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In estimating cost, the state of knowledge is imperfect when uncertainties arise from incomplete design information. Motivated by needs in design reliability for estimating costs at the time of Feasibility Report, this research and development proposal addresses methods for quantifying the uncertainty in the design arising from the imperfect state of knowledge. A model of design reliability for estimating cost of pile foundations will be developed.

As the design process is now developing within the Army Corps of Engineers, it is becoming necessary to estimate costs with high levels of accuracy at the time of the Feasibility Report. Experience has shown that the accuracy requirements may not be met by current estimating practices. Discussions with designers reveal that the primary cause of poor cost estimates is not a failure to correctly assess unit construction costs but rather a poor judgment of the structure itself. For example, if the

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foundation type is incorrectly anticipated a large change in cost occurs.

In this Report of Research (Phase I), a conceptual model for design reliability was developed. A regression analysis method was used to develop a linear cost function using statistical data. Fuzzy set theory was then utilized to get both the design fuzzy probability and the design membership function. In addition, sensitivity analysis was used to identify the critical variables of the problem. A computer program was developed. The program listing and the user manual are given in the appendices.

The conceptual model for design reliability proposed in this report will be used to estimate the cost of pile foundations. A particular project will be selected and design uncertainties will be assessed by the researchers working in cooperation with engineers from the New Orleans District who will provide the required data. The membership design function data must be assembled.

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Cost accounting
Piling (Civil engineering)--Reliability
Piling (Civil engineering)--Costs
Reliability (Engineering)

Preface

The investigation described in this report was conducted by the Department of Civil, Environmental, and Architectural Engineering, College of Engineering and Applied Science, University of Colorado at Boulder (UC), for the Computer-Aided Engineering Division (CAED), Information Technology Laboratory (ITL), US Army Engineer Waterways Station (WES), under Contract No. DACW39-89-M-4488. This study was conducted as part of a project on design cost reliability for pile foundations sponsored under the Civil Works Guidance Update Program (CWGUP), Headquarters, US Army Corps of Engineers (HQUSACE). The CWGUP is managed by Mr. Thomas Mudd, Civil Works Guidance Update Center, CAED. The Technical Monitor for the project is Mr. Donald Dressler, Chief, Structures Branch, Engineering Division, Civil Works Directorate, HQUSACE.

This report was prepared by Drs. Dan M. Fangopol, George G. Goble, and Mr. Ahmed Banafa. The work was monitored by Mr. Reed L. Mosher, CAED, under the general supervision of Dr. Edward Middleton, Chief, CAED, Mr. Paul K. Senter, Assistant Chief, ITL, and Dr. N. Radhakrishnan, Chief, ITL.

COL Larry B. Fulton, EN, was Commander and Director of WES during publication of this report. Dr. Robert W. Whalin was Technical Director.

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1 Introduction

The accuracy of a design and a cost estimate depend upon the stage at which they are performed. This accuracy ranges from 5% at the definitive stage, when almost all the engineering data and design details about the project are available, to more than 35% at the feasibility stage, when there is less information available about the project [9]. The objective here is to develop a model that produces a reliable basis for estimating at the feasibility stage. This stage is very important for both the owner and the designer. It is necessary to have a reliable basis for design and an accurate cost estimate at this stage in order to decide whether to proceed with the project or not.

To accomplish this, a review of the types of estimation methods available is first presented. Two methods, using regression, that deal with the estimate at the predesign stages, are also reviewed. In such estimates there are many subjective factors that can be handled by using the fuzzy sets concept. A model for analyzing design reliability based on fuzzy sets theory is proposed. Numerical examples, including sensitivity analysis, are also presented.

2 Types of Estimation

Cost estimates range from a feasibility study to complete design of project. The estimate and type of design data desired may differ radically from one project to another. In this section, a review of the available methods at all stages will be given and the pitfalls of the estimates will be outlined. A detailed review of the preliminary estimate will follow.

2.1 Estimates and their accuracies

As mentioned previously, several cost estimates are usually performed at different stages of any project. The accuracy of an estimate depends upon the reliability of the design which is performed. Usually, the stages and their corresponding accuracies are as follows:

1. Definitive $\Longrightarrow$ accuracy $\pm 5\%$
2. Appropriation $\Longrightarrow$ accuracy $\pm 10 - 15\%$

3. Feasibility $\Longrightarrow$ accuracy $> \pm 35\%$

It is very important to note that the estimate in all stages is an approximation based on judgment and experience. Some of the reasons that lead to inaccurate estimates are pointed out in Section 2.2.

2.2 Pitfalls of the Estimation

The pitfalls of the estimation may vary from one project to another. The following pitfalls may be common to many projects [9].

1. Misinterpretation of the statement of work.
2. Omissions or improperly defined scope.
3. Poorly defined or overly optimistic schedules.
4. Inadequate project design data and engineering properties.
5. Inaccurate work breakdown structure.
6. Inappropriate analytical techniques or modeling.
7. Applying improper skill levels to tasks.
8. Failure to account for risks, including:
   - safe and reliable design concepts
   - site geology
   - labor productivity
   - unproven technology
   - regulatory, environmental factors
   - equipment productivity
   - changed conditions
   - permit approval
   - weather
   - accidents

- quality control during design and construction

9. Failure to understand or account for cost escalation and inflation.

10. Failure to use correct estimating techniques.

11. Failure to use forward pricing rates for overhead, general and administrative, and indirect costs.

2.3 Preliminary Estimate

A preliminary estimate is generally presented before all the information required for the project is available. Such estimates are performed by the owner or his consultant during a feasibility study, by the designer to evaluate possible design alternatives, and by contractors for bidding and budgeting. A cost estimate at the feasibility study stage cannot be as precise since it is based on an approximate design. However, an accurate method is required. To develop such a method, the identification of available methods, allocation of risk involved as well as when it is likely to occur, and identification of design and cost variables responsible for large variations in the project cost are required.

2.3.1 Types of Preliminary Estimate

Most of the existing conceptual and preliminary estimating methods fall into one or more of the following categories [3]:

1. Time referenced indices.

2. Capacity factors.

3. Component ratios.

4. Parameters.

These methods will be explained in detail in sections 2.3.2 to 2.3.5.

2.3.2 Indices

Indices show changes in estimated quantities over time. These indices are divided into two types. The first type updates the data which serve as inputs to the project. The second one is based on the completed construction project, which is based on the output of the project.

2.3.3 Capacity Factors

Capacity factors apply to changes in size, scope, or capacity of projects of similar type, as opposed to indices which focus on changes over time. Time-independent factors are expressed in the following equation [3]:

$$C_2 = C_1(\frac{Q_2}{Q_1})^x \tag{1}$$

where C_2 is the estimated quantity of the new project of capacity Q_2, C_1 is known quantity of facility of capacity Q_1, and x is the capacity factor for this type of work.

Capacity factors and indices can be combined to take into account changes in both time and capacity. The modified time-dependent formula is [3]:

$$C_2^\circ = C_1(\frac{I_2}{I_1})(\frac{Q_2}{Q_1})^x \tag{2}$$

where C_2° is the estimated quantity of the new project and I_1 and I_2 are the relative indices for the time associated with known and proposed facilities, respectively.

2.3.4 Component Ratios

The component ratio method is based on information concerning the major equipment to be installed in the project. This can be found by multiplying the equipment characteristic or property by an empirically documented factor.

2.3.5 Parameters

The parameter approach relates all design aspects of a project to just a few physical measures of parameters that reflect the size or scope of that project.

For example, the gross enclosed floor area would be a typical parameter for a structure such as a warehouse.

3 Regression Methods

3.1 Kouskoulas Method[20]

In using this method, the building cost function is defined as a function of six variables, namely, locality index, price index, building type, building height, building quality and building technology. The data used in this method are also used in the proposed model, therefore, this method will be explained later on in Section 4.

3.2 Karshenas Method[17]

In the Karshenas (1984) [17] model, the equation selected is a power equation. The cost in this model is a function of height and floor area only, however, these are not the only variables that affect the cost.

3.2.1 Data Collection

Engineering News Record(ENR) data since 1968 are used in Karshenas' study. Table 1 shows the data for office building construction costs. Since the year of construction and the location of each building varies, their cost cannot be directly compared to what was reported by ENR. In order to make the building costs comparable, all costs were converted to March 1982, New York City costs, by using the cost index for steel-framed buildings, published by Marshall and Swift Publishing Company[17]. The converted costs are shown in the second column of Table 2.

3.2.2 Cost Function

The data in Table 1 may be used to derive a cost function of the general form

$$C = f(A, H) \tag{3}$$

in which A and H are the typical floor area and height of the building, respectively.

One method for investigating the shape of this cost function is to plot contours of constant C using A and H as abscissa and ordinate, respectively. Figure 1 shows an example of such contours fitted "by eye". The least squares method was used to select the type of function and power function was chosen. The general form of the power function is [17]:

$$f(A, H) = A^{\beta} H^{\gamma} \tag{4}$$

in which β and γ are constants. Therefore, Eq. (4) would be

$$C = \alpha A^{\beta} H^{\gamma} \tag{5}$$

in which α is a constant.

Taking the natural logarithm on both sides, Eq.(5) may be equivalently expressed as

$$\ln C = \ln \alpha + \beta \ln A + \gamma \ln H \tag{6}$$

Using the data given in Table 1 and the least squares method, the parameters of the preceding equation were estimated. Eq (6) with the estimated parameters is

$$\ln \hat{C} = -0.0235 + 1.1045 \ln A + 1.1268 \ln H \tag{7}$$

or

$$\hat{C} = A^{1.1045} H^{1.1268} e^{-0.0235} \tag{8}$$

in which $\hat{C}$ is the predicted cost of a building with the typical floor area A (sq. ft), and height of H (ft). Examples of the contours of Eq.(8) superimposed on the scatter plot of the observed building costs are shown in Figure 2. The costs predicted by Eq.(8) are tabulated in Column 3 of Table 2 .

3.2.3 Examination of the Formula

The following questions are considered in this section:

1. Whether the data are adequately described by the regression equation?

2. How accurately Eq.(8) predicts the cost of a building?

The ability of the model to explain the variations of the cost data can be measured by investigating the coefficient of determination, defined as

$$r^2 = \frac{S_t - S_r}{S_t} \tag{9}$$

where

$$S_t = \sum_{i=1}^{n} (C_i - \bar{C})^2$$

and

$$S_r = \sum_{i=1}^{n} (C_i - a_0 - a_1 X_{1i} - \ldots\ldots - a_m X_{mi})^2$$

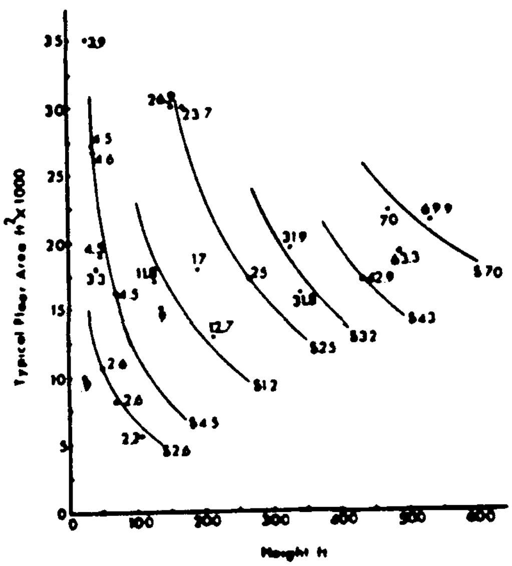

Figure 1: Scattergram of Observed Costs and Contours of Constant Cost Fitted "by eye"(in millions of dollars) [17]

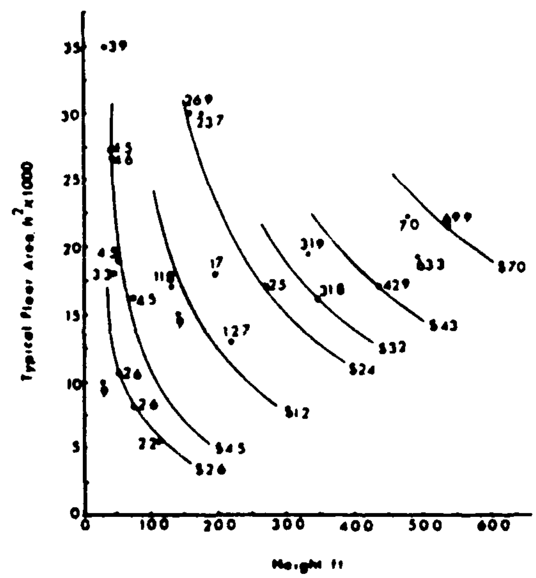

Figure 2: Scattergram of Observed Costs and Contours of Estimated Regression Plane (in millions of dollars) [17]

where $\bar{C}$ is the mean value of the cost and n is the number of observed data.

If Eq. (8) perfectly predicts the cost of the building, (i.e.,$S_r = 0$) . then $r = 1$. On the other hand, if the proposed model does not explain the variations in the building cost, $r = 0$.

4 Feasibility Estimate

The design of any project is a function of many variables. An important problem to consider is the selection of a set of independent variables that defines a project design and its cost. Another problem is the selection of the cost function in terms of the design variables. The criterion for selecting the variables is availability of data concerning such variables from previously completed projects. Regarding data availability, the cost function may be defined as:

$$C = f(V_1, ..., V_m) \tag{10}$$

in which C is a cost measure of a project and V_i are independent design variables.

The design variables may include:

1. Location, including site geology.
2. Quality of work.
3. Labor productivity.
4. Material.
5. Equipment productivity.
6. Technology.
7. Weather.
8. Project type.
9. Structure height.

4.1 Cost Function

The selection of independent design variables and the construction of the cost function depend on the availability of data. The design variables used by Kouskoulas [20] are the data for a building. His independent design variables include, building locality index, price index, building type index, building height, building quality index and technology index.

4.2 Measurement of Design Variables

In preliminary cost estimating, one may identify a building by its location, time of realization, function or type, height, quality and technology. Consider the variable $V_1 = L$, identifying the differences in construction costs as a consequence of differences in the life style between different cities. Table 3 presents typical locality indices for ten cities. The price index variable $V_2 = P$ is time-dependent. The problem, therefore, is that the price index has to be projected into the future or to the time when the construction takes place in order to construct the function.

$$P = g(t) \tag{11}$$

Utilizing historical data, Table 4 provides a sample time history of average construction costs in dollars per cubic foot for various apartment buildings.

The third variable, $V_3 = T$, specifies the types of buildings. A good measure for this variable is provided by the average cost per cubic foot for different types of buildings. Table 5 shows a range of classes of buildings, with their corresponding relative cost values, as provided by [20].

The height index, $V_4 = H$, indicates the number of stories.

The quality variable, $V_5 = Q$, stands for what it specifies. It is a measure of:

1. The quality of workmanship and materials used in the construction process.
2. The building use.
3. The design effort.
4. The material type and quality used in various building components.

To define the measure for such a variable, let $C_i, i = 1, \ldots k$, denote the average cost portions of total building cost distributed among k building components. Assign an integer index, I, from 1 to 4 (corresponding to fair, average, good and excellent) to each component, and compute the values of the variable, V_5, from [20]:

$$V_5 = \frac{1}{k} \sum_{i=1}^{k} I_i C_i \tag{12}$$

A more subjective way might be to identify the components of the building, assigning each a value of 1 to 4 and, on this basis, pass a judgment on the quality of the total building by assigning it one of the integer values 1 to 4. It must be realized that the range of integer value, 1 to 4, is arbitrarily selected and is only good as long as it can provide a reliable estimation function.

Table 6 identifies general building components and, on the basis of their qualitative description, rates them accordingly, from fair to excellent.

The technology index variable, $V_6 = TE$, takes into account the extra cost that is incurred for special types of buildings or the labor and material savings that result from the use of new techniques in the construction process. Some data regarding this variable are given in Table 7.

4.3 Derivation of Cost Function

The cost function, $C = f(V_1, \ldots V_m)$, may take many possible forms. The one chosen here is the linear expression:

$$\hat{C} = a_0 + a_1 V_1 + a_2 V_2 + \ldots\ldots\ldots + a_m V_m + e \tag{13}$$

where $a_k (k = 0, \ldots, m)$ are constants to be determined from the available data and $(e = C_i - \hat{C}_i)$ is the error associated with the selected function. Once the coefficients a_k are known, $\hat{C}$ can be computed for any new project identified by a set of values V_k . The coefficients a_k $(k = 0, 1, \ldots m)$ are found by multiple linear regression analysis using the data in Table 8 .

Table 1: Office Building Design Data and Costs [17]

Number	Location	Time of construction	Number of floors	Building height (feet)	Typical floor area (square feet)	Total Cost (dollars)
1	Lexington,Mass	Nov.77/Jan. 79	3	36	27,000	3,133,100
2	Southfield,Mich.	Apr.76/June 78	15	187.5	17,690	11,645,000
3	Pocatello,Idaho	May 76/Sept.77	3	36.6	26,690	2,969,800
4	Dallas,Tex	Apr.76/May 77	21	262.5	17,000	12,958,000
5	Glendale,Calif.	Dec.75/Nov.76	6	72	15,800	2781,480
6	Seattle,Wash.	Feb.74/Dec.76	36	468	22,200	36,470,000
7	Scottsdale,Ariz.	Dec.74/May 76	2	28	140,332	12,729,900
8	Knoxville,Tenn.	Aug.74/Apr.75	2	25.3	9,986	446,500
9	Troy,Mich.	Aug.73/Oct.75	26	330	19,400	16,822,00
10	Birmingham,Ala.	Oct.74/Jan.76	18	216	12,616	6,104,140
11	Franklin, Ill.	Mar.74/dec.74	5	62.6	8000	1,396,200
12	Beverly Hills,Calif.	Nov.73/July 75	8	105.6	5,500	1,204,100
13	Houston,Tex.	July 73/Jan.75	13	175.5	29,920	10,408,000
14	Chicago, Ill.	Dec. 73/Dec. 74	2	28	35,280	1,951,175
15	Detroit, Mich.	Aug. 71/Apr.73	4	48	17,700	1,731,800
16	Warren,Mich.	June 72/Oct.73	11	137.5	15,000	4,435,000
17	Wellesley, Mass.	Dec. 69/Sept. 70	4	48	18,800	1,763,000
18	Central,N.J.	Nov. 70/Feb.72	12	153	30,134	11,129,000
19	San Francisco,Calif.	Oct.66/May 68	33	429	17,212	14,455,000
20	New York, N.Y.	Oct.61/Nov.63	42	483	18,893	16,820,900
21	Cleveland, Ohio	Feb. 63/Nov. 64	41	533	21,600	20,116.000
22	Columbus, Ohio	Dec. 63/Feb. 65	26	338	16,000	8,683,000
23	Pittsburgh, Pa.	Apr 66/Apr 68	9	126	16,833	3,871,000
24	Houston,Tex.	Sept. 65/Aug 66	4	50	10,500	656,100

Table 2: Predicted Cost and Residuals

Number	Adjusted total cost, c, (dollars)	Predicted total cost, $\hat{c}$, (dollars)	Residuals, $\epsilon = c - \hat{c}$	Residual as a percentage, (ϵ/c)
1	4,542,000	4,377,454	164,545	0.036
2	17,271,000	17,619,844	-348,844	-0.020
3	4,699,000	4,441,512	257,487	0.054
4	25,254,000	24,636,604	617,396	0.024
5	4,506,000	5,289,436	-783,436	-0.170
6	70,022,000	63,469,742	6,552,258	0.093
7	21,050,000	20,362,490	687,510	0.032
8	928,720	980,603	-51,883	-0.055
9	31,942,800	36,890,823	-4,948,023	-0.150
10	12,704,000	14,226,597	-1,522,597	-0.120
11	2,619,110	2,126,700	492,410	0.190
12	2,175,000	2,538,924	-363,924	-0.160
13	23,722,000	29,222,684	5,500,684	-0.230
14	3,902,350	4,431,366	-529,016	-0.130
15	3,300,000	3,797,100	-497,100	-0.150
16	9,275,000	10,353,680	-1,078,680	-0.110
17	4,531,000	4,058,570	472,429	0.100
18	26,932,000	25,234,518	1,697,482	0.06
19	42,931,000	43,442,324	-511,324	-0.012
20	63,346,000	55,034,241	8,311,759	0.130
21	69,946,000	71,296,732	-1,350,732	-0.019
22	31,817,000	30,634,697	1,182,303	0.037
23	11,818,000	10,657,469	1,160,531	0.098
24	2,646,100	2,233,291	412,809	0.15

Table 3: Typical Locality Indices for 10 Cities [20]

City	Index.L
Boston,Mass.	1.03
Buffalo,N.Y.	1.10
Dallas,Tex.	0.87
Dayton,Ohio	1.05
Detroit,Mich.	1.13
Erie,Pa.	1.02
Houston,Tex.	0.90
Louisville,Ky.	0.95
New York,N.Y.	1.16
Omaha,Neb.	0.92

Table 4: Sample Time History of Average Construction Costs, in Dollars per Cubic Foot,for Various Apartment Buildings [20]

Year	t	Index.P
1963	0	1.66
1964	1	1.71
1965	2	1.76
1966	3	1.8
1967	4	1.93
1968	5	2.09
1969	6	2.30
1970	7	2.49
1971	8	2.76
1972	9	2.95

Table 5: Range of Classes of Buildings with Corresponding Relative Cost Values [20]

Type	Index.I
Apartment	2.97
Hospitals	3.08
Schools	2.59
Hotels	3.08
Office building(fireproof)	2.95
Office building(not fireproof)	1.83
Stores	2.43
Garages	1.99
Factories	1.2
Foundries	1.49

Table 6: Quality Index Q[20]

Component	Fair	Average	Good	Very Good
Use	Multitenancy	Mixed. single tenant, and Multitenancy	Single tenant	Single tenant with custom requirements
Design	Minimum design load	Average design load	Above average design load	Many extra design load
Exterior Wall	Masonry	Glass or masonry	Glass,curtain wall,precast concrete	Monumental (marble)
Plumbing	Below Average quality	Average quality	Above average quality	Above average quality
Flooring	Resilient, ceramics	Resilient, ceramics and terrazzo	Vinyl,ceramic terrazzo	Rug, terrazzo marble
Electrical	Fluorescent light,poor quality ceiling	Fluorescent light,average quality, suspended ceiling	Fluorescent light,above average quality ceiling	Fluorescent light,excellent quality ceiling
Heating, Ventilating and Air Conditioning	Below average quality	Average quality	Above average quality	Above average quality
Elevator	Minimum required	Above required	High speed	High speed deluxe

Table 7: Data Regarding Magnitude of V_6[20]

Technology	Index.TE
Bank-monumental work	1.75
Renovation building	0.50
Special school building	1.10
Chemistry laboratory building	1.45
Telephone building-blast resistant	1.60
County jails	1.20
Dental school	1.15
Hospital addition	1.05
Correctional center	1.20
Home for aged	1.10

The basic idea of the least squares fitting method is to minimize the sum-of-squares of the residuals, S_r, as in

$$S_r = \sum_{i=1}^{n} \epsilon^2 = \sum_{i=1}^{n} (C_i - a_0 - a_1 \dot{V}_{1i} - \ldots\ldots\ldots - a_m \dot{V}_{mi})^2 \tag{14}$$

Then the function is differentiated with respect to each of the unknown coefficients.

$$\frac{\partial S_r}{\partial a_k} = -2 \sum_{i=1}^{n} \dot{V}_{ki}(C_i - a_0 - a_1 \dot{V}_{1i} - \ldots\ldots\ldots - a_m \dot{V}_{mi}) \tag{15}$$

$\dot{V}_{0i} = 1,\ k = 0, \ldots\ldots\ldots m\ ,\ \ i = 1, \ldots\ldots\ldots n$

The coefficients yielding the minimum sum of squares of the residuals are obtained by setting the partial derivatives equal to zero and expressing Eq. (15) as a set of simultaneous linear equations.

$$\sum_{k=0}^{m} \left(\sum_{i=1}^{n} \dot{V}_{ki} a_k \right) = \sum \dot{V}_{ki} C_i \tag{16}$$

Or, in matrix form:

$$\begin{bmatrix} n & \sum \dot{V}_{1i} & \ldots\ldots & \sum \dot{V}_{mi} \\ \sum \dot{V}_{1i} & \sum \dot{V}_{1i}^2 & \ldots\ldots & \sum \dot{V}_{1i}\dot{V}_{mi} \\ \sum \dot{V}_{2i} & \sum \dot{V}_{2i}\dot{V}_{1i} & \ldots\ldots & \sum \dot{V}_{2i}\dot{V}_{mi} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \sum \dot{V}_{mi} & \sum \dot{V}_{mi}\dot{V}_{1i} & \ldots\ldots & \sum \dot{V}_{mi}^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \cdot \\ \cdot \\ a_m \end{bmatrix} = \begin{Bmatrix} \sum C_1 \\ \sum \dot{V}_{1i} C_i \\ \sum \dot{V}_{2i} C_i \\ \cdot \\ \cdot \\ \sum \dot{V}_{mi} C_i \end{Bmatrix} \tag{17}$$

where C_i is the cost corresponding to data point i and $\dot{V}_{ki}$ is the variable corresponding to variable k and data point i.

Constants were computed using a regression subroutine, which is included in the program (as listed in Appendix A), with the historical data in Table 8 (from [20]).

According to [20], the resulting required function is:

$$\dot{C} = -84.97694 + 26.67051\dot{V}_1 + 11.23471\dot{V}_2 - 6.23531\dot{V}_3$$
$$+.201221\dot{V}_4 + 5.03921\dot{V}_5 - 31.219091\dot{V}_6 \tag{18}$$

The coefficients in Eq. (19) are slightly different from Kouskoulas's, however, this might be due to different numerical methods used in the analysis.

Table 8: Building Historical Data

Cost C in dollars per square foot	Locality Index V_1	Price Index V_2	Type Index V_3	Height V_4	Quality Index V_5	Technology Index V_6	Variable combinations
36.00	0.90	2.76	2.95	40	2	1.00	1
25.00	0.87	2.49	2.95	18	1	1.00	2
68.50	1.02	2.76	2.95	6	4	1.75	3
31.90	1.03	2.49	2.97	8	2	1.00	4
36.50	1.10	2.49	2.59	11	3	1.00	5
23.30	1.13	2.95	2.95	5	2	0.50	6
40.00	0.95	2.30	3.08	14	4	1.00	7
56.00	1.13	1.93	2.95	3	4	1.60	8
40.00	1.05	2.09	3.08	5	4	1.00	9
21.70	1.13	2.09	1.99	1	2	1.00	10
42.00	1.00	2.76	2.95	4	4	1.00	11
45.81	1.16	2.49	2.59	1	4	1.10	12
62.00	1.16	2.95	2.59	7	4	1.45	13
85.00	1.00	2.95	3.08	6	4	2.25	14
47.50	1.00	2.49	3.08	7	4	1.15	15
34.30	1.13	1.93	3.08	3	3	1.10	16
37.00	1.13	2.76	2.95	24	1	1.00	17
31.90	1.13	2.30	2.95	10	2	1.00	18
40.00	1.13	2.30	2.95	22	3	1.00	19
49.50	1.13	2.95	2.95	27	3	1.00	20
36.20	1.13	2.09	3.08	10	3	1.00	21
24.00	1.13	2.76	1.83	1	1	1.00	22
38.80	1.13	2.09	3.08	1	4	1.05	23
20.00	1.08	1.93	2.95	4	1	1.00	24
18.80	1.13	1.93	2.59	2	1	1.00	25
34.70	1.13	2.09	2.95	5	3	1.00	26
15.10	1.13	1.93	1.83	2	1	1.00	27
18.10	1.13	1.22	2.59	3	2	1.00	28
39.00	1.13	2.3	3.08	4	2	1.20	29
36.00	1.13	2.09	3.08	2	2	1.20	30
21.10	1.07	1.66	2.59	6	2	1.00	31
24.30	1.07	1.93	2.5	6	2	1.00	32
30.00	1.13	1.93	2.59	6	3	1.00	33
27.50	1.13	2.30	3.08	2	1	1.00	34
11.30	1.00	2.09	1.49	1	1	1.00	35
14.50	1.02	2.09	1.20	1	2	1.00	36
10.00	1.05	2.09	1.20	1	1	1.00	37
14.75	0.92	2.3	1.2	1	2	1.00	38

5 Fuzzy Model

Design reliability depends upon the subjective evaluation and judgments of experienced designers. Available data can be used to develop the functions which relate the dependent to the independent design variables. With the regression model, values of independent design variables can be substituted, for example, into the cost function to predict the cost of new designs. This can be done if we have precise information about these variables. Unfortunately, this is not always the case. In many cases, precise information is not always available. The only thing known about some variables may be what could be called "verbal descriptions," such as the lateral loading may be "high," the location is "good." and so on. Normally variables like these are omitted from the model, but this just results in more errors.

The objective here is to try to use the cost function developed by Kouskoulas Eq. (18) and, rather than eliminating independent design variables which are not precisely known, use fuzzy set theory to handle these types of variables in a regression model.

5.1 Fuzzy Set Theory

A conventional, or non-fuzzy set was defined by the use of an indicator function, for which any element of the sample space was 1 or 0, depending upon whether the element was a member of the set or not. The indicator function (or characteristic function) gives us a clear borderline between membership and non-membership for classical set theory. In fuzzy sets, the indicator function is allowed to vary over the range [0, 1] and was retermed by Zadeh as "the membership function" [23]. In this way, the uncertainty as to whether an object belongs to a given set or not is expressed.

If the membership level is 1, the element or object is definitely a member of the set. If the membership is 0, then it is definitely not a member. However, if the membership is an intermediate value, between 0 and 1, it indicates the degree of confidence that the object is a member of the set. A fuzzy set, therefore, is a class that admits the possibility of partial membership. Let $x = \{x\}$ denote a space of objects. Then, a fuzzy set A in X is a set of ordered pairs

$$A = \{x, \mu_A(x)\} \quad x \epsilon X$$

where $\mu_A(x)$ is termed "the grade of membership x to A".

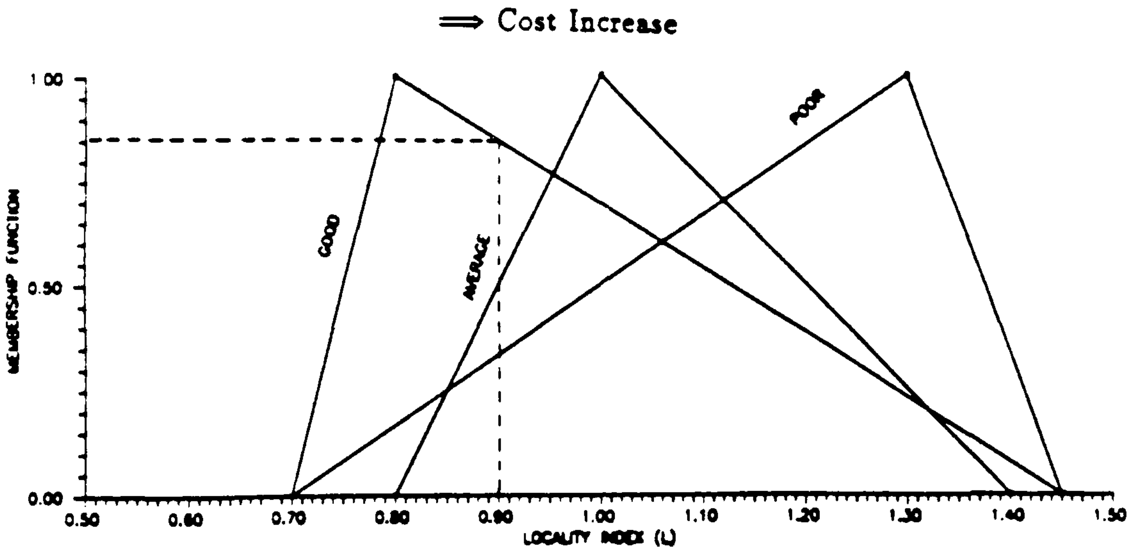

Figure 3: Locality Index Membership Functions

5.2 Application Example

The fuzzy theory will be applied to the design reliability problem using the cost function developed using regression analysis on the building variables. Five variables, including locality index (L), price index (P), building type index (T), quality index (Q) and technology index (TE) will be considered as fuzzy variables, with the membership functions shown in Figs. 3-7. These membership functions are assumed to be given.

5.3 Extension Principle

To find the membership function the extension principle will be used, as defined below [10].

Let X be a Cartesian product of universes, $X = x_1 \times .... \times x_r$, and $A_1, ..., A_r$ be r fuzzy sets in $X_1, ..., X_r$ respectively. The Cartesian product of $A_1, ..., A_r$ is defined as

$$A_1 \times ...... \times A_r = \int_{x_1 \times ... \times x_r} min(\mu_{A_1}(x_1), ..., \mu_{A_r}(x_r))|(x_1, ..., x_r)) \qquad (19)$$

Let f be a mapping from $X_1 \times .... \times X_r$ to universe Y such that $y = f(x_1, ..., x_r)$.

⟹ Cost Increase

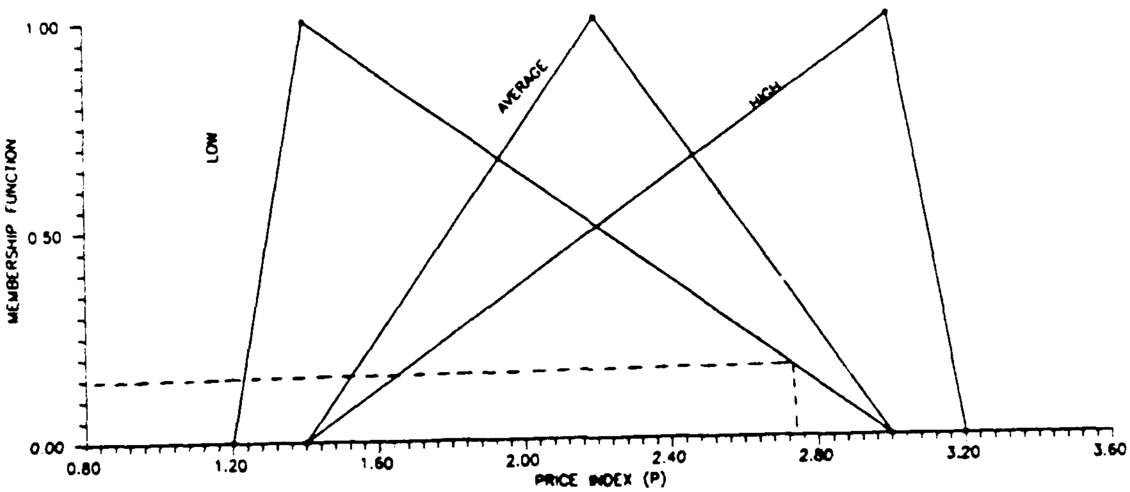

Figure 4: Price Index Membership Functions

⟹ Cost Increase

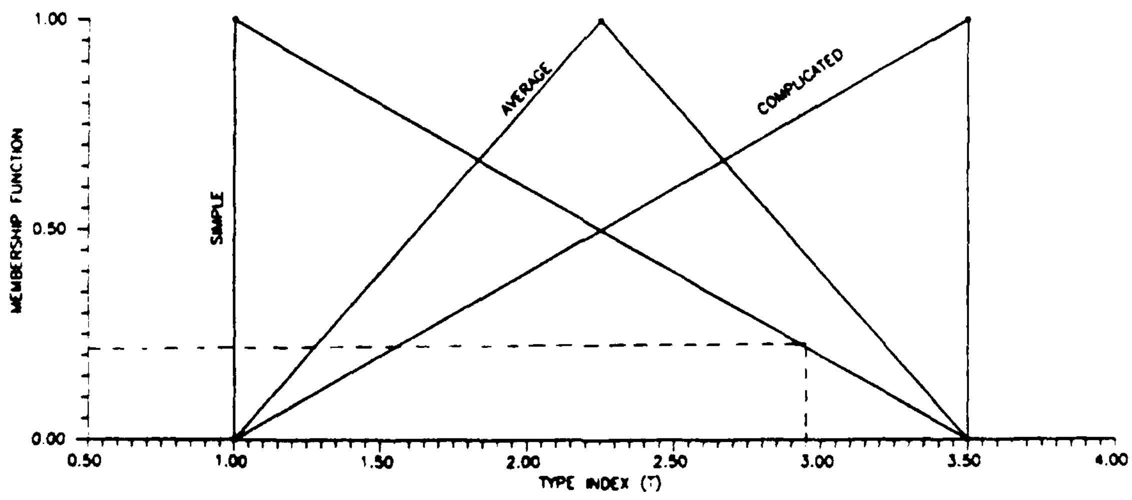

Figure 5: Type Index Membership Functions

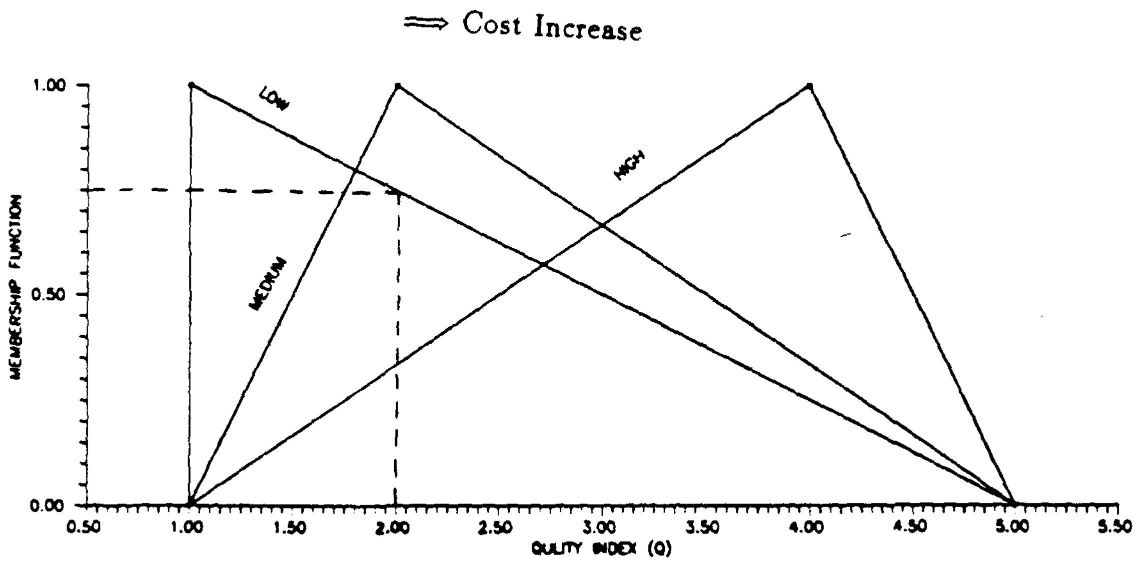

Figure 6: Quality Index Membership Functions

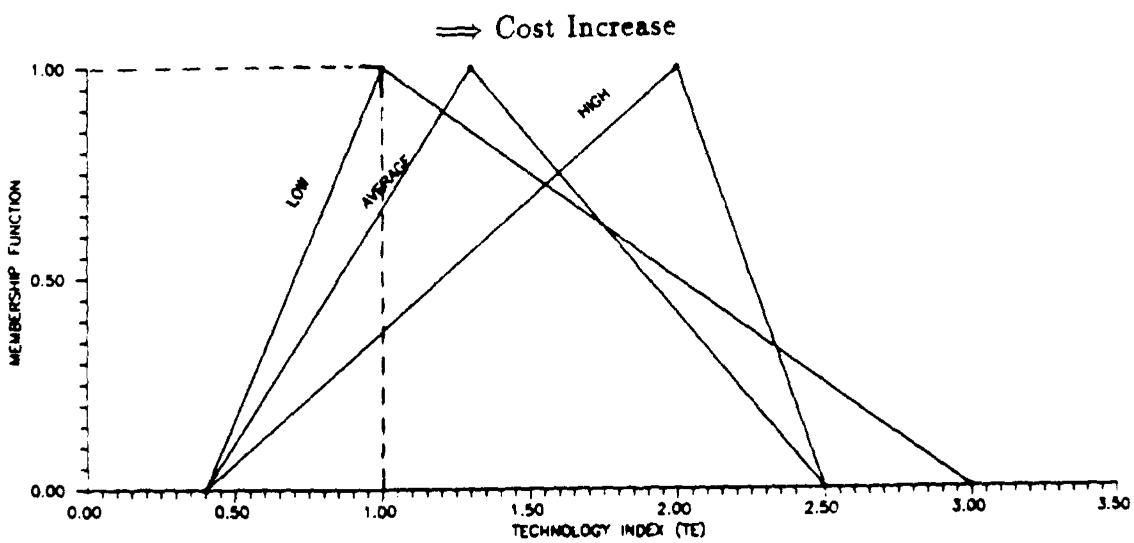

Figure 7: Technology Index Membership Functions

The extension principle allows us to induce from r fuzzy sets A, a fuzzy set B on Y through f, such that

$$\mu_B(y) = sup\ [min(\mu_{A_1}(x_1), \ldots\ldots, \mu_{A_r}(x_r))] \qquad (20)$$
$$\mu_B(y) = 0 \qquad if\ \ f^{-1} = \phi$$

where $f^{-1}(y)$ is the inverse image of y and ϕ denotes an empty set. $\mu_B(y)$ is the greatest among the membership values $\mu_{A_1} \times \ldots\ldots \times \mu_{A_r}(X_1, \ldots, X_r)$ of the realization of y using r-tuples $(\mathbf{x}_1, \ldots \mathbf{x}_r)$.

As an example, consider a variable combination 1 (see Table 8) with $L = .9, P = 2.76, T = 2.95, H = 40, Q = 2$ and $TE = 1.0$, and consider variable membership functions using figures 3-7, (e.g., good locality index, low price index, low type index, low quality index and low technology index), the membership values are $\mu_L = .85, \mu_P = .15$, $\mu_T = .22, \mu_H = 1$. (non-fuzzy variable), $\mu_Q = .75$, and $\mu_{TE} = 1$. Based on the extension principle, $\mu_{C(.9,2.76,2.95,40,2,1.0)} = \min(.85, .15, .22, 1., .75, 1.) = .15$. The corresponding cost is calculated using the cost function Eq. (18). That is, $C = f(L, P, T, H, Q, TE) = f(.9, 2.76, 2.95, 40, 2, 1.0) = 37.7$.

Each design variable has three possible membership functions. Cost function is a function of these design variables. Therefore, different combinations of the design membership functions will result in different cost membership functions. In this example, three design membership combinations were considered: (a) good locality index, low price index, simple type index, low quality index, and low technology index, (b) average indices for all five variables, (c) poor locality index, high price index, complicated type index, high quality index, and high technology index. These three combinations result in three different cost membership functions as shown in Table 9. The membership functions μ_{LC} (see Fig. 8), μ_{MC} (see Fig. 9) and μ_{HC} (see Fig. 10), correspond to combinations (a), (b) and (c), respectively.

6 Fuzzy Probability Approach

The cost function is used to find the estimated cost for each set of design variables . The data for these calculations are shown in Table 9, with the cost membership value of all three combinations. Based on these data, a probability mass function is found and shown in Figure 11. From the mass probability function and the event membership function, the probability of a fuzzy event can be found as:

$$P(A) = \sum \mu_i p_i \qquad (21)$$

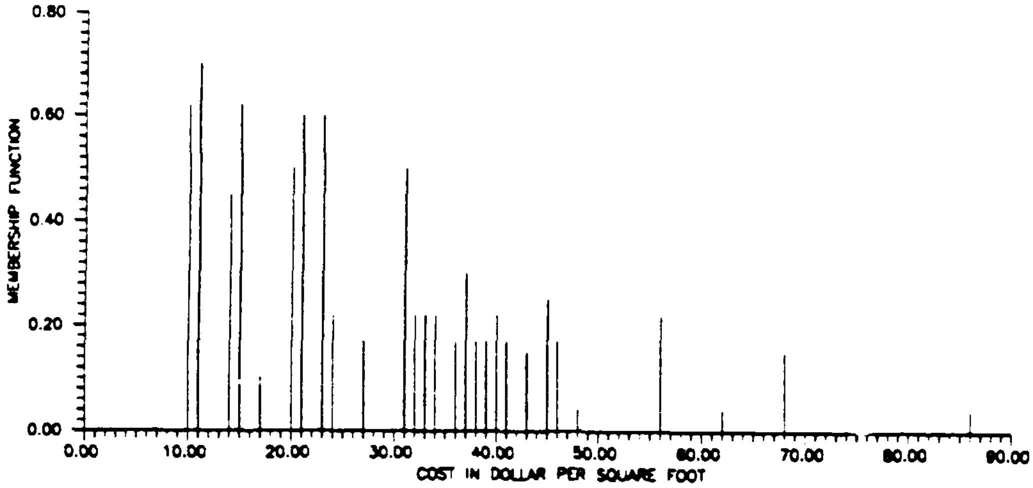

Figure 8: Low Cost Membership Function

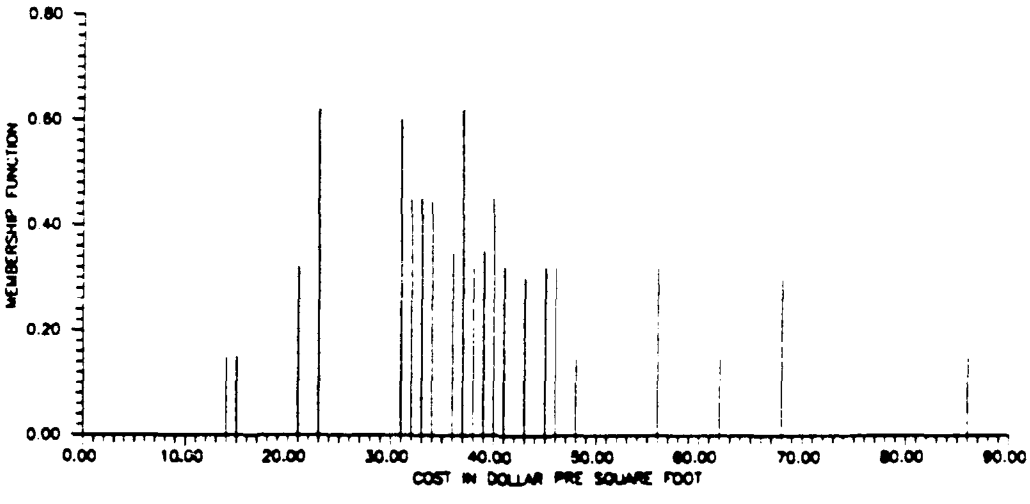

Figure 9: Medium Cost Membership Function

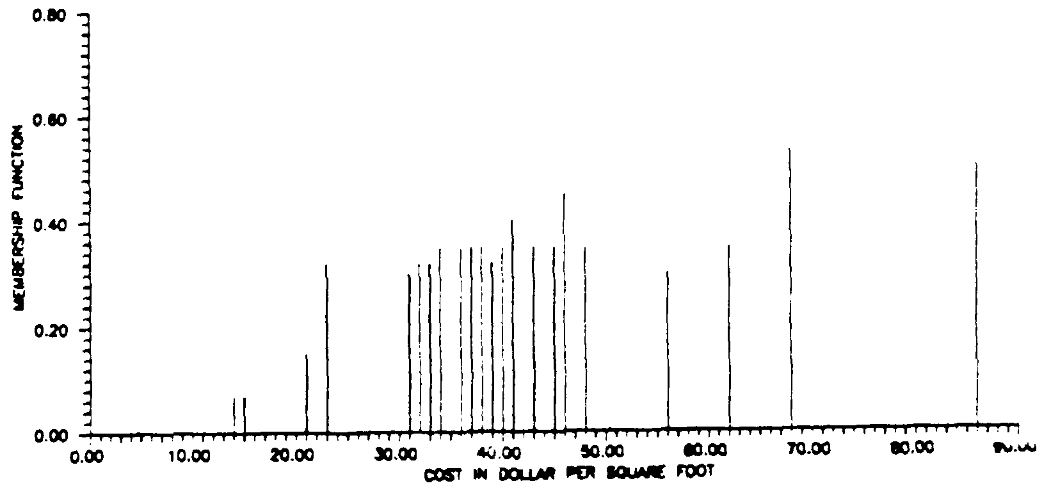

Figure 10: High Cost Membership Function

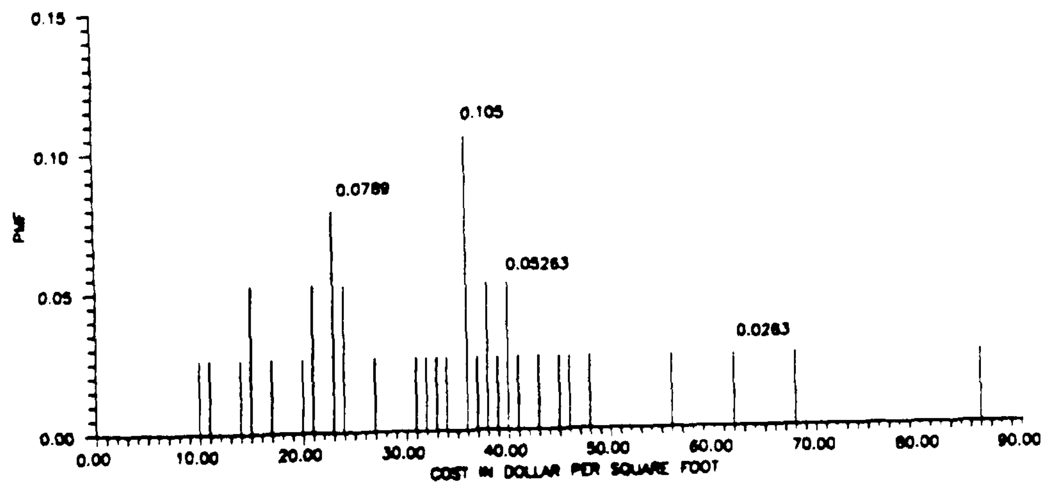

Figure 11: Cost Probability Mass Function

Accordingly,
the probability of low cost event is

$$P(C_L) = .0263(.62 + .7 + .45 + .5 + .5 + .17 + .5 + .3 + .22 + .22 + .3 + .17 + .17 + .15 + .25 + .17 + .04 + .22 + .04 + .15 + .04) + .05263(.62 + .6 + .22 + .17 + .22 + .22) + .0789 \times .6 + .105 \times .17 = 0.3145 \quad (22)$$

the probability of medium cost event is

$$P(C_M) = .0263(.15 + .6 + .15 + .6 + .45 + .45 + .45 + .62 + .35 + .32 + .3 + .32 + .32 + .15 + .32 + .15 + .45 + .15) + 05263(.15 + .6 + .32 + .45) + .0789 \times .62 + .105 \times .35 = .3681 \quad (23)$$

and the probability of high cost event is

$$P(C_H) = .0263(.07 + .07 + .32 + .3 + .32 + .32 + .35 + .23 + .32 + .4 + .35 + .35 + .45 + .35 + .3 + .35 + .32 + .5) + .05263(.07 + .32 + .35 + .35) + .0789 \times .35 + .105 \times .35 = .271 \quad (24)$$

The probability of cost C_i can be found by

$$P(C_i) = \frac{\mu_i}{\sum \mu_i} \quad (25)$$

For example, if low cost event is considered, probability of $C = 40$ will be:

$$P(C = 40) = \frac{\mu_i}{\sum \mu_i} = \frac{.22}{8.48} = .0259 \quad (26)$$

If all cost ranges are considered, Figure 12 shows fuzzy probabilities for cost values for the case where all variables have average membership values.

7 Sensitivity Analysis

Sensitivity analysis is used to calculate the sensitivity of the cost membership function and cost fuzzy probability to change in the design membership functions. The cost is a function of several variables, V_i. Cost may be sensitive to change in one variable but not to change in another. In this section, a study of the sensitivity of the cost membership function and fuzzy probability to change in the independent design membership functions is presented. Ten different cases of design membership functions are considered. These cases are as follows.

1. Good locality index (L)
2. Low price index (P)
3. Simple type index (T)
4. Low quality index (Q)
5. Low technology index (TE)
6. Poor locality index (L)
7. High price index (P)
8. Complicated type index (T)
9. High quality index (Q)
10. High technology index (TE)

The ten cases mentioned above are chosen to note the effect that independent design variables will have on the cost membership functions and on the fuzzy probability; for example as they change from average to low or high values (case 2, above) or (case 7, above), respectively.

Figures 9 and 12 represent cost membership function, and fuzzy probability, respectively, when all variables are at their average values. Appendix B contains the Output Data for fuzzy probability and membership values for the average case.

To investigate the effects that the design membership function change will have on the cost membership function, a comparison is presented in the following section.

7.1 Analysis of the Results

Figures 13 and 23 show the cost fuzzy probability and the cost membership function, respectively, that correspond to case 1, good locality index. These two Figures show a slight decrease in one intermediate value, but no increase in the low cost or decrease in the high cost membership values. Figures 18 and 28 are the cost fuzzy probability and the cost membership function respectively, that correspond to poor locality index (case 6, above). This case has no effect at all on the final cost. Figures 14 and 24 are the cost fuzzy probability and cost membership function. respectively that correspond to low price index(case 2. above). Price index. as shown in the Figures.has more effect than any other variable. The study of Figures 14 and 24 and the Output Data (Appendix C), corresponding to low price index show. for example. that the membership value of a low cost (e.g.. cost=$21) was increased from 0.32 to 0.67. and fuzzy probability increased from 0.042 to 0.09544: also. the membership value of a high cost(e.g.. cost=$68) was decreased from 0.32 to 0.15. and fuzzy probability was decreased from 0.0394 to 0.02136. On the other hand Figures 19 and 29 are the cost fuzzy probability and membership function, respectively, that correspond to high price index (case 7. above). This case has a high effect on the cost membership function. Figure 29 and the Output Data (Appendix C) show the effect of high price index on the final cost membership function. For example, the membership value of low cost(e.g.. cost=$21) was decreased from 0.32 at the average to 0.15 at the high price index. and fuzzy probability was decreased from 0.042 to 0.01976.

These two examples show that the effects of the design variables are not the same. consequently. the treatment of these variables should not be the same. The first variable. locality index. has less effect than the the second one. price index. which requires more attention than locality index. Another important point worth mentioning here is that. what is true for this particular building example might not be true for another project. meaning that if locality index is not critical in this project it may be critical in another one. Another example of different effect is a variable. which has almost the same effect on all variable combination values . for example technology index. This variable. as shown in Figure 22(high technology index) will decrease almost all cost fuzzy probability values except the high values. which were slightly increased (e.g.. cost=$68) where the fuzzy probability was decreased from 0.039 to 0.0467. Based on sensitivity analysis the designer will identify the most critical variable. The designer will give both emphasis and attention to this based on his interest and the amount of risk he is willing to take. For example. if he is conservative. he will emphasize those variables that decrease high-cost membership and fuzzy probability values. and vice versa.

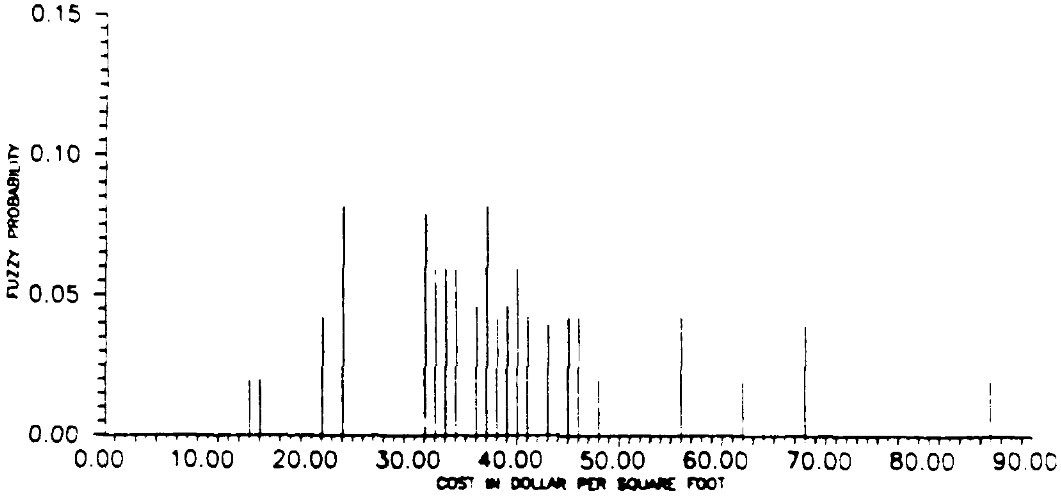

Figure 12: Fuzzy Probability of Cost Corresponding to Average Values

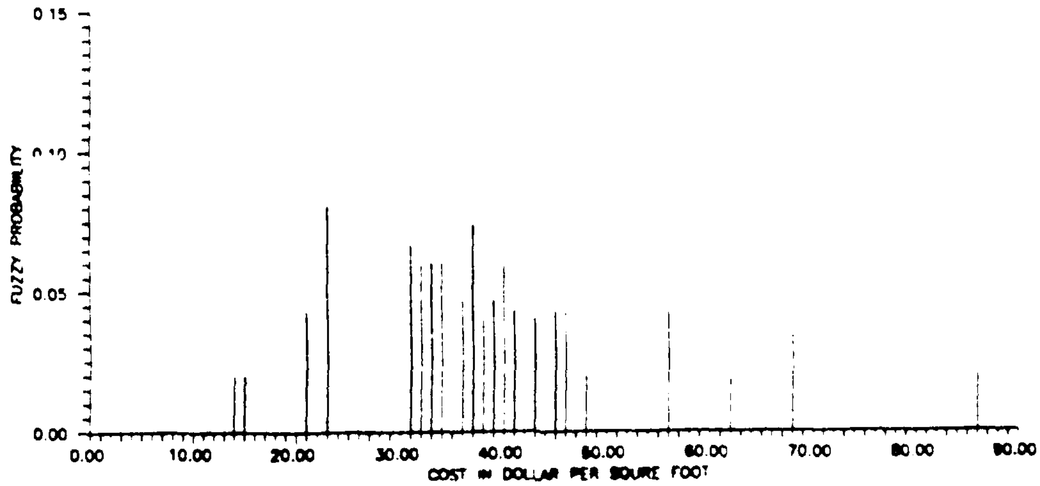

Figure 13: Fuzzy Probability of Cost Corresponding to Good Locality Index(L)

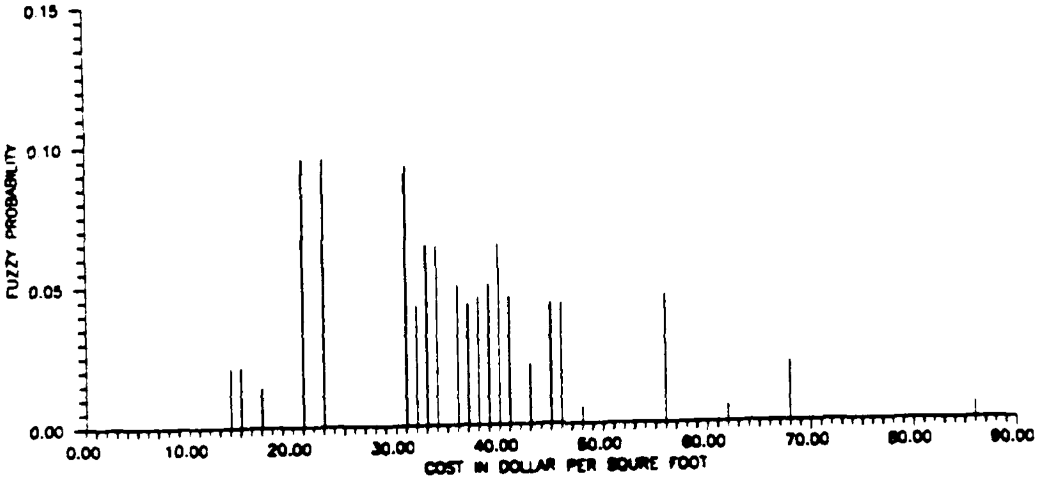

Figure 14: Fuzzy Probability of Cost Corresponding to Low Price Index(P)

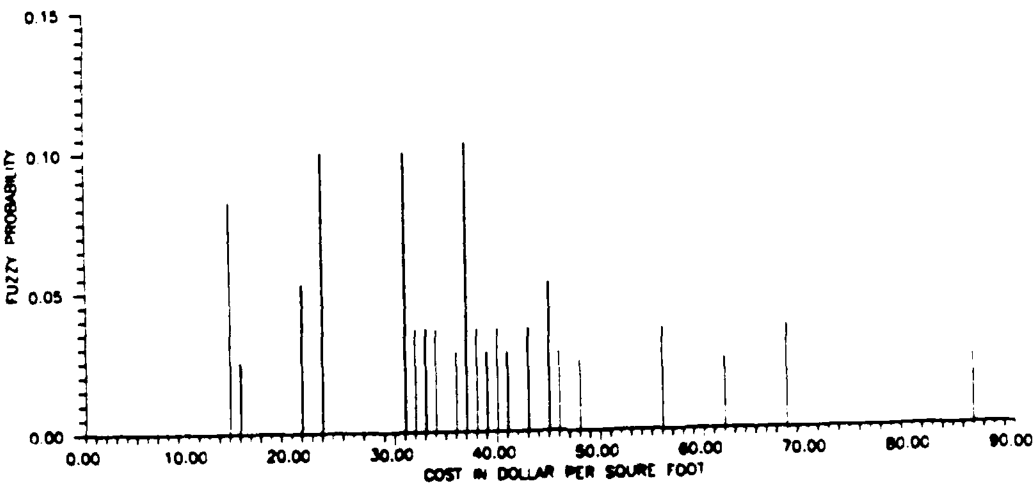

Figure 15: Fuzzy Probability of Cost Corresponding to Simple Type Index(T)

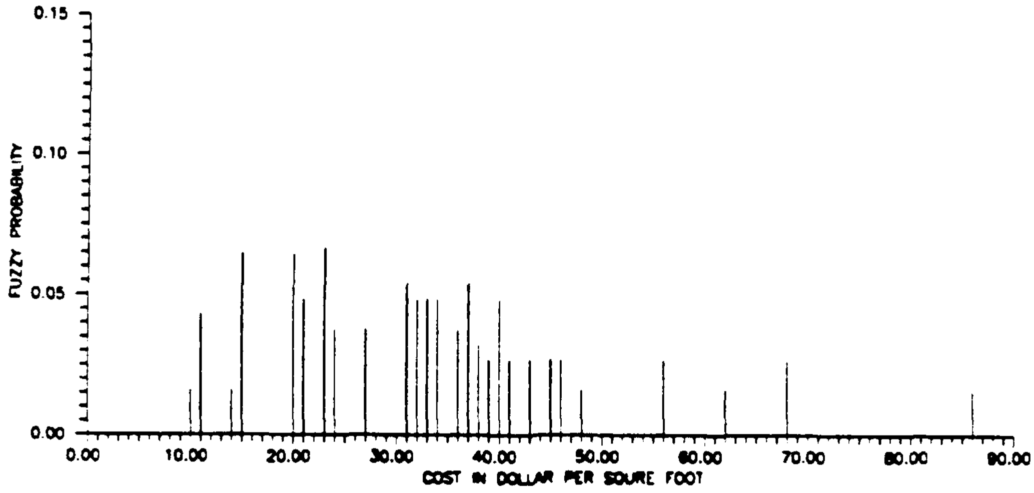

Figure 16: Fuzzy Probability of Cost Corresponding to Low Quality Index(Q)

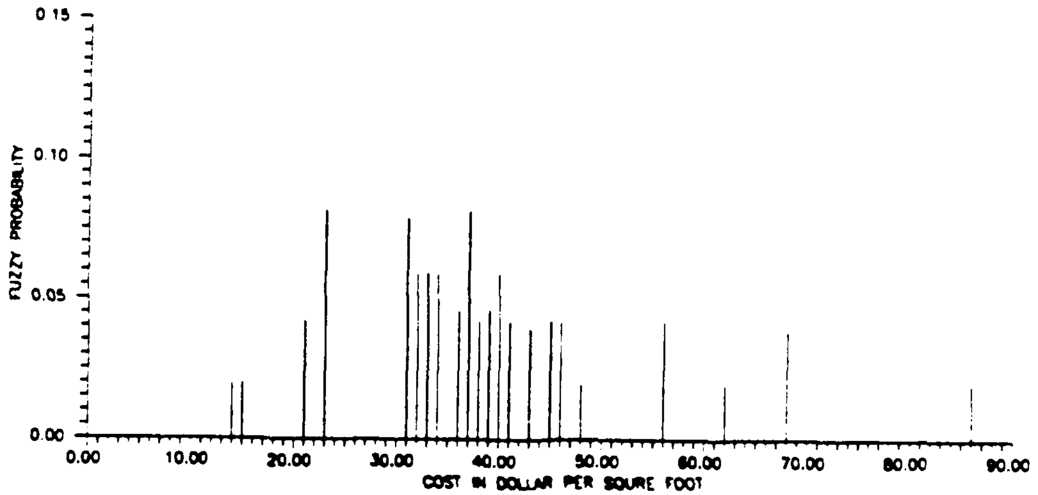

Figure 17: Fuzzy Probability of Cost Corresponding to Low Technology Index(TE)

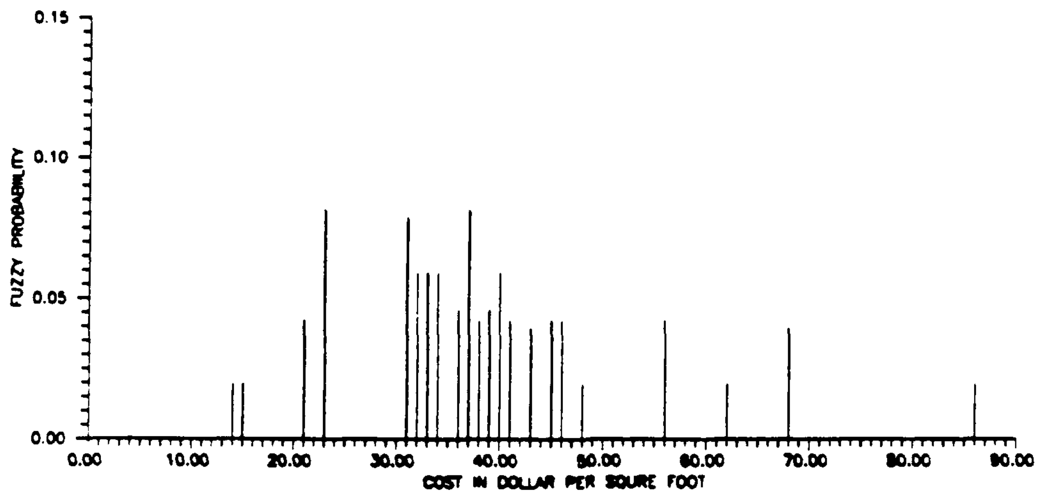

Figure 18: Fuzzy Probability of Cost Corresponding to Poor Locality Index(L)

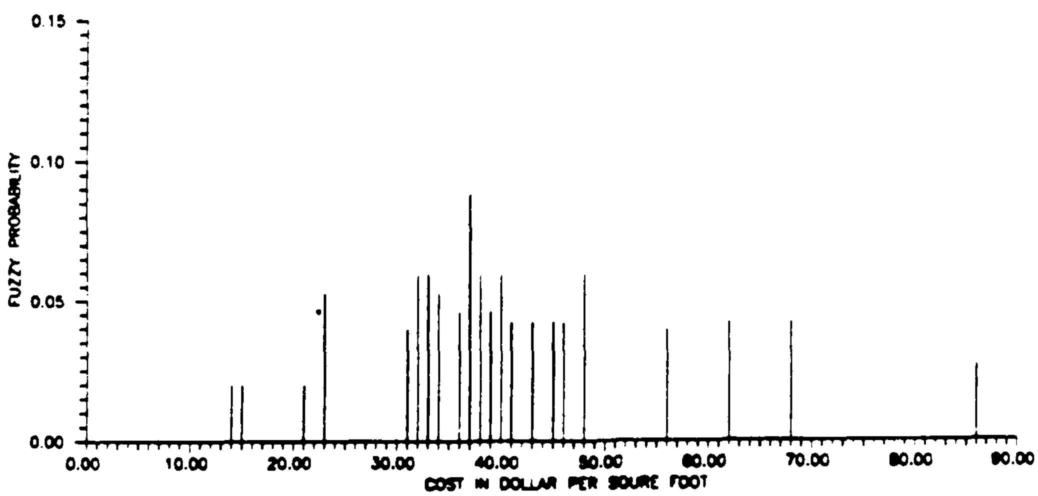

Figure 19: Fuzzy Probability of Cost Corresponding to High Price Index(P)

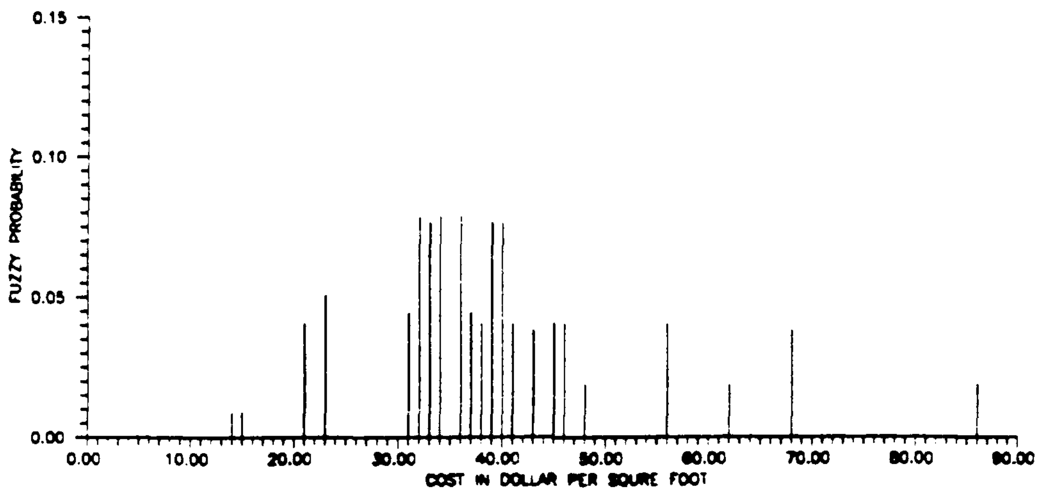

Figure 20: Fuzzy Probability of Cost Corresponding to Complicated Type Index(T)

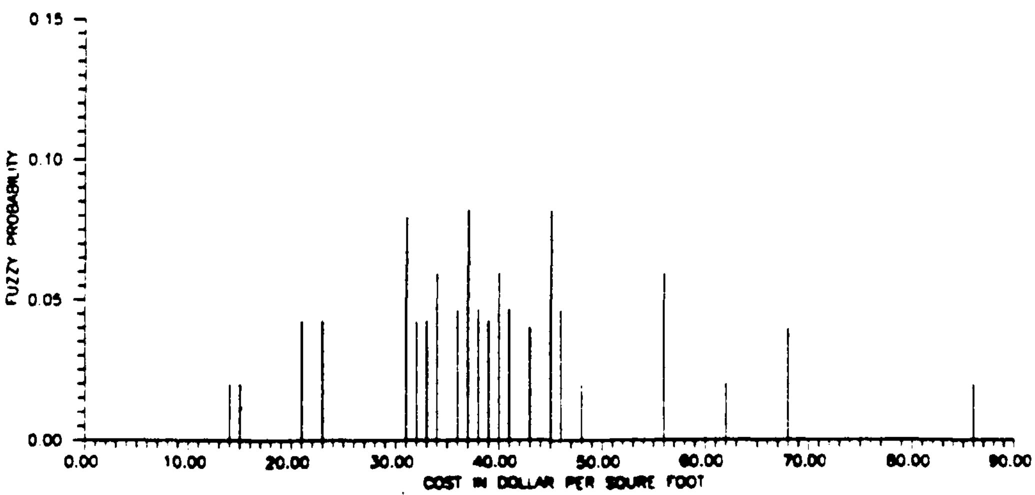

Figure 21: Fuzzy Probability of Cost Corresponding to High Quality Index(Q)

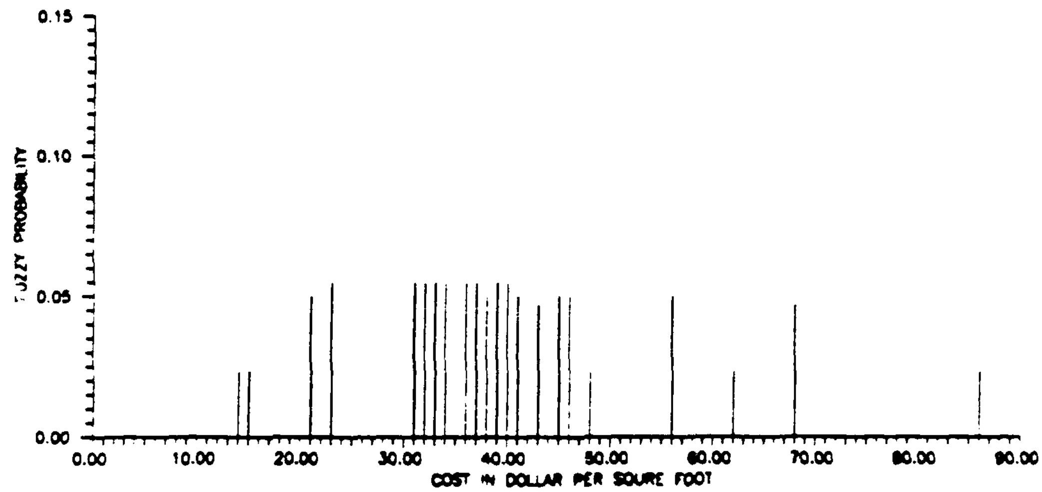

Figure 22: Fuzzy Probability of Cost Corresponding to High Technology Index(TE)

Table 9: Cost Membership Functions and Probabilities

Cost in dollars / square foot	Number of observations No.	Probability P	Membership Functions LVM^1 μ_{LC}	MVM^2 μ_{MC}	HVM^3 μ_{HC}	Variable Combinations
10	1	0.02631	0.62	0.00	0.00	37
11	1	0.02631	0.70	0.00	0.00	35
14	1	0.02631	0.45	0.15	0.07	38
15	2	0.05263	0.62	0.15	0.07	36.27
17	1	0.02631	0.10	0.00	0.00	28
20	1	0.02631	0.50	0.00	0.00	25
21	2	0.05263	0.60	0.32	0.15	24.31
23	3	0.07890	0.60	0.62	0.32	6.10.32
24	2	0.05263	0.22	0.00	0.00	2.22
27	1	0.02631	0.17	0.00	0.00	34
31	1	0.02631	0.50	0.60	0.30	33
32	1	0.02631	0.22	0.45	0.32	4
33	1	0.02631	0.22	0.45	0.32	18
34	1	0.02631	0.22	0.45	0.35	26
36	4	0.10526	0.17	0.35	0.35	16.17.21.30
37	1	0.02631	0.30	0.62	0.35	5
38	2	0.05263	0.17	0.32	0.35	1.9
39	1	0.02631	0.17	0.35	0.32	29
40	2	0.05263	0.22	0.45	0.35	7.19
41	1	0.02631	0.17	0.32	0.40	23
43	1	0.02631	0.15	0.30	0.35	11
45	1	0.02631	0.25	0.32	0.35	12
46	1	0.02631	0.17	0.32	0.45	15
48	1	0.02631	0.04	0.15	0.35	20
56	1	0.02631	0.22	0.32	0.30	8
62	1	0.02631	0.04	0.15	0.35	13
68	1	0.02631	0.15	0.30	0.53	3
86	1	0.02631	0.04	0.15	0.50	14

1-Low Variable Membership Combination
2-Medium Variable Membership Combination
3-High Variable Membership Combination

8 Conclusion

The actual cost deviation from the feasibility estimate is largely due to a lack of information concerning the project and its many design variables. This problem demonstrates the necessity of having a model that can handle it and devise a model that can derive more accurate results at this stage. Regression analysis is used to find the relation between historical data and design variables. This relation can be helpful if we know the values of the independent variables precisely, however, in the real world this is not always the case. In the process of design, there are many cases in which the available information concerning the value of the variable in the future will be known only by some verbal expression, such as "high," "low," "good," or "bad." These expressions cannot be used directly in such a function. Fuzzy set theory is applied by assigning membership functions to these design variables according to design experience and knowledge concerning them. The cost will be found using the cost function, and using extension principle, the cost membership value will be assigned based on the independent variable membership function. The example given was that of a building, but this methodology can be applied to any kind of project as long as the design variables are known and the data for dependent and independent variables are available. A computer program was developed to find the cost function, cost probability mass function and membership function (see Appendix A). The cost function for any project can be found with both the same program and procedure.

8.1 Required Information to Use the Design Reliability Model

The developed method to estimate the project cost at the feasibility stage is not limited to a building projects, as given in the report, but can be applied to any type of project, including pile-driving projects if the following data are provided:

1. Dependent variable (cost variable), this variable can represent cost per pile, cost per square foot, etc.

2. Independent design variables that affect the cost ($V_i, i = 1, \ldots, m$): the value of these variables depend on cost variable definition and how they will contribute in that cost specified in (1).

3. Historical data for such design variables.

4. Future prediction of variables.

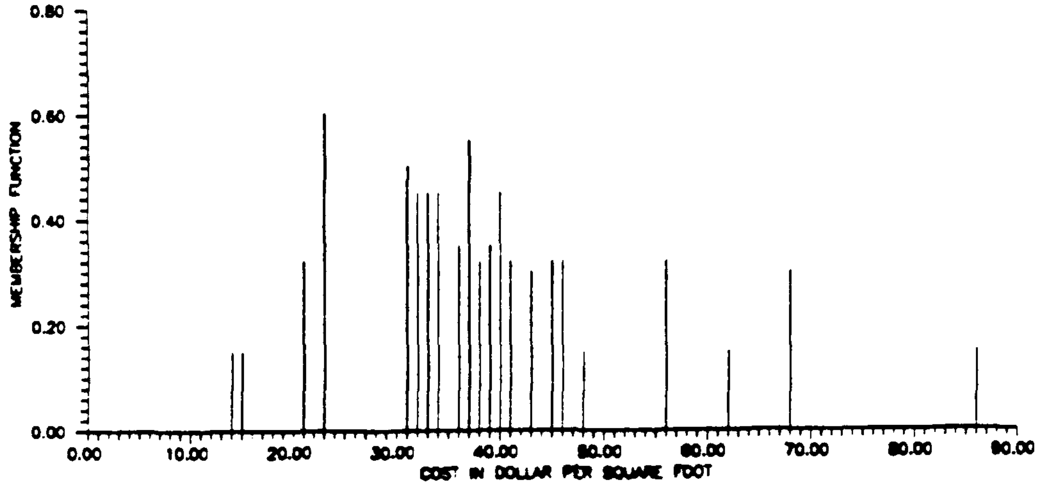

Figure 23: Membership Function of Cost Corresponding to Good locality Index(L)

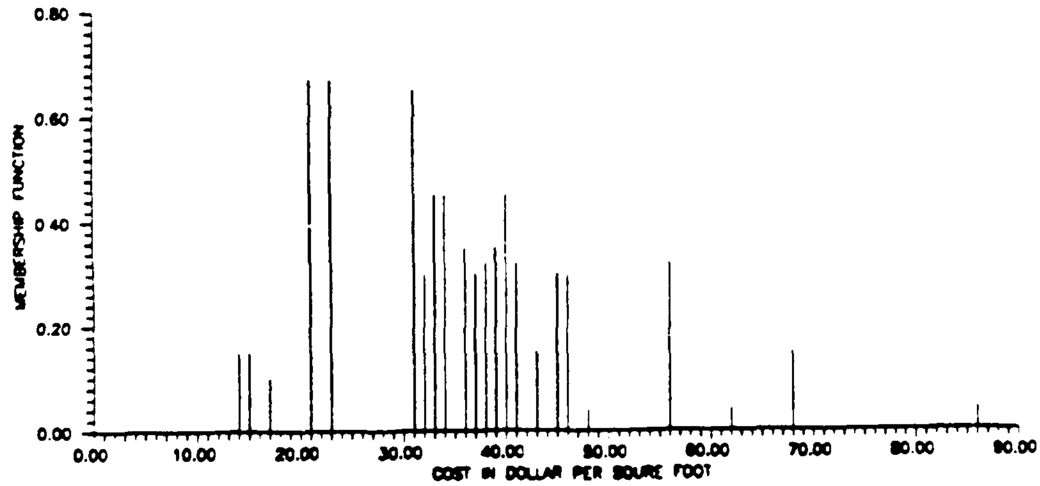

Figure 24: Membership Function of Cost Corresponding to Low Price Index(P)

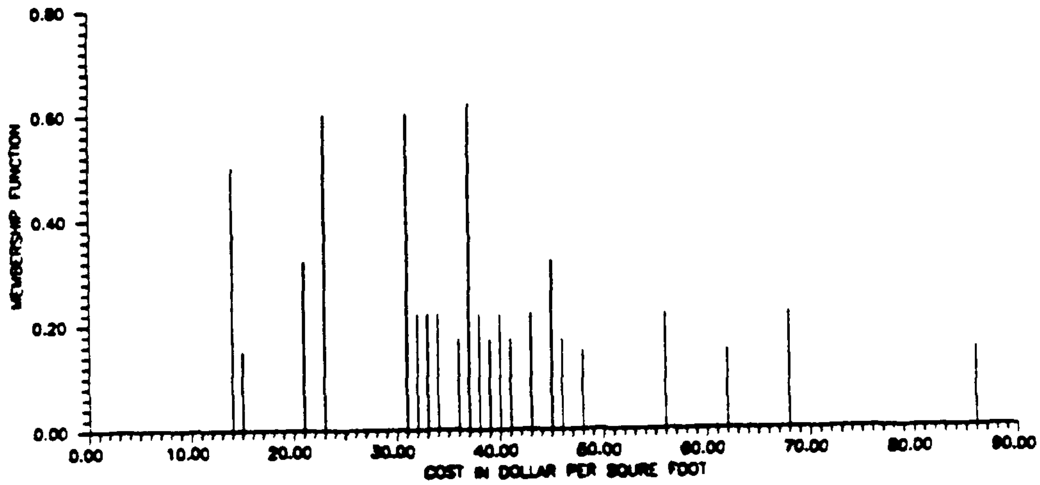

Figure 25: Membership Function of Cost Corresponding to Simple Type Index(T)

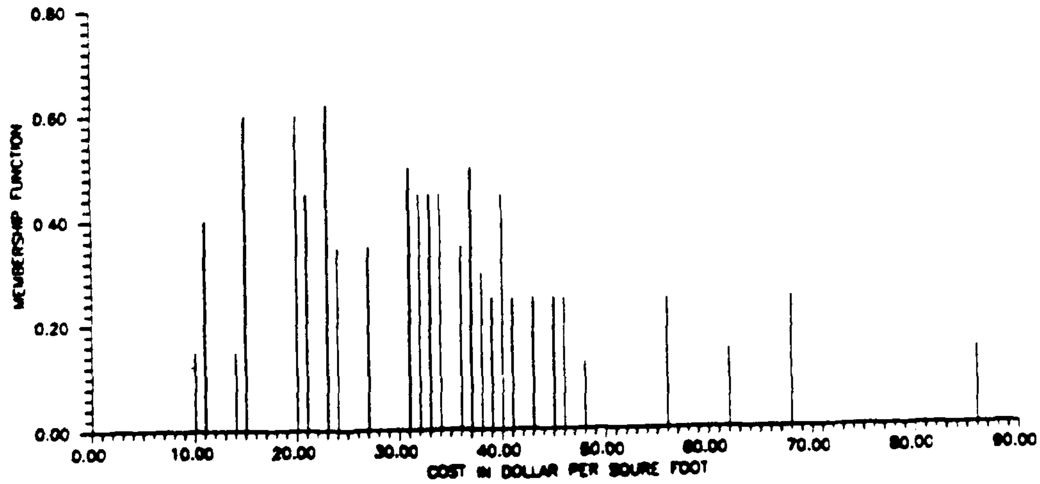

Figure 26: Membership Function of Cost Corresponding to Low Quality Index(Q)

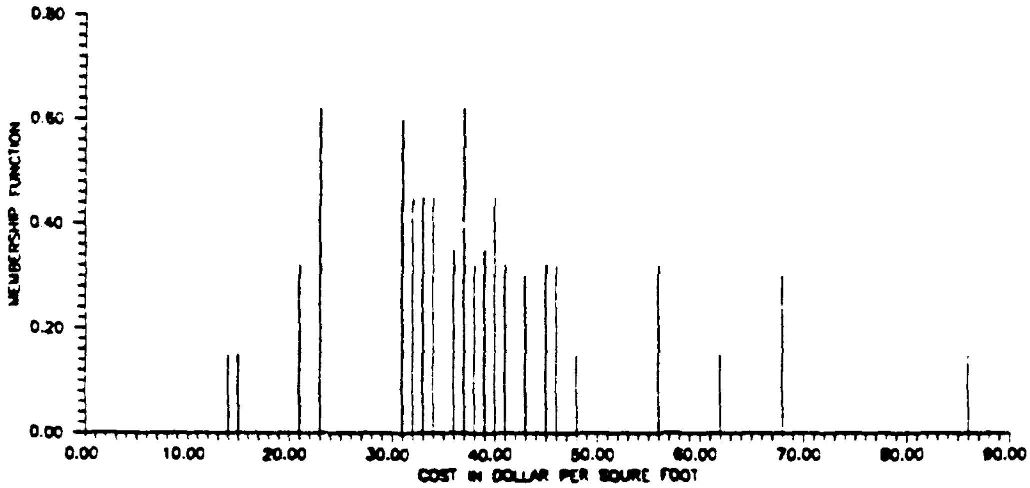

Figure 27: Membership Function of Cost Corresponding to Low Technology Index(TE)

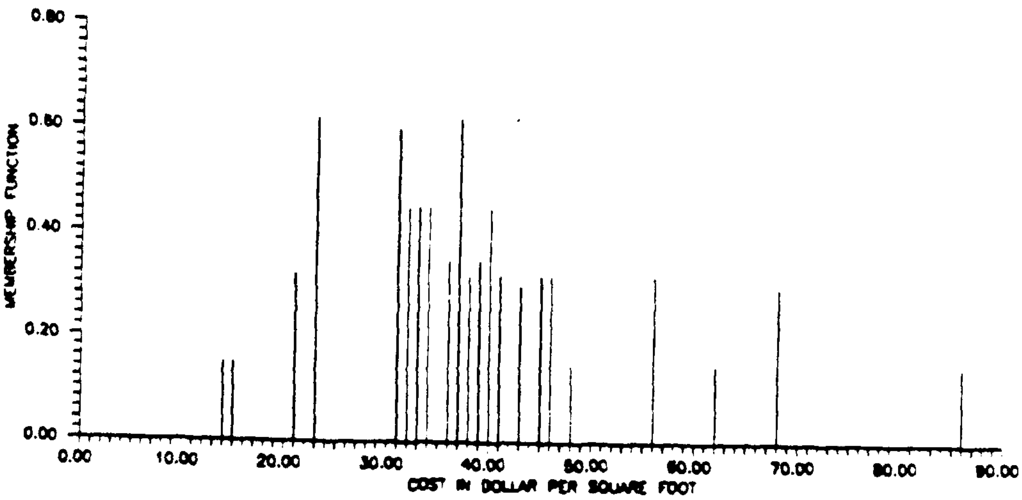

Figure 28: Membership Function of Cost Corresponding to Poor locality Index(L)

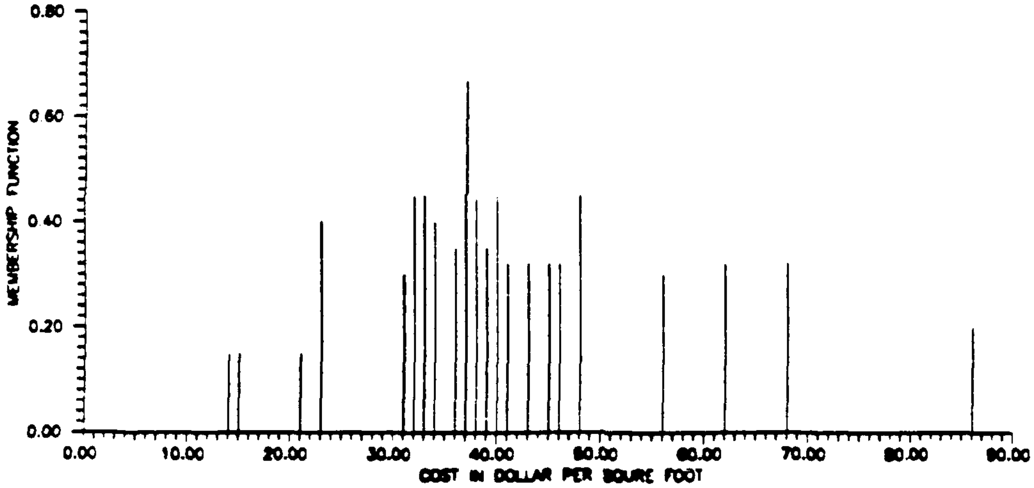

Figure 29: Membership Function of Cost Corresponding to High Price Index(P)

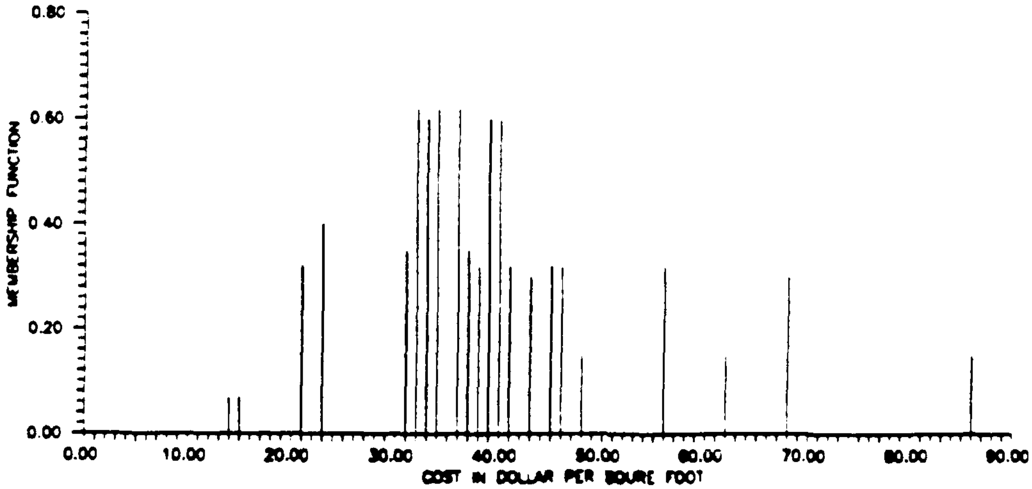

Figure 30: Membership Function of Cost Corresponding to Complicated Type Index(T)

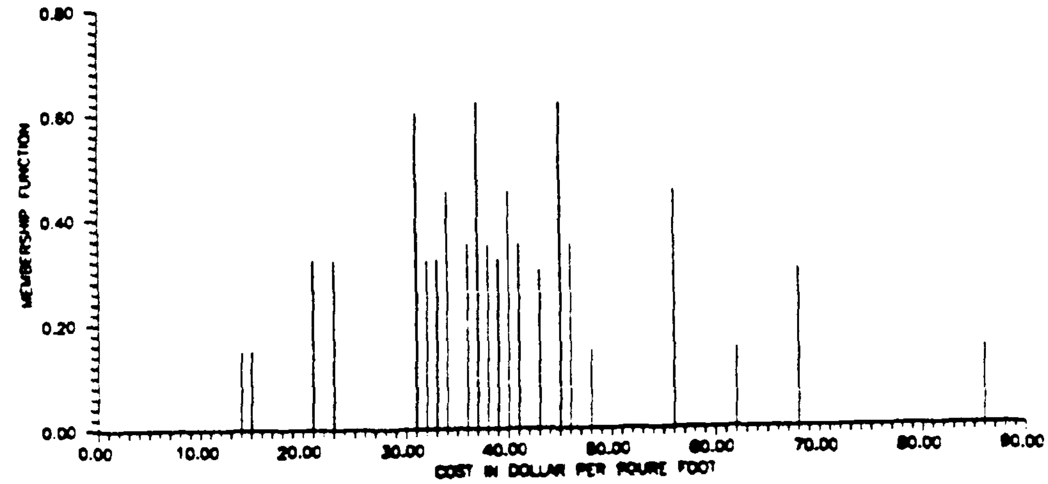

Figure 31: Membership Function of Cost Corresponding to High Quality Index(Q)

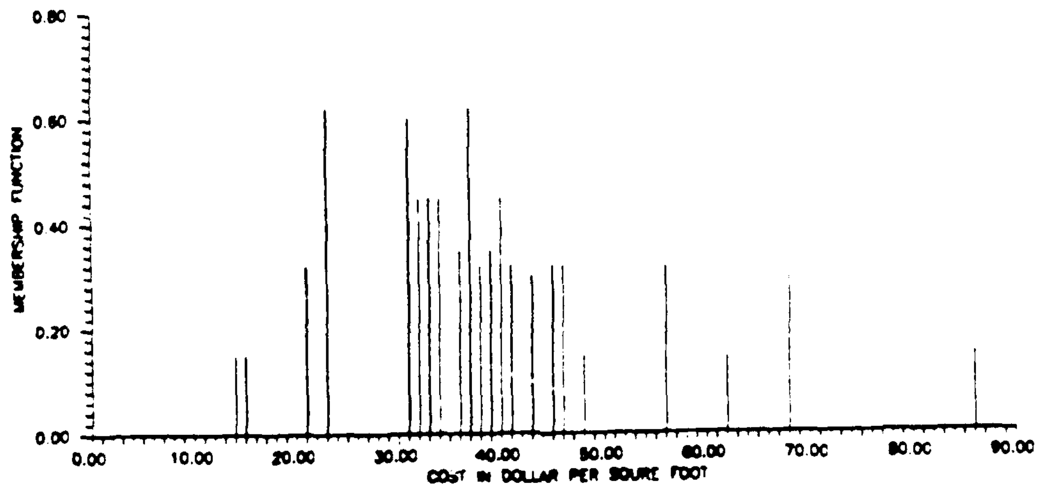

Figure 32: Membership Function of Cost Corresponding to High Technology Index(TE)

The last item is needed to develop membership functions of the variables. It is enough to know verbal information, such as "good," "bad," "low," and so on. It is also helpful to give a range of variables for maximum and minimum, to help assign membership values for such variables within that range.

To verify the model, a historical of actual and predicted design variables of past projects will be of great help.

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APPENDICES

APPENDIX A

Program Listing

```
********************** MAIN PROGRAM ******************************

      DIMENSION A(50,50),C(50),X(50),XP(50,50),YP(50),SUM(50),SUM1(50)
      DIMENSION SUM2(50),SUM3(50),EMU(50,50),CMU(50),CP(50),y1(50)
      INTEGER O(50),CO(50)
      INTEGER H,line,i2,i3


      character*40  title
      character*12  infn
      character*12  outfn1
      character*12  outfn2
      character*12  outfn3
C      character*12  outfn4

******The program will prompt user to input the following files
*       1-Input data file
*       2-file that contains regression analysis output which includes
*       3-file that contains cost membership function and cost PMF
*       and graphs of the data
*       4-file that contains input data
*


       print*,'please enter the output title max 40 characters'
      read(*,905)title
905   format(A40)
700     print*,'please enter input file'
      read(*,900,ERR=700)infn
900   format(A12)

      OPEN(UNIT=50,FILE=infn,STATUS='OLD')
      print*,'please enter regression analysis output file '
      read(*,900)outfn1
      print*,'please enter probabilities and membership functions
     .output file '
      read(*,900)outfn2
C
      print*,'please enter data output file '
      read(*,900)outfn3

C      print*,'please enter data output file '
C      read(*,900)outfn4

      OPEN(UNIT=60,FILE=outfn1,STATUS='UNKNOWN')
      OPEN(UNIT=65,FILE=outfn2,STATUS='UNKNOWN')
      OPEN(UNIT=66,FILE=outfn3,STATUS='UNKNOWN')
C      OPEN(UNIT=67,FILE=outfn4,STATUS='UNKNOWN')
C      WRITE(60,905)title
      WRITE(65,905)title
      WRITE(66,905)title
C      WRITE(67,905)title
***************************INPU DATA*********************************
*Read number of variables and number of data points

C         PRINT*,'PLEASE ENTER THE NUMBER OF VARIABLES'
         READ(50,*)M
C11       PRINT*,'PLEASE ENTER NUMBER OF DATA POINTS'
11       READ(50,*)L
         N=M+1

         IF(L.LE.M)THEN
```

```
            PRINT*,'           N MUST BE >M'
            GO TO 11
         ELSE

* *Read dependent variable ,independent variables and variables membership
*  values
              DO  12  I=1,L
                   READ(50,*)YP(I),(XP(J,I),J=1,M)
                   READ(50,*)(EMU(J,I),J=1,M)
                   WRITE(66,*)YP(I),(XP(J,I),J=1,M)
                   WRITE(66,*)(EMU(J,I),J=1,M)
12            CONTINUE

         ENDIF
*Call subroutines to construct simultaneus equation and solve for coefficients

         CALL MULTLR(A,XP,YP,M,L,N,C)

         CALL ORDER(S,O,A,N)

         CALL ELIM(S,A,O,N)

         CALL SOLVE(A,C,X,O,N)

*Write the resulted cost function

c         WRITE(65,204)
204      format(2x,75('='))

C         WRITE(65,200)

C         WRITE(65,252)
C252      FORMAT(30X,'THE COST FUNCTION'/)

C         WRITE(65,204)
C         WRITE(65,200)

C         WRITE(65,299)X(1),(X(J),(J-1),J=2,M+1)
C299      FORMAT(1X,' Y=',F10.5,'+',10('(',F10.5,')',1X,'X',I2,'+'))

*Call membership subroutine to find cost membereship function based on
*the input independent variables
*
         CALL MEMBERSHIP(XP,CO,X,L,M,N,EMU,CMU)

*Call sorting subroutine to sort the output data to simplify grouping
*subroutine to group the equal points and have one membership value for
*each point

         CALL SORTING(CO,CMU,L)

         CALL GROUPING(CO,CMU,L)

*Write the cost cofficient values

         WRITE(60,200)
200      format(2x,75('=')/)

         WRITE(60,205)
205      format(30X,'COST FUNCTION COEFFICIENTS'/)

         WRITE(60,200)

         WRITE(60,97)X(1)
```

```
97        FORMAT(5X,'a 0=',F10.6/)

          DO 14 I=2,N

             WRITE(60,99)(I-1),X(I)
99           FORMAT(5X,'a',I2,'=',F10.6/)

14        CONTINUE

*Quantify Error associated with the selected linear function

          ST=0.0
          SR=0.0

          DO 16 I=1,L

               SUM(I)=0.0
               SUM1(I)=0.0
               ST=ST+(YP(I)-YMEAN)**2

               DO 18 J=1,N

                  IF(J.EQ.1)THEN

                      SUM(I)=X(1)

                   ELSE

                       SUM1(I)=SUM1(I)+X(J)*(XP((J-1),I))

                   ENDIF

18              CONTINUE

                SUM2(I)=SUM(I)+SUM1(I)
                SR=SR+(YP(I)-SUM2(I))**2

16        CONTINUE

*Write the result of the error quantification

          WRITE(60,200)

          WRITE(60,210)
210       format(25x,'QUANTIFICATION OF ERROR OF LINEAR REGRESSION'/)

          WRITE(60,200)

          WRITE(60,225)ST
225       format(2x,'THE TOTAL OF SUM OF SQUARES OF RESIDUALS St=',F10.3)

          WRITE(60,230)SR
230       FORMAT(2X,'THE SUM ODF SQUARES OF THE RESIDUALS     Sr=',F10.3)

          SXY=SQRT(SR/(L-(M+1)))
          R2=(ST-SR)/ST
          R=SQRT(R2)


          WRITE(60,235)SXY
235       FORMAT(2X,'THE STANDARS ERROR OF ESTIMATE          Sx/y=',f10.3)

          WRITE(60,240)R2
240       FORMAT(2X,'THE COEFFICIENT OF DETERMINATION         r^2=',F6.4)

          WRITE(60,245)R
```

```
245     FORMAT(2X,'THE CORRELATION COEFFICIENT                  r=',F6.4)

        WRITE(60,207)
207     format('1'2x,75('='))

        WRITE(60,200)

        WRITE(60,250)
250     FORMAT(30X,'THE FINAL COST FUNCTION'/)

        WRITE(60,204)

        WRITE(60,200)

        WRITE(60,98)X(1),(X(J),(J-1),J=2,M+1)
98      FORMAT(1X,' Y=',F10.5,'+',10('(',F10.5,')',1X,'X',I2,'+'))


30         CONTINUE

      STOP
      END
***********************MULTILINEAR REGRESSION SUBROUTINE*******************

      SUBROUTINE MULTLR(A,XP,YP,M,L,N,C)
      DIMENSION A(50,50),XP(50,50),YP(50),C(50)
      INTEGER H

      DO 10 I=1,N
         DO 20 J=1,I

            IF(I.EQ.1)THEN
             A(1,1)=L
            ELSE

               SUM=0

               DO 30 H=1,L
                  XP(0,H)=1
                  SUM=SUM+(XP(I-1,H))*(XP(J-1,H))

30             CONTINUE

               A(I,J)= SUM
               A(J,I)= SUM
            ENDIF

20       CONTINUE

         SUM=0.0
         IF(I.EQ.1)THEN

            DO 25 H=1,L

               SUM=SUM+YP(H)

25          CONTINUE
            C(1)=SUM

         ELSE

           DO 40 H=1,L

              SUM=SUM+YP(H)*(XP(I-1,H))

40         CONTINUE
```

```
         C(I)=SUM

       ENDIF
10    CONTINUE

               WRITE(60,360)
360            FORMAT(2X,75('*')/)

               WRITE(60,370)
370            FORMAT(40X,'OUTPUT '/)

               WRITE(60,380)
380            FORMAT(2X,75('*')/)

      DO 60 I =1,N

          WRITE(60,100)(A(I,J),J=1,N),C(I)
100       FORMAT(2X,10(2X,F14.6),2X,' C=',F14.6)

60    CONTINUE

      RETURN
      END


****************************ORDER SUBROUTINE**********************************

      SUBROUTINE ORDER(S,O,A,N)
      DIMENSION A(50,50),S(50)
      INTEGER O(50)

      DO 10  I=1,N
         O(I)=I
         S(I)=ABS(A(I,1))

         DO 20  J=2,N

            IF (ABS(A(I,J)).GT.S(I)) THEN
               S(I)=ABS(A(I,J))
            ENDIF

20       CONTINUE
10    CONTINUE
      RETURN
      END

**************************SUBROUTINE ELIM***********************************

      SUBROUTINE ELIM(S,A,O,N)

      DIMENSION A(50,50),C(50),S(50),F(50,50)
      INTEGER O(50)
      INTEGER*2 I,J,K

      DO 10 K=1,N-1

         CALL PIVOT(S,A,O,N,K)

         DO 20 I=K+1,N

            F(O(I),K)=A(O(I),K)/A(O(K),K)

            DO 30 J=K+1,N
```

```
               A(O(I),J)=A(O(I),J)-F(O(I),K)*A(O(K),J)
30          CONTINUE
20       CONTINUE
10    CONTINUE
      DO 40 K=1,N-1
         DO 50 I=K+1,N
         A(O(I),K)=F(O(I),K)
50       CONTINUE
40    CONTINUE
      RETURN
      END
*************************** SOLVE SUBROUTINE *********************************
      SUBROUTINE SOLVE(A,C,X,O,N)
      DIMENSION A(50,50),C(50),X(50)
      INTEGER O(50)
       X(1)=C(O(1))
      DO 10 I=2,N
         SUM=0.0
         DO 20 J=1,I-1
            SUM=SUM+A(O(I),J)*X(J)
20       CONTINUE
         X(I)=C(O(I))-SUM
10    CONTINUE
      X(N)=X(N)/A(O(N),N)
      DO 30 I=N-1,1,-1
         SUM=0.0
         DO 40 J=I+1,N
            SUM=SUM+A(O(I),J)*X(J)
40       CONTINUE
         X(I)=(X(I)-SUM)/A(O(I),I)
30    CONTINUE
      RETURN
      END

**************************** SUBROUTINE PIVOT ******************************
      SUBROUTINE PIVOT(S,A,O,N,K)
```

```
      DIMENSION A(50,50),S(50)
      INTEGER O(50)
      INTEGER*2 K,PIVIT,II,IDUM

      PIVIT = K

      BIG=ABS(A(O(K),K)/S(O(K)))

      DO 10 II=K+1,N

         DUMMY=ABS(A(O(II),K)/S(O(II)))

         IF (DUMMY.GT.BIG) THEN

            BIG=DUMMY
            PIVIT=II

         ENDIF

10    CONTINUE

      IDUM=O(PIVIT)
      O(PIVIT)=O(K)
      O(K)=IDUM

      RETURN
      END

*******************SUBROUTINE MEMBERSHIP FUNCTION****************************

      SUBROUTINE MEMBERSHIP(XP,CO,X,L,M,N,EMU,CMU)
      DIMENSION  XP(50,50),X(50),SUM3(50),CO1(50),DIF(50),CMU(50)
      DIMENSION EMU(50,50)
      INTEGER CO(50)

*Find the  cost corresponding to each independent variable set and find cost
*membership value

       DO 15 I=1,L

          SUM3(I)=0

          DO 110 J=2,N

             SUM3(I)=SUM3(I)+X(J)*XP((J-1),I)

110       CONTINUE

          CO(I)=X(1)+SUM3(I)
          CO1(I)=X(1)+SUM3(I)
          DIF(I)=CO1(I)-CO(I)
          CMU(I)=EMU(1,I)

          DO 114 J=1,M-1

             IF(CMU(I).LT.EMU((J+1),I))THEN
                 CMU(I)=CMU(I)
             ELSE
                CMU(I)=EMU((J+1),I)
             ENDIF

114       CONTINUE

          IF(DIF(I).GT.0.5)THEN
```

```
            CO(I)=CO(I)+1

          ELSE

          ENDIF

15      CONTINUE

        RETURN
        END

******************************SUBROUTINE SORTING**********************************

        SUBROUTINE SORTING(CO,CMU,L)
        DIMENSION CO(50),CMU(50)
        INTEGER L,M,CO

         LOGICAL SORTED

         M=L-1
         SORTED=.FALSE.

30        IF(.NOT.SORTED) THEN

             SORTED=.TRUE.

             DO 40 I=1,M
                IF(CO(I).GT.CO(I+1))THEN
                   TEMP=CO(I)
                   TEMPM=CMU(I)
                   CO(I)=CO(I+1)
                   CMU(I)=CMU(I+1)
                   CO(I+1)=TEMP
                   CMU(I+1)=TEMPM
                   SORTED=.FALSE.

                ENDIF

40           CONTINUE

                GO TO 30

          ENDIF

C            WRITE(65,255)
C255         FORMAT('1'2X,75('='))

C            WRITE(65,257)
C257         FORMAT(2X,75('=')/)

C            WRITE(65,260)
C260         FORMAT(15X,'THE COSTS AND THEIR CORRESPONDING MEMBERSHIP
C       1 FUNCTIONS'/)

C            WRITE(65,258)
C258         FORMAT(2X,75('='))

C            WRITE(65,256)
C256         FORMAT(2X,75('=')//)

C            WRITE(65,265)
C265         FORMAT(12X,' COST',2X,'|',1X,'MEMBERSHIP VALUES    |'/)

C            WRITE(65,275)
C275         FORMAT(2X,75('-'))
```

```
C          DO 60 I=1,L

C             WRITE(65,270)CO(I),CMU(I)
C270          FORMAT(9X,I6,4X,'|',6X,F7.5,10X,'|')

C             WRITE(65,275)

C60        CONTINUE

         RETURN
         END

****************************SUBROUTINE GROUPING**************************

      SUBROUTINE GROUPING(CO,CMU,L)
      DIMENSION CO(50),CMU(50),NUM(50),AP(50),CP(50)
      INTEGER CO,L,K,K1,I1,NUM,line
      CHARACTER*60 GTIT

      K=L-1
      K1=L-1
      I1=1

      DO 20 I=1,K

         CO(I)=CO(I1)
         NUM(I)=1
         AP(I)=REAL(NUM(I))/REAL(L)
         CMU(I)=CMU(I1)

 11         IF(CO(I1).EQ.CO(I1+1))THEN

                CO(I)=CO(I1)
                NUM(I)=NUM(I)+1
                AP(I)=REAL(NUM(I))/REAL(L)

                IF(CMU(I1).GT.CMU(I1+1))THEN

                     CMU(I)=CMU(I1)

                ELSE

                     CMU(I)=CMU(I1+1)

                ENDIF

                I1=I1+1
C               K=K-1
                K1=K1-1
                IF(I1.EQ.L)THEN

                    GO TO 21

                ELSE
                ENDIF
                GO TO 11
            ELSE
                IF(I1.EQ.L)THEN

                     GO TO 21

                ELSE
                ENDIF

          ENDIF
```

```
            IF(I1.EQ.L)THEN
               GO TO 21
            ELSE
            ENDIF
            I1=I1+1
20      CONTINUE
21      CONTINUE

C        WRITE(67,*)'cost  membeship values'
C       do 407 i=1,k1+1
C          WRITE(67,*)co(i),cmu(i)
C407    continue

c****This part will find probability of Ci's
c
C    CP=COST PROBABILITY
C    TCMU =TOTAL SUM OF MEMBERSHIP FUNCTIONS FOR COST EVENT

        TCMU=0.0

        DO 400 I=1,K1+1

           TCMU=TCMU+CMU(I)

400     CONTINUE

C        WRITE(65,*)'total=',tcmu
C         WRITE(67,*)'cost and cost fuzzy probability'
        DO  402 I=1,K1+1
            CP(I)=CMU(I)/TCMU
C             WRITE(67,*)CO(I),CP(I)
402     CONTINUE

* Prompt the user to enter line length ,this is required for the plot of the
*data this length will be the maximumum length in the graph and all the data
* will be scaled according to that length.


C         WRITE(67,*)'cost probability '

*call subroutine graph to plot the cost vs their probabilities

        WRITE(65,280)
280     FORMAT(2X,75('='))

        WRITE(65,290)
290     FORMAT(30X,'THE FINAL RESULTS')

        WRITE(65,289)
289     FORMAT(2X,75('='))

        WRITE(65,293)
293     FORMAT(2X,75('-'))

        WRITE(65,295)
295     FORMAT(5X,'COST',2X,'|',1X,'No. OF OBS.',1X,'|',3X,'PMF',3X,'|'
     1,1X,'MEMBERSHIP VALUES','|','FUZZY PROBABILITIES')

        WRITE(65,293)

        DO 30 I=1,K1+1

            WRITE(65,298)CO(I),NUM(I),AP(I),CMU(I),CP(I)
298         FORMAT(5X,I4,1X,'|',5X,I3,5X,'|',1X,F7.6,1X,'|',5X,F7.6,6X,
     1'|',F7.6)
```

```
               WRITE(65,293)

30         CONTINUE

C          WRITE(67,*)'PMF DATA '

C          do 409 i=1,kl+1
C            WRITE(67,*)co(i),ap(i)
C409       continue

*plot
        print*,' Plot Option '
414     print*,'Please enter (1) if plot is needed'
        print*,'                 or(2) if plot is not needed'
        read*,i2
        if(i2.eq.1)then
            print*,'Line max length Option'
411         print*,'Please enter (1) To use default (Max. L=40)'
            print*,'                 (2) To use new value for Max. L'
            read*,i3
                if(i3.eq.1)then
                    line=40
                 else
                    if(i3.eq.2)then
                        print*,'Please enter line max. length'
                        read*,line
                     else
                        go to 411
                     endif
                 endif
                 GTIT='PMF'
                 call graph(co,ap,kl+1,line,gtit,i3)


                 GTIT='Cost Membership Function'
                 call graph(co,cmu,kl+1,line,gtit,i3)

                 GTIT='Cost Fuzzy Probabilities'
                 call graph(co,cp,kl+1,line,GTIT,i3)
              else
                 if(i2.eq.2)then
                    RETURN
                 else
                    go to 414
                 endif
              endif
        return
        END


* This subroutine prints a bar graph using an array of
*N elements with maximum line size

        subroutine graph(co,value,n,line,gtit,i3)

       integer n,line,i,k,co(50),mccst,i3
       real value(n),max,scale,y1(50)
       logical  negnum
       character*200 bar
       CHARACTER*60 GTIT

*D***A****R*****E*****E*****N**
*
* Find maximum and check for error condition.
*
```

```
        if(line.gt.200.or.line.lt.1)then
           print*,'Line length error',line
        else
           negnum=.false.
           max=value(1)
           do  10 i=1,n
                if(value(i).lt.0.0)then
                   negnum=.true.
               else
                   If(value(i).gt.max)max=value(i)
               endif
10          continue
*
* Fill bar with dashs
*
       if(.not.negnum)then
           do  20 i=1,line
             bar(i:i)='-'
20         continue
*
* Scale data values and print bar.
*
        scale=real(line)/max
           WRITE(65,22)GTIT
22         format('1'30x,'plot of',1x,A60//)
           WRITE(65,23)gtit
23         format(30x,A60)
           if(i3.eq.1)then
             yl(1)=real(max)/5
              yl(2)=yl(1)*2
             yl(3)=yl(1)*3
             yl(4)=yl(1)*4
             yl(5)=max
           WRITE(65,24)(yl(i),i=1,5)
24           format(20x,5(4x,f4.3))
           WRITE(65,26)
26         format(18x,'|-------|-------|-------|-------|-------|------')
            else
            endif
* to print xtitle
              mcost=n/2
           do 30 i=1,n
              k=nint(value(i)*scale)
              if(i.eq.mcost)then
                 WRITE(65,*)'cost',co(i),'  |',bar(1:k)
               else
                 WRITE(65,*)'    ',co(i),'  |',bar(1:k)
              endif
30         continue
         endif
       endif
       return
       end
```

APPENDIX B
Output Data Corresponding to Low. Average. and High Design Variable Membership Combinations

```
****************************************************************************

                               OUTPUT

****************************************************************************

      38.000000      40.919991      87.049995     100.539986     280.00000
0     94.000000      41.350002    1288.060059
      40.919991      44.281006      93.509293     108.509201     294.39007
6    101.269989      44.457493    1388.371704
      87.049995      93.509293     205.269714     231.988861     693.93011
5    219.219986      95.615997    3078.763672
     100.539986     108.509201     231.988861     278.519623     806.48999
0    259.260040     110.477974    3611.197266
     280.000000     294.390076     691.930115     806.489990    4746.00000
0    697.000000     297.149994   10626.958984
      94.000000     101.269989     219.219986     259.260040     697.00000
0    280.000000     107.500000    3699.840332
      41.350002      44.457493      95.615997     110.477974     297.14999
4    107.500000      47.762501    1527.611084
============================================================================

                     COST FUNCTION COEFFICIENTS

============================================================================

   a 0=-84.976936

   a 1= 26.670523

   a 2= 11.234761

   a 3=  6.235301

   a 4=  0.201219

   a 5=  5.039278

   a 6= 31.219091

============================================================================

                 QUANTIFICATION OF ERROR OF LINEAR REGRESSION

============================================================================

THE TOTAL OF SUM OF SQUARES OF RESIDUALS St= 53183.535
THE SUM ODF SQUARES OF THE RESIDUALS    Sr=     30.700
THE STANDARS ERROR OF ESTIMATE         Sx/y=     0.995
THE COEFFICIENT OF DETERMINATION        r^2=0.9994
THE CORRELATION COEFFICIENT               r=0.9997
```

==
==

THE FINAL COST FUNCTION

==
==

Y= -84.97694+(26.67052) X 1+(11.23476) X 2+(6.23530) X 3+(0.20122) X 4+(5.03928) X 5+(31.21909) X 6

OUTPUT CORRESPONDING TO VARIABLES AT THEIR LOW MEMBERSHIP VALUES

THE FINAL RESULTS

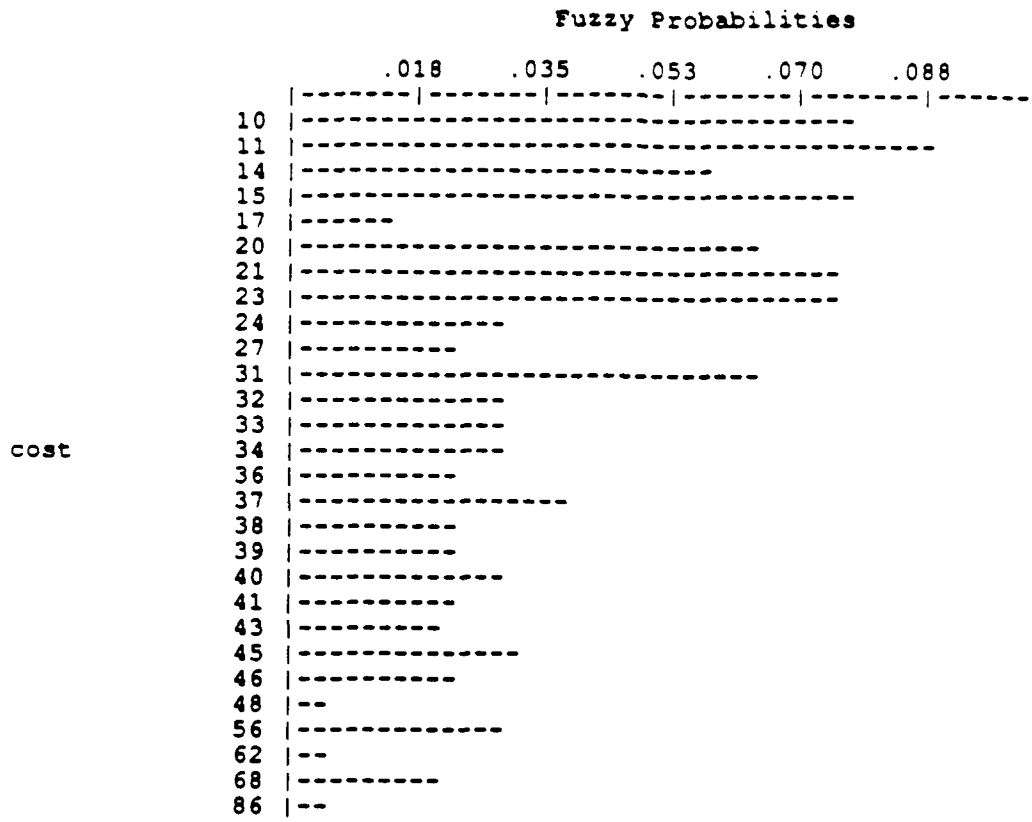
COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.620000	.077500
11	1	.026316	.700000	.087500
14	1	.026316	.450000	.056250
15	2	.052632	.620000	.077500
17	1	.026316	.100000	.012500
20	1	.026316	.500000	.062500
21	2	.052632	.600000	.075000
23	3	.078947	.600000	.075000
24	2	.052632	.220000	.027500
27	1	.026316	.170000	.021250
31	1	.026316	.500000	.062500
32	1	.026316	.220000	.027500
33	1	.026316	.220000	.027500
34	1	.026316	.220000	.027500
36	4	.105263	.170000	.021250
37	1	.026316	.300000	.037500
38	2	.052632	.170000	.021250
39	1	.026316	.170000	.021250
40	2	.052632	.220000	.027500
41	1	.026316	.170000	.021250
43	1	.026316	.150000	.018750
45	1	.026316	.250000	.031250
46	1	.026316	.170000	.021250
48	1	.026316	.040000	.005000
56	1	.026316	.220000	.027500
62	1	.026316	.040000	.005000
68	1	.026316	.150000	.018750
86	1	.026316	.040000	.005000

```
                          PMF

                       .021     .042     .063     .084     .105
                    |--------|--------|--------|--------|--------|------
               10   |-----------
               11   |-----------
               14   |-----------
               15   |----------------------
               17   |-----------
               20   |-----------
               21   |----------------------
               23   |---------------------------------
               24   |----------------------
               27   |-----------
               31   |-----------
               32   |-----------
               33   |-----------
cost           34   |-----------
               36   |--------------------------------------------
               37   |-----------
               38   |----------------------
               39   |-----------
               40   |----------------------
               41   |-----------
               43   |-----------
               45   |-----------
               46   |-----------
               48   |-----------
               56   |-----------
               62   |-----------
               68   |-----------
               86   |-----------
```

```
                                 Membership Function

                        .140     .280     .420     .560     .700
                   |-------|-------|-------|-------|-------|------
              10   |-----------------------------------
              11   |----------------------------------------
              14   |--------------------------
              15   |-----------------------------------
              17   |------
              20   |-----------------------------
              21   |----------------------------------
              23   |----------------------------------
              24   |-------------
              27   |----------
              31   |-----------------------------
              32   |-------------
              33   |-------------
cost          34   |-------------
              36   |----------
              37   |-----------------
              38   |----------
              39   |----------
              40   |-------------
              41   |----------
              43   |---------
              45   |--------------
              46   |----------
              48   |--
              56   |-------------
              62   |--
              68   |---------
              86   |--
```



Fuzzy Probabilities
.018
.035
.053
.070
.088
cost
10
11
14
15
17
20
21
23
24
27
31
32
33
34
36
37
38
39
40
41
43
45
46
48
56
62
68
86

OUTPUT CORRESPONDING TO VARIABLES AT THEIR MEDIUM MEMBERSHIP VALUES

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.150000	.019711
15	2	.052632	.150000	.019711
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.320000	.042050
23	3	.078947	.620000	.081472
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.600000	.078844
32	1	.026316	.450000	.059133
33	1	.026316	.450000	.059133
34	1	.026316	.450000	.059133
36	4	.105263	.350000	.045992
37	1	.026316	.620000	.081472
38	2	.052632	.320000	.042050
39	1	.026316	.350000	.045992
40	2	.052632	.450000	.059133
41	1	.026316	.320000	.042050
43	1	.026316	.300000	.039422
45	1	.026316	.320000	.042050
46	1	.026316	.320000	.042050
48	1	.026316	.150000	.019711
56	1	.026316	.320000	.042050
62	1	.026316	.150000	.019711
68	1	.026316	.300000	.039422
86	1	.026316	.150000	.019711

```
                             PMF

                     .021    .042    .063    .084    .105
                 |-------|-------|-------|-------|-------|------
              10 |----------
              11 |----------
              14 |----------
              15 |--------------------
              17 |----------
              20 |----------
              21 |--------------------
              23 |------------------------------
              24 |--------------------
              27 |----------
              31 |----------
              32 |----------
              33 |----------
cost          34 |----------
              36 |----------------------------------------
              37 |----------
              38 |--------------------
              39 |----------
              40 |--------------------
              41 |----------
              43 |----------
              45 |----------
              46 |----------
              48 |----------
              56 |----------
              62 |----------
              68 |----------
              86 |----------
```

```
                                   Membership Function

                         .124     .248     .372     .496     .620
                     |-------|--------|--------|--------|--------|------
                  10 |
                  11 |
                  14 |----------
                  15 |----------
                  17 |
                  20 |
                  21 |-------------- -------
                  23 |----------------------------------------
                  24 |
                  27 |
                  31 |---------------------------------------
                  32 |-----------------------------
                  33 |-----------------------------
cost              34 |-----------------------------
                  36 |-----------------------
                  37 |----------------------------------------
                  38 |---------------------
                  39 |-----------------------
                  40 |-----------------------------
                  41 |---------------------
                  43 |-------------------
                  45 |---------------------
                  46 |---------------------
                  48 |----------
                  56 |---------------------
                  62 |----------
                  68 |-------------------
                  86 |----------
```

```
                              Fuzzy Probabilities

                  .016     .033     .049     .065     .081
               |-------|--------|--------|--------|--------|------
          10   |
          11   |
          14   |----------
          15   |----------
          17   |
          20   |
          21   |---------------------
          23   |----------------------------------------
          24   |
          27   |
          31   |---------------------------------------
          32   |-----------------------------
          33   |-----------------------------
cost      34   |-----------------------------
          36   |-----------------------
          37   |----------------------------------------
          38   |---------------------
          39   |-----------------------
          40   |-----------------------------
          41   |---------------------
          43   |-------------------
          45   |---------------------
          46   |---------------------
          48   |----------
          56   |---------------------
          62   |----------
          68   |-------------------
          86   |----------
```

OUTPUT CORRESPONDING TO VARIABLES AT THEIR HIGH MEMBERSHIP VALUES

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.070000	.010219
15	2	.052632	.070000	.010219
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.150000	.021898
23	3	.078947	320000	.046715
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.300000	.043796
32	1	.026316	.320000	.046715
33	1	.026316	.320000	.046715
34	1	.026316	.000000	.000000
36	4	.105263	.350000	.051095
37	1	.026316	.350000	.051095
38	2	.052632	.350000	.051095
39	1	.026316	.320000	.046715
40	2	.052632	.350000	.051095
41	1	.026316	.400000	.058394
43	1	.026316	.350000	.051095
45	1	.026316	.350000	.051095
46	1	.026316	.450000	.065693
48	1	.026316	.350000	.051095
56	1	.026316	.300000	.043796
62	1	.026316	.350000	.051095
68	1	.026316	.530000	.077372
86	1	.026316	.500000	.072993

```
                        PMF

                  .021     .042     .063     .084     .105
               |--------|--------|--------|--------|--------|------
       10      |-----------
       11      |-----------
       14      |-----------
       15      |-----------------------
       17      |-----------
       20      |-----------
       21      |-----------------------
       23      |----------------------------------
       24      |-----------------------
       27      |-----------
       31      |-----------
       32      |-----------
       33      |-----------
cost   34      |-----------
       36      |---------------------------------------------
       37      |-----------
       38      |-----------------------
       39      |-----------
       40      |-----------------------
       41      |-----------
       43      |-----------
       45      |-----------
       46      |-----------
       48      |-----------
       56      |-----------
       62      |-----------
       68      |-----------
       86      |-----------
```

```
                              Membership Function

                         .106     .212     .318     .424     .530
                     |-------|-------|-------|-------|-------|------
                  10 |
                  11 |
                  14 |-----
                  15 |-----
                  17 |
                  20 |
                  21 |-----------
                  23 |------------------------
                  24 |
                  27 |
                  31 |-----------------------
                  32 |------------------------
                  33 |------------------------
cost              34 |
                  36 |--------------------------
                  37 |--------------------------
                  38 |--------------------------
                  39 |------------------------
                  40 |--------------------------
                  41 |------------------------------
                  43 |--------------------------
                  45 |--------------------------
                  46 |----------------------------------
                  48 |--------------------------
                  56 |-----------------------
                  62 |--------------------------
                  68 |----------------------------------------
                  86 |--------------------------------------
```

```
                        Fuzzy Probabilities

                     .015      .031      .046      .062      .077
                |--------|--------|--------|--------|--------|------
             10 |
             11 |
             14 |-----
             15 |-----
             17 |
             20 |
             21 |-----------
             23 |------------------------
             24 |
             27 |
             31 |-----------------------
             32 |------------------------
             33 |------------------------
cost         34 |
             36 |--------------------------
             37 |--------------------------
             38 |--------------------------
             39 |------------------------
             40 |--------------------------
             41 |------------------------------
             43 |--------------------------
             45 |--------------------------
             46 |----------------------------------
             48 |--------------------------
             56 |-----------------------
             62 |--------------------------
             68 |----------------------------------------
             86 |--------------------------------------
```

APPENDIX C
Sensitivity Analysis Output Data

OUTPUT CORRESPONDING TO GOOD LOCALITY

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.150000	.020216
15	2	.052632	.150000	.020216
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.320000	.043127
23	3	.078947	.600000	.080863
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.500000	.067385
32	1	.026316	.450000	.060647
33	1	.026316	.450000	.060647
34	1	.026316	.450000	.060647
36	4	.105263	.350000	.047170
37	1	.026316	.550000	.074124
38	2	.052632	.320000	.043127
39	1	.026316	.350000	.047170
40	2	.052632	.450000	.060647
41	1	.026316	.320000	.043127
43	1	.026316	.300000	.040431
45	1	.026316	.320000	.043127
46	1	.026316	.320000	.043127
48	1	.026316	.150000	.020216
56	1	.026316	.320000	.043127
62	1	.026316	.150000	.020216
68	1	.026316	.300000	.040431
86	1	.026316	.150000	.020216

```
                     PMF

                 .021      .042      .063      .084      .105
              |--------|--------|--------|--------|--------|------
          10  |-----------
          11  |-----------
          14  |-----------
          15  |----------------------
          17  |-----------
          20  |-----------
          21  |----------------------
          23  |---------------------------------
          24  |----------------------
          27  |-----------
          31  |-----------
          32  |-----------
          33  |-----------
cost      34  |-----------
          36  |--------------------------------------------
          37  |-----------
          38  |----------------------
          39  |-----------
          40  |----------------------
          41  |-----------
          43  |-----------
          45  |-----------
          46  |-----------
          48  |-----------
          56  |-----------
          62  |-----------
          68  |-----------
          86  |-----------
```

Membership Function

```
                .120     .240     .360     .480     .600
          |-------|-------|-------|-------|-------|------
       10 |
       11 |
       14 |----------
       15 |----------
       17 |
       20 |
       21 |---------------------
       23 |----------------------------------------
       24 |
       27 |
       31 |---------------------------------
       32 |------------------------------
       33 |------------------------------
cost   34 |------------------------------
       36 |-----------------------
       37 |-------------------------------------
       38 |---------------------
       39 |-----------------------
       40 |------------------------------
       41 |---------------------
       43 |--------------------
       45 |---------------------
       46 |---------------------
       48 |----------
       56 |---------------------
       62 |----------
       68 |--------------------
       86 |----------
```

```
                    Fuzzy Probabilities

              .016    .032    .049    .065    .081
          |-------|-------|-------|-------|-------|------
       10 |
       11 |
       14 |----------
       15 |----------
       17 |
       20 |
       21 |---------------------
       23 |----------------------------------------
       24 |
       27 |
       31 |---------------------------------
       32 |------------------------------
       33 |------------------------------
cost   34 |------------------------------
       36 |-----------------------
       37 |-------------------------------------
       38 |---------------------
       39 |-----------------------
       40 |------------------------------
       41 |---------------------
       43 |--------------------
       45 |---------------------
       46 |---------------------
       48 |----------
       56 |---------------------
       62 |----------
       68 |--------------------
       86 |----------
```

OUTPUT CORRESPONDING TO LOW PRICE

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.150000	.021368
15	2	.052632	.150000	.021368
17	1	.026316	.100000	.014245
20	1	.026316	.000000	.000000
21	2	.052632	.670000	.095442
23	3	.078947	.670000	.095442
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.650000	.092593
32	1	.026316	.300000	.042735
33	1	.026316	.450000	.064103
34	1	.026316	.450000	.064103
36	4	.105263	.350000	.049858
37	1	.026316	.300000	.042735
38	2	.052632	.320000	.045584
39	1	.026316	.350000	.049858
40	2	.052632	.450000	.064103
41	1	.026316	.320000	.045584
43	1	.026316	.150000	.021368
45	1	.026316	.300000	.042735
46	1	.026316	.300000	.042735
48	1	.026316	.040000	.005698
56	1	.026316	.320000	.045584
62	1	.026316	.040000	.005698
68	1	.026316	.150000	.021368
86	1	.026316	.040000	.005698

```
                              PMF

                          .021     .042     .063     .084     .105
                     |--------|--------|--------|--------|--------|------
                 10  |----------
                 11  |----------
                 14  |----------
                 15  |--------------------
                 17  |----------
                 20  |----------
                 21  |--------------------
                 23  |------------------------------
                 24  |--------------------
                 27  |----------
                 31  |----------
                 32  |----------
                 33  |----------
cost             34  |----------
                 36  |----------------------------------------
                 37  |----------
                 38  |--------------------
                 39  |----------
                 40  |--------------------
                 41  |----------
                 43  |----------
                 45  |----------
                 46  |----------
                 48  |----------
                 56  |----------
                 62  |----------
                 68  |----------
                 86  |----------
```

```
                      Membership Function

          .134     .268     .402     .536     .670
       |-------|--------|--------|--------|--------|------
    10 |
    11 |
    14 |---------
    15 |---------
    17 |------
    20 |
    21 |----------------------------------------
    23 |----------------------------------------
    24 |
    27 |
    31 |---------------------------------------
    32 |------------------
    33 |---------------------------
cost 34 |---------------------------
    36 |---------------------
    37 |------------------
    38 |-------------------
    39 |---------------------
    40 |---------------------------
    41 |-------------------
    43 |---------
    45 |------------------
    46 |------------------
    48 |--
    56 |-------------------
    62 |--
    68 |---------
    86 |--
```

```
                              Fuzzy Probabilities

                         .019    .038    .057    .076    .095
                     |--------|--------|--------|--------|--------|------
                  10 |
                  11 |
                  14 |---------
                  15 |---------
                  17 |------
                  20 |
                  21 |-----------------------------------------
                  23 |-----------------------------------------
                  24 |
                  27 |
                  31 |----------------------------------------
                  32 |------------------
                  33 |---------------------------
cost              34 |---------------------------
                  36 |---------------------
                  37 |------------------
                  38 |-------------------
                  39 |---------------------
                  40 |---------------------------
                  41 |-------------------
                  43 |---------
                  45 |------------------
                  46 |------------------
                  48 |--
                  56 |-------------------
                  62 |--
                  68 |---------
                  86 |--
```

OUTPUT CORRESPONDING TO SIMPLE TYPE

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.500000	.083333
15	2	.052632	.150000	.025000
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.320000	.053333
23	3	.078947	.600000	.100000
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.600000	.100000
32	1	.026316	.220000	.036667
33	1	.026316	.220000	.036667
34	1	.026316	.220000	.036667
36	4	.105263	.170000	.028333
37	1	.026316	.620000	.103333
38	2	.052632	.220000	.036667
39	1	.026316	.170000	.028333
40	2	.052632	.220000	.036667
41	1	.026316	.170000	.028333
43	1	.026316	.220000	.036667
45	1	.026316	.320000	.053333
46	1	.026316	.170000	.028333
48	1	.026316	.150000	.025000
56	1	.026316	.220000	.036667
62	1	.026316	.150000	.025000
68	1	.026316	.220000	.036667
86	1	.026316	.150000	.025000

```
                              PMF

                        .021      .042      .063      .084      .105
                    |--------|---------|---------|---------|---------|------
                 10 |----------
                 11 |----------
                 14 |----------
                 15 |--------------------
                 17 |----------
                 20 |----------
                 21 |--------------------
                 23 |--------------------- ---------
                 24 |--------------------
                 27 |----------
                 31 |----------
                 32 |----------
                 33 |----------
cost             34 |----------
                 36 |----------------------------------------
                 37 |----------
                 38 |--------------------
                 39 |----------
                 40 |--------------------
                 41 |----------
                 43 |----------
                 45 |----------
                 46 |----------
                 48 |----------
                 56 |----------
                 62 |----------
                 68 |----------
                 86 |----------
```

```
                            Membership Function

                 .124    .248    .372    .496    .620
           |-------|-------|-------|-------|-------|------
        10 |
        11 |
        14 |--------------------------------
        15 |----------
        17 |
        20 |
        21 |---------------------
        23 |---------------------------------------
        24 |
        27 |
        31 |---------------------------------------
        32 |--------------
        33 |--------------
cost    34 |--------------
        36 |-----------
        37 |----------------------------------------
        38 |--------------
        39 |-----------
        40 |--------------
        41 |-----------
        43 |--------------
        45 |---------------------
        46 |-----------
        48 |----------
        56 |--------------
        62 |----------
        68 |--------------
        86 |----------
```

```
                                 Fuzzy Probabilities

                    .021     .041     .062     .083     .103
              |--------|--------|--------|--------|--------|------
          10  |
          11  |
          14  |------------------------------------
          15  |-----------
          17  |
          20  |
          21  |------------------------
          23  |--------------------------------------------
          24  |
          27  |
          31  |--------------------------------------------
          32  |----------------
          33  |----------------
cost      34  |----------------
          36  |------------
          37  |---------------------------------------------
          38  |----------------
          39  |------------
          40  |----------------
          41  |------------
          43  |----------------
          45  |------------------------
          46  |------------
          48  |-----------
          56  |----------------
          62  |-----------
          68  |----------------
          86  |-----------
```

OUTPUT CORRESPONDING TO LOW QUALITY

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.150000	.016094
11	1	.026316	.400000	.042918
14	1	.026316	.150000	.016094
15	2	.052632	.600000	.064378
17	1	.026316	.000000	.000000
20	1	.026316	.600000	.064378
21	2	.052632	.450000	.048283
23	3	.078947	.620000	.066524
24	2	.052632	.350000	.037554
27	1	.026316	.350000	.037554
31	1	.026316	.500000	.053648
32	1	.026316	.450000	.048283
33	1	.026316	.450000	.048283
34	1	.026316	.450000	.048283
36	4	.105263	.350000	.037554
37	1	.026316	.500000	.053648
38	2	.052632	.300000	.032189
39	1	.026316	.250000	.026824
40	2	.052632	.450000	.048283
41	1	.026316	.250000	.026824
43	1	.026316	.250000	.026824
45	1	.026316	.250000	.026824
46	1	.026316	.250000	.026824
48	1	.026316	.150000	.016094
56	1	.026316	.250000	.026824
62	1	.026316	.150000	.016094
68	1	.026316	.250000	.026824
86	1	.026316	.150000	.016094

```
                              PMF

                      .021      .042      .063      .084      .105
                 |--------|--------|--------|--------|--------|------
             10  |----------
             11  |----------
             14  |----------
             15  |--------------------
             17  |----------
             20  |----------
             21  |--------------------
             23  |------------------------------
             24  |--------------------
             27  |----------
             31  |----------
             32  |----------
             33  |----------
cost         34  |----------
             36  |----------------------------------------
             37  |----------
             38  |--------------------
             39  |----------
             40  |--------------------
             41  |----------
             43  |----------
             45  |----------
             46  |----------
             48  |----------
             56  |----------
             62  |----------
             68  |----------
             86  |----------
```

```
                                         Membership Function

                                  .124     .248     .372     .496     .620
                           |--------|--------|--------|--------|--------|------
                       10  |-----------
                       11  |-----------------------------
                       14  |-----------
                       15  |--------------------------------------------
                       17  |
                       20  |--------------------------------------------
                       21  |--------------------------------
                       23  |---------------------------------------------
                       24  |--------------------------
                       27  |--------------------------
                       31  |------------------------------------
                       32  |--------------------------------
                       33  |--------------------------------
cost                   34  |--------------------------------
                       36  |--------------------------
                       37  |------------------------------------
                       38  |---------------------
                       39  |------------------
                       40  |--------------------------------
                       41  |------------------
                       43  |------------------
                       45  |------------------
                       46  |------------------
                       48  :  ----------
                       56  |------------------
                       62  |-----------
                       68  |------------------
                       86  |-----------
```

```
                                 Fuzzy Probabilities

                   .013    .027    .040    .053    .067
              |-------|-------|-------|-------|-------|------
          10  |----------
          11  |--------------------------
          14  |----------
          15  |---------------------------------------
          17  |
          20  |---------------------------------------
          21  |-----------------------------
          23  |----------------------------------------
          24  |-----------------------
          27  |-----------------------
          31  |--------------------------------
          32  |-----------------------------
          33  |-----------------------------
cost      34  |-----------------------------
          36  |-----------------------
          37  |--------------------------------
          38  |-------------------
          39  |----------------
          40  |-----------------------------
          41  |----------------
          43  |----------------
          45  |----------------
          46  |----------------
          48  |----------
          56  |----------------
          62  |----------
          68  |----------------
          86  |----------
```

OUTPUT CORRESPONDING TO LOW TECHNOLOGY

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.150000	.019711
15	2	.052632	.150000	.019711
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.320000	.042050
23	3	.078947	.620000	.081472
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.600000	.078844
32	1	.026316	.450000	.059133
33	1	.026316	.450000	.059133
34	1	.026316	.450000	.059133
36	4	.105263	.350000	.045992
37	1	.026316	.620000	.081472
38	2	.052632	.320000	.042050
39	1	.026316	.350000	.045992
40	2	.052632	.450000	.059133
41	1	.026316	.320000	.042050
43	1	.026316	.300000	.039422
45	1	.026316	.320000	.042050
46	1	.026316	.320000	.042050
48	1	.026316	.150000	.019711
56	1	.026316	.320000	.042050
62	1	.026316	.150000	.019711
68	1	.026316	.300000	.039422
86	1	.026316	.150000	.019711

```
                                   PMF

                                .021     .042     .063     .084     .105
                           |--------|--------|--------|--------|--------|------
                        10 |-----------
                        11 |-----------
                        14 |-----------
                        15 |----------------------
                        17 |-----------
                        20 |-----------
                        21 |----------------------
                        23 |---------------------------------
                        24 |----------------------
                        27 |-----------
                        31 |-----------
                        32 |-----------
                        33 |-----------
cost                    34 |-----------
                        36 |--------------------------------------------
                        37 |-----------
                        38 |----------------------
                        39 |-----------
                        40 |----------------------
                        41 |-----------
                        43 |-----------
                        45 |-----------
                        46 |-----------
                        48 |-----------
                        56 |-----------
                        62 |-----------
                        68 |-----------
                        86 |-----------
```

```
                         Membership Function

                    .124     .248     .372     .496     .620
                 |-------|-------|-------|-------|-------|------
          10     |
          11     |
          14     |----------
          15     |----------
          17     |
          20     |
          21     |---------------------
          23     |----------------------------------------
          24     |
          27     |
          31     |---------------------------------------
          32     |-----------------------------
          33     |-----------------------------
cost      34     |-----------------------------
          36     |-----------------------
          37     |----------------------------------------
          38     |---------------------
          39     |-----------------------
          40     |-----------------------------
          41     |---------------------
          43     |-------------------
          45     |---------------------
          46     |---------------------
          48     |----------
          56     |---------------------
          62     |----------
          68     |-------------------
          86     |----------
```

```
                              Fuzzy Probabilities

                  .016     .033     .049     .065     .081
                |-------|--------|--------|--------|--------|------
              10 |
              11 |
              14 |----------
              15 |----------
              17 |
              20 |
              21 |----------------------
              23 |-------------------------------------------
              24 |
              27 |
              31 |------------------------------------------
              32 |-------------------------------
              33 |-------------------------------
cost          34 |-------------------------------
              36 |-------------------------
              37 |-------------------------------------------
              38 |----------------------
              39 |-------------------------
              40 |-------------------------------
              41 |----------------------
              43 |--------------------
              45 |----------------------
              46 |----------------------
              48 |----------
              56 |----------------------
              62 |----------
              68 |--------------------
              86 |----------
```

OUTPUT CORRESPONDING TO POOR LOCALITY

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.150000	.019711
15	2	.052632	.150000	.019711
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.320000	.042050
23	3	.078947	.620000	.081472
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.600000	.078844
32	1	.026316	.450000	.059133
33	1	.026316	.450000	.059133
34	1	.026316	.450000	.059133
36	4	.105263	.350000	.045992
37	1	.026316	.620000	.081472
38	2	.052632	.320000	.042050
39	1	.026316	.350000	.045992
40	2	.052632	.450000	.059133
41	1	.026316	.320000	.042050
43	1	.026316	.300000	.039422
45	1	.026316	.320000	.042050
46	1	.026316	.320000	.042050
48	1	.026316	.150000	.019711
56	1	.026316	.320000	.042050
62	1	.026316	.150000	.019711
68	1	.026316	.300000	.039422
86	1	.026316	.150000	.019711

```
                                      PMF

                                   .021     .042     .063     .084     .105
                              |-------|-------|-------|-------|-------|------
                         10   |----------
                         11   |----------
                         14   |----------
                         15   |--------------------
                         17   |----------
                         20   |----------
                         21   |--------------------
                         23   |------------------------------
                         24   |--------------------
                         27   |----------
                         31   |----------
                         32   |----------
                         33   |----------
     cost                34   |----------
                         36   |----------------------------------------
                         37   |----------
                         38   |--------------------
                         39   |----------
                         40   |--------------------
                         41   |----------
                         43   |----------
                         45   |----------
                         46   |----------
                         48   |----------
                         56   |----------
                         62   |----------
                         68   |----------
                         86   |----------
```

```
                              Membership Function

                   .124     .248     .372     .496     .620
              |-------|-------|-------|-------|-------|------
          10  |
          11  |
          14  |----------
          15  |----------
          17  |
          20  |
          21  |---------------------
          23  |-----------------------------------------
          24  |
          27  |
          31  |----------------------------------------
          32  |------------------------------
          33  |------------------------------
cost      34  |------------------------------
          36  |------------------------
          37  |-----------------------------------------
          38  |---------------------
          39  |------------------------
          40  |------------------------------
          41  |---------------------
          43  |-------------------
          45  |---------------------
          46  |---------------------
          48  |----------
          56  |---------------------
          62  |----------
          68  |-------------------
          86  |----------
```

```
                         Fuzzy Probabilities

                   .016     .033     .049     .065     .081
               |--------|--------|--------|--------|--------|------
          10   |
          11   |
          14   |-----------
          15   |-----------
          17   |
          20   |
          21   |------------------------
          23   |--------------------------------------------
          24   |
          27   |
          31   |--------------------------------------------
          32   |---------------------------------
          33   |---------------------------------
cost      34   |---------------------------------
          36   |---------------------------
          37   |----------------------------------------------
          38   |------------------------
          39   |---------------------------
          40   |---------------------------------
          41   |------------------------
          43   |----------------------
          45   |------------------------
          46   |------------------------
          48   |-----------
          56   |------------------------
          62   |-----------
          68   |----------------------
          86   |-----------
```

OUTPUT CORRESPONDING TO HIGH PRICE

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.150000	.019763
15	2	.052632	.150000	.019763
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.150000	.019763
23	3	.078947	.400000	.052701
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.300000	.039526
32	1	.026316	.450000	.059289
33	1	.026316	.450000	.059289
34	1	.026316	.400000	.052701
36	4	.105263	.350000	.046113
37	1	.026316	.670000	.088274
38	2	.052632	.450000	.059289
39	1	.026316	.350000	.046113
40	2	.052632	.450000	.059289
41	1	.026316	.320000	.042161
43	1	.026316	.320000	.042161
45	1	.026316	.320000	.042161
46	1	.026316	.320000	.042161
48	1	.026316	.450000	.059289
56	1	.026316	.300000	.039526
62	1	.026316	.320000	.042161
68	1	.026316	.320000	.042161
86	1	.026316	.200000	.026350

```
                              PMF

                     .021      .042      .063      .084      .105
                 |--------|--------|--------|--------|--------|------
           10    |-----------
           11    |-----------
           14    |-----------
           15    |----------------------
           17    |-----------
           20    |-----------
           21    |----------------------
           23    |---------------------------------
           24    |----------------------
           27    |-----------
           31    |-----------
           32    |-----------
           33    |-----------
cost       34    |-----------
           36    |--------------------------------------------
           37    |-----------
           38    |----------------------
           39    |-----------
           40    |----------------------
           41    |-----------
           43    |-----------
           45    |-----------
           46    |-----------
           48    |-----------
           56    |-----------
           62    |-----------
           68    |-----------
           86    |-----------
```

```
                                  Membership Function

                          .134     .268     .402     .536     .670
                       |--------|--------|--------|--------|--------|------
                    10 |
                    11 |
                    14 |----------
                    15 |----------
                    17 |
                    20 |
                    21 |----------
                    23 |---------------------------
                    24 |
                    27 |
                    31 |--------------------
                    32 |------------------------------
                    33 |------------------------------
cost                34 |---------------------------
                    36 |-----------------------
                    37 |---------------------------------------------
                    38 |------------------------------
                    39 |-----------------------
                    40 |------------------------------
                    41 |---------------------
                    43 |---------------------
                    45 |---------------------
                    46 |---------------------
                    48 |------------------------------
                    56 |--------------------
                    62 |---------------------
                    68 |---------------------
                    86 |-------------
```

```
                                        Fuzzy Probabilities

                              .018      .035      .053      .071      .088
                          |--------|--------|--------|--------|--------|------
                       10 |
                       11 |
                       14 |----------
                       15 |----------
                       17 |
                       20 |
                       21 |----------
                       23 |---------------------------
                       24 |
                       27 |
                       31 |--------------------
                       32 |------------------------------
                       33 |------------------------------
  cost                 34 |---------------------------
                       36 |-----------------------
                       37 |---------------------------------------------
                       38 |------------------------------
                       39 |-----------------------
                       40 |------------------------------
                       41 |---------------------
                       43 |---------------------
                       45 |---------------------
                       46 |---------------------
                       48 |------------------------------
                       56 |--------------------
                       62 |---------------------
                       68 |---------------------
                       86 |-------------
```

OUTPUT CORRESPONDING TO COMPLICATED TYPE

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.070000	.008895
15	2	.052632	.070000	.008895
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.320000	.040661
23	3	.078947	.400000	.050826
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.350000	.044473
32	1	.026316	.620000	.078780
33	1	.026316	.600000	.076239
34	1	.026316	.620000	.078780
36	4	.105263	.620000	.078780
37	1	.026316	.350000	.044473
38	2	.052632	.320000	.040661
39	1	.026316	.600000	.076239
40	2	.052632	.600000	.076239
41	1	.026316	.320000	.040661
43	1	.026316	.300000	.038119
45	1	.026316	.320000	.040661
46	1	.026316	.320000	.040661
48	1	.026316	.150000	.019060
56	1	.026316	.320000	.040661
62	1	.026316	.150000	.019060
68	1	.026316	.300000	.038119
86	1	.026316	.150000	.019060

```
                              PMF

                     .021      .042      .063      .084      .105
                |--------|--------|--------|--------|--------|------
             10 |-----------
             11 |-----------
             14 |-----------
             15 |---------------------
             17 |-----------
             20 |-----------
             21 |---------------------
             23 |-------------------------------
             24 |---------------------
             27 |-----------
             31 |-----------
             32 |-----------
             33 |-----------
cost         34 |-----------
             36 |-----------------------------------------
             37 |-----------
             38 |---------------------
             39 |-----------
             40 |---------------------
             41 |-----------
             43 |-----------
             45 |-----------
             46 |-----------
             48 |-----------
             56 |-----------
             62 |-----------
             68 |-----------
             86 |-----------
```

Membership Function

```
                              .124     .248     .372     .496     .620
                         |--------|--------|--------|--------|--------|------
                     10  |
                     11  |
                     14  |-----
                     15  |-----
                     17  |
                     20  |
                     21  |-----------------------
                     23  |-----------------------------
                     24  |
                     27  |
                     31  |--------------------------
                     32  |---------------------------------------------
                     33  |--------------------------------------------
cost                 34  |---------------------------------------------
                     36  |---------------------------------------------
                     37  |--------------------------
                     38  |-----------------------
                     39  |--------------------------------------------
                     40  |--------------------------------------------
                     41  |-----------------------
                     43  |---------------------
                     45  |-----------------------
                     46  |-----------------------
                     48  |-----------
                     56  |-----------------------
                     62  |-----------
                     68  |---------------------
                     86  |-----------
```

```
                         Fuzzy Probabilities

                   .016     .032     .047     .063     .079
                 |-------|--------|--------|--------|--------|------
            10   |
            11   |
            14   |-----
            15   |-----
            17   |
            20   |
            21   |----------------------
            23   |---------------------------
            24   |
            27   |
            31   |------------------------
            32   |-----------------------------------------
            33   |----------------------------------------
cost        34   |-----------------------------------------
            36   |-----------------------------------------
            37   |------------------------
            38   |----------------------
            39   |----------------------------------------
            40   |----------------------------------------
            41   |----------------------
            43   |--------------------
            45   |----------------------
            46   |----------------------
            48   |-----------
            56   |----------------------
            62   |-----------
            68   |--------------------
            86   |-----------
```

OUTPUT CORRESPONDING TO HIGH QUALITY

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.150000	.019894
15	2	.052632	.150000	.019894
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.320000	.042440
23	3	.078947	.320000	.042440
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.600000	.079576
32	1	.026316	.320000	.042440
33	1	.026316	.320000	.042440
34	1	.026316	.450000	.059682
36	4	.105263	.350000	.046419
37	1	.026316	.620000	.082228
38	2	.052632	.350000	.046419
39	1	.026316	.320000	.042440
40	2	.052632	.450000	.059682
41	1	.026316	.350000	.046419
43	1	.026316	.300000	.039788
45	1	.026316	.620000	.082228
46	1	.026316	.350000	.046419
48	1	.026316	.150000	.019894
56	1	.026316	.450000	.059682
62	1	.026316	.150000	.019894
68	1	.026316	.300000	.039788
86	1	.026316	.150000	.019894

```
                         PMF

             .021     .042     .063     .084     .105
         |--------|--------|--------|--------|--------|------
      10 |----------
      11 |----------
      14 |----------
      15 |--------------------
      17 |----------
      20 |----------
      21 |--------------------
      23 |------------------------------
      24 |--------------------
      27 |----------
      31 |----------
      32 |----------
      33 |----------
cost  34 |----------
      36 |----------------------------------------
      37 |----------
      38 |--------------------
      39 |----------
      40 |--------------------
      41 |----------
      43 |----------
      45 |----------
      46 |----------
      48 |----------
      56 |----------
      62 |----------
      68 |----------
      86 |----------
```

```
                        Membership Function

               .124     .248     .372     .496     .620
            |--------|--------|--------|--------|--------|------
       10   |
       11   |
       14   |-----------
       15   |-----------
       17   |
       20   |
       21   |------------------------
       23   |------------------------
       24   |
       27   |
       31   |--------------------------------------------
       32   |------------------------
       33   |------------------------
cost   34   |---------------------------------
       36   |--------------------------
       37   |---------------------------------------------
       38   |--------------------------
       39   |------------------------
       40   |---------------------------------
       41   |--------------------------
       43   |----------------------
       45   |---------------------------------------------
       46   |--------------------------
       48   |-----------
       56   |---------------------------------
       62   |-----------
       68   |----------------------
       86   |-----------
```

```
                              Fuzzy Probabilities

                  .016     .033     .049     .066     .082
              |--------|--------|--------|--------|--------|-----
         10   |
         11   |
         14   |-----------
         15   |-----------
         17   |
         20   |
         21   |-----------------------
         23   |-----------------------
         24   |
         27   |
         31   |-------------------------------------------
         32   |-----------------------
         33   |-----------------------
cost     34   |--------------------------------
         36   |-------------------------
         37   |--------------------------------------------
         38   |-------------------------
         39   |-----------------------
         40   |--------------------------------
         41   |-------------------------
         43   |---------------------
         45   |--------------------------------------------
         46   |-------------------------
         48   |-----------
         56   |--------------------------------
         62   |-----------
         68   |---------------------
         86   |-----------
```

OUTPUT CORRESPONDING TO HIGH TECHNOLOGY

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES	FUZZY PROBABILITIES
10	1	.026316	.000000	.000000
11	1	.026316	.000000	.000000
14	1	.026316	.150000	.023364
15	2	.052632	.150000	.023364
17	1	.026316	.000000	.000000
20	1	.026316	.000000	.000000
21	2	.052632	.320000	.049844
23	3	.078947	.350000	.054517
24	2	.052632	.000000	.000000
27	1	.026316	.000000	.000000
31	1	.026316	.350000	.054517
32	1	.026316	.350000	.054517
33	1	.026316	.350000	.054517
34	1	.026316	.350000	.054517
36	4	.105263	.350000	.054517
37	1	.026316	.350000	.054517
38	2	.052632	.320000	.049844
39	1	.026316	.350000	.054517
40	2	.052632	.350000	.054517
41	1	.026316	.320000	.049844
43	1	.026316	.300000	.046729
45	1	.026316	.320000	.049844
46	1	.026316	.320000	.049844
48	1	.026316	.150000	.023364
56	1	.026316	.320000	.049844
62	1	.026316	.150000	.023364
63	1	.026316	.300000	.046729
86	1	.026316	.150000	.023364

PMF

```
              .021     .042     .063     .084     .105
             |--------|--------|--------|--------|--------|------
        10   |-----------
        11   |-----------
        14   |-----------
        15   |-----------------------
        17   |-----------
        20   |-----------
        21   |-----------------------
        23   |----------------------------------
        24   |-----------------------
        27   |-----------
        31   |-----------
        32   |-----------
        33   |-----------
cost    34   |-----------
        36   |----------------------------------------------
        37   |-----------
        38   |-----------------------
        39   |-----------
        40   |-----------------------
        41   |-----------
        43   |-----------
        45   |-----------
        46   |-----------
        48   |-----------
        56   |-----------
        62   |-----------
        68   |-----------
        86   |-----------
```

Membership Function

```
           .070    .140    .210    .280    .350
     |-------|-------|-------|-------|-------|------
  10 |
  11 |
  14 |-----------------
  15 |-----------------
  17 |
  20 |
  21 |--------------------------------------
  23 |-----------------------------------------
  24 |
  27 |
  31 |-----------------------------------------
  32 |-----------------------------------------
  33 |-----------------------------------------
  34 |-----------------------------------------
  36 |-----------------------------------------
  37 |-----------------------------------------
  38 |--------------------------------------
  39 |-----------------------------------------
  40 |-----------------------------------------
  41 |--------------------------------------
  43 |-----------------------------------
  45 |--------------------------------------
  46 |--------------------------------------
  48 |-----------------
  56 |--------------------------------------
  62 |-----------------
  68 |-----------------------------------
  86 |-----------------
```

cost

```
                              Fuzzy Probabilities

                   .011    .022    .033    .044    .055
                |-------|-------|-------|-------|-------|------
             10 |
             11 |
             14 |-----------------
             15 |-----------------
             17 |
             20 |
             21 |-------------------------------------
             23 |----------------------------------------
             24 |
             27 |
             31 |----------------------------------------
             32 |----------------------------------------
             33 |----------------------------------------
cost         34 |----------------------------------------
             36 |----------------------------------------
             37 |----------------------------------------
             38 |-------------------------------------
             39 |----------------------------------------
             40 |----------------------------------------
             41 |-------------------------------------
             43 |----------------------------------
             45 |-------------------------------------
             46 |-------------------------------------
             48 |-----------------
             56 |-------------------------------------
             62 |-----------------
             68 |----------------------------------
             86 |-----------------
```

APPENDIX D

User Manual

User Manual

1 Introduction

The preliminary cost estimate is a primary consideration of both owners and designers of construction projects. The estimation at this stage involves many design variables. Usually the designer does not have enough engineering data to predict the future behavior of these variables other than verbal description, such as "good," "bad," "large," "small," and so on.

Using regression analysis, this program uses previous data from the past projects to develop a cost function.

The opinion of the experienced designer regarding the value of the design variables is used to find cost membership function.

The variable membership values range from 0 to 1 depending upon the degree of confidence that the variable is a member of the set. Applying the extension principle, membership functions of independent design variables are used to find the cost membership function. The computer program (COSTEST) developed in this study enables the estimator to obtain:

1. A membership function which explains the degree that a cost value belongs to a cost event.

2. The probability of each cost value and event. (The second one is computed using the probability mass function (PMF) of the cost.)

An example is given below to show the input and output of the computer program.

2 Example Problem

Assume that the independent design variables are V_1, V_2, V_3, and V_4 and C is the dependent design variable representing the unit cost of the project. The linear function:

$$C = a_0 + a_1 V_1 + a_2 V_2 + a_3 V_3 + a_4 V_4 \qquad (1)$$

represents the relation between the independent and dependent design variables.

Table 1: Historical Data

C_i	V_{1i}	V_{2i}	V_{3i}	V_{4i}
55.50	4.5	3.2	2.2	5.2
58.00	5.2	4.2	3.0	4.0
54.00	4.7	3.2	1.5	6.5
64.00	5.8	4.9	1.6	6.9
60.00	4.0	4.6	2.5	5.7
49.00	4.9	3.0	1.9	4.8
57.00	4.5	3.9	2.8	4.7
57.00	5.7	4.2	1.3	6.3
61.00	5.3	4.6	2.4	5.4
54.00	4.1	3.2	2.7	5.0

Table 1 shows the values of dependent design variable C_i and independent design variables V_{ki}, $k = (1, \ldots, 4)$, $i = (1, \ldots, 10)$ associated with ten past projects.

These variables can be indices or unit prices of any component per unit of the project. This unit can be in cubic feet, square feet or other units, such as cost per output product, for example, cost per student in school projects.

The next step is to develop a membership function for each design variable. This will be based on the experienced designer opinion of the behavior of these design variables in the future. These membership functions can be discrete or continuous. In this example, assume the following membership functions for the four design variables.

Assume V_1 represents the weather index, V_2 represents the productivity index, V_3 the price index and V_4 the quality index.

Suppose, in this example, the prediction of design variables is as shown in Table 2.

Figures 1, 2, 3 and 4 represent the assumed membership functions of variables V_1, V_2, V_3 and V_4, respectively.

According to the designer opinion in Table 2 for the prediction of design variables V_i (see Table 2) and using Figures 1-4 and Table 1, the membership values of these design variables are computed and shown in Table 3. How to use these membership values is explained in the following section.

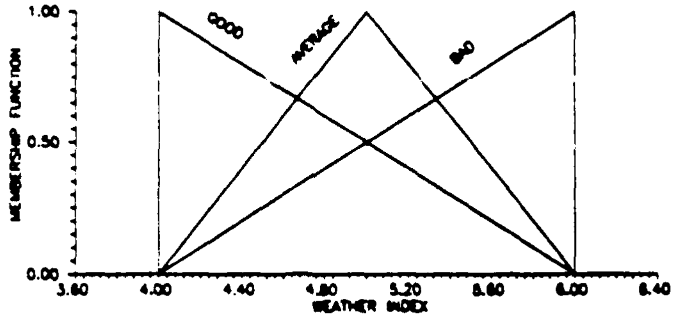


Figure 1: Weather Index Membership Function

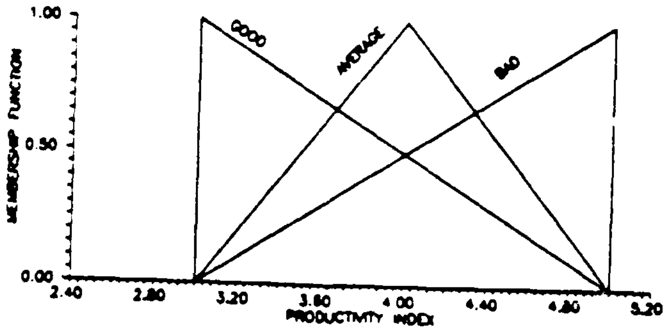


Figure 2: Productivity Index Membership Function

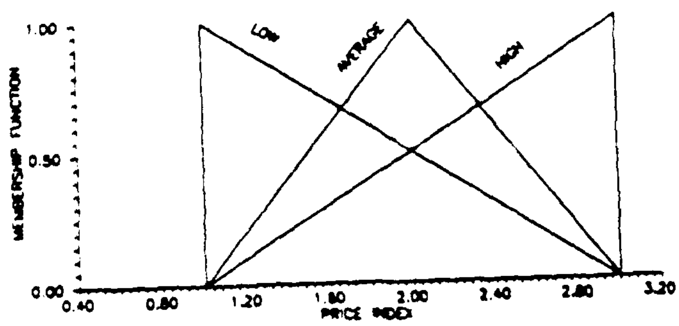


Figure 3: Price Index Membership Function

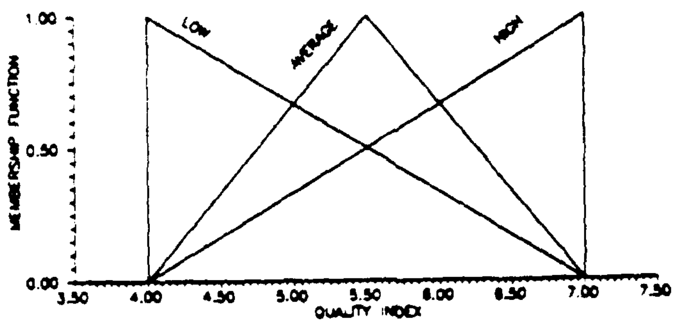


Figure 4: Quality Index Membership Function

Table 2: Designer Expectation of the Variables

Variable	V_i	Future Prediction
Weather Index	V_1	Good
Productivity Index	V_2	Medium
Price Index	V_3	High
Quality Index	V_4	High

3 Input

This program is developed to read the data from a file. The program indicates the name of the input file.

3.1 Input File

The input file should contain the dependent design variable values, C_i, independent design variable values, V_{ki}, and membership values, μ_{ki}, in the format as shown in Table 4.

Once the input file is established, the program can be run. The program contains an interactive part to prompt the user to input some information, as in the following steps, while running the example problem.

```
A > COSTEST
```

To run the program from the PC.

```
$Run COSTEST
```

To run the program from the VAX.

```
Please enter the output title max 40 characters.
```

Prompts the user to input the output files title. This title will appear at the top of all output files.

```
Example Problem
```

This is the title entered by the user.

```
Please enter the input file.
```

Table 3: Design Variables and Their Corresponding Membership Values

C_i	V_{1i}	$\mu_{V_{1i}}$ ($V_{1i} = Good$)	V_{2i}	$\mu_{V_{2i}}$ ($V_{2i} = Med$)	V_{3i}	$\mu_{V_{3i}}$ ($V_{3i} = High$)	V_{4i}	$\mu_{V_{4i}}$ ($V_{4i} = High$)
55.50	4.5	.75	3.2	.2	2.2	0.6	5.2	.433
58.00	5.2	.4	4.2	.8	3.0	1.0	4.0	0.0
54.00	4.7	.65	3.2	.2	1.5	.25	6.5	.833
64.00	5.8	.1	4.9	.1	1.6	.3	6.9	.967
60.00	4.0	1.0	4.6	.4	2.5	.75	5.7	.567
49.00	4.9	.55	3.0	0.0	1.9	.45	4.8	.267
57.00	4.5	.75	3.9	.9	2.8	.9	4.7	.233
57.00	5.7	.15	4.2	.8	1.3	.15	6.3	.767
61.00	5.3	.35	4.6	.4	2.4	.7	5.4	.467
54.00	4.1	0.95	3.2	.20	2.7	.85	5.0	.333

Prompts the user to enter the input file that is explained in the input section which contains the dependent design variables, independent design variables and membership function values.

EXCOST.DAT

This is the name of the input file.

Please enter regression analysis output file

Prompt the user to enter output file that contains the regression matrix, cost function coefficient, quantification of error of linear regression and the final cost function.

REGEX.OUT

This file is the regression output file for the example.

Please enter probabilities and membership functions output file.

Prompts the user to enter the output file that contains cost function, table that contains cost, number of observations of that cost, probability mass function, and corresponding membership function and cost fuzzy probability.

This program will generate three plots. They are: probability mass function (PMF), cost membership function and cost fuzzy probability. All of three plots will also be in this output file.

CMEX.OUT

This is the name of the example output entered by the user.

Please enter data output file.

Prompts the user to input an output file, which contains the input data, for review.

INEX.OUT

This is the output file.

Plot option.

Pleas e enter (1) if plot is needed.
(2) if plot is not needed.

As mentioned earlier, this program will generate three plots, but these plots are optional. So the program will prompt the user to choose between plotting or not. If (2) is selected, the program will stop. But if (1) is selected, then the following message will appear:

Line max length option.

Please enter (1) to use default (max. L = 40)
(2) to use new value for max. L

This option is to chose the maximum line length in the plot. If (1) is selected, the program will stop and the maximum line in the plot will be 40 and the other lines will be scaled accordingly. But if (2) is chosen, the program will ask for a value.

Please enter line max. length.

30.

30 will be the maximum length in the plots and as in (1) the other values will be scaled accordingly.

Table 4: Inpt Data File Format

A-	1-	Number of Variables						
B-	1-	Number of Data Points						
C-	1-	C_1	V_{11}	V_{12}	V_{13}	......	V_{1m}
	2-		μ_{11}	μ_{12}	μ_{13}	......	μ_{1m}
	3-	C_2	V_{21}	V_{22}	V_{23}	......	V_{2m}
	4-		μ_{21}	μ_{22}	μ_{23}	......	μ_{2m}
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	$2n-1$-	C_n	V_{n1}	V_{n2}	V_{n3}	......	V_{nm}
	$2n$-		μ_{n1}	μ_{n2}	μ_{n3}	......	μ_{nm}

The three computer output files are shown in the next section.

4 Output

The output is contained in three files, as explained in the previous section. Computer output of these three files is added to this manual for illustration.

OUTPUT FILES

Rgression Analysis Output File

EXAMPLE PROBLEM

```
********************************************************************************

                                    OUTPUT

********************************************************************************

          10.000000      48.700001      39.000000      21.900000      54.50000
4        559.500000
          48.700001     240.669998     191.790009     104.940002     267.27002
0       2782.549805
          39.000000     191.790009     156.540009      85.540009     214.30000
3       2245.700195
          21.900000     104.940002      85.540009      51.090000     115.41000
4       1248.500000
          54.500004     267.270020     214.300003     115.410004     304.37002
6       3116.800049
================================================================================

                          COST FUNCTION COEFFICIENTS

================================================================================

    a 0= -3.817445

    a 1=  2.330793

    a 2=  2.661444

    a 3=  7.042204

    a 4=  4.332922

================================================================================

                   QUANTIFICATION OF ERROR OF LINEAR REGRESSION

================================================================================

THE TOTAL OF SUM OF SQUARES OF RESIDUALS St= 32592.250
THE SUM ODF SQUARES OF TH RESIDUALS        Sr=     6.889
THE STANDER ERROR OF ESTIMATE            Sx/y=     1.174
THE COEFFICIENT OF DETERMINATION          r^2=0.9998
THE CORRELATION COEFFICIENT                 r=0.9999



================================================================================
================================================================================

                           THE FINAL COST FUNCTION

================================================================================
================================================================================

Y=  -3.81745+(    2.33079) X 1+(    2.66144) X 2+(    7.04220) X 3+(    4.33292) X
4+(
```

Membership Function and Fuzzy Probability Output File

Membership Function and Fuzzy Probability
Output File

EXAMPLE PROBLEM

==
==

THE COST FUNCTION

==
==

Y= -3.81745+(2.33079) X 1+(2.66144) X 2+(7.04220) X 3+(4.33292) X 4

THE COSTS AND THEIR CORRESPONDING MEMBERSHIP FUNCTIONS

COST	MEMBERSHIP VALUES
50	0.00000
53	0.20000
54	0.20000
55	0.20000
57	0.23300
57	0.15000
58	0.00000
60	0.40000
61	0.35000
64	0.10000

THE FINAL RESULTS

COST	No. OF OBS.	PMF	MEMBERSHIP VALUES
50	1	.100000	.000000
53	1	.100000	.200000
54	1	.100000	.200000
55	1	.100000	.200000
57	2	.200000	.233000
58	1	.100000	.000000
60	1	.100000	.400000
61	1	.100000	.350000
64	1	.100000	.100000

```
                              PMF

                     .040      .080      .120      .160      .200
                   |--------|--------|--------|--------|--------|------
                50 |---------------------
                53 |---------------------
                54 |---------------------
cost            55 |---------------------
                57 |-------------------------------------------
                58 |---------------------
                60 |---------------------
                61 |---------------------
                64 |---------------------
```

```
                              Membership Function

                        .080      .160      .240      .320      .400
                  |--------|--------|--------|--------|--------|------
               50 |
               53 |---------------------
               54 |---------------------
cost           55 |---------------------
               57 |------------------------
               58 |
               60 |------------------------------------------
               61 |-------------------------------------
               64 |----------
```

```
                                     Fuzzy Probabilities

                              .048    .095    .143    .190    .238
                           |-------|-------|-------|-------|-------|------
                        50 |
                        53 |--------------------
                        54 |--------------------
    cost                55 |--------------------
                        57 |-----------------------
                        58 |
                        60 |----------------------------------------
                        61 |-----------------------------------
                        64 |----------
```