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SUBSONIC WIND TUNNEL TESTING HANDBOOK

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This handbook is predominantly structured for subsonic (non-compressible flow), force and moment wind tunnel testing. Its purpose is to provide to the aerodynamic testing engineer equations, concepts, illustrations, and definitions that would aid him or her in wind tunnel testing. The information in this handbook is an amalgamation of numerous sources and observations. By design, this handbook is a living document readily expandable to include personal notes and additional sections. This handbook has not and cannot encompass every wind tunnel testing technique or principle.

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Foreword

This technical report was prepared by Captain Michael G. Alexander from Wright Laboratory, Flight Dynamics Directorate, Aeromechanics Division, Wright-Patterson AFB, Ohio, 45433-6553. This work was accomplished under work project 240410A2, Advanced Tactical Transport. This technical report provides the wind tunnel testing engineer a handbook with useful equations, definitions, and concepts to aid in wind tunnel testing.

The author wishes to thank the exceptional insights, observations, expertise and help from Mr. Tom Tighe, Mr. Jim Grove, Mr. Bob Guyton and Mr. Larry Rogers all from WL/FIMMC (Wright Laboratory, Airframe Aerodynamics Group).

This report has been reviewed and is approved.

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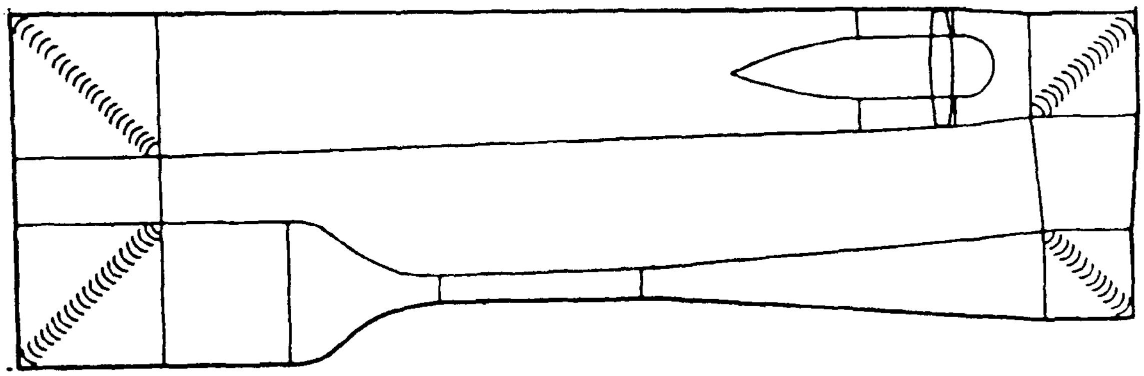
Testing

Handbook



By Captain Michael G. Alexander, USAF

I INTRODUCTION

I Introduction

The aerodynamic engineer who participates in wind tunnel testing often has a need for reference material to aid him/her in his/her testing. It is nearly impossible for the engineer to recall every equation, concept, definition, and to carry reference material to the wind tunnel site. This handbook, Subsonic Wind Tunnel Testing Handbook, attempts to provide to the testing engineer in one reference some of those equations, concepts, and definitions that he or she might find helpful during testing. This handbook is a quick reference for the testing engineer and is an amalgamation of information from numerous reference materials and personal observations. It is designed to be a living document readily expandable and to fit in a briefcase to accompany the testing engineer to the test site. Also by design, this aid has not encompassed every wind tunnel technique or principle, but offers enough information to facilitate and help ease wind tunnel testing. This handbook is predominantly structured for subsonic (non-compressible flow), force and moment, wind tunnel testing. For compressible flow testing, an excellent reference aid is the NACA 1135 (ref. 1). However, it is no longer in print by the government, but, reprints of the NACA 1135 can be procured from reference 2.

II AERODYNAMIC DEFINITIONS

II Aerodynamic Definitions

Symbols

a.c.	Aerodynamic Center
cg	Center of Gravity
C_L	Lift Coefficient
C_n	Yawing Moment Coefficient
C_M	Pitching Moment Coefficient
C_{M_o}	Pitching Moment Coefficient at zero angle-of-attack
cp	Center of Pressure
MAC	Mean Aerodynamic Chord
N_o	Neutral Point
SM	Static Margin
X_{np}	Distance from the MAC leading edge to the aerodynamic center
X_{cg}	Distance from the MAC leading edge to the center of gravity
$\frac{\partial C_m}{\partial \alpha}$	Pitching Moment Coefficient slope
$\frac{\partial C_{mcg}}{\partial C_L}$	Slope of the cg pitching moment and lift curve
$\frac{\partial C_n}{\partial \varphi}$	Slope of yawing moment coefficient and the yaw angle
$\frac{\partial C_n}{\partial \beta}$	Slope of yawing moment coefficient and the sideslip angle
$\frac{\partial C_l}{\partial \beta}$	Slope of rolling moment coefficient and the sideslip angle
α	Angle-of-Attack
β	Sideslip Angle
φ	Yaw Angle

Aerodynamic Center (a.c.) - The point where the pitching moment does not vary with the angle of attack.

Center of Pressure (cp)- A point where the pitching moment vanishes. The cp location varies with angle of attack and Mach number.

Mean Aerodynamic Chord (MAC) - The chord of an imaginary, untwisted, unswept, equal span non-tapered wing which would have force vectors throughout the flight range identical with those of the actual wing or wings.

Neutral Point (x_{np} or N_o) - As the cg is moved aft, the slope of $\partial C_{mcg} / \partial C_L$ becomes less negative. When there is no change of the pitching moment with lift coefficient ($\partial C_{mcg} / \partial C_L = 0$), that x_{cg} position is the neutral point ($N_o = x_{cg}$).

Static Margin - The distance from the center of gravity to the neutral point expressed as a fraction of the mean aerodynamic chord. The 'X' position of x_{np} is measured positive aft from the c.g..

$$\boxed{SM = \frac{X_{np} - X_{cg}}{\bar{c}} = \frac{\partial C_{mcg}}{\partial C_L}}$$

$$+SM = \frac{\partial C_{mcg}}{\partial C_L} < 0 \quad \text{(Stable; } N_o \text{ is behind c.g.)}$$

$$-SM = \frac{\partial C_{mcg}}{\partial C_L} > 0 \quad \text{(Unstable; } N_o \text{ is ahead of c.g.)}$$

Longitudinal Static Stability - Is determined by the sign and the magnitude of the slope of C_m versus α curve. $\frac{\partial C_m}{\partial \alpha}$ must be < 0 and C_{m_o} must be positive for trimmed, positive longitudinal static stability. Also, C_m versus C_L can be used to determine positive longitudinal stability.

Directional Static Stability - Is determined by the sign and magnitude of the slope of C_n versus β curve, where β is the sideslip angle. $\frac{\partial C_n}{\partial \beta}$ must be positive (> 0) for directional stability. This is also known as weathercock stability. Also, C_n versus φ can be used where φ is the yaw angle. $\frac{\partial C_n}{\partial \varphi}$ must be negative (< 0) for directional stability.

Lateral Static Stability - It is determined by the sign and magnitude of the slope of the C_l versus β curve. $\frac{\partial C_l}{\partial \beta}$ must be negative (< 0) for positive lateral stability. This is also known as the dihedral effect. An increase in lateral stability causes dutch roll; too little stability causes spiral instability.

Dynamic Stability - Is the time history of the movements of a body in response to its static stability tendencies following an initial disturbance from equilibrium.

Trimmed Flight - For balanced flight, the aircraft flies at a given elevator angle and angle of attack that produces $C_m = 0$.

Critical Mach Number - Is the Mach number where there is a sharp increase in drag (drag divergence, approximately Mach = 1.0).

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III FORCE AND MOMENT EQUATIONS

III Force and Moment Equations

General Aerodynamic Symbols and Equations

A	Axial Force (lbf) -- ($A = D\cos\alpha - L\sin\alpha$)
a	Speed of Sound (f/s) -- $\sqrt{(\gamma R \mathcal{T})} = 49\sqrt{\mathcal{T}} = \sqrt{(\gamma P/\rho)}$; $\mathcal{T} = {}^{o}R$
	Speed of Sound (mph) -- $33.42\sqrt{\mathcal{T}}$; $\mathcal{T} = {}^{o}R$
a	Lift Curve Slope -- C_{L_α}
C_A	Axial Force Coefficient ($C_A = -C_L\sin\alpha + C_D\cos\alpha$)
C_D	Drag Coefficient ($C_D = C_N\sin\alpha + C_A\cos\alpha$)
C_{D_i}	Induced Drag
C_{D_o}	Zero Lift Drag; Parasite Drag (profile, friction, pressure)
C_L	Lift Coefficient ($C_L = C_N\cos\alpha - C_A\sin\alpha$)
C_L'	Lift at minimum drag (generally zero)
C_l	Rolling Moment
C_{L_α}	Lift Curve Slope -- $\dfrac{\delta C_L}{\delta\alpha}$
C_m	Pitching Moment
C_{m_α}	Pitching moment curve slope -- $\dfrac{\delta C_m}{\delta\alpha}$
C_N	Normal Force Coefficient ($C_N = C_L\cos\alpha + C_D\sin\alpha$)
C_n	Yawing Moment
C_{n_β}	Yawing moment curve slope -- $\dfrac{\delta C_n}{\delta\beta}$
C_T	Thrust Coefficient -- $\dfrac{\text{Thrust}}{q\ S_w}$
cp	Center of Pressure -- $\dfrac{x_{cp}}{\bar{c}} = -\dfrac{C_m}{C_N}$

General Aerodynamic Symbols and Equations (continued)

C_Y Side Force Coefficient

D Drag Force (lbf) -- ($D = N\sin\alpha + A\cos\alpha$)

Drag Polar -- $C_D = C_{D_o} + \dfrac{\left(C_L - C_L'\right)^2}{\pi e AR}$

e Oswald's wing efficiency factor

g Gravity Constant

K Induced drag factor -- $\dfrac{1}{\pi e AR}$

L Lift Force (lbf) -- ($L = N\cos\alpha - A\sin\alpha$)

N Normal Force -- ($L\cos\alpha + D\sin\alpha$) (lbf)

N_o Neutral point -- $\dfrac{\delta C_m}{\delta C_L} = \dfrac{x_{cg} - N_o}{\bar{c}}$

n Load Factor -- $\dfrac{L + T\sin\alpha_T}{W} = \dfrac{L}{W}$ (for α_T small)

M Mach number

PM Pitching Moment (dimensional)

P Pressure

R Gas Constant

T Thrust

$\mathscr{T}$ Temperature; (^{o}Rankine)

q Dynamic Pressure -- $0.5\rho V^2$ or $0.7(P_s)M^2$

RM Rolling Moment (dimensional)

S Surface Area

S Side Force (lbf)

$\bar{V}$ Tail/Canard Volume Coefficient = $\left[\dfrac{l_t \; S_t}{q_\infty S_w \bar{c}}\right]$

General Aerodynamic Symbols and Equations (concluded)

X_{np}	Neutral point -- $\dfrac{\delta C_m}{\delta C_L} = \dfrac{x_{cg} - N_o}{\bar{c}}$
Y	Yawing Force (side force)
YM	Yawing Moment (dimensional)
α_T	Thrust vector angle
α	Geometric angle of attack (measured from V_∞ to the wing chord)
α_a	Absolute angle of attack ($\alpha - \alpha_{ZL}$)
α_{ZL} ; α_o	Zero lift angle of attack (measured from the wing chord to the wing zero lift line)
β	Angle of sideslip
ϕ	Angle of roll
η	Tail efficiency factor (q_t/q_∞)
φ	Angle of yaw
ε	Downwash angle
θ	local angle of inclination of surface to V_∞

Subscripts

np -- neutral point	w -- wing	t -- tail
s -- static pressure	T -- Thrust	c -- canard
cg -- center of gravity	ac -- aerodynamic center	
inc -- incompressible	∞ -- freestream conditions	

Table 1

Force and Moment Definitions

Forces

Body Axis		Wind Axis		Stability Axis	
Force	Coefficient	Force	Coefficient	Force	Coefficient
N	$C_N = N/qS_w$	L	$C_L = L/qS_w$	L	$C_L = L/qS_w$
A	$C_A = A/qS_w$	D	$C_D = D/qS_w$	D	$C_D = D/qS_w$
S	$C_S = S/qS_w$	Y	$C_Y = Y/qS_w$	Y	$C_Y = Y/qS_w$

Moments

Body Axis		Wind Axis		Stability Axis	
Moment	Coefficient	Moment	Coefficient	Moment	Coefficient
PM	$C_m = PM/qS_w\bar{c}$	PM	$C_m = PM/qS_w\bar{c}$	PM	$C_m = PM/qS_w\bar{c}$
RM	$C_l = RM/qS_wb$	RM	$C_l = RM/qS_wb$	RM	$C_l = RM/qS_wb$
YM	$C_n = YM/qS_wb$	YM	$C_n = YM/qS_wb$	YM	$C_n = YM/qS_wb$

Parameter legend

q = Dynamic pressure b = Span S_w = Wing area $\bar{c}$ = MAC

Aerodynamic Equations

Trim Pitching Moment Equations (ref. 3)

Conventional Horizontal Tailed Aircraft (figure III-1)

$$C_{M_{cg}} = \left[C_{M_{ac}}\right]_w + C_L\left[\frac{X_w}{\bar{c}}\right] + C_D\left[\frac{z}{\bar{c}}\right] + \left[\frac{T\ \ Z_T}{q_\infty S_w \bar{c}}\right] - C_{L_t}(\bar{V})\eta_t - \left[C_{M_{cg}}\right]_{inlet}$$

For an all flying tail....

$$C_{L_t} = a_t\,(\alpha_t + \Delta\alpha)$$

$$= a_t[(1 - \frac{\delta\varepsilon}{\delta\alpha})\alpha + \Delta\alpha]$$

$\Delta\alpha = \alpha$ created by control column input

For fixed stabilizer and a movable elevator...

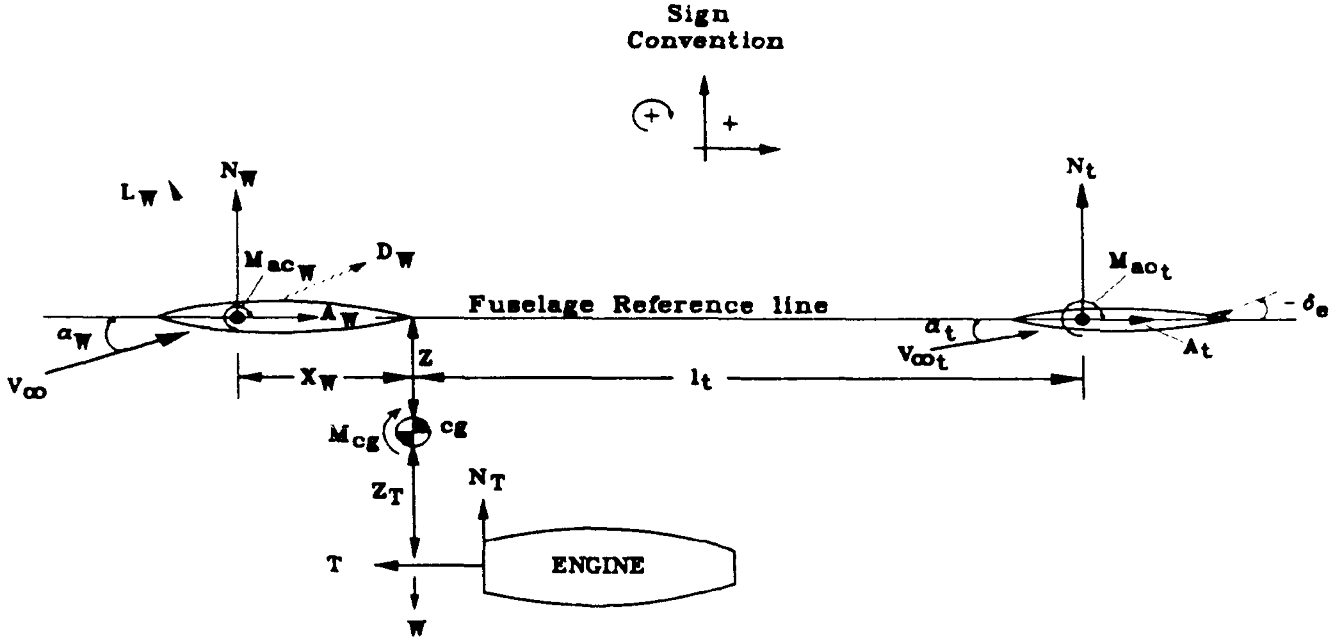
$$C_{L_t} = a_t[(1 - \frac{\delta\varepsilon}{\delta\alpha})\alpha - (\alpha_{ZL})_t]$$

Tailless Aircraft (figure III-2)

$$C_{M_{cg}} = \left[C_{M_{ac}}\right]_w - C_L\left[\frac{X_w}{\bar{c}}\right] + C_D\left[\frac{z}{\bar{c}}\right] + \left[\frac{T\ \ Z_T}{q_\infty S_w \bar{c}}\right] - \left[C_{M_{cg}}\right]_{inlet}$$

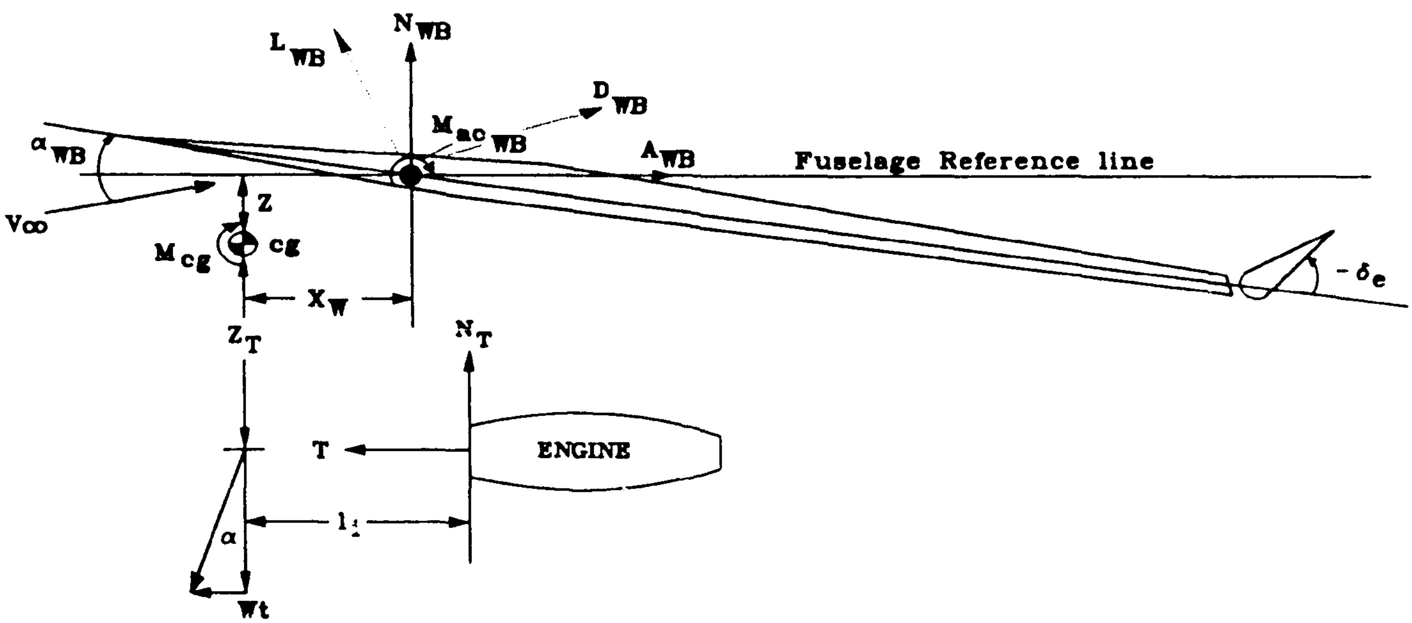
Canard Aircraft (figure III-3)

$$C_{M_{cg}} = \left[C_{M_{ac}}\right]_w - C_L\left[\frac{X_w}{\bar{c}}\right] + C_D\left[\frac{z}{\bar{c}}\right] + \left[\frac{T\ \ Z_T}{q_\infty S_w \bar{c}}\right] - C_{L_c}\left[\bar{V}_c\right] - \left[C_{M_{cg}}\right]_{inlet}$$



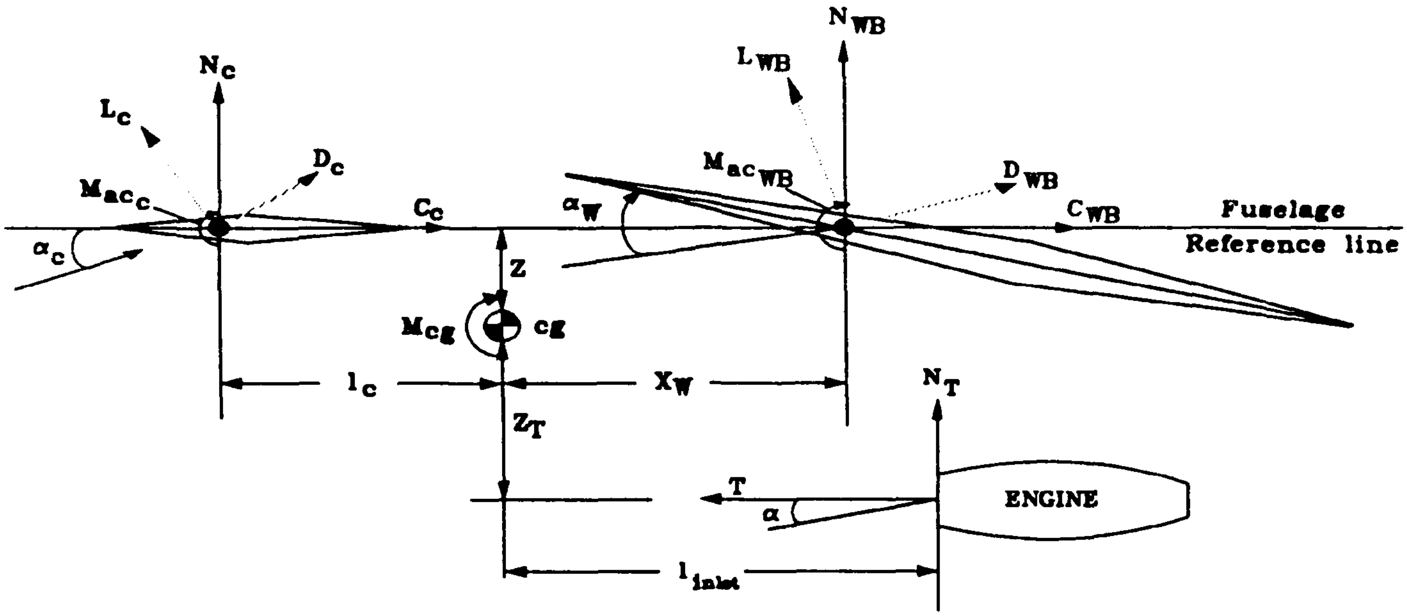
Aft Tailed Aircraft

Figure III-1 Force and Moment Vectors
Aft Tailed Aircraft



Tailless Aircraft

Figure III-2 Force & Moment Vectors
Tailless Aircraft



Canard Aircraft

Figure III-3 Force and Moment Vectors
Canard Aircraft

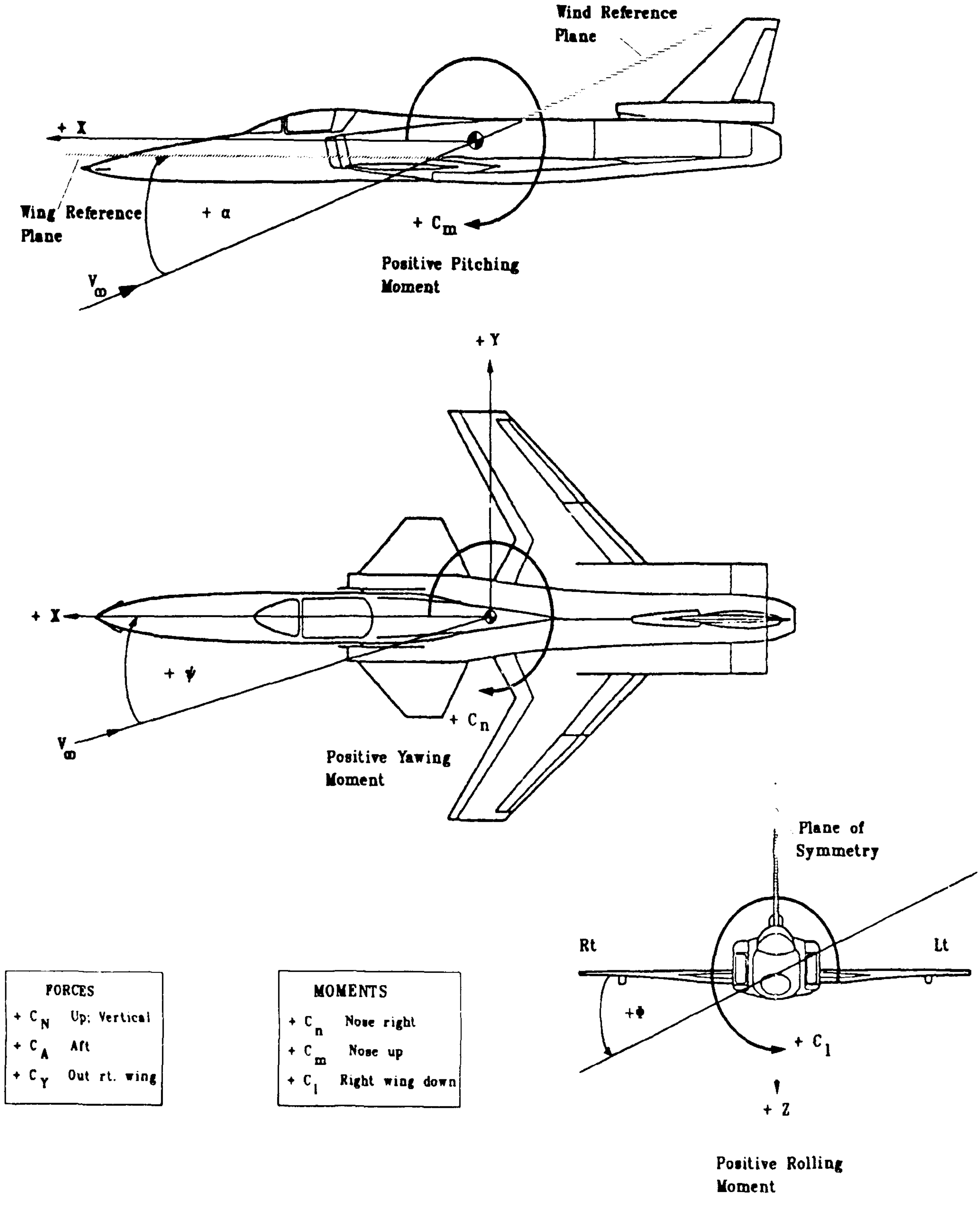


Figure III-4 Positive Angles, Moments, and Body Axis System

Stick-Fixed Neutral Point

$$N_o = X_{cg} \qquad ;\text{when } (\delta C_M / \delta C_L) = 0$$

$$N_o = X_{ac} - \left[\delta C_M / \delta C_L\right] + (a_t / a_w)\bar{V}\eta_t[1 - (\delta\varepsilon / \delta\alpha)]$$

$$\delta C_M / \delta C_L = \frac{x_{cg} - N_o}{\bar{c}}$$

Two-Dimensional Lift

Subsonic

$$C_{L_{2D_{inc}}} = \frac{a_{inc} \; (\alpha_a)}{\sqrt{1 - M^2}} \qquad (\alpha \text{ in rads})$$

Supersonic

$$C_{L_{2D}} = \frac{C_{L_{2D_{inc}}}}{\sqrt{M^2 - 1}} \qquad\qquad C_{L_{2D}} = \frac{4\alpha}{\sqrt{M^2 - 1}} \quad \text{(flat plate)} \; (\alpha \text{ in rads})$$

Pressure Coefficient

Incompressible

$$C_{p_{inc}} = 1 - \left[\frac{V}{V_\infty}\right]^2 \qquad C_{P_{inc}} = \frac{P - P_\infty}{q_\infty} \qquad C_P = \frac{C_{P_{inc}}}{\sqrt{(1-M^2)}} \quad \text{(thin-airfoil theory)}$$

Compressible

$$C_p = \frac{2}{\gamma M_\infty^{\;2}} \left[\left[\frac{1 + .5(\gamma - 1)M_\infty^{\;2}}{1 + .5(\gamma - 1)M^2} \right]^{\frac{\gamma}{\gamma-1}} - 1 \right]$$

$$C_P = \frac{2\theta}{\sqrt{M^2 - 1}} \qquad \begin{array}{ll} \theta = >0 & \text{compression} \\ \theta = <0 & \text{expansion} \end{array} \qquad (\theta \text{ in rads})$$

Center of Pressure Location

$$X_{cp} = -\frac{C_M}{C_N} \text{ (units of length)}$$

Aerodynamic Center (a.c.) Determination

Assume the moment reference center is at a distance 'x' from the L.E. and then take moments about the a.c. ....

$$C_{M_{ac}} q\bar{c}S = \left[C_{M_x} q\bar{c}S\right] + \left[C_L qS\right](x_{ac} - x)\cos\alpha + \left[C_D q\bar{c}S\right](x_{ac} - x)\sin\alpha$$

Solving for x_{ac}. . .

$$\frac{x_{ac}}{\bar{c}} = \frac{x}{\bar{c}} - \frac{C_{M_x} - C_{M_{ac}}}{C_L\cos\alpha + C_D\sin\alpha}$$

for $\alpha << 1$

$$\frac{x_{ac}}{\bar{c}} = \frac{x}{\bar{c}} - \frac{C_{M_x} - C_{M_{ac}}}{C_L}$$

Mass Flow (ref. 27)

$$M_o = \rho AV$$

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IV AXIS SYSTEM DEFINITIONS

IV Axis System Definitions

(Ref. 6)

Tunnel Axis - An orthogonal, right-handed axis system which remains fixed with respect to the tunnel in pitch, roll and yaw.

Body Axis - An orthogonal, right-handed axis system which remains fixed with respect to the model and rotates with it in pitch, roll, and yaw (figure IV-1). All forces, moments, and axis systems are referenced from the Body axis.

x_b - Longitudinal body axis in the plane of symmetry of the aircraft; positive forward.

y_b - Lateral body axis perpendicular to the plane of symmetry of the aircraft, usually taken in the plane of the wing; positive toward right wing tip.

z_b - Vertical body axis in the plane of symmetry of aircraft, perpendicular to the longitudinal and lateral axes; positive down.

Wind Axis - An orthogonal, right-handed axis system which is obtained by rotating through pitch and yaw, but not roll with respect to the Body axis. This system defines lift as perpendicular to the relative wind, drag as parallel to the relative wind, and side force as perpendicular to the plane of lift and drag (figure IV-2).

Stability Axis - An orthogonal, right-handed axis system which remains fixed with respect to the relative wind in pitch, but rotates with the model in yaw and roll. Lift is defined as perpendicular to the relative wind with drag and side force yawing with the model (figure IV-3). The only difference between the Stability axis and the Body axis is α.

x_s - Longitudinal stability axis, parallel to the projection of the total velocity vector ($\bar{V}$) on the plane of symmetry of the aircraft; positive forward

y_s - Lateral stability axis, coincident as the lateral body axis; positive out the right wing tip.

z_s - Vertical stability axis in the plane of symmetry of the aircraft, perpendicular to the longitudinal and lateral stability axes; positive down.

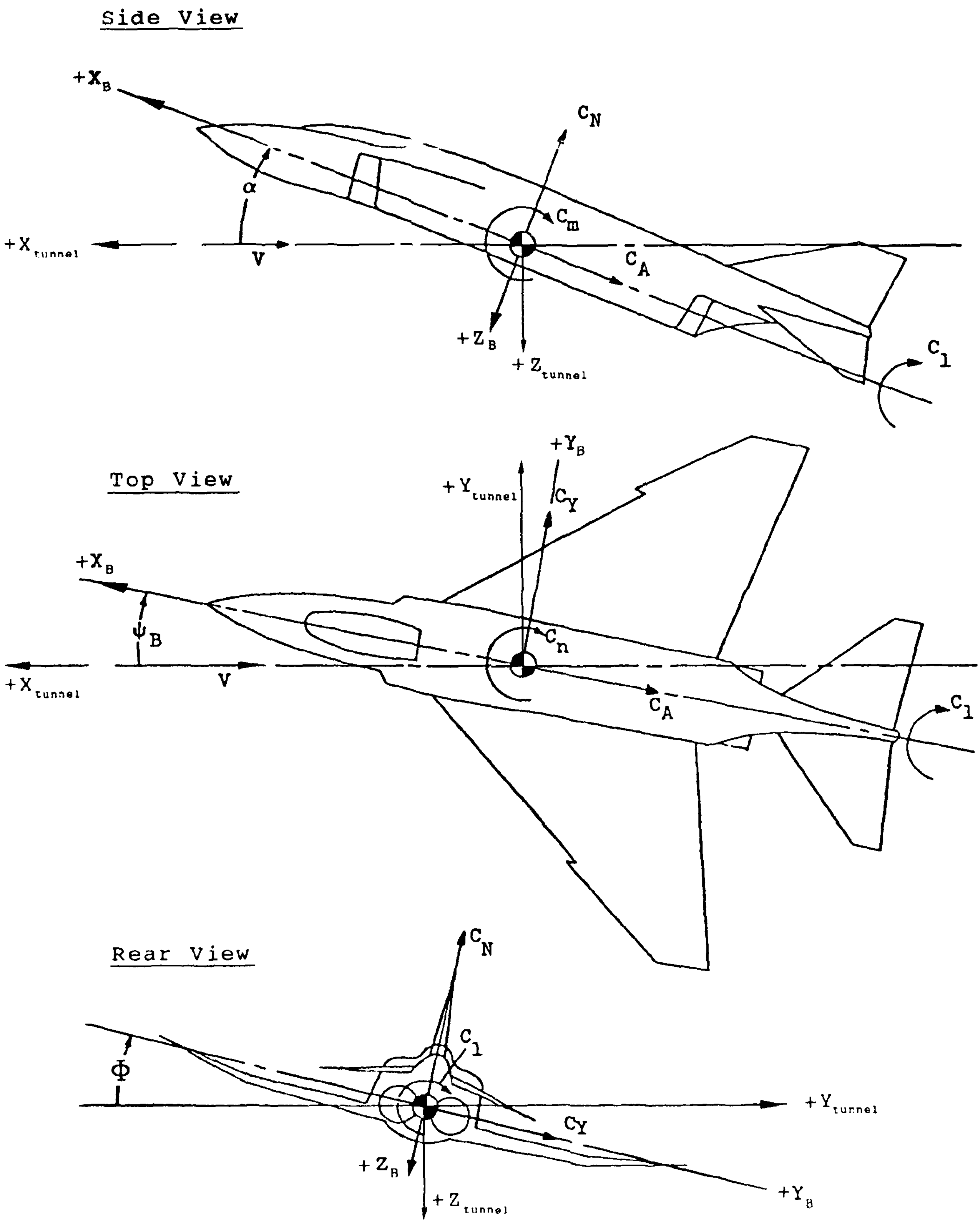

Figure IV-1 Body Axis System

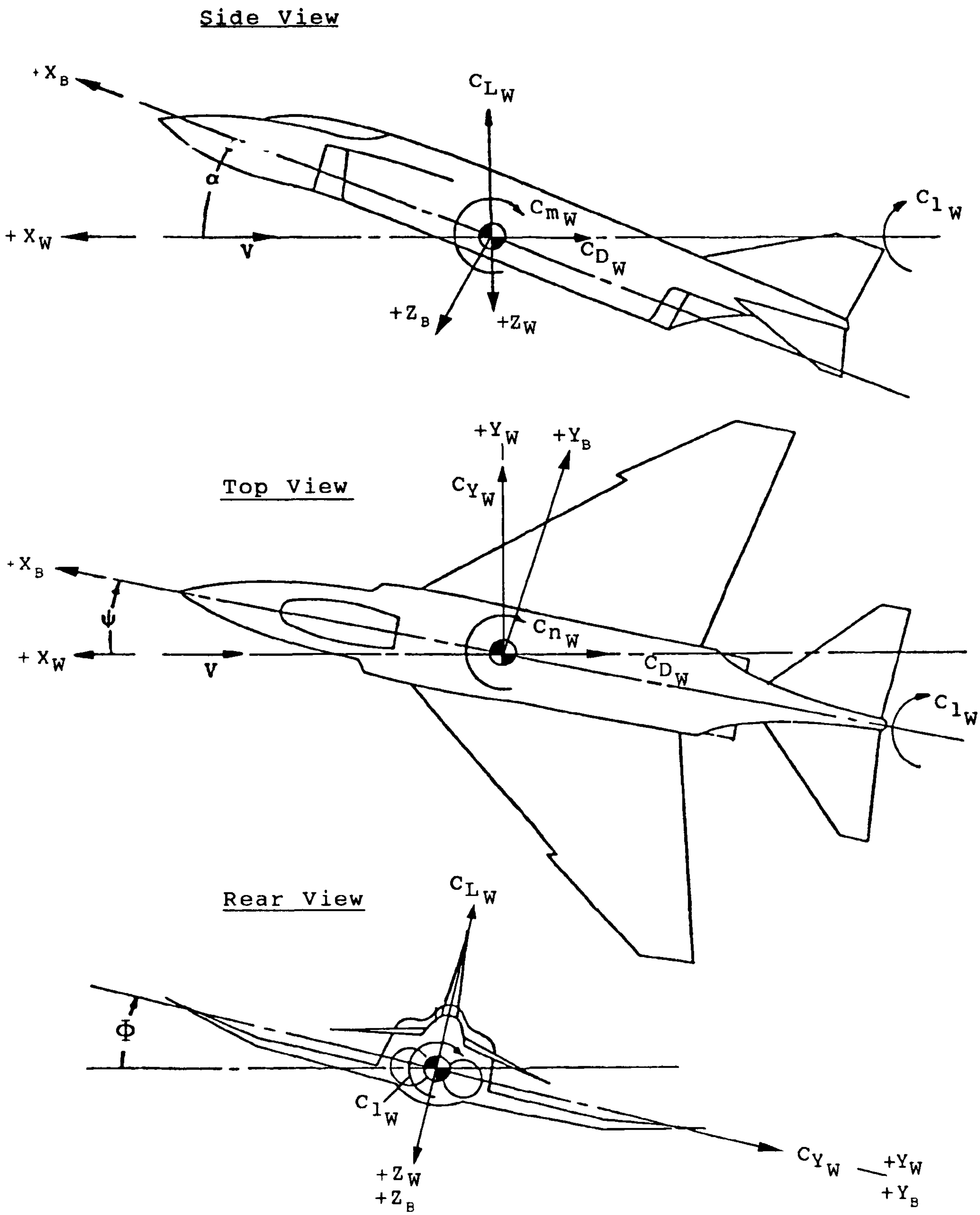

Figure IV-2 Wind Axis System

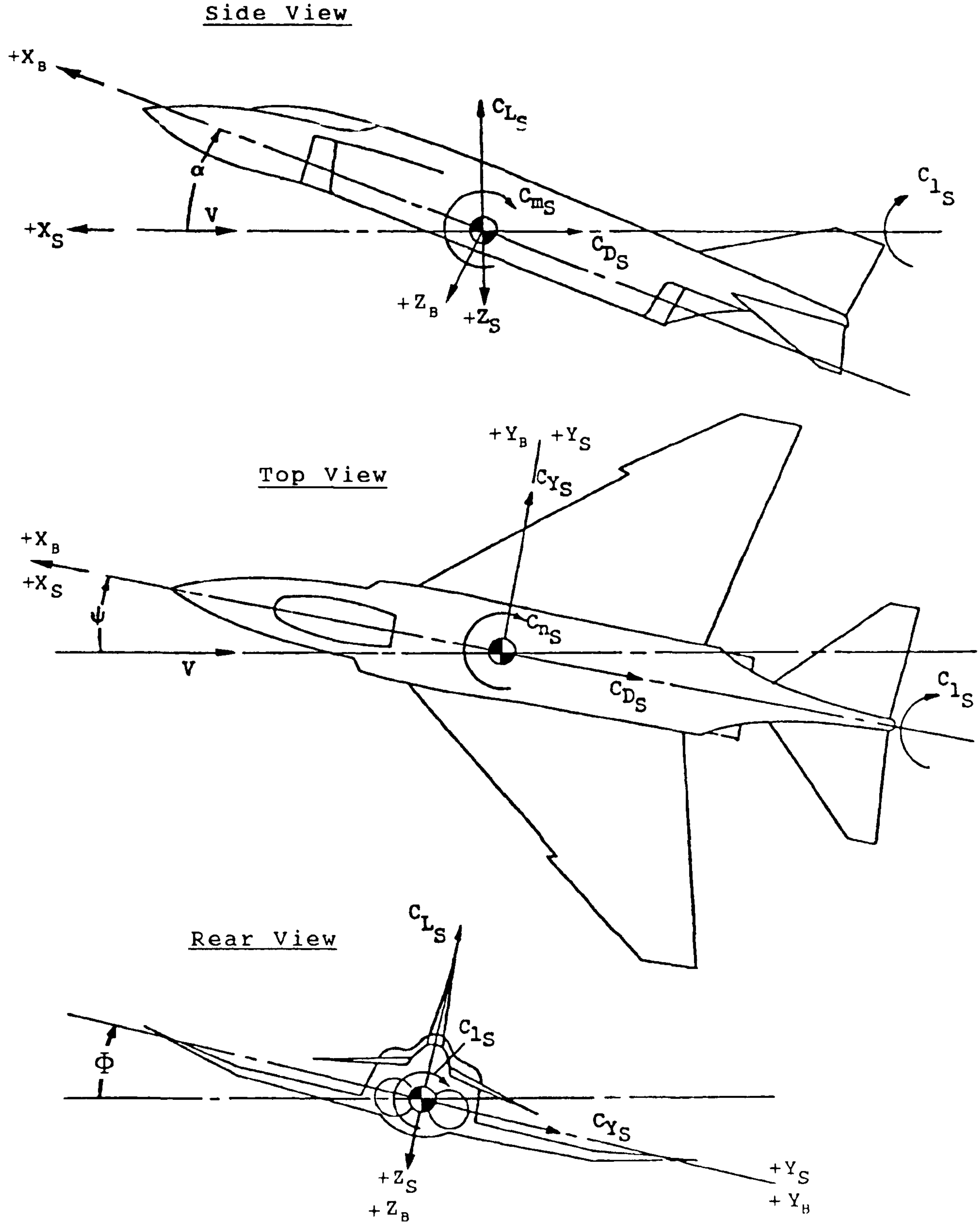

Figure IV-3 Stability Axis System

Angle Definitions (figure IV-4)

Aerodynamic Angles

α - Pitch angle-of-attack, angle between the x_b axis and the projection of $(\bar{V})$ on the plane of symmetry. A positive α is pitch up or rotates the ^+z_b axis into ^+x_b axis. $\alpha = \tan^{-1}(w/u)$

φ - Yaw angle, angle between the projected total velocity vector $(\bar{V})$ on the $x_b y_b$ plane and the ^+x_b axis. A positive direction rotates the ^+x_b axis into the ^+y_b axis (positive nose right). $\varphi = \tan^{-1}(v/u)$.

β - Angle-of-sideslip, angle between the total velocity vector $(\bar{V})$ and its projection on the $x_b z_b$ plane. A positive rotation rotates the ^+y_b axis into ^+x_b axis (positive nose left). $\beta = \sin^{-1}(v/\bar{V})$.

Orientation Angles (looking from origin; figure IV-4)

θ - Pitch angle, positive clockwise about the ^+y_i axis direction (+ nose up; ^+z_i into ^+x_i).

φ - Yaw angle, positive clockwise about the ^+z_i axis direction (+ nose right; ^+x_i into ^+y_i).

ϕ - Roll angle, positive clockwise about the ^+x_i axis direction (+ rt wing down; ^+y_i into ^+z_i).

φ, θ, and ϕ form a system of three angles which defines the orientations of the Body axis with the respect to the Tunnel axis system (inertial reference system). Any orientation of the Body axis system is obtained by rotationally displacing it from the Tunnel axis system through each of the three angles in turn. The order of rotation is important (figure IV-5) and is defined to be yaw-pitch-roll.

A long discussion ensued over the definition of yaw (φ) and sideslip (β). After many arguments and three dimensional velocity box drawings, $\varphi = -\beta$ (after yawing and pitching the model. Roll = 0). When the model is yawed, pitched, and rolled, then yaw does not equal minus sideslip ($\varphi \neq -\beta$). The difference is obviously roll.

Angle Transformation form Tunnel Axis (Inertial Axis) to Body Axis
(figure IV-4; +Z down, +X out nose, +Y out rt wing)

$$\begin{bmatrix} X_b \\ Y_b \\ Z_b \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ \begin{bmatrix}\sin\phi\sin\theta\cos\varphi \\ -\cos\phi\sin\varphi\end{bmatrix} & \begin{bmatrix}\sin\phi\sin\theta\sin\varphi \\ +\cos\phi\cos\varphi\end{bmatrix} & \sin\phi\cos\theta \\ \begin{bmatrix}\cos\phi\sin\theta\cos\varphi \\ +\sin\phi\sin\varphi\end{bmatrix} & \begin{bmatrix}\cos\phi\sin\theta\sin\varphi \\ -\sin\phi\cos\varphi\end{bmatrix} & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}$$

b = body i = inertial

Wing Reference Plane (ref. 4)

It is the plane which passes through the wing tips and is parallel to the longitudinal axis.

Wind Reference Plane (ref. 4)

It is the plane that passes through the relative wind vector and intersects the wing reference plane along a line which is perpendicular to the plane of symmetry.

Plane of Symmetry (ref. 4)

It is the plane that passes through the fuselage axis and is perpendicular to both the wing and wind reference planes.

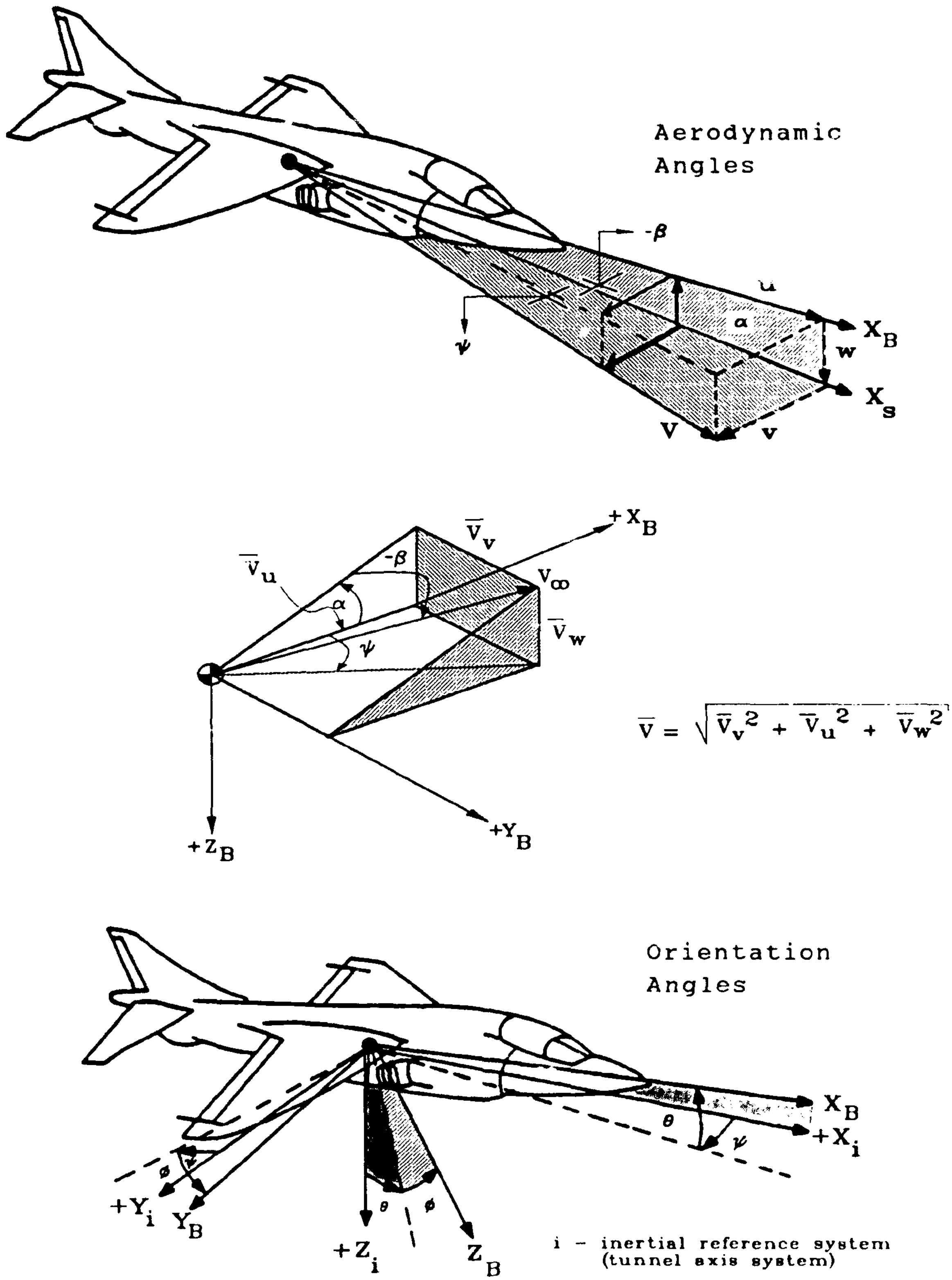

Figure IV-4 Angle Definitions

Coordinate Transformation Equations

Most external balances measure about the wind axis system and most internal balances measure about the body axis system. Thus it becomes necessary to transfer from one axis system to another. If the model and balance are fixed to the sting, with no relative motion between the model and the balance, and the sting is capable of movement in yaw, pitch, and roll then body axis coincides with the balance axis. Therefore, all forces and moments indicated by the balance are body forces and moments.The axis can pass through either the balance moment reference center or the desired model c.g.. However, if the model and balance axes do not coincide, additional transformations must be considered.

The coordinate transformations below assume data have been transferred to the desired c.g. location. Also, α is α_{body} and has not been corrected for the wind tunnel upflows and wall effects.

Balance Axis to Body Axis

(Ref. 28)

Order of rotation: yaw, pitch, roll (See figure IV-6)

Note: α and β are balance to body axis angles and not aerodynamic angles.

Forces (roll = 0)

$$C_{N_b} = C_{N_{bal}}\cos\alpha + C_{A_{bal}}\cos\beta\sin\alpha - C_{Y_{bal}}\sin\beta\sin\alpha$$

$$C_{A_b} = C_{A_{bal}}\cos\beta\cos\alpha - C_{Y_{bal}}\sin\beta\cos\alpha - C_{N_{bal}}\sin\alpha$$

$$C_{Y_b} = C_{Y_{bal}}\cos\beta + C_{A_{bal}}\sin\beta$$

Forces (roll = 0)

$$\begin{bmatrix} C_{N_b} \\ C_{A_b} \\ C_{Y_b} \end{bmatrix} = \begin{bmatrix} \cos\alpha & \cos\beta\sin\alpha & \sin\beta\sin\alpha \\ -\sin\alpha & \cos\alpha\cos\beta & \cos\alpha\sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} C_{N_{bal}} \\ C_{A_{bal}} \\ C_{Y_{bal}} \end{bmatrix}$$

Forces (roll $\neq$ 0)

$$\begin{bmatrix} C_{N_b} \\ C_{A_b} \\ C_{Y_b} \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\phi & \begin{matrix}\sin\alpha\cos\beta\cos\phi \\ -\sin\beta\sin\phi\end{matrix} & \begin{matrix}\sin\alpha\sin\beta\cos\phi \\ +\cos\beta\sin\phi\end{matrix} \\ -\sin\alpha & \cos\alpha\cos\beta & \cos\alpha\sin\beta \\ -\cos\alpha\sin\phi & \begin{matrix}-\sin\alpha\cos\beta\sin\phi \\ -\sin\beta\cos\phi\end{matrix} & \begin{matrix}-\sin\alpha\sin\beta\sin\phi \\ +\cos\beta\cos\phi\end{matrix} \end{bmatrix} \begin{bmatrix} C_{N_{bal}} \\ C_{A_{bal}} \\ C_{Y_{bal}} \end{bmatrix}$$

Moments (roll $\neq$ 0)

(Note: Body c.g. is located +X, +Y, +Z from balance c.g. See figure IV-6)

$$\begin{bmatrix} C_{l_b} \\ C_{m_b} \\ C_{n_b} \end{bmatrix} = \begin{bmatrix} c\alpha c\beta & (CB)c\alpha s\beta & -s\alpha & 0 & (ZB) & (YB) \\ \begin{array}{l}-(BC)s\alpha c\beta s\phi \\ -(BC)s\beta c\phi\end{array} & \begin{array}{l}-s\alpha s\beta s\phi \\ +c\beta c\phi\end{array} & -(BC)c\alpha s\phi & (ZC) & 0 & -(XC) \\ s\alpha c\beta c\phi - s\beta s\phi & \begin{array}{l}(CB)s\alpha s\beta c\phi \\ +(CB)c\beta s\phi\end{array} & c\alpha c\phi & -(YB) & -(XB) & 0 \end{bmatrix} \begin{bmatrix} C_{l_{bal}} \\ C_{m_{bal}} \\ C_{n_{bal}} \\ C_{A_b} \\ C_{Y_b} \\ C_{N_b} \end{bmatrix}$$

Legend

$(CB) = \frac{\bar{c}}{b}$ $\quad (BC) = \frac{b}{\bar{c}}$ $\quad (ZC) = \frac{Z}{\bar{c}}$

$(ZB) = \frac{Z}{b}$ $\quad (YB) = \frac{Y}{b}$ $\quad (XC) = \frac{X}{\bar{c}}$ $\quad (XB) = \frac{X}{b}$

b = span $\quad$ s = sin $\quad$ c = cos

$\bar{c}$ = M.A.C $\quad$ X, Y, Z = body transfer distances

Generally, roll (ϕ) is considered as zero in the balance to body transformation.

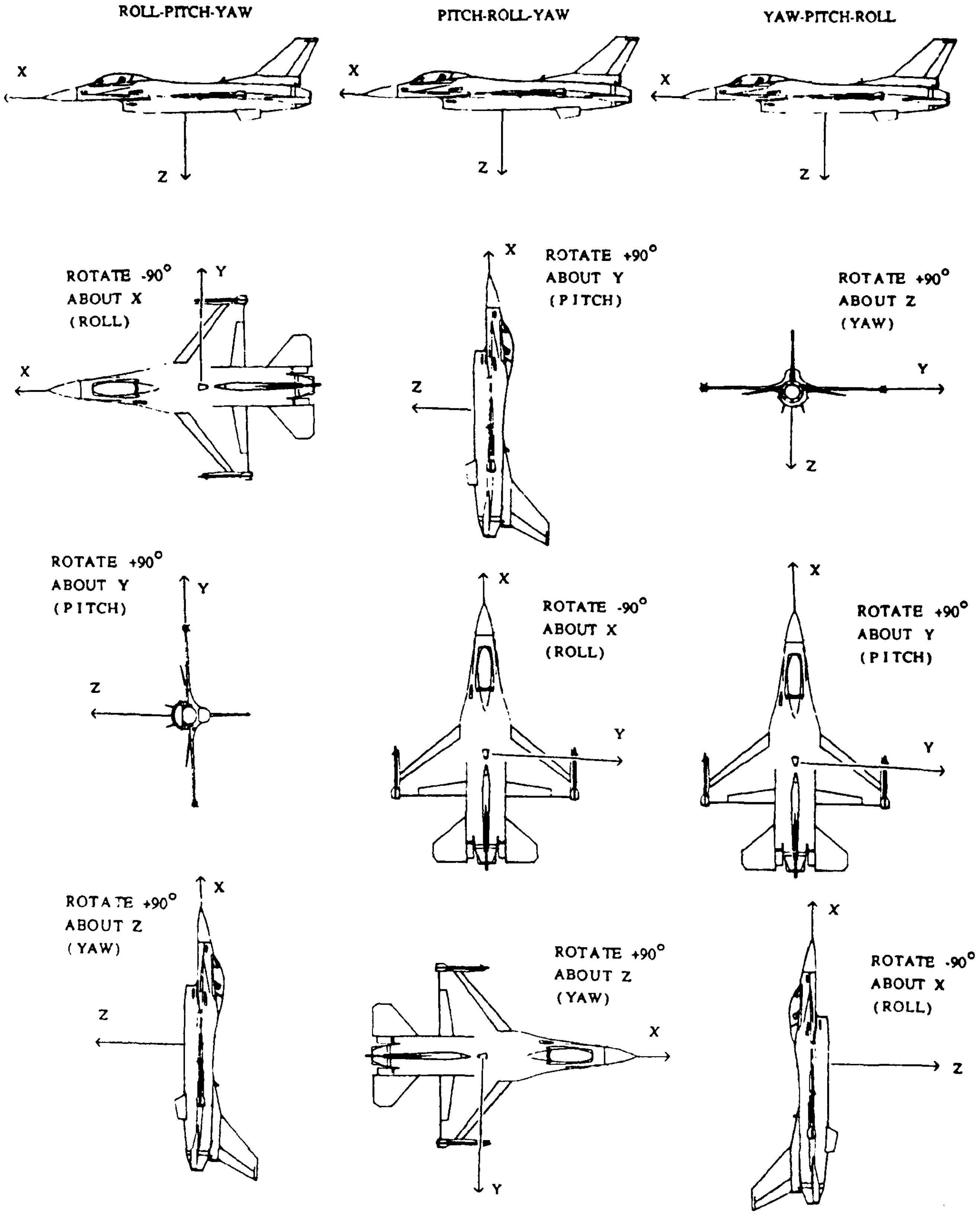

Figure IV-5 Rotation Order

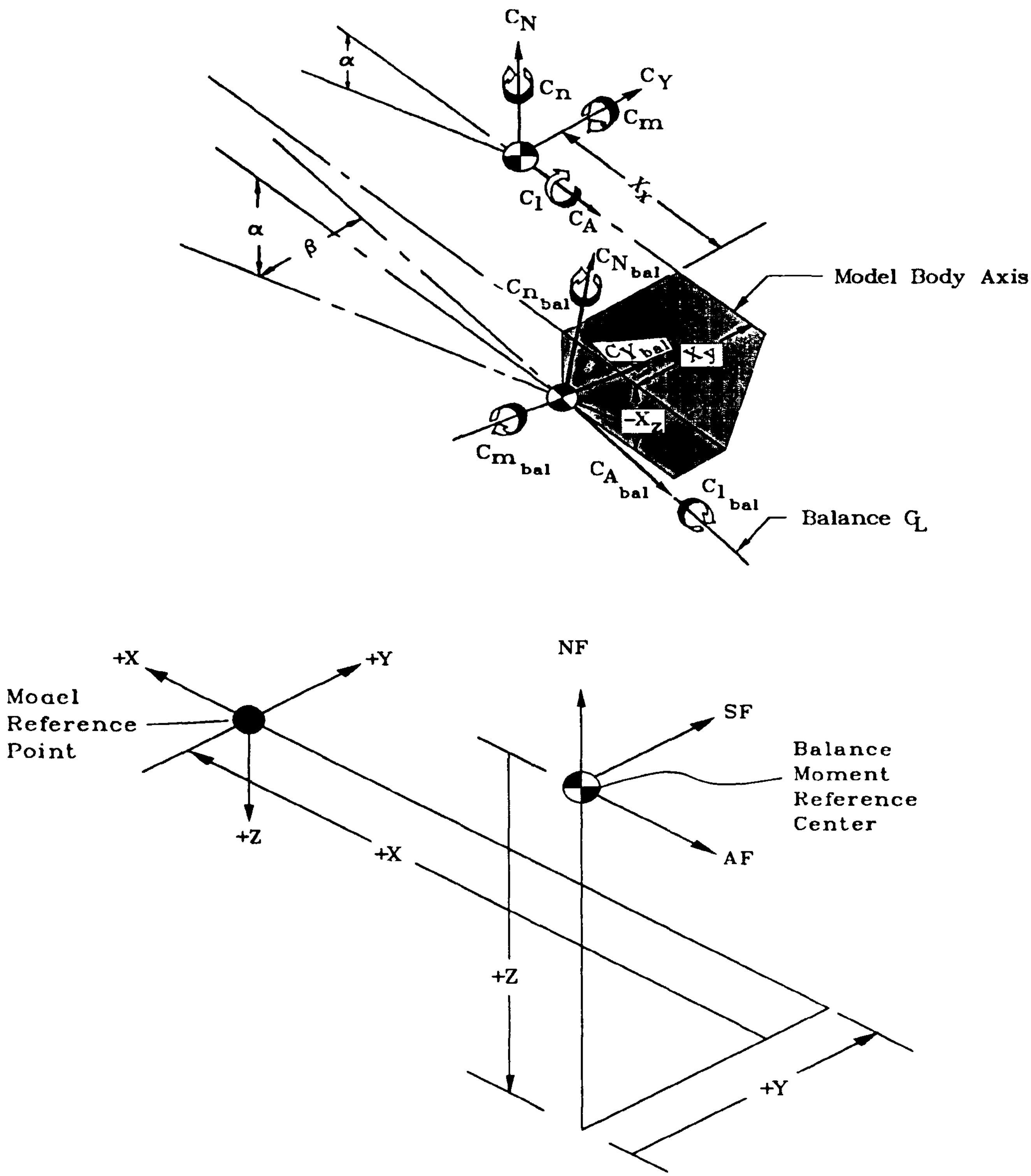

Figure IV-6 Balance Axis

Body Axis to Wind Axis (ref. 4 & 8)

Note: α and φ are aerodynamic angles

$(\bar{c}/B)$ or $(B/\bar{c})$ is used to add moment coefficients together (apples to apples).

Forces

$$C_{L_w} = C_{N_b}\cos\alpha - C_{A_b}\sin\alpha$$

$$C_{D_w} = C_{A_b}\cos\alpha\cos\varphi + C_{Y_b}\sin\varphi + C_{N_b}\sin\alpha\cos\varphi$$

$$C_{Y_w} = C_{Y_b}\cos\varphi - C_{A_b}\cos\alpha\sin\varphi - C_{N_b}\sin\alpha\sin\varphi$$

Forces

$$\begin{bmatrix} C_{L_w} \\ C_{D_w} \\ C_{Y_w} \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha\cos\varphi & \cos\alpha\cos\varphi & \sin\varphi \\ -\sin\alpha\sin\varphi & -\cos\alpha\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} C_{N_b} \\ C_{A_b} \\ C_{S_b} \end{bmatrix}$$

Moments

$$C_{m_w} = C_{m_b}\cos\varphi + (B/\bar{c})C_{l_b}\cos\alpha\sin\varphi + (B/\bar{c})C_{n_b}\sin\alpha\sin\varphi$$

$$C_{l_w} = C_{l_b}\cos\alpha\cos\varphi - (\bar{c}/B)C_{m_b}\sin\varphi + C_{n_b}\sin\alpha\cos\varphi$$

$$C_{n_w} = C_{n_b}\cos\alpha - C_{l_b}\sin\alpha$$

Moment

$$\begin{bmatrix} C_{m_w} \\ C_{l_w} \\ C_{n_w} \end{bmatrix} = \begin{bmatrix} \cos\varphi & (B/\bar{c})\cos\alpha\sin\varphi & (B/\bar{c})\sin\alpha\sin\varphi \\ -(\bar{c}/B)\sin\varphi & \cos\alpha\cos\varphi & \sin\alpha\cos\varphi \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} C_{m_b} \\ C_{l_b} \\ C_{n_b} \end{bmatrix}$$

B = wing span

$\bar{c}$ = M.A.C.

Body Axis to Stability Axis (ref. 4)

Forces

$$C_{L_s} = C_{N_b}\cos\alpha - C_{A_b}\sin\alpha$$

$$C_{D_s} = C_{N_b}\sin\alpha + C_{A_b}\cos\alpha$$

$$C_{Y_s} = C_{Y_b}$$

Forces

$$\begin{bmatrix} C_{L_s} \\ C_{D_s} \\ C_{Y_s} \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{N_b} \\ C_{A_b} \\ C_{S_b} \end{bmatrix}$$

Moments

$$C_{m_s} = C_{m_b}$$

$$C_{l_s} = C_{l_b}\cos\alpha + C_{n_b}\sin\alpha$$

$$C_{n_s} = C_{n_b}\cos\alpha - C_{l_b}\sin\alpha$$

Moments

$$\begin{bmatrix} C_{m_s} \\ C_{l_s} \\ C_{n_s} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} C_{m_b} \\ C_{l_b} \\ C_{n_b} \end{bmatrix}$$

Wind Axis to Stability Axis (ref. 4 & 8)

Forces

$$C_{L_s} = C_{L_w}$$

$$C_{D_s} = C_{D_w}\cos\varphi - C_{Y_w}\sin\varphi$$

$$C_{Y_s} = C_{Y_w}\cos\varphi + C_{D_w}\sin\varphi$$

Force

$$\begin{bmatrix} C_{L_s} \\ C_{D_s} \\ C_{Y_s} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} C_L \\ C_{D_w} \\ C_{Y_w} \end{bmatrix}$$

Moments

$$C_{m_s} = C_{m_w}\cos\varphi - (B/\bar{c})C_{l_w}\sin\varphi$$

$$C_{l_s} = C_{l_w}\cos\varphi + (\bar{c}/B)C_{m_w}\sin\varphi$$

$$C_{n_s} = C_{n_w}$$

Moments

$$\begin{bmatrix} C_{m_s} \\ C_{l_s} \\ C_{n_s} \end{bmatrix} = \begin{bmatrix} \cos\varphi & -(B/\bar{c})\sin\varphi & 0 \\ (\bar{c}/B)\sin\varphi & \cos\varphi & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_{m_w} \\ C_{l_w} \\ C_{n_w} \end{bmatrix}$$

B = wing span

$\bar{c}$ = M.A.C

Since all of the fore mentioned coordinate transformation equations are orthogonal matrices (square matrices), one can obtain the 'other' axis transformation by simply transposing the matrix.

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V TRIP STRIPS

V Trip Strips

Boundary Layer Symbols

R_k	Reynolds number based on roughness height, velocity, and kinematic viscosity at top of roughness ($V_k k / \upsilon_k$)
R_x	Reynolds number based on freestream conditions and distance of roughness from leading edge ($V_\infty x / \upsilon_\infty$)
k	Roughness height
x	Distance of roughness from leading edge
u	Local streamwise velocity component outside the boundary layer
u_k	Local streamwise velocity component inside the boundary layer at the top of the roughness particle
V_∞	Freestream velocity
x	Distance from leading edge to roughness strip (trip strip)
δ	Boundary layer thickness
υ_k	Kinematic viscosity at top of the roughness particle
υ_∞	Freestream kinematic viscosity
T_t	Total temperature (^{o}F)

Boundary Layer Discussion (ref. 7)

Due to the effects of viscosity, the velocity near a surface is gradually slowed from the freestream velocity, V_∞, to zero velocity. The region where this velocity change occurs is called the boundary layer. A boundary layer in which the velocity varies approximately linearly from the surface is called laminar and a boundary layer whose velocity varies approximately exponentially from the surface is called turbulent. Generally speaking, the upper Reynolds number (Rn) limit for a laminar boundary layer is 1 X 10^6 per ft. However, transition from a laminar to a turbulent boundary layer most often occurs at a much lower Reynolds number (5 x 10^5 per ft).

Boundary Layer Thickness

The boundary layer thickness is defined as the distance from the surface to a point where the velocity in the boundary layer is 99% of the velocity of V_∞. The boundary layer thickness can be approximated by

Laminar

$$\delta = 5.2\ (l^2/Rn)^{0.5}$$

Turbulent

$$\delta = 0.37\ l/(Rn)^{0.2}$$

where

l = distance from body leading edge

Rn = Reynolds number = $(\rho V_\infty l\ /\ \mu)$

ρ = air density * 32.1741 (ft/sec^2)

V_∞ = freestream velocity

l = some reference length

μ = absolute viscosity

= (1.2024 X 10^{-5} lbm/ft-sec)

= (3.7373 X 10^{-7} slugs/ft)

Trip Strips (ref. 8)

A trip strip is an artificial roughness added to the model to fix the location of transition from laminar to a turbulent boundary layer. The reason a trip strip is added to a wind tunnel model is to increase the local effective Reynolds number and to "duplicate" the boundary layer to that of a full scale test article. Some general guidelines that are applicable to all grit-type trips are listed below (ref. 25):

1) The roughness bands should be narrow (0.125 to 0.25 inches).

2) The roughness should be sparsely distributed. According to reference 8, approximately one grain per every 2mm (.080 in) along the trip strip is the closest desirable spacing; grains could possibly be spaced as far as 5mm (.200 in) apart and still cause transition. However, if the trip strip is either too high above the surface or is too densely packed with particles, it can affect the model drag, maximum lift and not fix transition.

3) Two-dimensional trips are unsatisfactory because reasonable heights do not fix transition at the trip location.

4) Care should be taken not to build up layers of adhesive which can form spanwise ridges at the edge of the trip. These ridges also tend to make the trip act as a two-dimensional step.

Trip Strip Types (ref. 8)

Grit

The traditional trip strip is a finite width strip of grit. Two commercially available grit materials that are used are Carborundum and Ballotine micro beads or balls. Trip strip width is usually 0.100 to 0.25 inches. Clear lacquer or double sided tape can be used as a gluing agent.

Two-Dimensional Tape

These consist of 0.125 inch height printed circuit drafting tape or chart tape. Also, cellophane type tape can be used. The trip strip is built up by multiple layers of tape.

Epoxy Dots

A vinyl tape of varying thicknesses that has holes of 0.05 inch diameter and 0.10 inch center to center displacement is used. This tape is applied to the model surface and an epoxy compound is forced into the holes. Once the epoxy has harden remove the tape.

Thread or String

Thread or string is glued to the model. This technique is not used much any more.

Location of Trip Strips

Lifting Surfaces

This includes wings, tails, and fins. The trip strip is applied to both sides of the lifting surface. For four and five digit airfoils and conventional wing constructions, the full scale transition will occur approximately 10% of the chord at cruise conditions.

Fuselage

The trip strip is often located where the local diameter is one-half of the maximum diameter. Care should be taken to ensure that the flow has not reattached or that laminar flow has been reestablished aft of the trip strip.

Nacelles

For flow through nacelles, the trip strip is placed inside the nacelle and is located approximately 5% aft of the inlet L.E. Also, additional trip strip is located on the outside at 5-10% aft of the inlet L.E.

From reference 8, testing of models through a range of Reynolds and Mach numbers, it is desirable to eliminate the need for changing grit sizes for each condition. This elimination is accomplished by using a grit (roughness) size determined for the combination of test Reynolds number and Mach number which require the largest grit size - usually at the smallest Reynolds number and largest Mach number condition. For tapered wings (ref. 10), apply grit at a constant percentage of the local chord (nominally 5%). In order to permit the use of a single grit size across the span, the grit size is calculated for the largest chord. For wings with a sharp supersonic leading edge, calculate the grit size using the Mach number and unit Reynolds number based on the flow outside the boundary layer on the upper surface at the maximum test angle of attack. Apply grit to both upper and lower wing surfaces. For sharp subsonic leading-edge wing or round leading-edge wing, whether or not swept behind the Mach line, the freestream Mach number and unit Reynolds number are used to determine the grit size. Using the above criteria, there will be test conditions when grit particles are larger than the minimum required to cause transition. However, careful application of a sparse distribution of grit particles on a narrow strip will minimize the drag contribution of the grit itself.

Determination of Trip Strip Height

Grit height determination is a black art. The testing engineer, having an understanding of boundary layer build-up/profile (at low q, the boundary layer height is larger than at high q), can make a determination of the grit height based on a proven method. Also, don't forget the helpful hints from the the "old guard" that has determined grit height many times previously. The correct method in determining the grit height for establishing boundary layer transition is to accomplish a drag study based on grit height. But reality (money and time) dictates the test and the use of reference 25 is generally adequate. If the test has variable dynamic pressure (q) runs, the proper method is to have a new grit height at each q. However, economics ($) will dictate if a grit study will be accomplished.

Two methods of determining boundary layer and grit heights based on experimental data are presented. The first method is for atmospheric tunnels only. The second method can be used for either atmospheric or pressure tunnels.

Method 1 (Atmospheric tunnels; ref. 11)

This method of determining boundary layer and grit height is for wind tunnels whose test section is roughly at one (1) atmosphere. In other words, there is no control over the total pressure or total temperature and consequently Reynolds number per foot is a function of velocity or Mach number. This method is based upon reference 11 (flat plate) and each figure (figures V-1 thru V-3) is represented by two total temperature curves, T_t= 40°F and T_t= 120°F.

The effect of increasing the temperature is to increase the boundary layer thickness at each Mach number. Also by increasing the Mach number (Reynolds number per foot), the boundary layer thickness decreases at any station (on a flat plate).

It is desirable to prevent the transition point from moving on the model during the test. By applying the correct grit at the correct distance, the boundary layer will transition from laminar to turbulent near the grit grains at a fixed position. The conditions which these grains cause transition and remain fixed at subsonic speed, are generally dependent upon the length Reynolds number, R_x, and the height Reynolds number, R_k.

Reference 25 (using ref. 11 as a reference) points out not to try to cause artificial transition below a length Reynolds number of approximately 100,000. The first restriction is line "A" on figure V-1. This restriction has the practical effect of moving the transition strip aft of the wing leading edge to a fixed dimension rather than a fixed percentage of the chord. Using the lowest Mach number on figure V-1, the minimum distance is approximately one (1) inch. Another restriction based upon the length Reynolds number, R_x, is the location on the surface where transition from laminar to turbulent first starts to occur naturally. Natural transition occurs on a flat plate at an approximate length Reynolds number of 680,000 and the corresponding distance is represented by line "B" on figure V-1. Since transition starts naturally at line "B", it's desirable to place the trip strip ahead of "B" at a given set of test conditions. This action precludes the possibility of the natural transition point moving ahead of the trip strip which could possibly be attributed to an adverse pressure gradient.

A curve of grit height verse Mach number is shown in figure V-3. Also shown is a curve of boundary layer height at a length Reynolds number R_x = 100,000. The critical grit height is shown to be well within the boundary layer at each Mach number. Reference 25 has shown that transition may be achieved by the range of grit size above the minimum. However, when the grit protrudes from the boundary layer, measurable drag is created. Satisfactory artificial transition of boundary layer from laminar to turbulent may be caused at a given Mach number by grit sizes which lie between the two curves in figure V-3.

Shown in figure V-4 is a curve of Grit height verses Carborundum Grit Number. This figure can be used to determine the use of nominal grit sizes that will provide enough grains to cause transition provided the spacing of the individual grains is correct.

In order to satisfy the condition for minimum influence of the particles, some consideration must be given to particle density and to the width of the grit strip. It has been demonstrated experimentally in reference 25 that approximately one grit per every 2mm (.080 in) along the transition line is the closest desirable spacing; grains could possibly be spaced as far as 5mm (.200 in) apart and still cause transition (sublimation chemicals can help determine the transition point). The width of the trip strip should be on the order of 1mm (0.04 in).

The maximum forward location of the trip strip required to cause transition is shown in figure V-2 at two (2) different total temperatures. Figures V-2 and V-3 are used to determine the trip strip grain size and trip location.

Method 2 (Atmospheric or Pressure tunnels)

This method for determining the height of the trip strip originated from reference 10. Figure V-5 is derived from reference 10 and is included in this report.

Below is an example using Method 2 to determine the grit height required to start transition and the corresponding carborundum grit number.

Example

Wing (2-D curves)

X = 0.5 inches (position of grit from LE)

M = 3.0

RN/FT = 4 x 10^6

$$X * \frac{R_\infty / ft}{1 \times 10^6} = X * \frac{4 \text{ X } 10^6}{1 \times 10^6} = 4X$$

$$X * \frac{R_\infty / ft}{1 \times 10^6} = (.5)(4) = 2 \text{ inches}$$

From figure V-5 (2-D, Mach 3.0) at 2.0

$$K * \frac{R_\infty / ft}{1 \times 10^6} = 0.0264$$

$$K * \frac{4 \ X \ 10^6}{1 \ x \ 10^6} = 4K = 0.0264$$

$$\boxed{K \ = \ 0.0066 \ \text{inches}}$$

From figure V-3 or table V-1 at K = 0.0066 a grit of 90 is to be used.

Table V-1

Nominal Grit Size

Grit Number	Nominal Grit Size (in)
10	0.0937
12	0.0787
14	0.0661
16	0.0555
20	0.0469
24	0.0331
30	0.0280
36	0.0232
46	0.0165
54	0.0138
60	0.0117
70	0.0098
80	0.0083
90	0.0070
100	0.0059
120	0.0049
150	0.0041
180	0.0035
220	0.0029
320	0.0017

Application of Grit types of Trip Strips (ref. 8)

Masking tape is typically used to lay out the trip strips (at a certain width apart). Then shellac, lacquer, artist's clear acrylic, or even superhold hair spray is painted or sprayed over the trip strip area. Once the adhesive material has covered the surface, the grit material is dusted or blown on the wet adhesive. Grit usually is difficult to apply to vertical and lower surfaces. To aid in applying grit to those surfaces, a piece of paper or cardboard can be shaped into a 'V' and with skill can be blown onto the surface. If the grit is too densely packed, use a tooth-pick to pick off selected grit particles.

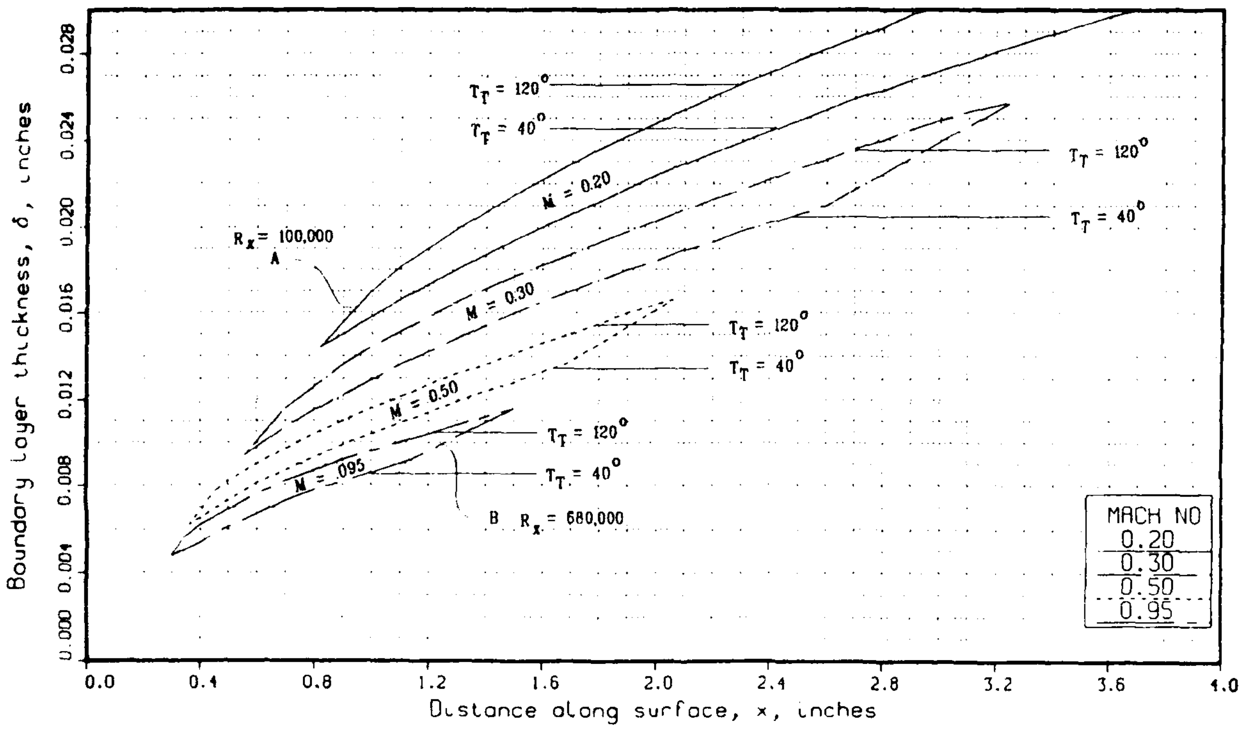


Figure V-1 Boundary Layer Thickness

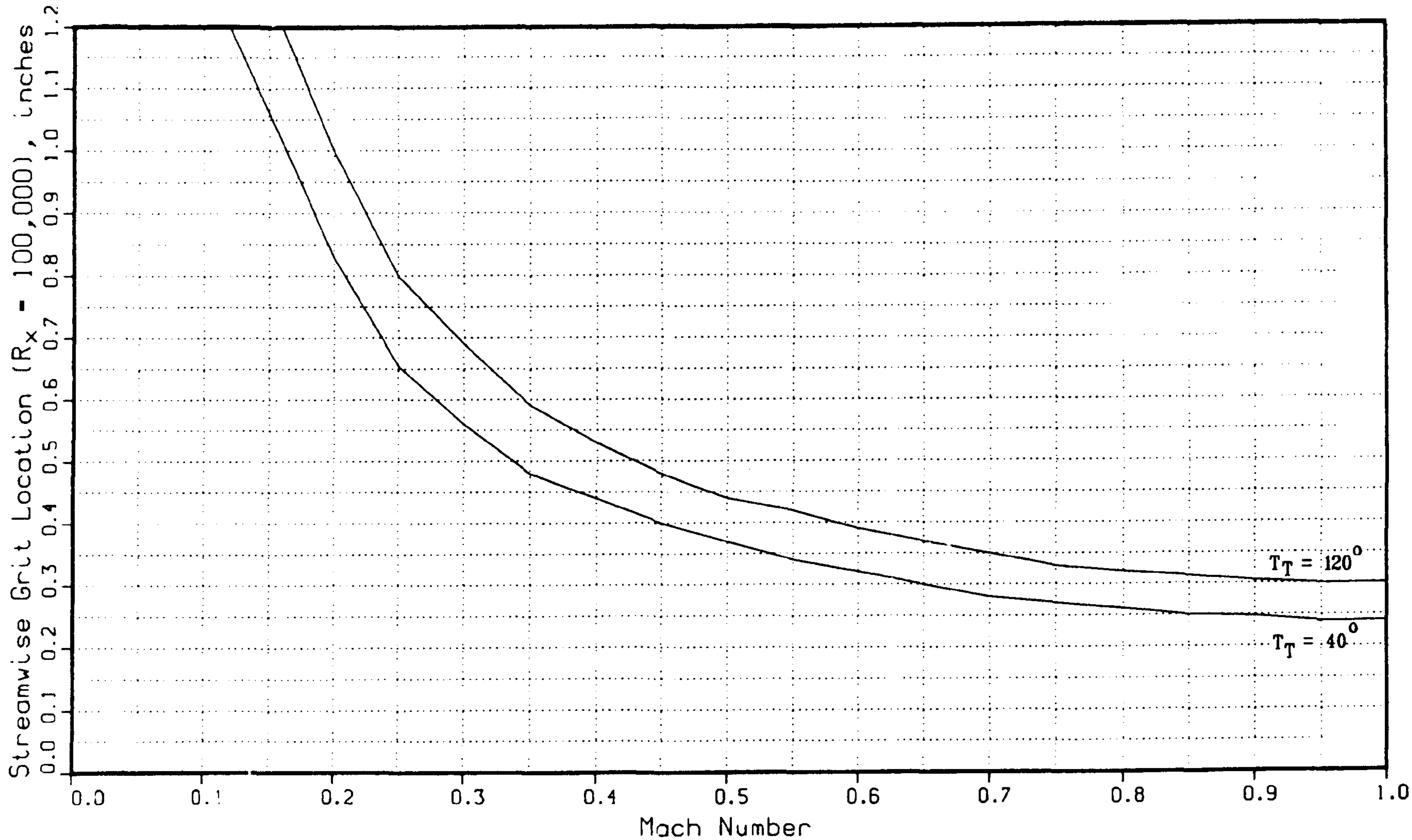


Figure V-2 Streamwise Grit Location

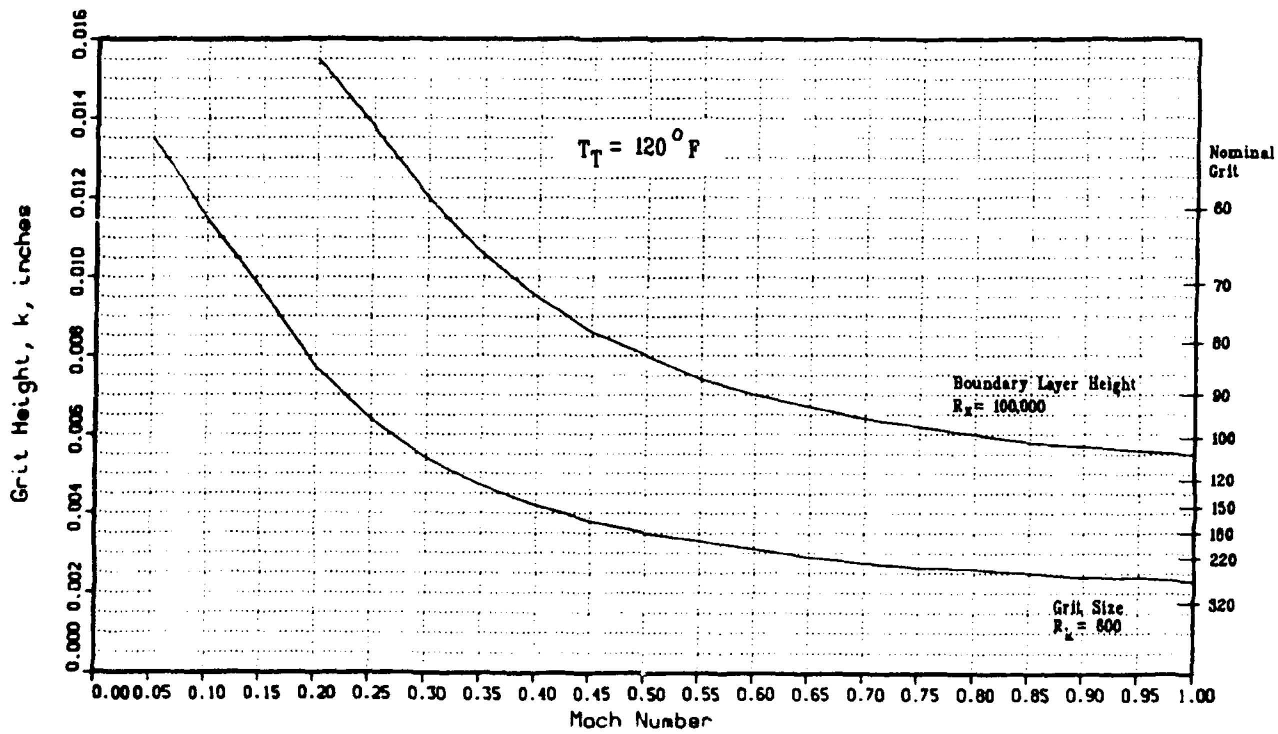


Figure V-3 Grit Height

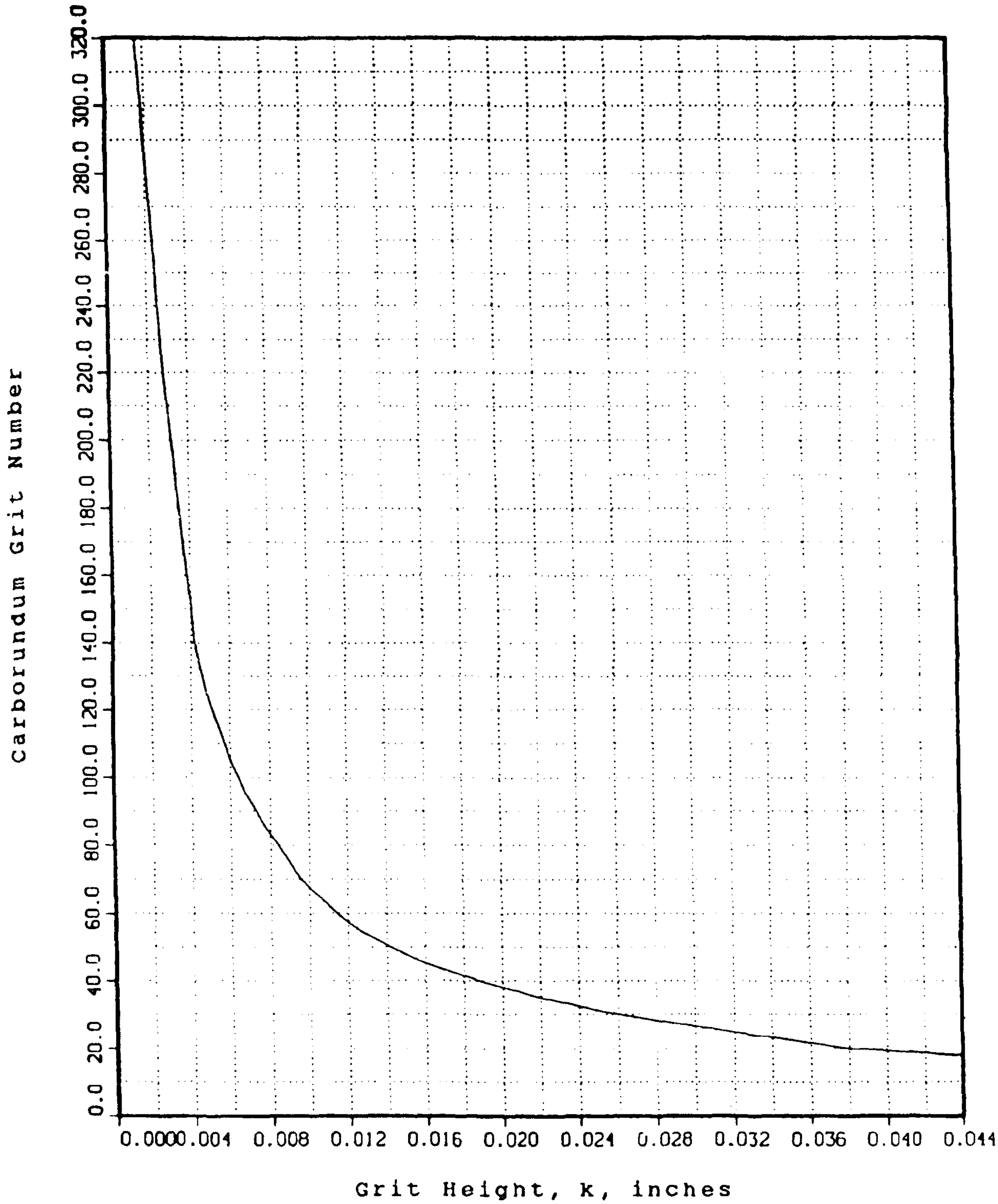


Figure V-4 Carborundum Grit Number

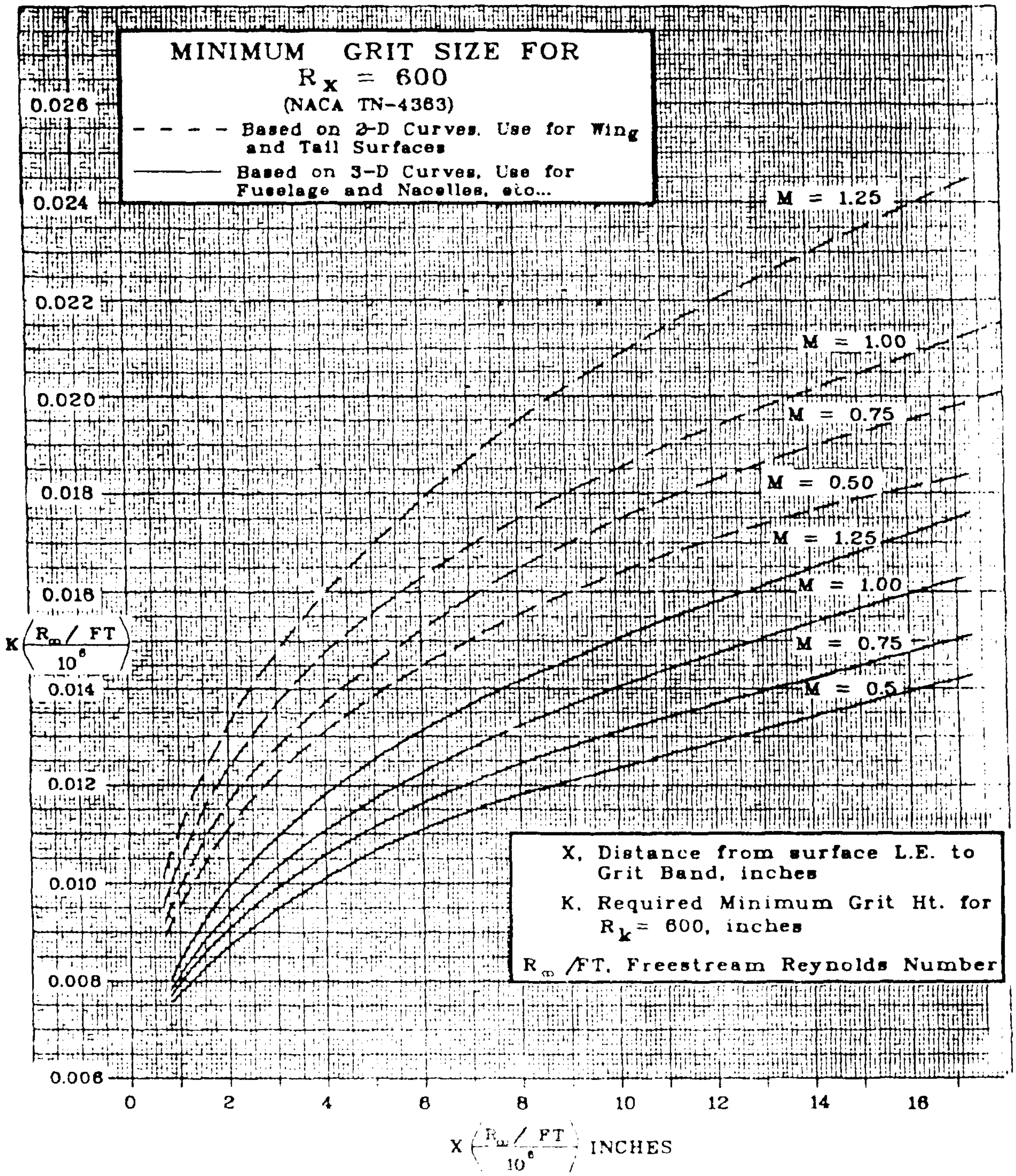


Figure V-5
Minimum Grit Size
(Method 2)

Example

Wing with X = 1.125"

$M_\infty = 0.75$; $R_\infty = 6 \times 10^6$/ft

$$X\left(\frac{R_\infty}{1 \times 10^6}\right) = \left(\frac{6 \times 10^6}{1 \times 10^6}\right) = 6X$$

(1.125)(6) = 6.75 inches

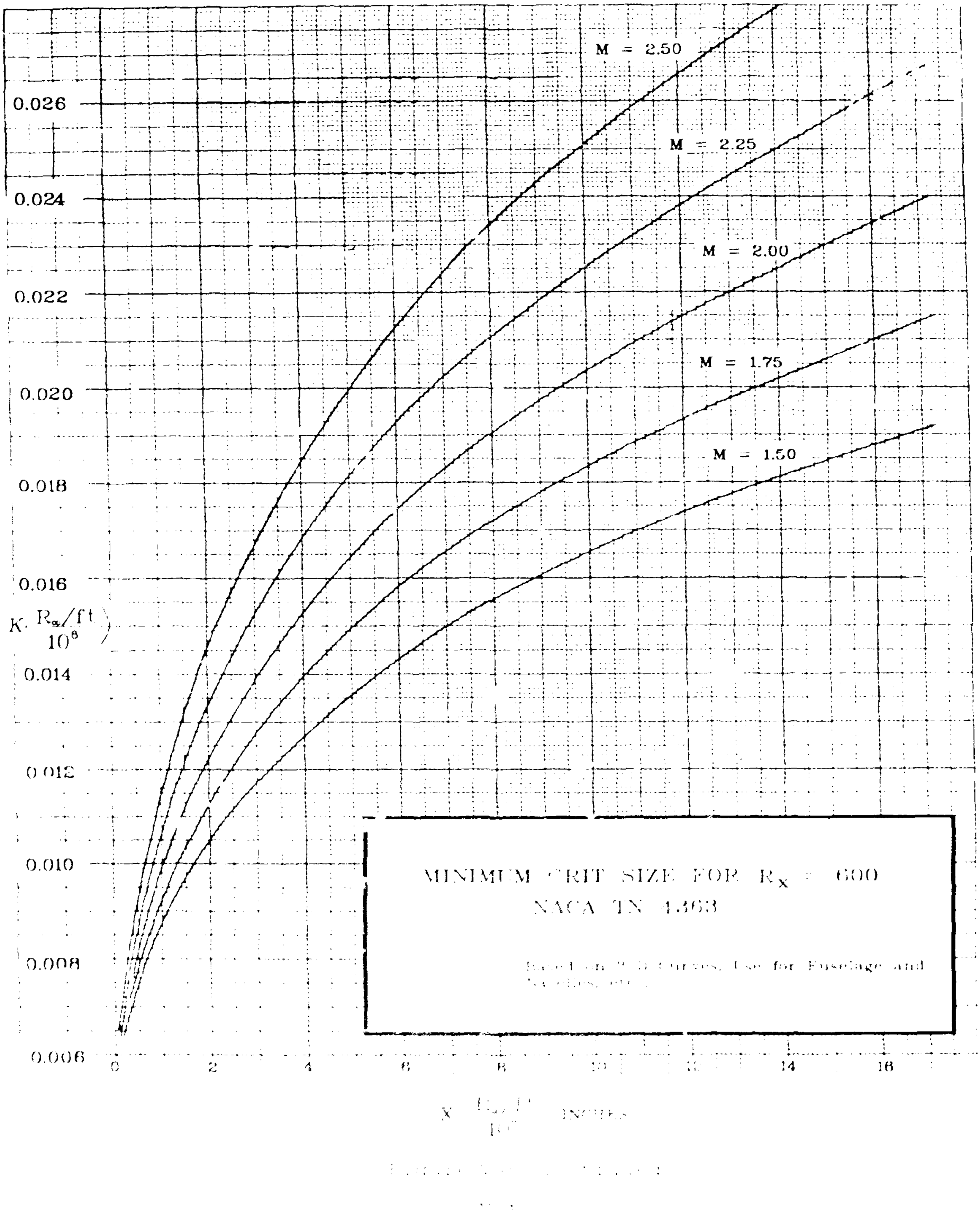
From above chart at 6.75

$M_\infty = 0.75$; 2D

$$K\left(\frac{R_\infty/ft}{10^6}\right) = 0.0157$$

6K = 0.0157

$\boxed{K = 0.0026"}$



M = 2.50
M = 2.25
M = 2.00
M = 1.75
M = 1.50
0.026
0.024
0.022
0.020
0.018
0.016
K (Rₓ/ft / 10⁶)
0.014
0.012
0.010
0.008
0.006
0
2
4
6
8
10
12
14
16
MINIMUM GRIT SIZE FOR Rₓ = 600
NACA TN 4363
X INCHES

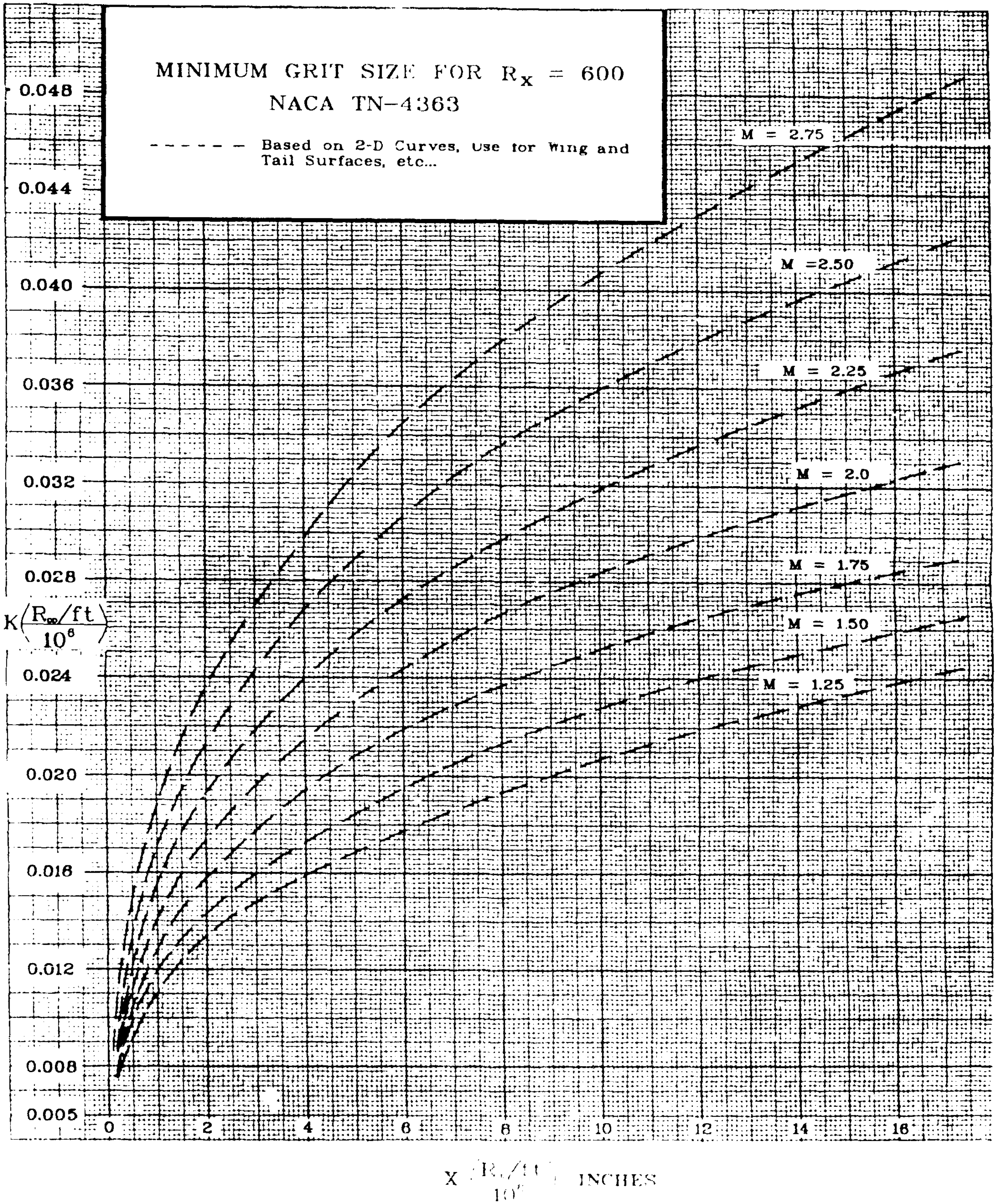


Figure V-5 continued

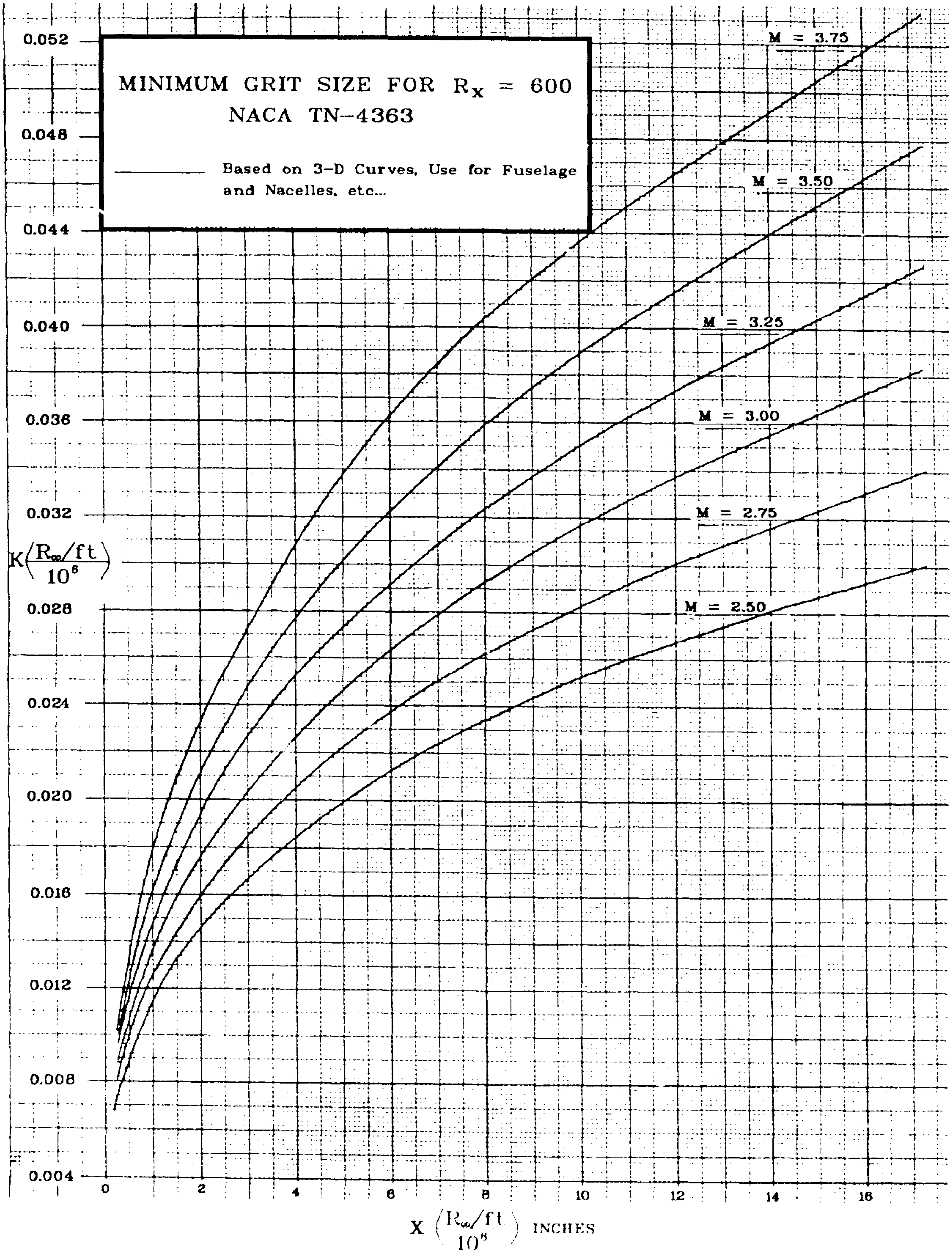


Figure V-5 continued

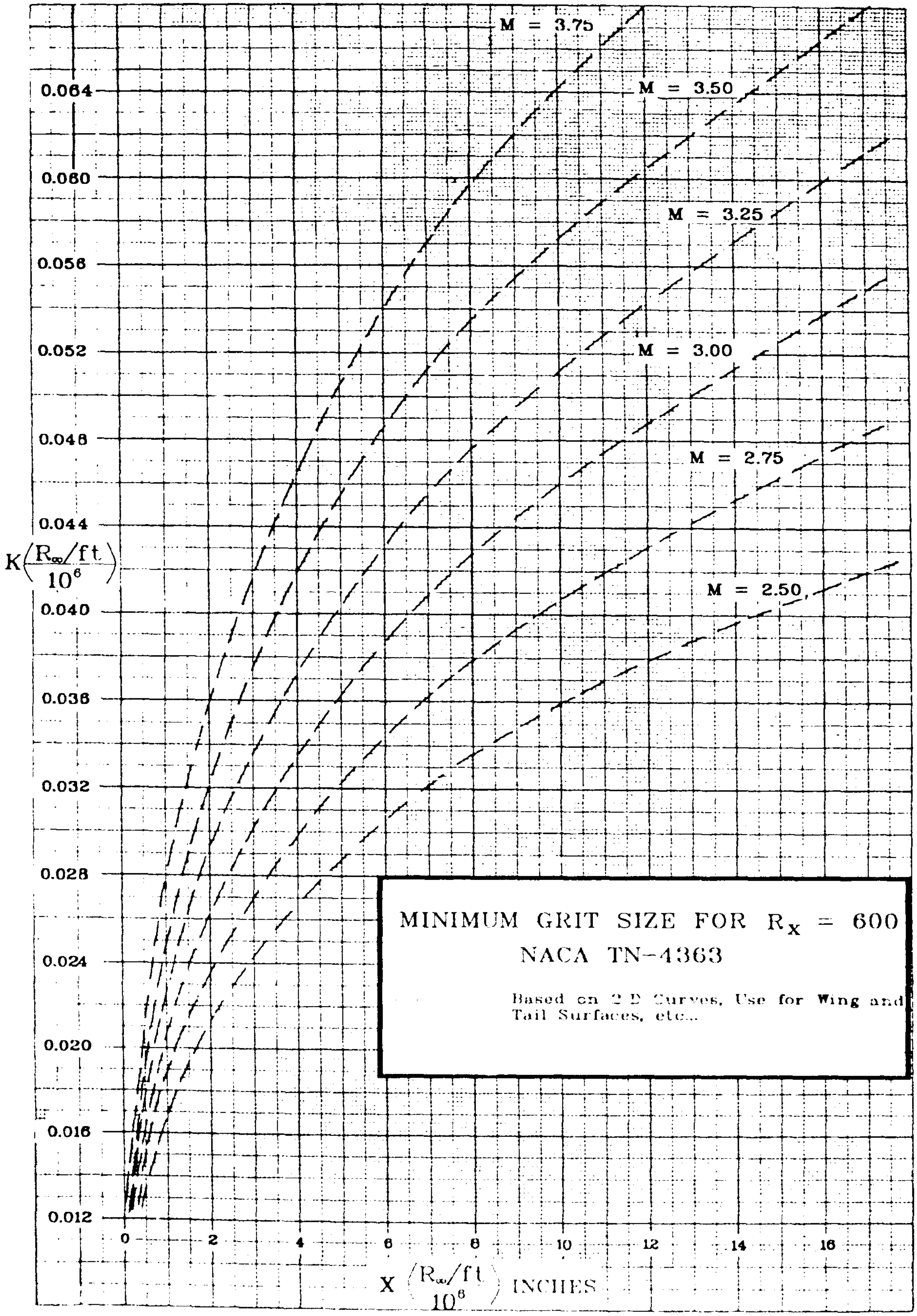


Figure V-5 concluded

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VI PLANFORM CHARACTERISTICS

VI Planform Characteristics

Planform Symbols

AR	Aspect ratio
a	Cutout factor
b	Full wing span
b/2	Half wing span
b/(2l)	Wing-slenderness parameter
b_i	Span of inboard planform formed by two panels
b_o	Span of outboard planform formed by two panels
c	Chord (parallel to axis of symmetry) at any given span station y
$\bar{c}$	Mean Aerodynamic Chord (MAC)
c_B	Chord at span break station
c_r or C_r	Root chord
c_t or C_t	Tip chord
l	Over-all length from wing apex to most aft point on the trailing edge
m,n	Non-dimensional chordwise stations in terms of c
p	Planform-shape parameter
S	Wing area
S_i	Total area of inboard panels
S_o	Total area of outboard panels
S_W	Wing area affected by trailing edge deflection
ΔS	Incremental wing area
x	Chordwise location of leading edge at span station y
$x_{centroid}$	Chordwise location of centroid of area (chordwise distance from apex to c/2)
x_{MAC} or $\bar{x}$	Chordwise location of mean aerodynamic chord
y_{MAC} or $\bar{y}$	Spanwise location of MAC (equivalent to spanwise location of centroid of area)
λ	Taper ratio
Λ_{LE}	Sweep angle of leading edge
Λ_{TE}	Sweep angle of trailing edge
Λ_n, Λ_m	Sweep angles of arbitrary non-dimensional chordwise locations
η	Non-dimensional span station

α	Angle-of-attack
η_i, η_o	Non-dimensional span stations at inboard and outboard edges of control, respectively
η_B	Non-dimensional spanwise location of leading edge break in wing
σ	Ratio of chordwise position of leading edge at tip to root chord length
ζ_B	Chordwise location of break in leading edge sweep(s) in terms of chordwise distance to leading edge at tip = x_B/x_t

Subscripts

B	Refers to span station were leading/trailing edge change sweep angle
MAC	Mean aerodynamic chord
i,o	Inboard and outboard panels respectively
ZL	Zero lift

Wing Parameters Definitions (figure VI-1)

Chord Line — A straight line between the leading edge and the trailing edge of the airfoil in the streamwise direction

Mean Camber Line — A line described by the points which are equidistant from the upper and lower wing surfaces.

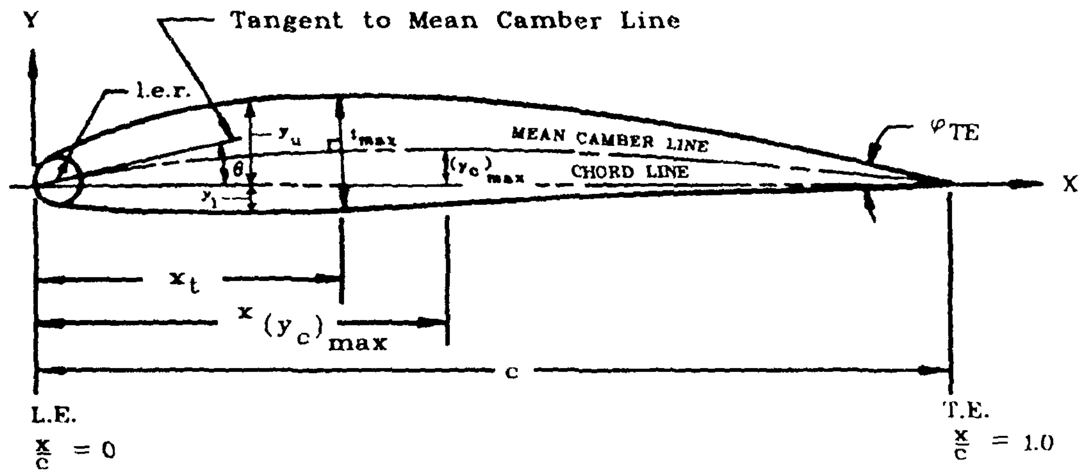
Camber — Is a measure of curvature of an airfoil and is measured as the maximum distance between the mean camber line and the chord line and is measured perpendicular to the chord line. Camber is typically measured in % chord. Positive camber is when the camber line is above the chord line and negative when the camber line is below the chord line.

Angle-of-Attack — Angle between the relative wind (V_∞) and the chord line.

Zero Lift Line — A line parallel to the relative wind (V_∞) and passing through the trailing edge of the airfoil when the airfoil is at zero lift.

Effective Angle of Attack — The angle of attack of the zero lift line measured from the the relative relative wind.

α_{ZL} — Approximately equal to the amount of camber in percent chord for airfoils and untwisted wings with constant section.



X = distance along the chord measured from L.E.

Y = ordinate at some X

c = chord

l.e.r. = leading-edge radius

$x_{(y_c)_{max}}$ = position of maximum camber

$(y_c)_{max}$ = maximum ordinate of mean camber line

t_{max} = maximum thickness

y_u = ordinate of upper surface

y_l = ordinate lower surface

$\frac{x}{c}$ = non-dimensionalized chord length

θ = slope of l.e.r

φ_{TE} = trailing-edge angle

x_t = position of maximum thickness

(reference 12)

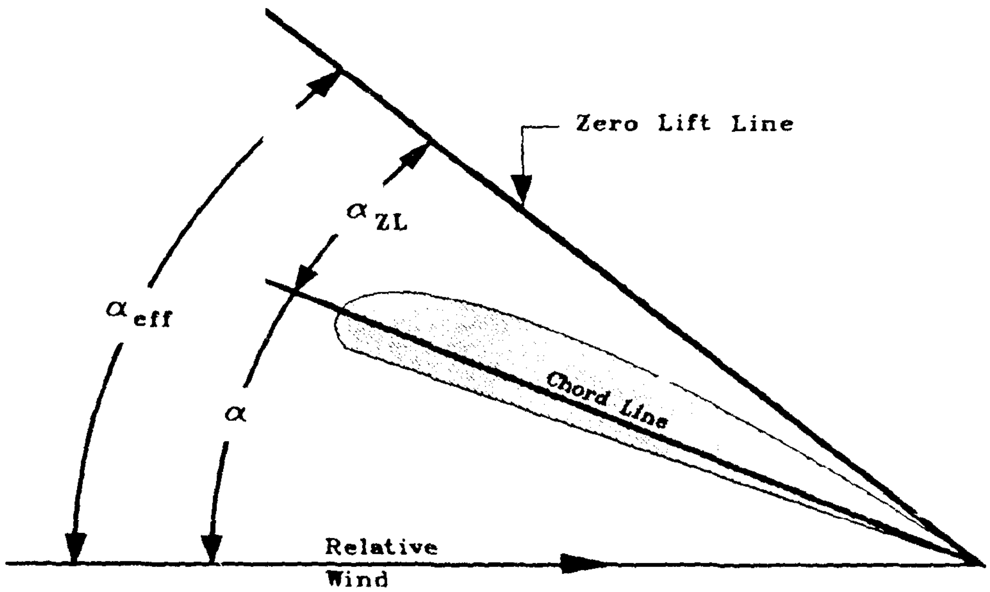


Figure VI-1 Airfoil Nomenclature

Planform Parameters

General Planforms

(ref. 12 & 14)

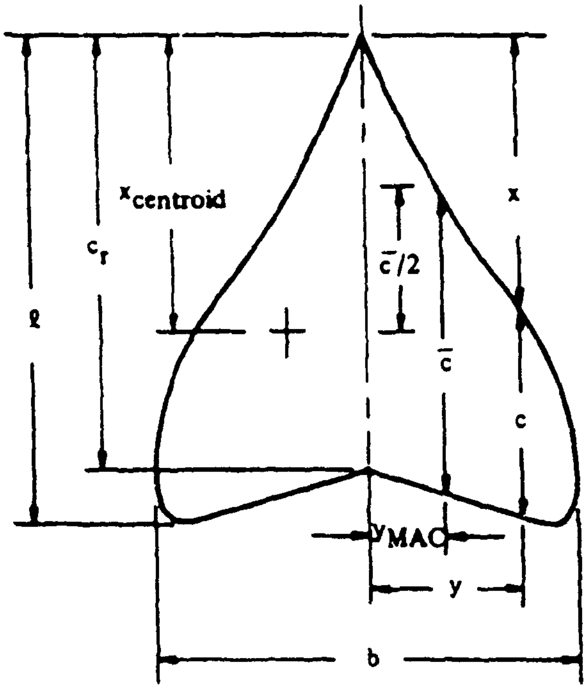


Figure VI-2 General Planform Parameters

$$S = 2 \int_0^{b/2} c \, dy \qquad \bar{c} = \frac{2}{S} \int_0^{b/2} c^2 dy$$

$$\bar{x} = \frac{2}{S} \int_0^{b/2} cx \, dx \qquad \bar{y} = \frac{2}{S} \int_0^{b/2} cy \, dy$$

$$\bar{x}_{LE} = \bar{x} - \frac{\bar{c}}{2} \qquad x_{centroid} = \frac{2}{S} \int_0^{b/2} c(x + \frac{c}{2}) \, dy$$

$$\eta = \frac{(2y)}{B}; \; p = \frac{S}{(bl)}; \; \lambda = C_t / C_r \qquad \left[\frac{t}{c}\right]_{RMS} = \sqrt{\frac{1}{(b/2) - C_r} \int_{C_r}^{b/2} \left[\frac{t}{c}\right]^2 dy}$$

Conventional, Straight-Tapered

(ref. 12 & 14)

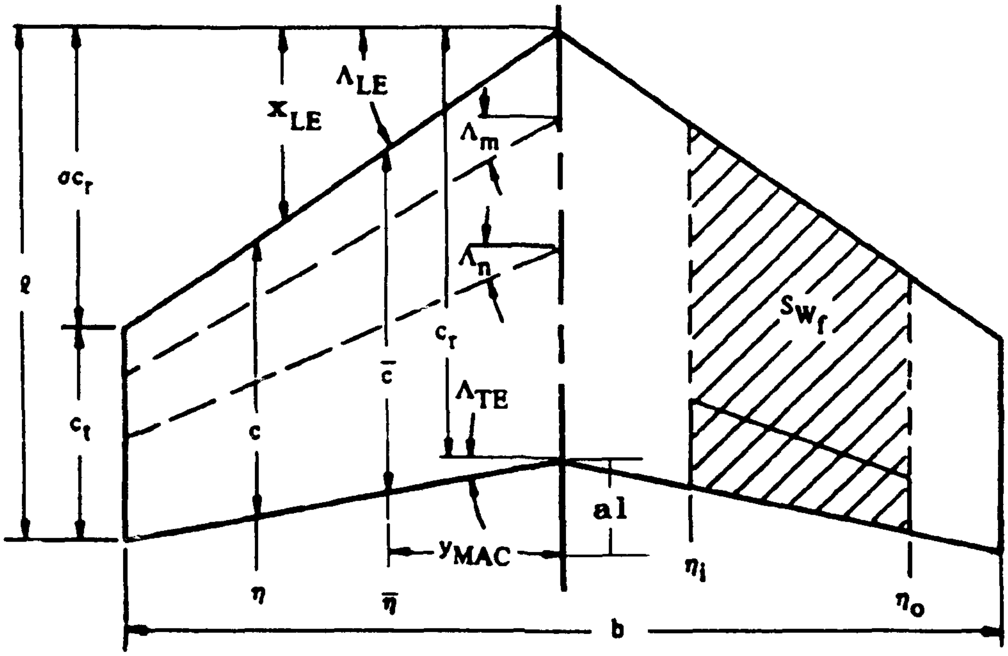


Figure VI-3 Conventional, Straight Tapered Planform

$$AR = \frac{b^2}{S} = \frac{2b}{C_r(1 + \lambda)} = \frac{4(1 - \lambda)}{(1 - a)(1 + \lambda)\tan\Lambda_{LE}}$$

$$S = (b/2)C_r(1 + \lambda) = \frac{b^2}{AR}$$

$$\Delta S = \frac{b}{2} C_r[2-(1-\lambda)(\eta_1+\eta_2)]$$

$$\bar{c} = \frac{2}{3} C_r \frac{1 + \lambda + \lambda^2}{1 + \lambda}$$

$$\bar{y} = \frac{b}{2}\left[\frac{1}{3}\left[\frac{1 + 2\lambda}{1 + \lambda}\right]\right]$$

$$\bar{x}_{LE} = \left[\frac{b}{2C_r}\right]\bar{\eta}\tan\Lambda_{LE} = \bar{y}\tan\Lambda_{LE}$$

$$\bar{\eta} = (2\bar{y})/b = \frac{1}{3}\left[\frac{1 + 2\lambda}{1 + \lambda}\right]$$

$$x_{centroid} = \frac{C_r}{3}\left[\lambda + \sigma + \left[\frac{1 + \lambda\sigma}{1 + \lambda}\right]\right]$$

$$c = C_r[1 - \eta(1 - \lambda)] = C_r - y\left[\frac{C_r - C_t}{(b/2)}\right]$$

$$C_r = \frac{S}{(b/2)(1+\lambda)} = \frac{4(b/2)}{AR(1+\lambda)}$$

$$S_{W_f} = b\,\frac{[\eta_o - \eta_i]}{2}\,C_r[2 - (1 - \lambda)(\eta_o + \eta_i)]$$

$$a = \frac{\tan\Lambda_{TE}}{\tan\Lambda_{LE}} = 1 - \frac{C_r(1 - \lambda)}{(b/2)\tan\Lambda_{LE}} = 1 - \frac{4(1 - \lambda)}{AR(1 + \lambda)\tan\Lambda_{LE}}$$

$$\eta = (2y)/b$$

$$\tan\Lambda_n = \tan\Lambda_m - \frac{4}{AR}\left[(n-m)\left[\frac{1 - \lambda}{1 + \lambda}\right]\right] \quad n > m$$

$$\tan\Lambda_m = \tan\Lambda_{LE}[1-(1-a)m]$$

$$\tan\Lambda_{LE} = \frac{1}{a}\tan\Lambda_{TE} = \frac{4\tan\Lambda_{c/4}}{3 + a}$$

$$\tan\Lambda_{TE} = \frac{4}{AR} - \tan\Lambda_{TE} \quad (\lambda = 0)$$

$$\sigma = \frac{AR}{4}(1 + \lambda)\tan\Lambda_{LE} = \left[\frac{b}{2C_r}\right]\tan\Lambda_{LE}$$

Double Delta

and

Cranked Wings

(ref. 12 & 14)

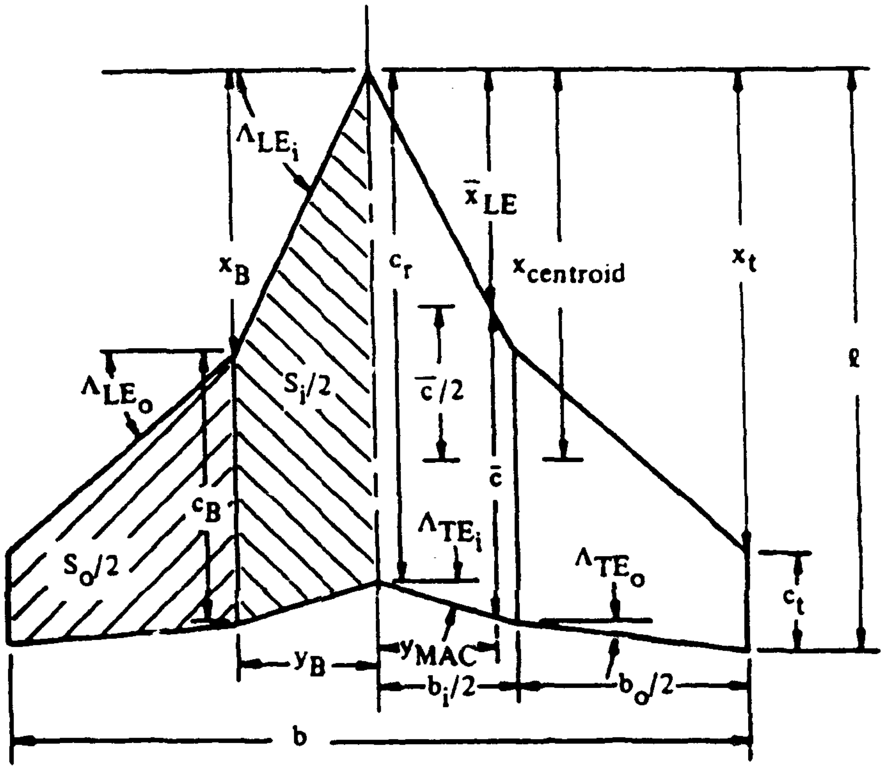


Figure VI-4 Double Delta and Cranked Wing Planform

$$AR = \frac{b^2}{S} = \frac{2b}{C_r[(1 - \lambda)\eta_B + \lambda_i + \lambda]}$$

$$S = S_i + S_o = \frac{b^2}{AR} = (b/2)C_r[(1 - \lambda)\eta_B + \lambda_i + \lambda]$$

$$\bar{c} = \frac{\bar{c}_i S_i + \bar{c}_o S_o}{S_i + S_o}$$

$$\bar{x} = \bar{x}_{LE} + \bar{c}/2$$

$$\bar{y} = \frac{\bar{y}_i S_i + [y_B + \bar{y}_o] S_o}{S_i + S_o}$$

$$\bar{x}_{LE} = \frac{\left[\bar{y}_i \tan\Lambda_{LE_i}\right] S_i + \left[y_B \tan\Lambda_{LE_i} + \bar{y}_o \tan\Lambda_{LE_o}\right] S_o}{S_i + S_o}$$

$$\bar{\eta} = \frac{\bar{y}}{b/2} = \frac{b_i \bar{\eta}_i S_i + \left[b_i + b_o \bar{\eta}_o\right] S_o}{b[S_i + S_o]}$$

$$\eta_B = \frac{b_i}{b} = \left[\frac{1}{1-\lambda}\right]\left[\frac{2S}{bC_r} - \lambda_i - \lambda\right]$$

$$\zeta_B = \frac{1}{1 + \dfrac{b_o \tan\Lambda_{LE_o}}{b_i \tan\Lambda_{LE_i}}}$$

$$\lambda = C_t/C_r = \lambda_i \lambda_o$$

$$\lambda_i = c_B/C_r$$

$$\lambda_o = C_t/c_B$$

Planform Example

This is an example using the equations for a cranked wing to determine the MAC, c/4, and other items of interest.

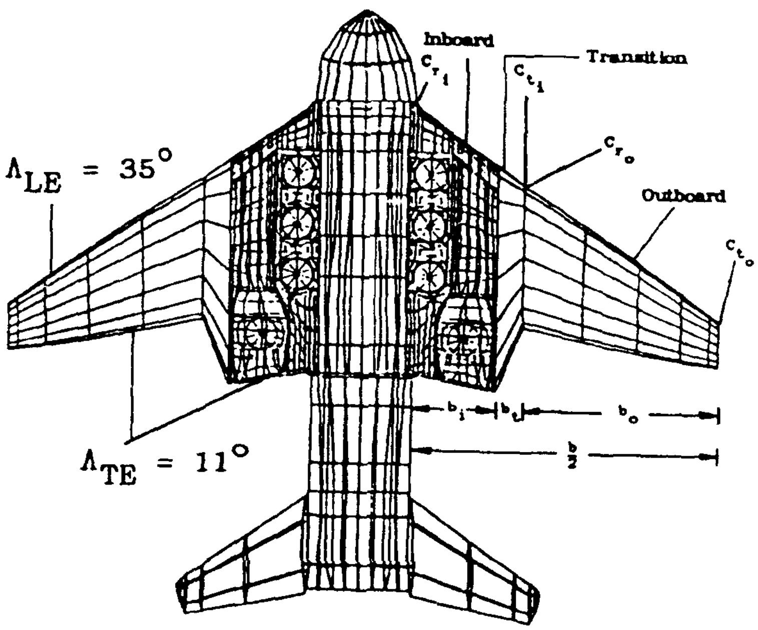


Figure VI-5 Planform Example

Inboard Section (i)

$$C_{r_i} = 50.45 \text{ in}$$

$$C_{t_i} = 36.51 \text{ in}$$

$$\lambda_i = C_{t_i} / C_{r_i} = 36.51/50.45 = 0.724$$

$$b/2 = 26.96 \text{ in}$$

$$S_i/2 = \frac{(50.45 + 36.51)}{2}(26.96) = 1172.2 \text{ in}^2 = 8.14 \text{ ft}^2$$

$$\bar{c}_i = \frac{2}{3}(50.45)(1.30) = 43.86 \text{ in}$$

$$\bar{y}_i = \left[\frac{b_i}{2}\right]\left[\frac{1}{3}\right]\left[\frac{1 + 2\lambda_i}{1 + \lambda_i}\right] = (26.96)(0.473) = 12.76 \text{ in}$$

$$\bar{x}_i = \left[\frac{b_i}{2}\right]\left[\frac{1}{3}\right]\left[\frac{1 + 2\lambda_i}{1 + \lambda_i}\right]\tan\Lambda_{LE} = (12.76)\tan 35$$

$$= 8.93 \text{ in (from LE of } C_{r_i})$$

Outboard Section (o)

$$C_{r_o} = 24.58 \text{ in}$$

$$C_{t_o} = 7.51 \text{ in}$$

$$\lambda_o = 7.51 / 24.58 = 0.306$$

$$b_o/2 = 33.15 \text{ in}$$

$$S_o = \frac{24.58 + 7.51}{2}(33.15) = 531.89 \text{ in}^2 = 3.69 \text{ ft}^2$$

$$\bar{c}_o = \frac{2}{3}(24.58)(1.072) = 17.56 \text{ in}$$

Measured from LE of C_{r_o}

$$\bar{y}_o = \left[\frac{b_o}{2}\right]\left[\frac{1}{3}\right]\left[\frac{1 + 2\lambda_o}{1 + \lambda_o}\right] = (33.15)(.4114) = 13.64 \text{ in}$$

$$\bar{x}_o = \bar{y}_o \tan 35 = (13.64)(0.7) = 9.55 \text{ in}$$

Measured from LE of C_{r_i}

$$\bar{y}_o = 13.64 + 26.96 = 40.6 \text{ in}$$

$$\bar{x}_o = \bar{y}_o \tan 35 = (40.6)(0.7) = 28.42 \text{ in}$$

Total Wing

$$S = 8.14 + 3.69 = 11.83 \text{ ft}^2$$

$S_i/S = 0.688$ $\quad\quad$ $S_o/S = 0.312$

$\bar{y}$ = 12.76(0.688) + 40.6(0.312) = 21.45 in (1.79 ft)

$\bar{c}$ = 43.86(0.688) + 17.56(0.312) = 35.65 in (2.97 ft)

Total Aircraft Aerodynamic Center Location

Inboard Section

$\bar{x}_{\bar{c}/4} = \bar{x}_i + \bar{c}_{c/4}$ = 8.93 + (43.86)/4 = 19.9 in

$\bar{y}_{\bar{c}/4}$ = 12.76 in (measured from C_{r_i})

Outboard Section

$\bar{x}_{\bar{c}/4} = \bar{x}_o + b_i \tan\Lambda_{LE} + \bar{c}_o$

= 26.96 tan 35 + 9.55 + (17.56)/4 = 32.82 in

$\bar{y}_{\bar{c}/4}$ = 13.64 + 26.96 = 40.6 in

Total Aircraft

Measured from LE of C_{r_i}

$x_{\bar{c}/4}$ = 19.9(0.688) + 32.82(0.312) = 23.93 in

Measured from C_{r_i}

$\bar{y}_{\bar{c}/4}$ = 21.146 in

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VII DRAG

VII Drag

List of Symbols

AR	Aspect Ratio
C_D	Total Drag Coefficient
C_{D_o}	Zero Lift Drag Coefficient (parasite drag)
$C_{D_{min}}$	Minimum Drag Coefficient
ΔC_D	Minimum Drag Coefficient change due to camber
C_D^*	Drag Coefficient at $(L/D)_{max}$
C_L	Total Lift Coefficient
C_L'	Lift Coefficient at minimum drag
$C_{L_{pb}}$	Lift Coefficient at the Polar Break
C_L^*	Lift Coefficient at $(L/D)_{max}$
e	Span efficiency of actual aircraft drag polar
e_p	Span efficiency of a parabolic drag polar
g	Gravitational constant
K	Induced Drag factor
$(L/D)_{max}$	Maximum lift-to-drag ratio
Rn	Reynolds number
V_∞	Freestream velocity

Drag (ref. 13)

Drag is a part of a resultant aerodynamic force produced by the tangential (skin friction) and normal (pressure) forces acting along the vehicle's surface due to the relative fluid motion. This resultant force is resolved into the lift and drag components in the vehicle's plane of symmetry. Drag, the component of the total force that opposes motion in the equilibrium flight path direction, approximately follows a parabolic variation with lift and the angle of attack. Total drag can be resolved into various components (figure VII-1).

Subsonic Drag

The aircraft drag, when taken below the divergent Mach number, is traditionally decomposed into lift-induced and minimum drag.

Minimum Drag

Minimum drag can be divided into profile (skin friction, pressure, and base drag) and interference drag (figure VII-1). In general, about two-thirds of subsonic minimum drag may be attributed to profile (skin friction) drag (figure VII-2).

Profile Drag/Skin Friction Drag (ref. 7)

The skin friction drag is due to the momentum transfer between the fluid particles adjacent to the vehicle surface and the vehicle. It (in incompressible flow) can be established for a laminar and turbulent boundary layer. This drag is based upon the "wetted" surface area of a flat plate, on ONE surface.

Laminar

$$C_{D_{lam}} = D_{lam} / (q\ S_{wet}) = 1.328 / (Rn)^{0.5}$$

$$1 \times 10^3 > Rn < 1 \times 10^6$$

Turbulent

$$C_{D_{turb}} = D_{turb} / (q\ S_{wet}) = 0.455 / (\log\ Rn)^{2.58}$$

$$1 \times 10^6 > Rn < 1 \times 10^9$$

Turbulent

$$C_{D_{cone}} = \frac{2}{\sqrt{3}} * C_{D_{flat\ plate}}$$

Pressure Drag (Form Drag)

Pressure Drag at subsonic speeds is due to boundary layer displacement effects and separation effects on aft-facing slopes.

Interference Drag

This drag is the result of mutual interaction of the flow fields developed by the major configuration components (ie...wing-body).

Miscellaneous Drag

Miscellaneous drag is caused by aircraft protuberances and surface irregularities (ie... gaps, fasteners).

Drag Due to Lift

Drag due to lift is almost entirely the result of the lift-produced circulation as well as the vortex shedding from the wing tip.

Zero Lift Drag

Zero lift drag, C_{D_o}, is the drag at $C_L = 0$ and can be easily found on a drag polar curve. The primary contributor to this drag is from skin friction. From figure X-6, the effect of wing sweep and Mach number on zero lift drag (C_{D_o}) can be seen.

Base Drag

Base drag contribution to minimum drag is caused by a rapid expansion of the flow into a base region which causes significantly reduced pressures to act upon a finite base (area).

Internal Duct Drag

This drag is associated with a momentum loss in a flow through nacelle. This drag is measured by having a nozzle pitot static pressure rake (assume inlet conditions are at freestream conditions) measuring the momentum loss within the nacelle (duct). Once a $\Delta C_{D_{duct}}$ is obtained, subtract it out of the total aircraft C_D.

Wave Drag

Wave drag is primarily due to the lack of pressure recovery on the surface due to a total pressure loss through the shock wave.

Drag Polar (Subsonic)

A drag polar can be represented by two curve types, C_L vs C_D or C_D vs C_L^2. Both types reveal aerodynamic parameters that are important to the performance of an aircraft configuration. These two drag polars can be seen in figures VII-2 through VII-5. Figures VII-2 & VII-3 display some of the major contributors to the overall total drag.

(C_L vs C_D) Polar

This drag polar, as seen in figures VII-2, VII-3, VII-4, is parabolic (a quadratic function) by nature. The eccentricity of the parabola and its origin is affected mainly by wing camber, twist, and flow separation. A few performance parameters that can be established by this curve are $C_{D_{min}}$, C_{D_o}, $(L/D)_{max}$, C_L, $C_{L_{pb}}$. A drag polar has the characteristic equations listed below.

Parabolic

$$C_D = C_{D_{min}} + \frac{C_L^{\,2}}{\pi ARe_p}$$

NON-Parabolic (camber, twist, etc.. effects)

$$C_D = C_{D_{min}} + \frac{(C_L - C_L')^2}{\pi ARe}$$

(C_D vs $C_L^{\,2}$) Polar

From figure VII-5, the slope of this line is K, the induced drag factor. And from K, Oswald's wing efficiency factor, e, can be obtained. Also, from figure VII-5's cutout, the intersection of the C_D axis and the curve reveals C_{D_o}.

Polar Break (ref. 14)
(Subsonic)

When the wing leading edge suction has lost its force (separation has occurred), the polar shape departs drastically from the typical parabolic shape. The drag polar is said to "break" at this point (figure VII-2). The break point is sensitive to Mach number, Reynolds number, leading edge radius, and wing geometry.

Camber Effects (ref. 14)

Camber (and twist) essentially shifts the drag polar. The effect of camber (and twist) can be seen in $\Delta C_{D_{min}}$ resulting in a higher C_{D_o} and $C_{D_{min}}$. The parabolic extent of the polar is increased and its shape is improved through the use of camber (and twist).

Drag and Performance Equations (ref. 14)

$$C_D = C_{D_{min}} + K(C_L - C_L')^2$$

$$C_{D_{min}} = C_{D_{min_{parabolic}}} + \Delta C_{D_{min}}$$

$$C_L' = (1 - \frac{e}{e_p}) * C_{L_{pb}}$$

$$\Delta C_{D_{min}} = \left[\frac{e_p - e}{\pi A R e_p^2}\right] * C_{L_{pb}}^2 = \frac{C_L'^2}{\pi A R(e_p - e)}$$

$$e = \frac{(C_L - C_L')^2}{\pi A R(C_D - C_{D_o})}$$

NON-Parabolic Polar

$$C_L^* = \sqrt{\frac{C_{D_o}}{K}}$$

$$C_D^* = 2(C_{D_o} - C_L'\sqrt{KC_{D_o}})$$

$$(L/D)_{max} = \frac{1}{2(C_{D_o} - C_L'\sqrt{KC_{D_o}})}$$

Parabolic Polar

$$C_L^* = \sqrt{\frac{C_{D_o}}{K}} \qquad (L/D)_{max} = \frac{1}{2\sqrt{KC_{D_o}}}$$

$$C_D^* = 2C_{D_o}$$

Analytically Determined Drag Polar (ref. 15)

Aerodynamic wind tunnel data (drag) is never exactly parabolic. But, the data can be approximated by a parabolic relationship. The approach is as follows.

$$C_D = C_{D_{min}} + K(C_L - C_L')^2 \tag{1}$$

$$C_{D_o} = C_{D_{min}} + K(C_L')^2$$

where $C_{D_{min}}$, K and C_L' are unknown; expanding equation 1

$$C_D = C_{D_o} - 2KC_L' C_L + KC_L^2$$

$$C_D = a + bC_L + cC_L^2$$

Method of Determining a Drag Polar

Assume "N" matched points of [CL(i), CD(i)]

$$\boxed{\begin{array}{llllllll}
aN & + & b\sum_{i=1}^{N} C_L(i) & + & c\sum_{i=1}^{N} C_L(i)^2 & = & \sum_{i=1}^{N} C_D(i) \\
a\sum_{i=1}^{N} C_L(i) & + & b\sum_{i=1}^{N} C_L(i)^2 & + & c\sum_{i=1}^{N} C_L(i)^3 & = & \sum_{i=1}^{N} C_L(i)*C_D(i) \\
a\sum_{i=1}^{N} C_L(i)^2 & + & b\sum_{i=1}^{N} C_L(i)^3 & + & c\sum_{i=1}^{N} C_L(i)^4 & = & \sum_{i=1}^{N} C_L(i)*C_D(i)^2
\end{array}}$$

Once $\sum_{i=1}^{N} C_L(i)$'s & $\sum_{i=1}^{N} C_D(i)$'s are determined, use a Gaussian Elimination technique to solve for the coefficients a, b, c where $a = C_{D_o}$, $b = -2KC'$, $c = K$

Example

$N = 5$

i	C_L	C_D
1	0.0	0.0190
2	0.1	0.0202
3	0.2	0.0234
4	0.3	0.0313
5	0.4	0.0456

Resulting augmented matrix

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & .3 & .1 \\ .3 & .1 & .0354 \end{bmatrix} \begin{bmatrix} 0.1395 \\ 0.03433 \\ 0.11251 \end{bmatrix}$$

$a = 0.01945$

$b = -0.02398$

$c = 0.2207$

$$\boxed{\begin{aligned} C_{D_o} &= 0.0194 \\ K &= 0.2207 \\ C_L' &= 0.0543 \end{aligned}}$$

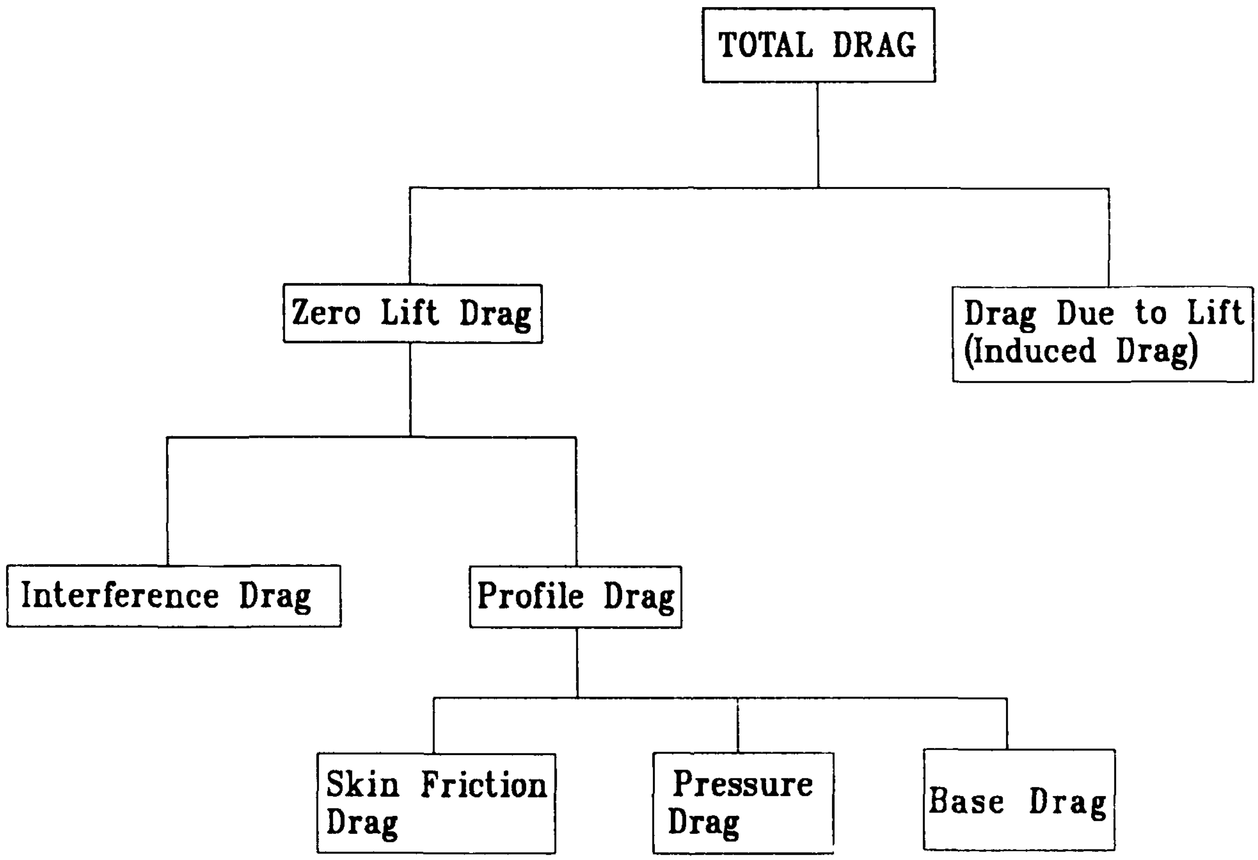

Figure VII-1 Drag Tree

Subsonic

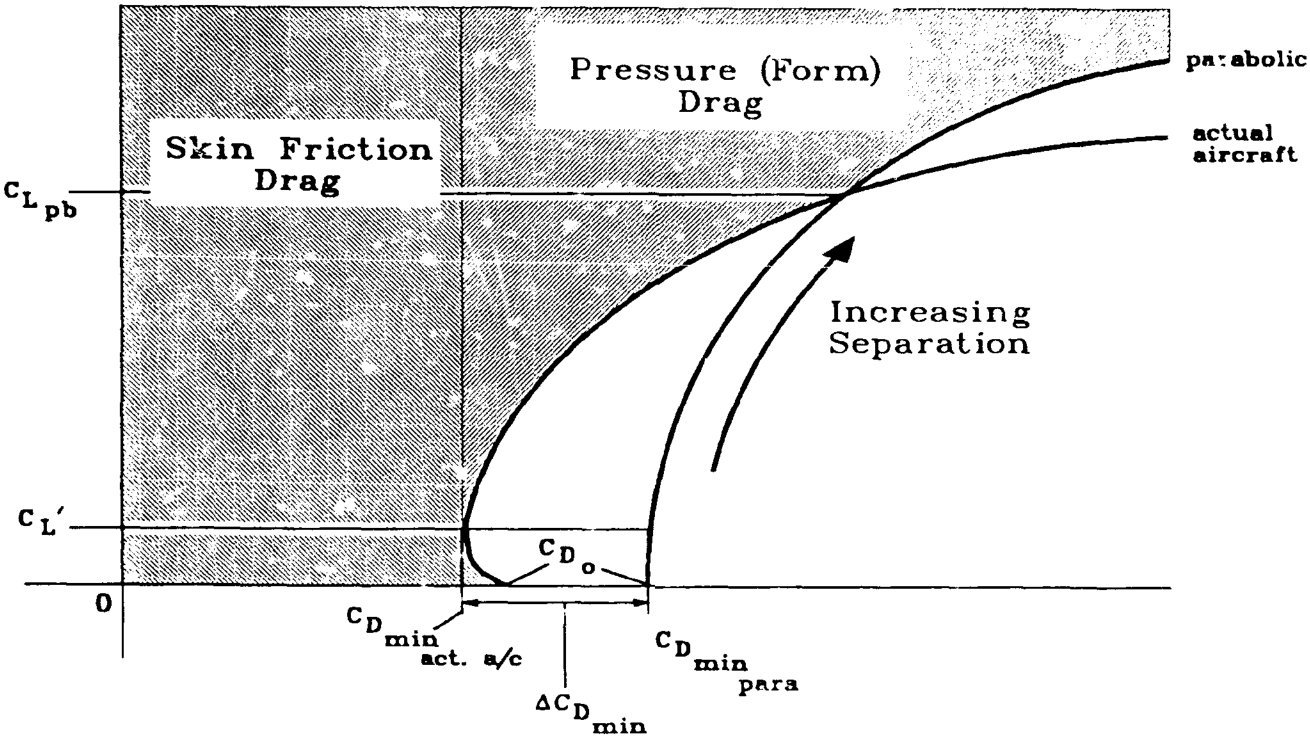

Figure VII-2 (C_L vs C_D) Drag Polar

Supersonic

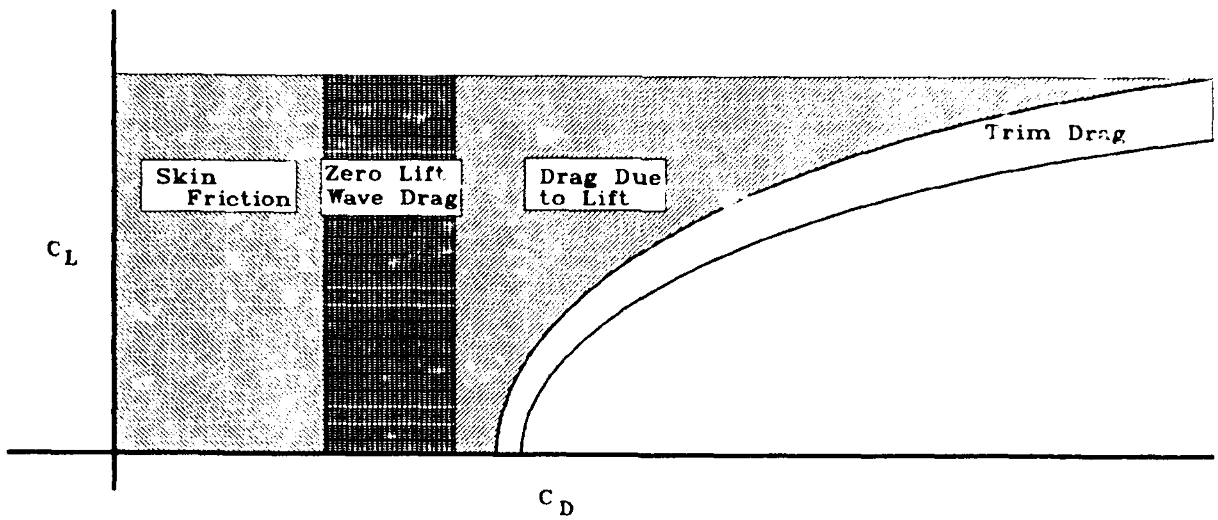

Figure VII-3 (C_L vs C_D) Drag Polar

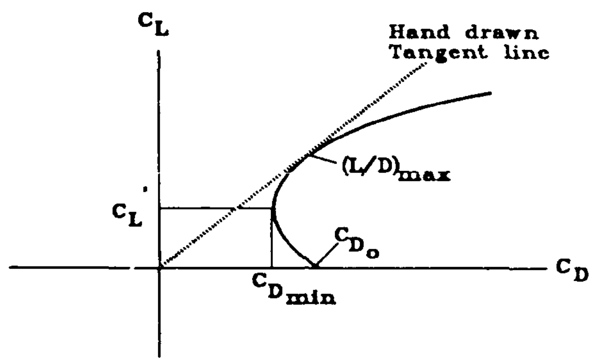

Figure VII-4 (C_L vs C_D) Subsonic Drag Polar

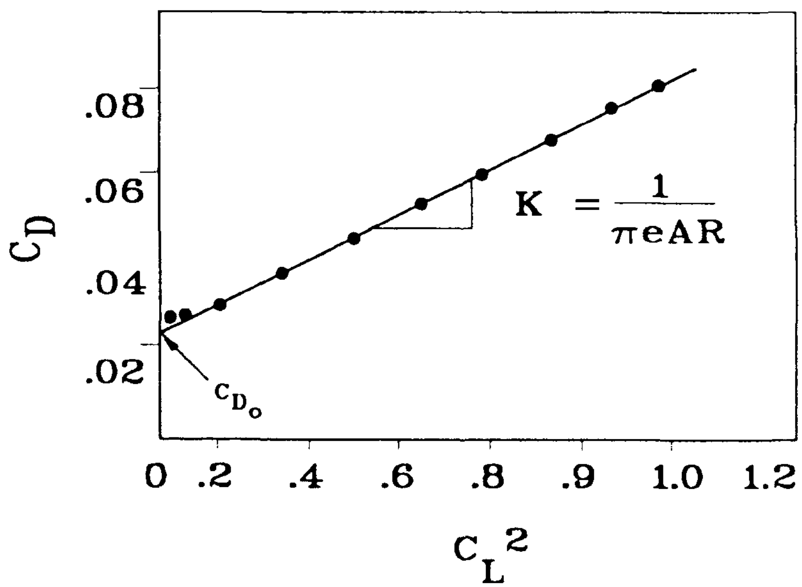

Figure VII-5 (C_D vs C_L^2) Drag Polar

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VIII EXPERIMENTAL TESTING AND INTERPRETATION

VIII Experimental Testing and Interpretation

(Ref. 8)

For each wind tunnel test accomplished, the items of interest for the engineer will vary. Below is a list of items and facts that are usually of interest to the engineer. Reference 8 is an excellent aid for finding those items and for subsonic wind tunnel testing. This handbook and reference 8 should be the two reference materials that accompany the testing engineer to the wind tunnel site.

Aircraft (wing-body-tail)

$C_{L_{max}}$	C_{L_α}	C_{M_α}	C_{M_o}
C_D	$C_{D_{min}}$	C_{D_o}	α_o
$(L/D)_{max}$	C_{L_o}	$C_{M_{ac}}$	C_{n_β}
C_{l_β}	C_{Y_β}		

Flaps

The purpose of flaps is to reduce the wing area through increasing $C_{L_{max}}$ and thus reduce the parasite (skin friction) drag in cruise. Flap systems usually generate large negative pitching moments and require a large horizontal tail to develop adequate down loads for trim, which reduce the total (wing-body-tail) $C_{L_{max}}$.

Trailing Edge Flaps (figure VII-9)

- Reduce α for α_o (α zero lift)
- Increase $C_{L_{max}}$
- Increases α for stall

Leading Edge Flaps

- Extend lift curve to increase α_{stall}
- Extend $C_{L_{max}}$

Lift Curve (Flaps Up)

- Used to determine flap-up stalling velocity
- $C_{L_{max}}$ range from 0.6 to 1.7 (unpowered)
- Wing $C_{L_{max}}$ runs 85% to 90% of airfoil values (with no high lift devices)
- $C_{L_{max}}$ usually increases with Reynolds number (Rn)
- L.E. slats are insensitive to Rn, but slats reduce the effect of Rn on $C_{L_{max}}$
- $C_{L_{max}}$ for flaps retracted is less than $C_{L_{max}}$ flaps extended
- Substantial variations of C_M often occur at $C_{L_{max}}$ or (α_{stall})
- Model should be as close to trim as possible
- Nacelles usually reduce $C_{L_{max}}$
- To be usable, $C_{L_{max}}$ must be for trimmed flight
- α at 0.9 $C_{L_{max}}$ is of interest for landing gear length consideration
- Stall α should be taken in very small steps so that its shape (non-linear portion of C_L curve) and $C_{L_{max}}$ and α_{stall} can be determined accurately
- To increase the span of the wing affected by the flaps (increasing $C_{L_{max}}$) the ailerons can be drooped

Lift Curve (Flaps Down)

- $C_{L_{max}}$ flaps down range generally from 1.2 to 3.5 (unpowered)
- Should have approximately same slope (C_{L_α}) as flaps up
- Should have same location for the aerodynamic center as flaps up
- Used to find $\Delta C_{L_{max}}$ due to flap deflection

- There is usually little need to take the flap- down lift curve as low as α_o

Drag Curve (Flaps Up and Down)

- Near $C_{D_{min}}$, step α in one degree increments
- $C_{D_{min}}$ (clean fighter) ≅ O 0.0120 (120 counts of drag)
- The shape of the drag curve is important for climb and cruise with small changes in drag due to lift being desired during these portions of the missions.
- The value of C_D at $C_{L_{max}}$ is needed for takeoff and landing calculations.

Pitching Moment

- C_{m_α} must be negative for longitudinal stability.
- Generally, the pitching moment curves are used to determine if the aircraft has static stability through the desired C.G. range at all flight conditions (trimmed flight).

Elevator or Stabilizer Power Curve

- Plotting ΔC_M versus δ_e, elevator deflection, (stab. incidence) is made at several C_L's. This plot indicates the amount of elevator or stabilizer deflection is needed to produce a certain moment coefficient.
- Stabilizer (elevator) effectiveness ($dC_M/d\delta_e$) is obtained by holding α_{wing} constant and varying the tail incidence.
- Plot $C_{M_{cg}}$ versus C_L for several elevator angles. The intersections of the curves with the axis indicate trim conditions (figure VII-7). By holding α_{wing} constant and varying stabilizer incidence, the pitching moment about the tail is $M_t = -l_t q_t S_t C_{L_t}$. When l_t, S_t are known, then C_{L_t} can be found. Use $q_t = 0.85q_\infty$. From C_{L_t} and known stabilizer angles,

the slope of the tail lift curve ($dC_{L_t}/d\alpha_t$) can be established.

- Elevator power must be sufficient to balance ($C_{M_{cg}}$ = 0.0) the airplane at maximum lift. The critical condition is gear and landing flaps down and in ground effects.
- On low aspect ratio configurations, with short tails, tail effectiveness varies with α as the local dynamic pressure changes.
- On swept wing configurations, attention must be paid to pitch-up (reversal of C_M curve) in the C_L vs C_M curve as it can limit usable C_L.

Aileron Power Curves

- Aileron criteria are usually determined at $\varphi = 0°$ and plots of C_l vs C_n, C_l vs δ_a, and C_l vs δ_a are used.
- Good qualities of ailerons are high rolling moments and low hinge moments.
- Maximum roll rate and maximum helix angles are determined from $C_{l_{max}}$. Generally, $C_{l_{max}}$ of 0.03 is adequate for one aileron.
- When yaw (φ) equals 0, and there is a slight rolling or yawing moment or side force when controls are neutral, be sure to subtract them out before reporting the data. This delta can be attributed to asymmetrical tunnel flow or model asymmetry.
- $\partial C_l/\partial_\varphi$ = 0.0002 is equivalent to 1^0 of effective dihedral.

If there is enough time and money (which usually there is not), do the aileron effectiveness test with tail off (ref. 8). The first reason to do this is when the ailerons are deflected in flight, the aircraft normally rolls and the inboard aileron trailing vortices are swept away from the horizontal tail by the helix angle. In the wind tunnel, these vortices stream back close to the horizontal tail and induce a load on the tail that does not occur in flight. Secondly, it saves effort in data reduction since tunnel wall effects on a horizontal tail is then non-existent.

Rudder Power Curves

- Rudder power on high-performance multi-engine aircraft must possess sufficient directional stability to prevent excessive yaw angles (φ).
- Rudder power must be able to be balanced directionally at best climb speeds with asymmetric power (engine out).
- Rudder equilibrium is a plot of rudder deflection, δ_r, versus yaw angle φ.
- When doing a rudder study, deflect the rudder and yaw the model only in one direction. You are allowed to do this due to model symmetry.
- Rudder Equilibrium: $\partial\varphi / \partial\delta_r$ ranges from -1.2 to -0.5 (maneuverable to stable)
- Rudder Power: $\partial C_n / \partial\delta_r = -0.001$ is reasonable.

Determine Center of Pressure Shift (C.P.)

To determine the C.P. shift the derivative $\partial C_M / \partial \mathrm{Mach}$ is used. The main factor that contributes to this derivative is the backward shift of the wing center of pressure (C.P.) which occurs in the transonic range. On two-dimensional symmetrical wings, for example, the C.P. moves from approximately 0.25c to approximately 0.5c as the Mach number increases from subsonic to supersonic values.

C.G. Shift

Moving the C.G. forward reduces α_{trim} or C_L resulting in an increase in trim speed.

Lift Curve Slope C_{L_α}

- C_{L_α} ranges approximately

$$\left.\begin{array}{l} 0.110 \text{ per degree for thin airfoils} \\ 0.115 \text{ per degree for thick airfoils} \end{array}\right\} Rn > 10^6$$

- As a rule of thumb; $C_{L_{\alpha_t}}$ is usually < 90% of C_{L_α}.
- C_{L_α} makes an important contribution to the dampening of the longitudinal short period mode.

Determining Aircraft Parameters from Wind Tunnel Data (ref. 29)

Wing-Body wind tunnel data:

α	-1.5	5.0
C_L	0.0	0.52

α = geometric angle-of-attack

α	1.0	7.88
C_M	-0.01	0.05

$C_M = C_{Mcg_{wb}}$

Static Margin

Lift curve slope

$$a_{wb} \equiv \frac{\partial C_L}{\partial \alpha} = \frac{0.52-0}{5-(-1.5)} = 0.08 \text{ per degree}$$

$$C_{Mcg_{wb}} = C_{Mac_{wb}} + a_{wb}\alpha_{wb}(h-h_{ac_{wb}}) \tag{1}$$

α_{wb} = absolute angle-of-attack

at $\alpha = 1.0°$

$$-0.01 = C_{Mac_{wb}} + 0.08\,(1 + 1.5)(h - h_{ac_{wb}}) \tag{2}$$

at $\alpha = 7.88^{o}$

$$0.05 = C_{Mac_{wb}} + 0.08(7.88 + 1.5)(h - h_{ac_{wb}}) \tag{3}$$

Equations 2 & 3 can be solved simultaneously. Subtracting equation 3 from equation 2...

$$-0.06 = 0 - 0.55(h - h_{ac_{wb}})$$

$$(h - h_{ac_{wb}}) = \frac{-0.06}{-0.55}$$

$$\boxed{(h - h_{ac_{wb}}) = 0.11} \quad \text{(static margin)}$$

Aerodynamic Center Location

$$h = 0.35\bar{c}$$

$$(h - h_{ac_{wb}}) = 0.11$$

$$h_{ac_{wb}} = 0.35 - 0.11$$

$$\boxed{h_{ac_{wb}} = 0.24} \quad \text{(a.c. location \% } \bar{c}\text{)}$$

Aerodynamic Center Pitching Moment

Using equation 1 ...

$$-0.01 = C_{Mac_{wb}} + 0.08(1 - 1.5)(0.11)$$

$$\boxed{C_{Mac_{wb}} = -0.032}$$

Center-of-Gravity Pitching Moment

For a given wing-body combination, the aerodynamic center lies 0.05 (5%) of a chord length ahead of the c.g.. The moment coefficient about the aerodynamic center is -0.016. If the lift coefficient is 0.45, calculate the moment coefficient about the c.g..

$$C_{Mcg_{wb}} = C_{Mac_{wb}} + C_{L_{wb}}(h - h_{ac_{wb}}) \tag{4}$$

$$(h - h_{ac_{wb}}) = 0.05 \qquad C_{L_{wb}} = 0.45 \qquad C_{Mac_{wb}} = -0.016$$

$$C_{Mcg_{wb}} = -0.016 + 0.45(0.05)$$

$$\boxed{C_{Mcg_{wb}} = 0.0065}$$

Wing-Body-Tail

Consider the wing-body data above, the area and the M.A.C. of the wing are 1.076 ft^2 and 0.328 ft respectively. The distance from the airplane c.g. to the tail a.c. is 0.557 ft, the tail area is 0.215 ft^2, the tail setting angle is 2.7^o, the tail lift-slope is 0.1 per degree, and from experimental measurement, $\varepsilon_o = 0.0$ and $\partial\varepsilon/\partial\alpha = -.35$. If $\alpha = 7.88^o$, calculate $C_{M_{cg}}$.

From the information above ..

$$C_{Mcg} = C_{Mac_{wb}} + a\alpha_a\left[(h - h_{ac_{wb}}) - V_H\frac{a_t}{a}\left[1 - \frac{\partial\varepsilon}{\partial\alpha}\right]\right] + V_H a_t(i_t + \varepsilon_o) \qquad (5)$$

$C_{Mac_{wb}} = -\ 0.032$ $\qquad\qquad$ $\alpha_a = 7.88 + 1.5 = 9.38^o$

$a_{wb} = 0.08$ $\qquad\qquad$ $(h - h_{ac_{wb}}) = 0.11$

$$V_H = \frac{l_t S_t}{\bar{c} S_w} \approx \frac{(.557)(.215)}{(.328)(1.076)} = 0.34$$

$a_t = 0.1/\text{deg}$ $\qquad\qquad$ $\partial\varepsilon/\partial\alpha = 0.35$

$i_t = 2.7^o$ $\qquad\qquad$ $\varepsilon_o = 0.0$

Using Equation 5

$$C_{M_{cg}} = -0.032+(.08)(9.38)\left[0.11-.34\left[\frac{.1}{.08}\right](1-.35)\right]+.34(.1)(2.7+0.0)$$

$$C_{M_{cg}} = -0.032 - 0.125 + 0.092$$

$$C_{M_{cg}} = \boxed{-0.065}$$

Longitudinal Static Stability

Does the aircraft (wing-body-tail) above have longitudinal static stability and balance ?

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = a\left[(h - h_{ac_{wb}}) - V_H\left[\frac{a_t}{a}\right]\left[1 - \frac{\partial \varepsilon}{\partial \alpha}\right]\right] \tag{6}$$

$a_{wb} = 0.08$ $\quad$ $(h - hac_{wb}) = 0.11$

$V_H = 0.34$ $\quad$ $a_t = 0.1$ per degree

$\partial\varepsilon/\partial\alpha = 0.35$

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = 0.08\left[(0.11) - 90.34)\left[\frac{.1}{0.08}\right](1 - 0.35)\right]$$

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = \boxed{-0.0133}$$

Since the slope is negative, thus the aircraft is statically stable.

Longitudinal Balance

$$C_{M_o} = C_{Mac_{wb}} + V_H a_t (i_t + \varepsilon_o) \tag{7}$$

$C_{Mac_{wb}} = -0.032$ $\quad$ $i_t = 2.7^o$

$$C_{M_o} = -0.032 + (0.34)(0.1)(2.7)$$

$$C_{M_o} = \boxed{0.06}$$

Trim angle-of-attack

Remember that $\frac{\partial C_{M_{cg}}}{\partial \alpha_{wb}}$ is the slope of a straight line. Therefore by setting $C_{M_{cg}}$ to zero, and writing the equation of a straight line

$$y = mx + b \tag{8}$$

$$y = C_{M_{cg}} \qquad m = \frac{\partial C_{M_{cg}}}{\partial \alpha_a}$$

$$x = \alpha_{trim} \qquad b = C_{M_o}$$

α_{trim} can be found. From the above example...

$$0.0 = 0.06 - 0.0133\alpha_{trim}$$

$$\boxed{\alpha_{trim} = 4.5^{o}}$$

Clearly, this angle-of-attack falls within the reasonable flight range. Therefore, the aircraft is longitudinally balanced and statically stable.

Finding Trimmed Flight Parameters (ref. 16)
(Unpowered, No thrust components)

Trimmed flight parameters can be easily found from a C_M vs C_L and C_L vs α plot. One of the model test parameters an engineer should always consider testing is the elevator. From this data, trim conditions can be found. When testing the elevator for its effectiveness, the angle-of-attack for the model configuration should be held constant while only a change of the elevator deflection angle (δ_e) is accomplished. Holding the configuration at a constant angle-of-attack is not necessary, but it helps when manipulating the data for presentation. Make as many runs as necessary at different δ_e's while the horizontal tail incident angle is held constant. The resultant plot can be see in figure VII-7. On the C_M vs C_L plot, where each δ_e curve crosses $C_M = 0$, that point is considered a trim point and the corresponding C_L is the trim C_L. To find α_{trim}, a line (this line equates to a constant C_L) is drawn from the C_M vs C_L plot to the corresponding δ_e on the C_L vs α plot. That intersection on the C_L vs α plot α_{trim} can be found. If a series of trim points are found on the C_L vs α plot a $C_{L_{\alpha_{trim}}}$ can also be found.

Determining any C.G. Location
(unpowered, gliding flight)

To find the trim envelope (where $C_M = 0$) for any c.g. location, a line of any slope is drawn by rotating that line from $C_M = C_L = 0$ on the C_M vs C_L curve to any position on the C_M vs C_L curve. The slope of the newly drawn line is equal to the c.g. shift (refs. 16 and 8).

$$\left.\frac{\partial C_M}{\partial C_L}\right|_{\text{new c.g. position}} = \left.\frac{\partial C_M}{\partial C_L}\right|_{\text{old c.g. position}} - \Delta$$

where

$$\Delta = [\text{old c.g. position (\%MAC) - new c.g. position (\%MAC)}]$$

or

$$\Delta = \left[(x_{cg} - x_{ac})_{old} - (x_{cg} - x_{ac})_{new}\right]$$

Example:

Move c.g. from 0.35 to 0.2;

$$\left.\frac{\partial C_M}{\partial C_L}\right|_{0.35} = 0.2$$

$$\left.\frac{\partial C_M}{\partial C_L}\right|_{0.2} = 0.2 - (0.35 - 0.2)$$

$$\left.\frac{\partial C_M}{\partial C_L}\right|_{0.2} = 0.2 - 0.15$$

$$\left.\frac{\partial C_M}{\partial C_L}\right|_{0.2} = 0.05$$

It appears that to acquire this transfer a subtraction of moment arms (Δ) was accomplished. However, in reference 16, a thorough explanation is discussed and it shows that it is not just subtraction of moment arms but rather a subtraction of moments.

Determining the Average Downwash Angle (ref. 8)

To determine the average downwash behind the wing .est the model configuration with the tail-off. Then test the configuration with the tail-on at different tail incident angles. The tail incident angle (i_t) is the angle between the wing-body zero-lift line (generally the longitudinal axis) and the tail zero-lift line. If the tail is a symmetric airfoil, the tail zero lift line and the tail chord line are the same. Also, if the tail is an all movable tail, the tail incident angle would generally be the same as δ_e. Once the tail-on and tail-off data are acquired, plot the data similar to figure VIII-1 (α_w vs C_M) at different i_t's and then plot the tail-off data. The intersection of the tail-on curve with the tail-off curve are points where, at a given α_w, the tail-on C_M equals the tail-off C_M. Also, at those intersections, the tail is at zero lift.

(For a symmetrical airfoil section)

$$\alpha_t = \alpha_w + i_t - \varepsilon_w = 0$$

ε_w = wing downwash angle

α_t = Tail angle-of-attack

$$\boxed{\varepsilon_w = \alpha_w + i_t}$$

Once ε_w is found then the parameter $\frac{\partial \varepsilon}{\partial \alpha}$ can easily be obtained by plotting ε_w vs α.

<u>Determining Induced Drag Factor</u> (K)
<u>and Oswald's Wing Efficiency Factor</u> (e)

Draw a plot of C_D vs C_L^2. The slope of this line is K, the induced drag factor (see figure VII-5). The intercept is C_{d_o}.

Since $K = \frac{1}{\pi ARe}$

Oswald's wing efficiency factor (e) is... $e = \frac{1}{\pi ARK}$

<u>Base Pressure</u> (ref. 4)

A base pressure correction is applied to remove the base pressure drag from the total drag in order to correct the force drag. Such a correction is required because of the base drag is unknown without the interference or interactions of the sting or a jet. The measured base pressure is corrected to the reference ambient static pressure which is considered to act over the entire base of the model. Occasionally more than one static pressure is measured on the base of the model and averaged to arrive at the base pressure. This average is multiplied by the area of the base to obtain the base-pressure axial force correction. In equation form:

$$F_A = (C_{P_{ave}})(A_B)(q_\infty)$$

Where

F_A = Base pressure axial force to be added to the measured axial force

$C_{P_{ave}}$ = Average pressure coefficient on base,

P_∞ = Free stream static pressure (in test section)

q_∞ = Free stream dynamic pressure

A_B = Base area

Pressure Transducer Selection

To determine the appropriate differential pressure transducer, three items are needed. The first being the maximum expected pressure coefficient (CP), the second being the minimum expected CP and finally the dynamic pressure (q). If obtaining the maximum and minimum CP's are difficult, then use +0.5 and -2.0 for the first iteration (these suggested CP's are good numbers for wing pressures). Then just "plug and chug" the equation below.

$$CP = \frac{p - p_\infty}{\frac{1}{2}\rho V^2}$$

$$\Delta p = (p - p_\infty) = CP(q^*)$$

* Differential pressure transducers are usually rated in psi $(\frac{lbf}{in^2})$. Conversion of 'q' to psf $(\frac{lbf}{ft^2})$ may be required.

From the maximum or minimum CP determine the greatest magnitude of Δp and then acquire the appropriate pressure transducer.

Example:

$CP1 = +0.5 \qquad CP2 = -2.0 \qquad q = 2.11 \text{ psi}$

$$\Delta p1 = 1.055 \text{ psi} \qquad \boxed{\Delta p2 = -4.22 \text{ psi}}$$

Based upon $\Delta p2$, a differential pressure transducer of 5 psi will be adequate.

Flow Visualization

Flow visualization offers the testing engineer a unique way to observe the local flow fields. Flow visualization can be separated into two categories, surface flow visualization and off-body flow field visualization. Techniques to observe surface flow fields are tufts and oil flows. Laser light sheet, smoke, and tufts are used for off-body flow fields.

Surface Flow Visualization

Yarn

Tufts do affect the aerodynamic forces.

Yarn should be used when the testing engineer is not concerned with the model forces and moments. Yarn has the greatest adverse effect on lift and drag compared to other surface flow visualization techniques. For yarn tufts, use 0.75 inch length of No. 6 floss crochet yarn (any color). Have plenty of yarn available since it does not last a long time in the wind tunnel. Before applying tufts on the model, clean the model with naphtha or other solvents to remove any oils. There are two methods used to apply the yarn tufts to the model. The first method is to use a tuft board. A tuft board is a piece of scrapwood with two nails in it. The distance between the two nails is the length of the tuft (generally 0.75 to 1.0 inch). Wrap the tufts around the nails and then cut the tufts on the backside of the nail to give you the correct length that is needed. To attach the yarn to the model use cellophane tape or "super glue" and apply the yarn to the model in a symmetric pattern (0.75 inch x 0.75 inch). Ensure that at least 0.75 inch of the tuft material is available for the flow to manipulate freely. The second method in applying tufts to the model is to tape the yarn at two opposite ends of the item of interest on the model (ie... leading to the trailing edge) and glue/tape the tuft at the desired interval lengths (0.75 in) then cut the yarn.

Fluorescent Mini-tufts (ref. 26)

Fluorescent mini-tufts allow a large number of tufts to be applied to a model surface in a manner that produces negligible interference with model forces and pressures. Comparisons of tufts-on and tufts-off from Mach 0.5 to 2.4 showed differences of two to three drag counts. Mini-tufts are an extremely fine nylon mono-filament fiber that has been treated with a fluorescent dye that renders it visible during fluorescence (ultraviolet) photography. There are two sizes of mini-tufts generally used during wind tunnel testing. Those sizes are 3 denier (0.02mm, 0.0007 inch diameter) and 15 denier (0.04mm, 0.0017 inch diameter). Free moving length (measured from the gluing point) of the mini-tufts should range from 0.5 to 0.75 inches.

Mini-Tuft Installation

The procedures listed below have proven to be adequate for most applications. Tuft application utilizing these procedures are able to withstand many hours of testing at transonic and supersonic Mach numbers without appreciable adhesion failure.

Surface Preparation Steps

1. Wipe model surface with a solvent to remove grease and oil. Use clean paper towels or tissues rather than shop rags. Preferred solvents are naphtha or any chlorinated hydrocarbons such as trichloroethylene or trichloroethane. Methyl ethyl ketone or Freon are less suitable because of rapid evaporation rates. Do not use alcohols.
2. If possible, lightly abrade the surface with a fine grit carborundum pad or aluminum oxide paper. Do not use silicon carbide paper on aluminum. Care should be taken not to compromise the aerodynamic smoothness requirements for the model.
3. After abrading the surface, thoroughly clean the surface with a solvent using clean tissues. Then wipe that area with another clean tissue using that tissue only once. Wiping more than once with the same tissue redeposits the contaminants. Continue to wipe the surface with solvent until the tissue shows no stains. Avoid letting the solvent evaporate completely before it is wiped.
4. Apply an Alodine solution to the surface with a swab or a brush. Allow it to remain wet on the surface for three to five minutes. Any areas that resist wetting should be scrubbed with carborundum pad wet with the Alodine solution (Alodine is a chromate conversion solution that promotes adhesion of the gluing material to the surface. It

is available from aircraft paint suppliers. In this application, add a wetting agent such as Kodak Photo-Flo® to Alodine to make it a one percent concentration.).

5. Rinse the Alodine from the surface with clean water and then dry the surface with clean tissues. Be careful to limit skin contact with the Alodine and to wash thoroughly after use.
6. Cover the cleansed area with paper to avoid recontamination until after the tuft adhesive is applied. Realize that the above procedures are involved and can be reduced in scope. Steps one through six are the correct way to prepare the model surface. However, there is a correct way of doing things and the getting it done way of doing things. Be flexible in this area.

Mini-Tuft Attachment Steps

1. Lay out alignment marks with a soft pencil (roughly 0.75 inch grid) so that the tufts can be applied in a symmetrical pattern.
2. Place oversized lengths of tuft material over the extreme ends of the model (i.e... L.E. to T.E. if a wing) surface by wrapping the tuft around the model and then taping it to the lower surface.
3. Be careful to use only slight tension to avoid stretching the filament.
4. Use a "super glue" type of gluing agent to apply the tufts to the surface. Apply a small drop of glue to the strand of tufting material in intervals of approximately 0.5 to 0.75 inches.
5. Be sure glue drops are dried before cutting the tuft material. Cut tufts just ahead of each adhesive drop with a new razor blade.

Experience has shown that the visual appearance of these tuft results are greatly enhanced by carefully keeping the tuft spacing uniform. If the tuft pattern is asymmetrical, it is still possible to describe the flow, but it is difficult to interpret. To illuminate the mini-tufts, an ultraviolet light source is required. The ultraviolet illumination excites the fluorescent material to radiate in the visible spectrum (i.e.. 400-600 nanometers). Video tape or black and white still photographs are the normal surface flow visualization data medium. It also helps if a fluorescent felt tip marker is used to hi-lite/outline the model. This will allow the model lines to be seen in the photograph or video tape.

Oil Flow

Oil flow visualization shows flow separation lines, vortex reattachment lines, shock lines, and complete boundary layer activity on the surface. The oil flow runs should be held for the last runs of the test since the oil will clog static pressure taps and is generally a messy procedure. The model is painted a flat color with the oil a color that will contrast the model color (generally a black model and white oil). The oil viscosity is an important parameter to be considered. If the oil is too viscous it will not display the true surface flow. If the oil is not viscous enough, it will run off the model and not display the flow on the surface. There are several formulas that can be used for the oil. The formulas will differ from tunnel-to-tunnel and from test engineer-to-test engineer based on his or her experience. One formula that is used and works well is one teaspoon of titanium dioxide (a white color), one teaspoon of STP Oil Treatment®, and five drops of Oleic acid. Mix this amalgamation thoroughly and apply it on the model using a syringe with a 18 or 20 gauge needle. When applying the oil to the model it should have a logical and somewhat symmetrical pattern.

Off Body Flow Visualization

Laser Light Sheet (Vapor Screen)

The vapor screen technique is a simple, yet effective, flow visualization tool to study the off-body flows about aerodynamic shapes at subsonic, transonic, and supersonic speeds. In recent years, this technique has frequently been employed in wind tunnel experiments to improve the understanding and control of the vortices shed from slender bodies of missiles, fuselage forebodies, and wings of fighter aircraft at high angles-of-attack. The technique features the injection of sufficient water into the tunnel circuit to create a condensation in the test section. An intense sheet of light is generated, usually with a laser (18-watt Argon-ion laser is sufficient), that can be oriented in any selected plane relative to the test model. The light is scattered as the water particles pass through the sheet, which enable the off-body flow to be visualized. However, the tunnel operator/owner might become perturbed at putting water in the tunnel. To alleviate his/her fear and to remove the water from the tunnel after the vapor screen runs, pump the tunnel down to a P_{total} of 500 psf, start the tunnel(subsonic) and turn the driers on. Do this approximately twice for 15 minutes each time. This will evaporate a majority of the water. After doing this have the tunnel technicians enter the tunnel and wipe up what little water is remaining.

Smoke Seeded Flow

Flow visualization using smoke is an excellent tool for observing off-body (external) flow fields that are dominated by vortical flows. If done correctly, superb qualitative data will result at a Reynolds number that might make it feasible to extrapolate the flow field to flight conditions.

Typically, smoke is generated by several methods. Such methods include pyrotechnic smoke devices, chemical (titanium tetrachloride and tin tetrachloride), petroleum products (kerosene, Type 1962 Fog Juice) and Rosco smoke generating fluid. The technique of producing smoke will be determined at the wind tunnel site.

A low turbulence wind tunnel is extremely helpful when trying to accomplish a flow visualization study using smoke. The surrounding air in the test section needs to be as undisturbed as possible in order that an accurate analysis of the model's flow field can be made.

Generally, the smoke enters the wind tunnel via a smoke wand that is typically located in the stilling chamber of the wind tunnel. The density of the smoke emitting from the wand should range from a mild to moderate fog (qualitatively speaking). The smoke should then traverse down to the test section and enter as a filament sheet that spans most of the section. The smoke filament sheet should be set (vertically) in such a manner that the smoke is "caught up" in the upwash of the wing, chine, or item of interest. It might be necessary to change the position of the the smoke wand to achieve an optimum smoke filament sheet position. Once the smoke filament sheet is caught in the upwash, the external flow field can be observed using a laser light sheet to expose the vortical systems.

Tufts

To use tufts for off-body flow visualization, a tuft wand or a tufted, framed, wire grid is used. A tuft wand is a pole with a long tuft on the end of it. The wand is hand held in the tunnel near the model to observe off-body flow fields. This technique is a qualitative procedure since the person and the pole create a flow disturbance in the tunnel. A tufted, framed, wire grid gives the testing engineer a planar view of the off-body flow field. The wire grid contains symmetrical, square (1 inch by 1 inch) wire mesh with tufts (usually yarn) glued at the corners of each mesh. The wire grid is placed downstream of the model and is useful in examining wing tip vortices.

Determining Aerodynamic Angles from the Model Support (sting) Angles

Aerodynamic angles α and β are calculated using the model support (sting) angles γ, ψ, θ, and ϕ.

Where

γ = prebend angle of sting (only in pitch)

φ = yaw angle of the support

θ = pitch angle of the support

ϕ = roll angle of the support

Knowing

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right) \qquad \beta = \sin^{-1}\left(\frac{v}{\bar{V}}\right)$$

Where

u = longitudinal velocity component

v = lateral velocity component

w = vertical velocity component

$\bar{V}$ = total freestream velocity

from reference 28 and 38

$$u = \left[\cos\varphi\left(\cos\gamma\cos\theta - \sin\gamma\sin\theta\cos\phi\right) - \sin\gamma\sin\phi\sin\varphi\right]V_\infty$$

$$v = \left[\sin\phi\sin\theta\cos\varphi - \cos\phi\sin\varphi\right]V_\infty$$

$$w = \left[\cos\varphi\left(\sin\gamma\cos\theta + \cos\gamma\cos\phi\sin\theta\right) + \cos\gamma\sin\phi\sin\varphi\right]V_\infty$$

Solving for the model angles α and β

$$\tan\alpha = \frac{\left[\cos\varphi\left(\sin\gamma\cos\theta + \cos\gamma\cos\phi\sin\theta\right) + \cos\gamma\sin\phi\sin\varphi\right]}{\left[\cos\varphi\left(\cos\gamma\cos\theta - \sin\gamma\sin\theta\cos\phi\right) - \sin\gamma\sin\phi\sin\varphi\right]}$$

$$\sin\beta = \left[\sin\phi\sin\theta\cos\varphi - \cos\phi\sin\varphi\right]$$

$$\phi = 0$$

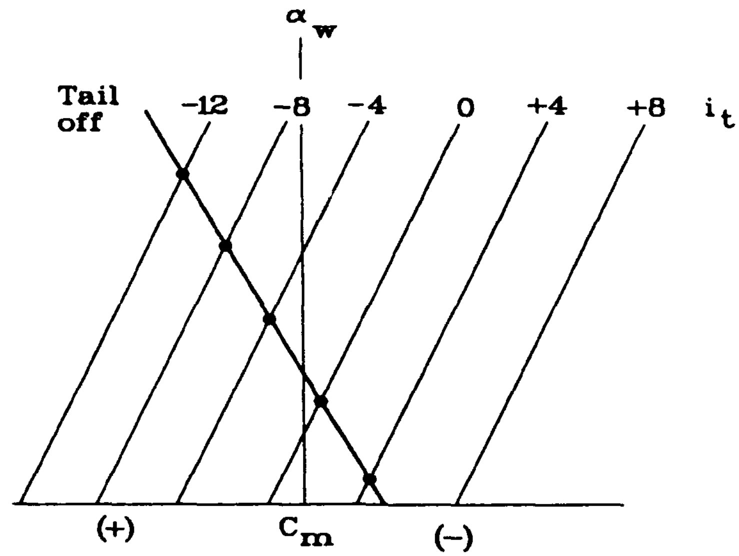

Figure VIII-1 Downwash Determination

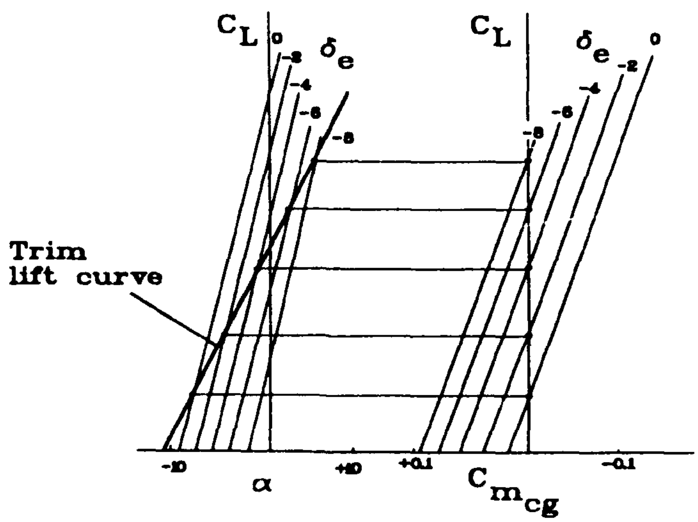

Figure VIII-2 Trim Determination Plot

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IX STRESS ANALYSIS

IX Stress Analysis

List of Symbols

A	Area
d	Diameter
c	Distance from Neutral Axis
E	Modulus of Elasticity
f_{tu}	Ultimate (allowable) Tensile Stress
f_{su}	Ultimate (allowable) Stress in pure shear
f_{ty}	Tensile Yield Stress (point)
f_{tp}	Tensile Proportional limit
G	Modulus of Rigidity (Shear Modulus of Elasticity)
I	Moment of Inertia
J	Polar Moment of Inertia
L	Total length of element (shaft length etc...)
l	Length of element (not shaft length)
ΔL	Change in length after deformation
M	Moment
NF	Normal Force
P	Load
R	Average radius
r	Distance to point of interest for torsion
r_o	Shaft radius
Δs	Change in arc length after deformation
T	Torque
t	Thickness
y	Distance to point of interest for stress
α	Constant based on l/t (non dimensional)
β	Constant based on l/t (non dimensional)
ε	Strain
σ	Stress
τ	Shear

List of Symbols continued

μ	Poisson's ratio
γ	Angle of shear strain

Subscripts

i -- inside	b -- bending	n -- normal
o -- outside	s -- shear	

Definitions

Stress — Stress implies a force per unit area and is a measure of the intensity of the force acting on a definite plane passing through a given point. The stress distribution may or may not be uniform, depending on the nature of the loading conditions. Tensile load is considered (+) and a compressive load is considered (-).

Strain — It is the change in length per unit length. The strain distribution may or may not be uniformed depending on the member and loading conditions.

Normal Stress — A unit stress which acts normal to the cross section of the structural element. These stresses are created by bending moments and axial forces.

Shear Stress — A unit stress which acts parallel and in the plane of the cross section. These stresses are caused by torsional moments and shear forces.

Normal Strain — Strain associated with a normal stress; it takes place in the direction in which its associated normal stress acts. Increase in length are (+) strains, decrease in lengths are (-) strains.

Shearing Strains — Those strains related to relative changes in angles.

Yield Point — Where elongation increases with no increase in load. The stress at this point is known as Tensile Yield Stress.

Proportional Limit — Where the stress-strain curve first becomes non-linear. That point is the maximum stress where the strain remains directly proportional to stress.

Elastic Limit — The maximum stress to which a material maybe subjected and still upon the removal of the load, return to its original dimensions.

Neutral Axis — When a beam is deflected, one surface is in compression while the other surface is under tensile stress. There is a plane (neutral) where the stress will be zero. Where this neutral

Neutral Axis (continued)	plane intersects any perpendicular cross section (or plane of loading) this location is the neutral axis for that cross section.
Ultimate Tensile Stress	This is the maximum allowable stress of the material.
Factor of Safety	Ultimate load / limit load
Modulus of Elasticity	Ratio of stress to strain; slope of the straight portion of the stress-strain diagram.
Shear Modulus of Elasticity	Ratio of shear stress to shear strain at low loads. The initial slope of the stress-strain diagram for shear.
Radius of Gyration of an Body	The distance from the inertia axis that the entire mass would be concentrated in order to give the same moment of inertia.
Radius of Gyration of an Area	The distance from the inertia axis to the point where the area would be concentrated in order to produce the same moment of inertia.
Shear Center	A point where a load produces no torsion on a asymmetric beam cross section.

STRESS FORMULAS

Normal Stress

$$\sigma_n = \frac{P}{A}$$

Normal Strain

$$\varepsilon_n = \frac{\Delta L}{L}$$

Modulus of Elasticity

$$E = \frac{\sigma_n}{\varepsilon_n}$$

Poisson's Ratio

$$\mu = \frac{\text{lateral deformation}}{\text{axial deformation}}$$

Shear Strain (thin wall, circular cylinder: twcc)

$$\varepsilon_s = \frac{\Delta s}{L}$$

Shear Modulus of Elasticity (twcc)

$$G = \frac{\tau_s}{\gamma} = \frac{\sigma_s}{\varepsilon_s}$$

ANGLE OF TWIST

Rectangular shaft

$$\phi = \frac{TL}{\beta \, l \, G \, t^3}$$

Circular shaft

$$\phi = \frac{TL}{JG}$$

Split tube

$$\phi = \frac{3TL}{2\pi R t^3 G}$$

POLAR MOMENT OF INERTIA

Solid, Circular Shaft

$$J = \frac{\pi d^4}{32}$$

Tubular, Circular cross-section

$$J = \frac{\pi}{32}(d_o^{\,4} - d_i^{\,4})$$

Solid, Circular Shaft

$$J = \frac{\pi}{2} r^4$$

Tubular, Circular cross-section

$$J = \frac{\pi}{2}(r_o^4 - r_i^4)$$

RADIUS OF GYRATION

Body

$$\rho = \sqrt{\frac{I}{M}}$$

Area

$$\rho = \sqrt{\frac{I}{A}}$$

BENDING STRESS

$$f_b = \sigma_x = \frac{Mc}{I}$$

$$f_b = \frac{6M}{bh^2} \text{ (rectangular)}$$

SHEAR STRESS

Rectangular cross section (beam)

$$f_s = \tau_{max} = \frac{3P}{2A}$$

Circular cross section (beam)

$$f_s = \tau_{max} = \frac{4P}{3A}$$

TORSIONAL FORMULAS

Solid, circular shaft

$$f_s = \tau_{max} = \frac{16T}{\pi d^3} = \frac{Tr}{J}$$

Tubular, circular cross section

$$f_s = \tau_{max} = \frac{16\pi d_o}{\pi(d_o^4 - d_i^4)}$$

$$= \frac{2\pi r_o}{\pi(r_o^4 - r_i^4)}$$

Split tube

$$f_s = \tau_s = \frac{3T}{2\pi R t^2}$$

Rectangular) cross section

(flat plate)

$$f_s = \tau_s = \frac{3T}{l\,t^2}\ ; \quad l >> t \qquad\qquad f_s = \tau_s = \frac{T}{\alpha\ l\ t^2}\ ; \quad l = t$$

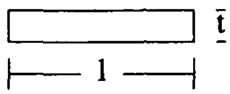

Table IX-1 Stress Constants (ref. 31)

l/t	1.0	1.5	1.75	2.0	2.5	3.0	4.0	6.0	8.0	10.0	∞
α	.208	.231	.239	.246	.258	.267	.282	.299	.307	.313	.333
β	.141	.196	.214	.229	.249	.263	.281	.299	.307	.313	.333

COMBINED STRESS

Shafts under bending and torsional loads (ref. 33)

$$f_T = \frac{f_b}{2} + \sqrt{\left(\frac{f_b}{4}\right)^2 + f_s^{\,2}}$$

Stresses on an incline plane (ref. 34)

$$\sigma = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos\theta + \tau_{xy}\sin 2\theta$$

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principle Stress

$$\sigma_{\substack{max \\ min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Angle on which principle stresses occurs
(measured from x axis)

$$\tan 2\theta = \frac{-\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

Maximum and Minimum Shear stresses

$$\tau_{\substack{max \\ min}} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Angle on which max and min Shear stress occurs
(measured from x axis)

$$\tan 2\theta = \frac{\frac{\sigma_x + \sigma_y}{2}}{\tau_{xy}}$$

FACTOR OF SAFETY

$$SF = \frac{f_{tu}}{stress} \qquad SF = \frac{f_{tu}}{f_T}$$

$$SF = \frac{1}{\sqrt{\left(\frac{f_b}{f_{tu}}\right)^2 + \left(\frac{f_s}{f_{tu}}\right)^2}}$$

Calculate Centroid of a Planform Area

Elem.	Dim (in)	Area (in^2)	$\bar{y}$ (in)	$A\bar{y}$ (in^3)	$A\bar{y}^2$ (in^4)
1	1.45 X 4.20	6.1	10.3+2.1= 12.4	75.5	937.9
2	2.3 X 10.3	23.7	10.3/2 = 5.14	122.0	628.6
3	8.98 X 2.0 X 0.5	8.98	8.98/3= 3.0	26.9	80.82
4	8.98 X 2.0 X 0.5	8.98	8.98/3= 3.0	26.9	80.82
5	0.83 X 2.0	1.66	0.83/2= .42	0.7	0.29
6	0.83 X 2.0	1.66	0.83/2= .42	0.7	0.29
7	0.85 X 4.22/2	1.8	4.22/3+10.3= 11.7	21.1	246.4
8	0.85 X 4.22/2	1.8	4.22/3+10.3= 11.7	21.1	246.4
		Σ 54.7 in^2		Σ 294.9	Σ 2221.5

$$\bar{y} = \frac{\Sigma A\bar{y}}{A} = \frac{294.9}{54.7} = 5.4 \text{ in}$$

$$I_X = 2221.5 \text{ in}^4$$

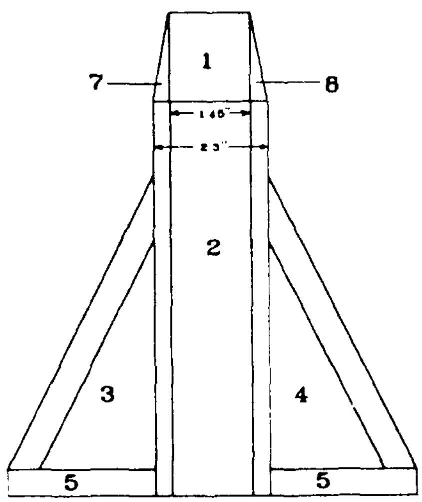

Figure IX-1 Stress Analysis Example 1
(centr id determination)

Example: Stress Analysis

Shear and bending at section F-F (figure IX-2)

Assuming a rectangular section of average thickness...

$$f_b = \frac{6M}{l\,t^2}\ ; \quad l = \text{base} \quad t = \text{height (thickness)}$$

$$(f_b = \frac{Mc}{I};\ I = \frac{l\,t^3}{12};\ c = \frac{t}{2}\ )$$

$$f_s = \frac{T}{\alpha\, l\, t^2}$$

T = torsion in beam
α = constant dependent upon l/t

$$l = 0.84 \text{ in}$$

$$t_{ave} = \frac{0.044 + 0.030}{2} = 0.037 \text{ in}$$

$$l/t = 22.7\ ;\ \alpha = 0.333$$

$$M = 6.0(0.380) = 2.28 \text{ in-lb}$$

$$T = 6\left[0.473 + \frac{0.84}{2}\right] = 5.358 \text{ in-lb}$$

Bending

$$f_b = \frac{6(2.28)}{(0.840)(0.037)^2} = 11{,}896 \text{ psi}$$

$$\text{S.F.} = \frac{89100}{11896} = 7.48$$

Shear

$$f_s = \frac{5.358}{(0.333)(0.840)(0.037)^2} = 13{,}992 \text{ psi}$$

$$S.F. = \frac{1}{\sqrt{\left[\frac{11896}{89100}\right]^2 + \left[\frac{13992}{57000}\right]^2}} = 3.57$$

Shear and bending at first set of attachment holes at section E-E (figure IX-2)

Assuming a rectangular section of average thickness...

$$f_b = \frac{6M}{l\,t^2};$$

$$(f_b = \frac{Mc}{I};\ I = \frac{l\,t^3}{12};\ c = \frac{t}{2})$$

$$f_s = \frac{T}{\alpha\, l\, t^2}$$

T = torsions in beam
α = constant dependent upon l/t

$$l = 0.840 - 2(0.138) = 0.564 \text{ in}$$

$$t_{ave} = \frac{0.062 + 0.030}{2} = 0.046 \text{ in}$$

$$l/t = 12.3\ ;\ \alpha = 0.333$$

$$M = 6.0(0.380 + 0.210) = 3.54 \text{ in-lb}$$

$$T = 6\left[0.473 + \frac{0.84}{2}\right] = 5.358 \text{ in-lb}$$

<u>Bending</u>

$$f_b = \frac{6(3.54)}{(0.564)(0.046)^2} = 17{,}798 \text{ psi}$$

$$S.F. = \frac{89100}{17798} = 5.01$$

<u>Shear</u>

$$f_s = \frac{5.358}{(0.333)(0.564)(0.046)^2} = 13{,}482 \text{ psi}$$

$$S.F. = \frac{1}{\sqrt{\left[\frac{17798}{89100}\right]^2 + \left[\frac{13482}{57000}\right]^2}} = 3.23$$

Tension in screws at wing tip missile attachment...

Summing moments about A-A

$$\Sigma M_{A\text{-}A} = 0 = 1.57NF - 0.706R_1 - 0.558R_2 - 0.154R_3 - 0.302R_4$$

Solving screw loads in terms of R_1

$$R_2 = \frac{0.558}{0.706}R_1 \qquad R_3 = \frac{0.154}{0.706}R_1 \qquad R_4 = \frac{0.302}{0.706}R_1$$

Substitute into the $M_{A\text{-}A}$ equation above...

$$1.57NF = R_1\left[0.706 + \frac{0.558^2 + 0.154^2 + 0.302^2}{0.706}\right]$$

$$1.57NF = 1.31R_1$$
$$R_1 = 1.199NF$$

From above...

$$R_2 = 0.948NF \qquad R_3 = 0.262NF \qquad R_4 = 0.308NF$$

From figure IX-2, NF = 6.0 lbs...

$$\boxed{R_1 = 7.2 \text{ lbs}} \quad \boxed{R_2 = 5.69 \text{ lbs}} \quad \boxed{R_3 = 1.57 \text{ lbs}} \quad \boxed{R_4 = 3.08 \text{ lbs}}$$

f_{tu} for 0-80 flat-head socket screws is 265 lbs

$$S.F. = \frac{265}{7.2} = 36.8$$

Thread pullout in wing tip missile attachment screws...

For screw R_1 (0-80 flat-head screws)...

Shear Area = π(screw pitch dia.)(screw length)(0.5)

(0.5) is and arbitrary fudge factor for conservatism

$$= \pi(0.519)(0.58)(0.5)$$

$$\text{Shear Area} = 0.0094(0.5) = 0.0047 \text{ in}^2$$

Shear

$$f_s = \text{load / shear area}$$
$$f_s = 7.2 / 0.0047$$
$$\boxed{f_s = 1532 \text{ psi}}$$

For flat-head screw 0-80

$$f_{su} = 96000 \text{ psi}$$

$$S.F. = \frac{96000}{1532} = 62.7$$

For the wing; 17-4PH stainless steel screws...

$$f_{su} = 120000 \text{ psi}$$

$$S.F. = \frac{120000}{1532} = 78.4$$

Screw head pullout in wing tip missile attachment...

$$\text{shear area} = \pi(\text{screw head dia.})(\frac{t}{2} - \text{head depth})$$

$$= \pi(0.117)(0.058 - 0.045) = 0.00478 \text{ in}^2$$

Shear force...(for 4130 steel)

$$f_s = \frac{7.2}{0.00478} = 1506.3 \text{ psi}$$

$$S.F. = \frac{89100}{1506.3} = 59.2$$

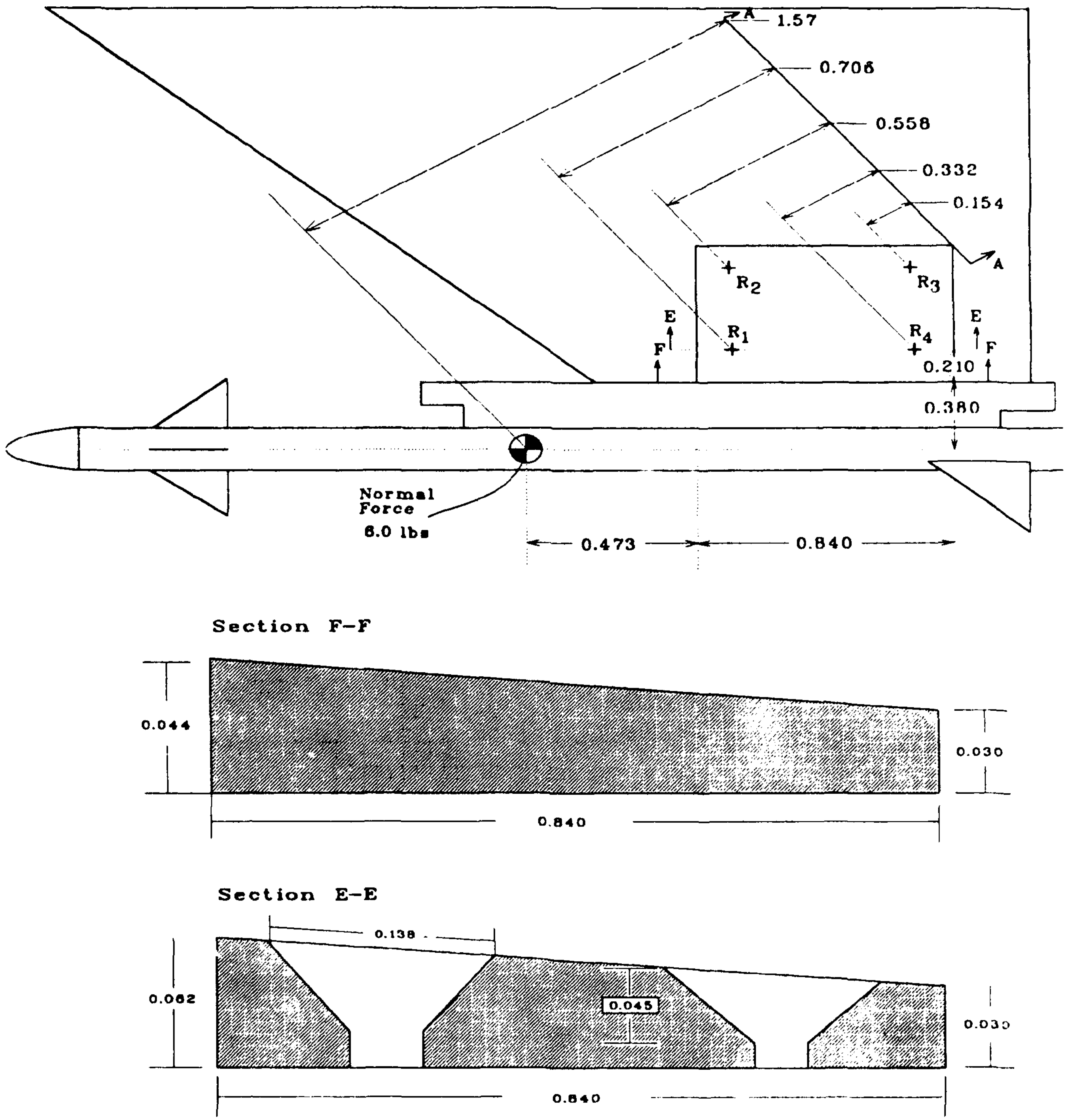

Figure IX-2 Stress Analysis Example 2
(wing tip missile)

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X TRENDS

X Trends

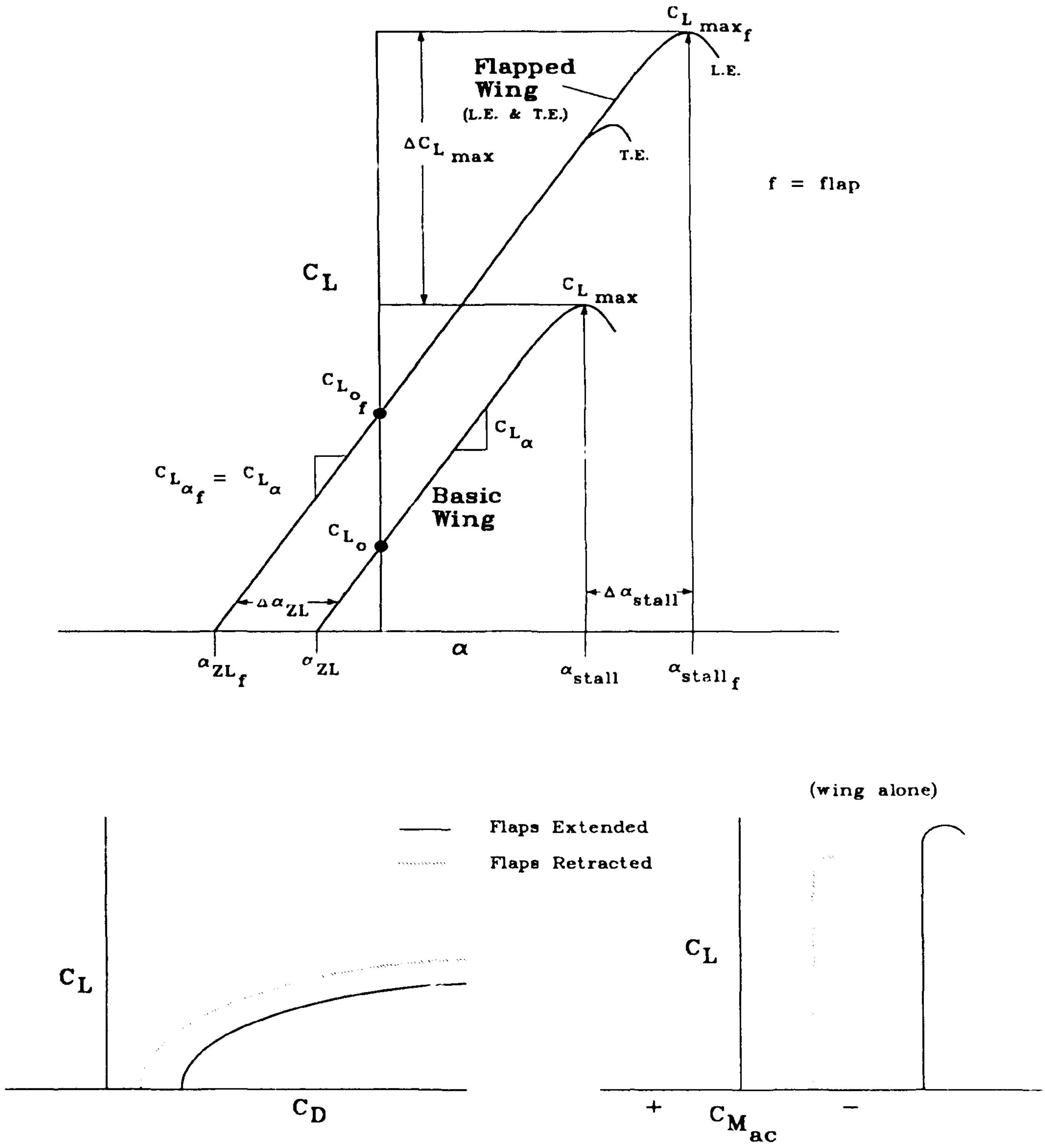


Figure X-1 Flap Characteristics

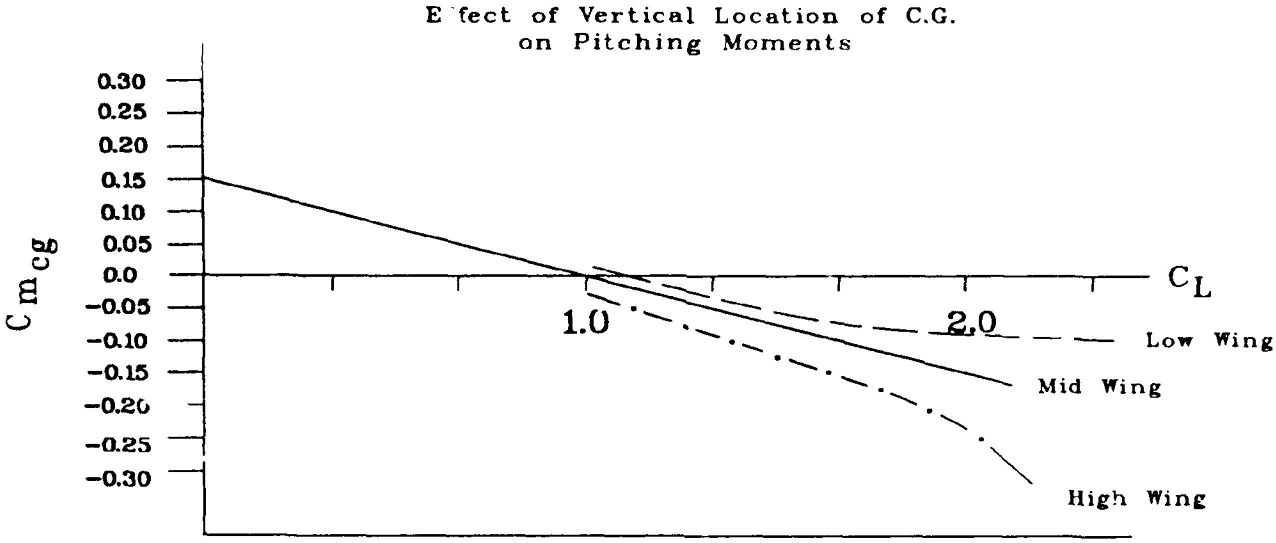


Figure X-2 Effect of Vertical Location of C.G. on Pitching Moments

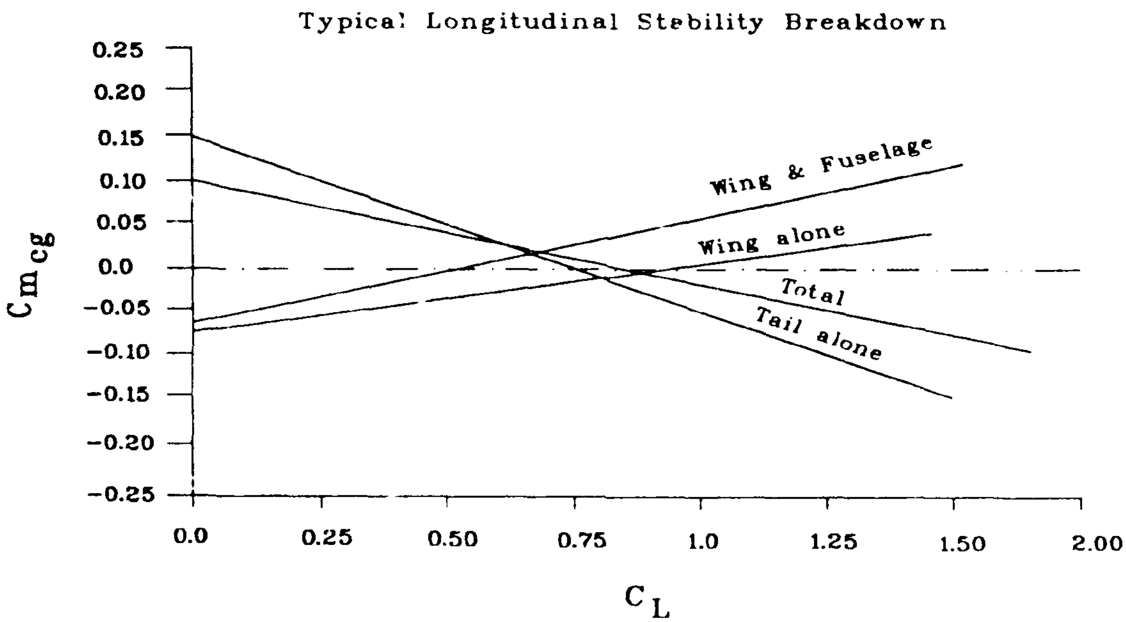


Figure X-3 Typical Component Longitudinal Stability Breakdown

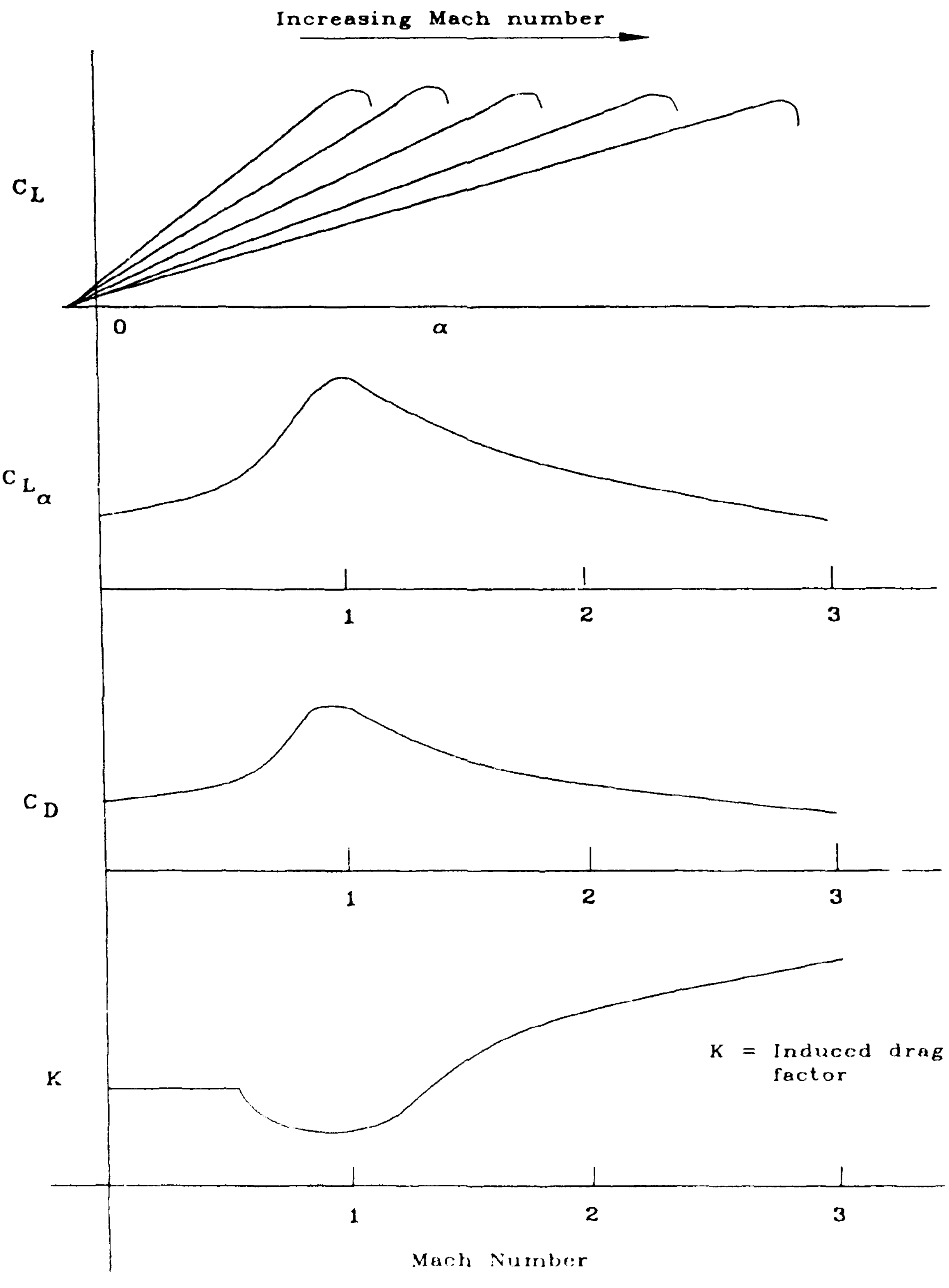


Figure X-4 Mach Number Trends (Effects)

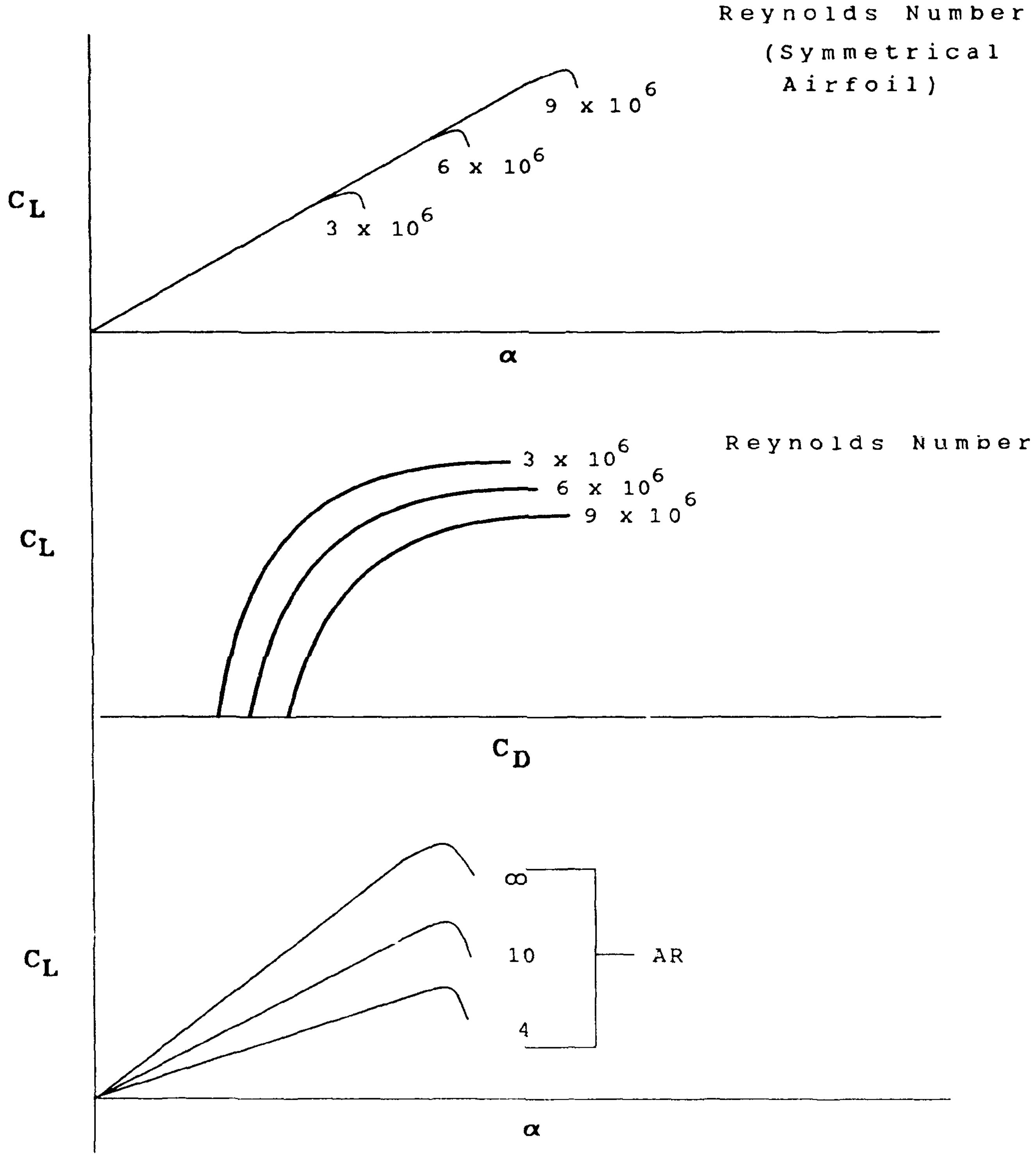


Figure X-5 Reynolds Number and Aspect Ratio Trends (Effects)

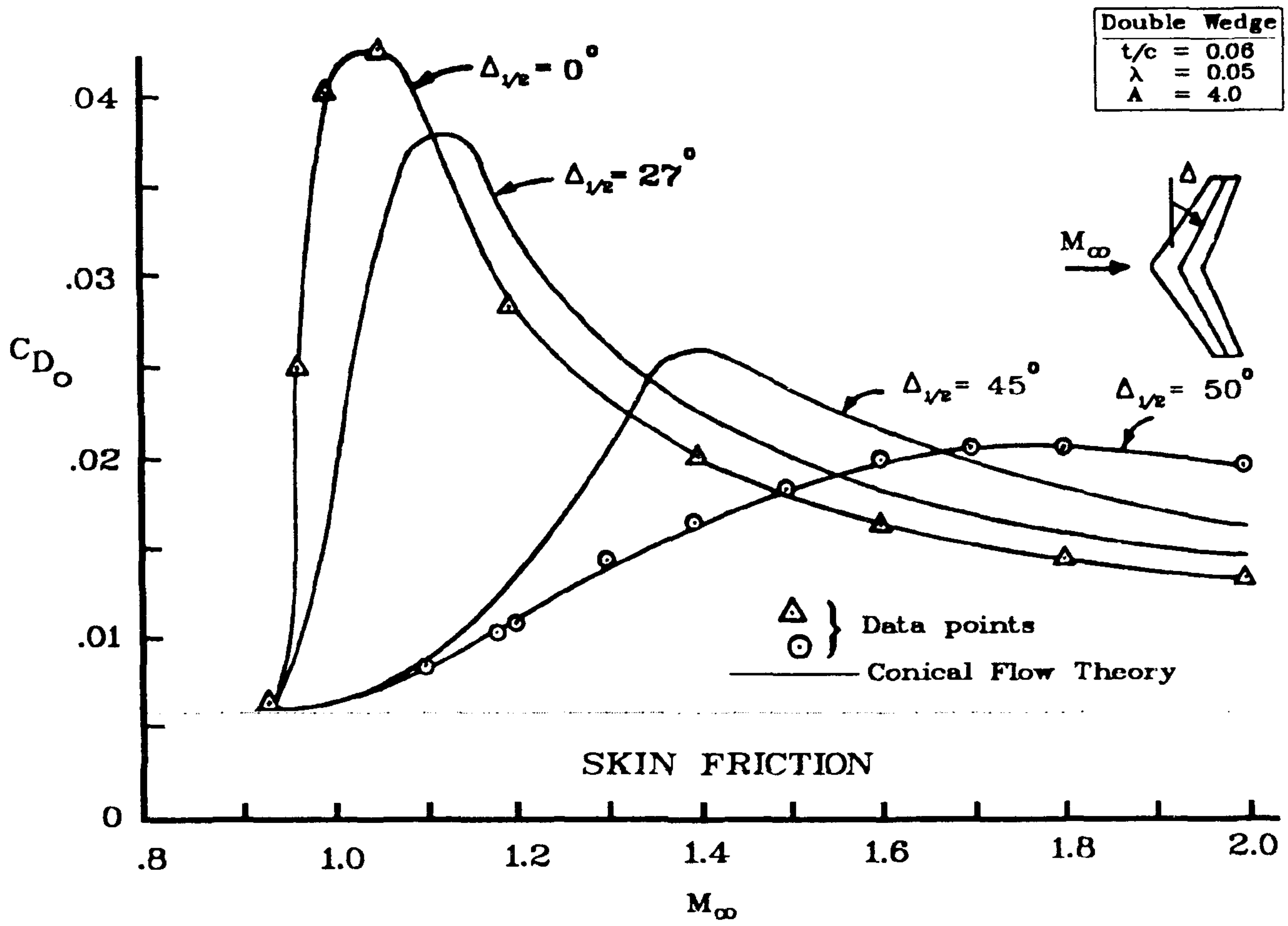
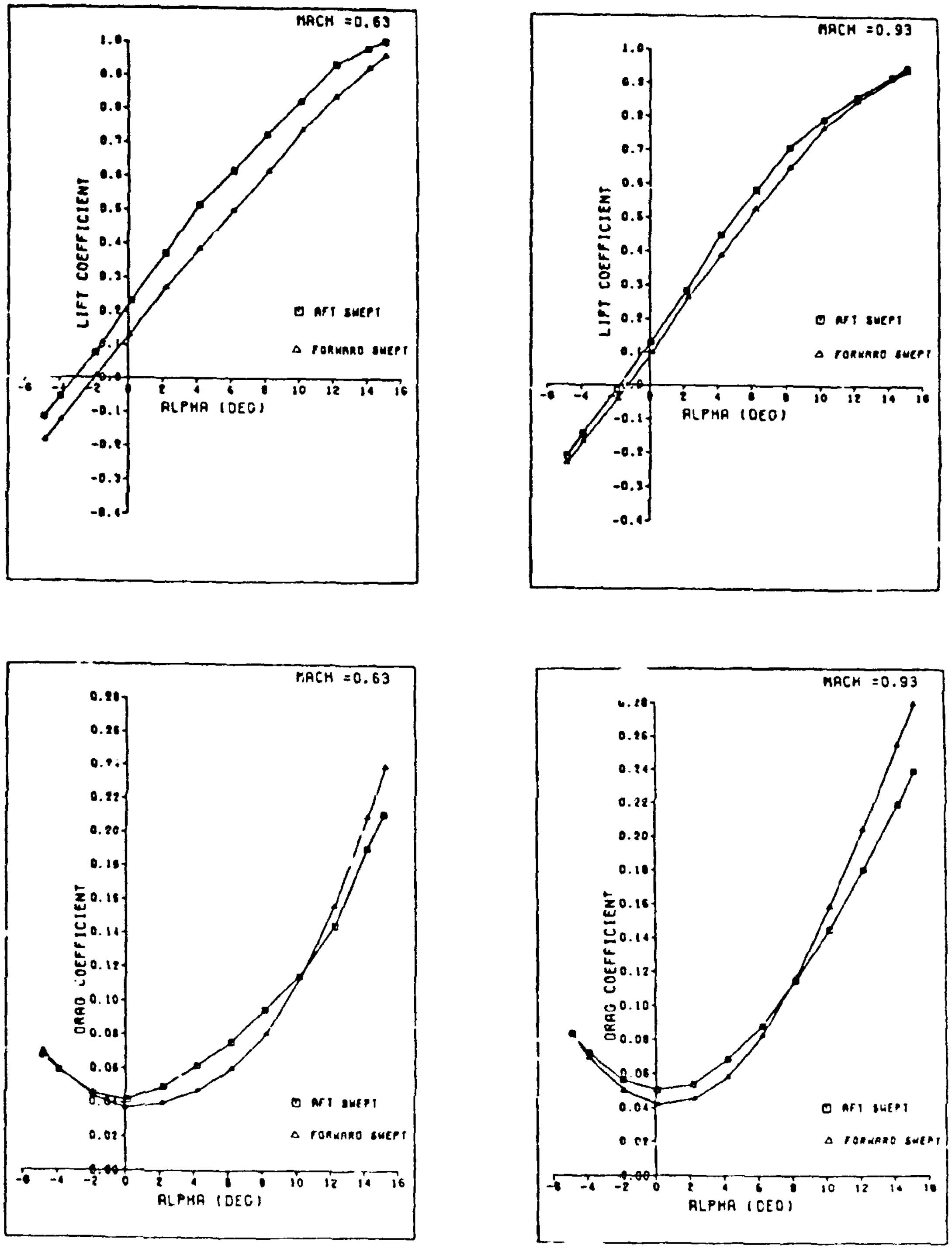


Figure X-6 Effect of Wing Sweep on C_{D_O}



$AR = 3.34$	$\Lambda_{c/4} = \pm 45^0$	$S_w = 25.7 \text{ in}^2$
$MAC = 5.62''$	$\lambda = 0.4$	$t/c = 5.5\%$
$b/2 = 7.58''$	$C_r = 6.1''$	$C_t = 2.45''$

Figure X-7 Aft and Forward Swept Wing-Fuselage Effects (Ref. 18)

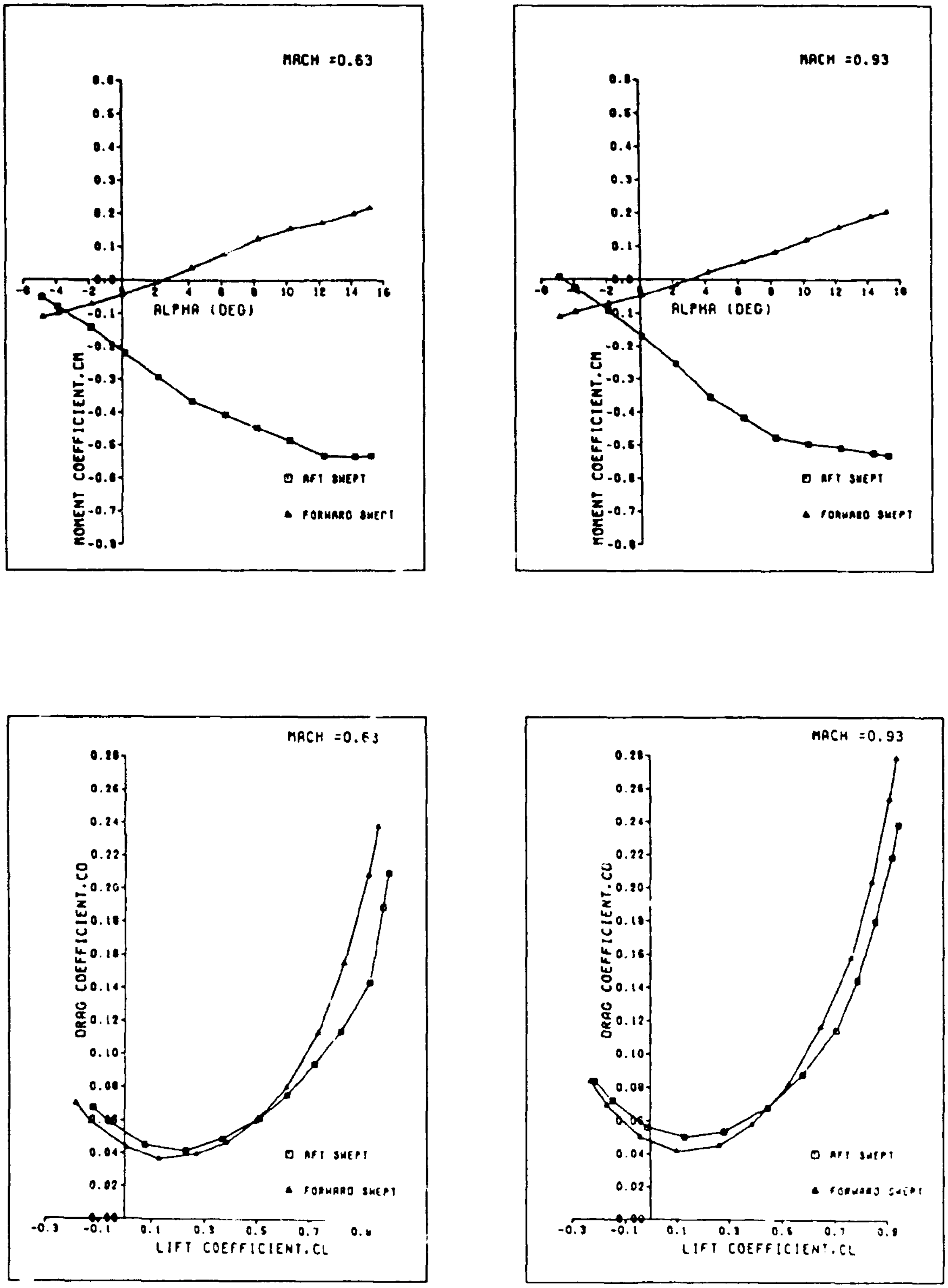
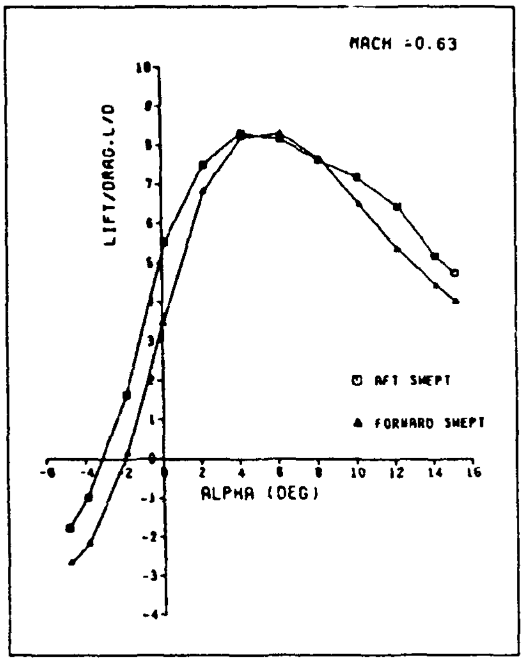
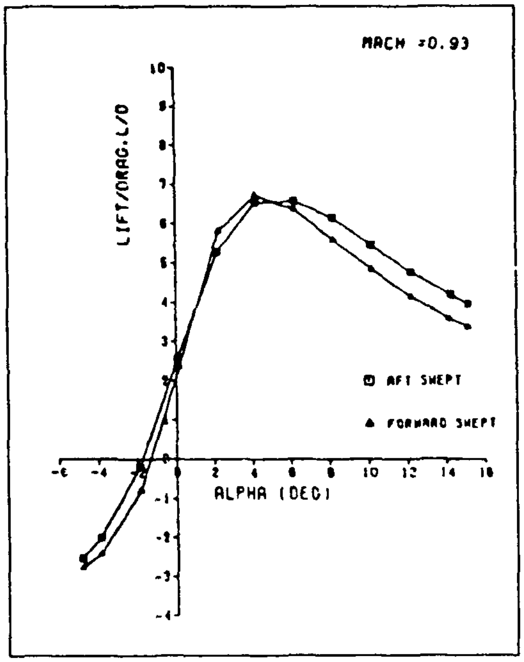


Figure X-7 continued





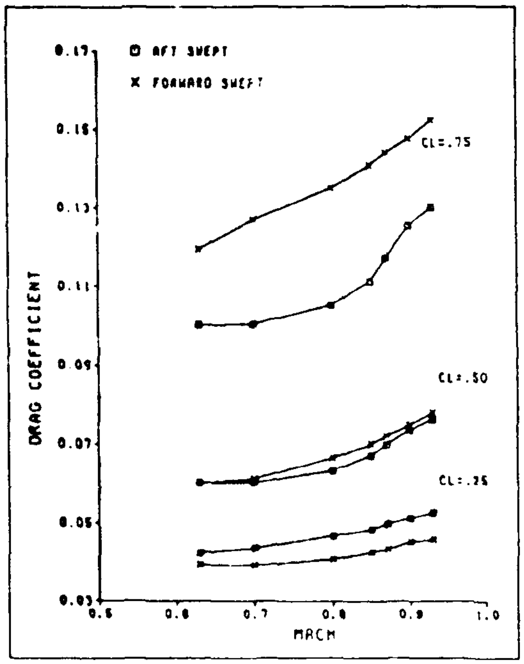
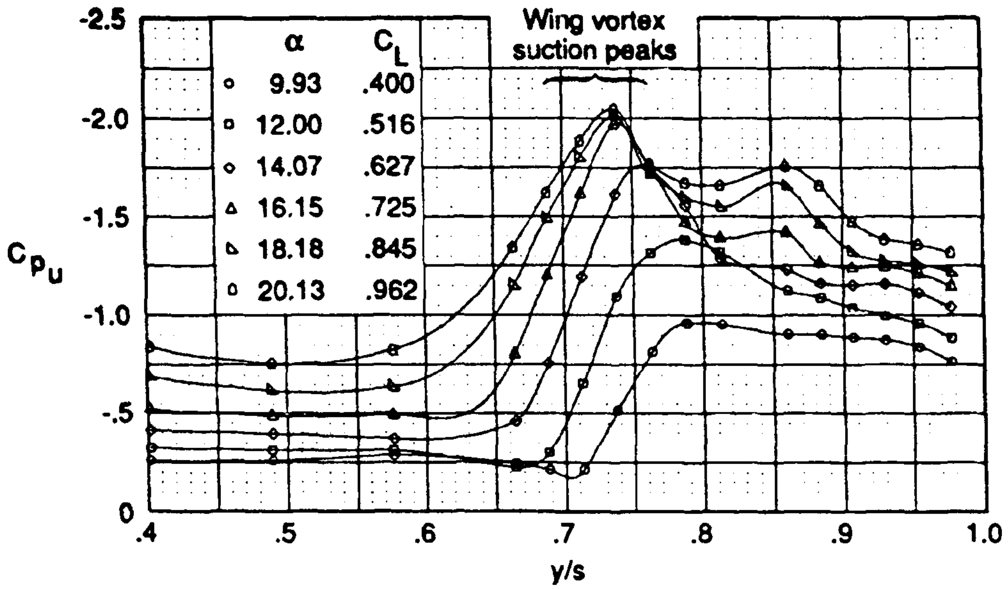
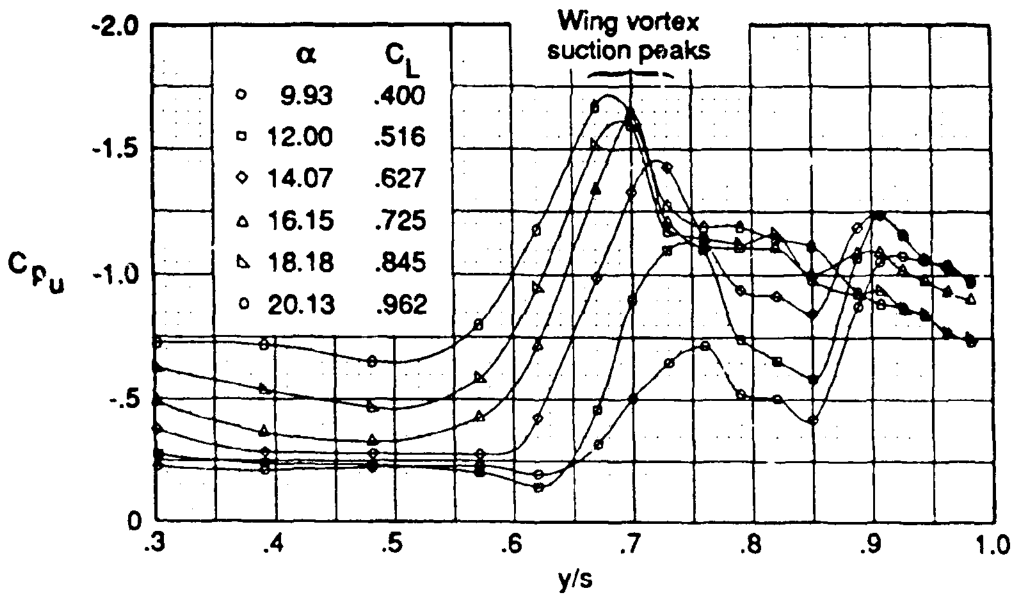


Figure X-7 concluded

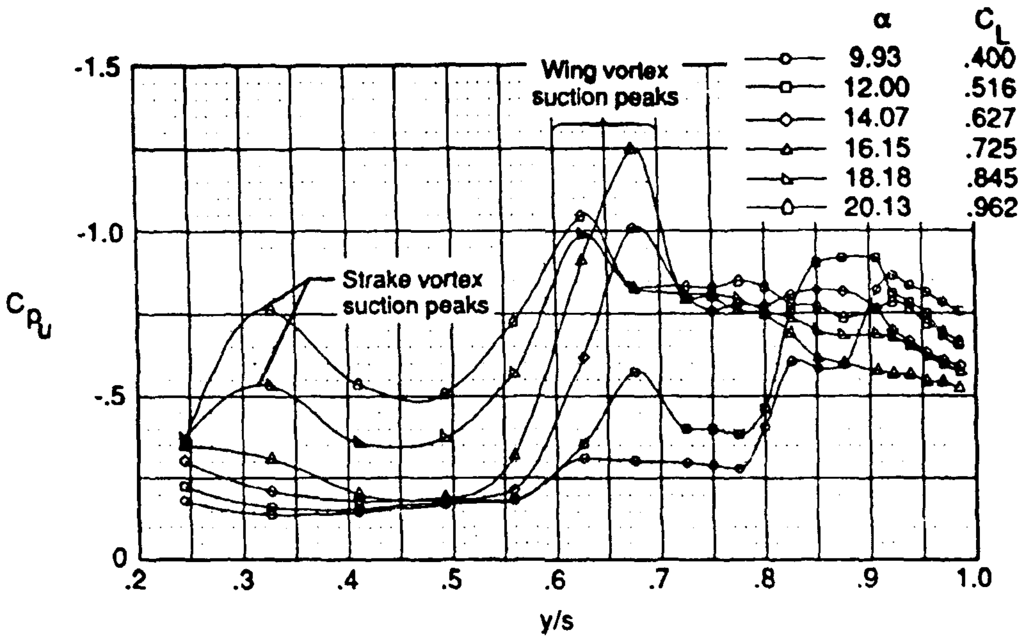


(a) x/c=0.50



(c) x/c=0.62

Figure X-8 Wing Pressure Distribution in the presence of a Coupled Chine (ref. 19)



(e) x/c=0.75

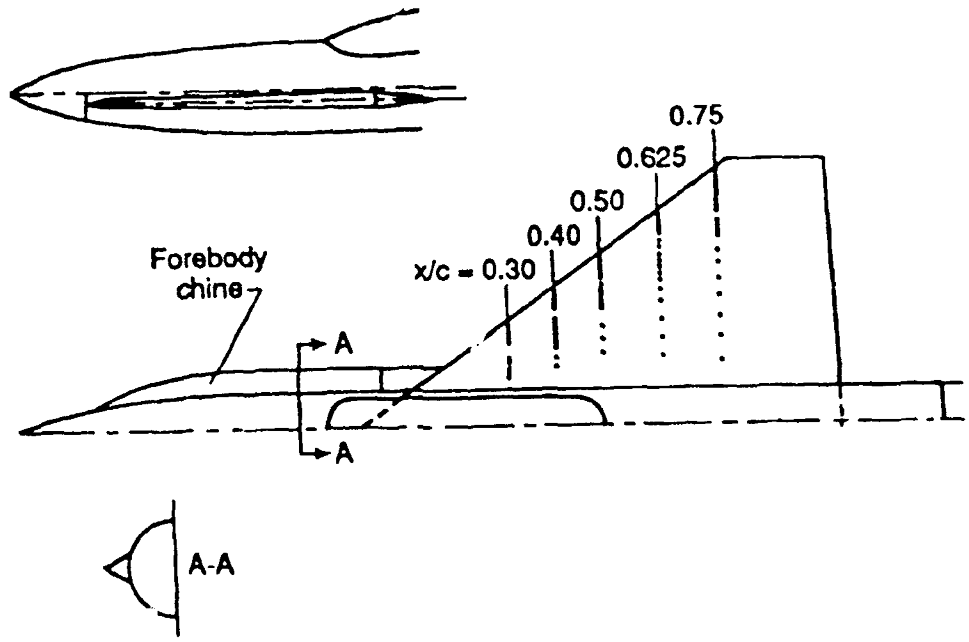


Figure X-8 concluded

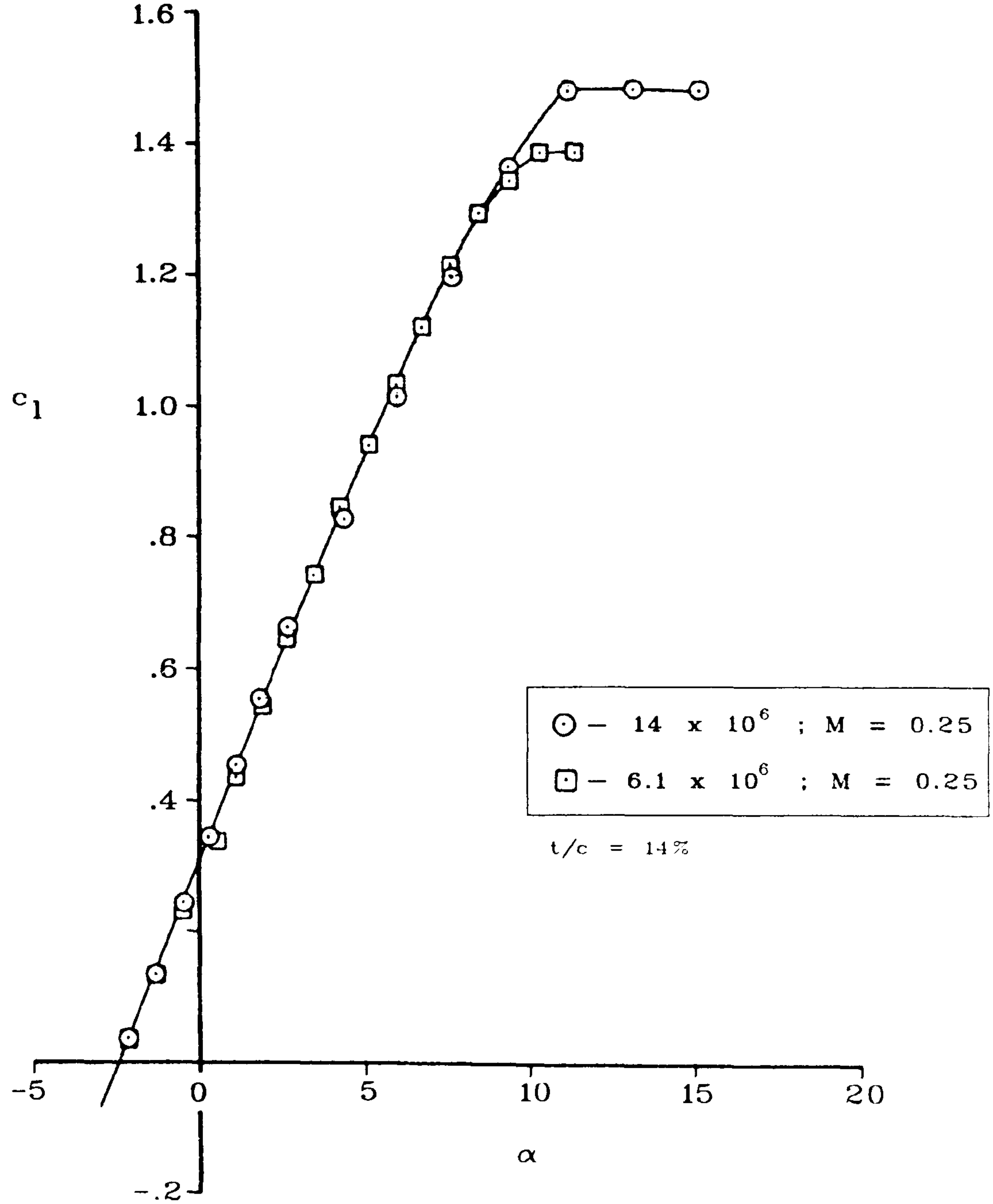


Figure X-9 Reynolds Number Effect on $c_{l_{max}}$ (wing alone)

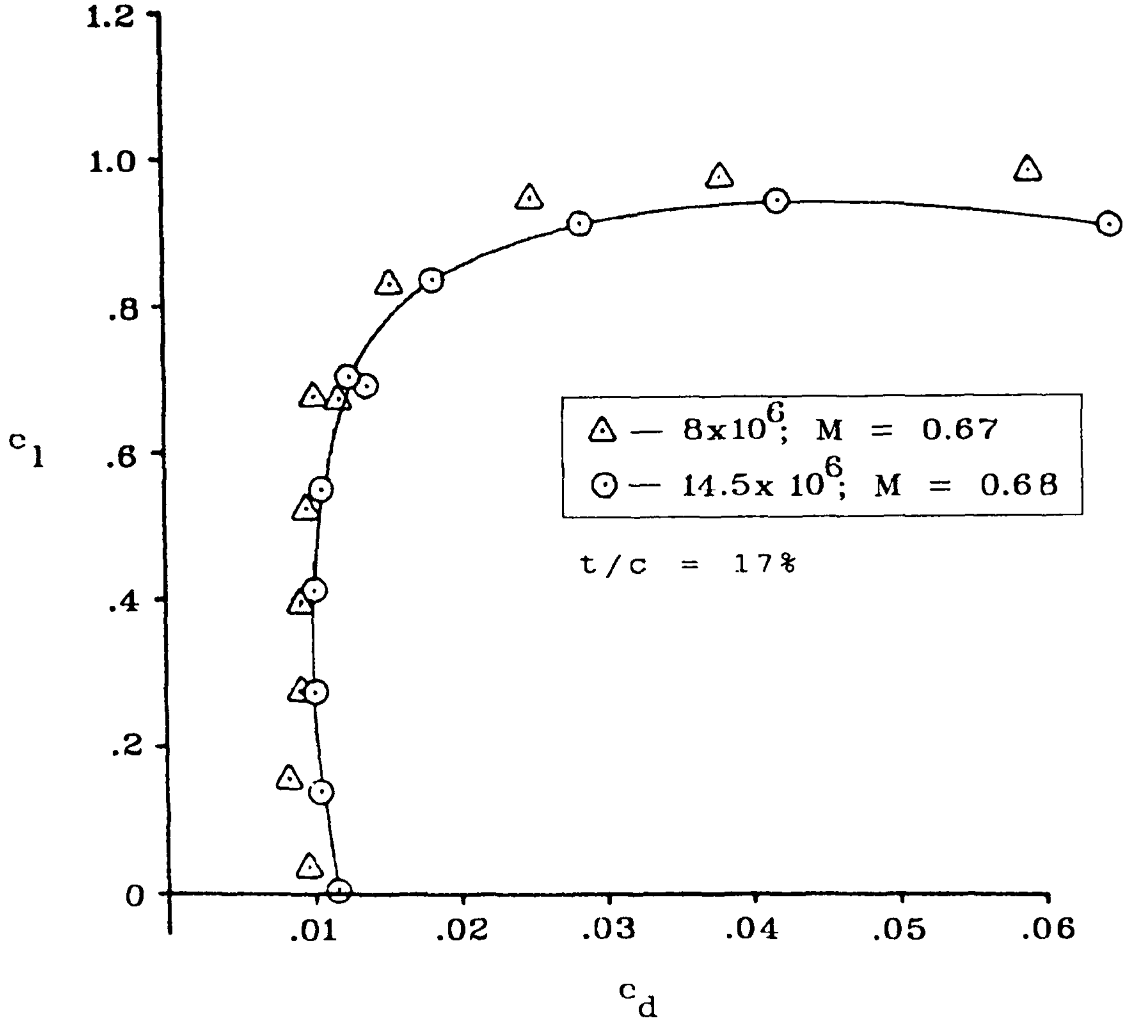


Figure X-10 Reynolds Number Effect on Drag (wing alone)

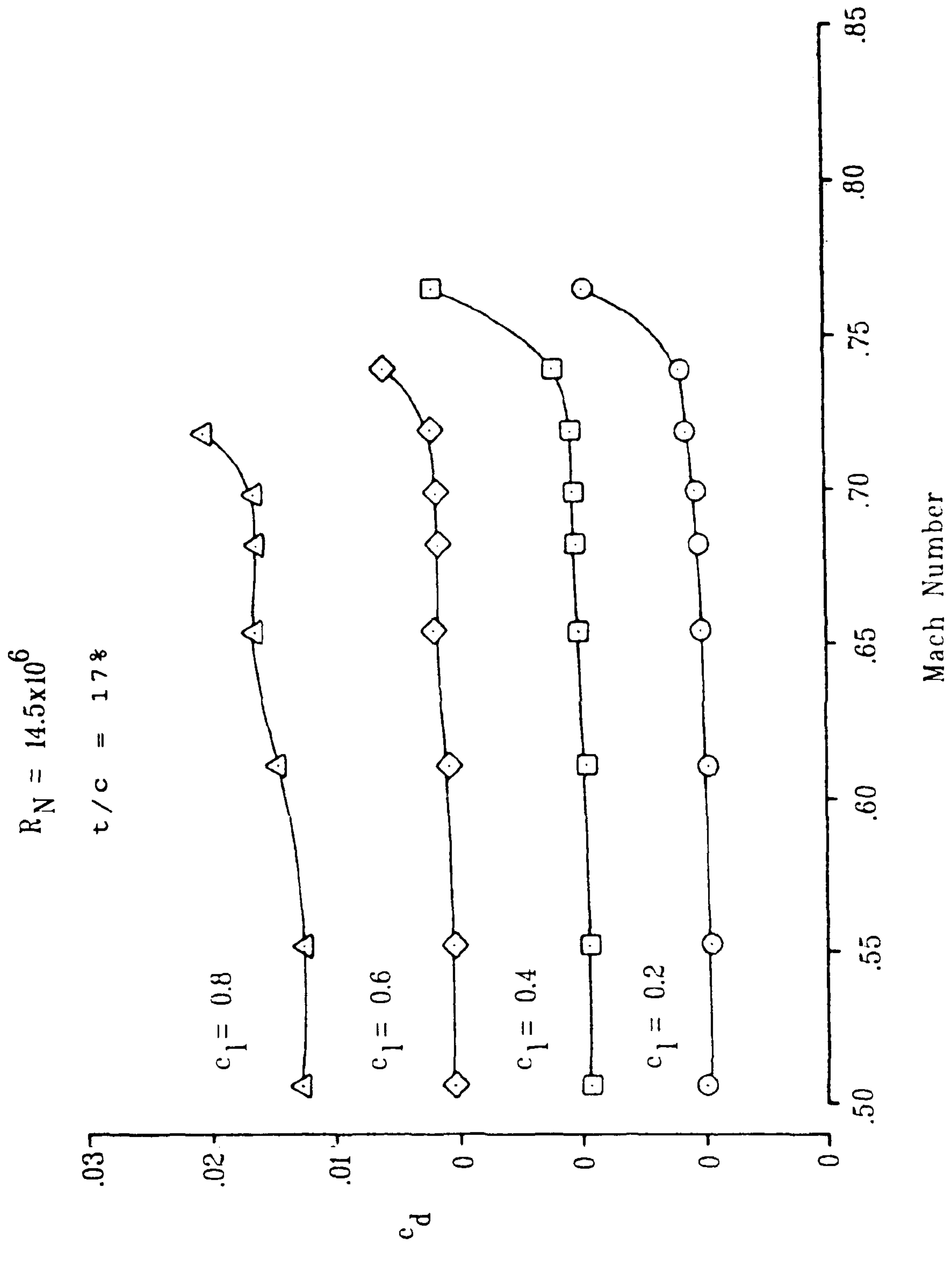


Figure X-11 Drag Rise Characteristics
(wing alone)

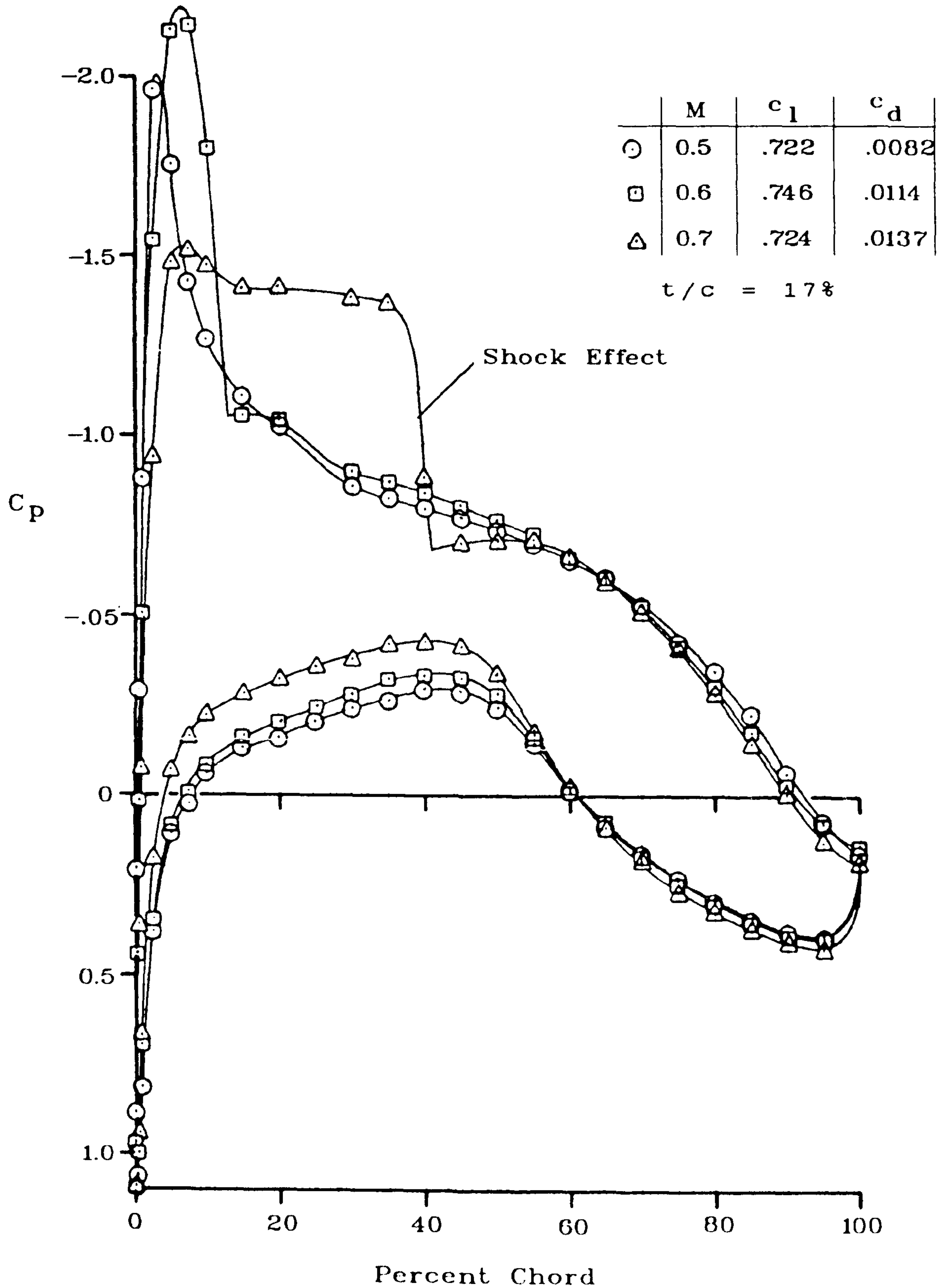


Figure X-12 Mach Effect on Airfoil Pressure Distribution

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XI INTERNAL STRAIN GAGE BALANCES

XI Internal Strain Gage Balances

(Ref. 36)

List of Symbols

A	Cross-sectional area
E	Voltage
F	Gage Factor
$\{F_A\}$	Applied load to balance matrix
$\{F_o\}$	Output (mv) force matrix
$[K_{ij}]^{-1}$	Inverted balance calibration matrix
L	Length
l	Rolling moment
m	Pitching moment
mv	Milli-volts
n	Yawing moment
R	Resistance
{SC}	Sensitivity constant matrix
wt	Weight
X	Axial force
Y	Side force
Z	Normal force
ε	Axial (local) strain
Δ	Incremental
μ	Poisson's ratio
ρ	Resistivity of the gage material

Subscripts

A - Applied load	AF - Axial force	NF - Normal force
O - Output (mv)	PM - Pitching moment	SF - Side Force
RM - Rolling moment	YM - Yawing moment	

Strain Gages

In strain measurement. the simplest device to use is the resistance type strain gage. Its construction and operation are simple, but it is so precise that strains on the order of 0.1% may be measured. The gage is about the size of a postage stamp but only slightly heavier. It consists of a metallic wire or strip of foil whose electrical resistance varies linearly with strain. The gage is securely bonded to the member to be strained so that any strain in the member due to a load is transmitted to the wire. There are literally hundreds of types of strain gages available commercially; each gage having been developed in response to a demand for a gage to meet a specific condition.

The most fundamental part of the gage is the wire itself. The resistance increase of a wire, when it is stretched, is due to an increase in the length and a decrease in its cross section, than to actual change in specific resistance. Typically, an electrical conductor (wire) is bonded to the specimen (balance) with an insulating cement under a no-load condition. A load is then applied, which produces a deformation in both the specimen and resistance element (wire). This deformation is indicated through a measurement of the change in resistance of the element (wires).

Three common types of resistance strain gages used in internal balances are wire, foil, and semi-conductor. The bonded wire is most commonly used with the wire diameter varying between 0.005 and 0.001 inches. The foil gage usually employs a foil less than 0.001 inches thick. The semi-conductor gage employs a silicon base material that is strain sensitive and has the advantage of a very large gage factor (F~100). (The gage factor relates the electrical resistance to the physical properties of the gage.) The material is usually produced in brittle wafers having a thickness of 0.01 inches.

Wire and foil gages may be manufactured in various ways, but the important point is that the resistance element (wire/foil) be securely bonded to its mounting. Most wire strain gages employ either a nitrocellulose cement or a phenolic resin for the bonding agent with a thin paper backing to maintain the wire's configuration. Such gages may be used up to $300^{\circ}F$. A Bakelite mounting is usually employed for temperatures up to $500^{\circ}F$. Foil gages are manufactured by an etching process and use base material of paper, Bakelite, and epoxy film. Epoxy cement is also used on wire and foil gages.

Temperature Effects

The major source of strain gage error is the fact that the resistance of most wires changes with temperature. This variation is not only a function of the change in temperature but, may also be a function of the number of heating cycles to which the gages has been subjected and of the time elapsed between cycles. Temperature compensation may easily be accomplished by installing a second strain gage, often known as a dummy gage, on an unstrained piece of the same metal that the active gage is bonded to. There are two changes in gage resistance which are caused by temperature changes. The first effect is the temperature coefficient of resistivity which is a random effect that may be considered as independent of the gage factor. The second effect is the difference in linear expansion between the gage and the member (material) on which it is mounted. The latter effect is one in which the error in the strain gage output is significant. Semi-conductor gages offer the advantage that they have a lower expansion coefficient then either wire or foil gages. It's best to make sure the strain gages are temperature compensated.

Deformation Theory and Calculation

Basic relations for the resistance strain gage are

$$R = \rho\left(\frac{L}{A}\right) \tag{1}$$

L = length

A = cross-sectional area

ρ = resistivity of the material

Differentiating (1)

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \tag{2}$$

The area may be related to the square of some transverse dimension, such as the diameter (D) of the resistance wire.

$$\frac{dA}{A} = 2\left(\frac{dD}{D}\right) \tag{3}$$

The unit axial strain ε is defined as

$$\varepsilon = \frac{dL}{L} \tag{4}$$

Poisson's ratio is defined as

$$\mu = \frac{dD/D}{dL/L} \tag{5}$$

Substituting equations (3), (4) and (5) into (2)...

$$\frac{dR}{R} = \varepsilon(1 + 2\mu) + \frac{d\rho}{\rho} \tag{6}$$

The gage factor 'F' is defined as

$$F = \frac{dR/R}{\varepsilon} \tag{7}$$

Substituting (7) into (6)

$$F = 1 + 2\mu + \left(\frac{1}{\varepsilon}\right)\left(\frac{d\rho}{\rho}\right) \tag{8}$$

Rearranging (7)

Local Strain

$$\boxed{\varepsilon = \frac{1}{F}\left(\frac{\Delta R}{R}\right)} \tag{9}$$

The value of the gage factor 'F' and the resistance (R) are usually specified by the manufacturer so that the user need only measure the change of resistance (ΔR) in order to determine the local strain due to a load.

Measurement of ΔR/R

Consider the basic Wheatstone bridge in figure XI-1. The voltage at the detector is given by

$$E_D = E\left[\frac{R_1}{R_1+R_4} - \frac{R_2}{R_2+R_3}\right] \tag{10}$$

If the bridge is balanced then $E_D = 0.0$ volts.

Let the strain gage represent R_1 in the circuit in figure XI-2 and a voltage readout is used such that the bridge operates as a voltage sensitive circuit. We assume that the bridge is balanced at zero strain conditions and that a strain , ε, on the gage results in the change in the resistance ΔR_1 and a change of voltage ΔE on the bridge. R_1 will be used to represent the resistance of the gage at zero strain conditions.

The voltage due to strain is...

$$\frac{\Delta E_D}{E} = \frac{R_1+\Delta R_1}{R_1+\Delta R_1+R_4} - \frac{R_2}{R_2+R_3} \tag{11}$$

Solving for the resistance change...

$$\boxed{\frac{\Delta R_1}{R_1} = \frac{(R_4/R_1)[\Delta E_D/E + R_2/(R_2+R_3)]}{1 - \Delta E_D/E - R_2/(R_2+R_3)} - 1} \tag{13}$$

Balance Calibration (interaction) Matrix

(ref. 8 and 37)

No internal balance is able to measure the pure loads that it was intended to measure. This is due to errors within the balance. There are generally two types of errors. The first type of 'balance' errors are the linear (first degree) errors. This type of errors are primarily due to construction errors, improperly positioned gages, variation in gage factors, and electrical circuits. The second type of error is the non-linear (2nd degree) error. This error is primarily attributed to elastic deformation (deflections) of various balance parts. Both types of errors create interaction of forces and moments.

To account for these interactions (errors) in the balance, the balance is calibrated. The purpose of this calibration is to acquire a set of interaction equations that can be used to determine the loads applied by the model through the balance output (voltage) signals. This type of calibration will not (generally) be accomplished by the testing engineer. All the testing engineer will receive will be a sheet(s) of paper with the linear and non-linear interaction coefficients on it (figure XI-3). For a six component (3 forces and 3 moments) balance, the linear coefficients will be used as a 6x6 matrix with near unity (1) on the diagonal. The non-linear coefficients will be used as a 6x21. For a six component balance, there is a total of 27 interaction terms. Generally speaking, the 6x6 matrix will need to be <u>inverted</u> to be used in the data acquisition computer. This can be seen below in the theory of how the computer acquires force/moment data from a balance.

$$\{F_O\} = [K_{ij}]\{F_A\}$$

$$\{F_A\} = [K_{ij}]^{-1}\{SC\}\{F_O\} \tag{14}$$

<u>Example</u>:

Original Balance Calibration Matrix
(non-dimensional)

$$\begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix} = \begin{bmatrix} 1.0000 & .0336 & -.1764 & -.0549 & .0059 & -.0004 \\ -.0169 & 1.0000 & 0.0000 & .0131 & .0000 & .0000 \\ -.0059 & -.0022 & 1.0000 & -.0032 & .0000 & .0002 \\ -.0004 & -.0006 & .0078 & 1.0000 & .1012 & -.0044 \\ 0000 & .0002 & .0000 & -.0412 & 1.0000 & -.0064 \\ .0012 & .0049 & .0294 & -.1555 & -.0384 & 1.0000 \end{bmatrix} \begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix}$$

Inverted Original Balance Calibration Matrix

$$\begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix} = \begin{bmatrix} 1.0000 & -.0332 & -.176 & .0555 & -.0115 & .0005 \\ .0169 & .9994 & .0031 & -.0121 & .0011 & -.0000 \\ .006 & .002 & 1.001 & .0034 & -.0004 & -.0002 \\ .0004 & .0006 & -.0078 & .9964 & -.1007 & .0037 \\ .0000 & -.0002 & -.0005 & .0421 & .9959 & .0065 \\ -.0013 & -.0048 & -.0309 & .1564 & .0225 & 1.0008 \end{bmatrix} \begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix}$$

The interactions of the forces and moments can be seen in the inverted matrix. For an example, there is a -17.6% interaction on the normal force due to the the pitching moment and a 3.7% interaction on the rolling moment due to the side force.

The testing engineer needs to know if the balance coefficients are indeed non-dimensional or dimensional (ie... $(\frac{\text{load}}{\text{volts}})$). If the interaction matrix is non-dimensional, then the testing engineer will have to acquire a sensitivity constant $(\frac{\text{load}}{\text{volts}})$ for each force and moment. The acquisition of the sensitivity constants is acquired during the check loading of the balance (see page XI-10).

Calibration Body

In order to load a balance accurately, it is first necessary to construct a fixture. This fixture is generally called a calibration body (cal. body). The cal. body simulates the model to be tested which will allow loads to be accurately transmitted to the balance in the same manner as the model will during testing. It should be attached in the same manner as the model is attached to the balance (generally a single pin) and should have reference surfaces which will remain fixed with respect to the balance reference center (usually the balance electrical moment center)

The cal. body is an accurately machined 'rectangle box' with precisely located V-shaped grooves in which the loads are applied during calibration or check loadings. It is important that both the cal. body and the model be aligned with the balance in exactly the same manner. Also, the balance should fit tightly (no relative motion between the two items) in the cal. body and in the model. All fixtures (moment arms, weight pans etc...)

should be manufactured as light as possible and as stiff as possible.

Loads applied to the balance (via the cal. body) components are considered absolute loads. Therefore, the weight of the cal. body and fixtures and even parts of the balance must be considered as applied loads. However, the weight of the cal. body and fixtures are taken out in the 'zero' reading. The zero reading is the reference voltage to be subtracted out from the load reading to acquire a 'true' reading.

Check Loading

The purpose of check loading a balance serves several purposes.

1) Acquire sensitivity constants (if needed)
2) Proof (check) load the balance
3) Check gaging and wiring
4) Determine hysteresis and repeatability
5) Determine component sensitivity
6) Determine deflections when a load is applied
7) Determine accuracy

It is important to check load the balance over the entire anticipated load and center-of-pressure range and that sufficient points are taken to assure accurate and repeatable loadings. The minimum check load to hang would be 10% of the balance limit.

Check Loading Procedure

This procedure will assume that a dead weight (ie... disks) will be used to acquire the loadings required. However, a load is a load whether it is generated by a disk or hydraulic actuator or some other source. Just make sure it's a point load and not a distributed load on the calibration body.

Hang the weights on the calibration body at CONSTANT INCREMENTS, as an example, $\Delta 50$ lbf or $\Delta 100$ lbf or whatever weight increment you choose to use. Be sure to maintain that increment at all times. Don't forget to take a 'zero' voltage reading prior to hanging weights. A zero reading includes the weights of the cal. body, weight pan or any other weight that's considered a 'zero' weight. After loading a weight on the weight pan, stop the weight pan from swaying. Place an inclinometer on the cal. body and raise or lower the cal. body (via the sting) until it is level (or $< \pm 3$ mins of a degree). This takes out the weight component due to the deflection of the sting. Also, sting deflection measurements can be taken at this time. After the weight has been added and the swaying has been stopped, have the control room take a data point and record the output (normally in milli-volts [mv], and then add the next incremental weight and repeat the above procedure. Do this until the maximum load is achieved. Once the maximum load is applied and voltage taken, remove the weights at the same increment as when you added the weights and take a data point at each weight going down. Do this until all the weights have been removed and then take another zero. This up and down procedure of check loading will show accuracy, repeatability and a zero shift of the balance. Do this for all force and moment check loadings.

Dead-Weight Loading System

When the dead-weight system is employed, the weights are generally in the form of cast iron disks (accurate to 0.1%) hanging on a weight pan supported by the cal. body or moment arm fixture. For a highly accurate check loading, one can weigh the cast iron disks prior to check loading.

When hanging weights for the normal force and side force, at a minimum, hang the weights at the balance electrical moment center and use this check loading to determine the normal and side forces sensitivity constant. Taking the sensitivity constant for the forces at the balance electrical moment center, allows the contribution (interactions) of the principle moments (pitching and yawing) to become negligible.

Hanging the weights for the pitching and yawing moments, at a minimum, at one moment arm ahead and behind the electrical moment center but at different moment arms. This will result in a positive and a negative moment. When check loading for the rolling moment, place the rolling moment fixture at the balance electrical moment center and at a minimum, apply loads at two different moment arms on the rolling moment fixture (expect to see a normal force reading).

Axial Force Check Loading

Axial force check loading is generally accomplished with a pulley rig. A pulley rig has two wires that are connected to the cal. body that run down the side of the sting and over the pulleys down to a 'T' weight pan where the weights are placed. The 'T' weight pan must be absolutely level. Any angle in the 'T' weight pan will produce undesirable interactions and an erroneous axial force will be acquired. Also, it might be desirable to check load in the negative axial force direction. If this is desirable, set the pulley rig in front of the cal. body and hang weights. Be sure to check the balance limits to see if a negative axial force limit exists. Normally, a negative axial force calibration is not done.

Sensitivity Constants

Sensitivity constants are required when the balance calibration matrices are non-dimensional. They are acquired during check loading and are nothing but slopes of a line $\left[\frac{\Delta \text{load}}{\Delta \text{volts}}\right]$ and are required for each force and moment. The sensitivity matrix is a diagonal matrix.

To acquire sensitivity constants...

1) Put balance calibration matrix in the data acquisition computer.
2) Initially set sensitivity constants to unity (1).
3) Accomplish the check loading in all planes and at multiple positions within that plane.
4) When the check loadings are complete, obtain the 'Raw Balance Data' (figure XI-4). Be sure and account for the zero reading. Usually it needs to be subtracted out of the output readings. There should be 'n' forces and moments usually titled X, Y, Z, l, m, n. Where X = axial force, Y = side force, Z = normal force, l = rolling moment, m = pitching moment, and n = yawing moment.
5) At this point use a linear regression program to acquire the slope of the line with the x-axis as the voltage and the y-axis as the load. The slope of this line is the sensitivity constant for that force or moment. Do this for each position and for each force and moment. The force and moment sensitivity constants should not vary much from one another when taken at different positions on that plane. For the force sensitivity constants, use the sensitivity constants at the balance electrical moment center. This eliminates, theoretically, the influence of the moment of that plane.
6) If a linear regression program is not available at the test site, take the first weight reading (mv) and subtract it from second weight reading (mv). Then take the second weight reading and subtract it from the third weight reading and so on. The

Δmilli-volts (Δmv) should be close to one another since the weight increment (Δwt) is held constant.

7) Obtain an average Δmv.
8) Divide the Δwt by the average Δmv. This is the sensitivity constant for that position on the plane.
9) Repeat steps 6 3 for each position on the plane.
10) Steps 6-8 are not as accurate as using a linear regression program on the computer.
11) For the moments, add your sensitivities together and divide by the number of sensitivities to obtain and average. This average sensitivity constant for the moment is the one given to the software/tunnel personnel.
12) After the sensitivity constants have been acquired, re-hang selected loads on the cal. body once again to verify the validity of the constants. The applied load to the output load should be within 0.5% of one another.

Example Sensitivity Constant Matrix

$$
\begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix}
=
\begin{bmatrix}
2.3184 & 0 & 0 & 0 & 0 & 0 \\
0 & -.7844 & 0 & 0 & 0 & 0 \\
0 & 0 & -.1867 & 0 & 0 & 0 \\
0 & 0 & 0 & .1324 & 0 & 0 \\
0 & 0 & 0 & 0 & -.07780 & \\
0 & 0 & 0 & 0 & 0 & -1.188
\end{bmatrix}
\begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix}
$$

Obtaining Force and Moments from Raw Balance Data

1) Take the balance output (mv) and subtract out the zero reading (mv).
2) Take that increment (Δmv) and multiply it by that position on the balance calibration coefficient (ie... if you are checking the normal force, on the nxn matrix (which is usually inverted) go to the Z,Z position), and multiply it by the its sensitivity constant.However, don't forget the interaction terms. They'll need to be added or subtracted to the force or moment you are looking at for an accurate reading.

F or M = Δmv(balance calibration)(sensitivity constant) + interaction terms

F = force; M = moment

Don't forget that each interaction term also has a sensitivity factor associated with it. Keep a close eye on the units. Don't mix apples and oranges.

Balance Calibration Equation Example (ref. 8)

The example below uses a 6 component balance calibration matrix. The calibration matrix below is considered non-dimensional. If the calibration matrix was dimensional, then consider the sensitivity constants on the diagonal of the sensitivity constant matrix as unity (1).

From equation 14

$$\{F_O\}\{SC\} = [K_{ij}]\{F_A\}$$

$$\{F_A\} = [K_{ij}]^{-1}\{SC\}\{F_O\}$$

Example Original Balance Calibration Matrix
(non-dimensional)

$$\begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix} = \begin{bmatrix} 1.0000 & .0336 & -.1764 & -.0549 & .0059 & -.0004 \\ -.0169 & 1.0000 & .0000 & .0131 & .0000 & .0000 \\ -.0059 & -.0022 & 1.0000 & -.0032 & .0000 & .0002 \\ -.0004 & -.0006 & .0078 & 1.0000 & .1012 & -.0044 \\ .0000 & .0002 & .0000 & -.0412 & 1.0000 & -.0064 \\ .0012 & .0049 & .0294 & -.1555 & -.0384 & 1.0000 \end{bmatrix} \begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix}$$

Example Inverted Balance Calibration Matrix

$$\begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix} = \begin{bmatrix} 1.0000 & -.0332 & -.176 & .0555 & -.0115 & .0005 \\ .0169 & .9994 & .0031 & -.0121 & .0011 & -.0000 \\ .006 & .002 & 1.001 & .0034 & -.0004 & -.0002 \\ .0004 & .0006 & -.0078 & .9964 & -.1007 & .0037 \\ .0000 & -.0002 & -.0005 & .0421 & .9959 & .0065 \\ -.0013 & -.0048 & -.0309 & .1564 & .0225 & 1.0008 \end{bmatrix} \begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix}$$

Example Sensitivity Constant Matrix

$$\begin{bmatrix} Z_A \\ X_A \\ m_A \\ l_A \\ n_A \\ Y_A \end{bmatrix} = \begin{bmatrix} 2.3184 & 0 & 0 & 0 & 0 & 0 \\ 0 & .7844 & 0 & 0 & 0 & 0 \\ 0 & 0 & -.1867 & 0 & 0 & 0 \\ 0 & 0 & 0 & .1324 & 0 & 0 \\ 0 & 0 & 0 & 0 & -.0778 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.188 \end{bmatrix} \begin{bmatrix} Z_o \\ X_o \\ m_o \\ l_o \\ n_o \\ Y_o \end{bmatrix}$$

A typical load (normal force) equation can be seen below.

$$Z_A = Z_O K_{11} SC_{NF} + X_O K_{12} SC_{AF} + Y_O K_{13} SC_{SF} + l_O K_{14} SC_{LM} + m_O K_{15} SC_{PM} + \ldots$$
$$\ldots + n_O K_{15} SC_{YM} \qquad (15)$$

Using the inverted and sensitive constant matrix above, the total normal force (Z) and pitching moment (m) equations are listed below.

$$Z = Z(1)(2.3184) + X(-.0332)(-.7844) + m(-.176)(-.1867) + \ldots$$
$$\ldots + l(.0555)(.1324) + n(-.0115)(-.0778) + Y(.0005)(-1.188)$$

$$m = Z(.006)(2.3184) + X(.002)(-.7844) + m(1.001)(-.1867) + \ldots$$
$$\ldots + l(.0034)(.1324) + n(-.0004)(-.0778) + Y(-.002)(-1.188)$$

The rest of the forces and moments equations are derived in the same manner.

Balance Placement in the Model

The physical placement of the balance in a wind tunnel model can vary. Usually, the balance electrical moment center is placed at the center of pressure of the configuration at the maximum aerodynamic load. Since the center of pressure position varies with angle-of-attack, make re the aerodynamic load is not outside the load of the balance.

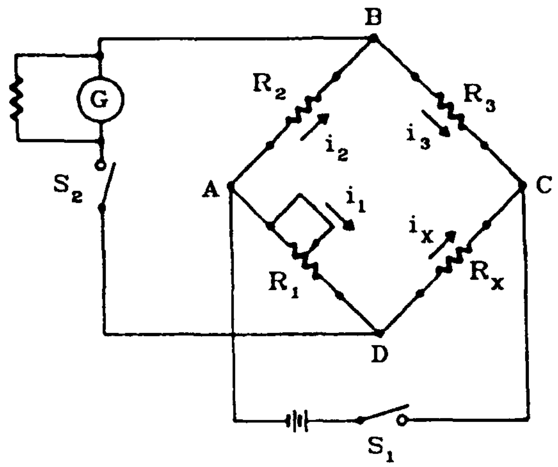

Figure XI-1 Basic Wheatstone Bridge

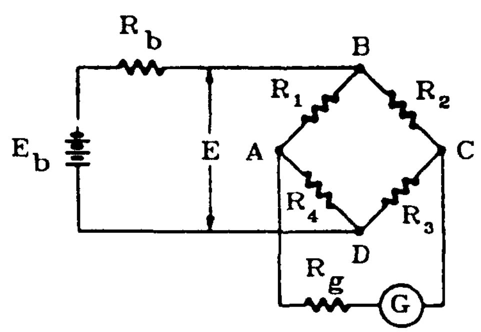

Figure XI-2 Unbalanced Wheatstone Bridge

Normal Force

CARD SEQUENCE	COEFFICIENT SYMBOL	VALUE ENGLISH	VALUE S.I.U.	COMPONENT(S) OPERATED ON BY COEFFICIENTS
	LINEAR INTERACTION COEFFICIENTS			
1	K(1)	----------	----------	NORMAL
2	K(2)	-1.6926E-02	-1.6926E-02	AXIAL
3	K(3)	-1.4982E-02	-5.8995E-03	PITCH
4	K(4)	-1.1272E-03	-4.4377E-04	ROLL
5	K(5)			YAW
6	K(6)	1.1570E-03	1.1570E-03	SIDE
	NONLINEAR COEFFICIENTS			
7	K(7)			NORMAL SQUARED
8	K(8)			NORMAL X AXIAL
9	K(9)			NORMAL X PITCH
10	K(10)			NORMAL X ROLL
11	K(11)			NORMAL X YAW
12	K(12)	-5.7771E-07	-1.2987E-07	NORMAL X SIDE
13	K(13)			AXIAL SQUARED
14	K(14)	-5.1202E-07	-4.5319E-08	AXIAL X PITCH
15	K(15)			AXIAL X ROLL
16	K(16)			AXIAL X YAW
17	K(17)			AXIAL X SIDE
18	K(18)	4.0264E-08	1.4032E-09	PITCH SQUARED
19	K(19)			PITCH X ROLL
20	K(20)			PITCH X YAW
21	K(21)			PITCH X SIDE
22	K(22)	-3.0464E-08	-1.0617E-09	ROLL SQUARED
23	K(23)	-9.7235E-08	-3.3886E-09	ROLL X YAW
24	K(24)	-8.6340E-07	-7.6420E-08	ROLL X SIDE
25	K(25)			YAW SQUARED
26	K(26)	2.0256E-07	1.7929E-08	YAW X SIDE
27	K(27)			SIDE SQUARED

Figure XI-3 Example Balance Calibration Sheet

Normal Force (Z) Calibration
(Balance Electrical Moment Center)

Wt Inc.	Data Pt	X	Y	Z	l	m	n
0	1	-1329.8	960.6	0.0	-1112.7	705.4	536.3
100	2	-1308.8	962.6	191.1	-1110.7	710.4	536.3
200	3	-1289.8	964.6	384.2	-1108.7	714.4	537.3
300	4	-1268.8	965.6	575.3	-1105.7	718.4	536.3
400	5	-1250.8	966.6	766.5	-1104.7	722.4	539.3
500	6	-1230.7	966.6	958.6	-1103.7	726.4	536.3
600	7	-1208.0	966.8	1150.9	-1101.9	731.6	534.4
700	8	-1186.7	967.6	1342.8	-1100.7	735.4	534.3
600	9	-1211.0	967.8	1149.9	-1102.9	730.6	536.4
500	10	-1227.7	964.6	957.6	-1103.7	725.4	536.3
400	11	-1250.8	966.6	765.5	-1106.7	721.4	536.3
300	12	-1268.8	965.6	573.3	-1107.7	727.4	536.3
200	13	-1287.8	965.6	381.2	-1110.7	713.4	538.3
100	14	-1306.8	964.6	189.1	-1112.7	709.4	540.3
0	15	-1326.8	962.6	0.0	-1113.7	704.4	541.3

Pitching Moment (m) Calibration
(2 inch moment arm)

Wt Inc.	Data Pt	X	Y	Z	l	m	n
0	16	-1324.8	962.6	-2.0	-1113.7	688.4	543.3
100	17	-1310.8	963.6	182.1	-1110.7	567.3	544.3
200	18	-1298.8	964.6	369.2	-1108.7	446.3	545.3
300	19	-1286.8	964.6	555.3	-1105.7	326.2	545.3
400	20	-1271.8	965.6	742.4	-1102.7	206.1	545.3
500	21	-1260.8	965.6	928.6	-1100.7	85.1	546.3
600	22	-1248.7	966.6	1114.7	-1099.7	-36.0	545.3
700	23	-1233.7	966.6	1301.8	-1097.7	-157.1	545.3
600	24	-1247.7	965.6	1114.7	-1098.7	-35.0	547.3
500	25	-1256.8	966.6	928.6	-1100.7	86.1	545.3
400	26	-1273.8	965.6	742.4	-1102.7	208.1	545.3
300	27	-1287.0	965.8	556.4	-1103.9	329.3	545.4
200	28	-1298.0	964.8	370.3	-1106.9	450.4	545.4
100	29	-1309.8	964.6	183.1	-1108.7	571.3	547.3
0	30	-1322.8	963.6	0.0	-1110.7	692.4	547.3

All readings are in milli-volts (mv)
Wt Inc. is in lbf

Figure XI-4 Raw Balance Data

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APPENDIX A

APPENDIX A

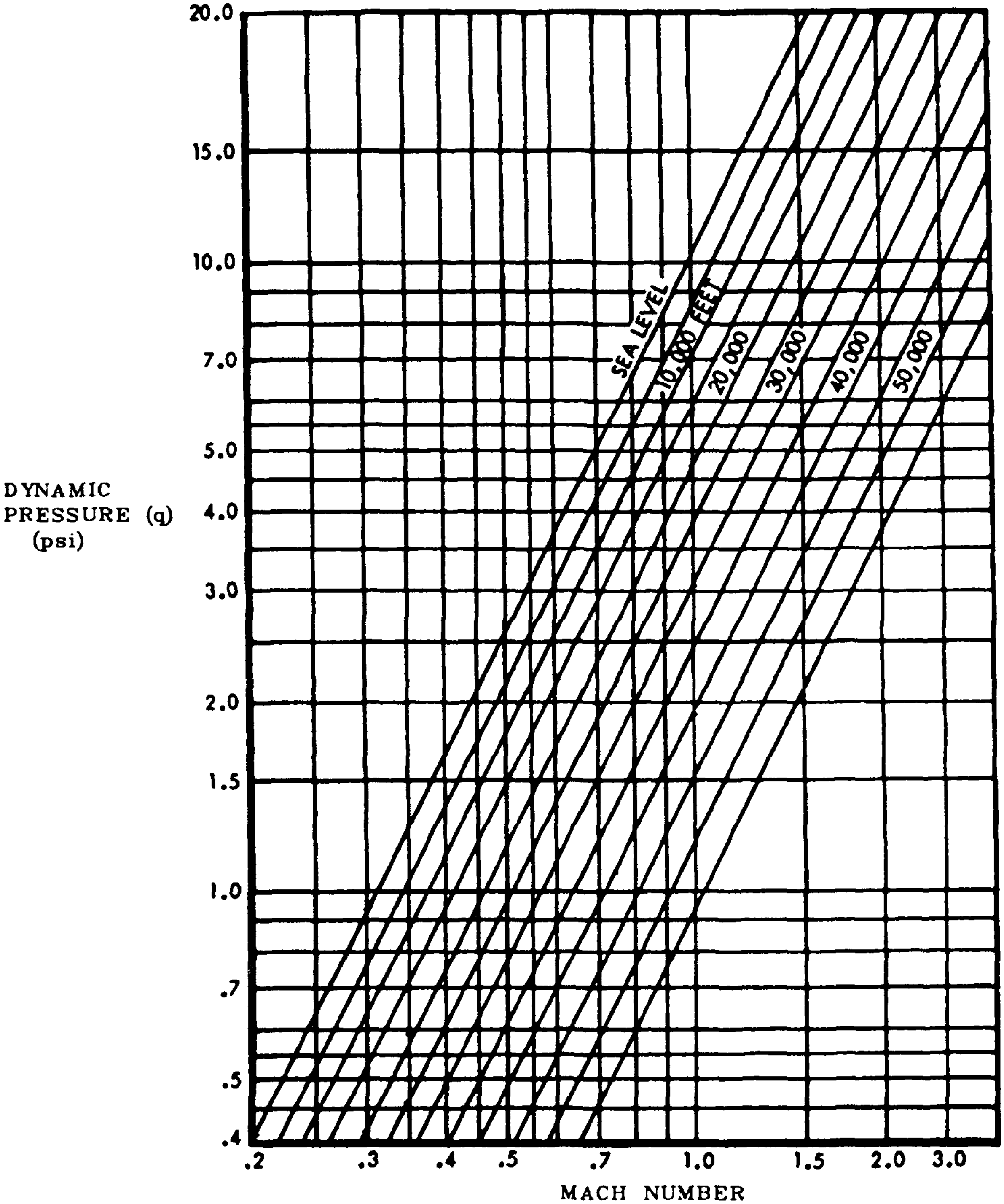


Figure A-1 Dynamic Pressure Determination (Standard Atmosphere)

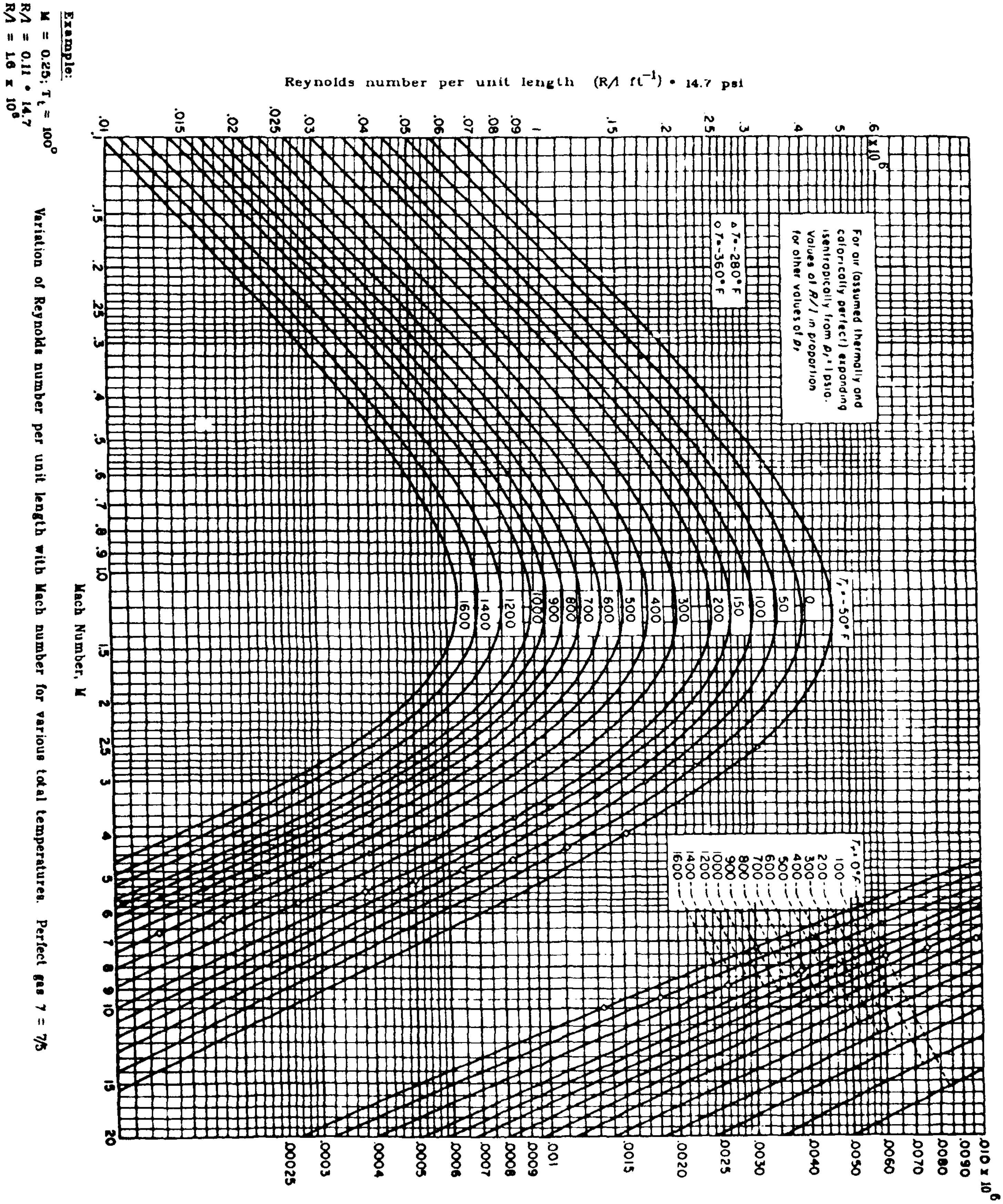


Figure A-2 Reynolds Number Determination (ref. 1)

Altitude ft	Temperature F	R	Pressure psf	psi	Density lbm/ft^3	ro/roØ	Speed of Sound ft/sec	mi/hr	Absolute Viscosity lbm/ft-s	Kinematic Viscosity ft^2/sec
0	58.7	518.4	2116.0	14.700	0.002378	1.0000	1107.64	760.9	3.725x10^-7	1.566x10^-4
2000	51.5	511.2	1968.0	13.672	0.002242	0.9428	1100.07	755.7	3.685	1.644
4000	44.4	504.1	1828.0	12.699	0.002112	0.8881	1092.36	750.4	3.644	1.725
6000	37.3	497.0	1696.0	11.782	0.001988	0.8358	1084.64	745.1	3.602	1.812
8000	30.2	489.9	1572.0	10.921	0.001869	0.7859	1076.78	739.7	3.561x10^-7	1.905x10^-4
10000	23.0	482.7	1455.0	10.108	0.001756	0.7384	1068.92	734.3	3.519	2.004
12000	15.9	475.6	1346.0	9.351	0.001648	0.6931	1060.91	728.8	3.476	2.109
14000	8.8	468.5	1243.0	8.635	0.001545	0.6499	1053.05	723.4	3.434	2.223
16000	1.6	461.3	1146.0	7.961	0.001448	0.6088	1046.21	718.7	3.391x10^-7	2.342x10^-4
18000	-5.5	454.2	1056.0	7.336	0.001355	0.5698	1036.75	712.2	3.348	2.471
20000	-12.6	447.1	972.1	6.753	0.001267	0.5327	1028.60	706.6	3.305	2.608
22000	-19.8	439.9	893.3	6.206	0.001183	0.4974	1020.59	701.1	3.261	2.756
24000	-26.9	432.8	819.8	5.695	0.001103	0.4640	1012.15	695.3	3.217x10^-7	2.916x10^-4
26000	-34.0	425.7	751.2	5.219	0.001028	0.4323	1003.71	689.5	3.173	3.086
28000	-41.2	418.5	687.4	4.775	0.000957	0.4023	995.26	683.7	3.128	3.268
30000	-48.3	411.4	628.0	4.363	0.000889	0.3740	986.82	677.9	3.083	3.468
32000	-55.4	404.3	572.9	3.980	0.000826	0.3472	978.23	672.0	3.038x10^-7	3.678x10^-4
34000	-62.5	397.2	521.7	3.624	0.000765	0.3218	969.50	666.0	2.992	3.911
36000	-67.3	392.4	474.4	3.296	0.000705	0.2963	963.67	662.0	2.962	4.204
38000	-67.3	392.4	431.1	2.995	0.000640	0.2692	963.67	662.0	2.962	4.625
40000	-67.3	392.4	391.9	2.723	0.000582	0.2448	963.67	662.0	2.962x10^-7	5.089x10^-4
42000	-67.3	392.4	356.2	2.475	0.000529	0.2225	963.67	662.0	2.962	5.599
44000	-67.3	392.4	323.7	2.249	0.000480	0.2021	963.67	662.0	2.962	6.161
46000	-67.3	392.4	294.2	2.044	0.000437	0.1838	963.67	662.0	2.962	6.778
48000	-67.3	392.4	267.4	1.858	0.000397	0.1670	963.67	662.0	2.962x10^-7	7.459x10^-4
50000	-67.3	392.4	243.1	1.689	0.000361	0.1518	963.67	662.0	2.962	8.206
52000	-67.3	392.4	220.9	1.535	0.000328	0.1379	963.67	662.0	2.962	9.028
54000	-67.3	392.4	200.8	1.395	0.000298	0.1254	963.67	662.0	2.962	9.933
56000	-67.3	392.4	182.5	1.268	0.000271	0.1140	963.67	662.0	2.962x10^-7	10.93x10^-4
58000	-67.3	392.4	165.9	1.153	0.000246	0.1036	963.67	662.0	2.962	12.02
60000	-67.3	392.4	150.8	1.048	0.000224	0.09415	963.67	662.0	2.962	13.23
62000	-67.3	392.4	137.1	0.952	0.000203	0.08557	963.67	662.0	2.962	14.56
64000	-67.3	392.4	124.6	0.866	0.000185	0.07777	963.67	662.0	2.962x10^-7	16.02x10^-4
65000	-67.3	392.4	118.7	0.825	0.000176	0.07414	963.67	662.0	2.962	16.8
70000	-67.3	392.4	93.53	0.650	0.000139	0.05839	963.67	662.0	2.962	21.33
75000	-67.3	392.4	73.66	0.512	0.000109	0.04599	963.67	662.0	2.962	27.09

Figure A-3 Standard Atmosphere

SUBSONIC FLOW

$\gamma = 7/5$

M	$\frac{p}{p_t}$	$\frac{\rho}{\rho_t}$	$\frac{T}{T_t}$	β	$\frac{q}{p_t}$	$\frac{A}{A_*}$	$\frac{V}{a_*}$	M	$\frac{p}{p_t}$	$\frac{\rho}{\rho_t}$	$\frac{T}{T_t}$	β	$\frac{q}{p_t}$	$\frac{A}{A_*}$	$\frac{V}{a_*}$
0	1.0000	1.0000	1.0000	1.0000	0	∞	0	0.50	0.8430	0.8852	0.9524	0.8660	0.1475	1.3398	0.53452
.01	.9999	1.0000	1.0000	1.0000	.7000 -4	57.8738	.01095	.51	.8374	.8809	.9506	.8602	.1525	1.3212	.54469
.02	.9997	.9998	.9999	.9998	.2799 -3	28.9421	.02191	.52	.8317	.8766	.9487	.8542	.1574	1.3034	.55483
.03	.9994	.9996	.9998	.9995	.6296 -3	19.3005	.03286	.53	.8259	.8723	.9468	.8480	.1624	1.2865	.56493
.04	.9989	.9992	.9997	.9992	.1119 -2	14.4815	.04381	.54	.8201	.8679	.9449	.8417	.1674	1.2703	.57501
.05	.9983	.9988	.9995	.9987	.1747 -2	11.5914	.05476	.55	.8142	.8634	.9430	.8352	.1724	1.2550	.58506
.06	.9975	.9982	.9993	.9982	.2514 -2	9.6659	.06570	.56	.8082	.8589	.9410	.8285	.1774	1.2403	.59507
.07	.9966	.9976	.9990	.9975	.3418 -2	8.2915	.07664	.57	.8022	.8544	.9390	.8216	.1825	1.2263	.60505
.08	.9955	.9968	.9987	.9968	.4460 -2	7.2616	.08758	.58	.7962	.8498	.9370	.8146	.1875	1.2130	.61501
.09	.9944	.9960	.9984	.9959	.5638 -2	6.4617	.09851	.59	.7901	.8451	.9349	.8074	.1925	1.2003	.62492
.10	.9930	.9950	.9980	.9950	.6951 -2	5.8218	.10944	.60	.7840	.8405	.9328	.8000	.1976	1.1882	.63481
.11	.9916	.9940	.9976	.9939	.8369 -2	5.2992	.12035	.61	.7778	.8357	.9307	.7924	.2026	1.1767	.64466
.12	.9900	.9928	.9971	.9928	.9979 -2	4.8643	.13126	.62	.7716	.8310	.9286	.7846	.2076	1.1657	.65448
.13	.9883	.9916	.9966	.9915	.1169 -1	4.4969	.14217	.63	.7654	.8262	.9265	.7766	.2127	1.1552	.66427
.14	.9864	.9903	.9961	.9902	.1353 -1	4.1824	.15306	.64	.7591	.8213	.9243	.7684	.2177	1.1452	.67402
.15	.9844	.9888	.9955	.9887	.1550 -1	3.9103	.16395	.65	.7528	.8164	.9221	.7599	.2227	1.1356	.68374
.16	.9823	.9873	.9949	.9871	.1760 -1	3.6727	.17482	.66	.7465	.8115	.9199	.7513	.2276	1.1265	.69342
.17	.9800	.9857	.9943	.9854	.1983 -1	3.4635	.18569	.67	.7401	.8066	.9176	.7424	.2326	1.1179	.70307
.18	.9776	.9840	.9936	.9837	.2217 -1	3.2779	.19654	.68	.7338	.8016	.9153	.7332	.2375	1.1097	.71268
.19	.9751	.9822	.9928	.9818	.2464 -1	3.1123	.20739	.69	.7274	.7966	.9131	.7238	.2424	1.1018	.72225
.20	.9725	.9803	.9921	.9798	.2723 -1	2.9635	.21822	.70	.7209	.7916	.9107	.7141	.2473	1.0944	.73179
.21	.9697	.9783	.9913	.9777	.2994 -1	2.8293	.22904	.71	.7145	.7865	.9084	.7042	.2521	1.0873	.74129
.22	.9668	.9762	.9904	.9755	.3276 -1	2.7076	.23984	.72	.7080	.7814	.9061	.6940	.2569	1.0806	.75076
.23	.9638	.9740	.9895	.9732	.3569 -1	2.5968	.25063	.73	.7016	.7763	.9037	.6834	.2617	1.0742	.76019
.24	.9607	.9718	.9886	.9708	.3874 -1	2.4956	.26141	.74	.6951	.7712	.9013	.6726	.2664	1.0681	.76958
.25	.9575	.9694	.9877	.9682	.4189 -1	2.4027	.27217	.75	.6886	.7660	.8989	.6614	.2711	1.0624	.77894
.26	.9541	.9670	.9867	.9656	.4515 -1	2.3173	.28291	.76	.6821	.7609	.8964	.6499	.2758	1.0570	.78825
.27	.9506	.9645	.9856	.9629	.4851 -1	2.2385	.29364	.77	.6756	.7557	.8940	.6380	.2804	1.0519	.79753
.28	.9470	.9619	.9846	.9600	.5197 -1	2.1656	.30435	.78	.6691	.7505	.8915	.6258	.2849	1.0471	.80677
.29	.9433	.9592	.9835	.9570	.5553 -1	2.0979	.31504	.79	.6625	.7452	.8890	.6131	.2894	1.0425	.81597
.30	.9395	.9564	.9823	.9539	.5919 -1	2.0351	.32572	.80	.6560	.7400	.8865	.6000	.2939	1.0382	.82514
.31	.9355	.9535	.9811	.9507	.6293 -1	1.9765	.33637	.81	.6495	.7347	.8840	.5864	.2983	1.0342	.83426
.32	.9315	.9506	.9799	.9474	.6677 -1	1.9219	.34701	.82	.6430	.7295	.8815	.5724	.3027	1.0305	.84335
.33	.9274	.9476	.9787	.9440	.7069 -1	1.8707	.35762	.83	.6365	.7242	.8789	.5578	.3069	1.0270	.85239
.34	.9231	.9445	.9774	.9404	.7470 -1	1.8229	.36822	.84	.6300	.7189	.8763	.5426	.3112	1.0237	.86140
.35	.9188	.9413	.9761	.9367	.7879 -1	1.7780	.37879	.85	.6235	.7136	.8737	.5268	.3153	1.0207	.87037
.36	.9143	.9380	.9747	.9330	.8295 -1	1.7358	.38935	.86	.6170	.7083	.8711	.5103	.3195	1.0179	.87929
.37	.9098	.9347	.9733	.9290	.8719 -1	1.6961	.39988	.87	.6106	.7030	.8685	.4931	.3235	1.0153	.88818
.38	.9052	.9313	.9719	.9250	.9149 -1	1.6587	.41039	.88	.6041	.6977	.8659	.4750	.3275	1.0129	.89703
.39	.9004	.9278	.9705	.9208	.9587 -1	1.6234	.42087	.89	.5977	.6924	.8632	.4560	.3314	1.0108	.90583
.40	.8956	.9243	.9690	.9165	.1003	1.5901	.43133	.90	.5913	.6870	.8606	.4359	.3352	1.0089	.91460
.41	.8907	.9207	.9675	.9121	.1048	1.5587	.44177	.91	.5849	.6817	.8579	.4146	.3390	1.0071	.92332
.42	.8857	.9170	.9659	.9075	.1094	1.5289	.45218	.92	.5785	.6764	.8552	.3919	.3427	1.0056	.93201
.43	.8807	.9132	.9643	.9028	.1140	1.5007	.46257	.93	.5721	.6711	.8525	.3676	.3464	1.0043	.94065
.44	.8755	.9094	.9627	.8980	.1187	1.4740	.47293	.94	.5658	.6658	.8498	.3412	.3500	1.0031	.94925
.45	.8703	.9055	.9611	.8930	.1234	1.4487	.48326	.95	.5595	.6604	.8471	.3122	.3534	1.0022	.95781
.46	.8650	.9016	.9594	.8879	.1281	1.4246	.49357	.96	.5532	.6551	.8444	.2800	.3569	1.0014	.96633
.47	.8596	.8976	.9577	.8827	.1329	1.4018	.50385	.97	.5469	.6498	.8416	.2431	.3602	1.0008	.97481
.48	.8541	.8935	.9560	.8773	.1378	1.3801	.51410	.98	.5407	.6445	.8389	.1990	.3635	1.0003	.98325
.49	.8486	.8894	.9542	.8717	.1426	1.3595	.52433	.99	.5345	.6392	.8361	.1411	.3667	1.0001	.99165
								1.00	.5283	.6339	.8333	.0000	.3698	1.0000	1.00000

TABLE II.—SUPERSONIC FLOW

$\gamma = 7/5$

M or M_1	$\frac{p}{p_t}$	$\frac{\rho}{\rho_t}$	$\frac{T}{T_t}$	β	$\frac{q}{p_t}$	$\frac{A}{A_*}$	$\frac{V}{a_*}$	ν	μ	M_2	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{t_2}}{p_{t_1}}$	$\frac{p_1}{p_{t_2}}$
1.00	0.5283	.6339	0.8333	0	0.3698	1.000	1.00000	0	90.00	1.000	1.000	1.000	1.000	1.000	0.5283
1.01	.5221	.6287	.8306	.1418	.3728	1.000	1.00831	.04473	81.93	.9901	1.023	1.017	1.007	1.000	.5221
1.02	.5160	.6234	.8278	.2010	.3758	1.000	1.01658	.1257	78.64	.9805	1.047	1.033	1.013	1.000	.5160
1.03	.5099	.6181	.8250	.2468	.3787	1.001	1.02481	.2294	76.14	.9712	1.071	1.050	1.020	1.000	.5100
1.04	.5039	.6129	.8222	.2857	.3815	1.001	1.03300	.3510	74.06	.9620	1.095	1.067	1.026	.9999	.5039
1.05	.4979	.6077	.8193	.3202	.3842	1.002	1.04114	.4874	72.25	.9531	1.120	1.084	1.033	.9999	.4980
1.06	.4919	.6024	.8165	.3516	.3869	1.003	1.04925	.6367	70.63	.9444	1.144	1.101	1.039	.9997	.4920
1.07	.4860	.5972	.8137	.3807	.3895	1.004	1.05731	.7973	69.16	.9360	1.169	1.118	1.046	.9996	.4861
1.08	.4800	.5920	.8108	.4079	.3919	1.005	1.06533	.9680	67.81	.9277	1.194	1.135	1.052	.9994	.4803
1.09	.4742	.5869	.8080	.4337	.3944	1.006	1.07331	1.148	66.55	.9196	1.219	1.152	1.059	.9992	.4746
1.10	.4684	.5817	.8052	.4583	.3967	1.008	1.08124	1.336	65.38	.9118	1.245	1.169	1.065	.9989	.4689
1.11	.4626	.5766	.8023	.4818	.3990	1.010	1.08913	1.532	64.28	.9041	1.271	1.186	1.071	.9986	.4632
1.12	.4568	.5714	.7994	.5044	.4011	1.011	1.09699	1.735	63.23	.8966	1.297	1.203	1.078	.9982	.4576
1.13	.4511	.5663	.7966	.5262	.4032	1.013	1.10479	1.944	62.25	.8892	1.323	1.221	1.084	.9978	.4521
1.14	.4455	.5612	.7937	.5474	.4052	1.015	1.11256	2.160	61.31	.8820	1.350	1.238	1.090	.9973	.4467
1.15	.4398	.5562	.7908	.5679	.4072	1.017	1.12029	2.381	60.41	.8750	1.376	1.255	1.097	.9967	.4413
1.16	.4343	.5511	.7879	.5879	.4090	1.020	1.12797	2.607	59.55	.8682	1.403	1.272	1.103	.9961	.4360
1.17	.4287	.5461	.7851	.6074	.4108	1.022	1.13561	2.839	58.73	.8615	1.430	1.290	1.109	.9953	.4307
1.18	.4232	.5411	.7822	.6264	.4125	1.025	1.14321	3.074	57.94	.8549	1.458	1.307	1.115	.9946	.4255
1.19	.4178	.5361	.7793	.6451	.4141	1.028	1.15077	3.314	57.18	.8485	1.485	1.324	1.122	.9937	.4204
1.20	.4124	.5311	.7764	.6633	.4157	1.030	1.15828	3.558	56.44	.8422	1.513	1.342	1.128	.9928	.4154
1.21	.4070	.5262	.7735	.6812	.4171	1.033	1.16575	3.806	55.74	.8360	1.541	1.359	1.134	.9918	.4104
1.22	.4017	.5213	.7706	.6989	.4185	1.037	1.17319	4.057	55.05	.8300	1.570	1.376	1.141	.9907	.4055
1.23	.3964	.5164	.7677	.7162	.4198	1.040	1.18057	4.312	54.39	.8241	1.598	1.394	1.147	.9896	.4006
1.24	.3912	.5115	.7648	.7332	.4211	1.043	1.18792	4.569	53.75	.8183	1.627	1.411	1.153	.9884	.3958

ν = Prandtl-Mayer angle

μ = Mach angle

$\beta = \sqrt{|M^2 - 1|}$

Figure A-4 Compressible Flow Parameters (ref. 1)

SUPERSONIC FLOW—Continued

$\gamma=7/5$

M or M_1	$\frac{p}{p_t}$	$\frac{\rho}{\rho_t}$	$\frac{T}{T_t}$	β	$\frac{q}{p_t}$	$\frac{A}{A_*}$	$\frac{V}{a_*}$	ν	μ	M_2	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{t_2}}{p_{t_1}}$	$\frac{p_1}{p_{t_2}}$
1.25	.3861	.5067	.7619	.7500	.4223	1.047	1.19523	4.830	53.13	.8126	1.656	1.429	1.159	.9871	.3911
1.26	.3809	.5019	.7590	.7666	.4233	1.050	1.20249	5.093	52.53	.8071	1.686	1.446	1.166	.9857	.3865
1.27	.3759	.4971	.7561	.7829	.4244	1.054	1.20972	5.359	51.94	.8016	1.715	1.463	1.172	.9842	.3819
1.28	.3708	.4923	.7532	.7990	.4253	1.058	1.21690	5.627	51.38	.7963	1.745	1.481	1.178	.9827	.3774
1.29	.3658	.4876	.7503	.8149	.4262	1.062	1.22404	5.898	50.82	.7911	1.775	1.498	1.185	.9811	.3729
1.30	.3609	.4829	.7474	.8307	.4270	1.066	1.23114	6.170	50.28	.7860	1.805	1.516	1.191	.9794	.3685
1.31	.3560	.4782	.7445	.8462	.4277	1.071	1.23819	6.445	49.76	.7809	1.835	1.533	1.197	.9776	.3642
1.32	.3512	.4736	.7416	.8616	.4283	1.075	1.24521	6.721	49.25	.7760	1.866	1.551	1.204	.9756	.3599
1.33	.3464	.4690	.7387	.8769	.4289	1.080	1.25218	7.000	48.75	.7712	1.897	1.568	1.210	.9738	.3557
1.34	.3417	.4644	.7358	.8920	.4294	1.084	1.25912	7.280	48.27	.7664	1.928	1.585	1.216	.9718	.3516
1.35	.3370	.4598	.7329	.9069	.4299	1.089	1.26601	7.561	47.79	.7618	1.960	1.603	1.223	.9697	.3475
1.36	.3323	.4553	.7300	.9217	.4303	1.094	1.27286	7.844	47.33	.7572	1.991	1.620	1.229	.9676	.3435
1.37	.3277	.4508	.7271	.9364	.4306	1.099	1.27968	8.128	46.88	.7527	2.023	1.638	1.235	.9653	.3395
1.38	.3232	.4463	.7242	.9510	.4308	1.104	1.28645	8.413	46.44	.7483	2.055	1.655	1.242	.9630	.3356
1.39	.3187	.4418	.7213	.9655	.4310	1.109	1.29318	8.699	46.01	.7440	2.087	1.672	1.248	.9607	.3317
1.40	.3142	.4374	.7184	.9798	.4311	1.115	1.29987	8.987	45.58	.7397	2.120	1.690	1.255	.9582	.3280
1.41	.3098	.4330	.7155	.9940	.4312	1.120	1.30652	9.276	45.17	.7355	2.153	1.707	1.261	.9557	.3242
1.42	.3055	.4287	.7126	1.008	.4312	1.126	1.31313	9.565	44.77	.7314	2.186	1.724	1.268	.9531	.3205
1.43	.3012	.4244	.7097	1.022	.4311	1.132	1.31970	9.855	44.37	.7274	2.219	1.742	1.274	.9504	.3169
1.44	.2969	.4201	.7069	1.036	.4310	1.138	1.32623	10.146	43.98	.7235	2.253	1.759	1.281	.9476	.3133
1.45	.2927	.4158	.7040	1.050	.4308	1.144	1.33272	10.438	43.60	.7196	2.286	1.776	1.287	.9448	.3098
1.46	.2886	.4116	.7011	1.064	.4306	1.150	1.33917	10.731	43.23	.7157	2.320	1.793	1.294	.9420	.3063
1.47	.2845	.4074	.6982	1.077	.4303	1.156	1.34558	11.023	42.86	.7120	2.354	1.811	1.300	.9390	.3029
1.48	.2804	.4032	.6954	1.091	.4299	1.163	1.35195	11.317	42.51	.7083	2.389	1.828	1.307	.9360	.2996
1.49	.2764	.3991	.6925	1.105	.4295	1.169	1.35828	11.611	42.16	.7047	2.423	1.845	1.314	.9329	.2962
1.50	.2724	.3950	.6897	1.118	.4290	1.176	1.36458	11.905	41.81	.7011	2.458	1.862	1.320	.9298	.2930
1.51	.2685	.3909	.6868	1.131	.4285	1.183	1.37083	12.200	41.47	.6976	2.493	1.879	1.327	.9266	.2898
1.52	.2646	.3869	.6840	1.145	.4279	1.190	1.37705	12.495	41.14	.6941	2.529	1.896	1.334	.9233	.2866
1.53	.2608	.3829	.6811	1.158	.4273	1.197	1.38322	12.790	40.81	.6907	2.564	1.913	1.340	.9200	.2835
1.54	.2570	.3789	.6783	1.171	.4266	1.204	1.38936	13.086	40.49	.6874	2.600	1.930	1.347	.9166	.2804
1.55	.2533	.3750	.6754	1.184	.4259	1.212	1.39546	13.381	40.18	.6841	2.636	1.947	1.354	.9132	.2773
1.56	.2496	.3710	.6726	1.197	.4252	1.219	1.40152	13.677	39.87	.6809	2.673	1.964	1.361	.9097	.2744
1.57	.2459	.3672	.6698	1.210	.4243	1.227	1.40755	13.973	39.56	.6777	2.709	1.981	1.367	.9061	.2714
1.58	.2423	.3633	.6670	1.223	.4235	1.234	1.41353	14.269	39.27	.6746	2.746	1.998	1.374	.9026	.2685
1.59	.2388	.3595	.6642	1.236	.4226	1.242	1.41948	14.564	38.97	.6715	2.783	2.015	1.381	.8989	.2656
1.60	.2353	.3557	.6614	1.249	.4216	1.250	1.42539	14.861	38.68	.6684	2.820	2.032	1.388	.8952	.2628
1.61	.2318	.3520	.6586	1.262	.4206	1.258	1.43127	15.156	38.40	.6655	2.857	2.049	1.395	.8915	.2600
1.62	.2284	.3483	.6558	1.275	.4196	1.267	1.43710	15.452	38.12	.6625	2.895	2.065	1.402	.8877	.2573
1.63	.2250	.3446	.6530	1.287	.4185	1.275	1.44290	15.747	37.84	.6596	2.933	2.082	1.409	.8838	.2546
1.64	.2217	.3409	.6502	1.300	.4174	1.284	1.44866	16.043	37.57	.6568	2.971	2.099	1.416	.8799	.2519
1.65	.2184	.3373	.6475	1.312	.4162	1.292	1.45439	16.338	37.31	.6540	3.010	2.115	1.423	.8760	.2493
1.66	.2151	.3337	.6447	1.325	.4150	1.301	1.46008	16.633	37.04	.6512	3.048	2.132	1.430	.8720	.2467
1.67	.2119	.3302	.6419	1.337	.4138	1.310	1.46573	16.928	36.78	.6485	3.087	2.148	1.437	.8680	.2442
1.68	.2088	.3266	.6392	1.350	.4125	1.319	1.47135	17.222	36.53	.6458	3.126	2.165	1.444	.8640	.2417
1.69	.2057	.3232	.6364	1.362	.4112	1.328	1.47693	17.516	36.28	.6431	3.165	2.181	1.451	.8598	.2392
1.70	.2026	.3197	.6337	1.375	.4098	1.338	1.48247	17.810	36.03	.6405	3.205	2.198	1.458	.8557	.2368
1.71	.1996	.3163	.6310	1.387	.4085	1.347	1.48798	18.103	35.79	.6380	3.245	2.214	1.466	.8516	.2344
1.72	.1966	.3129	.6283	1.399	.4071	1.357	1.49345	18.397	35.55	.6355	3.285	2.230	1.473	.8474	.2320
1.73	.1936	.3095	.6256	1.412	.4056	1.367	1.49889	18.689	35.31	.6330	3.325	2.247	1.480	.8431	.2296
1.74	.1907	.3062	.6229	1.424	.4041	1.376	1.50429	18.981	35.08	.6305	3.366	2.263	1.487	.8389	.2273
1.75	.1878	.3029	.6202	1.436	.4026	1.386	1.50966	19.273	34.85	.6281	3.406	2.279	1.495	.8346	.2251
1.76	.1850	.2996	.6175	1.448	.4011	1.397	1.51499	19.565	34.62	.6257	3.447	2.295	1.502	.8302	.2228
1.77	.1822	.2964	.6148	1.460	.3996	1.407	1.52029	19.855	34.40	.6234	3.488	2.311	1.509	.8259	.2206
1.78	.1794	.2931	.6121	1.473	.3980	1.418	1.52555	20.146	34.18	.6210	3.530	2.327	1.517	.8215	.2184
1.79	.1767	.2900	.6095	1.485	.3964	1.428	1.53078	20.436	33.96	.6188	3.571	2.343	1.524	.8171	.2163
1.80	.1740	.2868	.6068	1.497	.3947	1.439	1.53598	20.725	33.75	.6165	3.613	2.359	1.532	.8127	.2142
1.81	.1714	.2837	.6041	1.509	.3931	1.450	1.54114	21.014	33.54	.6143	3.655	2.375	1.539	.8082	.2121
1.82	.1688	.2806	.6015	1.521	.3914	1.461	1.54626	21.302	33.33	.6121	3.698	2.391	1.547	.8038	.2100
1.83	.1662	.2776	.5989	1.533	.3897	1.472	1.55136	21.590	33.12	.6099	3.740	2.407	1.554	.7993	.2080
1.84	.1637	.2745	.5963	1.545	.3879	1.484	1.55642	21.877	32.92	.6078	3.783	2.422	1.562	.7948	.2060
1.85	.1612	.2715	.5936	1.556	.3862	1.495	1.56145	22.163	32.72	.6057	3.826	2.438	1.569	.7902	.2040
1.86	.1587	.2686	.5910	1.568	.3844	1.507	1.56644	22.449	32.52	.6036	3.870	2.454	1.577	.7857	.2020
1.87	.1563	.2656	.5884	1.580	.3826	1.519	1.57140	22.735	32.33	.6016	3.913	2.469	1.585	.7811	.2001
1.88	.1539	.2627	.5859	1.592	.3808	1.531	1.57633	23.019	32.13	.5996	3.957	2.485	1.592	.7765	.1982
1.89	.1516	.2598	.5833	1.604	.3790	1.543	1.58123	23.303	31.94	.5976	4.001	2.500	1.600	.7720	.1963
1.90	.1492	.2570	.5807	1.616	.3771	1.555	1.58609	23.586	31.76	.5956	4.045	2.516	1.608	.7674	.1945
1.91	.1470	.2542	.5782	1.627	.3753	1.568	1.59092	23.869	31.57	.5937	4.089	2.531	1.616	.7627	.1927
1.92	.1447	.2514	.5756	1.639	.3734	1.580	1.59572	24.151	31.39	.5918	4.134	2.546	1.624	.7581	.1909
1.93	.1425	.2486	.5731	1.651	.3715	1.593	1.60049	24.432	31.21	.5899	4.179	2.562	1.631	.7535	.1891
1.94	.1403	.2459	.5705	1.662	.3696	1.606	1.60523	24.712	31.03	.5880	4.224	2.577	1.639	.7488	.1873
1.95	.1381	.2432	.5680	1.674	.3677	1.619	1.60993	24.992	30.85	.5862	4.270	2.592	1.647	.7442	.1856
1.96	.1360	.2405	.5655	1.686	.3657	1.633	1.61460	25.271	30.68	.5844	4.315	2.607	1.655	.7395	.1839
1.97	.1339	.2378	.5630	1.697	.3638	1.646	1.61925	25.549	30.51	.5826	4.361	2.622	1.663	.7349	.1822
1.98	.1318	.2352	.5605	1.709	.3618	1.660	1.62386	25.827	30.33	.5808	4.407	2.637	1.671	.7302	.1806
1.99	.1298	.2326	.5580	1.720	.3598	1.674	1.62844	26.104	30.17	.5791	4.453	2.652	1.679	.7255	.1789
2.00	.1278	.2300	.5556	1.732	.3579	1.688	1.63299	26.380	30.00	.5774	4.500	2.667	1.688	.7209	.1773
2.01	.1258	.2275	.5531	1.744	.3559	1.702	1.63751	26.655	29.84	.5757	4.547	2.681	1.696	.7162	.1757
2.02	.1239	.2250	.5506	1.755	.3539	1.716	1.64201	26.929	29.67	.5740	4.594	2.696	1.704	.7115	.1741
2.03	.1220	.2225	.5482	1.767	.3518	1.730	1.64647	27.203	29.51	.5723	4.641	2.711	1.712	.7069	.1726
2.04	.1201	.2200	.5458	1.778	.3498	1.745	1.65090	27.476	29.35	.5707	4.689	2.725	1.720	.7022	.1710
2.05	.1182	.2176	.5433	1.790	.3478	1.760	1.65530	27.748	29.20	.5691	4.736	2.740	1.729	.6975	.1695
2.06	.1164	.2152	.5409	1.801	.3458	1.775	1.65967	28.020	29.04	.5675	4.784	2.755	1.737	.6928	.1680
2.07	.1146	.2128	.5385	1.812	.3437	1.790	1.66402	28.290	28.89	.5659	4.832	2.769	1.745	.6882	.1665
2.08	.1128	.2104	.5361	1.824	.3417	1.806	1.66833	28.560	28.74	.5643	4.881	2.783	1.754	.6835	.1651
2.09	.1111	.2081	.5337	1.835	.3396	1.821	1.67262	28.829	28.59	.5628	4.929	2.798	1.762	.6789	.1636
2.10	.1094	.2058	.5313	1.847	.3376	1.837	1.67687	29.097	28.44	.5613	4.978	2.812	1.770	.6742	.1622
2.11	.1077	.2035	.5290	1.858	.3355	1.853	1.68110	29.364	28.29	.5598	5.027	2.826	1.779	.6696	.1608
2.12	.1060	.2013	.5266	1.869	.3334	1.869	1.68530	29.631	28.14	.5583	5.077	2.840	1.787	.6649	.1594
2.13	.1043	.1990	.5243	1.881	.3314	1.885	1.68947	29.897	28.00	.5568	5.126	2.854	1.796	.6603	.1580
2.14	.1027	.1968	.5219	1.892	.3293	1.902	1.69362	30.161	27.86	.5554	5.176	2.868	1.805	.6557	.1567

Figure A-4 continued

SUPERSONIC FLOW

$\gamma = 7/5$

M or M_1	$\frac{p}{p_t}$	$\frac{\rho}{\rho_t}$	$\frac{T}{T_t}$	β	$\frac{q}{p_t}$	$\frac{A}{A_*}$	$\frac{V}{a_*}$	ν	μ	M_2	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{t_2}}{p_{t_1}}$	$\frac{p_1}{p_{t_2}}$
2.15	.1011	.1946	.5196	1.903	.3272	1.919	1.69774	30.425	27.72	.5540	5.226	2.882	1.813	.6511	.1553
2.16	.9956 -1	.1925	.5173	1.915	.3252	1.935	1.70183	30.689	27.58	.5525	5.277	2.896	1.822	.6464	.1540
2.17	.9802 -1	.1903	.5150	1.926	.3231	1.953	1.70589	30.951	27.44	.5511	5.327	2.910	1.831	.6419	.1527
2.18	.9649 -1	.1882	.5127	1.937	.3210	1.970	1.70992	31.212	27.30	.5498	5.378	2.924	1.839	.6373	.1514
2.19	.9500 -1	.1861	.5104	1.948	.3189	1.987	1.71393	31.473	27.17	.5484	5.429	2.938	1.848	.6327	.1502
2.20	.9352 -1	.1841	.5081	1.960	.3169	2.005	1.71791	31.732	27.04	.5471	5.480	2.951	1.857	.6281	.1489
2.21	.9207 -1	.1820	.5059	1.971	.3148	2.023	1.72187	31.991	26.90	.5457	5.531	2.965	1.866	.6236	.1476
2.22	.9064 -1	.1800	.5036	1.982	.3127	2.041	1.72579	32.250	26.77	.5444	5.583	2.978	1.875	.6191	.1464
2.23	.8923 -1	.1780	.5014	1.993	.3106	2.059	1.72970	32.507	26.64	.5431	5.636	2.992	1.883	.6145	.1452
2.24	.8785 -1	.1760	.4991	2.004	.3085	2.078	1.73357	32.763	26.51	.5418	5.687	3.005	1.892	.6100	.1440
2.25	.8648 -1	.1740	.4969	2.016	.3065	2.096	1.73742	33.018	26.39	.5406	5.740	3.019	1.901	.6055	.1428
2.26	.8514 -1	.1721	.4947	2.027	.3044	2.115	1.74125	33.273	26.26	.5393	5.792	3.032	1.910	.6011	.1417
2.27	.8382 -1	.1702	.4925	2.038	.3023	2.134	1.74504	33.527	26.14	.5381	5.845	3.045	1.919	.5966	.1405
2.28	.8251 -1	.1683	.4903	2.049	.3003	2.154	1.74882	33.780	26.01	.5368	5.898	3.058	1.929	.5921	.1393
2.29	.8123 -1	.1664	.4881	2.060	.2982	2.173	1.75257	34.032	25.89	.5356	5.951	3.071	1.938	.5877	.1382
2.30	.7997 -1	.1646	.4859	2.071	.2961	2.193	1.75629	34.283	25.77	.5344	6.005	3.085	1.947	.5833	.1371
2.31	.7873 -1	.1628	.4837	2.082	.2941	2.213	1.75999	34.533	25.65	.5332	6.059	3.098	1.956	.5789	.1360
2.32	.7751 -1	.1609	.4816	2.093	.2920	2.233	1.76366	34.783	25.53	.5321	6.113	3.110	1.965	.5745	.1349
2.33	.7631 -1	.1592	.4794	2.104	.2900	2.254	1.76731	35.031	25.42	.5309	6.167	3.123	1.974	.5702	.1338
2.34	.7512 -1	.1574	.4773	2.116	.2879	2.274	1.77093	35.279	25.30	.5297	6.222	3.136	1.984	.5658	.1328
2.35	.7396 -1	.1556	.4752	2.127	.2859	2.295	1.77453	35.526	25.18	.5286	6.276	3.149	1.993	.5615	.1317
2.36	.7281 -1	.1539	.4731	2.138	.2839	2.316	1.77811	35.771	25.07	.5275	6.331	3.162	2.002	.5572	.1307
2.37	.7168 -1	.1522	.4709	2.149	.2818	2.338	1.78166	36.017	24.96	.5264	6.386	3.174	2.012	.5529	.1297
2.38	.7057 -1	.1505	.4688	2.160	.2798	2.359	1.78519	36.261	24.85	.5253	6.442	3.187	2.021	.5486	.1286
2.39	.6948 -1	.1488	.4668	2.171	.2778	2.381	1.78869	36.504	24.73	.5242	6.497	3.199	2.031	.5444	.1276
2.40	.6840 -1	.1472	.4647	2.182	.2758	2.403	1.79218	36.746	24.62	.5231	6.553	3.212	2.040	.5401	.1266
2.41	.6734 -1	.1456	.4626	2.193	.2738	2.425	1.79563	36.988	24.52	.5221	6.609	3.224	2.050	.5359	.1257
2.42	.6630 -1	.1439	.4606	2.204	.2718	2.448	1.79907	37.229	24.41	.5210	6.666	3.237	2.059	.5317	.1247
2.43	.6527 -1	.1424	.4585	2.215	.2698	2.471	1.80248	37.469	24.30	.5200	6.722	3.249	2.069	.5276	.1237
2.44	.6426 -1	.1408	.4565	2.226	.2678	2.494	1.80587	37.708	24.19	.5189	6.779	3.261	2.079	.5234	.1228
2.45	.6327 -1	.1392	.4544	2.237	.2658	2.517	1.80924	37.946	24.09	.5179	6.836	3.273	2.088	.5193	.1218
2.46	.6229 -1	.1377	.4524	2.248	.2639	2.540	1.81258	38.183	23.99	.5169	6.894	3.285	2.098	.5152	.1209
2.47	.6133 -1	.1362	.4504	2.259	.2619	2.564	1.81591	38.420	23.88	.5159	6.951	3.298	2.108	.5111	.1200
2.48	.6038 -1	.1346	.4484	2.269	.2599	2.588	1.81921	38.655	23.78	.5149	7.009	3.310	2.118	.5071	.1191
2.49	.5945 -1	.1332	.4464	2.280	.2580	2.612	1.82249	38.890	23.68	.5140	7.067	3.321	2.128	.5030	.1182
2.50	.5853 -1	.1317	.4444	2.291	.2561	2.637	1.82574	39.124	23.58	.5130	7.125	3.333	2.138	.4990	.1173
2.51	.5762 -1	.1302	.4425	2.302	.2541	2.661	1.82898	39.357	23.48	.5120	7.183	3.345	2.147	.4950	.1164
2.52	.5674 -1	.1288	.4405	2.313	.2522	2.686	1.83219	39.589	23.38	.5111	7.242	3.357	2.157	.4911	.1155
2.53	.5586 -1	.1274	.4386	2.324	.2503	2.712	1.83538	39.820	23.28	.5102	7.301	3.369	2.167	.4871	.1147
2.54	.5500 -1	.1260	.4366	2.335	.2484	2.737	1.83855	40.050	23.18	.5092	7.360	3.380	2.177	.4832	.1138
2.55	.5415 -1	.1246	.4347	2.346	.2465	2.762	1.84170	40.280	23.09	.5083	7.420	3.392	2.187	.4793	.1130
2.56	.5332 -1	.1232	.4328	2.357	.2446	2.789	1.84483	40.509	22.99	.5074	7.479	3.403	2.198	.4754	.1122
2.57	.5250 -1	.1218	.4309	2.367	.2427	2.815	1.84794	40.736	22.91	.5065	7.539	3.415	2.208	.4715	.1113
2.58	.5169 -1	.1205	.4289	2.378	.2409	2.842	1.85103	40.963	22.81	.5056	7.599	3.426	2.218	.4677	.1105
2.59	.5090 -1	.1192	.4271	2.389	.2390	2.869	1.85410	41.189	22.71	.5047	7.659	3.438	2.228	.4639	.1097
2.60	.5012 -1	.1179	.4252	2.400	.2371	2.896	1.85714	41.415	22.62	.5039	7.720	3.449	2.238	.4601	.1089
2.61	.4935 -1	.1166	.4233	2.411	.2353	2.923	1.86017	41.639	22.53	.5030	7.781	3.460	2.249	.4564	.1081
2.62	.4859 -1	.1153	.4214	2.422	.2335	2.951	1.86318	41.863	22.44	.5022	7.842	3.471	2.259	.4526	.1074
2.63	.4784 -1	.1140	.4196	2.432	.2317	2.979	1.86616	42.086	22.35	.5013	7.903	3.483	2.269	.4489	.1066
2.64	.4711 -1	.1128	.4177	2.443	.2298	3.007	1.86913	42.307	22.26	.5005	7.965	3.494	2.280	.4452	.1058
2.65	.4639 -1	.1115	.4159	2.454	.2280	3.036	1.87208	42.529	22.17	.4996	8.026	3.505	2.290	.4416	.1051
2.66	.4568 -1	.1103	.4141	2.465	.2262	3.065	1.87501	42.749	22.08	.4988	8.088	3.516	2.301	.4379	.1043
2.67	.4498 -1	.1091	.4122	2.476	.2245	3.094	1.87792	42.968	22.00	.4980	8.150	3.527	2.311	.4343	.1036
2.68	.4429 -1	.1079	.4104	2.486	.2227	3.123	1.88081	43.187	21.91	.4972	8.213	3.537	2.322	.4307	.1028
2.69	.4362 -1	.1067	.4086	2.497	.2209	3.153	1.88368	43.405	21.82	.4964	8.275	3.548	2.332	.4271	.1021
2.70	.4295 -1	.1056	.4068	2.508	.2192	3.183	1.88653	43.621	21.74	.4956	8.338	3.559	2.343	.4236	.1014
2.71	.4229 -1	.1044	.4051	2.519	.2174	3.213	1.88936	43.838	21.65	.4949	8.401	3.570	2.354	.4201	.1007
2.72	.4165 -1	.1033	.4033	2.530	.2157	3.244	1.89218	44.053	21.57	.4941	8.465	3.580	2.364	.4166	.9996 -1
2.73	.4102 -1	.1022	.4015	2.540	.2140	3.275	1.89497	44.267	21.49	.4933	8.528	3.591	2.375	.4131	.9929 -1
2.74	.4039 -1	.1010	.3998	2.551	.2123	3.306	1.89775	44.481	21.41	.4926	8.592	3.601	2.386	.4097	.9860 -1
2.75	.3978 -1	.9994 -1	.3980	2.562	.2106	3.338	1.90051	44.694	21.32	.4918	8.656	3.612	2.397	.4062	.9792 -1
2.76	.3917 -1	.9885 -1	.3963	2.572	.2089	3.370	1.90325	44.906	21.24	.4911	8.721	3.622	2.407	.4028	.9724 -1
2.77	.3858 -1	.9778 -1	.3945	2.583	.2072	3.402	1.90598	45.117	21.16	.4903	8.785	3.633	2.418	.3994	.9658 -1
2.78	.3799 -1	.9671 -1	.3928	2.594	.2055	3.434	1.90868	45.327	21.08	.4896	8.850	3.643	2.429	.3961	.9591 -1
2.79	.3742 -1	.9566 -1	.3911	2.605	.2039	3.467	1.91137	45.537	21.00	.4889	8.915	3.653	2.440	.3928	.9526 -1
2.80	.3685 -1	.9463 -1	.3894	2.615	.2022	3.500	1.91404	45.746	20.92	.4882	8.980	3.664	2.451	.3895	.9461 -1
2.81	.3629 -1	.9360 -1	.3877	2.626	.2006	3.534	1.91669	45.954	20.85	.4875	9.045	3.674	2.462	.3862	.9397 -1
2.82	.3574 -1	.9259 -1	.3860	2.637	.1990	3.567	1.91933	46.161	20.77	.4868	9.111	3.684	2.473	.3829	.9334 -1
2.83	.3520 -1	.9158 -1	.3844	2.647	.1973	3.601	1.92195	46.368	20.69	.4861	9.177	3.694	2.484	.3797	.9271 -1
2.84	.3467 -1	.9059 -1	.3827	2.658	.1957	3.636	1.92455	46.573	20.62	.4854	9.243	3.704	2.496	.3765	.9209 -1
2.85	.3415 -1	.8962 -1	.3810	2.669	.1941	3.671	1.92714	46.778	20.54	.4847	9.310	3.714	2.507	.3733	.9147 -1
2.86	.3363 -1	.8865 -1	.3794	2.679	.1926	3.706	1.92970	46.982	20.47	.4840	9.376	3.724	2.518	.3701	.9086 -1
2.87	.3312 -1	.8769 -1	.3777	2.690	.1910	3.741	1.93225	47.185	20.39	.4833	9.443	3.734	2.529	.3670	.9026 -1
2.88	.3263 -1	.8675 -1	.3761	2.701	.1894	3.777	1.93479	47.388	20.32	.4827	9.510	3.743	2.540	.3639	.8966 -1
2.89	.3213 -1	.8581 -1	.3745	2.711	.1879	3.813	1.93731	47.589	20.24	.4820	9.577	3.753	2.552	.3608	.8906 -1
2.90	.3165 -1	.8489 -1	.3729	2.722	.1863	3.850	1.93981	47.790	20.17	.4814	9.645	3.763	2.563	.3577	.8848 -1
2.91	.3118 -1	.8398 -1	.3712	2.733	.1848	3.887	1.94230	47.990	20.10	.4807	9.713	3.773	2.575	.3547	.8790 -1
2.92	.3071 -1	.8307 -1	.3696	2.743	.1833	3.924	1.94477	48.190	20.03	.4801	9.781	3.782	2.586	.3517	.8732 -1
2.93	.3025 -1	.8218 -1	.3681	2.754	.1818	3.961	1.94722	48.388	19.96	.4795	9.849	3.792	2.598	.3487	.8675 -1
2.94	.2980 -1	.8130 -1	.3665	2.765	.1803	3.999	1.94966	48.586	19.89	.4788	9.918	3.801	2.609	.3457	.8619 -1
2.95	.2935 -1	.8043 -1	.3649	2.775	.1788	4.038	1.95208	48.783	19.81	.4782	9.986	3.811	2.621	.3428	.8563 -1
2.96	.2891 -1	.7957 -1	.3633	2.786	.1773	4.076	1.95449	48.980	19.75	.4776	10.06	3.820	2.632	.3398	.8507 -1
2.97	.2848 -1	.7872 -1	.3618	2.797	.1758	4.115	1.95688	49.175	19.68	.4770	10.12	3.829	2.644	.3369	.8453 -1
2.98	.2805 -1	.7788 -1	.3602	2.807	.1744	4.155	1.95925	49.370	19.61	.4764	10.19	3.839	2.656	.3340	.8398 -1
2.99	.2764 -1	.7705 -1	.3587	2.818	.1729	4.194	1.96162	49.564	19.54	.4758	10.26	3.848	2.667	.3312	.8345 -1
3.00	.2722 -1	.7623 -1	.3571	2.828	.1715	4.235	1.96396	49.757	19.47	.4752	10.33	3.857	2.679	.3283	.8291 -1
3.01	.2682 -1	.7541 -1	.3556	2.839	.1701	4.275	1.96629	49.950	19.40	.4746	10.40	3.866	2.691	.3255	.8238 -1
3.02	.2642 -1	.7461 -1	.3541	2.850	.1687	4.316	1.96861	50.142	19.34	.4740	10.47	3.875	2.703	.3227	.8186 -1
3.03	.2603 -1	.7382 -1	.3526	2.860	.1673	4.357	1.97091	50.333	19.27	.4734	10.54	3.884	2.714	.3200	.8134 -1
3.04	.2564 -1	.7303 -1	.3511	2.871	.1659	4.399	1.97319	50.523	19.20	.4729	10.62	3.893	2.726	.3172	.8083 -1

Figure A-4 continued

SUPERSONIC FLOW

$\gamma=7/5$

M or M_1	$\frac{p}{p_t}$	$\frac{\rho}{\rho_t}$	$\frac{T}{T_t}$	β	$\frac{q}{p_t}$	$\frac{A}{A_*}$	$\frac{V}{a_*}$	ν	μ	M_2	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{t_2}}{p_{t_1}}$	$\frac{p_1}{p_{t_2}}$
3.05	.2526 -1	.7226 -1	.3496	2.881	.1645	4.441	1.97547	50.713	19.14	.4723	10.69	3.902	2.738	.3145	.8032 -1
3.06	.2489 -1	.7149 -1	.3481	2.892	.1631	4.483	1.97772	50.902	19.07	.4717	10.76	3.911	2.750	.3118	.7982 -1
3.07	.2452 -1	.7074 -1	.3466	2.903	.1618	4.526	1.97997	51.090	19.01	.4712	10.83	3.920	2.762	.3091	.7932 -1
3.08	.2418 -1	.6999 -1	.3452	2.913	.1604	4.570	1.98219	51.277	18.95	.4706	10.90	3.929	2.774	.3065	.7882 -1
3.09	.2380 -1	.6925 -1	.3437	2.924	.1591	4.613	1.98441	51.464	18.88	.4701	10.97	3.938	2.786	.3038	.7833 -1
3.10	.2345 -1	.6852 -1	.3422	2.934	.1577	4.657	1.98661	51.650	18.82	.4695	11.05	3.947	2.799	.3012	.7785 -1
3.11	.2310 -1	.6779 -1	.3408	2.945	.1564	4.702	1.98879	51.835	18.76	.4690	11.12	3.955	2.811	.2986	.7737 -1
3.12	.2276 -1	.6708 -1	.3393	2.955	.1551	4.747	1.99097	52.020	18.69	.4685	11.19	3.964	2.823	.2960	.7690 -1
3.13	.2243 -1	.6637 -1	.3379	2.966	.1538	4.792	1.99313	52.203	18.63	.4679	11.26	3.973	2.835	.2935	.7642 -1
3.14	.2210 -1	.6568 -1	.3365	2.977	.1525	4.838	1.99527	52.386	18.57	.4674	11.34	3.981	2.848	.2910	.7595 -1
3.15	.2177 -1	.6499 -1	.3351	2.987	.1512	4.884	1.99740	52.569	18.51	.4669	11.41	3.990	2.860	.2885	.7549 -1
3.16	.2146 -1	.6430 -1	.3337	2.998	.1500	4.930	1.99952	52.751	18.45	.4664	11.48	3.998	2.872	.2860	.7503 -1
3.17	.2114 -1	.6363 -1	.3323	3.008	.1487	4.977	2.00162	52.931	18.39	.4659	11.56	4.006	2.885	.2835	.7457 -1
3.18	.2083 -1	.6296 -1	.3309	3.019	.1475	5.025	2.00372	53.112	18.33	.4654	11.63	4.015	2.897	.2811	.7412 -1
3.19	.2053 -1	.6231 -1	.3295	3.029	.1462	5.073	2.00579	53.292	18.27	.4648	11.71	4.023	2.909	.2786	.7367 -1
3.20	.2023 -1	.6165 -1	.3281	3.040	.1450	5.121	2.00786	53.470	18.21	.4643	11.78	4.031	2.922	.2762	.7323 -1
3.21	.1993 -1	.6101 -1	.3267	3.050	.1438	5.170	2.00991	53.648	18.15	.4639	11.85	4.040	2.935	.2738	.7279 -1
3.22	.1964 -1	.6037 -1	.3253	3.061	.1426	5.219	2.01195	53.826	18.09	.4634	11.93	4.048	2.947	.2715	.7235 -1
3.23	.1936 -1	.5975 -1	.3240	3.071	.1414	5.268	2.01398	54.003	18.03	.4629	12.01	4.056	2.960	.2691	.7192 -1
3.24	.1908 -1	.5912 -1	.3226	3.082	.1402	5.319	2.01599	54.179	17.98	.4624	12.08	4.064	2.972	.2668	.7149 -1
3.25	.1880 -1	.5851 -1	.3213	3.092	.1390	5.369	2.01799	54.355	17.92	.4619	12.16	4.072	2.985	.2645	.7107 -1
3.26	.1853 -1	.5790 -1	.3199	3.103	.1378	5.420	2.01996	54.529	17.86	.4614	12.23	4.080	2.998	.2622	.7065 -1
3.27	.1826 -1	.5730 -1	.3186	3.113	.1367	5.472	2.02196	54.703	17.81	.4610	12.31	4.088	3.011	.2600	.7023 -1
3.28	.1799 -1	.5671 -1	.3173	3.124	.1355	5.523	2.02392	54.877	17.75	.4605	12.38	4.096	3.023	.2577	.6982 -1
3.29	.1773 -1	.5612 -1	.3160	3.134	.1344	5.576	2.02587	55.050	17.70	.4600	12.46	4.104	3.036	.2555	.6941 -1
3.30	.1748 -1	.5554 -1	.3147	3.145	.1332	5.629	2.02781	55.222	17.64	.4596	12.54	4.112	3.049	.2533	.6900 -1
3.31	.1722 -1	.5497 -1	.3134	3.155	.1321	5.682	2.02974	55.393	17.58	.4591	12.62	4.120	3.062	.2511	.6860 -1
3.32	.1698 -1	.5440 -1	.3121	3.166	.1310	5.736	2.03165	55.564	17.53	.4587	12.69	4.128	3.075	.2489	.6820 -1
3.33	.1673 -1	.5384 -1	.3108	3.176	.1299	5.790	2.03356	55.734	17.48	.4582	12.77	4.135	3.088	.2468	.6781 -1
3.34	.1649 -1	.5329 -1	.3095	3.187	.1288	5.845	2.03545	55.904	17.42	.4578	12.85	4.143	3.101	.2446	.6741 -1
3.35	.1625 -1	.5274 -1	.3082	3.197	.1277	5.900	2.03733	56.073	17.37	.4573	12.93	4.151	3.114	.2425	.6702 -1
3.36	.1602 -1	.5220 -1	.3069	3.208	.1266	5.956	2.03920	56.241	17.31	.4569	13.00	4.158	3.127	.2404	.6664 -1
3.37	.1579 -1	.5166 -1	.3057	3.218	.1255	6.012	2.04106	56.409	17.26	.4565	13.08	4.166	3.141	.2383	.6626 -1
3.38	.1557 -1	.5113 -1	.3044	3.229	.1245	6.069	2.04290	56.576	17.21	.4560	13.16	4.173	3.154	.2363	.6588 -1
3.39	.1534 -1	.5061 -1	.3032	3.239	.1234	6.126	2.04474	56.742	17.16	.4556	13.24	4.181	3.167	.2342	.6550 -1
3.40	.1512 -1	.5009 -1	.3019	3.250	.1224	6.184	2.04656	56.907	17.10	.4552	13.32	4.188	3.180	.2322	.6513 -1
3.41	.1491 -1	.4958 -1	.3007	3.260	.1214	6.242	2.04837	57.073	17.05	.4548	13.40	4.196	3.194	.2302	.6476 -1
3.42	.1470 -1	.4908 -1	.2995	3.271	.1203	6.301	2.05017	57.237	17.00	.4544	13.48	4.203	3.207	.2282	.6439 -1
3.43	.1449 -1	.4858 -1	.2982	3.281	.1193	6.360	2.05196	57.401	16.95	.4540	13.56	4.211	3.220	.2263	.6403 -1
3.44	.1428 -1	.4808 -1	.2970	3.291	.1183	6.420	2.05374	57.564	16.90	.4535	13.64	4.218	3.234	.2243	.6367 -1
3.45	.1408 -1	.4759 -1	.2958	3.302	.1173	6.480	2.05551	57.726	16.85	.4531	13.72	4.225	3.247	.2224	.6331 -1
3.46	.1388 -1	.4711 -1	.2946	3.312	.1163	6.541	2.05727	57.888	16.80	.4527	13.80	4.232	3.261	.2205	.6296 -1
3.47	.1368 -1	.4663 -1	.2934	3.323	.1153	6.602	2.05901	58.050	16.75	.4523	13.88	4.240	3.274	.2186	.6261 -1
3.48	.1349 -1	.4616 -1	.2922	3.333	.1144	6.664	2.06075	58.210	16.70	.4519	13.96	4.247	3.288	.2167	.6226 -1
3.49	.1330 -1	.4569 -1	.2910	3.344	.1134	6.727	2.06247	58.370	16.65	.4515	14.04	4.254	3.301	.2148	.6191 -1
3.50	.1311 -1	.4523 -1	.2899	3.354	.1124	6.790	2.06419	58.530	16.60	.4512	14.13	4.261	3.315	.2129	.6157 -1
3.51	.1293 -1	.4478 -1	.2887	3.365	.1115	6.853	2.06589	58.689	16.55	.4508	14.21	4.268	3.329	.2111	.6123 -1
3.52	.1274 -1	.4433 -1	.2875	3.375	.1105	6.917	2.06759	58.847	16.51	.4504	14.29	4.275	3.343	.2093	.6089 -1
3.53	.1256 -1	.4388 -1	.2864	3.385	.1096	6.982	2.06927	59.004	16.46	.4500	14.37	4.282	3.356	.2075	.6056 -1
3.54	.1239 -1	.4344 -1	.2852	3.396	.1087	7.047	2.07094	59.162	16.41	.4496	14.45	4.289	3.370	.2057	.6023 -1
3.55	.1221 -1	.4300 -1	.2841	3.406	.1078	7.113	2.07261	59.318	16.36	.4492	14.54	4.296	3.384	.2039	.5990 -1
3.56	.1204 -1	.4257 -1	.2829	3.417	.1069	7.179	2.07426	59.474	16.31	.4489	14.62	4.303	3.398	.2022	.5957 -1
3.57	.1188 -1	.4214 -1	.2818	3.427	.1059	7.246	2.07590	59.629	16.27	.4485	14.70	4.309	3.412	.2004	.5925 -1
3.58	.1171 -1	.4172 -1	.2806	3.437	.1051	7.313	2.07754	59.784	16.22	.4481	14.79	4.316	3.426	.1987	.5892 -1
3.59	.1155 -1	.4131 -1	.2795	3.448	.1042	7.382	2.07916	59.938	16.17	.4478	14.87	4.323	3.440	.1970	.5861 -1
3.60	.1138 -1	.4089 -1	.2784	3.458	.1033	7.450	2.08077	60.091	16.13	.4474	14.95	4.330	3.454	.1953	.5829 -1
3.61	.1123 -1	.4049 -1	.2773	3.469	.1024	7.519	2.08238	60.244	16.08	.4471	15.04	4.336	3.468	.1936	.5798 -1
3.62	.1107 -1	.4008 -1	.2762	3.479	.1016	7.589	2.08397	60.397	16.04	.4467	15.12	4.343	3.482	.1920	.5767 -1
3.63	.1092 -1	.3968 -1	.2751	3.490	.1007	7.659	2.08556	60.549	15.99	.4463	15.21	4.350	3.496	.1903	.5736 -1
3.64	.1076 -1	.3929 -1	.2740	3.500	.9984 -1	7.730	2.08713	60.700	15.95	.4460	15.29	4.356	3.510	.1887	.5705 -1
3.65	.1062 -1	.3890 -1	.2729	3.510	.9900 -1	7.802	2.08870	60.851	15.90	.4456	15.38	4.363	3.525	.1871	.5675 -1
3.66	.1047 -1	.3852 -1	.2718	3.521	.9817 -1	7.874	2.09026	61.000	15.86	.4453	15.46	4.369	3.539	.1855	.5645 -1
3.67	.1032 -1	.3813 -1	.2707	3.531	.9734 -1	7.947	2.09180	61.150	15.81	.4450	15.55	4.376	3.553	.1839	.5615 -1
3.68	.1018 -1	.3776 -1	.2697	3.542	.9652 -1	8.020	2.09334	61.299	15.77	.4446	15.63	4.382	3.568	.1823	.5585 -1
3.69	.1004 -1	.3739 -1	.2686	3.552	.9570 -1	8.094	2.09487	61.447	15.72	.4443	15.72	4.388	3.582	.1807	.5556 -1
3.70	.9903 -2	.3702 -1	.2675	3.562	.9490 -1	8.169	2.09639	61.595	15.68	.4439	15.81	4.395	3.596	.1792	.5526 -1
3.71	.9767 -2	.3665 -1	.2665	3.573	.9410 -1	8.244	2.09790	61.743	15.64	.4436	15.89	4.401	3.611	.1777	.5497 -1
3.72	.9633 -2	.3629 -1	.2654	3.583	.9331 -1	8.320	2.09941	61.889	15.59	.4433	15.98	4.408	3.625	.1761	.5469 -1
3.73	.9500 -2	.3594 -1	.2644	3.593	.9253 -1	8.397	2.10090	62.036	15.55	.4430	16.07	4.414	3.640	.1746	.5440 -1
3.74	.9370 -2	.3558 -1	.2633	3.604	.9175 -1	8.474	2.10238	62.181	15.51	.4426	16.15	4.420	3.654	.1731	.5412 -1
3.75	.9242 -2	.3524 -1	.2623	3.614	.9098 -1	8.552	2.10386	62.326	15.47	.4423	16.24	4.426	3.669	.1717	.5384 -1
3.76	.9116 -2	.3489 -1	.2613	3.625	.9021 -1	8.630	2.10533	62.471	15.42	.4420	16.33	4.432	3.684	.1702	.5356 -1
3.77	.8991 -2	.3455 -1	.2602	3.635	.8945 -1	8.709	2.10679	62.615	15.38	.4417	16.42	4.439	3.698	.1687	.5328 -1
3.78	.8869 -2	.3421 -1	.2592	3.645	.8870 -1	8.789	2.10824	62.758	15.34	.4414	16.50	4.445	3.713	.1673	.5301 -1
3.79	.8748 -2	.3388 -1	.2582	3.656	.8796 -1	8.870	2.10968	62.901	15.30	.4410	16.59	4.451	3.728	.1659	.5274 -1
3.80	.8629 -2	.3355 -1	.2572	3.666	.8722 -1	8.951	2.11111	63.044	15.26	.4407	16.68	4.457	3.743	.1645	.5247 -1
3.81	.8512 -2	.3322 -1	.2562	3.676	.8649 -1	9.032	2.11254	63.186	15.22	.4404	16.77	4.463	3.758	.1631	.5220 -1
3.82	.8396 -2	.3290 -1	.2552	3.687	.8577 -1	9.115	2.11395	63.327	15.18	.4401	16.86	4.469	3.772	.1617	.5193 -1
3.83	.8283 -2	.3258 -1	.2542	3.697	.8505 -1	9.198	2.11536	63.468	15.14	.4398	16.95	4.475	3.787	.1603	.5167 -1
3.84	.8171 -2	.3227 -1	.2532	3.708	.8434 -1	9.282	2.11676	63.608	15.10	.4395	17.04	4.481	3.802	.1589	.5140 -1
3.85	.8060 -2	.3196 -1	.2522	3.718	.8363 -1	9.366	2.11815	63.748	15.06	.4392	17.13	4.487	3.817	.1576	.5114 -1
3.86	.7951 -2	.3165 -1	.2513	3.728	.8293 -1	9.451	2.11954	63.887	15.02	.4389	17.22	4.492	3.832	.1563	.5089 -1
3.87	.7844 -2	.3134 -1	.2503	3.739	.8224 -1	9.537	2.12091	64.026	14.98	.4386	17.31	4.498	3.847	.1549	.5063 -1
3.88	.7739 -2	.3104 -1	.2493	3.749	.8155 -1	9.624	2.12228	64.164	14.94	.4383	17.40	4.504	3.863	.1536	.5038 -1
3.89	.7635 -2	.3074 -1	.2484	3.759	.8087 -1	9.711	2.12364	64.302	14.90	.4380	17.49	4.510	3.878	.1523	.5013 -1
3.90	.7532 -2	.3044 -1	.2474	3.770	.8019 -1	9.799	2.12499	64.440	14.86	.4377	17.58	4.516	3.893	.1510	.4987 -1
3.91	.7431 -2	.3015 -1	.2464	3.780	.7952 -1	9.888	2.12634	64.576	14.82	.4375	17.67	4.521	3.908	.1497	.4962 -1
3.92	.7332 -2	.2986 -1	.2455	3.790	.7886 -1	9.977	2.12767	64.713	14.78	.4372	17.76	4.527	3.923	.1485	.4938 -1
3.93	.7233 -2	.2958 -1	.2446	3.801	.7820 -1	10.07	2.12900	64.848	14.74	.4369	17.85	4.533	3.939	.1472	.4913 -1
3.94	.7137 -2	.2929 -1	.2436	3.811	.7755 -1	10.16	2.13032	64.983	14.70	.4366	17.94	4.538	3.954	.1460	.4889 -1

Figure A-4 concluded

Force

Units	lbf	pdl	dyne	g	Newton
1 lbf =	1	32.174	4.44822x10^5	453.59237	4.448222
1 pdl =	0.0310809	1	1.38255x10^4	14.0981	0.138255
1 dyne =	2.24809x10^-6	7.23301x10^-5	1	1.01972x10-3	10^-5
1 g =	2.20462x10^-3	0.070931	980.665	1	9.80665x10^-3
1 Newton =	0.224809	7.23298	10^-5	101.972	1

Length

Units	inch	feet	centimeter	meter
1 in =	1	0.0833333	2.54	0.0254
1 ft =	12	1	30.48	0.3048
1 cm =	0.3937008	0.0328084	1	0.01
1 m =	39.37008	3.28084	100	1

Density

Units	lbm/ft^3	lbm/gal	g/cm^3	kg/m^3
1 lbm/ft^3 =	1	0.1[illegible]68056	0.01601847	16.01847
1 lbm/gal =	7.480519	1	0.11982646	119.82646
1 g/cm^3 =	62.42793	8.[illegible]45402	1	10^-3
1 kg/m^3 =	0.06242793	8.345[illegible]02x10^-3	10^-3	1

Pressure

Units	lbf/in^2	ft H20 at 60 F	atm	kg/cm^2	torr	bar	N/m^2
1 lbf/in^2 =	1	2.[illegible]0897	0.06804596	0.0703066	51.71495	0.06894757	6.894757x10^3
1 ft H20, 60 F =	0.43309	1	0.0294703	0.0304495	22.3974	0.0290608	2.98608x10^3
1 atm =	14.69595	33.9325	1	1.033227	760	1.01325	1.01325x10^5
1 kg/cm^2 =	14.22335	32.8413	0.9678411	1	735.5596	0.980665	9.80665x10^4
1 torr =	0.0193368	0.044648	1.315789x10^-3	1.359509x10^-3	1	1.333224x10^-3	133.3224
1 bar =	14.503775	33.4887	0.9869233	1.019716	750.0617	1	10^5
1 N/m =	1.450378x10^-4	3.34887x10^-4	9.869233x10^-6	1.019716x10^-5	7.500617x10^-3	10^-5	1

Figure A-5 Conversion Factors

Volume

Units	inch^3	ft^3	gal	liter	cm^3	m^3
1 in^3 =	1	5.787037x10^-4	4.329004x10^-3	0.01638706	16.38706	1.638706x10^-5
1 ft^3 =	1728	1	7.480519	28.31684	28316.84	0.02831684
1 gal =	231	0.1336806	1	3.785411	3785.411	3.785411x10^-3
1 liter =	61.02374	0.03531467	0.2641721	1	1000	10^-3
1 cm^3 =	0.06102374	3.531467x10^-5	2.641721x10^-4	10^-3	1	10^-6
1 m^3 =	61023.74	35.31467	264.1721	10^3	10^6	1

Mass

Units	lbm	gram (g)	kg	ton	metric ton
1 lbm =	1	453.59237	0.045359237	0.0005	4.5359237x10^-4
1 g =	2.204623x10^-3	1	10^-3	1.1023115x10^-6	10^-6
1 kg =	2.204623	1000	1	1.1023115x10^-3	10^-3
1 ton =	2000	907184.74	907.18474	1	0.9071846
1 metric ton =	2204.623	10^6	1000	1.1023115	1

Figure A-5 Continued

To Convert	Into	Multiply By
	A	
Atmospheres	Ton/sq inch	0.007348
Atmospheres	cm of mercury	76
Atmospheres	dynes/cm^2	1.013x10^6
Atmospheres	Ft of water (@ 4 degs C)	33.9
Atmospheres	In. of Mercury @ 0 degs C	29.92
Atmospheres	kgs/ sq cm	1.0333
Atmospheres	kgs/sq meter	10332
Atmospheres	lbf/sq ft (psf)	2116.4
Atmospheres	lbf/sq inch (psi)	14.7
Atmospheres	tons/ sq ft	1.058
	B	
Bars	atmospheres	0.9869
Bars	dynes/ sq cm	10^6
Bars	kgs/sq meter	1.020x10^4
Bars	lbf/sq ft (psf)	2089
Bars	lbf/sq in (psi)	14.5
	C	
Centigrade	fahrenheit	(C * 9/5) - 32
Centimeters (cms)	feet	3.281x10^-2
Centimeters	inches	0.3937
Centimeters	kilometers	10^-6
Centimeters	meters	0.01
Centimeters	miles	6.214x10^-6
Centimeters	millimeters	10
Centimeters	mils	393.7
Centimeters	yards	1.094x10^-2
Centimeters of Mercury	atmospheres	0.01316
Centimeters of Mercury	feet of water	0.4461
Centimeters of Mercury	kgs/sq meter	136
Centimeters of Mercury	lbf/sq ft (psf)	27.85
Centimeters of Mercury	lbf/sq inch (psi)	0.1934
Centimeters /sec	feet/min	1.9685
Centimeters /sec	feet/sec	0.03281
Centimeters /sec	kilometers/hr	0.036
Centimeters /sec	knots	0.1943

Figure A-5 Continued

To Convert	Into	Multiply By
Centimeters /sec	meter/min	0.6
Centimeters /sec	miles/hr	0.02237
Centimeters /sec	miles/min	3.728x10^-4
Centimeters/sec/sec	feet/sec/sec	0.03281
Centimeters/sec/sec	kms/sec/sec	0.036
Centimeters/sec/sec	meters/sec/sec	0.01
Centimeters/sec/sec	miles/hr/sec	0.02237
Cubic centimeters	cubic feet (ft^3)	3.531x10^-5
Cubic centimeters	cubic inches (in^3)	0.06102
Cubic centimeters	cubic meters (m^3)	10^-6
Cubic centimeters	cubic yards (yd^3)	1.308x10^-6
Cubic centimeters	gallons (gal; US liq.)	2.6242x10^-4
Cubic centimeters	liters	0.001
Cubic centimeters	pints (US liq.)	2.113x10^-3
Cubic centimeters	quarts (US liq.)	1.057x10^-3
Cubic feet	cubic centimeters	2832
Cubic feet	cubic inches	1.728
Cubic feet	cubic meters	0.02832
Cubic feet	cubic yards	0.03704
Cubic feet	gallon (US liq.)	7.48052
Cubic feet	liters	28.32
Cubic feet	pints (US liq.)	59.84
Cubic feet	quarts (US liq.)	29.92
Cubic inches	cubic centimeters	16.39
Cubic inches	cubic feet	5.787x10^-4
Cubic inches	cubic meters	1.639x10^-5
Cubic inches	cubic yards	2.143x10^-5
Cubic inches	gallons	4.329x10^-6
Cubic inches	liters	0.01639
Cubic inches	pints (US liq.)	0.03463
Cubic inches	quarts (US liq.)	0.01732
Cubic meters	cubic centimeter	10^6
Cubic meters	cubic feet	35.31
Cubic meters	cubic inches	61023
Cubic meters	cubic yards	1.308
Cubic meters	gallons (US liq.)	264.2
Cubic meters	liters	1000
Cubic meters	pints (US liq.)	2113
Cubic meters	quarts (US liq.)	1057

Figure A-5 Continued

To Convert	Into	Multiply By
Centimeters /sec	meter/min	0.6
Centimeters /sec	miles/hr	0.02237
Centimeters /sec	miles/min	3.728x10^-4
Centimeters/sec/sec	feet/sec/sec	0.03281
Centimeters/sec/sec	kms/sec/sec	0.036
Centimeters/sec/sec	meters/sec/sec	0.01
Centimeters/sec/sec	miles/hr/sec	0.02237
Cubic centimeters	cubic feet (ft^3)	3.531x10^-5
Cubic centimeters	cubic inches (in^3)	0.06102
Cubic centimeters	cubic meters (m^3)	10^-6
Cubic centimeters	cubic yards (yd^3)	1.308x10^-6
Cubic centimeters	gallons (gal; US liq.)	2.6242x10^-4
Cubic centimeters	liters	0.001
Cubic centimeters	pints (US liq.)	2.113x10^-3
Cubic centimeters	quarts (US liq.)	1.057x10^-3
Cubic feet	cubic centimeters	2832
Cubic feet	cubic inches	1.728
Cubic feet	cubic meters	0.02832
Cubic feet	cubic yards	0.03704
Cubic feet	gallon (US liq.)	7.48052
Cubic feet	liters	28.32
Cubic feet	pints (US liq.)	59.84
Cubic feet	quarts (US liq.)	29.92
Cubic inches	cubic centimeters	16.39
Cubic inches	cubic feet	5.787x10^-4
Cubic inches	cubic meters	1.639x10^-5
Cubic inches	cubic yards	2.143x10^-5
Cubic inches	gallons	4.329x10^-6
Cubic inches	liters	0.01639
Cubic inches	pints (US liq.)	0.03463
Cubic inches	quarts (US liq.)	0.01732
Cubic meters	cubic centimeter	10^6
Cubic meters	cubic feet	35.31
Cubic meters	cubic inches	61023
Cubic meters	cubic yards	1.308
Cubic meters	gallons (US liq.)	264.2
Cubic meters	liters	1000
Cubic meters	pints (US liq.)	2113
Cubic meters	quarts (US liq.)	1057

Figure A-5 Continued

To Convert	Into	Multiply By
Cubic yard	cubic centimeters	7.646x10^5
Cubic yard	cubic feet	27
Cubic yard	cubic inches	46656
Cubic yard	cubic meters	0.7646
Cubic yard	gallons (US liq.)	202
Cubic yard	liters	764.6
Cubic yard	pints (US liq.)	1615.9
Cubic yard	quart (US liq.)	807.9
	D	
Days	seconds	86400
Degree (angle)	quadrants	0.01111
Degree (angle)	radian	0.01745
Degree (angle)	revolutions	2.7778x10^-3
Degree (angle)	seconds	3600
Degree/sec	radian/sec	0.01745
Degree/sec	revolutions/min	0.1667
Degree/sec	revolutions/sec	2.778x10^3
Dyne sq cm	atmosphere	9.869x10^-7
Dyne sq cm	inches of Mercury at 0 degs C	2.953x10^-5
Dyne sq cm	inches of water at 4 degs C	4.015x10^-4
Dynes	grams	1.02x10^-3
Dynes	joules centimenter	10^-7
Dynes	joules meter (newton)	10^-5
Dynes	kilogram	1.02x10^-6
Dynes	poundals	7.233x10^-5
Dynes	pounds	2.248x10-6
Dynes	bar	10^-6
	E	
Ergs	Btu	9.84x10^-11
Ergs	dyne-centimeters	1
Ergs	foot-pounds	7.367x10^-8
Ergs	gram-calories	0.2389x10^-7
Ergs	gram-centimeters	1.020x10^-3
Ergs	horsepower-hrs	3.725x10^-14
Ergs	joules	10^-7

Figure A-5 Continued

To Convert	Into	Multiply By
Ergs	kg-calories	2.389x10^-11
Ergs	kg-meter	1.02x10^-8
Ergs	kilowatt-hrs	0.2778x10^-13
Ergs	watt-hrs	0.2778x10^-10
Ergs/sec	Btu/min	5.688x10^-9
Ergs/sec	ft-lbf/min	4.427x10^-6
Ergs/sec	ft-lbf/sec	7.3756x10^-8
Ergs/sec	horsepower	1.341x10^-10
Ergs/sec	kg-calories/min	1.433x10^-9
Ergs/sec	kilowatts	10^-10
	F	
Feet	centimeters	30.48
Feet	kilometers	3.048x10^-4
Feet	meters	0.3048
Feet	miles (nautical)	1.645x10^-4
Feet	miles (statute)	1.894x10^-4
Feet	millimeters	304.8
Feet	mils	1.2x10^4
Feet of water	atmosphere	0.0295
Feet of water	inches of Mercury	0.8826
Feet of water	kgs/sq centimeters	0.03048
Feet of water	kgs/ sq meter	304.8
Feet of water	lbf/sq feet (psf)	62.43
Feet of water	lbf/sq inch (psi)	0.4335
Feet/min	centimeters/sec	0.508
Feet/min	feet/sec	0.01667
Feet/min	kilometer/hr	0.01829
Feet/min	meters/min	0.3048
Feet/min	miles/hr	0.01136
Feet/sec	centimeters/sec	30.48
Feet/sec	kilometers/hr	1.097
Feet/sec	knots	0.5921
Feet/sec	meters/min	18.29
Feet/sec	miles/hr	0.6818
Feet/sec	miles/min	0.01136
Feet/sec/sec	cms/sec/sec	30.48
Feet/sec/sec	kms/hr/sec	1.097

Figure A-5 Continued

To Convert	Into	Multiply By
Feet/sec/sec	meters/sec/sec	0.3048
Feet/sec/sec	miles/hr/sec	0.6818
Foot-pounds	Btu	1.286x10^-3
Foot-pounds	ergs	1.356x10^7
Foot-pounds	gram-calories	0.3238
Foot-pounds	hp-hrs	5.05x10^-7
Foot-pounds	joules	1.356
Foot-pounds	kg-calories	3.24x10^-4
Foot-pounds	kg-meters	0.1383
Foot-pounds	kilowatt-hrs	3.766x10^-7
Foot-pounds	newton-meters	1.35582
Foot-pounds/min	foot-pound sec	0.01667
Foot-pounds/min	horsepower	3.03x10^-5
Foot-pounds/min	kg-calories/min	3.24x10^-4
Foot-pounds/min	kilowatts	2.26x10^-5
Foot-pounds/sec	Btu/hr	4.6263
Foot-pounds/sec	Btu/min	0.07717
Foot-pounds/sec	horsepower	1.818x10^-3
Foot-pounds/sec	kg-calories/min	0.01945
Foot-pounds/sec	kilowatts	1.356x10^-3
	G	
Gallons	cubic centimeters	3785
Gallons	cubic feet	0.1337
Gallons	cubic inches	231
Gallons	cubic meters	3.785x10^-3
Gallons	cubic yards	4.951x10^-3
Gallons	liters	3.785
Gallons (liq. Br Imp)	gallons (US liq)	1.20095
Gallons (US)	gallons (imp)	0.83267
Gallons of water	pounds of water	8.3453
Grams	dynes	980.7
Grams	joules centimeter	9.807x10^-5
Grams	joules meter (newtons)	9.807x10^-3
Grams	kilogram	0.001
Grams	milligrams	1000
Grams	ounces ounce (troy)	0.03215
Grams	poundals	0.07093

Figure A-5 Continued

To Convert	Into	Multiply By
Grams	pounds	2.205x10^-3
Grams/cm	pounds/inch	5.6x10^-3
	H	
Horsepower	Btu/min	42.44
Horsepower	foot-lbf/min	33000
Horsepower	foot-lbf/sec	550
Horsepower (metric)	hp (550 ft /lbf/sec)	0.9863
Horsepower	hp metric (542.5 ft lbf/sec)	1.014
Horsepower	kg-calories/min	10.68
Horsepower	kilowatts	0.7457
Horsepower	watts	745.7
Horsepower (boiler)	Btu/hr	33.479
Horsepower (boiler)	kilowatts	9.803
	I	
Inches	centimeters	2.45
Inches	meters	2.54x10^-2
Inches	miles	1.578x10^-5
Inches	millimeters	25.4
Inches	mils	1000
Inches	yards	2.778x10^-2
Inches of Mercury	atmospheres	0.033422
Inches of Mercury	feet of water	1.133
Inches of Mercury	kgs/sq cm	0.03453
Inches of Mercury	kgs/sq meter	345.3
Inches of Mercury	pounds/sq ft (psf)	70.73
Inches of Mercury	pounds /sq in (psi)	0.4912
Inches of Water @ 4 degs C	atmospheres	2.458x10^-3
Inches of Water @ 4 degs C	inches of Mercury	0.07355
Inches of Water @ 4 degs C	kgs/sq centimeter	2.54x10^-3
Inches of Water @ 4 degs C	ounces /sq inches	0.5781
Inches of Water @ 4 degs C	lbf/sq ft (psf)	5.204
Inches of Water @ 4 degs C	lbf/sq in (psi)	0.03613

Figure A-5 Continued

To Convert	Into	Mulitiply By
	J	
Joules	Btu	9.840x10^-4
Joules	ergs	10^7
Joules	foot-pounds	0.7376
Joules	kg-calories	2.389x10^-4
Joules	kg-meters	0.102
Joules	watts-hrs	2.778x10^-4
Joules/cm	grams	1.02x10^4
Joules/cm	dynes	10^7
Joules/cm	joules/meter (newtons)	100
Joules/cm	poundals	723.3
Joules/cm	pounds	22.48
	K	
Kilograms (kgs)	dynes	980665
Kilograms	grams	1000
Kilograms	joules/cm	0.09807
Kilograms	joules/meter (newtons)	9.807
Kilograms	poundals	70.93
Kilograms	pounds	2.205
Kilograms	ton (long)	9.842x10^-4
Kilograms	ton (short)	1.102x10^-3
Kilogram/cubic meter	grams/cu cm	0.001
Kilogram/cubic meter	pounds/cu ft	0.06243
Kilogram/cubic meter	pounds/cu inch	3.613x10^5
Kilograms/meter	pounds/ft	0.672
Kilogram/sq cm	dynes	980665
Kilogram/sq cm	atmosphere	0.9678
Kilogram/sq cm	feet of water	32.81
Kilogram/sq cm	inches of Mercury	28.96
Kilogram/sq cm	lbf/sq feet (psf)	2048
Kilogram/sq cm	lbf/sq inch (psi)	14.22
Kilogram/sq meter	atmospheres	9.678x10^-5
Kilogram/sq meter	bars	98.07x10^-6
Kilogram/sq meter	feet of water	3.281x10^-3
Kilogram/sq meter	inches of Mercury	2.896x10^-3
Kilogram/sq meter	lbf/sq feet (psf)	0.2048
Kilogram/sq meter	lbf/sq inch (psi)	1.422x10^-3
Kilometers (kms)	centimeters	10^-6

Figure A-5 Continued

To Convert	Into	Multiply By
Kilometers	feet	3281.0
Kilometers	inches	3.937x10^4
Kilometers	meters	1000.0
Kilometers	miles	0.6214
Kilometers	millimeters	10^6
Kilometers	yards	1094.0
Kilometers/hr	centimeters/sec	27.78
Kilometers/hr	feet/min	54.68
Kilometers/hr	feet/sec	0.9113
Kilometers/hr	knots	0.5396
Kilometers/hr	meters/min	16.67
Kilometers/hr	miles/hr	0.6214
Kilometers/hr/sec	cms/sec/sec	27.78
Kilometers/hr/sec	ft/sec/sec	0.9113
Kilometers/hr/sec	meters/sec/sec	0.2778
Kilometers/hr/sec	miles/hr/sec	0.6214
Kilowatts	Btu/min	56.92
Kilowatts	horsepower	1.341
Knots	feet/hr	6080
Knots	kilometers/hr	1.08532
Knots	mautical miles/hr	1.0
Knots	statute miles/hr	1.151
Knots	yards/hr	2027.0
	feet/sec	1.689
	L	
Liters	bushels (US dry)	0.02838
Liters	cubic cm	1000
Liters	cubic feet	0.03531
Liters	cubic inches	61.02
Liters	cubic meters	0.001
Liters	cubic yards	1.308x10^-3
Liters	gallons (US liq)	0.2642
Liters	pints (US liq)	2.113
Liters	quarts (US liq)	1.057
Lumen	spherical candle power	0.07958
Lumen	watt	0.001496

Figure A-5 Continued

To Convert	Into	Multiply By
	M	
Meters	centimeters	100
Meters	feet	3.281
Meters	inches	39.37
Meters	kilometers	0.001
Meters	miles (nautical)	5.396x10^-4
Meters	miles (statute)	6.214x10^-4
Meters	millimeters	1000
Meters	yards	1.094
Meters/min	cms/sec	1.667
Meters/min	feet/min	3.281
Meters/min	feet/sec	0.05468
Meters/min	kms/hr	0.06
Meters/min	knots	0.03238
Meters/min	miles/hr	0.03728
Meters/sec	feet/min	196.8
Meters/sec	feet/sec	3.281
Meters/sec	kilometers/hr	3.6
Meters/sec	kilometers/min	0.06
Meters/sec	miles/hr	2.237
Meters/sec	miles/min	0.03728
Meters/sec/sec	cms/sec/sec	100
Meters/sec/sec	ft/sec/sec	3.281
Meters/sec/sec	kms/hr/sec	3.6
Meters/sec/sec	miles/hr/sec	2.237
Microns	meters	1.0x10^-6
Miles (nautical)	feet	6080.27
Miles (nautical)	kilometers	1.852
Miles (nautical)	meters	1853
Miles (nautical)	miles (statute)	1.1516
Miles (nautical)	yards	2027
Miles (statute)	centimeters	1.609x10^5
Miles (statute)	feet	5280
Miles (statute)	inches	6.336x10^4
Miles (statute)	kilometers	1.609
Miles (statute)	meters	1609
Miles (statute)	miles (nautical)	0.8684
Miles (statute)	yards	1760
Miles/hr	cms/sec	44.7
Miles/hr	feet/min	88

Figure A-5 Continued

To Convert	Into	Multiply By
Miles/hr	feet/sec	1.467
Miles/hr	kms/hr	1.609
Miles/hr	kms/min	0.02682
Miles/hr	knots	0.8684
Miles/hr	meters/min	26.82
Miles/hr	miles/min	0.1667
Miles/hr/sec	cms/sec/sec	44.7
Miles/hr/sec	feet/sec/sec	1.467
Miles/hr/sec	kms/hr/sec	1.609
Miles/hr/sec	meters/sec/sec	0.447
Miles/min	cms/sec	2682
Miles/min	feet/sec	88
Miles/min	kms/min	1.609
Miles/min	knots/min	0.8684
Miles/min	miles/hr	60
Millimeters	centimeters	0.1
Millimeters	feet	3.281x10^-3
Millimeters	inches	0.03937
Millimeters	kilometers	10^-6
Millimeters	meters	0.001
Millimeters	miles	6.214x10^-7
Millimeters	mils	39.37
Millimeters	yards	1.094x10^-3
Mils	centimeters	2.54x10^-3
Mils	feet	8.333x10^-5
Mils	inches	0.001
Mils	kilometers	2.54x10^-8
Mils	yards	2.778x10^-5
Minutes (angles)	degrees	0.01667
Minutes (angles)	quadrants	1.852x10^-4
Minutes (angles)	radians	2.909x10^-4
Minutes (angles)	seconds	60
	N	
Newtons	Dynes	1.0x10^5
Newtons	pounds (lbf)	0.2248

Figure A-5 Continued

To Convert	Into	Multiply By
	O	
Ounce	drams	16
Ounce	grains	437.5
Ounce	grams	28.349527
Ounce	pounds	0.0625
Ounce	ounces (troy)	0.9115
Ounce	tons (long)	2.79x10^-5
Ounce	tons (metric)	2.835x10^-5
Ounce (fluid)	cubic inches	1.805
Ounce (fluid)	liters	0.02957
Ounces (troy)	grains	480
Ounces (troy)	grams	31.103481
Ounces (troy)	ounce (avdp.)	1.09714
Ounces (troy)	pennyweights (troy)	20
Ounces (troy)	pounds (troy)	0.08333
Ounce/sq inch	Dynes/sq cm	4309
Ounce/sq inch	lbf/sq in (psi)	0.0625
	P	
Parsec	miles	19 x10^12
Parsec	kilometers	3.084x10^13
Pennyweight (troy)	grams	1.55517
Pennyweight (troy)	pounds (troy)	4.1667x10^-3
Pints (dry)	cubic inches	33.6
Pints (liq)	cubic centimeters	473.2
Pints (liq)	cubic feet	0.01671
Pints (liq)	cubic inches	28.87
Pints (liq)	cubic meters	4.732x10^-4
Pints (liq)	cubic yards	6.189x10^-4
Pints (liq)	gallons	0.125
Pints (liq)	liters	0.4732
Pints (liq)	quarts (liq.)	0.5
Pounds (avoirdupois)	ounces (troy)	14.5833
Poundals	dynes	13826
Poundals	grams	14.1
Poundals	joules/cm	1.383x10^-3
Poundals	joules/meter (newtons)	0.1383
Poundals	kilograms	0.0141
Poundals	pounds	0.03108

Figure A-5 Continued

To Convert	Into	Multiply By
Pounds	drams	256
Pounds	dynes	44.4823x10^4
Pounds	grains	7000
Pounds	grams	453.5924
Pounds	joules/cm	0.04448
Pounds	joules/meter (newtons)	4.448
Pounds	kilograms	0.4536
Pounds	ounces	16
Pounds	ounces (troy)	14.5833
Pounds	poundals	32.17
Pounds	pounds (troy)	1.21528
Pounds (lbf)	slugs	3.1081x10^-2
Pounds	tons (short)	0.0005
Pounds (troy)	grains	5760
Pounds (troy)	grams	373.24177
Pounds (troy)	ounces (avdp.)	13.1657
Pounds (troy)	ounces (troy)	12
Pounds (troy)	pennyweights (troy)	240
Pounds (troy)	pounds (avdp)	0.822857
Pounds (troy)	tons (long)	3.6735 x 10^-4
Pounds (troy)	tons (metric)	3.7324x10^-4
Pounds (troy)	tons (short)	4.1143x10^-4
Pounds of water	cubic feet	0.01602
Pounds of water	cubic inches	27.68
Pounds of water	gallons	0.1198
Pounds of water/min	cubic feet/sec	2.670x10^-4
Pounds-feet	cm-dynes	1.356x10^7
Pounds-feet	cm-grams	13825
Pounds-feet	meter-kgs	0.1383
Pounds/cubic feet	grams/cubic cm	0.01602
Pounds/cubic feet	kgs/cubic meter	16.01847
Pounds/cubic feet	pounds/cubic inch	5.787x10^-4
Pounds/cubic inch	grams/cubic centimeter	27.68
Pounds/cubic inch	kgs/cubic meter	2.768x10^4
Pounds/cubic inch	pounds/cubic feet	1728
Pounds/sq foot (psf)	*atmospheres*	4.725x10^ 4
Pounds/sq foot (psf)	bars	4.788x10^4
Pounds/sq foot (psf)	dyne/sq cm	4.788x10^2
Pounds/sq foot (psf)	feet of water (4 degs C)	0.01602

Figure A-5 Continued

To Convert	Into	Multiply By
Pounds/sq foot (psf)	inches of Mercury @ 0 degs C	0.01414
Pounds/sq foot (psf)	kgs/sq meter	4.882
Pounds/sq foot (psf)	lbf/sq inch (psi)	6.944x10^-3
Pounds/sq foot (psf)	newton sq. meter	47.88
Pounds/sq inch (psi)	atmospheres	0.06804
Pounds/sq inch (psi)	dynes/sq cm	6.8948x10^4
Pounds/sq inch (psi)	feet of water @ 4 degs C	2.307
Pounds/sq inch (psi)	inches of water @ 4 degs C	27.681
Pounds/sq inch (psi)	inches of Mercury @ 0 degs C	2.036
Pounds/sq inch (psi)	newton sq meter	6.8948x10^3
Pounds/sq inch (psi)	kgs/sq meter	703.1
Pounds/sq inch (psi)	pounds/sq inch (psf)	144
	Q	
Quadrants (angle)	degree	90
Quadrants (angle)	minutes	5400
Quadrants (angle)	radians	1.571
Quadrants (angle)	seconds	3.24x10^5
Quarts (dry)	cubic inches	67.2
Quarts (liq.)	cubic centimeters	946.4
Quarts (liq.)	cubic feet	0.03342
Quarts (liq.)	cubic inches	57.75
Quarts (liq.)	cubic meters	9.464x10^-4
Quarts (liq.)	cubic yards	1.238x10^-3
Quarts (liq.)	gallons	0.25
Quarts (liq.)	liters	0.9463
	R	
Radians	degrees	57.3
Radians	minutes	3438
Radians	quadrants	0.6366
Radians	seconds	2.063x10^5
Radians/sec	degrees/sec	57.3
Radians/sec	revolutions/min	9.549
Radians/sec	revolutions/sec	0.1592
Radians/sec/sec	revs/min/min	573
Radians/sec/sec	revs/min/sec	9.549

Figure A-5 Continued

To Convert	Into	Multiply By
Radians/sec/sec	revs/sec/sec	0.1592
Revolutions	degrees	360
Revolutions	quadrants	4
Revolutions	radians	6.283
Revolutions/min	degree/sec	6
Revolutions/min	radians/sec	0.1047
Revolutions/min	revs/sec	0.01667
Revolutions/min/min	radians/sec/sec	1.745x10^-3
Revolutions/min/min	revs/min/sec	0.01667
Revolutions/min/min	revs/sec/sec	2.778x10^-4
	S	
Seconds (angle)	degress	2.778x10^-4
Seconds (angle)	minutes	0.01667
Seconds (angle)	quadrants	3.087x10^-6
Seconds (angle)	radians	4.848x10^-6
Slug	ibm	32.2
Slug	kgs	14.594
Square centimeters	circular mils	1.973x10^5
Square centimeters	square feet	1.076x10^-2
Square centimeters	square inches	0.155
Square centimeters	square meters	0.0001
Square centimeters	square miles	3.861x10^-11
Square centimeters	square millimeters	100
Square centimeters	square yards	1.196x10^-4
Square feet	acres	2.296x10^-5
Square feet	circular mils	1.833x10^8
Square feet	square centimeters	929
Square feet	square inches	144
Square feet	square meters	0.0929
Square feet	square miles	3.587x10^-8
Square feet	square millimeters	9.29x10^4
Square feet	square yards	0.1111
Square inches	circular mils	1.273x10^6
Square inches	square centimeters	6.452
Square inches	square feet	6.944x10^-3
Square inches	square millimeters	645.2
Square inches	square mils	10^6

Figure A-5 Continued

To Convert	Into	Multiply By
Square inches	square yards	7.716x10^-4
Square kilometers	acres	247.1
Square kilometers	square centimeters	10^10
Square kilometers	square feet	10.76x10^6
Square kilometers	square inches	1.55x10^9
Square kilometers	square meters	10^4
Square kilometers	square miles	0.3861
Square kilometers	square yards	1.196x10^6
Square meters	acres	2.471x10^-4
Square meters	square centimeters	10^4
Square meters	square feet	10.76
Square meters	square inches	1550
Square meters	square miles	3.861x10^-7
Square meters	square millimeters	10^6
Square meters	square yards	1.196
Square miles	acres	640
Square miles	square feet	27.88x10^6
Square miles	square kilometers	2.59
Square miles	square meters	2.59x10^6
Square miles	square yards	3.098x10^6
Square milimeters	circular mils	1973
Square milimeters	square centimeters	0.01
Square milimeters	square feet	1.076x10^-5
Square milimeters	square inches	1.55x10^-3
Square yards	acres	2.066x10^-4
Square yards	square centimeters	8361
Square yards	square feet	9
Square yards	square inches	1296
Square yards	square meters	0.8361
Square yards	square miles	3.288x10^-7
Square yards	square millimeters	8.361x10^5
	T	
Tons (long)	kilograms	1016
Tons (long	lbf	2240
Tons (long)	tons (short)	1.12
Tons (metric)	kilogram	1000
Tons (metric)	lbf	2205

Figure A-5 Continued

To Convert	Into	Multiply By
Tons (short)	kilogram	907.1848
Tons (short)	ounce	32000
Tons (short)	ounce (troy)	29166.66
Tons (short)	lbf	2000
Tons (short)	pounds (troy)	2430.56
Tons (short)	tons(long)	0.89287
Tons (short)	tons (metric)	0.9078
	W	
Watts	Btu/hr	3.4129
Watts	Btu/min	0.05688
Watts	ergs/sec	107
Watts	foot-lbf/min	44.27
Watts	foot-lbf/sec	0.7378
Watts	horsepower	1.341x10^-3
Watts	horsepower (metric)	1.36x10^-3
Watts	kg-calories/min	0.01433
Watts	kilowatts	0.001
	Y	
Yards	centimeters	91.44
Yards	kilometers	9.144x10^-4
Yards	meters	0.9144
Yards	miles (nautical)	4.934x10^-4
Yards	miles (statute)	5.682x10^-4
Yards	millimeters	914.4

Figure A-5 Concluded

NOTES

NOTES

APPENDIX B

APPENDIX B

Tid Bits

This section offers tid bits of information that can be useful. The information listed below is not in any particular order.

1) One count of drag is 0.0001.
2) A Δ drag count of 30 counts is a significant increase.
3) Wing sweep decreases C_{L_α} and $C_{L_{max}}$.
4) Critical Mach is increased by wing sweep.
5) In a powered test (V/STOL), vary q, dynamic pressure, to obtain the appropriate thrust coefficient.
6) α_{ZL} does not vary with aspect ratio.
7) Principal contributor to C_{l_β} (dihedral effect) is wing geometric dihedral, 'Z' position of wings and sweep angle.
8) Supercritical airfoils (wings) delay the drag rise due to shock formation.
9) When the Mach number is > 0.3, be sure to correct the dynamic pressure (density) for compressibility effects.

Wind Tunnel First Aid Kit

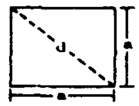
This section is a collection of suggested items that would be helpful to the testing engineer if they accompanied him/her to the testing site. It keeps the testing engineer from loosing an 'atta boy'.

1) Extra money for testing
2) Aspirin
3) Coffee mug
4) Triangle, ruler, symbol template, and other graphing equipment
6) Pencils with extra lead and erasers
7) Binders and dividers to put plots in
8) Labels & gummed tabs for binders and dividers
9) Paper clips
10) Calculator
11) Notebook for personal notes

12) Notebook for wind tunnel log. It is most important that the testing engineer write down everything that happens during the test. By the time the reduced data arrives back 'home', you will have forgotten what happened during the test.

13) Test Plan
14) Run Schedule
15) Stress report
16) Previous data for comparison
17) Model drawings
18) Reference material
19) Raucous reading material for the wee hours in the morning
20) Telephone numbers from back home and at the test site
21) Don't forget extra money for a post test party.

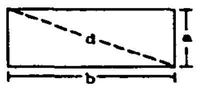
Square



$A = a^2$

$d = a\sqrt{2}$

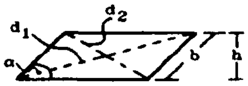
Rectangle



$A = ab \qquad I_x = \frac{ab^3}{12}$

$d = \sqrt{a^2 + b^2} \qquad I_y = \frac{ba^3}{2}$

Parallelogram

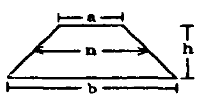


$A = ab$

$d_1 = \sqrt{a^2 + b^2 + 2ab\cos\alpha}$

$d_2 = \sqrt{a^2 + b^2 - 2ab\cos\alpha}$

Trapesium

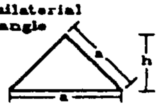


$A = \left[\frac{a + b}{2}\right]h = nh$

$n = \left[\frac{a + b}{2}\right]$

$\bar{y} = \frac{h}{3}\left[\frac{2a + b}{a + b}\right]$

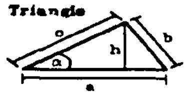
Equilaterial Triangle



$A = a^2\sqrt{3}$

$h = \frac{a}{2}\sqrt{3}$

Triangle

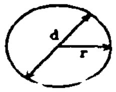


$A = \left[\frac{a + b}{2}\right] = \frac{1}{2}ab\sin\alpha$

$\bar{y} = \frac{h}{3} \qquad \bar{x} = \frac{a + b}{3}$

$I_x = \frac{ah^3}{36}$

Circle



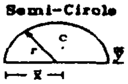
$A = \pi r^2 = \frac{\pi}{4}d^2 = 0.785d^2$

Circum $= 2\pi r$

$I_x = \frac{\pi r^4}{4}$

$\bar{y} = \bar{x} = 0$

Semi-Circle

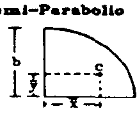


$A = \frac{\pi r^2}{2}$

$\bar{x} = \bar{y} = \frac{4r}{3\pi}$

$I_x = I_y = 0.1098r^4$

Semi-Parabolic

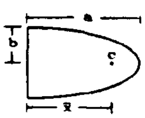


$A = \frac{2}{3}ab$

$\bar{x} = \frac{2}{5}a$

$y = \frac{3}{8}b$

Parabolic

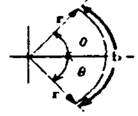


$A = \frac{4}{3}ab$

$\bar{x} = \frac{2}{5}a$

$\bar{y} = \frac{3}{8}b$

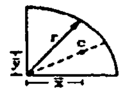
Circular Arc Sector



$A = \frac{\theta r^2}{2} \qquad b = 2\theta r$

$\bar{x} = \frac{2}{3}\frac{r\sin\theta}{\theta} \qquad \bar{y} = r\sin\theta$

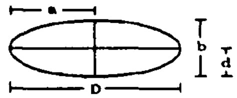
Quarter Circular



$\bar{x} = \bar{y} = \frac{4r}{3\pi}$

$I_x = I_y = 0.0549r^4$

Ellipse

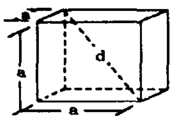


$A = \frac{\pi}{4}Dd = \pi ab$

Circum $= \pi\frac{D + d}{2}$

$I_x = \frac{\pi}{4}ad^3$

Cube

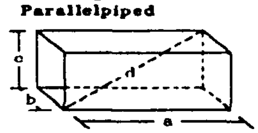


$V = a^3$

$A = 6a^2$

$d = a\sqrt{3}$

Rectangle Parallelpiped



m = mass

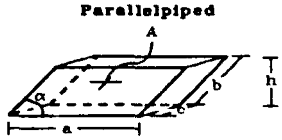
$V = abc$

$A = 2(ab + ac + bc)$

$d = \sqrt{a^2 + b^2 + c^2}$

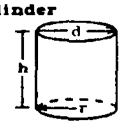
$I_{xx} = \frac{m}{12}(a^2 + b^2)$

Parallelpiped



$V = hA_{surf} = abc\sin\alpha$

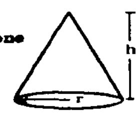
Cylinder



$V = \frac{\pi}{4}d^2h \qquad I_{xx} = \frac{mr^2}{4} + \frac{mh^2}{12}$

$A_{surf} = 2\pi r(r + h) \qquad I_{yy} = \frac{mr^2}{2}$

Cone



$V = \frac{\pi}{3}hr^2$

$A = \pi r\left[\sqrt{r^2 + h^2} + r\right]$

$\bar{y} = \frac{h}{4} \qquad \bar{x} = \frac{3}{4}h$

Sphere

$V = \frac{4}{3}\pi r^3 = \frac{\pi}{6}h^3$

$A_{surf} = 4\pi r^2$

Figure B-1 Geometric Equations

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APPENDIX C

APPENDIX C

Power On Symbols

Moment Arms

l_1 = Horizontal Ram Drag Arm

l_2 = Vertical Thrust Arm

l_3 = Vertical Ram Drag Arm

l_4 = Horizontal Thrust Arm

l_5 = Lateral Thrust Arm

Thrust Induced Aerodynamic Forces (Pwr ON - Pwr OFF)

ΔL Lift

ΔD Drag

ΔM Pitching moment

ΔRM Rolling moment

Total Coefficients

C_L Lift coefficient, L/qS

C_D Drag coefficient, D/qS

C_M Pitching moment, $M/qS\bar{c}$

C_{RM} Rolling moment, RM/qS

Incremental Thrust Coefficients

C_T Thrust coefficient, T/qS

ΔC_{L_T} Lift coefficient due to Thrust

ΔC_{D_T} Drag coefficient due to thrust

ΔC_{M_T} Pitching moment coefficient due to Thrust

Symbols continued

ΔC_{RM_T} Rolling moment coefficient due to Thrust

ΔC_{YM_T} Yawing moment coefficient due to Thrust

ΔC_{M_D} Pitching moment due to Ram drag

C_{D_R} Ram drag coefficient, $\frac{M_i V}{qS}$

Aerodynamic Coefficients (direct thrust effect removed)

$C_{L_{Aero}}$ or C_{L_A} Lift coefficient

$C_{D_{Aero}}$ or C_{D_A} Drag coefficient

$C_{M_{Aero}}$ or C_{M_A} Pitching moment coefficient

$C_{RM_{Aero}}$ or C_{RM_A} *Rolling moment coefficient*

Miscellaneous

x = Fan Number

θ_T = Total Thrust Vector (measured from longitudinal axis)

θ_x = Fan Thrust Vector

$\bar{c}$ = M.A.C.

b = Wing Span

h/D Non-dimensional height with respect to the nozzle diameter

Power On Aerodynamic Equations (ref. 5)

Incremental Aerodynamic Coefficient (x = fan number)

$$\theta_T = \sin^{-1}\Sigma T_x \sin\theta_x / \sqrt{(\Sigma T_x \cos\theta_x)^2 + (\Sigma T_x \sin\theta_x)^2}$$

$$\Delta C_{L_T} = C_T \sin(\theta_T + \alpha)$$

$$\Delta C_{D_T} = - C_T \cos(\theta_T + \alpha)$$

$$\Delta C_{M_{T_x}} = T_x(l_{4_x} \sin\theta_{T_x} + l_{2_x} \cos\theta_{T_x}) / qS\bar{c}$$

$$\Delta C_{M_T} = \Sigma\Delta C_{M_{T_x}} \quad \text{(Fan 1,2,3,...x)}$$

$$\Delta C_{M_{D_x}} = D_R(l_{3_x} \cos\alpha + l_{1_x} \sin\alpha) / qS\bar{c}$$

$$\Delta C_{M_D} = \Sigma\Delta C_{M_x} \quad \text{(Fan 1,2,3,...x)}$$

$$\Delta C_{RM_x} = T_x l_{5_x}\left[\sin\theta_x\cos\alpha + \cos\theta \sin\alpha\right] / qS\bar{c}$$

$$\Delta C_{YM_x} = T_x l_{5_x}\left[\cos\theta_x\cos\alpha - \sin\theta_x\sin\alpha\right] / qS\bar{c}$$

Aerodynamic Coefficients (total - thrust effects)
(figure III-5)

$$C_{L_{Aero}} = C_L - \Delta C_{L_T}$$

$$C_{D_{Aero}} = C_D - \Delta C_{D_T} - C_{D_R}$$

$$C_{M_{Aero}} = C_M - \Delta C_{D_T} - \Delta C_{M_D}$$

$$C_{RM_{Aero}} = C_{RM} - \Sigma\Delta C_{RM_x}$$

$$C_{YM_{Aero}} = C_{YM} - \Sigma\Delta C_{YM_x}$$

$$D_R = M_i V$$

$$C_{D_R} = \frac{D_R}{q\,S}$$

<u>Induced Force Coefficients</u>

$$\frac{\Delta L}{T} = \frac{C_{L_{A_{pwr\ on}}} - C_{L_{pwr\ off}}}{C_T}$$

$$\frac{\Delta D}{T} = \frac{C_{D_{A_{pwr\ on}}} - C_{D_{pwr\ off}}}{C_T}$$

$$\frac{\Delta M}{T\bar{c}} = \frac{C_{RM_{A_{pwr\ on}}} - C_{RM_{pwr\ off}}}{C_T}$$

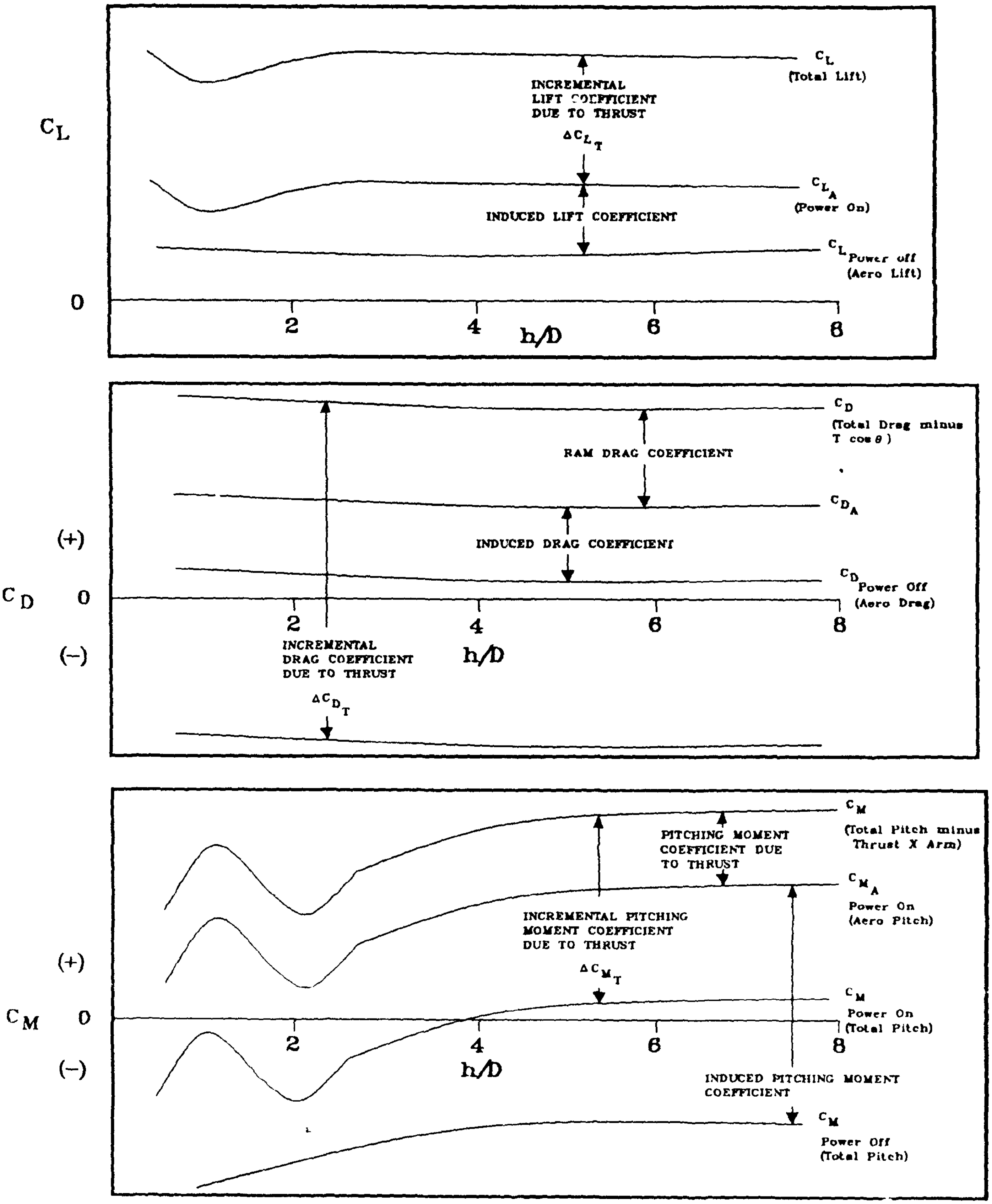

Figure C-1 Powered Effects

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SUPPLEMENTARY

INFORMATION

DEPARTMENT OF THE AIR FORCE

WRIGHT LABORATORY (AFMC)
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FROM: WL/DOA
Wright-Patterson AFB OH 45433-6523

7 January 1994

SUBJ: Notice of Changes in Technical Report(s) #WL-TR-91-3073/ADA240263

TO: Defense Technical Information Center
Attn: DTIC-OCC
Cameron Station
Alexandria VA 22304-6145

Please change subject report as follows:

Include the attached 2 corrected pages in subject Tech Report as an ERRATA SHEET.

Errata ADA240263

WM F. WHALEN
Chief, STINFO and Technical Editing Branch
Operations and Support Directorate

Cy to: WL/DOL (M. Kline)

Corrections to version 1.0 of <u>Subsonic Wind Tunnel Testing Handbook</u>
WL-TR-91-3073; DTIC AD A240263

1) Page II-2; Mean Aerodynamic Chord (MAC); Add a comma after 'span'.

2) Page IV-7; Figure IV-4, Top figure 'Aerodynamic Angles'; $-\beta$ is on the wrong plane. It should be on the $+\psi$ plane.

3) Page IV-9; Balance to Body Axis , force equations (roll=0) [not matrix]; on C_{N_b} term ... $-C_{Y_{bal}} \sin\beta\sin\alpha$ should be positive $+C_{Y_{bal}} \sin\beta\sin\alpha$.

4) Page IV-9; Balance to Body Axis , force equations (roll=0) [not matrix]; on C_{A_b} term ... $-C_{Y_{bal}} \sin\beta\cos\alpha$ should be positive $+C_{Y_{bal}} \sin\beta\cos\alpha$.

5) Page IV-9; Balance to Body Axis , force equations (roll=0) [not matrix]; on C_{Y_b} term ... $+C_{A_{bal}} \sin\beta$ should be negative $-C_{A_{bal}} \sin\beta$.

6) Page IV-13; Body Axis to Wind Axis, Force matrix; Change C_{S_b} to C_{Y_b}.

7) Page IV-14; Body Axis to Stability Axis, Force matrix; Change C_{S_b} to C_{Y_b}.

8) Page IV-15; Wind Axis to Stability Axis, Add a 'w' sub-subscript to C_L in the vector column of the force matrix.

9) Page V-2; change the units on 32.1741 to (lbm-ft/lbf-s^2) from (ft/sec^2).

10) Page VI-6; Figure VI-3; η_i is measured on this figure from the centerline to the inboard trailing edge flap chord.

11) Page VI-10; Inboard Section, b /2=26.96in; Change to $b_i/2$=26.96 in.

12) Page VII-7, Method of Determining a Drag Polar; Change $\sum_{i=1}^{N} C_L(i)*C_D(i)^2$ to $\sum_{i=1}^{N} C_L(i)^2*C_D(i)$.

13) Page VII-7, Last line; change $b=-2kC'$ to $b=-2KC_L'$

14) Page VII-8; Resulting augmented matrix; Change first line from '3 1 3' to '5 1 0.3'; Change 0.11251 to 0.011251

15) Page XI-9; 4th line; Change 'in milli-volts [mv]' to 'in milli-volts [mv])'

16) Page XI-12; Example Original Balance Calibration Matrix, **<u>no action is required</u>.** This is a **<u>point of clarification</u>.** Part of this matrix came from Figure XI-3. The original matrix was inadvertently entered and subsequently inverted in COLUMN order when in fact it should have been entered in ROW order. All the preceding calculations use the column order inverted matrix, but their ORDER of calculations and calculations is correct. If you "pretend" they were entered in row order everything works out fine. This was <u>not</u> corrected in this report.

17) Page A-3; Standard Atmosphere; The values used in this figure were obtained from a dated U.S. Standard Atmosphere source ie., speed of sound @ S.L. = 1107.64 when it should be 1116.1.

18) Delete Page A-12.

19) Page A-10...; Conversion Factors;

To Convert	Into	Multiply By	Change to
Centigrade	fahrenheit	(C*9/5)-32	(C*9/5)+32
Centimeters	kilometers	10^-6	10^-5
Cubic feet	cubic centimeters	2832	28,320.0
Cubic feet	*Cubic inches*	1.728	1728.0
Cubic inches	gallons	4.329x10^-6	4.329x10^-3
Degree/sec	revolutions/sec	2.778x10^3	2.778x10^-3
Ergs/sec	BTU/min	5.688x10^-9	5,688x10^-9
Hp (boiler)	BTU/hr	33.479	33,479.0
Knots	kilometers/hr	1.08532	1.8532
Knots	mautical mi/hr	1.0	nautical mi/hr

20) Page B-3; Figure B-1 Geometric Equations, Parallelogram; the length of 'a' is the base of the parallelogram.