors between the track's and object's angular momentum should not change!** Error propagation does not really apply at all. Table 7 contains the components of angular momentum for the original object at 5 different time steps. The time step labeled 'at error' is when the misassociation occurs. The errors observed in the angular momentum between the track and original object and previously discussed are reasonable and explainable, since the angular momentum and error between the track and object are conserved.

Application

The previous discussion has value only through its application to the tracking problem. Angular momentum may be useful in a number of ways for multi-track hypothesis, for track - report assignment, for cluster or group assignment and tracking, and for Kalman filtering.

For multi-track hypothesis, two or three reports are grouped together to initiate a track. For large numbers of objects, this method can result in numerous false tracks being initiated. These false tracks are later pruned when they are not re-enforced by additional reports which have passed an associated distance gate. The use of angular momentum can improve the problems of initiation and track pruning. Once a track hypothesis is formed, errors would be determined for position, velocity and angular momentum of the reports with respect to the track hypothesis. The position and velocity errors would be used to determine estimated angular momentum errors from error propagational theory. A statistical test can be made on the angular momentum errors compared to the estimated angular momentum error. This test

ANGULAR MOMENTUM ERRORS (km^2/sec)

Item	i	j	k	sqrt of sum of squares
original object	−1.13	0.30	0.12	1.18
misassociated object	244.87	−353.91	111.54	444.58

Table 5: Table of angular momentum components errors of objects with respect to track

ESTIMATED ANGULAR MOMENTUM ERRORS (km^2/sec)

Item	i	j	k	sqrt of sum of squares
original object	90.86	22.94	11.90	94.46
misassociated object	176.96	28.49	20.41	180.39

Table 6: Table of estimated angular momentum errors of objects with respect to track

ANGULAR MOMENTUM OF ORIGINAL OBJECT (km^2/sec)

Time Step	i	j	k
4^{th}	41346.26	2101.28	-6342.32
8^{th}	41346.63	2104.98	-6344.80
12^{th}	41347.68	2103.36	-6343.66
16^{th}	41347.83	2104.20	-6344.03
at error	41347.39	2103.84	-6344.08
average	41347.16	2103.53	-6343.78
std dev	0.6356	1.3915	0.8991

Table 7: Table of original object's angular momentum for several time steps

would determine if the angular momentum error for each report in the track hypothesis is less than the estimated angular momentum errors. If any report for the track hypothesis fails this test, the track hypothesis is rejected. If all the reports for the track hypothesis pass the test, then the track hypothesis is accepted and initiates a track. Track pruning is accomplished by using a angular momentum gate with the distance gate. False tracks would be less likely to be re-enforced and therefore, eventually would be eliminated.

At the beginning of this paper, drag and maneuvering were temporarily ignored. However, the previous discussion on multi-track hypotheses using angular momentum does not rule out drag and maneuvering if the track hypothesis and tracker includes these items. The angular momentum for the track hypothesis would not be conserved but the errors and test can still be performed. Of course care must be taken with the angular momentum tests for acceptance or rejection of hypotheses when acceleration is suspected, but the velocities and positions would also be changing in equally unpredictable ways. Thus, the angular momentum may be useful during boost phase, RV and decoy deployment as well as ballistic flight.

The tracking of groups or clusters may also benefit from using angular momentum. In the case of a bus deploying RVs and decoys, the angular momentum for the system's observable center of mass may remain close to a constant. An object in a group or cluster could be defined based on a positional distance from the "center of mass" and a deviation from the average momentum for the group in addition to using the group's or cluster's position and velocity. Because small differences in position and velocity can result in large angular momentum differences and because angular momentum usually will be conserved, it may be possible to identify individual objects within a group or cluster earlier using angular momentum..

The inclusion of angular momentum into a Kalman filter represents a modification of the work of E. W. Kamen and C. R. Sastry [4] [5] [6]. Kamen and Sastry consider not only the Kalman filter but the association problem. New variables, $y_1(k), y_2(k), \ldots, y_n(k)$, are defined that are symmetric functionals, h_i, of the target positions, x_i. Symmetric means that the value of the function, h_i, is unchanged for any permutation of x_i [4]. The new variables - functions considered by Kamen and Sastry are either scaled sums of products or scaled sums of powers of the measurements. These new variables, y_i, are then used to form an extended Kalman filter, which is nonlinear. Kamen and Sastry showed results similar to an associated filter, i. e., a filter using the correctly assigned reports to tracks. In addition, Daum [7] showed that Kamen's formulation is a Cramer-Rao bound on the estimation error covariance matrix.

If $z_i(k)$ are the measurements then the first two equations of the sum of products are:

$$y_1(k) = \sum_{i=1}^{N} z_i(k)$$

$$y_2(k) = z_1(k)z_2(k) + z_1(k)z_3(k) + \ldots + z_1(k)z_n(k) + \ldots + z_{n-1}(k)z_n(k)$$

The last equation is similar to the angular momentum equations where the $z_i(k)$ would be position and velocity. It would therefore appear possible to use the angular momentum equations to develop an extended Kalman filter. It should be noted that the angular momentum equation may not be symmetric by Kamen's definition. However, based on the gating of position, velocity and angular momentum, and Kamen's work, it may be possible to develop a simple assignment algorithm that would assign a single or a few reports to a track. Kamen's algorithm could then be used to complete the updating of the Kalman filter and association of report to track. The gating and assignment would reduce the N^2, operational requirements to a n^2, where N is the number of reports or tracks which ever is larger, and n would be limited to less than the maximum number of reports assigned to a track. This calculation can also parallelize easily to take full advantage of modern computer architectures.

Points of Discussion

From the example and discussion presented in this paper, the potential value of using angular momentum to track ballistic objects has been demonstrated. Angular momentum should help the assignment-gating algorithms of reports to tracks, since the angular momentum, and hence its errors, are conserved. Angular momentum should help multi-tracking hypothesis by providing an additional pruning factor, and thereby reduce the number of track hypotheses that must be analyzed. Angular momentum also detects if a target is maneuvering and therefore tells when a maneuvering filter should be used instead of a ballistic filter. However, numerous questions are posed that require farther study. Can angular momentum be incorporated in a Kalman filter similar to Kamen's work? What happens with clusters and groups? How will sensor errors influence angular momentum use? Can angular momentum be used to reduce errors in position and velocity estimates of the Kalman filter? Can angular momentum be used for discrimination? These issues will be addressed by the incorporation of angular momentum as a primary gating variable in the BEAST model.

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