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Multi-Mode Random Response Analysis Procedure

Prepared by

P. BROUSSINOS and A. M. KABE
Engineering Group

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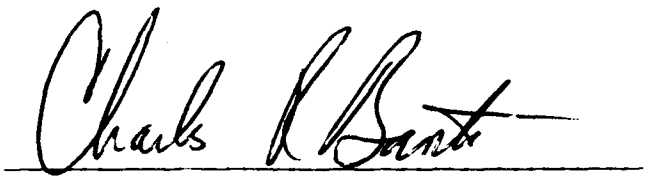

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A frequency domain procedure for solving large-scale, multi-mode, multi-force, random vibration problems is described. The study was motivated by the need to perform transonic buffet load analyses for the Delta II/Global Positioning System (GPS) vehicle, for which wind-tunnel tests had been performed to collect fluctuating pressure data. The procedure can be applied to random response problems which are described by a general set of parameters. These include random support-excitation ("base-shake") problems typical of component test situations. The main features of the procedure and the corresponding computer program, when compared to traditional procedures, are that, by taking advantage of modern computational capabilities, information such as frequency-dependent power spectral densities and cross-power densities can be used. To establish confidence in the implementation of the procedure, results from test problems were compared to values obtained by directly integrating the equations of motion in the time domain. These test problems, one of which was the Delta II/GPS, also demonstrate the feasibility of performing a buffet loads analysis in the time domain.

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CONTENTS

I. INTRODUCTION 5

II. RESPONSE TO RANDOM EXCITATION - A REVIEW 7

III. NUMERICAL IMPLEMENTATION 13

IV. TEST PROBLEMS 17

V. PRACTICAL CONSIDERATIONS 27

VI. SUMMARY/CONCLUSIONS 29

REFERENCES 31

NOMENCLATURE 33

APPENDICES:

A. Formulation of Equations of Motion for Base Excitation 37

B. Computer Program User's Guide 39

C. MATRIX Tape Format 53

D. Random Response Computer Program 55

FIGURES

1. Test Model Configuration 18

2. Pressure Transducer Distribution on Launch Vehicle Wind-Tunnel Model 20

3. Forcing Function Distribution 21

4. Typical Force Time History, Power Spectral Density, and Cross-Power Spectral Density 23

5. Typical Response Power Spectral Densities 24

TABLES

1. Test Problem Results 25

2. Typical Spacecraft Response Values 25

I. INTRODUCTION

There exists a large class of vibration problems for which the excitations, or forcing functions, are random in nature and, therefore, are described statistically. Typical problems include launch vehicle buffeting during the transonic time of flight, random base-shake excitation of structures or components during qualification and/or acceptance testing, and spacecraft solar panel response analysis for acoustic excitation. The elements which generally describe this class of problem are:

a. Multi-degree-of-freedom dynamic models, resulting in one or more normal modes required in any analysis.

b. Multiple, random forces acting on the system, with the forces described by frequency-dependent power spectral density functions (PSDs).

c. Some degree of correlation between the forces, with resulting force cross-correlations that are nonzero.

Traditional analysis procedures (e.g., Refs. 1-7) have employed simplifying assumptions about one or more of the above problem components. Typical assumptions have resulted in analyses which: (1) treat the system one mode at a time, thus ignoring phasing between the different modes, and/or (2) deal with only one force at a time, thus not including information about force phasing, and/or (3) ignore the frequency-dependence of the force spectra by treating them as constant throughout certain frequency bands. The primary reason for making these assumptions, it seems, has been limitations in computational capabilities. With the advent of larger and faster supercomputers, however, the development of a procedure for solving the aforementioned class of problems, without making the traditional simplifying assumptions, became possible.

The development presented herein is for linear systems subjected to forcing environments made up of ergodic, random forces, which are represented either as frequency-dependent auto- and cross-power spectral densities, force time histories, or a combination thereof. The analysis procedure is developed in a standard matrix formulation, and is in essence a combination of matrix

structural analysis methods and Gaussian random response theory. Several example problems will illustrate typical applications of the procedure, as well as provide comparisons with results from direct numerical solution of the equations of motion. Development of the procedure, and the accompanying trial runs with the example problems, indicated which problem parameters need to be carefully chosen to maximize accuracy as well as computational efficiency.

II. RESPONSE TO RANDOM EXCITATION - A REVIEW

The class of problems we are considering involves excitations that are random in nature and, therefore, must be treated by statistical means. These excitations are nearly ergodic and, therefore, can be described by their frequency-dependent auto- and cross-power spectral density functions (PSD and cross-PSD respectively). This makes it possible to solve for the structural response in the frequency domain.

The behavior of linear elastic structures, which include most launch vehicles and spacecraft, can be described by the matrix differential equation of motion

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \tag{1}$$

In the above equation [M], [C], and [K] are, respectively, the mass, damping, and stiffness matrices, {x(t)} is the displacement vector (dots denote differentiation with respect to time), and {F(t)} is the vector of externally applied forces.

Transforming to normal coordinates we obtain

$$[I]\{\ddot{q}(t)\} + [2\zeta_n\omega_n]\{\dot{q}(t)\} + [\omega_n^2]\{q(t)\} = [\phi]^T\{F(t)\} \tag{2}$$

where

$$\{x(t)\} = [\phi]\{q(t)\} \tag{3}$$

and [ϕ] is the matrix of mode shape vectors and {q(t)} is the vector of modal displacements. In the above equations, we have assumed uncoupled modal damping, and the mode shape vectors have been normalized with respect to the mass matrix such that

$$[\phi]^T[M][\phi] = [I] \tag{4}$$

By taking the Fourier transform of each term in Eq. (2) and using the standard relationships for Fourier transforms of derivatives we obtain

$$[(\omega_n^2 - \omega^2) + i(2\zeta_n\omega_n\omega)]\ \{q(\omega)\} = \{f(\omega)\} \tag{5}$$

In the above equation, $\{q(\omega)\}$ and $\{f(\omega)\}$ are the Fourier transforms of $\{q(t)\}$ and $[\phi]^T\{F(t)\}$, respectively. Solving for $\{q(\omega)\}$, we obtain

$$\{q(\omega)\} = [H(\omega)]\{f(\omega)\} \tag{6}$$

where

$$[H(\omega)] = [(\omega_n^2 - \omega^2) + i(2\zeta_n\omega_n\omega)]^{-1} \tag{7}$$

$[H(\omega)]$ is the structure's modal frequency response (or admittance) function. This matrix is diagonal with a typical term being

$$H_k(\omega) = \frac{1}{\omega_k^2 - \omega^2 + i2\zeta_k\omega_k\omega} \tag{8}$$

The physical response correlation matrix is given by

$$[R_x(\tau)] = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \{x(t)\}\{x(t+\tau)\}^T dt \tag{9}$$

Substituting $\{x(t)\} = [\phi]\{q(t)\}$ yields

$$[R_x(\tau)] = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [\phi]\{q(t)\}\{q(t+\tau)\}^T[\phi]^T dt \tag{10}$$

$$= [\phi]\left[\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \{q(t)\}\{q(t+\tau)\}^T dt\right][\phi]^T \tag{11}$$

$$= [\phi][R_q(\tau)][\phi]^T$$

where $[R_q(\tau)]$ is the modal response correlation matrix.

It is shown in Ref. 8 that the auto- and cross-spectral density functions of the responses, $[S_q(\omega)]$, are related to the auto- and cross-spectral density functions of the excitations, $[S_f(\omega)]$, by

$$[S_q(\omega)] = [H^*(\omega)][S_f(\omega)][H(\omega)] \tag{12}$$

In Eq. (12), $[H^*(\omega)]$ is the complex conjugate of $[H(\omega)]$, and $[S_q(\omega)]$ and $[S_f(\omega)]$ are the auto- and cross-power spectral density functions of the modal responses and modal forces, respectively. Since the Fourier transforms of the auto- and cross-correlation functions are the auto- and cross-power spectral density functions, respectively, the inverse Fourier transform of Eq. (12) is given by

$$[R_q(\tau)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} [H^*(\omega)][S_f(\omega)][H(\omega)]e^{i\omega\tau}\, d\omega \tag{13}$$

To obtain $[S_f(\omega)]$ we begin with the modal force correlation matrix given by

$$[R_f(\tau)] = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \{f(t)\}\{f(t+\tau)\}^T dt \tag{14}$$

Substituting $\{f(t)\} = [\phi]^T\{F(t)\}$ we obtain

$$[R_f(\tau)] = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [\phi]^T\{F(t)\}\{F(t+\tau)\}^T[\phi] dt \tag{15}$$

$$= [\phi]^T[\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \{F(t)\}\{F(t+\tau)\}^T dt]\ [\phi]$$

$$= [\phi]^T[R_F(\tau)][\phi] \tag{16}$$

where $[R_F(\tau)]$ is the physical force correlation matrix.

Since $[S_f(\omega)]$ is the Fourier transform of $[R_f(\tau)]$, we obtain

$$[S_f(\omega)] = \int_{-\infty}^{\infty} [\phi]^T [R_F(\tau)][\phi] e^{-i\omega\tau} d\tau$$

$$= [\phi]^T [S_F(\omega)][\phi] \tag{17}$$

where

$$[S_F(\omega)] = \int_{-\infty}^{\infty} [R_F(\tau)] e^{-i\omega\tau} d\tau \tag{18}$$

$[S_F(\omega)]$ is the physical force cross-PSD matrix. Substituting Eq. (17) into Eq. (13) yields

$$[R_q(\tau)] = \Phi^{-1}([H^*(\omega)][\phi]^T [S_F(\omega)][\phi][H(\omega)]) \tag{19}$$

where $\Phi^{-1}(\ )$ signifies the inverse Fourier transform of $(\ )$. Substituting Eq. (19) into Eq. (11) yields

$$[R_x(\tau)] = \Phi^{-1}([\phi][H^*(\omega)][\phi]^T [S_F(\omega)][\phi][H(\omega)][\phi]^T) \tag{20}$$

Taking the Fourier transform of both sides, we obtain

$$[S_x(\omega)] = [\phi][H^*(\omega)][\phi]^T [S_F(\omega)][\phi][H(\omega)][\phi]^T \tag{21}$$

This equation relates the physical response cross-PSD matrix to the physical force cross-PSD matrix.

The mean square values of the responses are derived by setting $\tau = 0$ in Eq. (9). Recalling that $[R_x(\tau)]$ and $[S_x(\omega)]$ are a Fourier transform pair, and considering only the diagonal elements of $[S_x(\omega)]$, we obtain

$$\overline{x_\ell^2} = \int_{-\infty}^{\infty} S_{\ell\ell}(\omega)\, d\omega \tag{22}$$

where $S_{\ell\ell}(\omega)$ signifies the ℓ-th diagonal element of $[S_x(\omega)]$.

III. NUMERICAL IMPLEMENTATION

For numerical (computer) implementation, we must deal with the real and imaginary elements of Eq. (21) separately. Thus, we factor the complex matrices into their coincident (real) and quadrature (imaginary) components as follows:

$$[S_F(\omega)] = [C_F(\omega)] + i[Q_F(\omega)] \tag{23}$$

and

$$[H(\omega)] = [D(\omega)] - i[E(\omega)] \tag{24}$$

where

$$D_k(\omega) = \frac{\omega_k^2 - \omega^2}{(\omega_k^2 - \omega^2)^2 + (2\zeta_k\omega_k\omega)^2} \tag{25}$$

and

$$E_k(\omega) = \frac{2\zeta_k\omega_k\omega}{(\omega_k^2 - \omega^2)^2 + (2\zeta_k\omega_k\omega)^2} \tag{26}$$

are the k-th elements along the diagonals of $[D(\omega)]$ and $[E(\omega)]$, respectively. Thus, Eq. (21) can be written as

$$[S_x(\omega)] = [\phi]\{[D(\omega)] + i[E(\omega)]\}[\phi]^T\{[C_F(\omega)] + i[Q_F(\omega)]\}[\phi]\{[D(\omega)] - i[E(\omega)]\}[\phi]^T \tag{27}$$

At this point, for convenience, we drop the matrix brackets, as well as the (ω), and keep in mind that all matrices except $[\phi]$ are frequency-dependent.

Thus, performing the required algebra, Eq. (27) becomes

$$S_x = [RC_FR + RQ_FT - TQ_FR + TC_FT] + i[RQ_FR - RC_FT + TC_FR + TQ_FT] \tag{28}$$

where

$$R = \phi D \phi^T \tag{29}$$

and

$$T = \phi E \phi^T \tag{30}$$

Equation (28) can be written as

$$[S_x(\omega)] = [C_x(\omega)] + i[Q_x(\omega)] \tag{31}$$

where $[C_x(\omega)]$ and $[Q_x(\omega)]$ are defined in Eq. (28) and we have reintroduced the matrix brackets.

The diagonal elements of $[C_x(\omega)]$ are the auto-spectra of the displacements. The cross-spectral information is contained in the off-diagonal elements of $[C_x(\omega)]$ and $[Q_x(\omega)]$ (the diagonal elements of $[Q_x(\omega)]$ are identically zero). Thus, mean square values of the responses of interest can be derived from the diagonal elements of $[C_x(\omega)]$.

The above development leads to displacement response power spectral densities. Typical applications can call for the derivation of related parameters such as accelerations, acceleration-based loads, or combinations thereof. The power spectral densities of the system accelerations can be computed by recalling the relations (Ref. 9)

$$\Phi(\dot{q}(t)) = i\omega\Phi(q(t)) \tag{32}$$

and

$$\Phi(\ddot{q}(t)) = -\omega^2\Phi(q(t)) \tag{33}$$

These allow us to write the acceleration-based frequency response functions as

$$[HA(\omega)] = -\omega^2[H(\omega)] \tag{34}$$

Equations (6) and (34) relate the Fourier transforms of the modal accelerations to the Fourier transforms of the modal forces, i.e.

$$\{\ddot{q}(\omega)\} = [HA(\omega)]\{f(\omega)\} \tag{35}$$

If power spectral densities of loads are required, and a loads transformation matrix, [LTM], which recovers the loads according to

$$\{L(t)\} = [LTM]\ \{\ddot{x}(t)\} \tag{36}$$

is available, then Eq. (21) becomes

$$[S_L(\omega)] = [LTM][\phi][HA^*(\omega)][\phi]^T[S_F(\omega)][\phi][HA(\omega)][\phi]^T[LTM]^T \tag{37}$$

or

$$[S_L(\omega)] = [LTM][S_{\ddot{x}}(\omega)][LTM]^T$$

Another common application is the "base-shake" vibration problem. The extension to this type of configuration is straightforward and is presented in Appendix A.

Typically, the number of forces, N_F, applied to a system, and the number of modes retained in the analysis, N_M, are significantly less than the number of degrees of freedom in the system. Using these facts when solving Eq. (21) can result in a significant increase in computational efficiency. This topic will be discussed at length later, when use of the computer program is described.

It is also noted that, although the preceding development deals with power spectra defined over both positive and negative frequency ranges, practical applications are limited to the use of spectra defined over a positive frequency range only. The procedure applies directly in these cases, as long as consistency is maintained throughout.

IV. TEST PROBLEMS

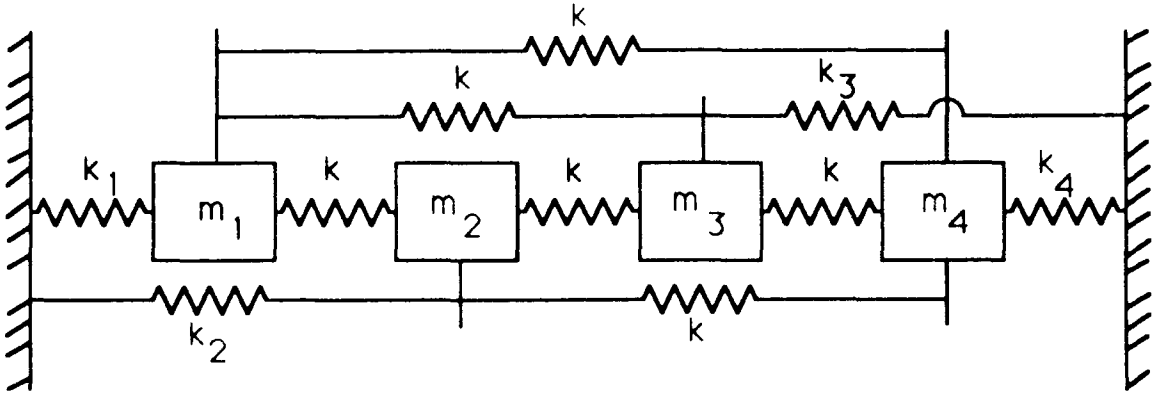
To establish confidence in the numerical implementation of the random response analysis procedure, several test problems were solved. The first test case consisted of a four-degree-of-freedom system (Fig. 1). The structure was subjected to identical random forces applied to the first two degrees of freedom. The force was derived from pressure measurements from a buffet wind tunnel test. The use of the cross-spectrum will provide some insight into the importance of using the cross-spectra in random response analyses. Furthermore, having access to the time histories of the forces that were used to produce the PSDs and cross-PSDs allowed for a direct comparison of the subject procedure against results derived in the time domain.

The time domain analysis results were obtained by integrating the equations of motion directly with a fourth-order Runga-Kutta procedure. The integration step was 0.001 sec, and the record length was 30 sec. The mean square values of the responses were then established as follows:

$$\overline{x_\ell^2} = \frac{1}{T} \int_0^T x^2(t)\, dt \tag{38}$$

where $x(t)$ is the calculated response time history, $\overline{x_\ell^2}$ is the mean square value and $T = 30$ sec. The frequency domain analysis used a frequency increment of 0.5 Hz, and the analysis range was from 0.0 to 200 Hz. The frequency increment was selected to yield at least four spectral lines between the half-power points of each mode.

Table 1 lists the root-mean-square (RMS) displacement values derived with three different methods: (1) direct numerical integration in the time domain, (2) random response procedure with cross-spectra of forces included, and (3) random response procedure without cross-spectra of forces; i.e., only the auto-spectra (diagonal elements of $[S_F(\omega)]$) were used. The results indicate that the responses derived with the frequency domain random response procedure are in good agreement with those from the time domain analysis, when the cross-spectrum of the forces was included in the analysis. In the case



$k_1 = 72000$

$k_2 = 108000$

$k_3 = 180000$

$k_4 = 144000$

$k = 36000$

$m_i = 1$

$\zeta_i = 0.02 \quad i = 1,2,3,4$

$f_1 = 53.99$ Hz

$f_2 = 76.94$ Hz

$f_3 = 83.29$ Hz

$f_4 = 89.09$ Hz

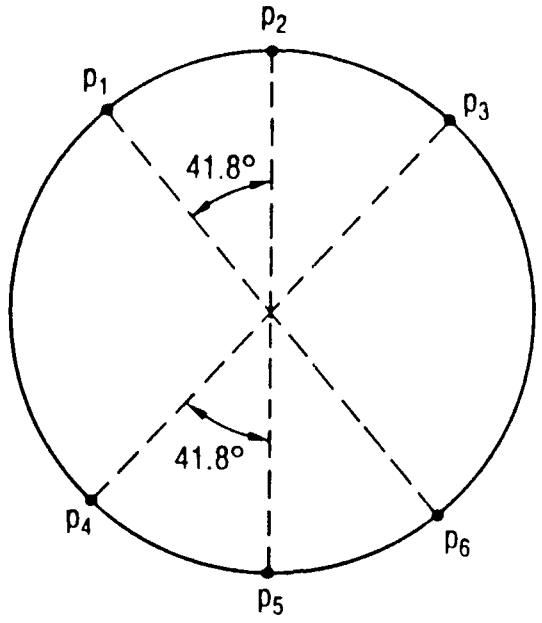
Figure 1. Test Model Configuration

where only the auto-spectra of the forces were used, however, we see that the responses differ significantly—in this case they are lower by up to 27%. In other cases, the results derived by using only the force auto-spectra over-predicted the time domain results (see test problem 2). In any case, the degree of discrepancy between responses derived using cross-spectra and responses derived without using the cross-spectra will depend on the degree of correlation between the various forces.

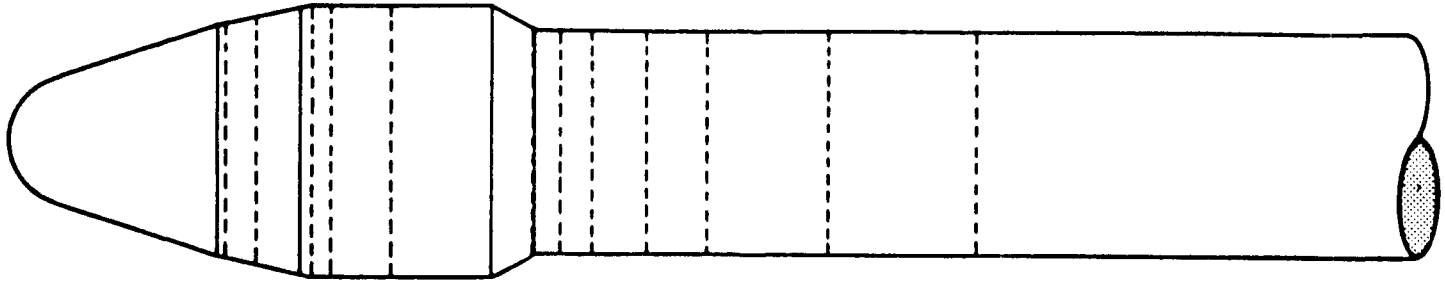
For the second test problem the dynamic model consisted of the GPS spacecraft coupled to the Delta II (6925) launch vehicle. Input forcing functions were formulated from pressure measurements collected in a buffet wind tunnel test which was performed using a rigid scale model of the launch vehicle. The test was performed at the Arnold Engineering Development Center in the 16T Propulsion Wind Tunnel. The scale model of the launch vehicle was designed and manufactured by McDonnell Douglas Astronautics Company under contract to the Air Force Space Systems Division. In the test, fluctuating pressure measurements were taken using transducers which were located at various stations along the launch vehicle model. These transducers were distributed circumferentially as well (see Fig. 2). The forcing functions were derived from these measured pressure readings by assigning an area to each transducer (depending on its location) and by accounting for the effect of model scale, which affects both the amplitude and frequency contents.

The coupled system dynamic model had 47 normal modes to 20 Hz*, of which 5 were rigid-body modes (torsional rigid-body motion was not modeled). The forces were applied to a portion of the length of the vehicle (see Fig. 3), and responses were recovered at all degrees of freedom defining the spacecraft dynamic model. For illustrative and comparative purposes, we will concern ourselves here with spacecraft acceleration results, although it is noted that the subject procedure was used, for this particular system, to recover spacecraft loads, relative displacements (clearance loss), and launch vehicle

*Delta II and Titan IV flight data obtained since these analyses were performed indicate that significant excitation above 20 Hz exists. Current analysis cutoff frequencies are near 40 Hz.

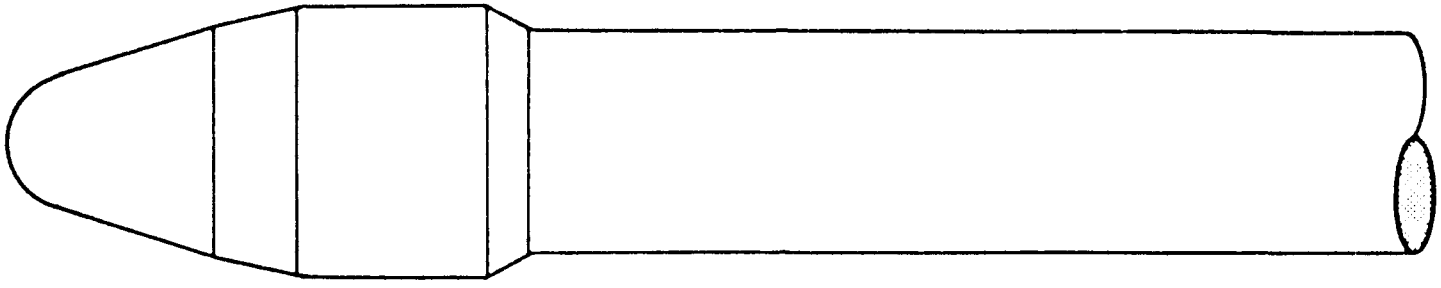


a. TYPICAL CROSS-SECTION OF LAUNCH VEHICLE WIND-TUNNEL MODEL SHOWING TRANSDUCER LOCATIONS IN RING PATTERN

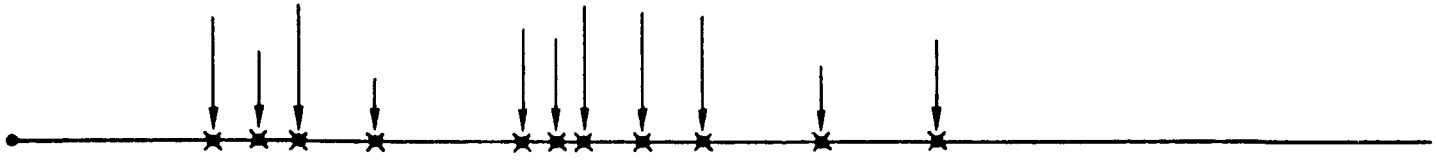


b. TRANSDUCER RING LOCATIONS ALONG FORWARD PORTION OF LAUNCH VEHICLE MODEL (indicated by dashed lines)

Figure 2. Pressure Transducer Distribution on Launch Vehicle Wind-Tunnel Model



a. CONFIGURATION OF LAUNCH VEHICLE EXTERIOR



b. DYNAMIC MODEL CENTERLINE REPRESENTATION SHOWING FORCE DISTRIBUTION

↓ PITCH PLANE FORCES

× YAW PLANE FORCES

Figure 3. Forcing Function Distribution

accelerations and loads as well. It is further noted that, although the total forcing environment applied to the coupled system was formulated from a combination of wind-tunnel-derived forces (time histories) and flight-data-based forces (power spectra), the random response analysis described herein was performed using the wind-tunnel-derived force spectra alone so as to provide a comparison with results derived in the time domain using the force time histories directly.

Figures 4a, 4b, and 4c show a typical force time history, its power spectral density, and one of its cross-power spectral densities, respectively. Figure 5 shows typical response power spectral densities at three locations on the spacecraft, including primary structure and appendage degrees of freedom. Table 2 lists a comparison of typical resulting RMS values. The first column shows time domain results obtained with Runge-Kutta integration, column 2 shows frequency domain results obtained with the random response analysis procedure using selected force cross-spectra (i.e., cross-spectra resulting from forces that were adjacent to each other were included in the force PSD matrix, with the assumption being that correlation between forces separated by more than one vehicle station would have minor effect). Column 3 shows results derived with only the force auto-spectra. In this case results show that omission of the force cross-spectra will cause an overprediction of responses at some spacecraft locations, and a slight underprediction at other locations.

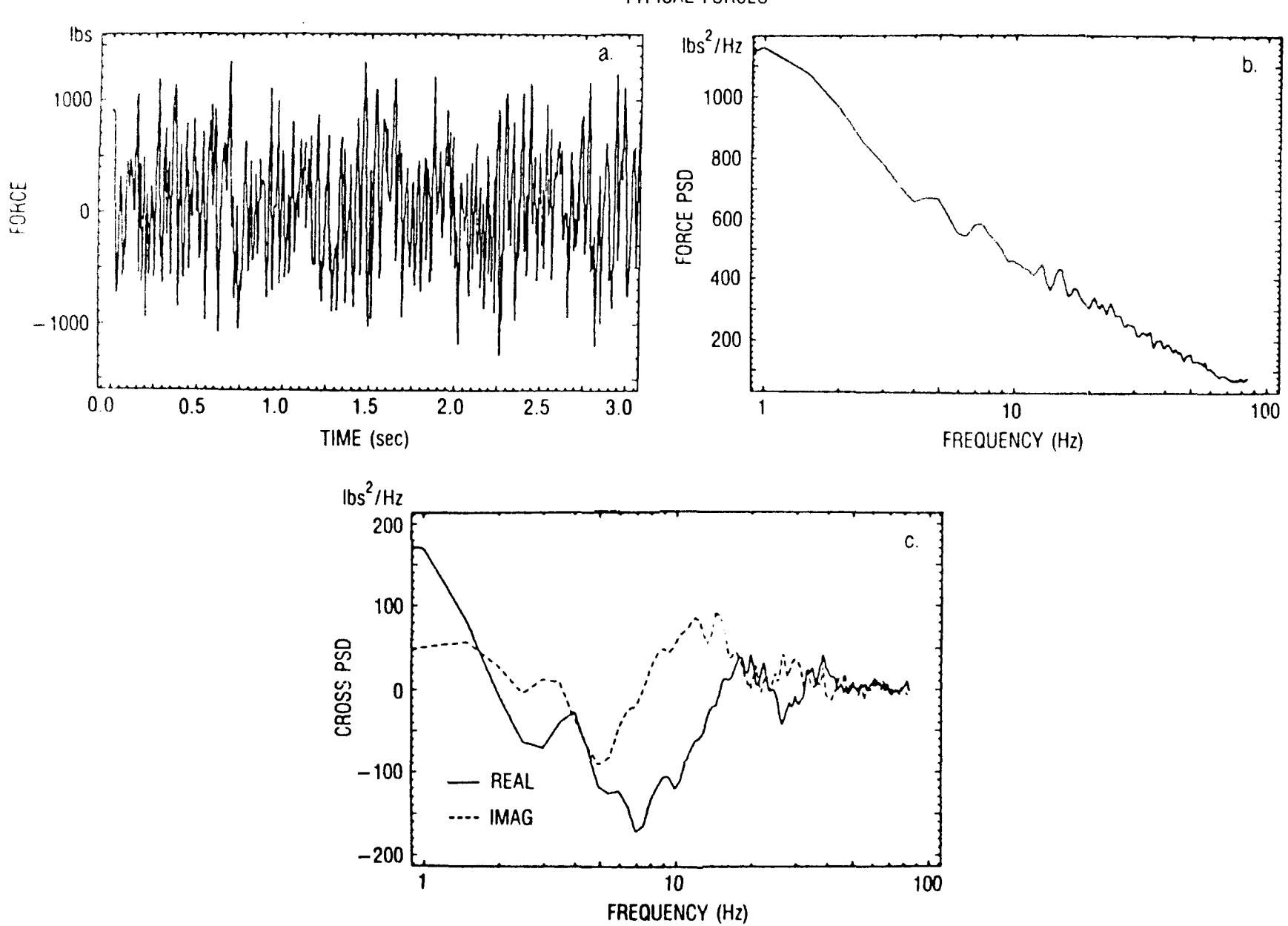


Figure 4. Typical Force Time History, Power Spectral Density and Cross-Power Spectral Density

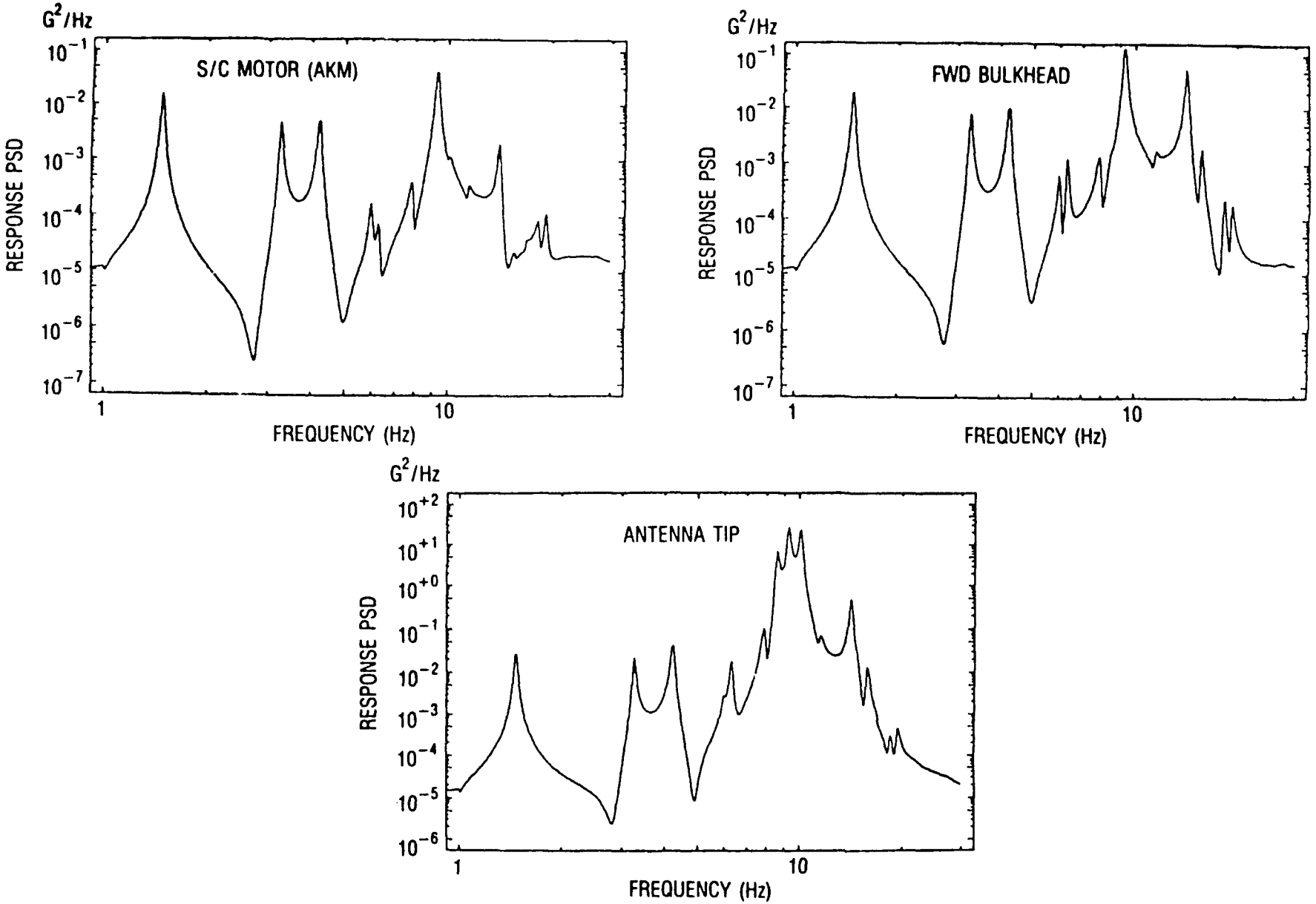


Figure 5. Typical Response Power Spectral Densities

Table 1. Test Problem Results

	R-K(1)	RRP(2)	RRP(3)
X_1	0.262	0.259	0.213
X_2	0.203	0.199	0.179
X_3	0.177	0.171	0.128
X_4	0.134	0.131	0.096

Note: Values are inches RMS.

(1)Time domain Runge-Kutta integration
(2)Frequency domain with force cross-spectra
(3)Frequency domain without force cross-spectra

Table 2. Typical Spacecraft Response Values

		R-K(1)	RRP(2)	RRP(3)
S/C Motor	X	0.133	0.134	0.137
	Y	0.145	0.157	0.162
Forward Bulkhead	X	0.268	0.276	0.280
	Y	0.286	0.308	0.316
Solar Array	X	0.253	0.258	0.233
	Y	0.139	0.149	0.166
Antenna Tip	X	4.65	4.59	4.20
	Y	3.97	3.98	3.94

Note: Values are G's RMS

(1)Time domain Runge-Kutta integration
(2)Frequency domain with selected force cross-spectra
(3)Frequency domain without force cross-spectra

V. PRACTICAL CONSIDERATIONS

Since the formulation and first application in late 1988 of the procedure presented herein, there have been several Delta II flights and two Titan IV flights. Data obtained from these flights and additional review of the wind tunnel data indicate that the buffet excitation energy content is still significant past the previously used analysis cutoff frequencies of 20 Hz. Studies with both vehicles and their payloads indicate that analysis cutoff frequencies in the 30 Hz to 40 Hz range are needed to obtain convergence of spacecraft responses. Convergence of launch vehicle loads is configuration dependent and typically occurs below 20 Hz. However, convergence studies should still be performed for each new vehicle configuration and its payload.

To assure an adequately conservative prediction of launch vehicle and spacecraft buffet loads, multiple Mach number and angle-of-attack cases need to be analyzed. This is particularly true for the transonic time of flight. It is not unusual to have different cases maximize loads in different areas of the launch vehicle and spacecraft. Therefore, this requires that wind tunnel data at multiple Mach numbers and angle-of-attack combinations be obtained and the appropriate analyses be performed.

An additional practical consideration is the analysis frequency increment. A frequency increment that is too coarse can result in a significant underprediction of loads. However, it is also possible that an overprediction of responses can occur. It has been found that adequately accurate predictions result if a frequency increment corresponding to four spectral lines between a mode's half-power points is used. That is:

$$\Delta f = \frac{1}{2} \zeta_n f_n$$

where Δf is the frequency increment, ζ_n is the critical damping ratio of the mode and f_n is the natural frequency of the mode.

As a final note, the work presented herein has introduced the concept and demonstrated the feasibility of performing buffet analyses in the time domain. This requires that the forcing functions be available in the form of time histories. This being the case, rather than converting the forcing functions into the frequency domain, one may integrate the equations of motion directly. The root-mean-square (RMS) values can then be computed from the response time histories. Work to date indicates that 20 to 30 seconds of response data are needed to establish RMS values that adequately characterize the statistics of buffet response. This is consistent with the data collected in both the Delta II and Titan IV wind tunnel tests. However, the required lengths of the time histories are a function of the frequency content of the time histories. Therefore, for each new configuration, convergence studies should be performed.

Several factors make the calculation of buffet response in the time domain an attractive proposition. First, the cost of calculating auto- and cross-power spectral densities is avoided. Second, numerical integration software packages that directly integrate the modal equations of motion are readily available. And finally, the calculation of loads is simplified since the time-phasing between the externally applied forces and the acceleration and displacement responses can be accounted for exactly and in a straightforward manner. Obviously, the choice of a time-domain or frequency-domain approach depends on the form of the forcing function representation and the respective costs. However, as demonstrated herein, both approaches are viable and each approach has its own advantages.

VI. SUMMARY/CONCLUSIONS

A multi-force, mode-superposition random response analysis procedure has been described. The procedure, which allows for the use of frequency-dependent force power spectral densities and cross-power spectral densities, is a combination of Gaussian random response theory and matrix structural analysis methods. It can be used to analyze a wide variety of random vibration problems which arise in the structural dynamics area. These include transonic buffet loads analyses for launch vehicle/spacecraft systems, and base excitation analyses typical of component vibration testing situations. Example problems, which help to establish confidence in the procedure as well as to demonstrate its use, were included. In addition, the concept of establishing buffet responses in the time domain was introduced and its feasibility was demonstrated on test problems, one of which was the Delta II/GPS system.

The accompanying FORTRAN computer program, as well as a guide describing its use, are included as a stand-alone appendix. The program has been written with sufficient flexibility so as to allow the user to adapt it to a wide variety of situations and needs; however, the main components of the code are written with an eye toward deriving maximum computational efficiency from various situations which often arise when treating the aforementioned types of problems.

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NOMENCLATURE

$[C]$ = damping matrix

$[C_F(\omega)]$ = real part of $[S_F(\omega)]$

$[C_x(\omega)]$ = real part of $[S_x(\omega)]$

$[D(\omega)]$ = real part of $[H(\omega)]$

$[E(\omega)]$ = imaginary part of $[H(\omega)]$

$\{f(t)\}$ = vector of modal forces

$\{F(t)\}$ = vector of physical forces

$[H(\omega)]$ = diagonal matrix of modal admittance functions for displacement

$[HA(\omega)]$ = diagonal matrix of modal admittance functions for acceleration

$[\hat{H}(\omega)]$ = matrix of physical admittance functions

$i = \sqrt{-1}$

$[I]$ = identity matrix

$[K]$ = stiffness matrix

$[M]$ = mass matrix

N_F = number of applied forces

N_M = number of modes retained in analysis

$\{q(t)\}$ = vector of modal displacements

$\{\dot{q}(t)\}$ = vector of modal velocities

$\{\ddot{q}(t)\}$ = vector of modal accelerations

$[Q_F(\omega)]$ = imaginary part of $[S_F(\omega)]$

$[Q_x(\omega)]$ = imaginary part of $[S_x(\omega)]$

$[R_f(\tau)]$ = correlation matrix of vector $\{f(t)\}$

$[R_F(\tau)]$ = correlation matrix of vector $\{F(t)\}$

$[R_q(\tau)]$ = correlation matrix of vector $\{q(t)\}$

$[R_x(\tau)]$ = correlation matrix of vector $\{x(t)\}$

$[S_F(\omega)]$ = cross-power spectral density matrix of physical forces

$[S_x(\omega)]$ = cross-power spectral density matrix of physical displacements

$\overline{x_\ell^2}$ = mean square value of x_ℓ

$\{x(t)\}$ = vector of physical displacements

$\{\ddot{x}(t)\}$ = vector of physical accelerations

ζ = critical damping ratio

σ^2 = variance (square of standard deviation)

$[\phi]$ = matrix of normal modes

$\{\phi^{(k)}\}$ = k-th normal mode

$[\phi_F]$ = mode shape matrix for force application degrees of freedom

$\Phi(\ )$ = Fourier transform of ( )

$\Phi^{-1}(\ )$ = inverse Fourier transform of ( )

ω = analysis frequency (rad/sec)

ω_n = natural frequency (rad/sec)

$\{\ \}^*$ = complex conjugate of { }

APPENDIX A

FORMULATION OF EQUATIONS OF MOTION FOR BASE EXCITATION

For base excitation where no relative motion between the support points is allowed, the equations of motion for a linear elastic structure are

$$[M]\{\ddot{x}_{ABS}\} + [C]\{\dot{x}_{REL}\} + [K]\{x_{REL}\} = \{0\} \qquad \text{(A-1)}$$

where

$\ddot{x}_{ABS}$ = absolute acceleration vector

$\dot{x}_{REL}$ = relative velocity

x_{REL} = relative displacement

and [M], [C], and [K] are the mass, damping, and stiffness matrices, respectively Substituting

$$\{\ddot{x}_{ABS}\} = \{\ddot{x}_{REL}\} + [L_{RB}]\{\ddot{x}_B\} \qquad \text{(A-2)}$$

yields

$$[M]\{\ddot{x}_{REL}\} + [C]\{\dot{x}_{REL}\} + [K]\{x_{REL}\} = -[M][L_{RB}]\{\ddot{x}_B\} \qquad \text{(A}$$

where

$\ddot{x}_B$ = base acceleration

and

L_{RB} = rigid body vectors referenced to base of structure

Using the base-fixed modes, $[\phi]$, to transform to normal coordinates, we obtain

$$[I]\{\ddot{q}\} + [2\zeta_n\omega_n]\{\dot{q}\} + [\omega_n^2]\{q\} = -[\Gamma]\{\ddot{x}_B\} \qquad \text{(A-4)}$$

where

$$\{x_{REL}\} = [\phi]\{q\} \tag{A-5}$$

$$[\Gamma] = [\phi]^T[M][L_{RB}] \tag{A-6}$$

and $[\phi]$ has been normalized such that

$$[\phi]^T[M][\phi] = [I] \tag{A-7}$$

To enable solution using the random response program, the base excitation vector is treated as part of a set of mixed coordinates, i.e.

$$\left[\begin{array}{c|c} I & \\ \hline & I \end{array}\right]\left\{\begin{array}{c}\ddot{x}_B \\ \ddot{q}\end{array}\right\} + \left[\begin{array}{c|c} 0 & \\ \hline & 2\zeta_n\omega_n \end{array}\right]\left\{\begin{array}{c}\dot{x}_B \\ \dot{q}\end{array}\right\} + \left[\begin{array}{c|c} 0 & \\ \hline & \omega_n^2 \end{array}\right]\left\{\begin{array}{c}x_B \\ q\end{array}\right\} = \left[\begin{array}{c} I \\ \hline -\Gamma \end{array}\right]\{\ddot{x}_B\} \tag{A-8}$$

Using Eq. (A-2), we can recover the absolute accelerations in terms of the response quantities from Eq. (A-8) as follows:

$$\{\ddot{x}_{ABS}\} = \left[\begin{array}{c|c} L_{RB} & \phi \end{array}\right]\left\{\begin{array}{c}\ddot{x}_B \\ \ddot{q}\end{array}\right\} = [ATM]\left\{\begin{array}{c}\ddot{x}_B \\ \ddot{q}\end{array}\right\} \tag{A-9}$$

Loads can now be recovered with an acceleration based loads transformation matrix, [LTM], as follows:

$$\{\text{Loads}\} = [LTM]\ [ATM]\left\{\begin{array}{c}\ddot{x}_B \\ \ddot{q}\end{array}\right\} \tag{A-10}$$

To recover statically-determinate base reaction forces, a force transformation matrix, [FTM], can be used, i.e.

$$[FTM] = [L_{RB}]^T[M]\ [ATM]$$

To assemble the complete recovery matrix, the separate recovery matrices can be augmented rowwise.

APPENDIX B

COMPUTER PROGRAM USER'S GUIDE

1. BASIC PROGRAM DESCRIPTION

The random response procedure (RRP) computer program was written in FORTRAN 77 and was designed to run on CRAY X/MP or CDC mainframes. The code consists of a main program, which does the bulk of the numerical manipulation, and two subroutines—one which assembles the force PSD matrices at each analysis frequency, and one which assembles the frequency response function matrices at each analysis frequency. The program accepts input model and forcing spectra data in a certain format, and solves the random response equations described in Sections II and III to produce output in the form of response power spectral density functions and root mean square (RMS) values.

2. INPUT DATA

A. General

The input data formats for the program were designed to be consistent with other programs and subroutine libraries available and in use at The Aerospace Corporation. These formats, although possibly in use only at Aerospace, are straightforward and should pose no problems for the analyst.

B. Dynamic Model

The model data requirements are as follows:

(i) $[\phi_F]^T$ — Force transformation matrix consisting of mode shape vectors transposed and collapsed to the degrees of freedom where forces are applied.

Size = NMODES x NFORCE

(ii) $[2\zeta_n\omega_n]$ — Diagonal modal damping matrix.

Size = NMODES x NMODES

(iii) $[\omega_n^2]$ Diagonal modal stiffness matrix.

Size = NMODES x NMODES

(iv) [TA] Recovery transformation matrix. Recovers parameters of interest (loads, physical accelerations, etc.) when post-multiplied by q or $\ddot{q}$. For example, if accelerations are desired, [TA] = [ϕ]; if loads are desired, [TA] = [LTM][ϕ].

Size = NLOAD x NMODES

The ordering of the modes should be from lowest natural frequency to highest. Therefore, the rigid-body modes should be the first in the modal matrices. The matrices should be on one permanent file or tape, specified locally as "unit 32" in the Job Control Language (JCL), in the order listed above, and with end-of-file markers between the individual matrices. The matrices must be written in that file in Aerospace Corporation "MATRIX" format (see Appendix C).

C. FORCING FUNCTIONS

The forcing functions must be available as force power spectral densities and cross-power spectral densities. These inputs should be on a tape or permanent file specified locally as "unit 33" in the JCL. The default read option assumes (for the case where cross-PSDs are used) that the cross-PSDs are written on the same file as the auto-PSDs. This can easily be altered by the user if he or she wishes to use auto-PSDs and cross-PSDs that are written on separate files; however, the frequency spacing of the two files must be consistent. The frequency band over which the PSDs are defined is arbitrary (but should, naturally, be kept in mind when the analysis bands are determined). The default force PSD interpolation option assumes that the PSDs are defined on a linear scale, however, this can be changed to accommodate input taken directly from log-log plots, if need be.

The cross-PSDs read from the input unit are mapped to the appropriate arrays representing the real and imaginary components of the S_F matrix, denoted "C_F" and "Q_F" in Equation (23) of the main text, and denoted simply as "C" and "Q" in the program. This mapping is accomplished in the "PSD"

subroutine through the use of the arrays IROW and ICOL, which establish a correspondence between the order that the cross-PSD data is written on the input tape and the appropriate locations in the "C" and "Q" arrays.

The operation of the "PSD" subroutine, at each analysis frequency, is as follows:

i) The main program passes the value of the analysis frequency to the subroutine.

ii) The subroutine determines whether or not the analysis frequency lies between the two previously defined interpolation limit frequencies. Based on this determination, either iiia or iiib is executed.

iiia) If the analysis frequency is within the limit frequencies, the input spectra at those frequencies are interpolated to the analysis frequency.

iiib) If the analysis frequency is greater than the previously defined upper limit, the next record of input spectra is read from the input tape, new lower and upper interpolation limit frequencies are defined, and the input spectra are interpolated to the analysis frequency.

iv) Using the resulting interpolated input spectra, arrays "C" and "Q" are assembled through the use of the mapping arrays IROW and ICOL.

v) The arrays "C" and "Q" are passed back to the main program.

3. OUTPUT SPECIFICATION

The output from the program consists of:

a. Power spectral density functions of the responses defined by the rows of the recovery matrix [TA].

b. Response RMS values derived from the response power spectral densities.

The output is written to a tape or permanent file specified locally as "unit 34" using a binary, unformatted WRITE statement. Thus, for each analysis frame defined by frequency F, the statement

```
WRITE (34) F, (R(M), M=1, NLOAD)
```

is executed. After the PSD functions have been written to the output file, an End-of-File marker is written. Following the EOF marker, the RMS values are written to a single record with the statement

```
WRITE (34) (S(J), J=1, NLOAD)
```

The RMS file is followed by another EOF marker.

The printed output consists of:

i) A summary of the modal frequencies and damping values defining the input dynamic model.

ii) The PSD values for the first 10 responses (this can be altered by the user) at each analysis frequency.

iii) A summary of the RMS values for all the responses.

PROGRAM NOMENCLATURE

1. USER-SET PARAMETERS

A. Integer

NMODES = total number of normal modes in dynamic model

NRIG = number of rigid body modes

NFORCE = number of applied forces

NCROSS = total number of independent force cross-PSDs

NFREQ = total number of spectral lines in analysis

NLOAD = number of recovered responses (equal to the number of rows in [TA])

NPRINT = number of response PSDs to be printed on output (default value = 10)

B. Floating Point

XMUF = model uncertainty factor (default = 1.0)

FZERO = frequency defining the first spectral line in analysis

F1 = cutoff frequency of first band

F2 = cutoff frequency of second band

.

.

F5 = cutoff frequency of fifth band(1)

DF1 = frequency increment to be used between FZERO and F1

DF2 = frequency increment to be used between F1 and F2

.

.

DF6 = frequency increment to be used between F4 and F5(1)

EPS = small parameter which directs program to update frequency increment if analysis frequency is within EPS of a band cutoff frequency (default value = 1E-6)

---

(1)More bands can be used simply by defining the band cutoff frequencies, the corresponding frequency increments, and by adding the appropriate number of lines in the portion of the program where the decision is made to update the frequency increment.

C. Arrays

AMP(NMODES) = modal amplification factors (default values = 1.0)

IROW(NCROSS) = cross-PSD row mapping array (used in conjunction with ICOL(NCROSS))

ICOL(NCROSS) = cross-PSD column mapping array (used in conjunction with IROW(NCROSS))

ARRAY DIMENSION GUIDE

I. Arrays whose size is based on NMODES:

AF
Z
D
E
DEN
TEMP
A2ZW
A1(NMODES, NMODES)
A2(NMODES, NMODES)
A3(NMODES, NMODES)
A4(NMODES, NMODES)
A5(NMODES, NMODES)

II. Arrays whose size is based on NFORCE:

SF
SFA
SFB
C(NFORCE, NFORCE)
Q(NFORCE, NFORCE)

III. Arrays whose size is based on NCROSS:

SR
SI
SRA
SRB
SIA
SIB
IROW
ICOL

IV. Arrays whose size is based on NLOAD:

X
S
SA
R

V. Arrays whose size is based on a combination of parameters:

PF(NFORCE, NMODES)
TA(NLOAD, NMODES)

Total = 32 arrays

23 are 1-dimensional
9 are 2-dimensional

ANALYSIS GUIDE

1. DETERMINING ANALYSIS BANDS AND FREQUENCY INCREMENTS (Parameters, NFREQ, FZERO, F1, F2,..., DF1, DF2,...)

Determining the analysis band cutoff frequencies and associated frequency increments is an important part of setting up a random response analysis run. As was previously mentioned, the analyst can use as many bands as he or she wishes, or, if constant frequency steps within bands are not required, a steadily-increasing step size can be used from one end of the analysis spectrum to the other, thus eliminating altogether the need for band-width and frequency increment determination. In any event, the parameters that will dictate the band cutoff frequencies and frequency increments will be the number of modes in the model, the relative separation of these modes, their associated damping, and the frequency spectrum over which the forces contain energy. In turn, the total number of analysis iterations, NFREQ, is determined by the value of the cutoff frequencies and the associated increments.

The value of NFREQ must be determined by the analyst after he or she has decided upon the appropriate cutoff frequencies and increments for the particular problem at hand. The primary contributor to analysis accuracy is the step size, in the neighborhood of a mode, relative to the half-power bandwidth of the mode. The user must make certain that the peaks in the response spectra are adequately characterized, thus allowing their contribution to the total area under the PSD curve to be accurately accounted for when RMS values are computed. This concern is completely analogous to the importance of using an adequate time increment when numerically integrating equations of motion in the time domain using Runge-Kutta or other algorithms (see Ref. 10). Of course, specifying frequency increments that are overly fine can result in values of NFREQ that are inefficient, and can be, in some cases, prohibitively expensive from a computational standpoint. This is the primary motivation for leaving the computation of NFREQ as a manual exercise for the user, as opposed to making it an automatic computation internal to the program.

In general, it has been found that an adequate frequency increment in the neighborhood of a mode allows for four spectral lines between the half-power points of the mode's frequency response function. Thus, since the half-power bandwidth for a lightly damped system is given by

$$\Delta f_{hp} = 2\zeta_n f_n$$

the analysis frequency increment should be no greater than

$$\Delta f_i = \zeta_n f_n / 2$$

The analyst should specify a sufficient number of spectral lines so that the analysis is performed far enough past the last mode of interest (i.e., the highest frequency mode of interest) to include any measurable contribution from the "tail" of that mode. This determination will also depend on the mode's frequency and associated damping.

Again, the requirements for any particular problem can vary significantly, and the above recommendations should not be construed as inflexible constraints, but rather as general guidelines based on experience gained in applications of the procedure to date.

2. DETERMINING FORCE CROSS-PSD MAPPING ARRAY COEFFICIENTS

(Parameter NCROSS and arrays IROW and ICOL)

The size of the force PSD matrix, and consequently the size of the "C" and "Q" arrays, is determined by the number of applied forces, NFORCE. Thus the $[\phi_F]$ matrix is collapsed to only those degrees of freedom at which forces are applied. The diagonal elements of "C" will contain the auto-PSDs (real) of the forces, whereas the off-diagonal elements of "C" and "Q" contain the real and imaginary components, respectively, of the force cross-PSDs. Since these PSDs and cross-PSDs are typically frequency-dependent, and each frame, or spectral line of data, is written onto the input unit as a separate record, a tractable method for assembling the "C" and "Q" arrays at each analysis frequency is needed. A scheme involving mapping arrays IROW and ICOL

(dimension NCROSS each) has been devised. Simply put, these arrays map the cross-PSD data from the individual records on the input unit to the appropriate ROW/COLUMN locations in arrays "C" and "Q". This occurs within the "PSD" subroutine, where the interpolation of the power spectra is also performed.

Presently, the integer elements of IROW and ICOL are defined in a DATA statement in the main program and are included in the common block CBPSD. This method of defining the mapping arrays can be altered by the user; for instance, the elements could be defined by supplemental data, which would be input to the program by corresponding READ statements. In any case, instructions for filling in the mapping arrays are more easily described after explicitly defining the format in which the auto-PSDs and cross-PSDs must be written to the input unit.

Each record must be written on the input unit using an unformatted binary WRITE statement, such as

```
WRITE(33) F, (SF(K),K=1,NFORCE), (SR(K),K=1,NCROSS), (SI(K),K=1,NCROSS)
```

where

F = frequency defining the frame

SF = array of force auto-PSDs at F, ordered in agreement with $[\phi_F]$

SR = array of real components of cross-PSDs at F, in an order consistent with IROW & ICOL

SI = array of imaginary components of cross-PSDs at F, in an order consistent with IROW & ICOL

If the analyst is using cross-PSD data for every possible force combination (i.e., no elements of "C" or "Q" are identically zero) then the maximum number of independent cross-PSD functions will be needed. This number, given by

$$NCROSS_{max} = NFORCE(NFORCE-1)/2$$

is defined by the number of elements in the upper triangle, not including the diagonal elements. The lower triangle elements are defined by the upper triangle elements due to the Hermitian property of the force PSD matrix.

In many circumstances, particularly those involving a large number of forces, the correlation, and thus the cross-PSDs, of forces distant from each other, or in some other way unrelated, can be assumed to be negligible. In cases such as these, the number of independent cross-PSD functions can be significantly less than $NCROSS_{max}$. Often, in these cases, the analyst will arrange the forces such that the resulting force PSD arrays are banded.

The mapping arrays IROW and ICOL are needed whenever <u>any</u> force cross-PSDs are used. The elements of the arrays are defined as follows:

a. IROW and ICOL are dimensioned as NCROSS (Be sure dimensions are defined both in the main program <u>and</u> in the subroutines.)

b. IROW(K) and ICOL(K) instruct subroutine "PSD" to insert SR(K) and SI(K) to the appropriate element locations in the "C" and "Q" array (thus all elements of IROW and ICOL are integers).

c. Index K runs from 1 to NCROSS.

Example:

i) NFORCE = 3

ii) NCROSS = 2

iii) Only cross-PSD functions between forces 1 and 2, and 2 and 3 are being used

The resulting mapping arrays would be defined as follows:

```
IROW(1) = 1     ICOL(1) = 2
IROW(2) = 2     ICOL(2) = 3
```

This would correspond to an input unit written as

```
WRITE(33) F,SF(1),SF(2),SF(3),SR(1),SR(2),SI(1),SI(2)
```

on a record-by-record basis.

3. A WORD ABOUT COMMON BLOCKS AND THEIR ASSOCIATED SUBROUTINES

The program uses two subroutines: Subroutine "PSD", which interpolates the input force PSD functions to the defined analysis frequencies and assembles the resulting force PSD arrays "C" and "Q", and subroutine "ADMIT", which assembles the diagonal admittance function arrays "D" and "E" at each analysis frequency. Associated with these subroutines are common blocks CBPSD and CBADMIT. CBPSD contains variables NFORCE, NCROSS, IROW, ICOL, and I. CBADMIT contains variables NMODES, NRIG, PI, AF, Z, and AMP.

The analyst is forewarned to make certain that

a. The COMMON statements in the main program and in the appropriate subroutine are identical.

b. The arrays used in common block memory (IROW, ICOL, AF, Z, AMP) are dimensioned correctly both in the main program and in the subroutines.

Failure to heed these reminders will result in variable addresses being assigned to inappropriate variables, resulting in an incorrect solution.

4. A WORD ABOUT RIGID BODY MODES, MODAL AMPLIFICATION, AND THE "ADMIT" SUBROUTINE

Often the dynamic model of a system includes one or more rigid body modes. The modal acceleration frequency response function for these modes is simply the inverse of their associated modal masses. However, the displacement frequency response functions will have a singularity at $\omega=0$, causing the computation to "blow-up" at that frequency. Ergo, it is recommended that only the elastic modes be used in the model when displacement frequency response functions are implemented.

Modifying the "ADMIT" subroutine to implement displacement frequency response functions involves changing the two lines that read

```
D(J) = AMP(J)*(-(2*PI*F)**2)*(FJ**2-F**2)/DEN(J)
E(J) = AMP(J)*(-(2*PI*F)**2)*(2*ZJ*F*FJ)/DEN(J)
```

to

```
D(J) = AMP(J)*(FJ**2-F**2)/DEN(J)
E(J) = AMP(J)*(2*ZJ*F*FJ)/DEN(J)
```

The analyst might also choose to amplify the effect of certain of the modes. This can be accomplished through the use of the AMP array. AMP simply provides a scalar multiplier for each modal frequency response function. As such, the default setting for all elements of AMP is 1.0. AMP is dimensioned to NMODES, and the analyst can alter any of the individual elements of AMP in the main program, as long as it is done after the section of the program where the arrays are initialized, and before the section of the program where the random response calculation loop begins. These sections are clearly denoted by comment cards.

5. TIMING ESTIMATES AND COMPILER CHOICE

Execution time is clearly dependent on the various parameters defining the problem at hand. An exact relationship cannot, at this time, be established. However, before running a full problem, it is suggested that the user run a small test case by setting NFREQ to a small number, and by setting the time request in the JCL to a small value. This will allow the user to verify that the problem has been set up correctly, and will also enable him or her to make a rough estimate of execution time for the full run. These check runs can usually be put through the computer relatively quickly due to the low time request.

At present, the two FORTRAN 5 compilers available at Aerospace, for use on the CRAY X/MP computer, are the "CFT" compiler and the "CFT77" compiler. The CFT77 compiler is newer and has more ANSI standard features. Both compilers have been used to run the random response code, and it has been found that, although the executable code produced by the CFT77 compiler runs faster than that produced by the CFT compiler, the actual compiling of the source code takes significantly longer with CFT77. This would indicate that it would be preferable to use the CFT77 compiler for larger problems, where compile

time is a small portion of overall CPU time, and the CFT compiler for smaller problems, where compile time could be a significant fraction of the overall execution time. Also, if the executable code is to be saved and used repeatedly, say with slightly different inputs in each case, then the CFT77 compiler would be preferable. It is noted that the program is written with an objective of exploiting the vectorization capabilities of the CRAY computer. Both of the aforementioned compilers allow for this vectorization feature to be utilized.

APPENDIX C

MATRIX TAPE FORMAT

General Description

The standard model file format used by the random response program is termed the MATRIX format. A MATRIX tape may have any number of files, each in similar format. Each file contains a single matrix in the format described below. The data are written in a standard default, unformatted form.

File Structure

Assume a matrix of size NROW by NCOL. It is written in MATRIX format as follows:

Record 1 = Header record; length = 7 words

Words 1, 2, 3, 4	= zero
Word 5	= NROW
Word 6	= NCOL
Word 7	= zero

Records 2 through (NROW + 1) = Data records; length = (NCOL + 3)

Word 1	= row number
Word 2	= NROW
Word 3	= NCOL
Words 4 through NCOL + 3	= row of data

Data followed by end-of-file

Example file for matrix of size (5 x 7)

Record 1	0	0	0	0	5	7	0
Record 2	1	5	7		Row 1 of data		
Record 3	2	5	7		Row 2 of data		
Record 4	3	5	7		Row 3 of data		
Record 5	4	5	7		Row 4 of data		
Record 6	5	5	7		Row 5 of data		
EOF							

APPENDIX D

RANDOM RESPONSE COMPUTER PROGRAM

```
JOB,JN=D$1PT$B,T=10,MFL=500000.
ACCOUNT,AC=453301,US=18332.
ACCESS,DN=FT32,PDN=**********.
ACCESS,DN=FT33,PDN=**********.
*ASSIGN,DN=FT34,RF=IW,FD=CDC,CV=ON,MBS=5120,RS=5120.
CFT.
SEGLDR,GO.
REWIND,DN=FT34.
*DISPOSE,DN=FT34,DC=ST,DF=BB,MF=MB,TEXT='CTASK.SAVEPF,FT34,'
*         'XXXXXXXX,ID=18332,ST=PF6.'.
*SAVE,DN=FT34,PDN=XXXX.
*EOR
      PROGRAM RANDOM
C*************************************************************************
C*************************************************************************
C*************************************************************************
C*************************************************************************
C*************************************************************************
C*************************************************************************
C*************************************************************************
C
C
C     THIS PROGRAM COMPUTES RESPONSE POWER SPECTRAL DENSITY FUNCTIONS
C     (PSD'S) FOR A LINEAR MULTI-DEGREE-OF-FREEDOM SYSTEM SUBJECTED TO
C     ONE OR MORE FREQUENCY-DEPENDENT FORCE PSD'S AND CROSS PSD'S.
C
C     A SUGGESTED REFERENCE IS ATM NO. 89(4533-01)-55:
C
C         'A MULTI-MODE RANDOM RESPONSE ANALYSIS PROCEDURE'
C
C     APPENDIX 'B' OF THIS REPORT SHOULD BE HELPFUL AS AN ANALYSIS GUIDE
C
C
C*************************************************************************
C
C                   PETER BROUSSINOS  AUGUST,1989
C
C*************************************************************************
C
C
C
C     THE INPUTS TO THE PROGRAM ARE:
C
C           1. A DYNAMIC MODEL OF THE SYSTEM, IN AEROSPACE CORPORATION
C              "MATRIX" FORMAT (SEE REPORT, APPENDIX C).  THE MODEL
C              CONSISTS OF
C
C                  A.  FORCE TRANSFORMATION MATRIX CONSISTING OF MODE
C                      SHAPE VECTORS TRANSPOSED AND COLLAPSED TO THE
C                      DEGREES OF FREEDOM WHERE FORCES ARE APPLIED.
C
C                  B.  DIAGONAL MODAL DAMPING MATRIX.
C
C                  C.  DIAGONAL MODAL STIFFNESS MATRIX.
C
C                  D.  LOAD TRANSFORMATION MATRIX WHICH RECOVERS
C                      PARAMETERS OF INTEREST (LOADS, PHYSICAL ACCELER-
C                      ATIONS, ETC.) WHEN POST-MULTIPLIED BY MODAL
C                      DISPLACEMENTS OR ACCELERATIONS.
```

```
C
C           2. FORCE PSD'S AND CROSS-PSD'S, IN AEROSPACE CORPORATION
C              "TRP" FORMAT, WHICH IS DESCRIBED IN THE REPORT.
C
C
C     THE OUTPUT FROM THE PROGRAM CONSISTS OF:
C
C           1. POWER SPECTRAL DENSITY FUNCTIONS OF THE RESPONSES
C              DEFINED BY THE ROWS OF THE RECOVERY MATRIX.
C
C           2. RESPONSE ROOT-MEAN-SQUARE (RMS) VALUES DERIVED FROM
C              THE RESPONSE POWER SPECTRAL DENSITIES.
C
C
C
C
C
C
C
C*****************************************************************************
C*****************************************************************************
C
C  THIS SECTION CONTAINS THE COMMON BLOCK AND ARRAY DIMENSION STATEMENTS
C
C     COMMON BLOCK CBPSD IS USED IN THE PSD SUBROUTINE.
C     COMMON BLOCK CBADM IS USED IN THE ADMIT SUBROUTINE.
C     THE USER MUST MAKE CERTAIN THAT THE LINES ARE IDENTICAL IN THE
C     SUBROUTINES.  ALSO, THE ARRAYS THAT ARE USED IN THE SUBROUTINES
C     MUST HAVE THEIR DIMENSIONS SPECIFIED EXACTLY AS THEY ARE IN THE
C     MAIN PROGRAM.
C
C
C
C
      COMMON /CBPSD/NFORCE,NCROSS,IROW,ICOL,I
      COMMON /CBADM/NMODES,NRIG,PI,AF,Z,AMP
      DIMENSION AF(111),Z(111),D(111),E(111),DEN(111),SF(22),
     &X(1356),S(1356),SA(1356),SR(46),SI(46),SFA(22),SFB(22),
     &C(22,22),Q(22,22),A1(111,111),A2(111,111),R(1356),
     &A3(111,111),A4(111,111),PF(22,111),TA(1356,111),
     &A5(111,111),SRA(46),SRB(46),SIA(46),
     &SIB(46),IROW(46),ICOL(46),AMP(111),TEMP(111),A2ZW(111)
C
C
C*****************************************************************************
C
C    THE FOLLOWING SECTION IS FOR DEFINING THE FORCE CROSS-PSD
C    MAPPING COEFFICIENTS. SEE THE REFERENCED REPORT FOR DETAILS.
C
C
C
      DATA IROW/1,1,1,2,2,3,3,3,4,4,5,5,5,6,6,7,7,7,8,8,9,9,9,10,10,
     &11,11,11,12,12,13,13,13,14,14,15,15,15,16,16,17,17,17,18,18,19/
      DATA ICOL/2,3,4,3,4,4,5,6,5,6,6,7,8,7,8,8,9,10,9,10,10,11,12,11,
     &12,12,13,14,13,14,14,15,16,15,16,16,17,18,17,18,18,19,20,19,20,20/
C
C*****************************************************************************
C
C    THE FOLLOWING SECTION IS FOR DEFINING THE BASIC PARAMETERS:
C
```

```
C
      NMODES=47
      NRIG=5
      NFORCE=20
      NCROSS=40
      NSEQ=5
      NLOAD=3.1
      NPRINT=10
      XMUF =1.000
C
C*****************************************************************
C
      THIS SECTION IS FOR DEFINING THE ANALYSIS BAND CUTOFF FREQUENCIES
      AND THE ASSOCIATED FREQUENCY INCREMENTS:


      FZERO=0.
      F1=.5
      F2=5.
      F3=10.
      F4=20.
      F5=30
C
      DF1=.5
      DF2=.01
      DF3= 025
      DF4=0.0500
      DF5=.10000
      DF6=.10000
C
C*****************************************************************
      PI=3.141592654
      EPS=.000001
C*****************************************************************
C*****************************************************************

C     INITIALIZE ARRAYS:

      DO 5 K=1,NMODES
      A(K)=0.
      B(K)=0.
      AN(K)=0.
      BN(K)=0.
      CM(K)=0.
      AMP(K)=0.
      PHA(K)=0.
      AMP(K)=1.0
    5 CONTINUE
C
      DO 6 K=1,NFORCE
      SF(K)=0.
      SFA(K)=0.
      SFB(K)=0.
    6 CONTINUE
      DO 7 K=1,NCROSS
      SRA(K)=0
      SRB(K)=0.
      SIA(K)=0.
      SIB(K)=0.
```

```
      SR(K)=0.
      SI(K)=0.
    7 CONTINUE
      DO 8 K=1,NLOAD
      X(K)=0.
      S(K)=0.
      SA(K)=0.
      R(K)=0.
    8 CONTINUE
      DO 9 K=1,NFORCE
      DO 9 L=1,NFORCE
      C(K,L)=0.
      Q(K,L)=0.
    9 CONTINUE
C
C
C*****************************************************************************
C
C     THE FOLLOWING SECTION ALLOWS THE USER THE OPTION OF SETTING THE
C     MODAL AMPLIFICATION COEFFICIENTS TO VALUES OTHER THAN 1.00:
C
C
C      DO 850 K=N1,N2
C      AMP(K)= ***
C  850 CONTINUE
C
C
C*****************************************************************************
C*****************************************************************************
C*****************************************************************************
C*****************************************************************************
C
C     PUT MODEL MATRICES, LTM IN APPROPRIATE ARRAYS:
C
C           1 =  PHIF                 .... FORCING MATRIX
C           2 =  2*ZETA*OMEGA         .... MODAL DAMPING
C           3 =  (OMEGA)**2           .... MODAL STIFFNESS
C           4 =  TA (OR TC)           .... ACCELS=TA*QDD   (LOADS=TC*QDD)
C
C
      READ(32)
      DO 800 K=1,NMODES
  800 READ(32) JK,JK,JK,(PF(J,K),J=1,NFORCE)
      READ(32,END=805)
  805 READ(32)
      DO 810 K=1,NMODES
      READ(32,END=810) JK,JK,JK,(TEMP(J),J=1,NMODES)
  810 A2ZW(K)=TEMP(K)
      READ(32,END=815)
  815 READ(32)
      DO 820 K=1,NMODES
      READ(32,END=820) JK,JK,JK,(TEMP(J),J=1,NMODES)
      AF(K)=SQRT(TEMP(K))/(2*PI)
      IF(K.LE.NRIG) THEN
        Z(K)=0.0
        ELSE
        Z(K)=A2ZW(K)/(2*2*PI*AF(K))
      ENDIF
  820 CONTINUE
      READ(32,END=825)
```

```
 825  READ(32)
      DO 830 K=1,NLOAD
 830  READ(32,END=835) JK,JK,JK,(TA(K,J),J=1,NMODES)
 835  CONTINUE
C
C
C
      DO 610 K=1,NMODES
  610 PRINT 7000, K,AF(K),Z(K)
C*********************************************************************
C*********************************************************************
C*********************************************************************
C
C       THIS LOOP DOES THE RANDOM RESPONSE CALCULATION.
C       EACH ITERATION CORRESPONDS TO A NEW SPECTRAL LINE.
C
C       PARAMETER NCASE CAN BE USED IF ONE WISHES TO RUN MULTIPLE
C       CASES PER RUN.
C
C
CCC     DO 700 NCASE=1,
        DO 12 ITAPE=1,4
  12    REWIND ITAPE
        DO 13 ITAPE=7,33
  13    REWIND ITAPE
C
      NLOAD1=NLOAD+1
      WRITE(1) 0,0,0,0,NFREQ,NLOAD1,0
      F=FZERO-DF1
      IF(NCROSS.EQ.0) GOTO 150
      DO 100 I=1,NFREQ
C
C     IF MORE ANALYSIS FREQUENCY BANDS ARE REQUIRED, CUTOFF FREQUENCIES
C     BEYOND F5 AND FREQUENCY INCREMENTS BEYOND DF6 MUST BE DEFINED IN
C     THE 'BASIC PARAMETERS' SECTION, AND THE APPROPRIATE ADDITIONAL
C     LINES MUST BE APPENDED TO THE FOLLOWING SECTION, OR TO THE
C     CORRESPONDING SECTION IN THE PORTION OF THE PROGRAM FOR NCROSS=0.
C
C
C
      DF=DF1
      IF(ABS(F-F1).LE.EPS .OR. F.GT.F1) DF=DF2
      IF(ABS(F-F2).LE.EPS .OR. F.GT.F2) DF=DF3
      IF(ABS(F-F3).LE.EPS .OR. F.GT.F3) DF=DF4
      IF(ABS(F-F4).LE.EPS .OR. F.GT.F4) DF=DF5
      IF(ABS(F-F5).LE.EPS .OR. F.GT.F5) DF=DF6
C
C
C
      F=F+DF
C     PRINT 1000,I,F
C
      DO 490 M=1,NMODES
      DO 490 K=1,NMODES
      A3(M,K)=0.
      A4(M,K)=0.
 490  A5(M,K)=0.
      DO 495 M=1,NLOAD
 495  R(M)=0.
C
```

```
      CALL PSD     (F,C,Q)
      CALL ADMIT   (F,D,E)
C
      IF(NFORCE.GT.NMODES) THEN
      DO 500 J=1,NMODES
      DO 500 M=1,NFORCE
         A1(M,J)=PF(M,J)*D(J)
 500     A2(M,J)=PF(M,J)*E(J)
      ELSE
      DO 510 M=1,NFORCE
      DO 510 J=1,NMODES
         A1(M,J)=PF(M,J)*D(J)
 510     A2(M,J)=PF(M,J)*E(J)
      ENDIF
C
      IF(NFORCE.GT.NMODES) THEN
      DO 520 M=1,NMODES
      DO 520 J=1,NMODES
      DO 520 K=1,NFORCE
      DO 520 L=1,NFORCE
         A3(M,J) = A3(M,J) + A1(K,M)*C(K,L)*A1(L,J)
         A4(M,J) = A4(M,J) + A2(K,M)*C(K,L)*A2(L,J)
 520     A5(M,J) = A5(M,J) + A1(K,M)*Q(K,L)*A2(L,J)
      ELSE
      DO 525 K=1,NFORCE
      DO 525 L=1,NFORCE
      DO 525 M=1,NMODES
      DO 525 J=1,NMODES
         A3(M,J) = A3(M,J) + A1(K,M)*C(K,L)*A1(L,J)
         A4(M,J) = A4(M,J) + A2(K,M)*C(K,L)*A2(L,J)
 525     A5(M,J) = A5(M,J) + A1(K,M)*Q(K,L)*A2(L,J)
      ENDIF
C
      IF(NMODES.GT.NLOADS) THEN
      DO 530 M=1,NLOAD
      DO 530 K=1,NMODES
      DO 530 L=1,NMODES
 530   R(M)=R(M)+TA(M,K)*( A3(K,L)+A4(K,L)+A5(K,L)+A5(L,K) )*TA(M,L)
      ELSE
      DO 535 K=1,NMODES
      DO 535 L=1,NMODES
      DO 535 M=1,NLOAD
 535   R(M)=R(M)+TA(M,K)*( A3(K,L)+A4(K,L)+A5(K,L)+A5(L,K) )*TA(M,L)
      ENDIF
C
      WRITE(1) I,NFREQ,NLOAD1,F,(R(M),M=1,NLOAD)
      WRITE(34) F,(R(M),M=1,NLOAD)
 100  CONTINUE
      GOTO 220
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C   LOOP FOR THE CASE WHERE NO CROSS-PSD'S ARE USED (NCROSS=0)
C
C
 150  CONTINUE
      DO 200 I=1,NFREQ
C
      DF=DF1
      IF(ABS(F-F1).LE.EPS .OR. F.GT.F1) DF=DF2
```

```
      IF(ABS(F-F2).LE.EPS .OR. F.GT.F2) DF=DF3
      IF(ABS(F-F3).LE.EPS .OR. F.GT.F3) DF=DF4
      IF(ABS(F-F4).LE.EPS .OR. F.GT.F4) DF=DF5
      IF(ABS(F-F5).LE.EPS .OR. F.GT.F5) DF=DF6
C
      F=F+DF
C     PRINT 1000,I,F
C
      DO 290 M=1,NMODES
      DO 290 K=1,NMODES
      A3(M,K)=0.
 290  A4(M,K)=0.
      DO 295 M=1,NLOAD
 295  R(M)=0.
C
      CALL PSD      (F,C,Q)
      CALL ADMIT    (F,D,E)
C
      IF(NFORCE.GT.NMODES) THEN
      DO 300 J=1,NMODES
      DO 300 M=1,NFORCE
         A1(M,J)=PF(M,J)*D(J)
 300     A2(M,J)=PF(M,J)*E(J)
      ELSE
      DO 315 M=1,NFORCE
      DO 315 J=1,NMODES
         A1(M,J)=PF(M,J)*D(J)
 315     A2(M,J)=PF(M,J)*E(J)
      ENDIF
C

      IF(NFORCE.GT.NMODES) THEN
      DO 320 M=1,NMODES
      DO 320 J=1,NMODES
      DO 320 K=1,NFORCE
         A3(M,J) = A3(M,J) + A1(K,M)*C(K,K)*A1(K,J)
 320     A4(M,J) = A4(M,J) + A2(K,M)*C(K,K)*A2(K,J)
      ELSE
      DO 325 K=1,NFORCE
      DO 325 M=1,NMODES
      DO 325 J=1,NMODES
         A3(M,J) = A3(M,J) + A1(K,M)*C(K,K)*A1(K,J)
 325     A4(M,J) = A4(M,J) + A2(K,M)*C(K,K)*A2(K,J)
      ENDIF
C
      IF(NMODES.GT.NLOAD) THEN
      DO 330 M=1,NLOAD
      DO 330 K=1,NMODES
      DO 330 L=1,NMODES
 330   R(M)=R(M)+TA(M,K)*(    A3(K,L)+A4(K,L)    )*TA(M,L)
      ELSE
      DO 335 K=1,NMODES
      DO 335 L=1,NMODES
      DO 335 M=1,NLOAD
 335   R(M)=R(M)+TA(M,K)*(    A3(K,L)+A4(K,L)    )*TA(M,L)
      ENDIF
C
      WRITE(1) I,NFREQ,NLOAD1,F,(R(M),M=1,NLOAD)
      WRITE(34) F,(R(M),M=1,NLOAD)
 200  CONTINUE
```

```
 220  CONTINUE
      ENDFILE 1
      ENDFILE 34
      REWIND 1
C*************************************************************************
C*************************************************************************
C*************************************************************************
C
C  THIS SECTION INTEGRATES THE RESPONSE PSD'S TO PRODUCE THE RMS VALUES
C
C  NOTE: THE FIRST TEN RESPONSE PSD'S WILL BE PRINTED AT EACH FREQUENCY
C  STEP.  THIS CAN BE CHANGED BY SETTING NPRINT TO THE DESIRED VALUE,
C  AND BY MODIFYING THE FORMAT STATEMENT (LINE 4000) BY CHANGING THE
C  INTEGER MULTIPLIER ON THE INSIDE PARENTHESES.
C
C  THE INTEGRATION ROUTINE USED IS 'TRAPEZOIDAL'.  THIS TOO CAN BE
C  ALTERED BY CHANGING THE WEIGHTING VALUES USED.  HOWEVER, THIS IS NOT
C  RECOMMENDED.
C
C
C
C
      READ(1) J1,J2,J3,J4,NROW,NCOL,J5
      READ(1) J1,J2,J3,F, (X(J),J=1,NLOAD)
      PRINT 6000
      PRINT 4000, 1,F ,(X(J),J=1,NPRINT)
      DO 30 J=1,NLOAD
 30   S(J)=X(J)*DF1/2.
C30   SA(J)=X(J)*DF1/(2.*(2*PI*F)**4)
      NRW=NROW-1
      IC=1
      DO 40  M=2,NRW
        FPREV=F
        READ(1) J1,J2,J3,F, (X(J),J=1,NLOAD)
        W=2.*PI*F
      PRINT 4000, M,F,(X(J),J=1,NPRINT)
C
       IF(IC.EQ.1) THEN
           WEIGHT=2.
           IC=2
         ELSE
           WEIGHT=2.
           IC=1
       ENDIF
C
        DF=F-FPREV
 33       DO 35  J=1,NLOAD
 35       S(J)= S(J)+X(J)*WEIGHT*DF/2.
C35       SA(J)=SA(J)+X(J)*WEIGHT*DF/(2*(W**4))
 40   CONTINUE
      READ(1) J1,J2,J3,F, (X(J),J=1,NLOAD)
        W=2.*PI*F
        DF=F-FPREV
      PRINT 4000, NROW,F,(X(J),J=1,NPRINT)
      DO 45 J=1,NLOAD
 45   S(J)= S(J)+ X(J)*DF/2
C45   SA(J)= SA(J)+ X(J)*DF/(2*(W**4))
      DO 50  J=1,NLOAD
 50   S(J)= XMUF*SQRT(S(J))
C50   SA(J)=XMUF*SQRT(SA(J))
```

```
C
      PRINT 2000,NCASE,NFREQ,NDOF,NMODES,NRIG,NFORCE,NLOAD,XMUF
      DO 60  J=1,NLOAD
      PRINT 3100, J,S(J)
   60 CONTINUE
      WRITE(34) (S(J),J=1,NLOAD)
      ENDFILE 34
  700 CONTINUE
C
C*********************************************************************
C*********************************************************************
C*********************************************************************
 1000 FORMAT(2X,'I= ',I5,' , F= ',F11.5)
 2000 FORMAT('1',2X,'NCASE = ',I3,',   NFREQ = ',I4,/,
     &2X,'NDOF = ',I4,',  NMODES = ',I3,',  NRIG = ',I3,',  NFORCE = ',
     &I3,',  NLOAD = ',I4,',   MUF =  ',F6.2,//)
 3000 FORMAT(2X,'RMS OF XDD ',I4,' = ',E12.5,' = ',F7.3,' G')
 3100 FORMAT(2X,'RMS OF DOF ',I4,' = ',E12.5)
C
C
C   TO PRINT MORE THAN 10 RESPONSE PSDS ON THE OUTPUT, MODIFY
C   THE FOLLOWING FORMAT LINE (LINE 4000) BY SETTING 'N' TO
C   AN APPROPRIATE NUMBER.
C
 4000 FORMAT(1X,I4,F8.2,10E12.4)
C4000 FORMAT(1X,I4,F8.2,10E12.4,/,N(12X,10E12.4))
C
C
C
 6000 FORMAT('1',3X,'N','    F',9X,'PSDS OF THE RESPONSES',//)
 7000 FORMAT(2X,'MODAL FREQUENCY NO.',I5,' = ',F10.3,' HZ,',',   DAMPING
     &RATIO = ',F7.4)
 9000 FORMAT('1')
      STOP
      END
C*********************************************************************
C*********************************************************************
C*********************************************************************
C*********************************************************************
C*********************************************************************
C*********************************************************************
C        SUBROUTINE TO CREATE PSD MATRICES AT EACH F
C
      SUBROUTINE PSD(F,C,Q)
      COMMON /CBPSD/NFORCE,NCROSS,IROW,ICOL,I
      DIMENSION SF(22),SFA(22),SFB(22),SR(46),SI(46),
     &SRA(46),SRB(46),SIA(46),SIB(46),C(22,22),Q(22,22),
     &IROW(46),ICOL(46)
C
      L=NFORCE
      LC=NCROSS
      ITAP=33
C
      IF(NCROSS.EQ.0)GOTO 200
C
      IF(I.GT.1)GOTO 10
        READ(ITAP) FA,(SFA(K),K=1,L),(SRA(K),K=1,LC),(SIA(K),K=1,LC)
        READ(ITAP) FB,(SFB(K),K=1,L),(SRB(K),K=1,LC),(SIB(K),K=1,LC)
   10 IF(F.LT.FB)GOTO 20
        GOTO 30
```

```
  20  DO 25 K=1,NFORCE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C       IF(SFA(K).EQ.0.00 .OR.SFB(K).EQ.0.00)THEN
C       SF(K)=SFA(K)+((F-FA)*(SFB(K)-SFA(K)))/(FB-FA)
C       ELSE
C       XXPON=LOG10(SFA(K))+( (LOG10(F)-LOG10(FA) )*
C     &   ( LOG10(SFB(K) )-LOG10(SFA(K) )))/( LOG10(FB)-LOG10(FA) )
C       SF(K)=10.**XXPON
C       ENDIF
C 25    CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
  25    SF(K)=SFA(K)+((F-FA)*(SFB(K)-SFA(K)))/(FB-FA)
        DO 27 K=1,LC
        SR(K)=SRA(K)+((F-FA)*(SRB(K)-SRA(K)))/(FB-FA)
  27    SI(K)=SIA(K)+((F-FA)*(SIB(K)-SIA(K)))/(FB-FA)
        GOTO 100
  30  FA=FB
        DO 35 K=1,NFORCE
  35    SFA(K)=SFB(K)
        DO 37 K=1,LC
        SRA(K)=SRB(K)
  37    SIA(K)=SIB(K)
        READ(ITAP) FB,(SFB(K),K=1,L),(SRB(K),K=1,LC),(SIB(K),K=1,LC)
        GOTO 10
 100  CONTINUE
C
        DO 40 K=1,NFORCE
  40  C(K,K)=SF(K)
C
        DO 45 K=1,LC
        C(IROW(K),ICOL(K))=SR(K)
        Q(IROW(K),ICOL(K))=SI(K)
        C(ICOL(K),IROW(K))=SR(K)
 45     Q(ICOL(K),IROW(K))=-SI(K)
        GOTO 999
C
C
C       THIS SECTION IS FOR THE CASE WHERE NO FORCE CROSS-PSDS ARE
C       BEING USED (NCROSS = 0).
C
  200   CONTINUE
        IF(I.GT.1)GOTO 510
        READ(ITAP) FA,(SFA(K),K=1,L)
        READ(ITAP) FB,(SFB(K),K=1,L)
  510 IF(F.LT.FB)GOTO 520
        GOTO 530
  520 DO 525 K=1,NFORCE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C       IF(SFA(K).EQ.0.00 .OR.SFB(K).EQ.0.00)THEN
C       SF(K)=SFA(K)+((F-FA)*(SFB(K)-SFA(K)))/(FB-FA)
C       ELSE
C       XXPON=LOG10(SFA(K))+( (LOG10(F)-LOG10(FA) )*
C     &   ( LOG10(SFB(K) )-LOG10(SFA(K) )))/( LOG10(FB)-LOG10(FA) )
C       SF(K)=10.**XXPON
C       ENDIF
C525    CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
 525    SF(K)=SFA(K)+((F-FA)*(SFB(K)-SFA(K)))/(FB-FA)
```

```
        GOTO 600
 530  FA=FB
      DO 535 K=1,NFORCE
 535    SFA(K)=SFB(K)
        READ(ITAP) FB,(SFB(K),K=1,L)
        GOTO 510
 600  CONTINUE
C
      DO 540 K=1,NFORCE
 540  C(K,K)=SF(K)
C
 999  RETURN
      END
C
C*************************************************************************
C*************************************************************************
C
C       SUBROUTINE TO CREATE ADMITTANCE MATRICES AT EACH F
C
      SUBROUTINE ADMIT(F,D,E)
      COMMON /CBADM/NMODES,NRIG,PI,AF,Z,AMP
      DIMENSION DEN(111),D(111),E(111),AF(111),Z(111),AMP(111)
C
      DO 100 J=1,NMODES
      FJ=AF(J)
      ZJ=Z(J)
      IF(J.LE.NRIG) GOTO 90
C     PRINT 999, J,FJ,ZJ
C 999 FORMAT(1X,'J= ',I4,' FJ= ',F7.3,'  ZJ= ',F6.3)
      DEN(J)=((FJ**2-F**2)**2+(2*ZJ*FJ*F)**2)*(2*PI)**2
      IF(J.EQ.9)DEN(J)=0.
              IF(DEN(J).EQ.0.) THEN
 110           PRINT  200,J,DEN(J)
               STOP
              ENDIF
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C     THE FOLLOWING 2 LINES SHOULD BE ADJUSTED ACCORDING TO THE REPORT
C     IF THE USER WISHES TO USE DISPLACEMENT FREQUENCY RESPONSE FUNCTIONS
C     INSTEAD OF ACCELERATION FREQUENCY RESPONSE FUNCTIONS.
C
C
      D(J)=AMP(J)*(-(2*PI*F)**2)*(FJ**2-F**2)/DEN(J)
      E(J)=AMP(J)*(-(2*PI*F)**2)*(2*ZJ*F*FJ)/DEN(J)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      GOTO 100
  90  D(J)=1.
      E(J)=0.
 100  CONTINUE
 125  CONTINUE
 200  FORMAT(2X,'J= 'I5,2X,'DEN(J)= 'F10.5,'    THE DENOMINATOR IN THE
     &FREQUENCY RESPONSE FUNCTION = 0',//,30X,'CHECK BASIC PARAMETERS',
     &///,23X,'*******  EXECUTION TERMINATED *******',/////)
      RETURN
      END
*EOR
```