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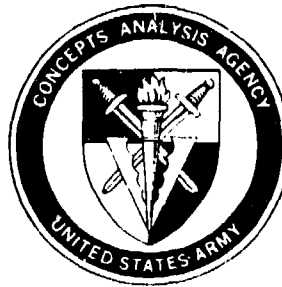
RESEARCH PAPER  
CAA-RP-90-1

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# RATES OF ADVANCE IN HISTORICAL LAND COMBAT OPERATIONS

JUNE 1990



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PREPARED BY  
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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE				Form Approved OMB No 0704-0182	
1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b. RESTRICTIVE MARKINGS None			
2a. SECURITY CLASSIFICATION AUTHORITY N/A		3. DISTRIBUTION/AVAILABILITY OF REPORT Unlimited			
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A					
4. PERFORMING ORGANIZATION REPORT NUMBER(S) CAA-RP-90-1		5. MONITORING ORGANIZATION REPORT NUMBER (S) Same			
6a. NAME OF PERFORMING ORGANIZATION U.S. Army Concepts Analysis Agency		6b. OFFICE SYMBOL (if applicable) CSCA-MVM	7a. NAME OF MONITORING ORGANIZATION Same		
6c. ADDRESS (City, State, and ZIP Code) 8120 Woodmont Avenue Bethesda, MD 20814-2797		7b. ADDRESS (City, State, and ZIP Code) Same			
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Secretary of the Army		8b. OFFICE SYMBOL (if applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N/A		
8c. ADDRESS (City, State, and ZIP Code) The Pentagon Washington, DC 20301		10. SOURCE OF FUNDING NUMBERS			
		PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) Rates of Advance in Historical Land Combat Operations					
12. PERSONAL AUTHOR(S) Robert L. Helmbold					
13a. TYPE OF REPORT Final		13b. TIME COVERED FROM 8/89 TO 6/90		14. DATE OF REPORT (Year, Month, Day) 90/06	15. PAGE COUNT 153
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Operations, operations research, rate of advance, maneuver, mobility, survey, review. (SP20)		
12	04				
15	06				
19. ABSTRACT (Continue on reverse if necessary and identify by block number) The U.S. Army Concepts Analysis Agency (CAA) is pleased to host and support our Secretary of the Army Research and Study Fellow for 1989-90, Dr. Robert L. Helmbold, who won this fellowship with an excellent proposal for research on rates of advance in land combat operations. That proposal called for the conduct of original research into the basic phenomena underlying advances by military forces in land combat operations. The basic approach used was to: a. Obtain through extensive personal visits, correspondence, and phone calls all of the noteworthy documents with statistical data on rates of advance; b. Compile, computerize, describe, critique, and comparatively review them, and then to; c. use this data to examine a wide range of alternative hypotheses about rates of advance. This research paper presents his analysis of the available statistical data on rates of advance, and provides his latest thoughts on rates of advance in land combat operations. It is the third and last paper to appear under this fellowship (the first was "A Survey of Past Work on Rates of Advance in Land Combat Operations," CAA-RP-90-03, February 1990, and the second was "A Compilation of Data on Rates of Advance in Land Combat Operations," CAA-RP-90-04, February 1990). We are not aware of any other work that covers this area as thoroughly as this Research Paper does. As such, it furnishes a valuable resource for current and future work in this important field of investigation. Wide dissemination will make this work available to others for study and analysis. <i>Keynotes</i>					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a. NAME OF RESPONSIBLE INDIVIDUAL Robert L. Helmbold		22b. TELEPHONE (Include Area Code) (202) 295-5245		22c. OFFICE SYMBOL CSCA-MV	

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RESEARCH PAPER

CAA-RP-90-1

**RATES OF ADVANCE IN HISTORICAL  
LAND COMBAT OPERATIONS**

June 1990

Prepared by

Dr. Robert L. Helmbold  
OFFICE, SPECIAL ASSISTANT FOR MODEL VALIDATION

US Army Concepts Analysis Agency  
8120 Woodmont Avenue  
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## PREFACE

The 1989-1990 Secretary of the Army Fellowship awarded to Dr. Robert L. Helmbold called for conducting an investigation into the distances, durations, and rates of advance of land combat forces. The work consisted of four phases. The initial phase was devoted to assembling as much of the existing statistical data on historical rates of advance as possible, and to computerizing it. The second phase consisted of a comprehensive survey and critical review of the assembled literature to identify trends, omissions, and gaps, and to provide a comparative analysis and assessment of its conclusions and findings. The third phase consisted of original statistical and other analyses of the assembled data. The fourth and final phase involves reporting the results. The planned schedule was as follows:

<u>Phase</u>	<u>Scheduled completion</u>
First	December 1989
Second	February 1990
Third	May 1990
Fourth	June 1990

This Research Paper gives our analyses of the available data on rates of advance in land combat operations. These analyses use a larger and more varied set of data than past analyses and consider a number of alternative hypotheses. It is the third and final paper to appear under this Fellowship. The first was A Survey of Past Work on Rates of Advance in Land Combat Operations, CAA-RP-90-3, February 1990. The second was A Compilation of Data on Rates of Advance in Land Combat Operations, CAA-RP-90-4, February 1990.

I am grateful to the Secretary of the Army and to the US Army Concepts Analysis Agency for the opportunity to work on this fascinating and important topic. I also thank all those who contributed facts and suggestions that smoothed my way and opened new lines of thought. It is my privilege to express admiration and respect for all those who, in the past, have striven to understand the nature of advances in land combat operations. Finally, I would like to dedicate this work to those who, in future years, will seek a scientific understanding of the dynamics of combat in general, and of rates of advance in particular.



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US ARMY CONCEPTS ANALYSIS AGENCY  
8120 WOODMONT AVENUE  
BETHESDA, MARYLAND 20814-2797



26 JUL 1990

MEMORANDUM FOR Deputy Under Secretary of the Army (Operations Research),  
ATTN: SAUS-OR, Washington, DC 20310

SUBJECT: Rates of Advance in Historical Land Combat Operations

1. The U.S. Army Concepts Analysis Agency (CAA) is pleased to host and support our Secretary of the Army Research and Study Fellow for 1989-90, Dr. Robert L. Helmbold, who won this fellowship with an excellent proposal for research on rates of advance in land combat operations. This research paper gives the findings and conclusions of his analyses of rates of advance. It is the third and final paper to appear under this fellowship. The first was "A Survey of Past Work on Rates of Advance in Land Combat Operations," CAA-RP-90-3, February 1990, and the second was "A Compilation of Data on Rates of Advance in Land Combat Operations," CAA-RP-90-4, February 1990.

2. The products of this fellowship include an extensive compilation of data (in both written form and in a form usable by microcomputers), a wide-ranging overview and critical survey of the state of the art, and several original analyses. We believe these products represent a substantial contribution to military operations analysis. Since this research paper provides the Army with a useful summary of them, it furnishes a valuable resource for current and future work in this important field of investigation. Wide dissemination will make this work available to others for further study and analysis. Questions or inquiries should be directed to the Office of Special Assistant for Model Validation, U.S. Army Concepts Analysis Agency, ATTN: CSCA-MVM, 8120 Woodmont Avenue, Bethesda, MD 20814-2797, phone (301) 295-1611.

E. B. VANDIVER III  
Director

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**RATES OF ADVANCE IN HISTORICAL  
LAND COMBAT OPERATIONS**

**STUDY  
SUMMARY  
CAA-RP-90-1**

**THE REASON FOR PERFORMING THIS STUDY** was that original analyses of the statistical data on rates of advance are called for. Past analyses have used more limited data bases and often a narrower set of alternative hypotheses. We are not aware of any other work that covers this area as thoroughly as this Research Paper does.

**THE STUDY SPONSOR** was the Secretary of the Army. This is the third and last paper to be prepared by Dr. Robert L. Helmbold under his Secretary of the Army Research and Study Fellowship.

**THE STUDY OBJECTIVE** was to provide the Army with original analyses of the available data on rates of advance, using a larger set of data than has been used in the past, and a range of alternative hypotheses. As such, it furnishes a valuable resource for further work in this important field.

**THE SCOPE OF THE STUDY** was intended to be broad, in the sense of using all of the available data to examine a wide range of worthy hypotheses. No doubt it is too much to hope that the paper used literally all of the available data or included all of the important hypotheses. Nevertheless, its analyses should be very helpful to military historians and operations researchers.

**THE MAIN ASSUMPTION** of this paper is that no data or hypothesis that would substantially alter its principal findings has been overlooked.

**THE BASIC APPROACHES** used in this study were to:

- a. Obtain through extensive personal visits, correspondence, and phone calls all of the noteworthy documents with statistical data on rates of advance.
- b. Compile, computerize, describe, critique, and comparatively review them, and then to
- c. Use these data to examine a wide range of alternative hypotheses about rates of advance.

**THE PRINCIPAL FINDINGS** of the work reported herein are:

- a. A lot of statistical data on rates of advance in land combat operations is available, but for a given purpose only a properly chosen part of it is useful. Sometimes none of it applies.
- b. Past work used a variety of subjective descriptors and *ad hoc* terminology. This subjectivity and lack of standardization makes systematic comparisons difficult, and sometimes impossible.
- c. Several sources, some intentionally and others unintentionally, tended to select cases of a successful advance by the attacker. This biases the data against successful defensive efforts and in favor of advances by attackers.
- d. Reported advance rates tend to be systematically biased toward lower values than are actually achieved. This bias can cause reported rates to be too low by factors around 3 to 5, and seriously distorts the apparent influence of size upon rate of advance.

e. Several epistemological weaknesses affect past work. Among the more important are:

(1) Inadequately caveating hasty and premature overgeneralizations based on only a small number of cases, or on a narrow sample of cases representing only a particular time and operational context.

(2) Theory and observation are seldom compared directly, quantitatively, and in detail.

(3) Despite their effectiveness in other contexts, advanced multivariate statistical methods have been singularly unsuccessful and often misused when dealing with advance rates.

f. Reported advance rates do not seem to have changed much over the last 400 years or so. But the data are widely scattered and highly variable.

g. Reported advance rates may be somewhat higher for battalion-sized units than for larger ones. But the data are widely scattered and highly variable.

h. For heavily engaged forces, reported advance rates of mechanized and armored units are about the same as for infantry units. But for lightly engaged forces reported advance rates of mechanized and armored units are somewhat higher than for infantry units. But again the data are widely scattered and highly variable.

i. Reported advance rates for lightly engaged forces are substantially higher than for heavily engaged forces. However, the evidence indicates that both lightly and heavily engaged forces stand still about 90 to 99 percent of the time. This observation suggests that the key to understanding advances by land combat forces may lie not with their periods of movement, but instead with their periods of standing still. As in other cases, the data are widely scattered and highly variable.

j. Reported advance rates are somewhat higher in summer than in winter—more so for mechanized and armored units than for infantry, but the data are widely scattered and highly variable.

k. Reported advance rates are not consistently lower for longer operations. In fact, on the average, extended operational advances proceed at a steady uniform pace. But the data are widely scattered and highly variable.

l. Reported advance rates are not normally distributed. They are highly skewed and follow a lognormal distribution much more closely than they do either a normal, exponential, Weibull, or gamma distribution.

m. Reported advance rates are practically independent of force ratios. They are much more strongly associated with other indexes of combat capability. But the data are widely scattered and highly variable.

n. Both our and past efforts to devise consistently accurate schemes for predicting advance rates have been unsuccessful. Accordingly, the hypothesis that advance rates are governed primarily by chance should receive serious consideration in future work. Also, the nonmovement phases should be studied in conjunction with the movement phases of land combat operations.

THE STUDY EFFORT was directed by Dr. Robert L. Helmbold, Office, Special Assistant for Model Validation.

COMMENTS AND SUGGESTIONS may be sent to the Director, US Army Concepts Analysis Agency, ATTN: CSCA-MV, 8120 Woodmont Avenue, Bethesda, Maryland, 20814-2797.

*Tear-out copies of this synopsis are at back cover.*

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# RATES OF ADVANCE IN HISTORICAL LAND COMBAT OPERATIONS

## CHAPTER 1

### EXECUTIVE SUMMARY

1-1. **BACKGROUND.** About 20 years ago, the MEFORD report<sup>1</sup> concluded that "Current wargame models are structured generally to produce two basic outputs—combat casualties and movement of the FEBA. Casualty rates have been the object of exhaustive analyses. ... There has been little, and certainly no comparable, research and analysis of advance rates. ... The present base for rates of advance is inadequate." The same statements are valid today. The MEFORD report also asked and answered the following rhetorical questions. The same dialogue is also valid today:

"Q1: Is it actually necessary to establish credibility for the wargame rates of advance inputs?"

"Q2: Is reference to the historical record the proper way to establish credibility?"

"A: The answer to both questions is affirmative."

1-2. **OBJECTIVE.** The present study is an effort to advance the state of the art in this area. This paper summarizes and expands upon the findings of two research papers published earlier in this Fellowship.<sup>2</sup> It also presents the results of some original statistical analyses of the data assembled and suggests directions for further research and analysis.

1-3. **SCOPE.** The Fellowship work was to be comprehensive, in the sense of including all the important statistical data on rates of advance, reviewing all the significant past work, and considering a variety of testable hypotheses regarding rates of advance. No doubt it is too much to hope that we have indeed managed to be this comprehensive. Nevertheless, more than 30 past works were reviewed and critiqued, over 30 primary data bases were compiled, and several major hypotheses were analyzed.

1-4. **ASSUMPTIONS.** The main assumption of this paper is that no data or hypothesis that would substantially alter its principal findings has been overlooked.

#### 1-5. **APPROACH.**

a. The first step was to obtain through extensive personal visits, correspondence, and phone calls all of the important past work on rates of advance, and all documents containing statistical data on rates of advance. A thorough search of the Defense Technical Information Center (DTIC) holdings was made for works keyed to such terms as advance, movement, Army operations, mobility, maneuver, and so forth. All works whose title, abstract, date of publication, and point of origin indicated relevance to this paper's objective were obtained. In addition, inputs were solicited in PHALANX (the

<sup>1</sup> Methodology for Force Requirements Determination (MEFORD), Research Analysis Corporation, RAC-R-121, May 1971.

<sup>2</sup> These are A Survey of Past Work on Rates of Advance in Land Combat Operations, CAA-RP-90-3, February 1990; and A Compilation of Data on Rates of Advance in Land Combat Operations, CAA-RP-90-4, February 1990.

US military operations research newsletter), on the FORUM computer bulletin board system (using both its Army-wide FORUMNET and the special military history HISTORYNET), the Army ORSA bulletin-board system, and personal contact with some 50-odd US and foreign government organizations, industrial firms, and educational institutions. These included, among others, such agencies as the Institute for Defense Analyses, the Center for Naval Analyses, The RAND Corporation, the Brookings Institution, the US Army Center of Military History, TRADOC Headquarters, TRAC, US Army Command and General Staff College, US Army War College, Air Force Center for Studies and Analyses, the various national laboratories, the major operational research establishments of SHAPE, the United Kingdom, Canada, and the Federal Republic of Germany, and the principal US military study and analysis contractors such as SAIC, PRC, SPC, etc., as well as many others. In each case, we asked them not only to furnish or suggest whatever relevant materials they themselves had, but also to direct us to any other points of contact they thought might be fruitful, and all such leads were followed up. Appendix A provides a bibliography of the works collected and consulted.

b. Having secured the relevant documents, we then proceeded to extract and computerize their relevant statistical data, and to describe, critique, and comparatively review their arguments and analyses.

c. Finally, we prepared some original statistical analyses of the data. These used the computerized data and microcomputers. They consisted of descriptive and exploratory statistical presentations; studies of trends and correlations; and selected hypothesis tests.

1-6. FINDINGS. The principal findings of this Fellowship are as follows:

a. A lot of statistical data on rates of advance in land combat operations is available, but for a given purpose only a properly chosen part of it is useful. Sometimes none of it applies.

b. Past work has used a variety of *ad hoc* descriptors and terminology. This lack of standardization makes comparisons difficult, and sometimes impossible.

c. Some past work made mistakes in figuring elapsed times from calendar dates and/or times of day; some even made mistakes in computing rates from distances and elapsed times.

d. Several sources, some intentionally and others unintentionally, tended to select cases of a successful advance by the attacker. This biases the data against successful defensive efforts and in favor of advances by attackers.

e. Reported advance rates tend to be systematically biased toward lower values than are actually achieved. This bias can cause reported rates to be too low by factors of around 3 to 5, and seriously distorts the apparent influence of size upon rate of advance. Where this bias is present, study results on advance rates are conditional on there having been a successful advance.

f. The literature is full of loud but wildly conflicting claims. Many of these "findings" are merely *post hoc* rationalizations or hasty and premature overgeneralizations with little objective basis in fact.

g. Appreciable difficulties for this and for future quantitative work have been created by the use of subjective/qualitative descriptors not defined in terms of objectively measurable quantities (e.g., the use of such subjective/qualitative descriptors as "intensity of enemy opposition," "degree of difficulty of the terrain," and the like).

h. Several epistemological weaknesses affect past work. Among the more important are:

(1) Inadequately caveating hasty and premature overgeneralizations based on only a small number of cases, or on a narrow sample of cases representing only a particular time and operational context.

(2) Theory and observation are seldom compared directly, quantitatively, and in detail.

(3) Despite their effectiveness in other contexts, powerful multivariate statistical methods have been singularly unsuccessful when dealing with advance rates. Some possible reasons for this are:

(a) There has been a tendency to use far too many variables. This, in turn, has led to excessive overfitting. The use of formal model selection criteria would help to control these excesses.

(b) Excessive focus on purely statistical "significance" at the expense of the practical importance of the results. There has also been too much focus on "trends" and too little attention to the variability of the data above and below the trend line.

(c) Normality has been assumed, sometimes tacitly, without adequate justification. Yet the data are highly skewed and poorly represented by normal distributions.

(d) The data have tacitly been assumed to follow a smooth distribution. Yet the reported data are very "grainy" because of the tendency to report certain simple values (e.g., 1 km/day).

(e) No allowance has been made for the successful advance bias and the reported versus actual bias mentioned in points 1-6d and 1-6e above.

(f) Correlations between events that are close together in time or space have been ignored.

i. Reported advance rates do not seem to have changed much over the last 400 years or so. But the data are widely scattered and highly variable.

j. Reported advance rates may be somewhat higher for small units than for large ones. However, in this context, a unit with 900 to 1,000 personnel is already "large," since its advance rates do not differ much from those of larger units. But the data are widely scattered and highly variable.

k. For heavily engaged forces, reported advance rates for heavily engaged mechanized and armored units are about the same as for infantry units. On the other hand, for lightly engaged forces, reported advance rates for mechanized and armored units are about 1.5 to 2 times higher than for infantry units. But the data are widely scattered and highly variable.

l. Reported advance rates for lightly engaged forces are substantially higher than for heavily engaged forces. However, the evidence indicates that both lightly and heavily engaged forces stand still about 90 to 99 percent of the time. This observation suggests that the key to understanding advances by land combat forces may lie not with their periods of movement, but instead with their periods of standing still. As in other cases, the data are widely scattered and highly variable.

m. Reported advance rates are higher in summer than in winter, more so for mechanized and armored units than for infantry, but the data are widely scattered and highly variable.

n. Reported advance rates are not consistently lower for longer operations. In fact, it appears that extended operational advances tend to proceed at a steady uniform pace. About as many of them speed up as slow down. But the data are widely scattered and highly variable.

o. Reported advance rates are not normally distributed. They follow a lognormal distribution much more closely than they do either a normal, exponential, Weibull, or gamma distribution.

p. Reported advance rates are practically independent of force ratios. They are much more strongly associated with other indexes of combat capability. But the data are widely scattered and highly variable.

q. Both this and past efforts to devise consistently accurate schemes for predicting advance rates have been unsuccessful. Accordingly, the hypothesis that advance rates are governed primarily by chance should receive serious consideration in future work.

### 1-7. OTHER OBSERVATIONS.

a. **Generalization and Prediction.** These are epistemological issues. It's time to seriously consider that usefully accurate generalization and prediction may not be possible.

(1) Rates of advance are not consistently or accurately predictable by current knowledge. The highest proportion of the variance accounted for by the epistemologically acceptable studies typically seems to cluster somewhere around 1/3. So despite enormous effort by several great analysts and historians, only weak general trends and broad tendencies have been brought to light. As Tukey<sup>3</sup> says, "The data may not contain the answer. The combination of some data and an aching desire for an answer does not ensure that a reasonable answer can be extracted from a given body of data. The data may not even contain the appearance of an answer... ."

(2) How many attempts to build a perpetual motion machine have to fail before we decide it is impossible? Surely at this point we must seriously consider the possibility that advance rates are largely due to inherently uncontrollable and unpredictable chance effects. If they are, we just have to face up to that and behave accordingly. One rational response to this would be to adopt Genichi Taguchi's advice on how to cope with irreducible variability. He says that "The most important quality of process or product design is its robustness against variation." I suggest that, in the future, we have a lot fewer studies looking for high fidelity predictors of advance rates, and a lot more on how to cope rationally with extreme variations in advance rates. In particular, how do we design our forces and operations to be robust in the face of the wide scatter and high variation in reported advance rates? We've been looking for the key, but what if there is no key? If there isn't, then we ought to implement Taguchi's advice in a practical way, so that our operations will be robust against unavoidable variability.

b. **The Burning Questions.** The sources concentrate on reporting when advances started and stopped, and what distances were covered. They give very little information on what *caused* the advances to start, stop, speed up, or slow down. So they are not very helpful in addressing such fundamental issues as:

- (1) What starts a force in motion?
- (2) Once started, what governs its speed and direction?
- (3) What eventually arrests or reverses the force's motion?

<sup>3</sup> John W. Tukey, *Sunset Salvo*, *The American Statistician*, 40(1986), 1(Feb), 72-76.

If we could give demonstrably good answers to these questions, then we would know just about all that was worth knowing about advance rates in land combat operations. I believe that this is the first time these questions have been stated so clearly and concisely. Unfortunately, no one has any good answers for them.

c. **Successful Advance Bias.** As mentioned earlier, some of our conclusions are conditional on there being a successful advance. That's because most of the data bases describe only cases of successful advances. I haven't yet found a good way of correcting for or dealing with this bias. Since it has been widely overlooked in the past, nobody has studied it enough to properly quantify its effects.

d. **Reported versus Actual Bias.** Past studies have completely missed this particular bias. Now that we know about it, we can start figuring out how to deal with it. My investigations to date show that it can bias advance rates downward by factors of at least 3 to 5.

e. **Check C<sup>3</sup> Delay Hypothesis.** I think it is very important to study the *non*-movement periods. Nobody's thought of that in the past. But we need to understand what's causing them and governing their durations. How long are they? How frequently do they occur? What causes them to start and end? What's going on while they last?

f. **Correlations.** How best to deal with correlations in time, space, and space-time?

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## CHAPTER 2

### DATA BASES

**2-1. OBJECTIVE.** This chapter summarizes what we found while compiling our data on advance rates in historical land combat operations.

**2-2. BACKGROUND.** A useful collection of statistical data is needed for work on advance rates. Unfortunately, these data are widely scattered and often hard to get. Accordingly, we produced a systematic compilation and description of data in a convenient form for use by military historians and operations researchers. It was published as *A Compilation of Data on Rates of Advance in Land Combat Operations*, CAA-RP-90-4, February 1990. As mentioned elsewhere, our intent was to be comprehensive, in the sense of including all of the important statistical data on advance rates. No doubt it is too much to hope that we have identified and obtained literally all of the useful data. Nevertheless, our data compilation should be very helpful to military historians and operations researchers. Counting earlier statistical tabulations that it includes or supersedes, over 30 data bases are encompassed by those we have assembled. We are not aware of any other work that covers this area as thoroughly as ours. We believe no data that would substantially alter our principal findings has been overlooked. As such, our compilation furnishes a valuable resource for further work in this important field of investigation.

#### 2-3. APPROACH

a. The data bases were compiled by obtaining through extensive personal visits, correspondence, and phone calls all of the important documents containing statistical data on advance rates. A thorough search of the Defense Technical Information Center (DTIC) holdings was made for works keyed to such terms as advance, movement, Army operations, mobility, maneuver, and so forth. All works whose title, abstract, date of publication, and point of origin indicated relevance to this paper's objective were obtained. In addition, inputs were solicited in PHALANX (the US military operations research newsletter), on the FORUM computer bulletin board system (using both its Armywide FORUMNET and the special military history HISTORYNET), the Army ORSA bulletin board system, and personal contact with some 50-odd US and foreign government organizations, industrial firms, and educational institutions. These included, among others, such agencies as the Institute for Defense Analyses, the Center for Naval Analyses, the RAND Corporation, the Brookings Institution, the US Army Center of Military History, TRADOC Headquarters, TRAC, US Army Command and General Staff College, US Army War College, Air Force Center for Studies and Analyses, the various national laboratories, the major operational research establishments of SHAPE, the United Kingdom, Canada, and the Federal Republic of Germany, and the principal US military study and analysis contractors such as SAIC, PRC, SPC, etc., as well as many others. In each case, we asked them not only to furnish or suggest whatever relevant materials they themselves had, but also to direct us to any other points of contact they thought might be fruitful, and all such leads were followed up. Appendix A provides a bibliography of the works collected and consulted.

b. Having secured the relevant documents, we then proceeded to extract and computerize their relevant statistical data, and to describe, critique, and comparatively review them.

**2-4. PRIMARY DATA BASES.** Two kinds of data bases were used in our analysis—the primary data bases and the derived data bases. The primary data bases are those described and documented in the research paper A Compilation of Data on Rates of Advance in Land Combat Operations, US Army Concepts Analysis Agency Research Paper CAA-RP-90-4, February 1990, prepared by Dr. Robert L. Helmbold. Derived data bases consist of data selected from the primary data bases and regrouped for use in special analytical investigations. This chapter is concerned only with the primary data bases. The derived data bases are described in Appendix B and in later chapters when they arise in connection with the analysis.

a. The chief criterion for including a work in the list of primary data bases was that it formally list or tabulate quantitative information on the advance rates experienced in several actual land combat operations. Although this criterion was not applied overly strictly, it served to exclude wargaming handbooks and similar works that make assertions about advance rates without presenting any substantial or specific supporting evidence. We have very particularly in mind assertions accompanied only by vague allusions to their being “based on historical data,” when no actual historical data are cited or in evidence.

b. Each primary data base was given a short name for reference purposes (often the author's last name in capitals or the principal commander, e.g., CAESAR). In CAA-RP-90-4 they are presented in alphabetical order of their short names. Each presentation begins with a brief introduction that identifies the data source used, describes any important features of the data or its background, offers our own comments and personal views, and explains any special notations or codes used. The statistical tabulations follow immediately after their introductions. This paper also uses the GIBBON data base that appeared as Table B-2 in CAA-RP-90-4, as explained in paragraph B-3a of Appendix B.

c. An effort was made to keep the statistical tabulations uniform in format. Thus, some tabulations contain many “NA” entries, indicating that the information was not given by the source. However, the variety of modes of presentation in the original sources made it impractical to adopt an absolutely uniform format. Conflicts in modes of presentation were resolved in favor of fidelity to the information given in the source.

d. Table 2-1 lists the short names of the primary data bases. Some of these short names may look a bit odd. This is because we shortened them to comply with the MS-DOS™ limitation of eight characters for file names. No data items were used from the OVERHOLT and RAND data bases. Since they presented their data only in graphical form, specific values are no longer readily recoverable and it was not feasible in the course of this work to convert them back into digital (tabular or numerical) form. However, they are listed for the sake of completeness.

## **2-5. STANDARDIZATION OF UNITS, NOTATION, AND TREATMENT OF COMPUTATIONAL MISTAKES**

a. Our sources used a variety of units. This made it very hard to compare values. For example, is an advance of 1,140 yards in 2.5 hours (456 yds/hr) faster or slower than one of 21.75 miles in 3.5 days (6.21 mi/day), and by what percentage? For our work, we decided to convert all distances to kilometers and all time intervals to days. Thus, we find for the example given above that the rates are practically the same, the first being 1.04 km in 2.5 hours (10.01 km/day) and the second being 35.00 km in 3.5 days (10.00 km/day). The appropriate conversion factors are shown in Table 2-2.



Table 2-1. Primary Data Bases

Number	Short name	No. of data items
1	ACSDB	3401
2	ALEXANDE	124
3	ANDREWS	55
4	BAORG	330
5	CAESAR	27
6	CDB90	660
7	DESANTIS	84
8	EASTFRON	212
9	ENGELS	19
10	GIBBON	182
11	GLANTZ	114
12	GUSTAVUS	8
13	HANNIBAL	4
14	HULSE	19
15	LONGMARX	371
16	MISCROAD	173
17	NAPOLEON	31
18	NORMANDY	50
19	ORALFORE	360
20	OVERHOLT	0
21	PARSONS	33
22	QUICKWIN	30
23	RADZIEV	40
24	RAND	0
25	RECORD	22
26	RMC	250
27	ROWLAND	288
28	SAVKIN	81
29	SIEGFRIE	40
30	WAINSTEI	237
31	<u>XENOPHON</u>	<u>38</u>
	TOTAL	7283

Table 2-2. Conversion Factors<sup>4</sup>

Multiply	By	To obtain
miles/day	1.609344	km/day
kiloyard/day	0.9144	km/day
yards/hour	$0.9144 \times 24 / 1000$	km/day

<sup>4</sup> Taken from E. A. Mechtly, The International System of Units: Physical Constants and Conversion Factors, NASA SP-7012, Second Revision, Scientific and Technical Information Office, National Aeronautics and Space Administration, Washington, DC, 1973.

b. In our data bases we used "NA" to indicate that the source did not provide this item of information, and "NU" to indicate that both we and the source do not use this item of information. Except in the ANDREWS and the GLANTZ data bases, we did not attempt to introduce our own values for information not actually given by the sources. Thus, the descriptors given in the data bases for such environmental and operational items as terrain, visibility, defensive posture/mission, level or intensity of opposition to the advance, size of unit(s) involved, etc., are in nearly all cases those used by the source. The introduction to each data base provides any description, discussion, or definition of these terms offered by the source of the data. We have not attempted to provide any additional clarification or explanation.

c. We found several errors in computation. Most of them involved computing elapsed times from starting and ending dates or hours of the day. For example, the source may state that a certain advance of 36 kilometers took place on 15-18 June, and give its elapsed time as 3 days and its rate as 12 km/day, without further explanation. But this is a mistake, since there are actually 4 days between 15 and 18 June inclusive (counting the starting and ending days because most operations start early on the first day and continue well into the last), so the rate is actually 9 km/day. Sometimes, even when the elapsed times are correct, the rates given do not agree with the tabulated distances and elapsed times, yet the source says nothing about these discrepancies. For our work, unless the source clearly indicated that such was *not* intended, we recomputed all its elapsed times by treating starting and ending dates as inclusive. Then all rates were recomputed using the new elapsed times with the distances given in the source. In effect, this approach treats the source's distances and dates as appreciably more reliable than either its elapsed times or its rates.

2-6. SPECIAL ISSUES. The sources rarely address certain important issues of concern to later analysis. For the most part, we can only point them out so that other users will be aware of the issues and alerted to find their own approach to them.

- a. How is distance defined? Do all the sources define it consistently?
  - (1) Initial to final position of some vehicle, or unit?
    - (a) Straight line distance?
    - (b) Along axis of main advance?
    - (c) Odometer distance?
    - (d) Following the actual route of some element?
    - (e) Initial to final location?
    - (f) Only forward of original FEBA (forward edge of the battle area) trace?
  - (2) FEBA trace to FEBA trace?
    - (a) Maximum displacement?
    - (b) Average displacement?
      1. Which sector of the front applies to this average?
      2. What time applies to this average?
- b. How is time interval defined? Do all the sources define it consistently?
  - (1) Start to finish (door to door)?
  - (2) Only time spent "in motion"?
  - (3) Only time spent moving toward goal?

c. What do such situational descriptors as the following really mean? How could they be defined more precisely and objectively? Do all the sources define them consistently?

- (1) Flat, hilly, rugged, mountainous, level
- (2) Open, medium, close terrain
- (3) Wooded, urban, brushy
- (4) Wet, rainy, snow-covered, muddy, stormy

d. Most of the reported data are biased toward successful advances. That is, most of our sources selected cases for examination based on whether or not the attacker actually succeeded in advancing. When the intent is to study unopposed advances, this may be entirely acceptable. However, most studies of opposed advances *also* use such selection practices. Perhaps they reasoned that "if we are going to study advance rates, hadn't we better study cases in which there *was* an advance?" Unfortunately, of course, such reasoning is specious, and the resulting selection process merely biases the sample. Ideally, all attempted advances would be included, and we would see how they turned out: large gain of ground, small gain of ground, no net gain or loss, small loss of ground, large loss of ground. This would certainly result in there being no biases whatever. However, this counsel of perfection is seldom attainable, and in practice there are almost sure to be some biases. So the practical issue is to gain at least an "appreciation" for the effects of whatever biases there are, and those effects can range from quite negligible to utterly disastrous. Unfortunately, very few sources even mention these selectivity and bias issues, and none discusses their impact on study findings. Perhaps these issues escaped their notice. When there is a selection bias toward attacks that actually succeeded in advancing and is sizable enough to have a serious impact on its findings, we say that the source suffers from successful advance bias. When the data are biased toward successful advances, then any conclusions based on that data are conditional on there *being* a successful advance. This is the case with most of the data used in our analyses, so most of our findings apply only after one has determined that a successful advance has taken place.

e. Most of the reported data are systematically biased toward lower advance rates than were actually achieved. This comes about as follows.

(1) Reported distances are shorter than actual. Most sources report only the straight-line displacement of a unit, and not the curvilinear distance it actually traveled. Even when actual unit displacements are reported, its elements generally move further and by more roundabout paths than the unit as a whole. Or the sources report the displacement of the FEBA parallel to itself rather than the movement of the units taking part, most of which displaced from some point behind the original FEBA location to some point in front of it. Also, the FEBA displacements reported by most sources are those taken and *held*—which, of course, cannot exceed the displacements *attained* (whether held or not).

(2) Reported times are longer than actual. Most studies report displacement times rounded up to the next nearest whole day. Even when the starting and ending times of a displacement are reported more precisely, they include short pauses and halts. Also, the maximum displacement may be attained long before the end of the reported time period.

(3) Reported rates are systematically biased toward lower values than actually achieved. This follows because reported distances are shorter than actual and reported times are longer than actual.

f. Of course, other biases are also possible. We did not explore them because so few sources give enough information to judge either their presence or impact. However, this should not be interpreted as suggesting that the sources do not exhibit other biases, or that they are not important.

2-7. DATA BASE FINDINGS. The principal findings regarding data bases given below are an expansion and refinement of those in CAA-RP-90-4.

a. A lot of statistical data on advance rates in land combat operations is available, but for a given purpose only a properly chosen part of it is useful. Hence, we will use the primary data bases as a resource and draw from them such derived data bases as are most suitable for various purposes. Sometimes only a tiny fraction of the data is suitable. Sometimes none of it applies.

b. Past work used a bewildering variety of *ad hoc* descriptors and terminology. Most of the sources use rather brief descriptors to indicate the tactical situation's environmental and operational conditions. But no one descriptor is used by all of the sources, nor is there any source that uses all of the descriptors. For example, one study reported natural obstacles such as rivers and canals. None of the other studies did that. Usually, it is hard to tell just what the descriptors actually mean in objectively measurable terms. Nor is it likely that a given descriptor is defined the same way by all sources. This lack of standardization makes comparisons always difficult, and sometimes impossible. In addition, appreciable difficulties for future quantitative work have been created by the use of subjective or qualitative descriptors not meaningfully defined in terms of objectively measurable quantities (e.g., the use of such subjective/qualitative descriptors as "intensity of enemy opposition," "degree of difficulty of the terrain," and the like).

c. Some past work made mistakes in figuring elapsed times from calendar dates and/or times of day; some even made mistakes in computing rates from distances and elapsed times. Also, a variety of units were used, including miles, kiloyards, hectometers, Persian parasangs (~ 3.5 miles), Chinese lis (~ 1/3 mile), Greek or Roman stadia (~ 607 feet), and French leagues (~ 3 miles). In all cases, we recomputed elapsed times from starting and ending times and converted all units to days and to kilometers. (Of course, a *rate* of 100 km/day does not *necessarily* mean that the distance advanced was 100 km or that the advance lasted a whole day.)

d. Several sources, some intentionally and others unintentionally, selected cases of a successful advance by the attacker. This biases the data against successful defensive efforts and in favor of advances by attackers. Where this bias is present, study results on advance rates are conditional on there having been a successful advance.

e. Reported advance rates are systematically biased toward lower values than are actually achieved. As will be seen in Chapter 4, this bias can cause reported rates to be too low by factors of around 3 to 5, and seriously distorts the apparent influence of size upon rate of advance.

## 2-8. OTHER OBSERVATIONS

a. **The Burning Questions.** The sources concentrate on reporting when advances started and stopped, and what distances were covered. They give very little information on what *caused* the advances to start, stop, speed up, or slow down. So they are not very helpful in addressing the following burning questions:

(1) What starts a force in motion? Usually this is the decision of the commander and his staff. But what makes them adopt one course of action over another? Also, retrograde motions are often triggered by enemy action. And there are instances of spontaneous advances by the troops—some of which were called back by order of the commander and some of which could not be recalled.

(2) Once started, what governs its speed and direction? Probably at least two cases need to be distinguished. Case 1: lightly engaged forces that meet only weak, disorganized, and/or scattered and fairly easily bypassed resistance. Case 2: heavily engaged forces that meet fairly strong, organized resistance that cannot easily be bypassed. Perhaps additional cases need to be considered as well. Some that come to mind are: pursuit operations, breakout after the enemy's main line of resistance has been breached, and administrative marches not in contact with enemy land combat forces—though perhaps attacked by air or missile artillery, etc.

(3) What eventually arrests or reverses the force's motion? This is not always determined by the commander and his staff!

If we could give good answers to these questions, then we would know just about all that was worth knowing about advance rates in land combat operations. So far as I know, this is the first time they have been stated so clearly and concisely. Unfortunately, no one has any good answers for them.

c. **Successful Advance Bias.** As mentioned earlier, some of our conclusions are conditional on there being a successful advance. That is because most of the data bases describe only cases of successful advances. I haven't yet found a good way of correcting for or dealing with this bias. Since it has been widely overlooked in the past, no one has studied it enough to properly quantify its effects.

d. **Reported versus Actual Bias.** Past studies have completely missed this particular bias. Now that we know about it, we can start figuring out how to deal with it.

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## CHAPTER 3

## SURVEY OF PAST WORK

**3-1. OBJECTIVE.** This chapter summarizes the results of our survey of past work on advance rates in land combat operations.

**3-2. BACKGROUND.** A short but systematic basic reference paper surveying and reviewing the current state of the art was needed to provide a sound basis for contemporary and future work on advance rates. Unfortunately, the literature on the quantitative analysis of advance rates is widely scattered and often hard to find. Accordingly, we prepared a critique and comparative survey of the noteworthy past quantitative analyses of the principal factors governing advance rates in land combat operations. It was published as A Survey of Past Work on Rates of Advance in Land Combat Operations, US Army Concepts Analysis Agency Research Paper CAA-RP-90-3, February 1990. We are not aware of any other work that covers the area as thoroughly as this research paper does. As such, it furnishes a valuable orientation and point of departure for further work in this important field of investigation. Our intent was to be comprehensive, in the sense of including all of the noteworthy work in this area. No doubt it is too much to hope that literally every work was indeed identified in time to be included. Nevertheless, this survey does provide an excellent overview of the current state of the art. It summarizes and reviews over 30 past works. We believe no works that would substantially alter its principal findings were omitted.

**3-3. APPROACH**

a. The source documents were compiled by obtaining through extensive personal visits, correspondence, and phone calls all of the noteworthy documents on advance rates. A thorough search of the DTIC holdings was made for works keyed to such terms as advance, movement, Army operations, mobility, maneuver, and so forth. All works whose title, abstract, date of publication, and point of origin indicated relevance to this paper's objective were obtained. In addition, inputs were solicited in PHALANX (the US military operations research newsletter), on the FORUM computer bulletin board system (using both its Armywide FORUMNET and the special military history HISTORYNET), the Army ORSA bulletin board system, and personal contact with some 50-odd US and foreign government organizations, industrial firms, and educational institutions. These included, among others, such agencies as the Institute for Defense Analyses, the Center for Naval Analyses, the RAND Corporation, the Brookings Institution, the US Army Center of Military History, TRADOC Headquarters, TRAC, US Army Command and General Staff College, US Army War College, Air Force Center for Studies and Analyses, the various national laboratories, the major operational research establishments of SHAPE, the United Kingdom, Canada, and the Federal Republic of Germany, and the principal US military study and analysis contractors such as SAIC, PRC, SPC, etc., as well as many others. In each case, we asked them not only to furnish or suggest whatever relevant materials they themselves had, but also to direct us to any other points of contact they thought might be fruitful, and all such leads were followed up. Appendix C provides a bibliography of the works collected and consulted.

b. Having secured the relevant documents, we then proceeded to study, analyze, critique, and comparatively review them.

**3-4. SPECIAL ISSUES.** The sources rarely address certain important issues of concern to later analysis. For the most part, we can only point them out so that other users will be aware of the issues and alerted to find their own approach to them. References to specific summaries are by their short names (e.g., CLAUSEWITZ-1832 refers to the summary of the same short name in Appendix A of CAA-RP-90-3).

a. Many operational descriptors are possible, and the studies use a variety of them. However, in this paragraph we focus on just the descriptors of terrain and of attacker and defender strengths. (The appendix cited in Tables 3-1 and 3-2 is Appendix A of CAA-RP-90-3.)

(1) Table 3-1 lists the terrain descriptors that were used.

(2) Table 3-2 lists the strength descriptors, where NR means that strength descriptions are not relevant to the study. Some studies used several measures of strength (e.g., firepower potentials, etc.), and all those used are listed. The entry "unit designation" means that the general size of the unit is indicated by its level in the usual hierarchy, i.e., battalion, regiment, etc. "Strength" refers to personnel strength. All allusions to "resistance" refer to characterizations of the defender's resistance. The entry QJM is an abbreviation for Quantified Judgment Model<sup>5</sup>.

(3) There is a special problem with descriptors that refer to the degree or intensity of enemy resistance, the quality of the leadership, the difficulty of the terrain, and similar notions. The problem is that these subjective descriptors often commit the following form of the vicious circle fallacy: (i) The advance rate was slow *because* resistance was high, and (ii) resistance must have been high *because* the advance rate was slow. At our present distance from the actual events, it is quite impossible to tell to what extent reported intensities of enemy resistance, leadership qualities, and like notions were affected by this sort of vicious circle.

b. In some studies, the successful advance bias is both obviously present and clearly has a serious impact on the findings. But often the successful advance bias is so subtle that assessing its impact is hard. Some sources provide too little information to assess its impact. Despite these problems, I somewhat rashly offer in Table 3-2, for whatever they may be worth, my tentative judgments of the seriousness of the successful advance bias. In this table, an entry of Y indicates that I think the attacker's bias is serious enough to impact study findings and feel pretty sure of that judgment; an entry of Y? indicates that I feel the successful advance bias is serious but am not so sure of that judgment; an entry of ?? indicates that the matter is so complex or the information so scanty that I decline to hazard a guess; an entry of N? indicates that I don't think the successful advance bias is serious but am not so sure of that judgment; and an entry of N indicates that I don't think the successful advance bias is serious and feel pretty sure of that judgment. An entry of NR indicates that successful advance bias is not relevant and is used for all studies dealing only with unopposed advances. It was also used for a few others where study findings are presented in a way that isn't affected by successful advance bias. It should be plainly understood that these judgments are tentative and reflect only my current opinions. They may be changed at any time.

c. The studies are not of one mind on what factors govern advance rates. The following list gives a condensation of each study's position on it. The only point to be made here is that there is no consensus. To keep the presentation short, I had to drastically simplify and abbreviate the study's position. I consider this loss of fidelity to the originals acceptable, since our sole aim is to illustrate the *range and variety* of viewpoints and *not* to detail any one study's particular perspective. The studies that inspired these condensations are indicated, but those who in the spirit of this paragraph

<sup>5</sup> See, e.g., Trevor N. Dupuy, Numbers, Predictions & War, The Bobbs-Merrill Co., Indianapolis, 1979.



Table 3-1. Examples of Terrain Descriptors Used

Study name	Terrain descriptors used
VEGETIUS-380	flat, mountains
CLAUSEWITZ-1832	flat, mountains
BAORG-1952	open, close (close = wooded or built up)
RAND-1953	none used
HULSE-1954	moderately open, close, mountainous
PARSONS-1954	open, moderately open, moderately close, mountainous
ANDREWS-1960	desert, mountainous, neither
BEKKER-1962	NR (at least for the part summarized in Appendix A)
BEST-1966	none used
OVERHOLT-1970	claims terrain used, but how is not indicated
MADER-1971	NR
MEFORD-1971	none used
PEARSALL-1972	flat, rolling, rugged
DESANTIS-1972	open, median, close
ORALFORE-1972	roadnet mobility characterized as: unlimited cross-country movement, good road net, fair road net, poor road net, impassable terrain. Exceptional obstacles characterized as: river, flooded area, fortified zone, exceptionally effective demolitions, urban area, sabotage by local populace, desert
RECORD-1973	Europe, desert
WAINSTEIN-1973a	open, mixed, close, or difficult
WAINSTEIN-1973b	NR
BARRIER-1974	obstacles described in some detail in narrative accounts of the action; shorthand descriptor terms not used
RMC-1974	no significant limitation on tank or infantry movement, tank movement canalized but infantry movement unaffected, tank movement severely canalized but infantry movement unaffected, tanks must breach an obstacle to advance but infantry movement not significantly limited, both tank and infantry movement canalized, infantry movement canalized and tank movement severely canalized, tanks must breach an obstacle to advance and infantry movement canalized, tanks must breach a difficult obstacle to advance and infantry movement canalized, both infantry and tanks must breach an obstacle to advance (e.g., a river), both infantry and tanks must breach a difficult obstacle to advance
FEBA-1975	all six combinations of [flat, rolling, rugged] with [bare, mixed]
MURPHY-1975	NR
QUICK WINS-1975	none used
LINDLEY-1976	NR
BREAKTHRU-1976	great advantage, advantage, disadvantage, great disadvantage
SCHÄFFER-1977	same as RMC-1974
FALLACY-1977	NR
IABG-1978	NR
DUPUY-1982	difficult terrain, rivers and canals, quality and density of roads
SIMPKIN-1984	good and bad roads
WAINSTEIN-1984	described in accompanying discussion; descriptors not used
ANTHONY-1987	NR
DUPUY-1987	NR
ROWLAND-1989	mean ridge height

Table 3-2. Examples of Strength Descriptors Used

Study name	Strength descriptors used
VEGETIUS-380 CLAUSEWITZ-1832 BAORG-1952 RAND-1953 HULSE-1954	NR divisions for attacker; defender NR number of attacking companies; defender strengths not used number of equivalent divisions on line for each side unit designation for attacker; very light, moderate, heavy for resistance
PARSONS-1954 ANDREWS-1960 BEKKER-1962 BEST-1966 OVERHOLT-1970	unit designation for attacker; none, light, heavy for resistance NR NR NR personnel strength for attacker and defender
MADER-1971 MEFORD-1971	NR personnel strength, firepower potential, QJM lethality for each side
PEARSALL-1972 DESANTIS-1972 ORALFORE-1972 RECORD-1973 WAINSTEIN-1973a WAINSTEIN-1973b BARRIER-1974 RMC-1974	strength, QJM lethality for each side JIFFY wargame combat power index for each side strength, QJM lethality for each side NR strength for attacker; light, moderate, heavy for resistance NR NR infantry platoon equivalents for each side
FEBA-1975 MURPHY-1975 QUICK WINS-1975 LINDLEY-1976 BREAKTHRU-1976	strength for each side NR none used same as RMC-1974 strength, firepower potential, QJM combat power, QJM effective combat power for each side
SCHÄFFER-1977 FALLACY-1977 IABG-1978 DUPUY-1982	same as RMC-1974 NR NR force ratio, combat power preponderance, combat effectiveness superiority
SIMPKIN-1984	NR (at least for the part summarized in Appendix A)
WAINSTEIN-1984 ANTHONY-1987 DUPUY-1987 ROWLAND-1989	unit designation for attacker; none for defender NR strength NR

Table 3-3. Examples of Suspected Successful Advance Bias

Study name	Author's judgmental assessment
VEGETIUS-380	NR
CLAUSEWITZ-1832	NR
BAORG-1952	Y?
RAND-1953	Y?
HULSE-1954	Y
PARSONS-1954	Y
ANDREWS-1960	NR
BEKKER-1962	NR
BEST-1966	NR
OVERHOLT-1970	N
MADER-1971	NR
MEFORD-1971	N?
PEARSALL-1972	N?
DESANTIS-1972	Y?
ORALFORE-1972	Y
RECORD-1973	NR
WAINSTEIN-1973a	Y
WAINSTEIN-1973b	NR
BARRIER-1974	N
RMC-1974	??
FEBA-1975	N?
MURPHY-1975	NR
QUICK WINS-1975	NR
LINDLEY-1976	NR
BREAKTHRU-1976	NR
SCHÄFFER-1977	??
FALLACY-1977	N?
IABG-1978	NR
DUPUY-1982	N
SIMPKIN-1984	NR (at least for the part summarized in Appendix A)
WAINSTEIN-1984	Y
ANTHONY-1987	NR
DUPUY-1987	N?
ROWLAND-1989	NR

are interested only in the range of views should ignore the sources and focus on the condensations. Besides illustrating the contradictory findings of past studies, these condensations and the summaries in Appendix A of CAA-RP-90-3 are good sources of hypotheses. As usual, NR means not relevant. ROA is an abbreviation for "rate(s) of advance." The phrase "defender posture" is synonymous with defender mission (e.g., defend in place, conduct a withdrawal, etc.). When the defender's state of physical protection is intended, we refer to the defender's fortifications.

- (1) VEGETIUS-380. Physical fitness makes for high ROA.
- (2) CLAUSEWITZ-1832. There is a sustainable rate of march. It can be exceeded for a time, but if pressed too hard will seriously damage the force.
- (3) BAORG-1952. ROA varies inversely with the intensity of opposition, size of the force, distance to the objective, night (*vs* day), and ruggedness of terrain.
- (4) RAND-1953. ROA varies directly with force ratio, but there may be a threshold force ratio below which advances are not possible.
- (5) HULSE-1954. Force ratios and terrain determine ROA.
- (6) PARSONS-1954. Force ratios and terrain determine ROA.
- (7) ANDREWS-1960. Unopposed advance rates decline if marches are sufficiently prolonged.
- (8) BEKKER-1962. There are some very fundamental physical or technological limits to how fast vehicles can go.
- (9) BEST-1966. While, statistically speaking, ROA varies inversely with casualty rate, it is doubtful that either causes the other. Instead, conditions favoring high ROA also favor low casualty rates, and *vice versa*.
- (10) OVERHOLT-1970. Casualties determine ROA. Force ratios have little to do with ROA.
- (11) MADER-1971. NR
- (12) MEFORD-1971. ROA are not determined by force ratios.
- (13) PEARSALL-1972. ROA depends on the defender's *posture*, and not on terrain or anything else.
- (14) DESANTIS-1972. ROA depends on the ratio of (the JIFFY wargame variety of) relative combat power indices, which in turn depend in a complicated way on the size and composition of the opposing forces, on the local terrain, and on the attacker's tactics. (But not on the weather or on day/night conditions.)
- (15) ORALFORE-1972. ROA varies inversely with casualty rates of the attacker and the presence of major obstacles. There may be a force ratio threshold below which advances are not possible. ROA also depends on the missions of the opposing forces, and perhaps on several other considerations.
- (16) RECORD-1973. Rates of advance actually achieved by large forces are far below those of their principal modes of transportation. This betokens some fundamental limitations in how fast large forces can advance, which will govern ROA regardless of what the doctrine and field regulations prescribe.
- (17) WAINSTEIN-1973a. Defensive posture and terrain difficulties tend to go hand in hand. (It may be that this reflects a tendency for the defender to select difficult terrain as the place to make his most determined stands.)
- (18) WAINSTEIN-1973b. Some of the most widely-used figures on advance rates may have little basis in historical fact.

(19) BARRIER-1974. ROA is not related to force ratio.

(20) RMC-1974. ROA depends on force ratio, but it is only 1 of 25 terms in the equation. Increases in any of these factors will eventually exhibit diminishing returns if the others are held fixed.

(21) FEBA-1975. ROA depends significantly on force ratio. Defense posture is also significant, but weather, season of year, and terrain are not.

(22) MURPHY-1975. It is not possible to determine *what* factors affect ROA.

(23) QUICK WINS-1975. A preponderance of effective force strength matters, but can to some extent be mitigated by clever tactics.

(24) LINDLEY-1976. Defensive posture, relative strengths, national character, and counterattacks seem to affect ROA. But all of these taken together influence ROA no more than other unknown and perhaps purely random factors.

(25) BREAKTHRU-1976. ROA are unrelated to force ratios.

(26) SCHÄFFER-1977. Force ratios are important, but only 1 of some 17 terms in the equation.

(27) FALLACY-1977. ROA are unrelated to force ratios.

(28) IABG-1978. ROA depends directly upon force ratios, and there is a threshold below which sustained advances are not possible. ROA also vary inversely with the attacker's losses. Five operational and four environmental factors also affect ROA.

(29) DUPUY-1982. Advances require combat power preponderance. But force ratios don't affect ROA. Yet combat effectiveness superiority enhances ROA. ROA varies directly (as opposed to inversely) with casualties, since all-out efforts increase ROA at the expense of casualties. ROA varies inversely with terrain difficulty, presence of rivers and canals, scarcity of good roads, bad weather, defender fortifications, nightfall, and duration of the operation.

(30) SIMPKIN-1984. For at least one class of operational movements, it's hard to see any dependence of ROA on day/night, good/bad roads, dry/wet weather, or number of routes used.

(31) WAINSTEIN-1984. ROA depends on many things, but enemy resistance, well-fortified and defended positions, obstacles (especially enemy emplaced), congestion, and logistic constraints seem to be rather consistently mentioned in unit records or histories.

(32) ANTHONY-1987. ROA may be fractal (or chaotic). If so, traditional methods of data analysis may not work very well.

(33) DUPUY-1987. ROA does not depend of force ratios.

(34) ROWLAND-1989. Unopposed ROA are higher for mechanized forces than for nonmechanized.

d. A variety of epistemological errors have found their way into past studies. These include the following.

(1) Inadequately caveating hasty and premature overgeneralizations based on only a few cases, or on a narrow sample of cases representing only a particular time and operational context.

(2) Seldom comparing (directly, quantitatively, and in detail) theory and observation. Few works attempt to compare directly the advance rates forecast using either an entire wargame or just its movement subroutine to those in an actual combat situation. This could be a fertile field for future investigations.

(3) Some past work doesn't describe clearly just what statistical methods were used. Sometimes past work blurs the distinction between the numerical results of statistical computations and the author's interpretation of them.

e. Despite their effectiveness in other contexts, powerful multivariate statistical methods have been singularly unsuccessful when dealing with advance rates. Some possible reasons for this are advanced below.

(1) There has been a habit of using far too many variables. This, in turn, has led to excessive overfitting. For example, one study boasted of using 40 independent variables and 8 independent variables. Yet it had only 60 cases to work with, all of which were from World War II. In fact, all were from the Italian theater, all involved heavily engaged forces, and all took place between the fall of 1943 and the spring of 1944. This study started off with factor analysis. It followed that with cluster analysis, then canonical correlation, and ended with multiple correlation. It claimed to have found some important correlations, but these probably were spurious. With 40 variables and 60 cases, about 70 percent of the variance in the data will *spuriously* appear to be accounted for by this amount of overfitting.<sup>6</sup> Since formal model selection criteria can help strike a judicious balance between the importance of a parsimonious representation of the data and the desirability of a good fit, they may help to reduce the overfitting habit. One such criterion is Akaike's Information Criterion.<sup>7</sup>

(2) Excessive focus on purely statistical "significance" at the expense of the practical importance of the results. There has also been too much focus on "trends" and too little attention to the variability of the data above and below the trend line. For example, the study referred to above, with but 60 cases, applied factor analysis, followed by cluster analysis, then canonical correlation was invoked, and finally multiple regression analyses were done.

(3) Normality has been assumed, sometimes tacitly, without adequate justification. For example, in the study of 60 cases, normality was taken for granted. But consider this study's distribution of advance rate values, as shown in Figure 3-1, where we have omitted its 3 cases of zero advance rate. The vertical scale is in units of standard deviation above and below the mean, and the horizontal scale is logarithmic. On such charts, lognormal distributions plot as straight lines. As can be seen, the data are highly skewed and poorly represented by a normal distribution. They are, in fact, much better represented by a lognormal distribution.

<sup>6</sup> See for example D. A. Freedman, A Note on Screening Regression Equations, *The American Statistician*, 37(1983), 152-155; and L. S. Freedman and David Pee, Return to a Note on Screening Regression Equations, *The American Statistician*, 43(1989), 279-282.

<sup>7</sup> See, among others: Y. M. Sakamoto, M. Ishiguro, and G. Kitagawa, *Akaike Information Criterion Statistics*, D. Reidel publishers, Dordrecht, Holland, 1986; H. Linhart and W. Zucchini, *Model Selection*, John Wiley, NY, 1986; H. Akaike, Information Theory and an Extension of the Maximum Likelihood Principle, in *Proceedings of the Second International Symposium on Information Theory*, eds. B. Petrov and F. Czakil, Budapest: Akademiai Kiado, pp 267-281; and H. Bozdogan, Model Selection and Akaike's Information Criterion (AIC): The General Theory and Its Analytical Extensions, *Psychometrika*, 52(1987), 3(Sep), pp 345-370.)

### DISTRIBUTION OF VALUES

#### FEBA Study Advance Rates

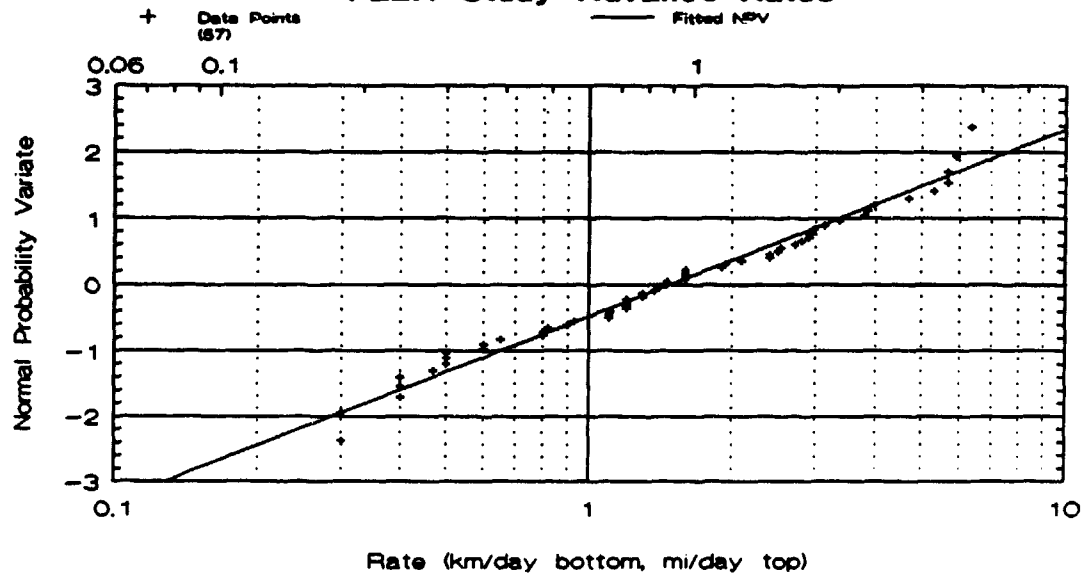


Figure 3-1. Distribution of Advance Rate Values

### AUTOCORRELATION ANALYSIS RESULTS

#### Multiple Databases

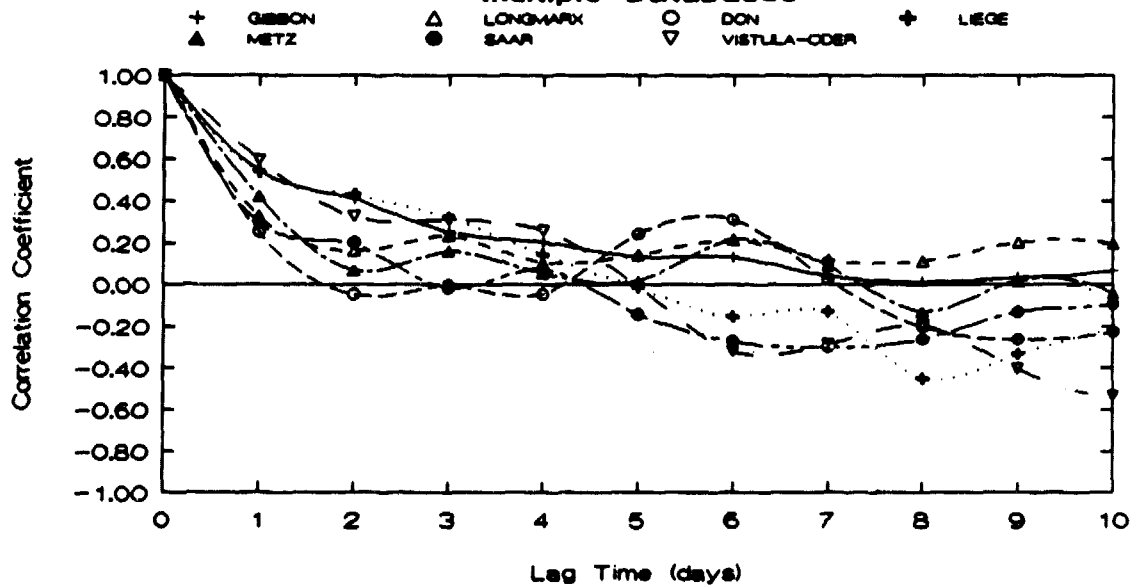


Figure 3-2. Autocorrelation Analysis Results

(4) The data have tacitly been assumed to follow a smooth distribution. Yet the reported data are very "grainy" because of the tendency to report certain simple values (e.g., 1 km/day, or to use units of 0.1 km/day). Such graininess is apparent in the data used for the study of 60 cases, as can be seen in Figure 3-2.

(5) No allowance has been made for the successful advance bias and the reported versus actual bias mentioned in Chapter 2.

(6) Correlations between events that are close together in time or space have been ignored. On general principles, one expects such correlations, provided events are sufficiently close in time and space. We did not attempt to compute correlations between events separated in space because the reported data on this are too skimpy. We did find seven cases for which the distance advanced on each of several consecutive days was reported. The correlation between advance distances as a function of separation in time can be computed for each of them. Those correlations are shown in Figure 3-2. It shows that the correlations are significant for consecutive days. However, within 3 to 5 days the correlation dies away to levels that are no longer statistically significant. (The derived data bases used for this figure are described in paragraph B-7 of Appendix B.)

3-5. SURVEY FINDINGS. The following principal findings regarding past work are an expansion and refinement of those in CAA-RP-90-3.

a. The literature is full of loud but wildly conflicting claims. Many of these "findings" are merely *post hoc* rationalizations or hasty overgeneralizations with little objective basis in fact.

b. Appreciable difficulties for this and for future quantitative work have been created by the use of subjective/qualitative descriptors not defined in terms of objectively measurable quantities (e.g., the use of such subjective/qualitative descriptors as "intensity of enemy opposition," "degree of difficulty of the terrain," and the like).

c. Several epistemological weaknesses affect past work. Among the more important are:

(1) Inadequately caveating hasty and premature overgeneralizations based on only a small number of cases, or on a narrow sample of cases representing only a particular time and operational context.

(2) Theory and observation are seldom compared directly, quantitatively, and in detail.

(3) Despite their effectiveness in other contexts, powerful multivariate statistical methods have been singularly unsuccessful when dealing with advance rates. Some possible reasons for this are:

(a) There has been a habit of using far too many variables. This, in turn, has led to excessive overfitting. The use of formal model selection criteria would help to control these excesses.

(b) Excessive focus on purely statistical "significance" at the expense of the practical importance of the results. There has also been too much focus on "trends" and too little attention to the variability of the data above and below the trend line.

(c) Normality has been assumed, sometimes tacitly, without adequate justification. Yet the data are highly skewed and poorly represented by normal distributions.



(d) The data have tacitly been assumed to follow a smooth distribution. Yet the reported data are very "grainy" because of the tendency to report certain simple values (e.g., 1 km/day).

(e) No allowance has been made for the successful advance bias and the reported versus actual bias discussed in Chapter 2.

(f) Correlations between events that are close together in time or space have been ignored.

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## CHAPTER 4

## ANALYSIS

4-1. **OBJECTIVE.** This chapter gives the results of our analyses of advance rates in land combat operations.

4-2. **BACKGROUND.** A systematic analysis of a wide range of hypotheses is needed to clarify the strength of the evidence for or against them. This chapter uses the primary and derived data bases to address systematically a range of hypotheses. Because our data are so extensive, we have been able to shed new light on some of these hypotheses. No doubt not all of the interesting hypotheses have been addressed. Indeed, some very interesting and plausible hypotheses cannot be studied adequately with the data at hand. For example, the influence of physical fitness on advance rates cannot be studied because the data we have compiled do not record that property of the forces.

4-3. **APPROACH.** After considering several candidates, we decided to address the hypotheses or propositions listed below. They were chosen because of their importance, because they can be studied with the data at hand, and because they could be addressed within the time remaining on this project. We will find that some of these propositions *are* supported by the reported data, some *are not*, and some are consistent with the reported data only after they are stated more precisely.

a. Rates of advance are about equal to the transport speed. That is, a force's advance rate is dictated primarily by the speed of its governing mode of transportation and not by other factors.

b. Rates of advance are much higher now than in the past. That is, modern developments have produced great increases in the speed with which forces move.

c. Rates of advance are much higher for small units. By this we mean that advance rates rapidly decline as the force size is increased.

d. Rates of advance are much higher for motor (and horse) than for foot. In other words, mechanized, motorized, and horse-mounted units advance far more rapidly than infantry units.

e. Rates of advance are higher for lightly engaged forces.

f. Rates of advance are higher in summer than in winter. This is a surrogate for a family of propositions to the effect that advance rates are higher in better weather, on ground with better footing, etc.

g. Rates of advance are lower for longer operations. That is, the advance rate declines the longer an operation lasts. Hence, the longer an operation lasts, the slower is its average advance rate.

h. Rates of advance are normally distributed.

i. Rates of advance are determined by force ratios. That is, advance rates are very sensitive to changes in force ratio, and force ratio is the most important single factor governing advance rates.

j. Rates of advance are predictable and generalizable. By this we mean that advance rates are sufficiently well understood that we can rather accurately and reliably estimate what the advance rate will be, both in future times (prediction) and in new tactical and operational situations (generalization).

#### 4-4. RATES OF ADVANCE AND SPEED OF THE GOVERNING MODE OF TRANSPORT

a. The proposition to be addressed is that advance rates are primarily dictated by, and in fact are about equal to (i.e., within a factor of 2 or 3 of) a unit's governing mode of transportation. By "governing mode of transportation" we mean the mode of transportation that governs or most strongly influences the reported advance rates. For example, the governing mode for pure infantry is foot, but for horse-mounted troops it is horse.

b. One view of this proposition is that a unit can move about as rapidly as its governing mode of transportation. If this view is correct, then we certainly expect that units whose governing mode is tank or truck will sustain much higher speeds than any human on foot can match. Another view is that the equipment makes little difference, and that advance rates are controlled mainly by factors other than the mechanical capability of the vehicles.

c. Figure 4-1 shows the maximum rated road speeds of several tanks or heavy trucks, according to Jane's.<sup>8</sup> The tank data include the French AMX-30 and -40; German Leopard I and II; Israeli Merkava; USSR T-80, T-72, and T-64B; United Kingdom Challenger, Chieftain, and Centurion; and US M1A1, M60, and M48. In each case the tanks are assumed to be combat-loaded and the trucks to be carrying their rated loads. Although the *maximum* road speeds are shown, it would seem that—even when loaded—almost all of these vehicles ought to be able to sustain road speeds of at least 20 to 40 km/hr. That would amount to about 200 to 400 km/day if we assume they operate an average of 10 hours per day. A figure of 300 km/day seems a good nominal figure. So if this paragraph's proposition is close to the truth, and these vehicles are a unit's governing mode of transportation, we anticipate that it should be able to sustain an advance rate of about 300 km/day—especially when it is unopposed and trying to go as fast as possible.

d. To address this issue, we created the new or derived data bases GUINFOOT, MISC, RECROW, and CDB90FT data bases. They are described briefly below. Figure 4-2 shows a chart of these data.

(1) The GUINFOOT data base is an extract of the male and female running, walking, hurdling, steeplechase, relay, and other cases of walks or runs afoot from recent issues of the Guinness Book of World Records<sup>tm</sup> (129 data points). The GUINFOOT data span about eight orders of magnitude in time ( $10^{-4}$  days  $\approx$  8.6 seconds;  $10^3$  days  $\approx$  2.75 years) and about  $6\frac{1}{2}$  orders of magnitude in distance. They include distances as short as the 50-meter dashes and 60-meter hurdles, and as long as from the northernmost point of Alaska to the southernmost tip of South America. Note that the GUINFOOT points all cluster relatively closely to their solid trend line (i.e., within a factor of 2 or so), despite the fact that they span such a large range of values. There is only a slight amount of diminishing returns to scale with increasing time.

(2) The MISC data base is a selection of 13 cases from recent issues of the Guinness Book of World Records<sup>tm</sup> and our MISCROAD data base for movements by car, wagon train, etc. The MISC data base includes the Marathon run. I figure a family can easily go 1,000 km in a day by modern automobile on modern interstate highways (about 620 miles). The Pony Express and the Iditarod dog sled race fall close together. Wagon trains on the Oregon Trail typically fell about where indicated. They generally included 1,000-1,500 people, 120 wagons, and 5,000 head of stock. The bathtub push is

<sup>8</sup> Christopher F. Foss (ed), Jane's Armour and Artillery, 1988-89, Jane's Information Group, 163 Brighton Road, Coulsdon, Surrey, CR3 2NX, UK, 1988. Christopher F. Foss and Terry J. Gander (eds), Jane's Military Logistics, 1989, Jane's Information Group, 163 Brighton Road, Coulsdon, Surrey, CR3 2NX, UK, 1989.

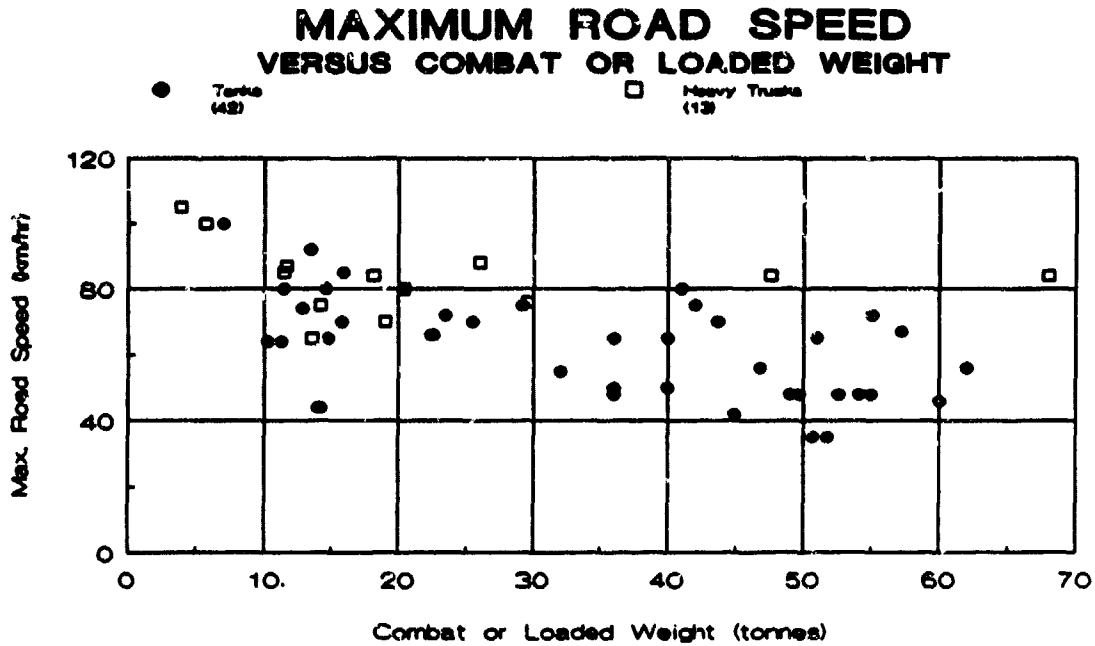


Figure 4-1. Maximum Rated Road Speeds of Tanks and Heavy Trucks

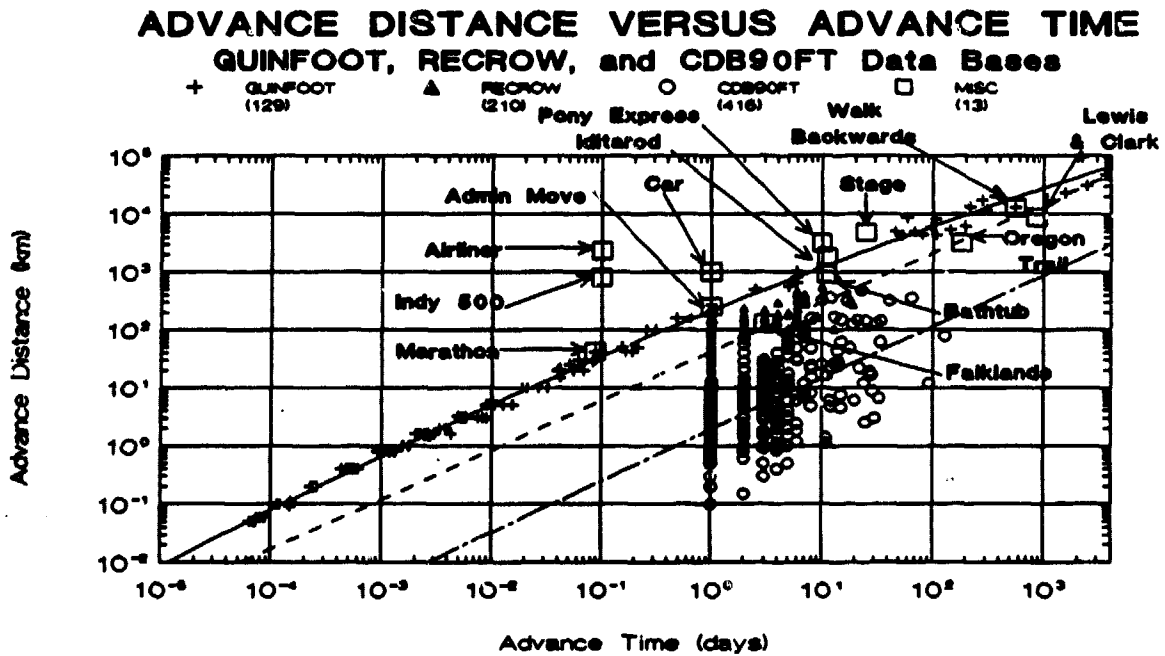


Figure 4-2. Examples of Advance Distances and Advance Times

done by a team pushing a bathtub mounted on wheels. One fellow managed to walk backwards a long way, and at a pretty good clip! The point for administrative moves at about 200 km/day may be a little high. The few cases of administrative advances reported in our WAINSTEI primary data base range from about 50 to 120 km/day.

(3) The RECROW data base has 210 cases of unopposed movements by motor selected from our RECORD and ROWLAND primary data bases, which were originally compiled by Jeffrey Record and David Rowland. Both Record and Rowland were seeking the upper limit of speed for movement by land combat forces. So these data are about as fast as land combat forces can go, even when they are unopposed and motor governs their speed. Note that the RECROW data fall about an order of magnitude below the Guinness foot curve. This suggests that, on the average, unopposed forces moving by motor typically are standing still about 90 percent of the time! They also have substantially more scatter (i.e., they are within a factor of ten of their dashed trend line, as opposed to a factor of two). The unopposed Falklands march by British commandos falls right in line with the RECROW unopposed movement data, even though it was by foot rather than by motor.

(4) The CDB90FT data base consists of 416 cases of heavily engaged forces fighting primarily on foot. It was derived from the CDB90 data base. Note that its dot-dashed trend line is another order of magnitude below RECROW's, or about two orders of magnitude below the Guinness foot line. This suggests that on the average heavily engaged forces fighting on foot are standing still about 99 percent of the time! It shows even more scatter than the RECROW data.

e. Could the delays be due primarily to stops to perform such command, control, and communications (C<sup>3</sup>) functions as: (i) assess the situation, (ii) decide what to do about it, (iii) do it, and (iv) be confronted with a new situation? These include coordination delays while waiting for support from artillery, armor, air, communications, supply or logistics, engineers, medical or medevac, etc. In any event, it is quite clear from Figure 4-2 that the advance rates of land combat forces are at least 1 or two orders of magnitude below that of their principal modes of movement. And this is true even for "best case" conditions of unopposed movement by units moving by motor. Opposed advance rates are at least another order of magnitude lower than that. Hence, for unopposed as well as for opposed conditions, advance rates of land combat forces are far below those of their principal modes of transport. Accordingly, it doesn't seem likely that improvements in the technological capabilities will greatly improve advance rates. No theory or wargame I know of explains this large discrepancy between the speeds of the principal modes of transport and those actually achieved by land combat forces. The best ones simply take it as a given. The worst ones don't even take it into consideration.

f. These considerations lead us to formulate a new HYPOTHESIS: The advance rate of large forces is largely dictated by a combination of logistics and C<sup>3</sup> factors that have little to do with the inherent mobility capability of the force's individual elements. A lot of time is spent deciding what to do next, waiting around for support (artillery, armor, air, engineer, medical, signal, ammo resupply, transportation, etc.), or collecting information (probes and reconnaissance). We did not have time to explore this hypothesis in depth during the course of the Fellowship. But we will return to it from time to time as we go through the analysis. As we take up the other propositions, our analyses will shed some additional light on it.

#### 4-5. RATES OF ADVANCE NOW AND IN THE PAST

a. The proposition to be addressed is that modern developments have produced great increases in the speed with which land combat forces move.

b. In view of the previous paragraph's findings, one might suspect that modern developments in transportation technology have little effect. However, there may be other factors that could cause an increase in advance rates over the years. For instance, if the hypothesis put forward at the end of the preceding paragraph is correct, then most of the delays might be due to coordination difficulties, and modern communications equipment might speed up coordination and thus accelerate advance rates. And if advance rates in the past are consistently lower than they are today, then past experience may be a poor guide to the future. However, some past advance rates have been cited as examples to be emulated in modern times. For example, the 1986 issue of Field Manual (FM) 100-5, *Operations*,<sup>9</sup> referring to GEN U. S. Grant's Vicksburg Campaign, states that "The same speed, surprise, maneuver, and decisive actions will be required in the campaigns of the future." The field manual notes that Grant set a pace so rapid his enemies couldn't keep track of him and covered 200 miles in 19 days (16.9 km/day). (Interestingly enough, though, the Civil Rights marchers walked about 50 miles from Selma to Montgomery at about 17.9 km/day, even though they included several middle-aged and elderly men and women.) This contrasts starkly with the reported rate of 13 km/day for "rapid movement" in Italy during the WW II Anzio Campaign of May-July 1944.<sup>10</sup> For the Soviet WW II Vistula-Oder Operation, which took place in January--February of 1945, it is reported<sup>11</sup> that "...rifle units, supported from the air by front aviation, moved ahead at the top possible speed--up to thirty kilometers a day." Roman legions were said to have marched along the famed Roman roads at up to 30 miles a day (48 km/day). They did about 3 times a month make 20 mile (32 km) training marches, fully armed and through rough and steep as well as level terrain, in about 8 hours.<sup>12</sup> The earliest military campaign of which we have a written account<sup>13</sup> was that of the Egyptian Pharaoh Thutmose III, the 22-year-old son of Thutmose I by a concubine. Believing him too young to maintain the empire created by his father, the Syrians rebelled. But he set off in the year of his accession and by marching through Kantara and Gaza at twenty miles a day [32 km/day for about 20 days, which falls *exactly* on the RECROW trend line of Figure 4-2!], confronted the rebel forces at Har-Megiddo (i.e., Mt. Megiddo). In the same pass where in 1918 the British defeated the Turks, Thutmose III, 3,397 years earlier, defeated the Syrians and their allies. For the next 6 months, he consolidated his victory by campaigning through western Asia, overcoming all opposition, taxing and levying tribute before returning triumphantly to

<sup>9</sup> US Army Field Manual (FM) 100-5, *Operations*, Headquarters, Dept. of the Army, May 1986, pp 91-94.

<sup>10</sup> J. Duncan Love, *Artillery Usage in World War II*, Vol II, Technical Memorandum ORO-T-375, Operations Research Office, Bethesda, MD, April 1959, pp 71 and 125. AD-208 021

<sup>11</sup> V. Larionov, N. Yeronin, B. Solovyov, and V. Timokhovich, *World War II: Decisive Battles of the Soviet Army*, Progress Publishers, Moscow, 1984, p 388. Translated from the Russian by William Biley.

<sup>12</sup> Vegetius, *The Military Institutions of the Romans*, circa 380 AD, Trans. by LT John Clark, Stackpole Books, Harrisburg, PA 1960. Reprinted July 1965 by the Telegraph Press, Harrisburg, PA.

<sup>13</sup> Will Durant, *Our Oriental Heritage*, The Story of Civilization, Vol I, Simon and Schuster, NY, 1954, p 154.

Thebes. In a footnote to this passage, Durant remarks that "Allenby took twice as long to accomplish a similar result; Napoleon, attempting it at Acre, failed." Apparently, FM 100-5 might have found Thutmose III's campaign an even better exemplar than Grant's.

c. Well, let's leave off these anecdotes and look more systematically at the data. For these analyses we created derived data bases for various combinations of governing mode of transportation, degree of combat engagement, and the use of daily as opposed to active time periods. These categories were chosen in part to be sufficiently coarse that (with rare exceptions) individual cases could unequivocally be assigned to them.

(1) The governing modes of transportation were foot, as opposed to horse or motor. Infantry units were grouped with the foot mode, as were cases of combined arms where the foot-mobile elements clearly dictated movements. Motorized, mechanized, and cavalry units were grouped with the horse or motor mode unless they clearly had to adjust their movements to foot-mobile elements. Horse and motor were combined because some preliminary analyses indicated that their rates, distances, and durations were statistically similar.

(2) The degrees of combat engagement were light as opposed to heavy. As a first cut at analyzing advance rates, this distinction seems to us to be more natural, more pertinent, and more likely to be fruitful than attempted distinctions based on quantitative indexes such as force ratios, on subjective or qualitative impressions of the degree of enemy resistance, or on measures of the resistance to maneuver posed by natural phenomena such as terrain and weather. The merit of this choice of categories is part of what we are trying to determine in this work. The criteria of success are whether this choice leads to a statistical population of data on advances that is:

- (a) Homogeneous within itself,
- (b) Relatively stable over time,
- (c) Relatively unaffected by further subdivision according to other possible distinctions, and
- (d) Subject to relatively simple statistical laws.

Hence, in this paper we divided the data into advances of lightly engaged forces and advances of heavily engaged forces in order that we may see by its results whether this distinction is fruitful and helpful in analyzing advance rate data. It is hard to give a precise intensive definition (i.e., one which uses other words to express the definition) of what is meant by a "lightly engaged" force. But in works on military history, the concept is often expressed by evocative phrases such as "lightly engaged," "light, scattered, or intermittent resistance," "relatively or practically unopposed," "met with slight resistance," "advanced against unprepared strong points," "exploitation of a breakthrough," "pursuit," etc. In contrast, heavily engaged forces are locked together in combat and neither side can advance autonomously. It is easier to give an extensive definition (i.e., to point out a number of examples illustrating situations in which the term applies, and contrasting them with other situations in which the term does not apply). Here it suffices to draw attention to Appendix B's definitions of the LGT... and HVY... derived data bases. A more precise definition of the distinction seems difficult to come by, although few would deny its reality. Advances of lightly engaged forces tend to be associated with operations or campaigns, rather than with battles or engagements, since the latter are the occasions where forces become engaged.



(3) At least two different kinds of advance distances, times and rates are of interest. The first is the daily and the second is the active kind.

(a) The daily (i.e., daily average) rate is computed from the total distance advanced and the total number of whole days elapsed during the advance. For example, an advance of 3 km between one Morning Report and the next would be counted as a 3 km/day daily average advance rate. So would an advance of 30 km in 10 days. These daily advance rates are of interest for many operational planning and wargaming purposes. Moreover, the available data often give only daily average rates (e.g., the source says only that some unit advanced 3 km on the 14th). When a daily time period is used the corresponding distance and duration will be called simply the advance distance and the advance duration (or, when particular emphasis is needed for clarity, the total or overall distance and duration). And the average advance rate based on total distance and total duration will be called the daily advance rate.

(b) However, the daily advance rate is clearly much lower than the rate actually achieved during the active period of the advance. For instance, if essentially all of the previous 3 km advance actually took place during a 3 hour period of combat activity, then the advance rate during that active period actually averaged 1 km/hour, or 24 km/day. We will refer to such a rate as an active advance rate, and to its corresponding distance and time as the active advance distance and duration. Such active advance distances, times, and rates are of interest for some operational planning and wargaming purposes. Even so, they are an incomplete description of the action, since the 3 hour period of combat activity presumably included many episodes of faster and slower advances, as well as pauses or short interruptions of the action. In general—were they obtainable—both the maximum instantaneous and the true average advance rate during periods of activity would be even higher than the average active advance rate computed from the active distance and duration for a general period of higher activity.

(4) The two categories for the mode of transportation, the two for the degree of engagement, and the two for the kind of time period, can be combined to make eight categories. Since we will have frequent occasion to refer to them, we adopted the abbreviations HVYFOOT for heavily engaged forces moving by foot using daily time periods, HVYHOMO for heavily engaged forces moving by horse or motor using daily time periods, LGTFOOT for lightly engaged forces moving by foot using daily time periods, LGTHOMO for lightly engaged forces moving by horse or motor using daily time periods, HVYFTACT for heavily engaged forces moving by foot using active periods, and HVYHOMOACT for heavily engaged forces moving by horse and motor using active periods. Appendix B describes how each of these derived data bases was created. The LGTFTACT and LGTHOMOACT categories had to be omitted because too few cases are available in our data, leaving six categories that were actually used in this analysis. However, rather than simply discarding the (ten or a dozen) lightly engaged cases for which active times were reported, we lumped them into the LGTFOOT and LGTHOMO data bases. Accordingly, the reader will see a few points for them that fall into the fractional day range of durations.

(5) The number of cases in each of these major derived data bases is shown in Table 4-1. A handful of these cases may be missing a value of the time or distance, so that not all of them can be used in all analyses. The total number of cases in these derived data bases is less than the total of over 7,000 in our primary data bases because we (i) eliminated some duplications, (ii) eliminated some primary data bases as unsuitable—such as the MISCRoad, LONGMARX, RADZIEV, WAINSTEI, XENOPHON, etc., (iii) eliminated some cases with missing values, and (iv) eliminated cases with reported zero advance rates. However, this still leaves us with an order of magnitude more cases than were used in any previous study on advance rates.

Table 4-1. Number of Cases in the Derived Data Bases

Degree of engagement	Foot	Horse	Motor	Total
LGT	526	122	736	1,384
HVY	1,504	16	451	1,971
TOTAL	2,030	138	1,187	3,355

d. Figure 4-3<sup>14</sup> shows daily advance rates versus calendar date for heavily engaged forces whose governing mode was foot (HVYFOOT), heavily engaged forces whose governing mode was horse or motor (HVYHOMO), lightly engaged forces whose governing mode was foot (LGTFOOT), and lightly engaged forces whose governing mode was horse or motor (LGTHOMO). This figure shows about 3,351 data points, which is slightly less than the total in Table 4-1 because some cases had missing values that prevented their inclusion in it. Note that the vertical scale in Figures 4-3c and 4-3d is an order of magnitude higher than in Figures 4-3a and 4-3b. Nothing in Figure 4-3 persuades me that daily advance rates have changed much over at least the last 400 years, or in fact even longer. Certainly any slight trends that may be present are completely overwhelmed by the reported data's wide scatter and high variability. The greatest apparent trend is for the HVYHOMO cases, but it clearly is strongly influenced by a handful of cases from the 1600s with high statistical "leverage." Besides, it is opposite in direction to the apparent trend for the LGTHOMO category. Since we will also find in paragraph 4-5f below that this trend disappears when active rather than daily rates are used, it appears to be more artifact than fact.

e. Figures 4-4 and 4-5 show that neither total advance distances nor durations have changed much over at least the 400 years, either. Figure 4-5<sup>15</sup> shows that total advance durations haven't changed much over at least the last 400 years either.

f. Figure 4-6 shows that active advance rates for heavily engaged forces haven't changed much for at least the last 400 years. It also shows that active advance rates for heavily engaged forces are about the same whether the force is moving by foot or by horse and motor. Figures 4-7 and 4-8 show that for heavily engaged forces, neither the active distances nor durations have changed much over at least the last 400 years. Note that the duration scale used in Figure 4-8 is an order of magnitude lower than that in Figure 4-5, showing that active durations are about an order of magnitude shorter than overall durations. However, by comparing Figure 4-7 to Figure 4-4, we see that the *distances* advanced are about the same whether we use total time or active time. Also, comparing Figure 4-6 to Figure 4-3c and 4-3d shows that when heavily engaged forces *do* enter an active time period, their advance rates are not vastly different from those of lightly engaged forces (i.e., they are within a factor of 2 or 3, rather than a factor of 10).

<sup>14</sup> On this and similar charts, the equations for the fitted trend line are usually in logarithms to the base 10. For example, the equation shown on Figure 4-3a should be interpreted to mean that:

$$\log_{10}(\text{Daily Advance Rate in km/day}) = -0.1299 + 0.0002(\text{Date}).$$

So when *Date* = 2000, this equation implies that *Daily Advance Rate* = 1.86 km/day.

<sup>15</sup> The appearance of Figure 4-5a is a little deceptive. The trend line doesn't *look* as if it goes through the center of the data points. But a *very* large number of data points for 1-day durations plot on top of one another. Even though they do not show separately on this chart, they are so numerous that they drag the trend line downward. The occasional plumes of points for longer durations arise through a combination of the inherent scatter in the data, and the very large number of data points from the American Civil War, WW I, and WW II.

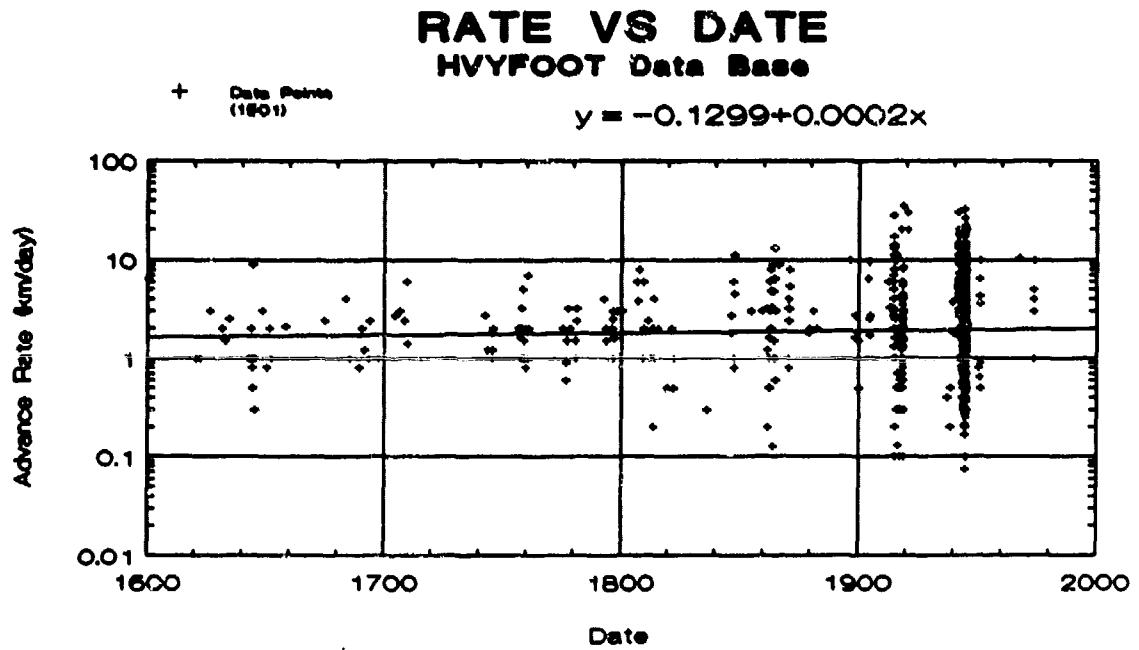


Figure 4-3a. Daily Advance Rate Versus Date for HVYFOOT Data

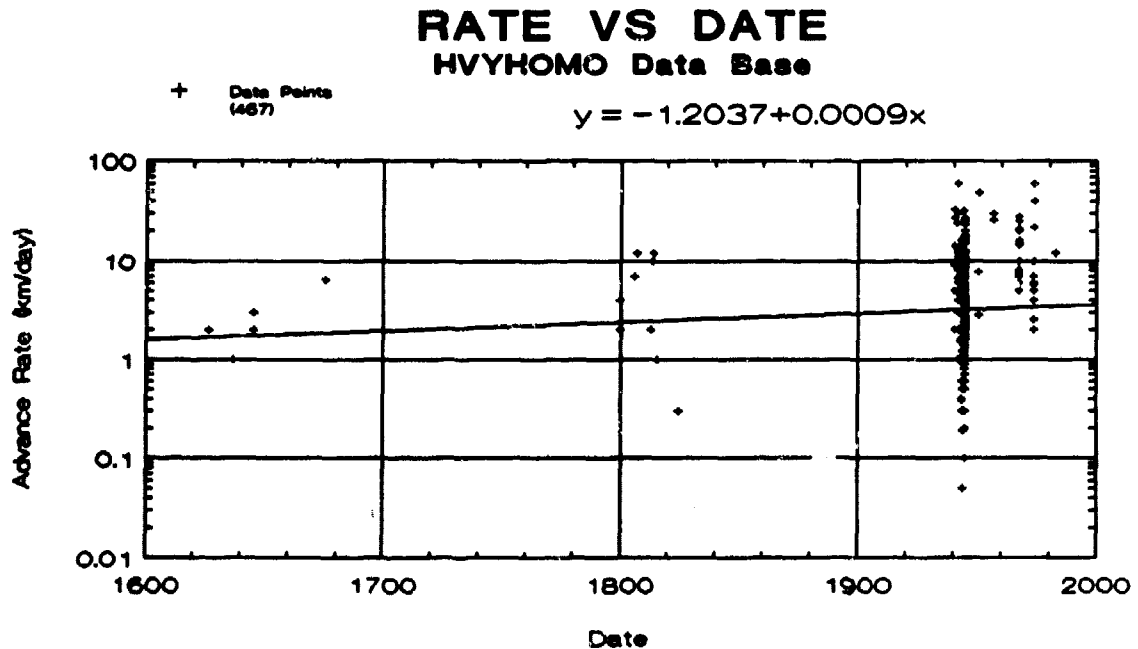


Figure 4-3b. Daily Advance Rate Versus Date for HVYHOMO Data

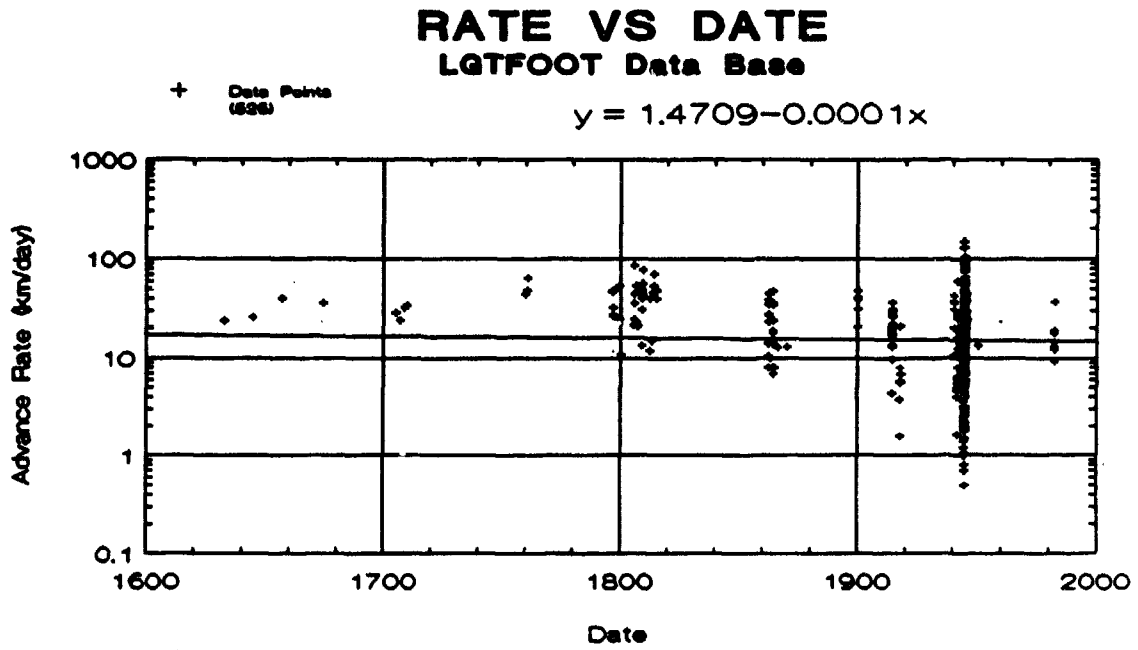


Figure 4-3c. Daily Advance Rate Versus Date for LGTFoot Data

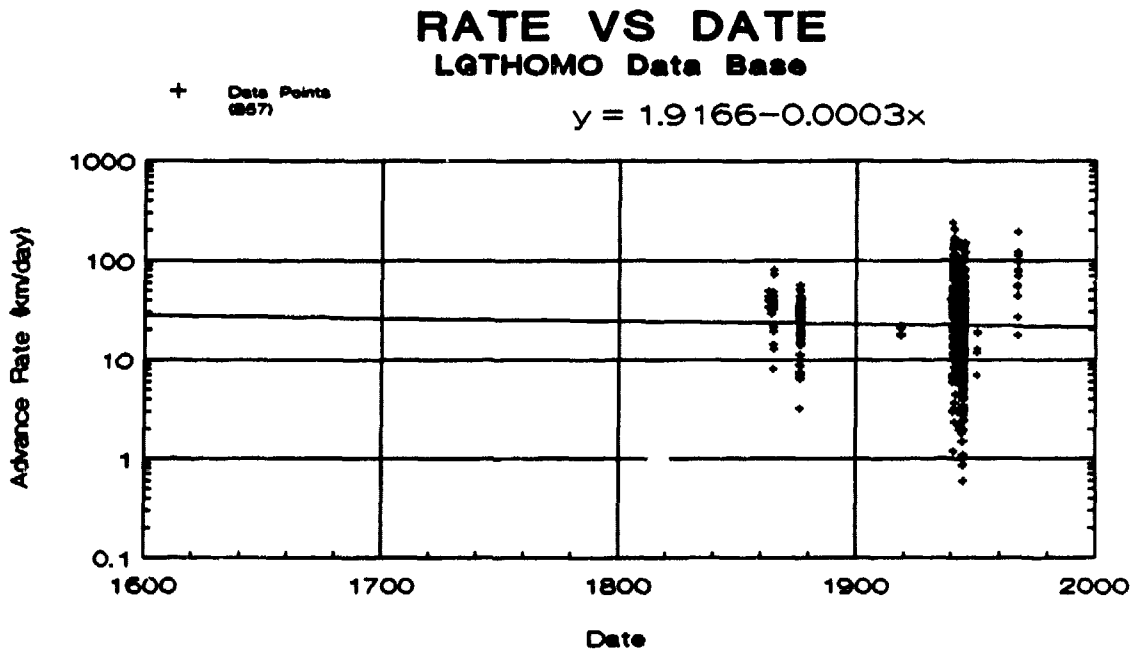


Figure 4-3d. Daily Advance Rate Versus Date for LGTHOMO Data

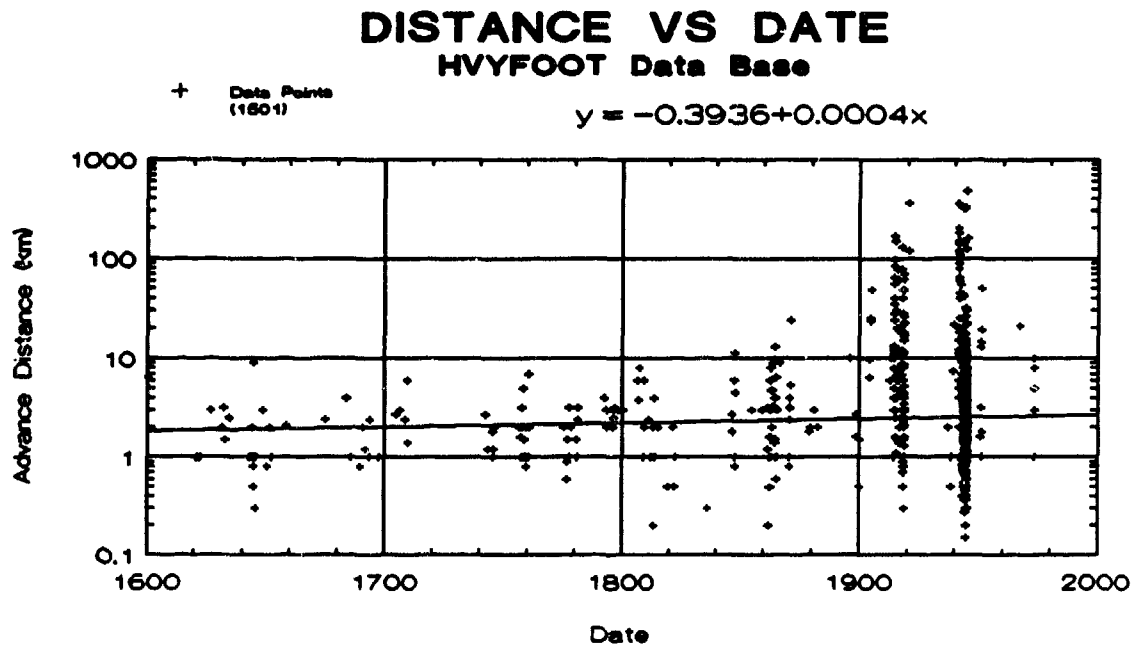


Figure 4-4a. Total Advance Distance Versus Date for HVMFOOT Data

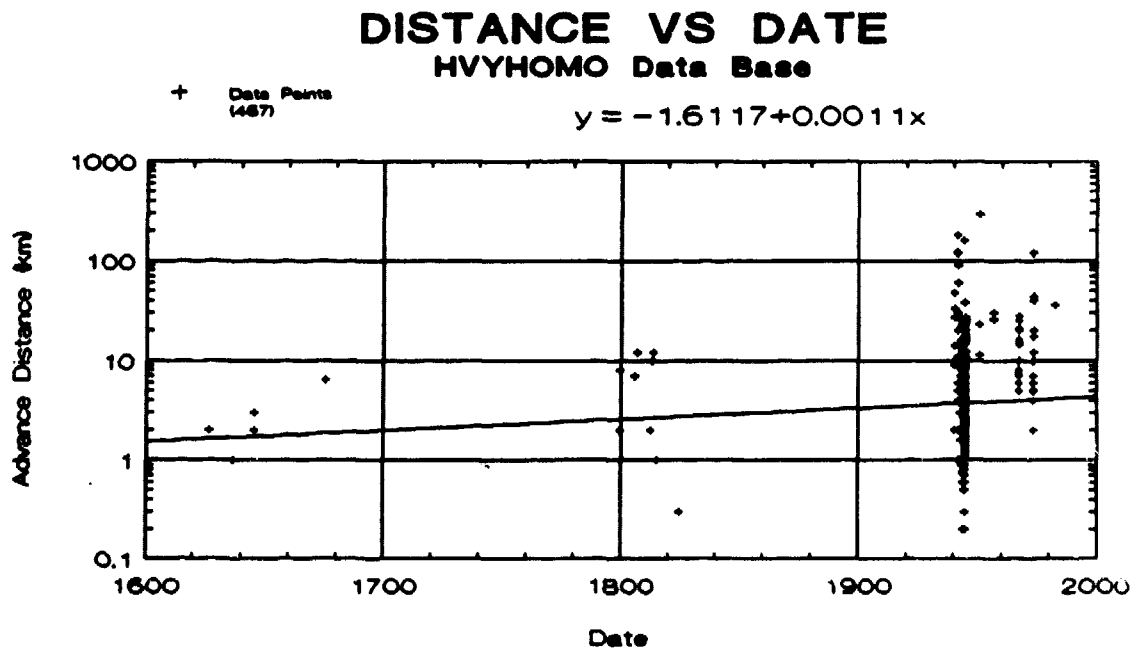


Figure 4-4b. Total Advance Distance Versus Date for HVMHOMO Data

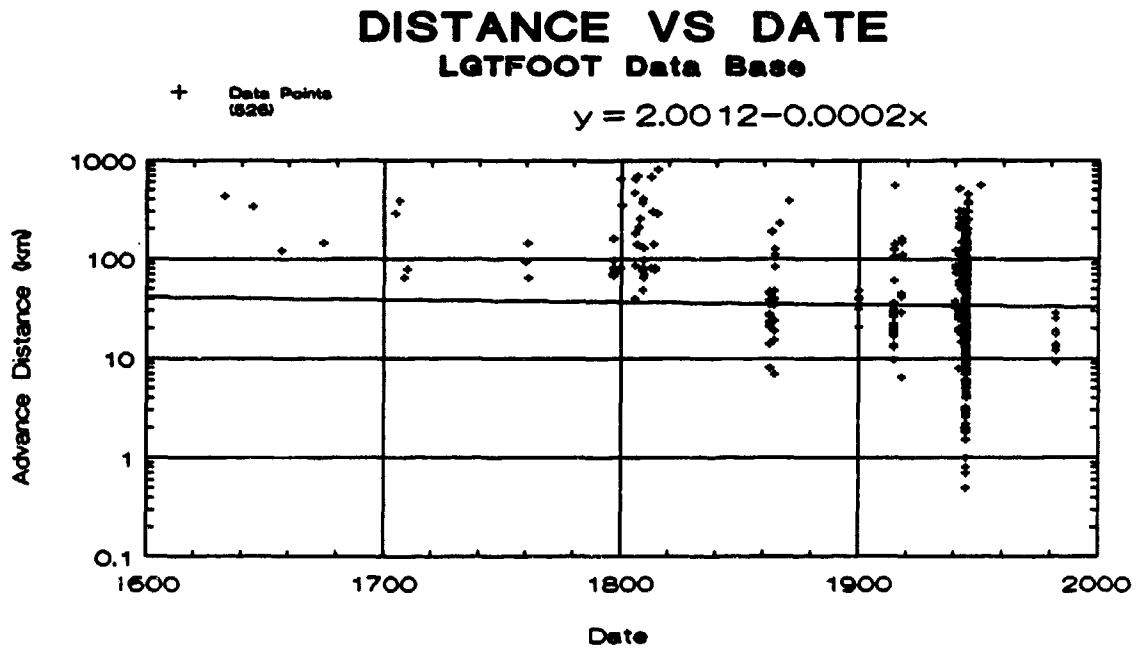


Figure 4-4c. Total Advance Distance Versus Date for LGTFooter Data

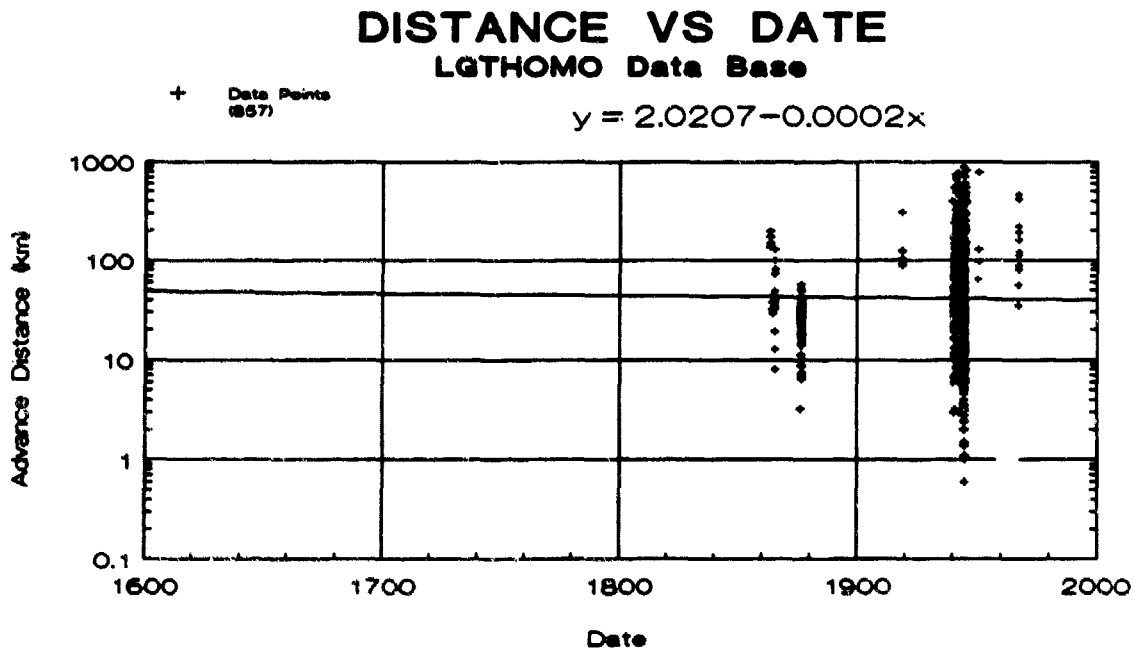


Figure 4-4d. Total Advance Distance Versus Date for LGTHOMO Data

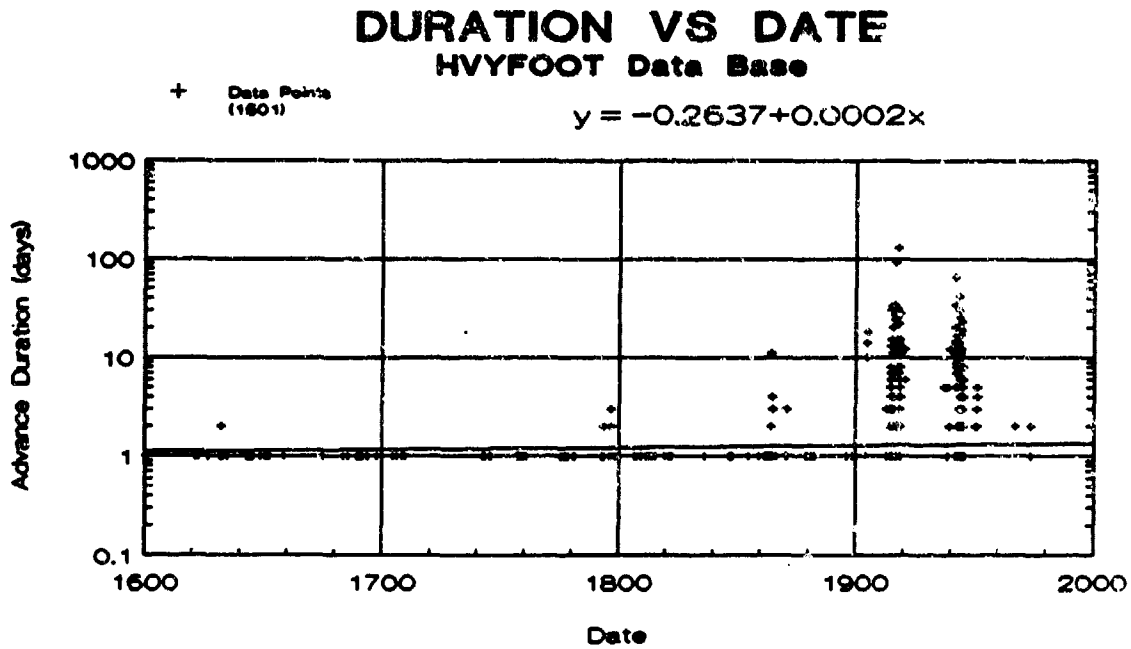


Figure 4-5a. Total Advance Duration Versus Date for HVYFOOT Data

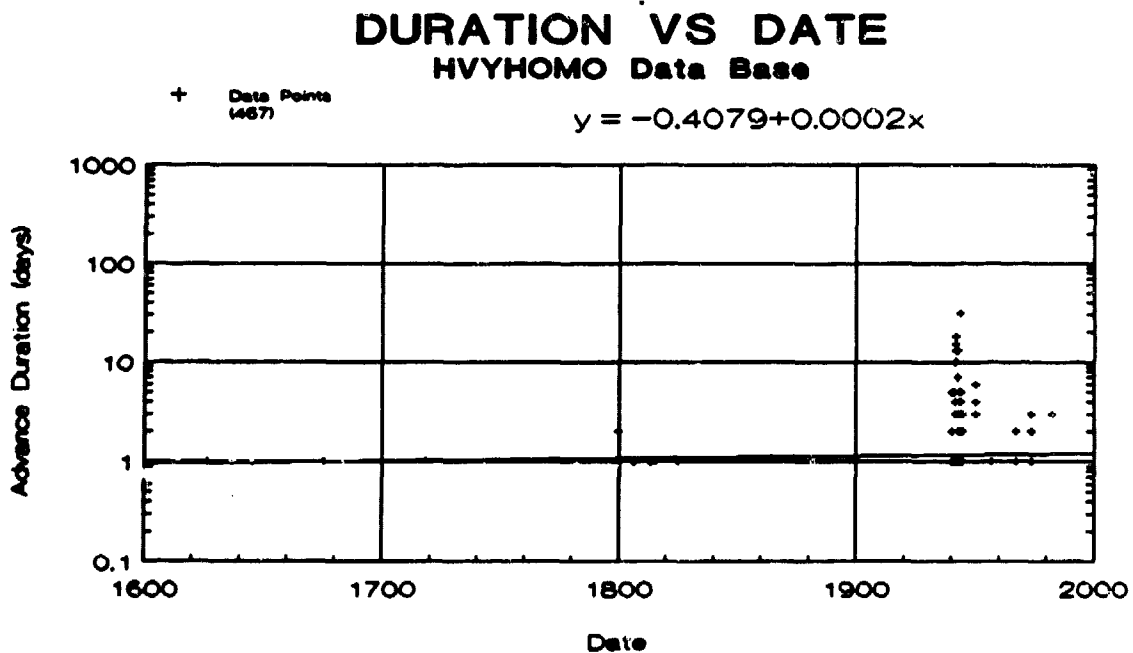


Figure 4-5b. Total Advance Duration Versus Date for HVYHOMO Data

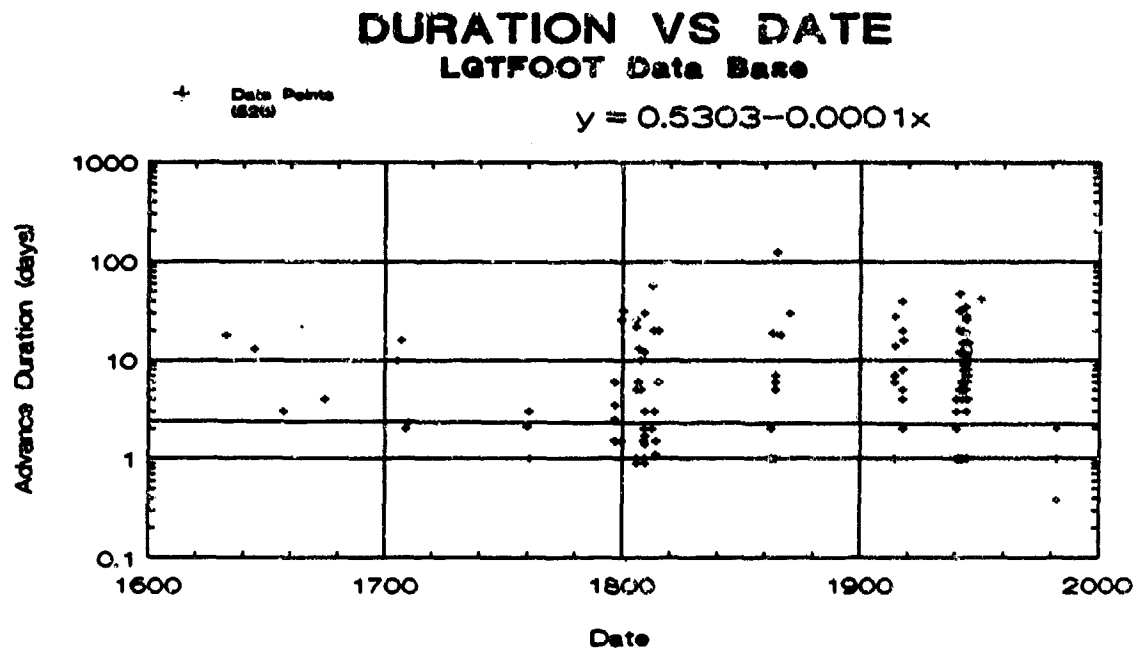


Figure 4-5c. Total Advance Duration Versus Date for LGTFOOT Data

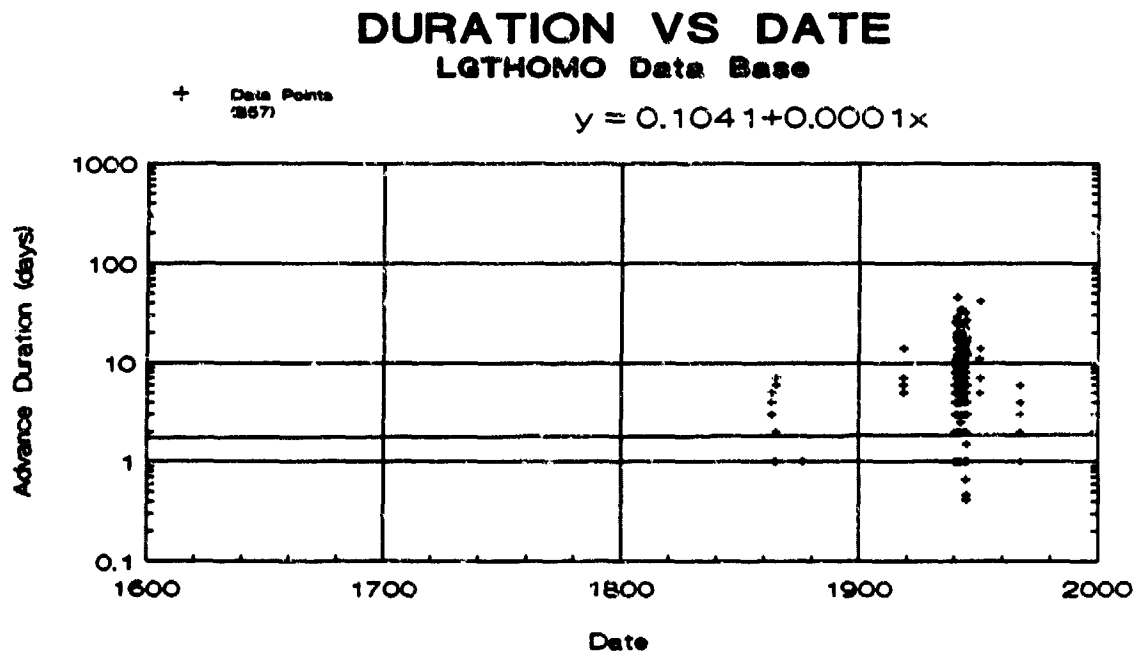


Figure 4-5d. Total Advance Duration Versus Date for LGTHOMO Data



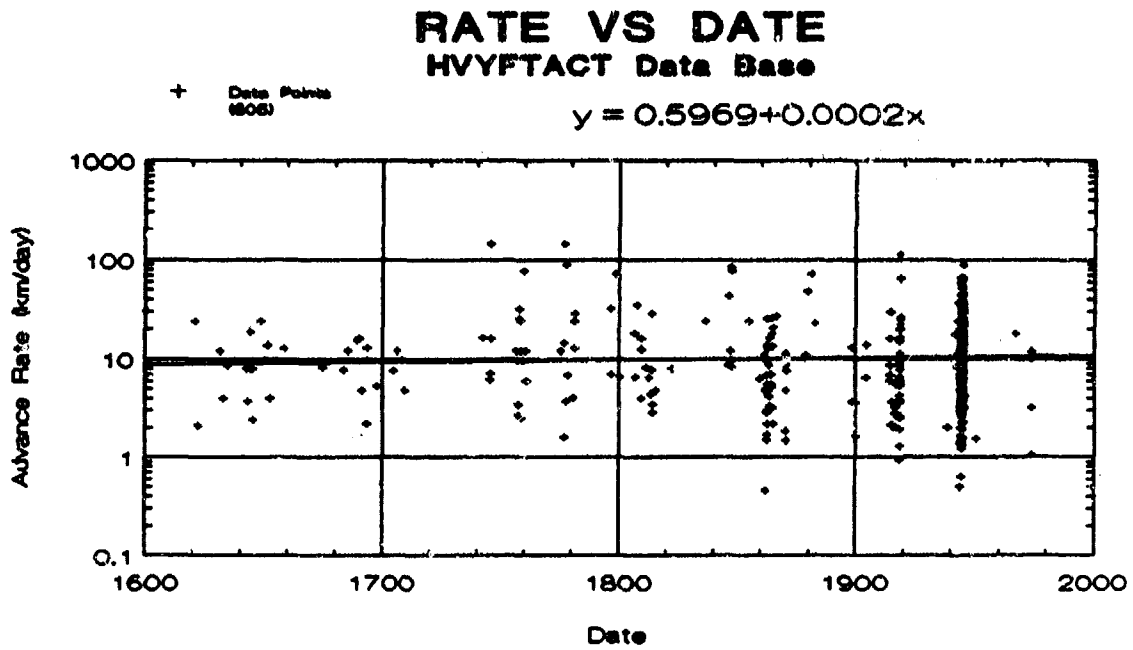


Figure 4-6a. Active Advance Rate Versus Date for HVYFOOT Data

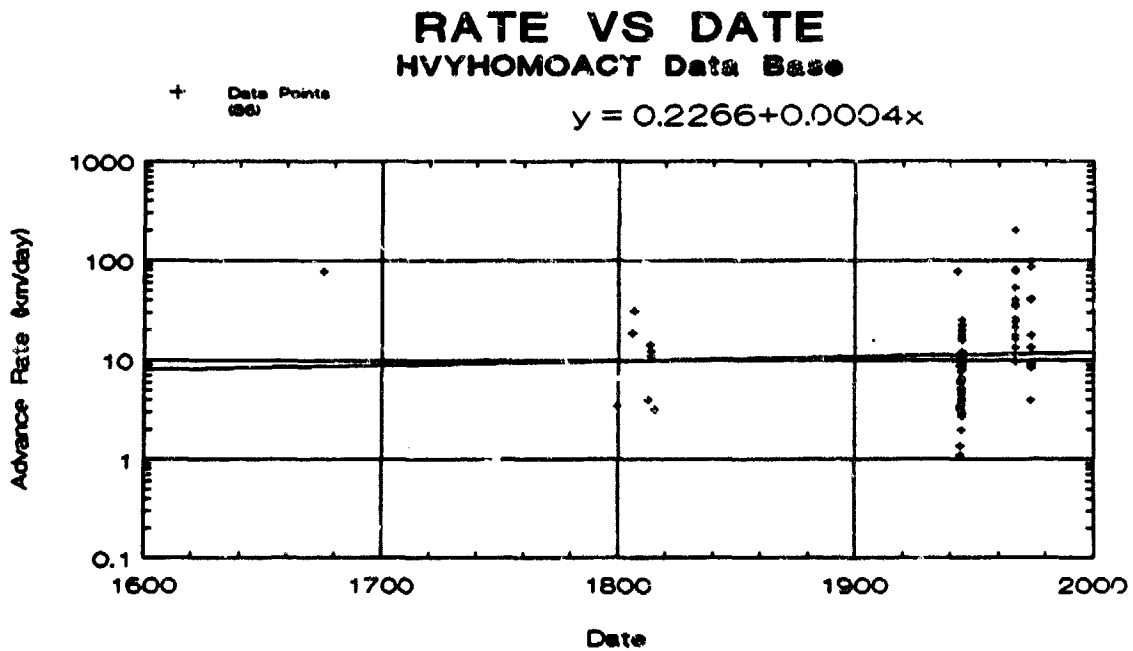


Figure 4-6b. Active Advance Rate Versus Date for HVYHOMO Data

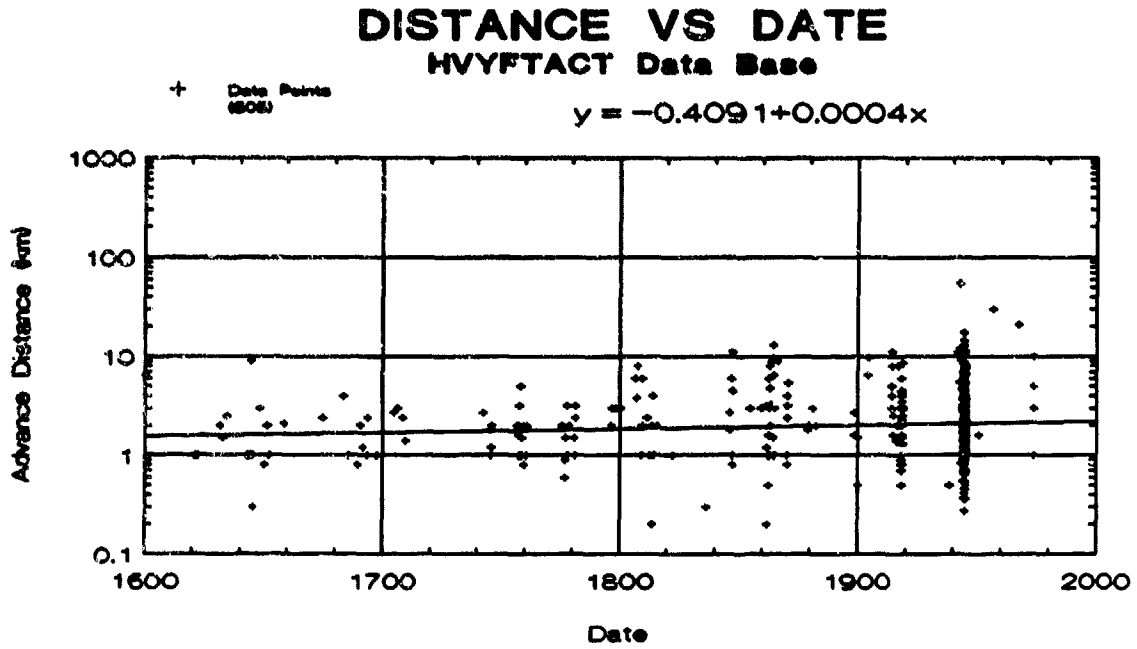


Figure 4-7a. Active Advance Distance Versus Date for HVYFOOT Data

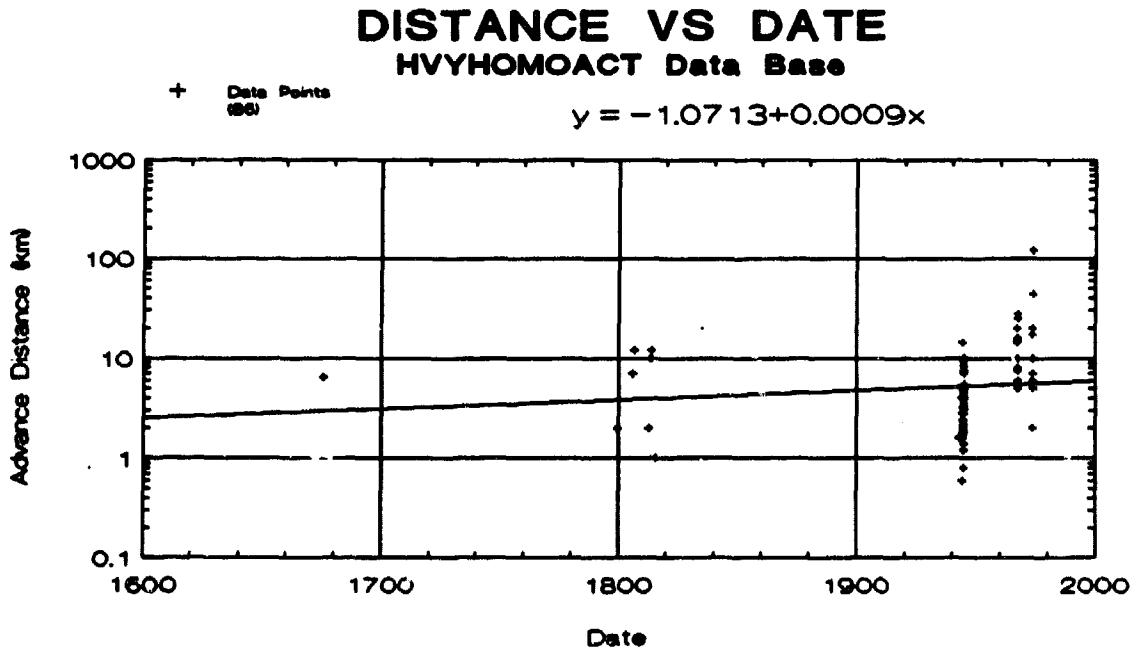


Figure 4-7b. Active Advance Distance Versus Date for HVYHOMO Data

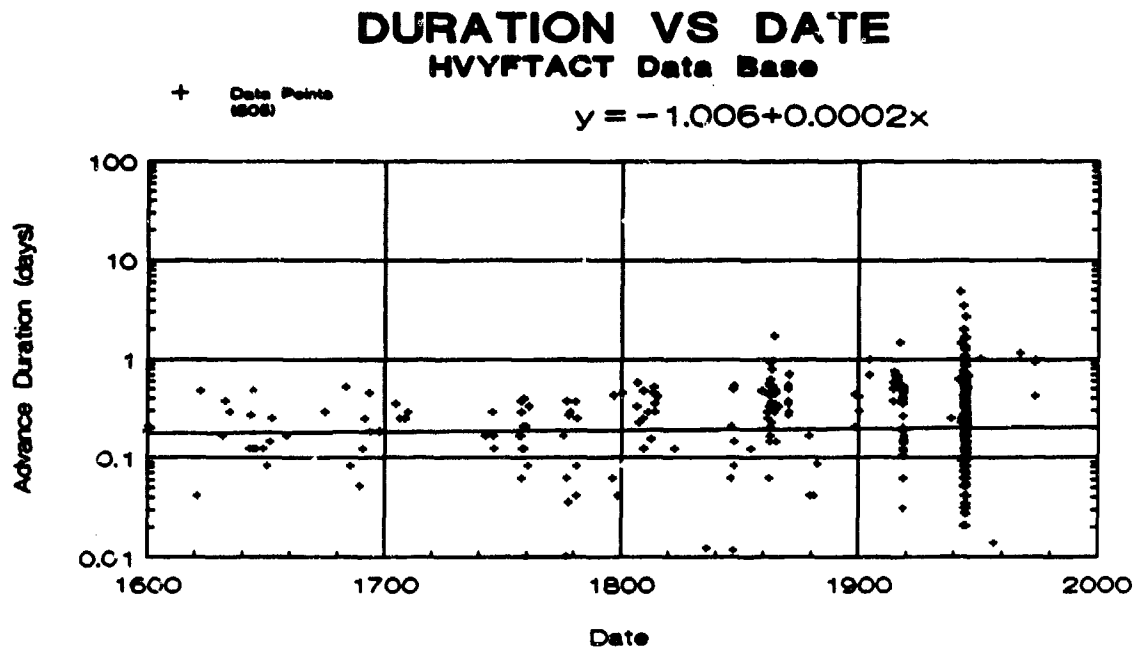


Figure 4-8a. Active Advance Duration Versus Date for HVYFOOT Data

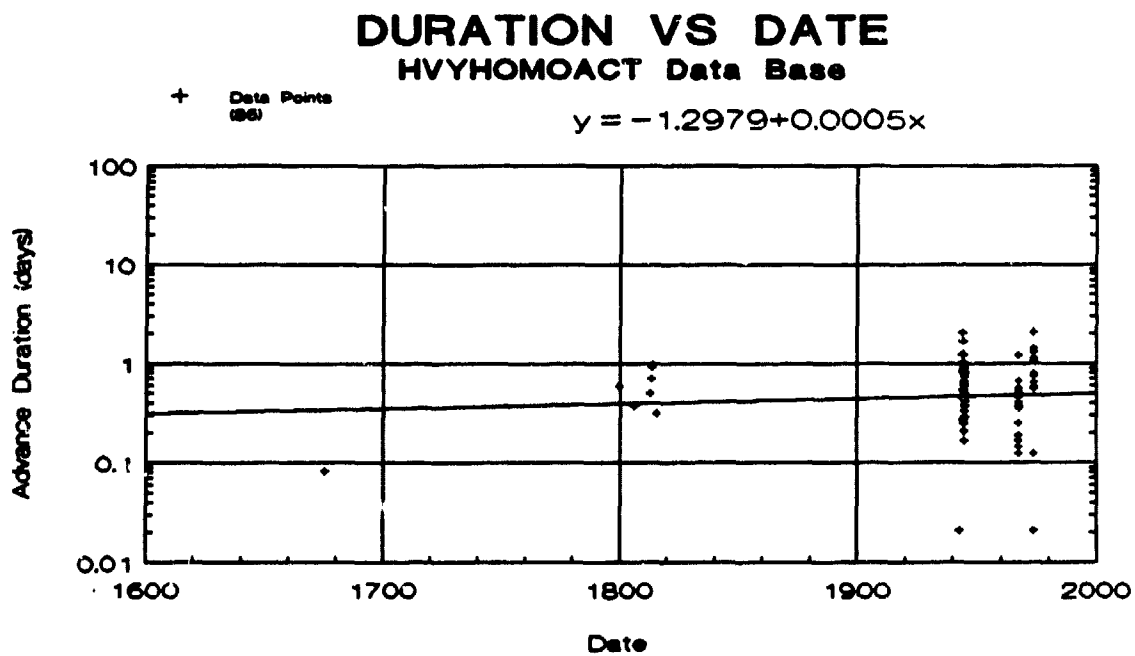


Figure 4-8b. Active Advance Duration Versus Date for HVYHOMO Data

g. The upshot of our analysis is that the reported data show essentially no evidence of any consistent trend in distance, duration or advance rate for at least the last 400 years. But the data scatter widely and are highly variable.

#### 4-6. RATES OF ADVANCE FOR SMALL AND LARGE UNITS

a. The proposition to be examined is that advance rates decline steeply as the force size increases.

b. If the hypothesis that advance rates are largely affected by  $C^3$  delays advanced in paragraph 4-4f above is correct, then one might argue that small units should advance more rapidly than large ones because they have fewer  $C^3$  delays. Moreover, it might be reasoned that  $C^3$  delays compound combinatorially as the force size increases, since there are more sub-units to coordinate and control. Certainly it is a commonplace observation that this is true as the number of travellers in a party increases from 1 or two to six or seven! On the other hand, one could argue that very large forces avoid this combinatorial growth by giving broad guidance and delegating authority for its implementation. (For instance, we recall GEN Eisenhower's one-sentence directive from the Combined Chiefs of Staff<sup>16</sup>: "You will enter the continent of Europe and, in conjunction with the other Allied Nations, undertake operations aimed at the heart of Germany and the destruction of her Armed Forces.") If that is the case, then at some point further increases in the size of the force may not produce further  $C^3$  delays.

c. Figure 4-9 shows daily advance rates versus force size for the HVYFOOT, HVYHOMO, LGTFOOT, and LGTHOMO derived data bases. The trend lines are parabolas fitted to the data points by least squares, and are included only to guide the eye to the predominant trends. Contrary to our expectations, reported daily advance rates actually *increase* with the size of the force, at least for heavily engaged forces! The situation with lightly engaged forces is not so clear, but it appears that within the range of the reported data their daily advance rates don't change much with force size. Again the reported data are widely scattered and highly variable.

d. Figure 4-10 shows total advance distances versus force size for the principal derived data bases. Observe that total advance distances don't change much for force sizes below about 10 or 20 thousand, but increase as the force size exceeds that level. Figure 4-11 shows that this is also true of total advance durations. Presumably this effect mirrors the tendency for corps and army objectives to be much deeper than those assigned to smaller units. Again the reported data are widely scattered and highly variable.

e. Figure 4-12 shows active advance rates versus force size for heavily engaged forces. As before, the trend lines are parabolas fitted to the data using least squares. This figure shows the anticipated trend toward declining active advance rates as the force size increases. However, most of the decline in active advance rates takes place for forces smaller than 1,000 or so. Active advance rates do not drop much further as force size grows from 1,000 to 100,000. Again the data are widely scattered and highly variable.

f. Figures 4-13 and 4-14 show active advance distances and durations versus size for heavily engaged forces. Note that the duration scale used is an order of magnitude lower than in Figure 4-11 (active durations are about an order of magnitude shorter than overall durations). Again the reported data are widely scattered and highly variable.

<sup>16</sup> Dwight D. Eisenhower, Crusade in Europe, Doubleday & Co., Inc., Garden City, NY, 1948, p 225.

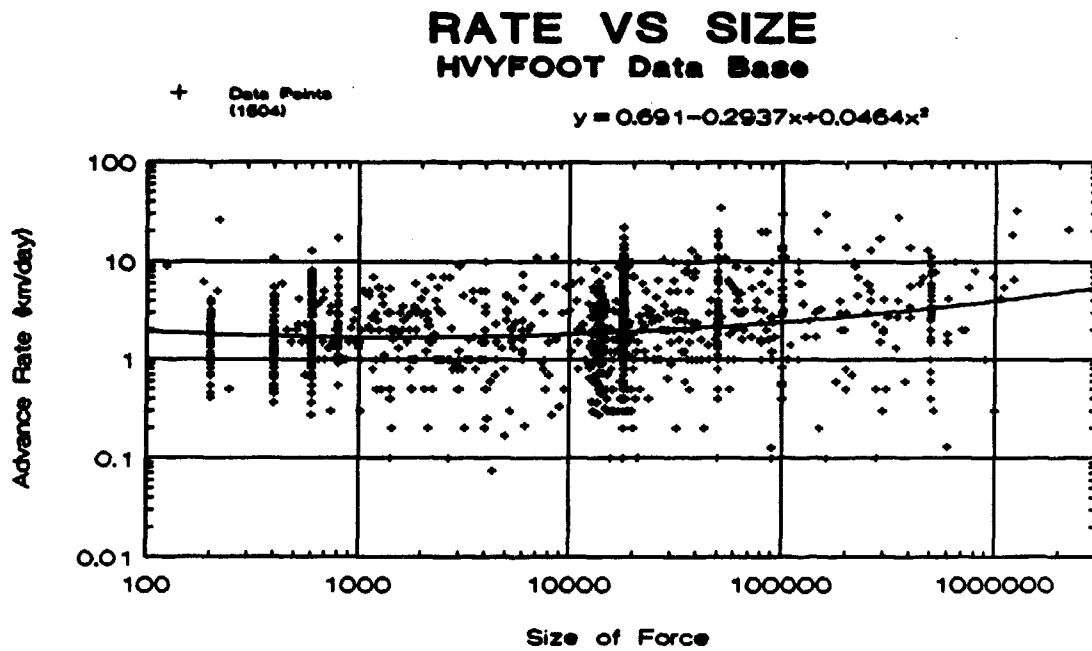


Figure 4-9a. Daily Advance Rate Versus Size for HVYFOOT Data

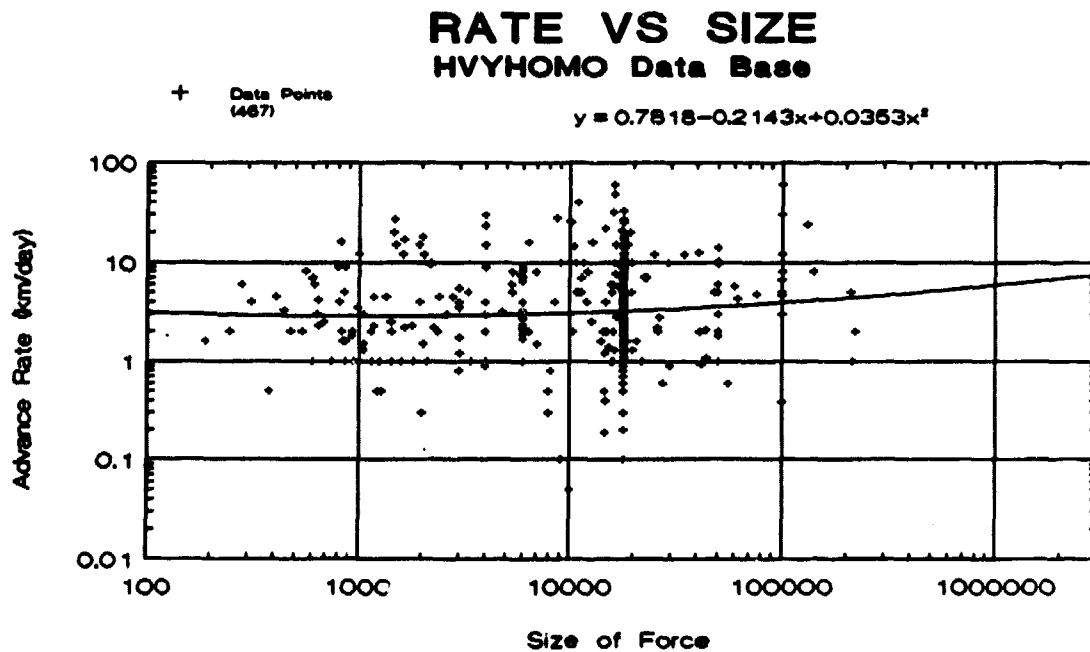


Figure 4-9b. Daily Advance Rate Versus Size for HVYHOMO Data

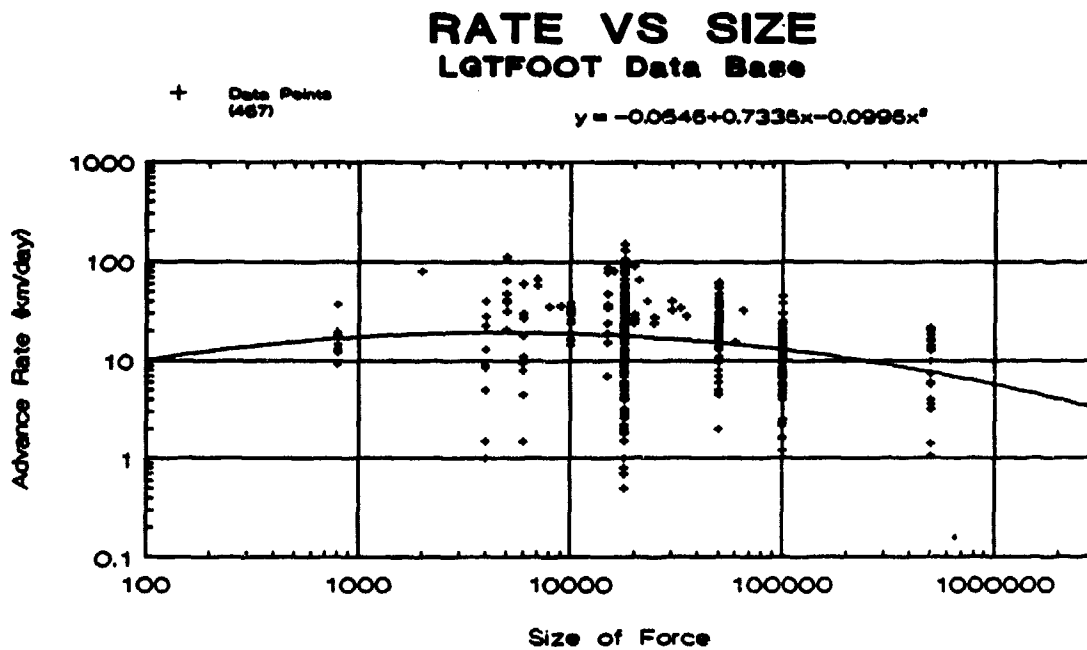


Figure 4-9c. Daily Advance Rate Versus Size for LGTFooter Data

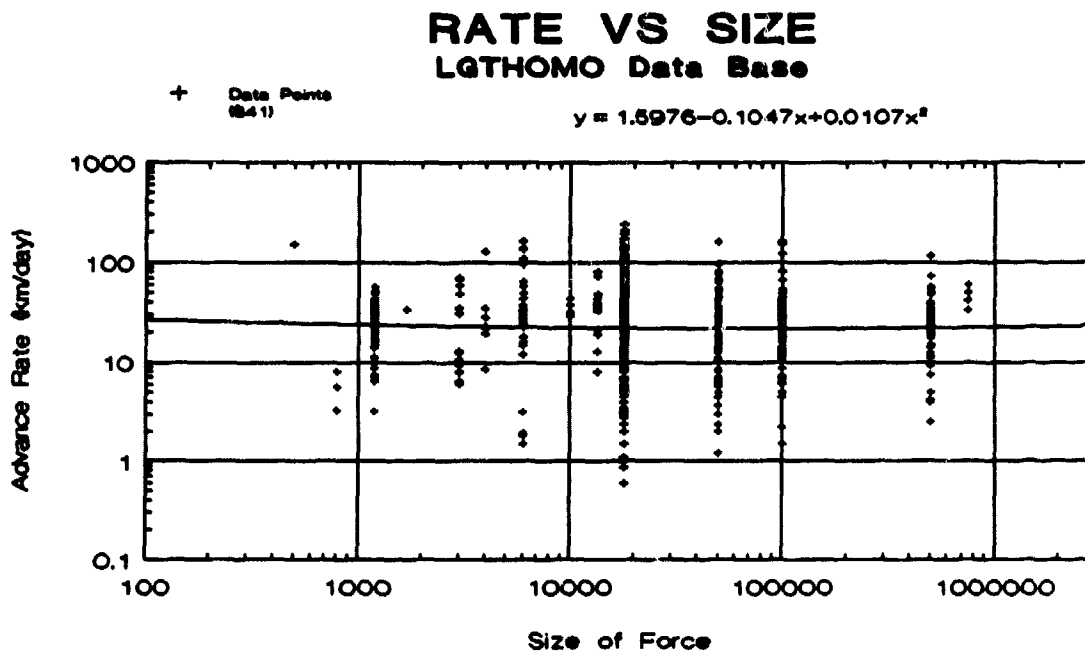


Figure 4-9d. Daily Advance Rate Versus Size for LGTHOMO Data

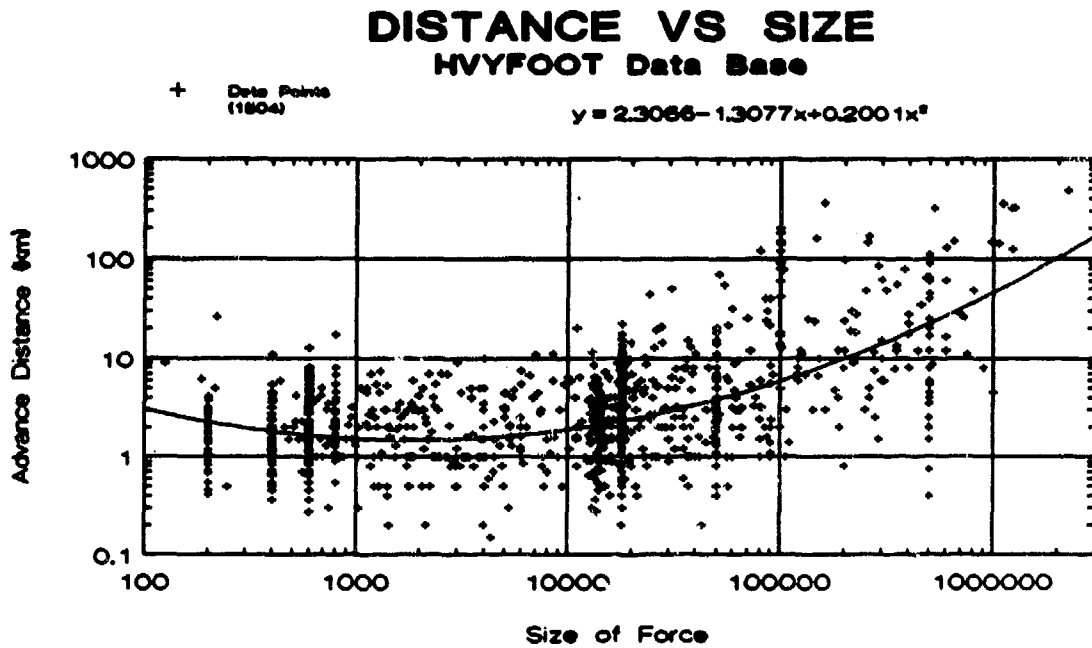


Figure 4-10a. Total Advance Distance Versus Size for HVYFOOT Data

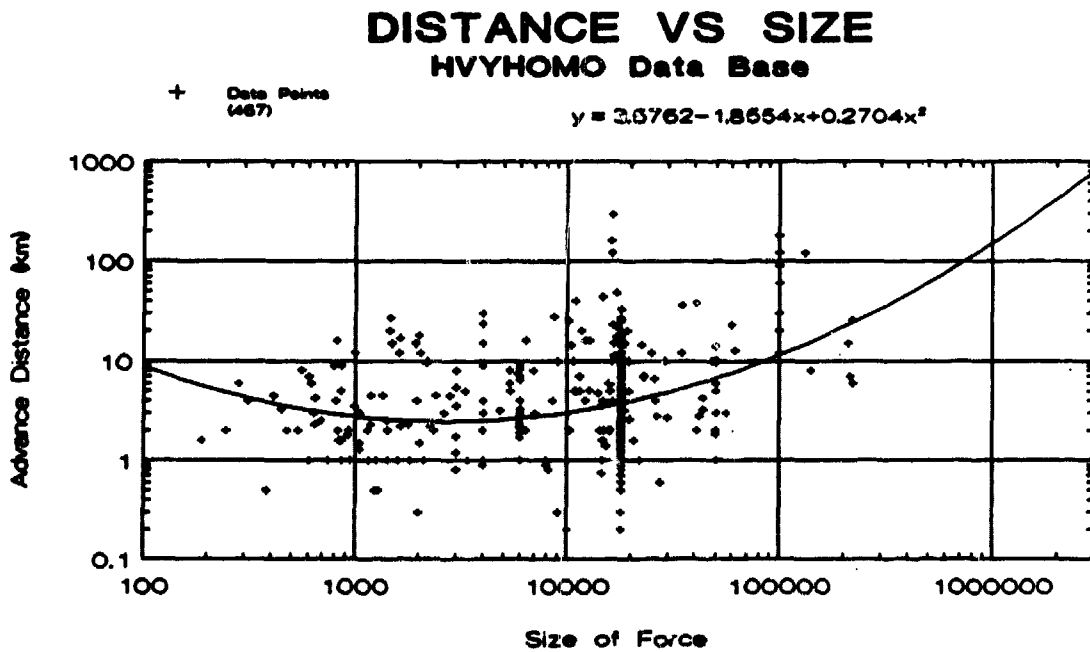


Figure 4-10b. Total Advance Distance Versus Size for HVYHOMO Data

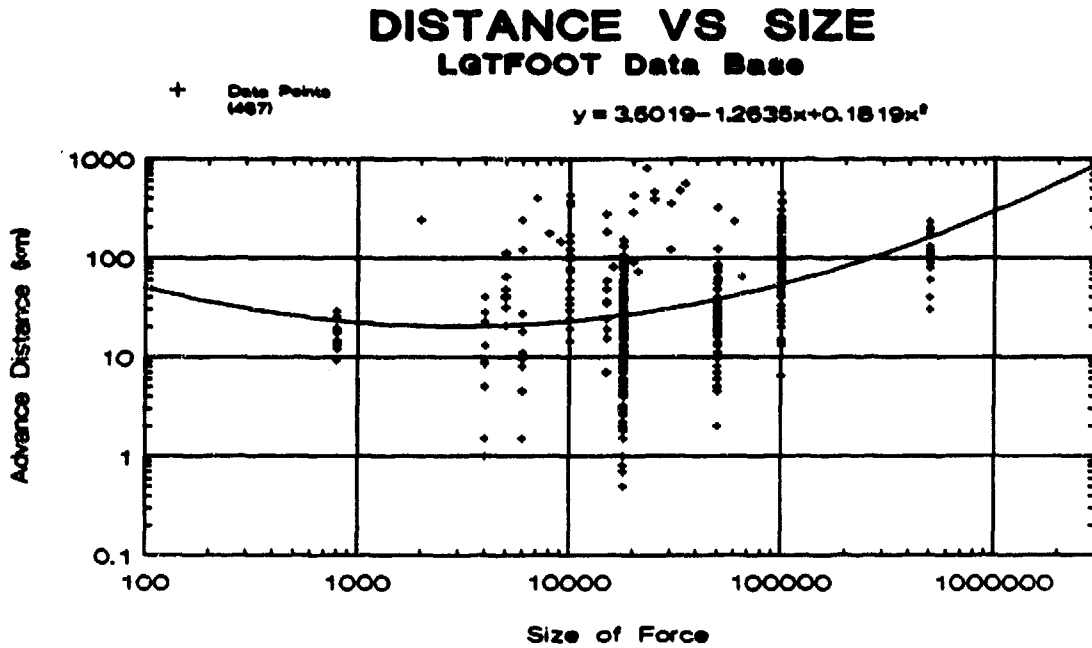


Figure 4-10c. Total Advance Distance Versus Size for LGTFoot Data

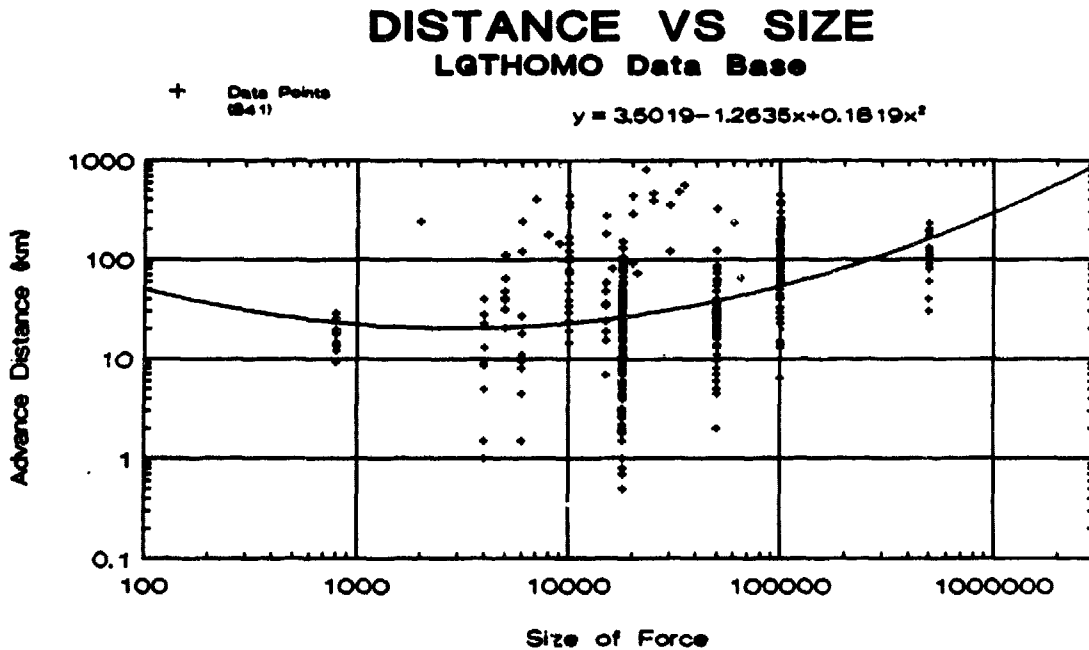


Figure 4-10d. Total Advance Distance Versus Size for LGTHOMO Data



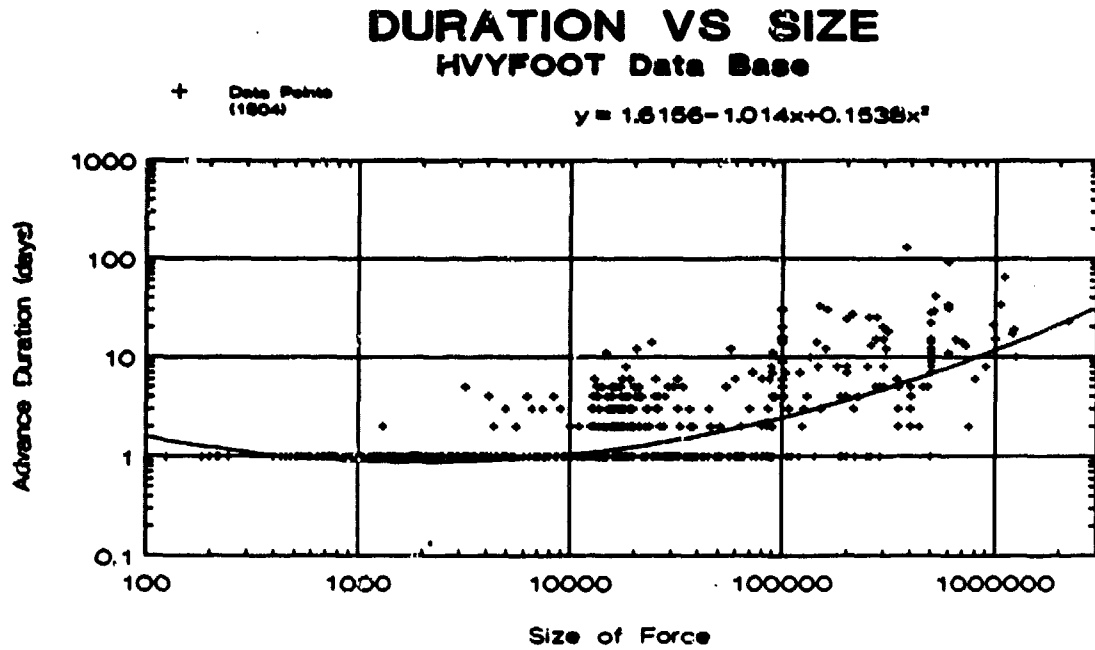


Figure 4-11a. Total Advance Duration Versus Size for HVYFOOT Data

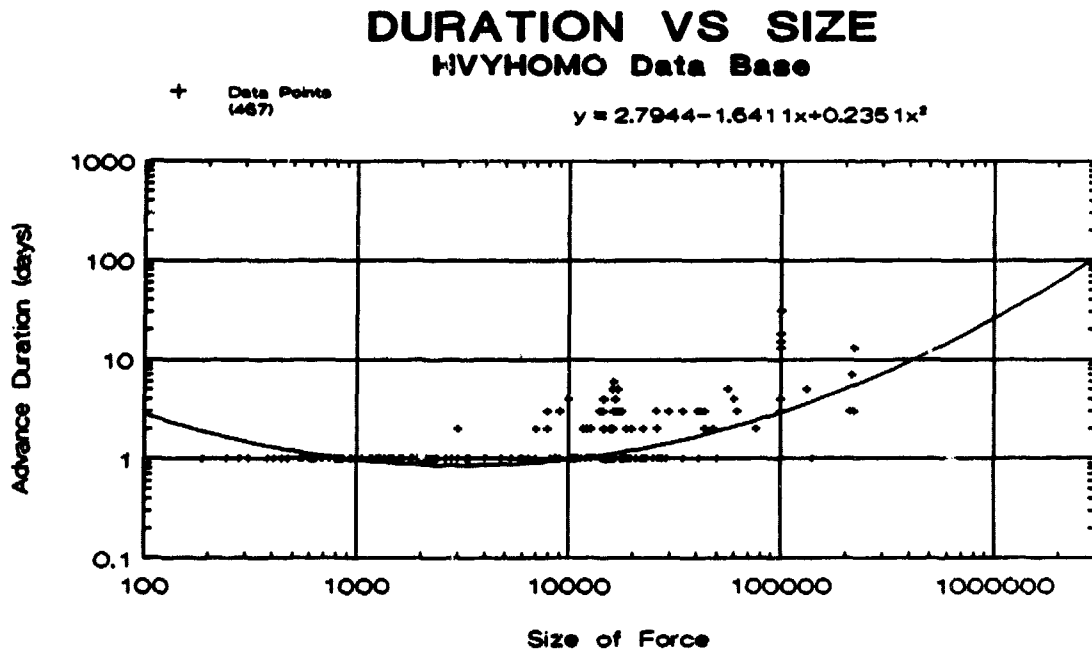


Figure 4-11b. Total Advance Duration Versus Size for HVYHOMO Data

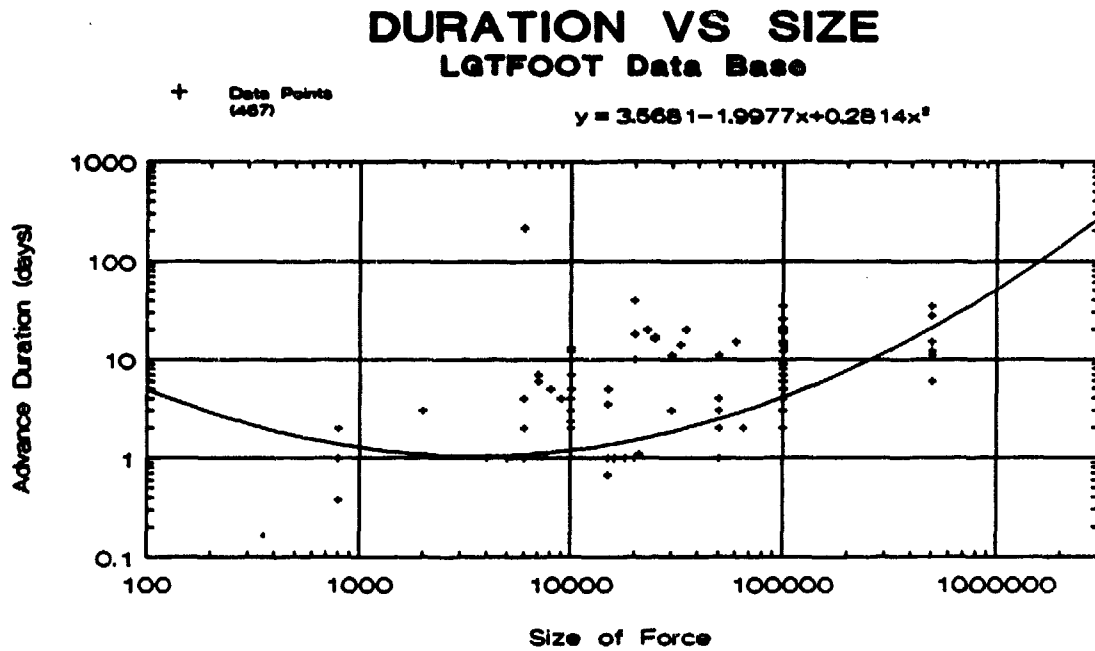


Figure 4-11c. Total Advance Duration Versus Size for LGTFOOT Data

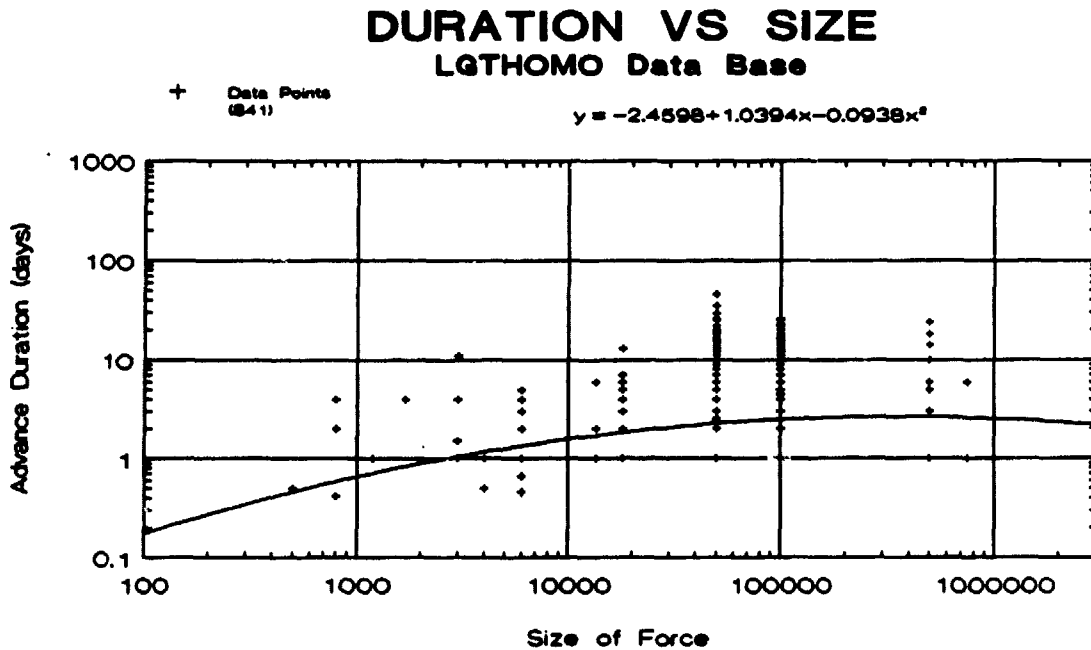


Figure 4-11d. Total Advance Duration Versus Size for LGTHOMO Data

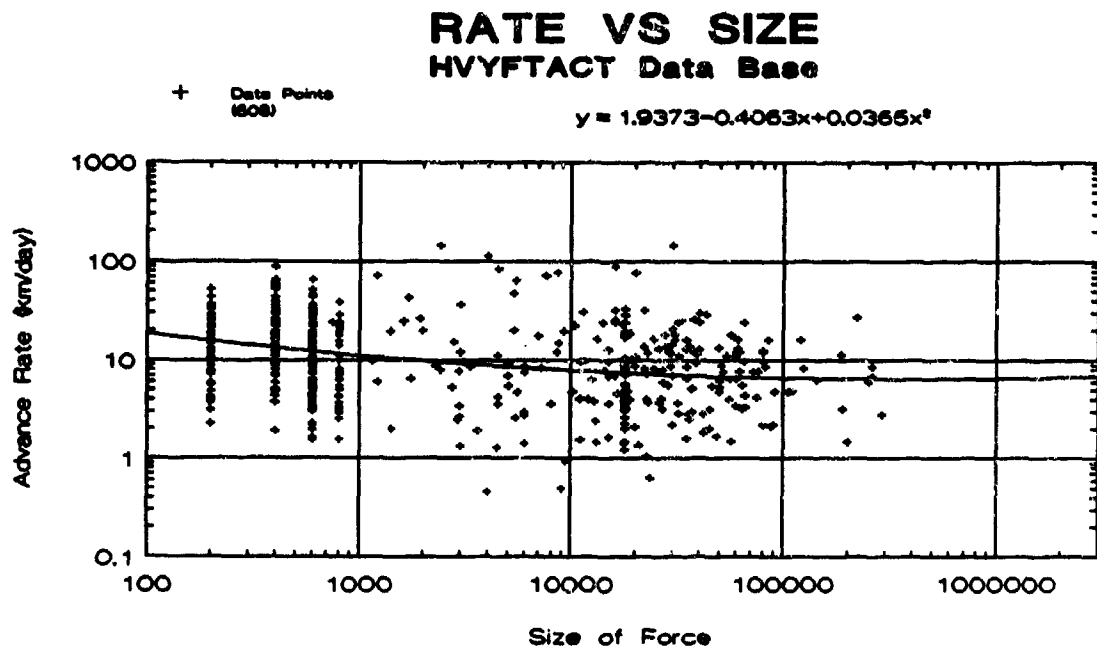


Figure 4-12a. Active Advance Rate Versus Size for HVYFOOT Data

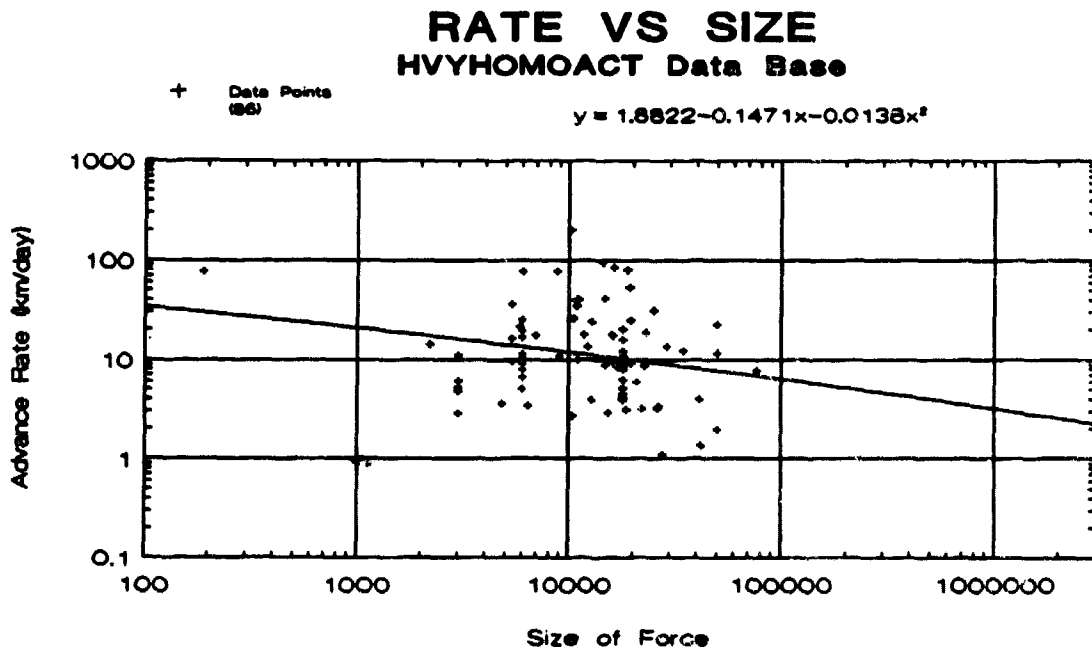


Figure 4-12b. Active Advance Rate Versus Size for HVYHOMO Data

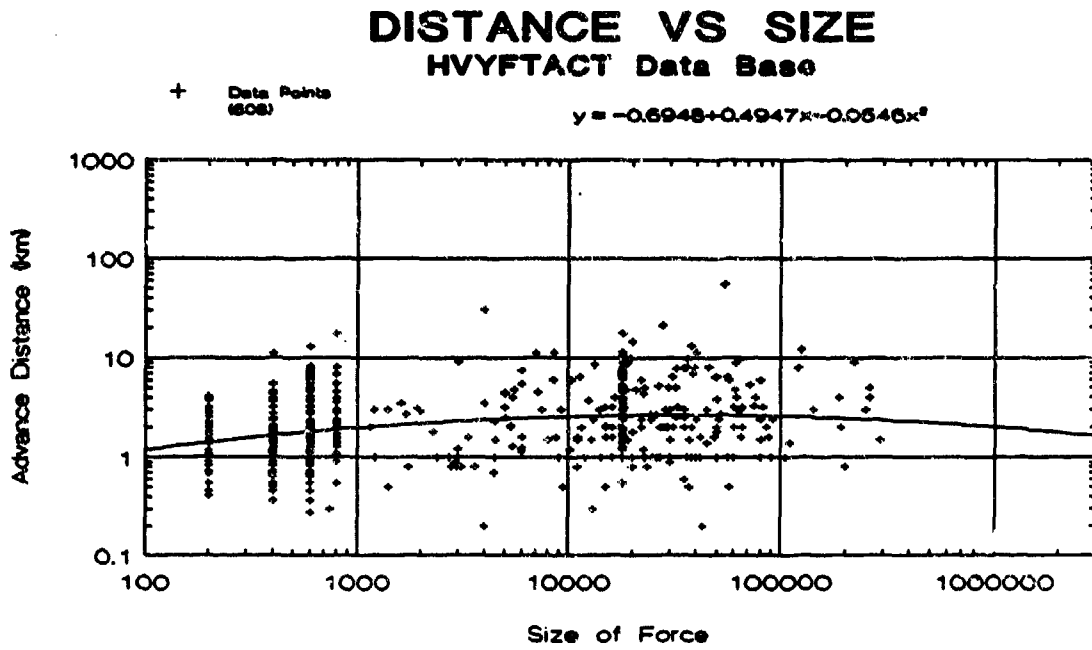


Figure 4-13a. Active Advance Distance Versus Size for HVVYFOOT Data

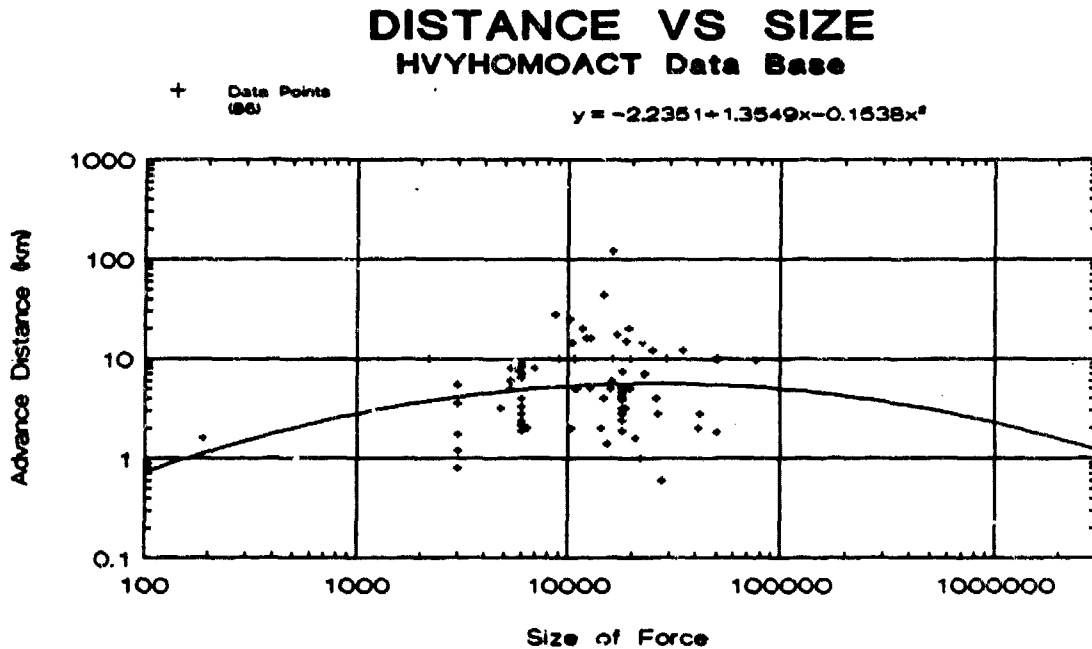


Figure 4-13b. Active Advance Distance Versus Size for HVVYHOMO Data

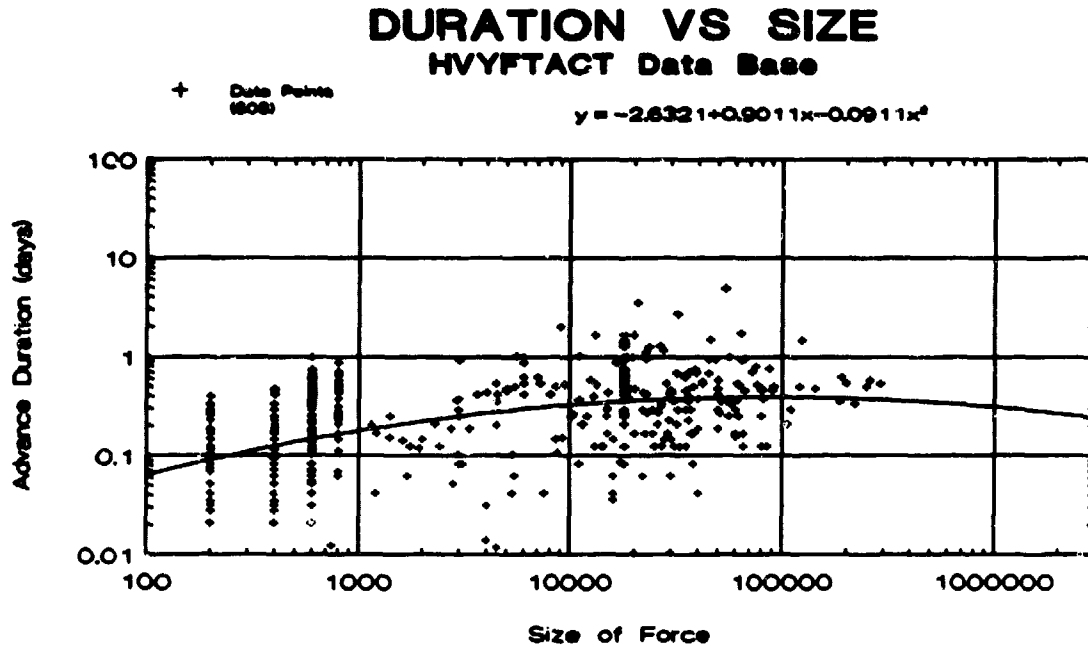


Figure 4-14a. Active Advance Duration Versus Size for HVYFOOT Data

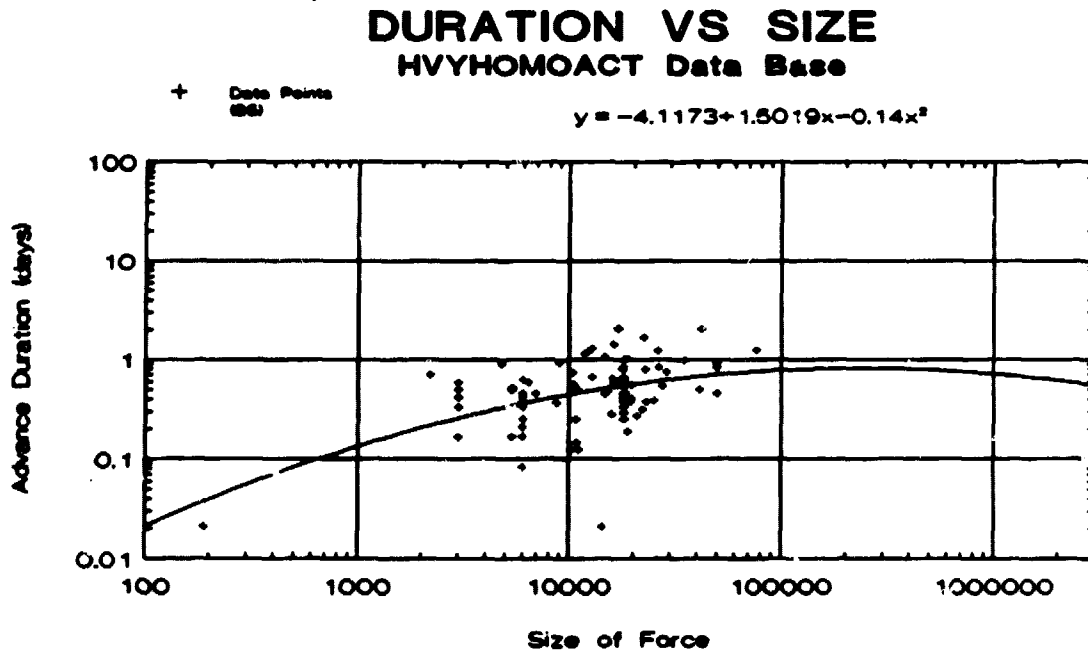


Figure 4-14b. Active Advance Duration Versus Size for HVYHOMO Data

g. The upshot of our analysis is that the reported data show that advance rates do *not* drop precipitously as force size increases. In fact, *daily* advance rates actually *increase* with force size. *Active* advance rates decline somewhat as force size increases to about 1,000 or so, but do not drop much further as force size increases from there to 100,000 or so. Again the reported data are widely scattered and highly variable.

#### 4-7. RATES OF ADVANCE FOR MOTOR (AND HORSE) AND FOOT

a. The proposition to be examined is that mechanized, motorized, and horse-mounted units advance much more rapidly than infantry units.

b. One would almost think this proposition must be a truism. Certainly horses and modern vehicles can travel much faster than people on foot. But we have seen in paragraph 4-4 that the speed of the major transport mode may not be a large factor. However, in paragraph 4-4 we compared horse and motor rates to those of World Record foot movements, and to rated maximum vehicle road speeds. In this paragraph we compare directly the rates for land combat forces moving by horse or motor to those for forces moving by foot.

c. Comparing Figure 4-3a to 4-3b and 4-3c to 4-3d, we see that daily advance rates for horse and motor are slightly higher--by a factor of about 1.5 or so--than those for foot. In fact, this factor seems to be about the same for lightly engaged forces as for heavily engaged ones. However, by comparing Figure 4-6a to 4-6b, we see that the advance rates of heavily engaged forces are about the same whether they move by foot or by motor. It may be thought that this is partly due to the tendency of heavily engaged forces to operate as combined arms teams whose components do not move independently. But this conjecture is a bit too hasty. Our derived data bases are based on whether the *governing* mode of transportation was by foot as opposed to by horse or motor. Consequently, the foot cases and the horse/motor cases are not even fighting the same battle, much less coordinating their operations. Besides, comparing Figure 4-7a to 4-7b or Figure 4-13a to 4-13b shows that the average distances moved are somewhat different for the different modes of transportation, which is hard to explain if they are closely coordinating their operations. Similarly, comparing Figure 4-8a to 4-8b or Figure 4-14a to 4-14b shows that the average times are also different for the different modes of transportation. (Cf. also paragraph 4-11.)

d. The upshot of our analysis is that forces moving by horse or motor do move slightly faster on a daily advance rate basis than forces moving by foot, and that this is true for both heavily and lightly engaged forces. However, during the active periods, heavily engaged forces advancing by horse or motor do so at about the same rate as forces advancing by foot.

#### 4-8. RATES OF ADVANCE FOR LIGHTLY AND HEAVILY ENGAGED FORCES

a. The proposition to be addressed is that advance rates are higher for lightly engaged forces.

b. Lightly engaged forces presumably spend less time in C<sup>3</sup> tasks than heavily engaged forces. If the hypothesis advanced earlier is correct, then lightly engaged forces should move more rapidly than heavily engaged ones. (There are, of course, many other hypotheses that would also predict that result.)

c. By comparing Figure 4-3a to 4-3c, 4-3b to 4-3d, 4-9a to 4-9c, and 4-9b to 4-9d (with due regard to the changes in vertical scale involved) we see that the reported *daily* advance rates of lightly engaged forces average about an order of magnitude higher than those for heavily engaged forces.

d. By comparing Figure 4-3c to 4-6a and 4-3d to 4-6b, we see that the *daily* advance rates of *lightly* engaged forces are not too different from the *active* advance rates of *heavily* engaged forces (i.e., they are within a factor of about 2 for horse or motor and within a factor of about 1.5 for foot, as opposed to a factor of 10). Comparing

Figures 4-3a to 4-6a and 4-3b to 4-6b shows that for heavily engaged forces, daily advance rates are substantially below active advance rates (by factors generally around 3 to 5). This means that when heavily engaged forces are actually on the move, they move quickly—almost as rapidly as lightly engaged forces. So the apparent slower speeds of heavily engaged forces must be due to the fact that they are standing still about 70 to 80 percent of the time, relative to lightly engaged forces. And that means that their average daily advance distances are determined largely by how much time they spend standing still. So in looking at advances, we may be looking in exactly the wrong place. Maybe we should instead be looking at the *non-movement* periods!

#### 4-9. RATES OF ADVANCE IN SUMMER AND IN WINTER

a. The proposition to be addressed is that advance rates are higher in summer than in winter. This is a proxy for a family of propositions to the effect that advance rates are higher in better weather, on ground with better footing, etc.

b. Comparing summer and winter is a substitute for more detailed comparisons of good and bad terrain, weather, footing, and so forth. Unfortunately, those more detailed comparisons cannot be made because our data don't support them. Only a few data bases report the weather, the terrain, or the footing. Moreover, some data bases report these items using only subjective descriptors (e.g., the terrain was "rough," or the weather was "poor"). Some report weather but not terrain, while others report terrain but not weather. Not only that, but those that report the weather don't all use the same terminology or the same definitions (e.g., one might report temperature while the other reports visibility). So it is not possible to make much out of such a hash. But nearly all the data bases report the date, from which we can determine at least the season of the year.

c. To do this analysis, we telescoped all calendar dates into a fractional year scale. On that scale, 0.00 is 0001 hours on 1 January, 1.00 is 2400 hours on 31 December, and 0.50 is 1 July. This is shown in Figure 4-15, where the tic marks are approximately at the start of each month (January, February, March, etc.). There is still a terrific amount of scatter in the data, but from Figure 4-15, it looks as though there is some increase in average daily advance rates at mid-year as compared to the winter months, especially for the horse and motor modes of movement. Figures 4-16 and 4-17 show that both the total distance and duration also increase during the summer months.

d. However, Figure 4-18 shows that the picture changes a bit for active advance rates. It is hard to see any annual change in active advance rates for heavily engaged forces moving by foot. It still looks as though there is an increase during the summer months in average active advance rates for heavily engaged forces moving by horse or motor, but the handful of points in February exerts a high "leverage" on the trend line and may be exaggerating the bow in the trend line.

e. Figures 4-19 and 4-20 are hard to interpret, but they suggest that on the average, the active advance *distances* increase during the summer months for heavily engaged forces, while active *durations* do not.

f. The upshot of our analysis is that (on the average) there is some increase in both daily and active advance rates during the summer months. But the data are widely scattered and highly variable. Also, the increase in the average is barely perceptible for forces advancing on foot, but more noticeable for forces advancing by horse or motor. In general, it seems that this increase in rates reflects mainly an increase in the average total and active advance distance, while total and active advance durations change but little.

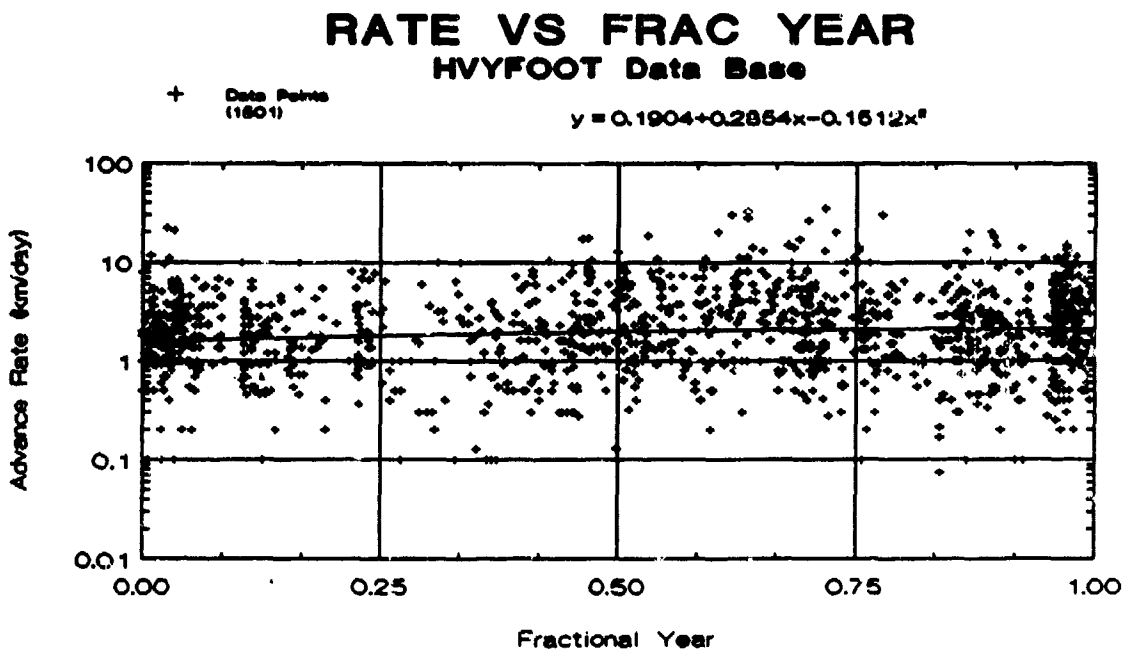


Figure 4-15a. Daily Advance Rate Versus Fractional Year for HVYFOOT Data

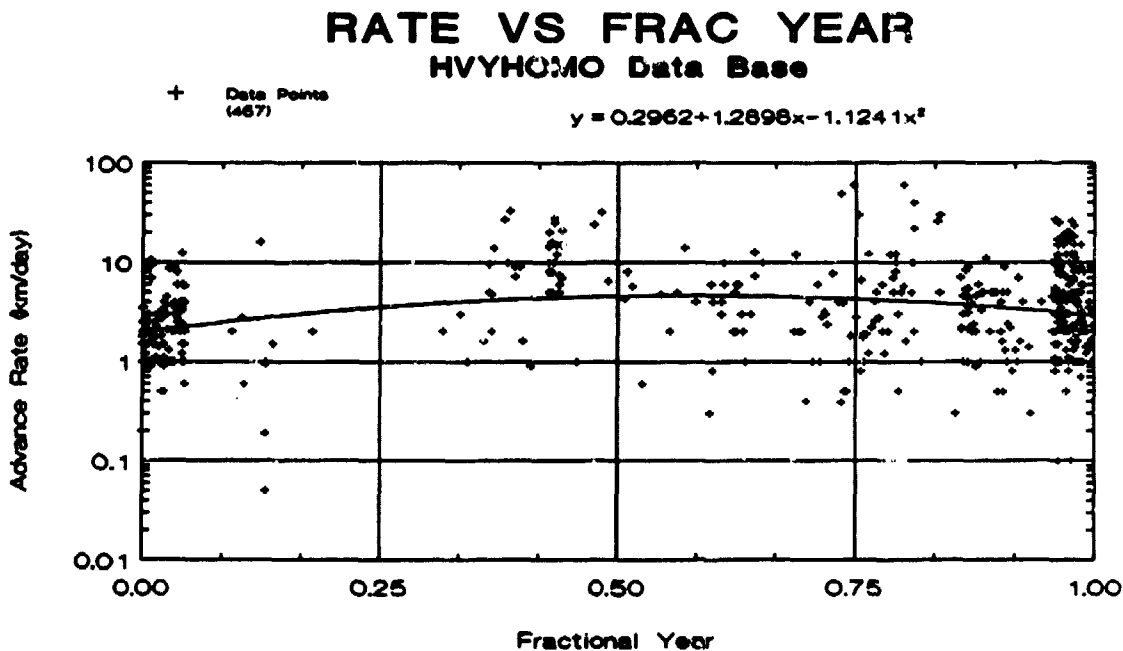


Figure 4-15b. Daily Advance Rate Versus Fractional Year for HVYHOMO Data



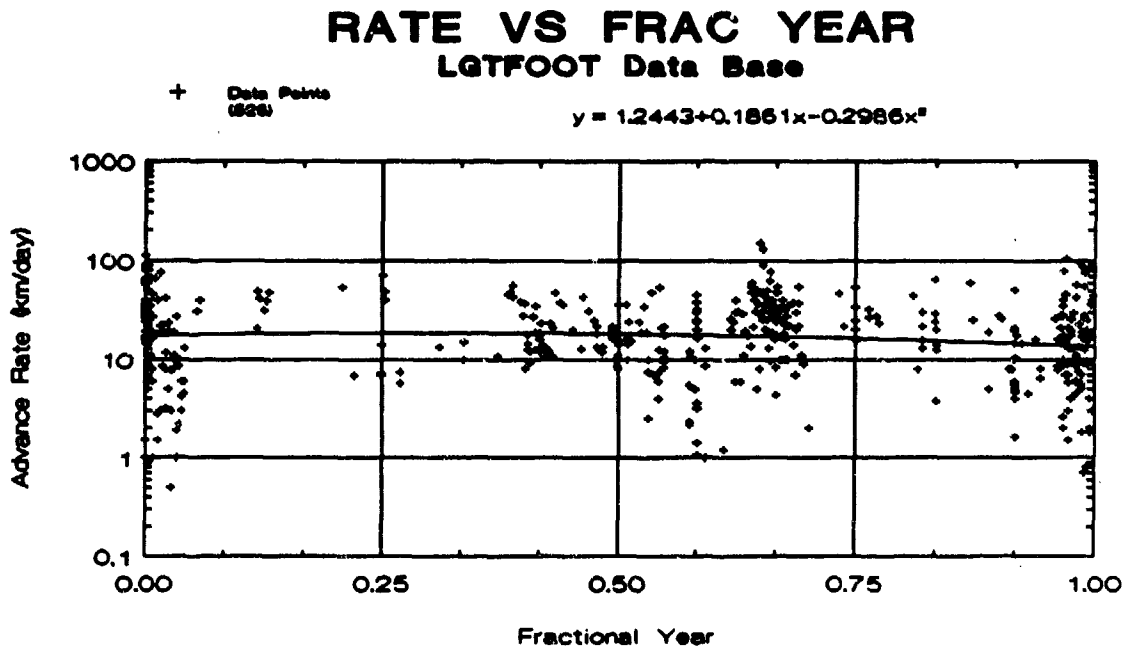


Figure 4-15c. Daily Advance Rate Versus Fractional Year for LGTFEOT Data

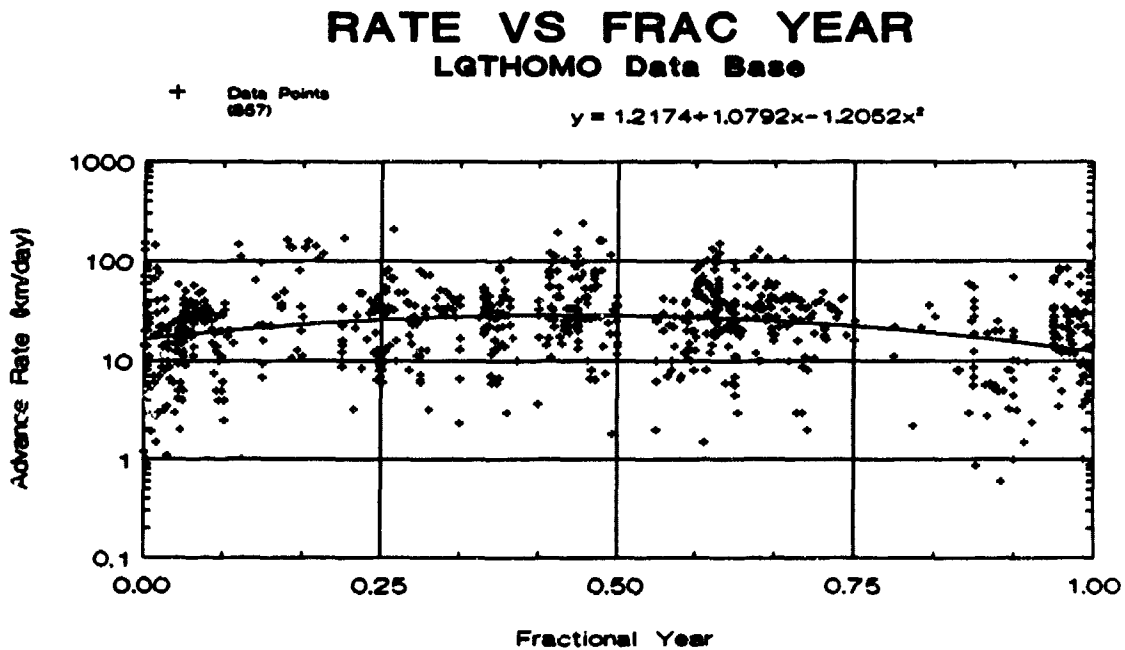


Figure 4-15d. Daily Advance Rate Versus Fractional Year for LGTHOMO Data

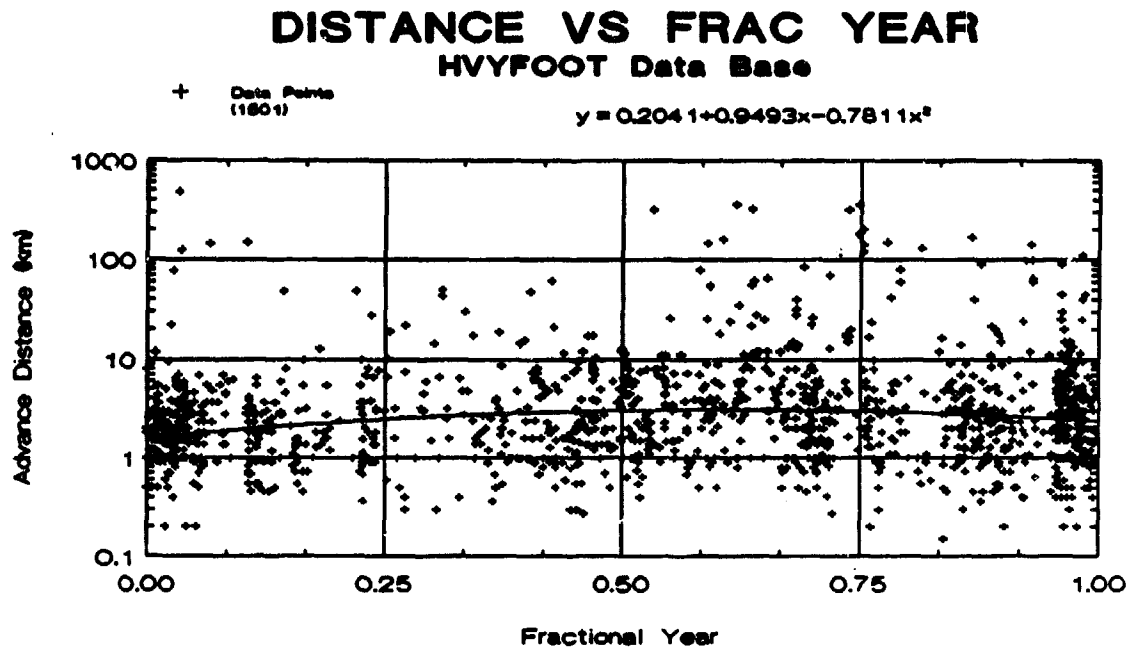


Figure 4-16a. Total Advance Distance Versus Fractional Year for HVYFOOT Data

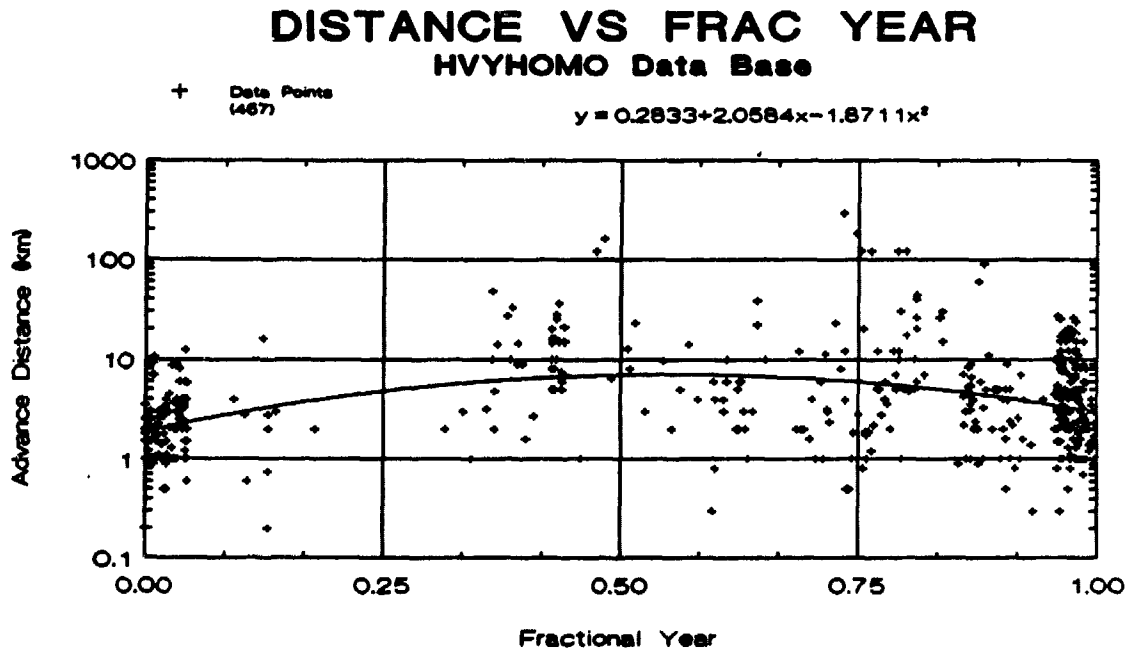


Figure 4-16b. Total Advance Distance Versus Fractional Year for HVYHOMO Data

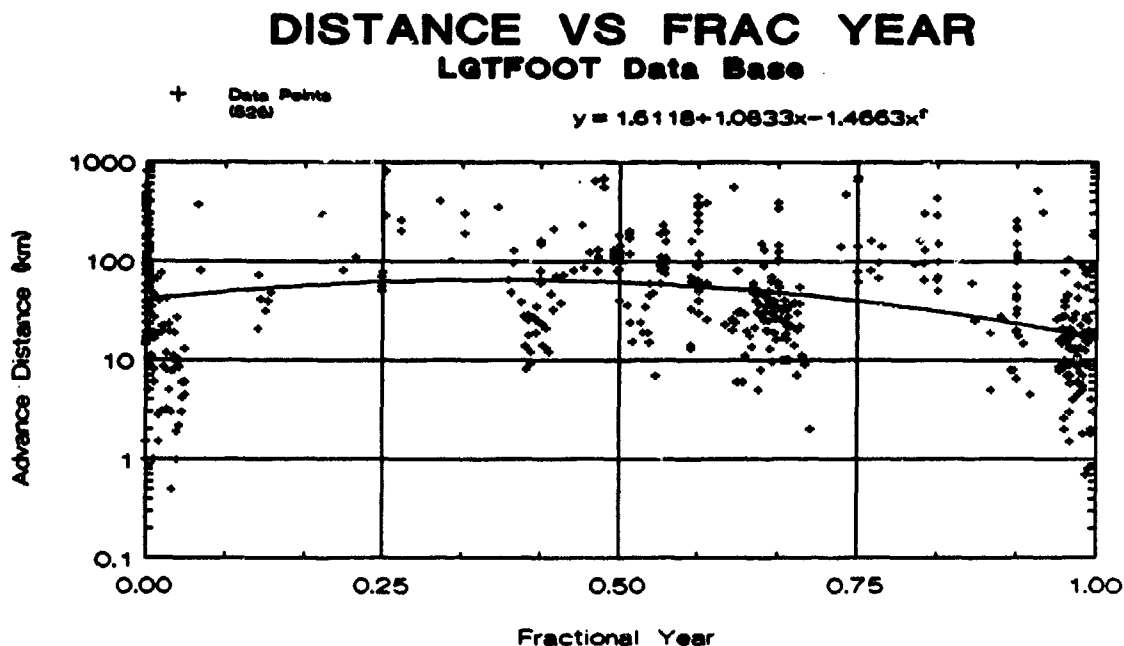


Figure 4-16c. Total Advance Distance Versus Fractional Year for LGTFoot Data

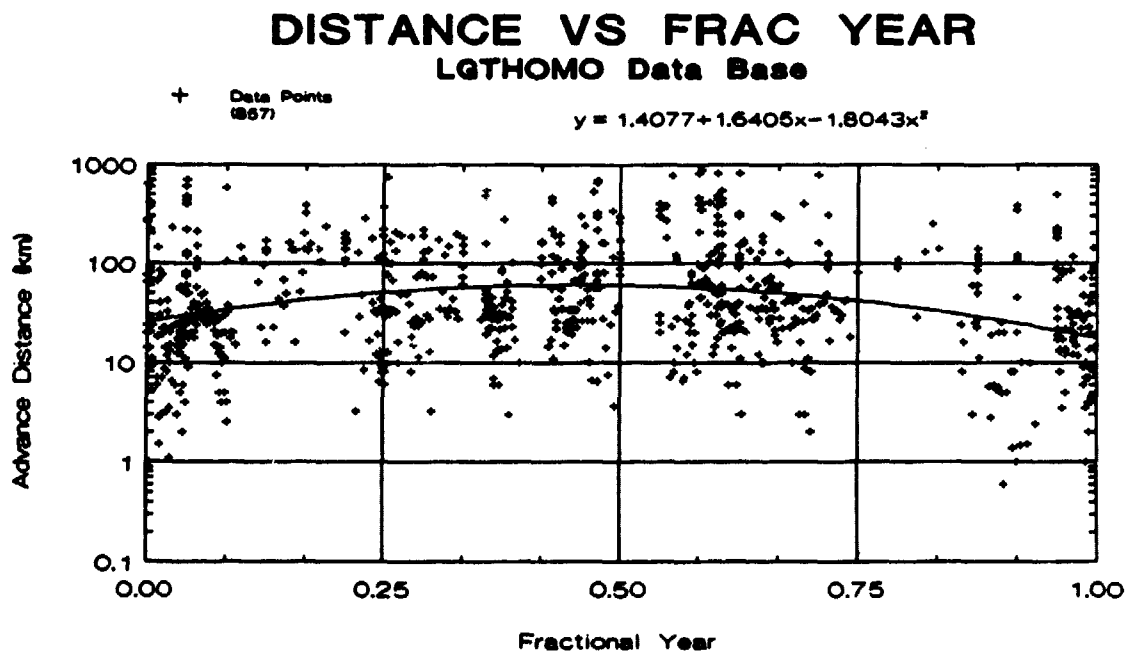


Figure 4-16d. Total Advance Distance Versus Fractional Year for LGTOMO Data

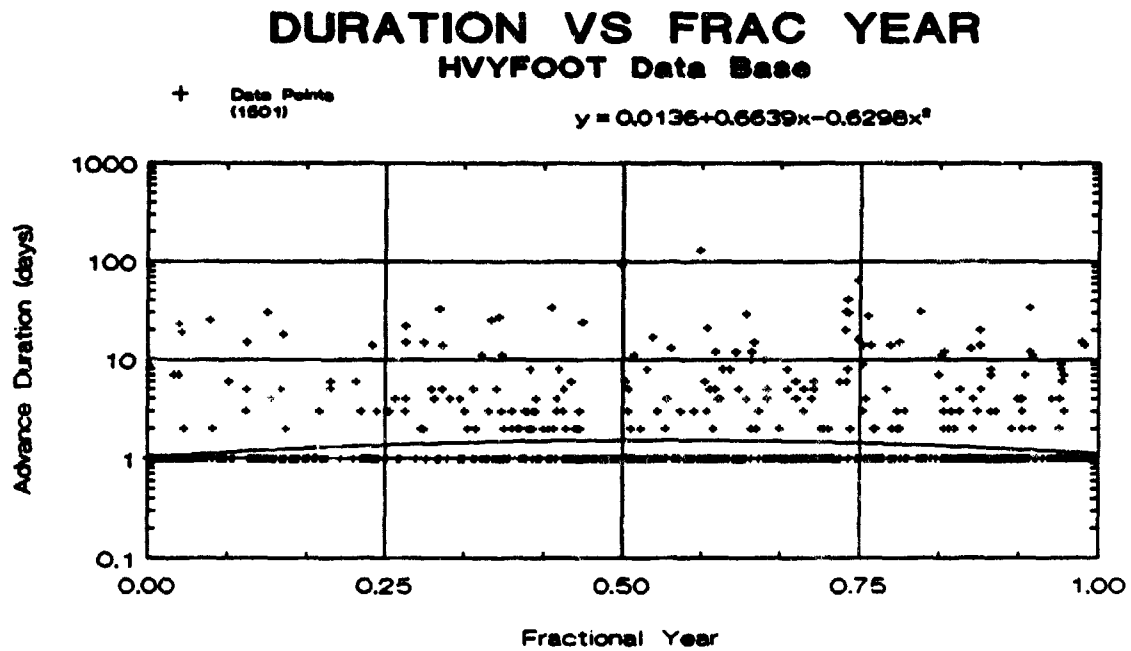


Figure 4-17a. Total Advance Duration Versus Fractional Year for HVYFOOT Data

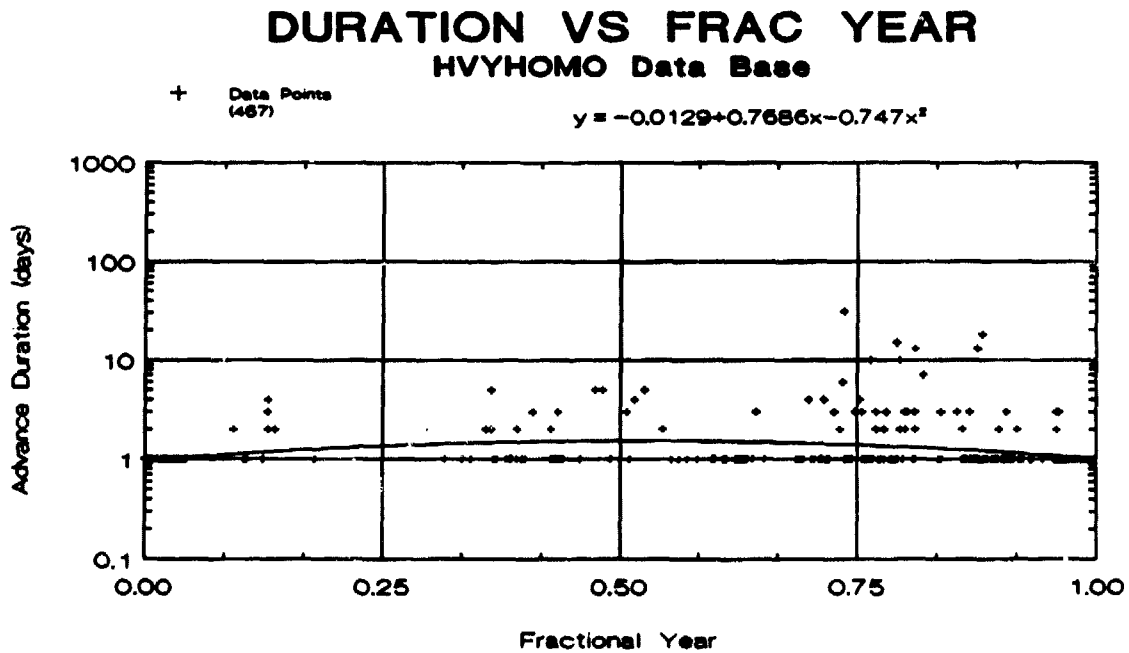


Figure 4-17b. Total Advance Duration Versus Fractional Year for HVYHOMO Data

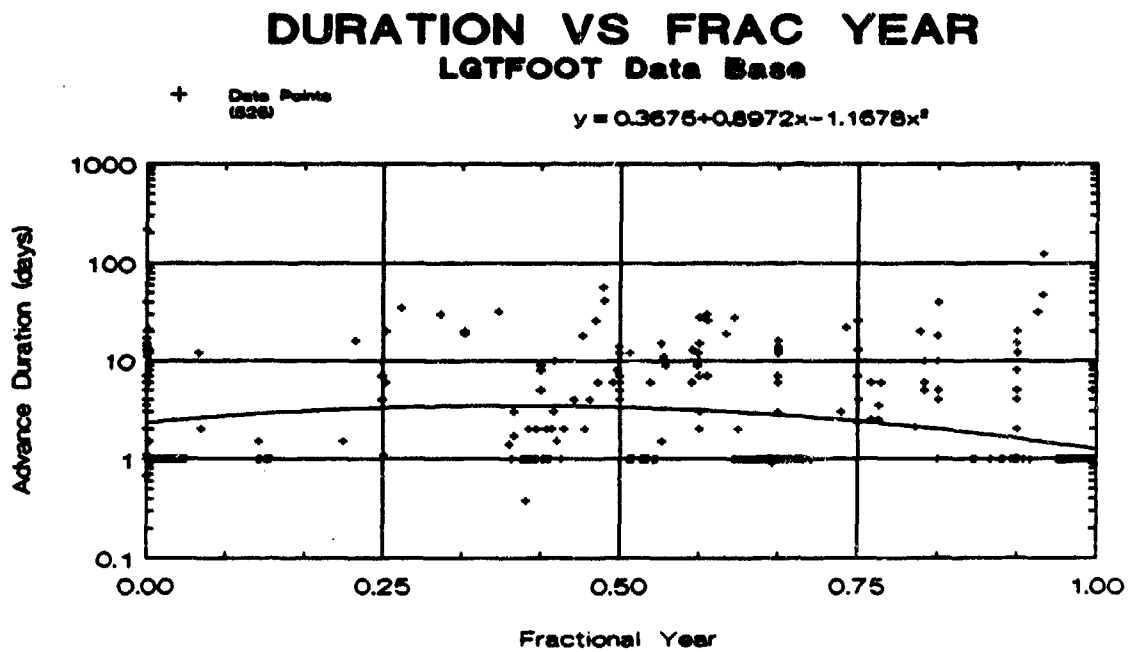


Figure 4-17c. Total Advance Duration Versus Fractional Year for LGTFoot Data

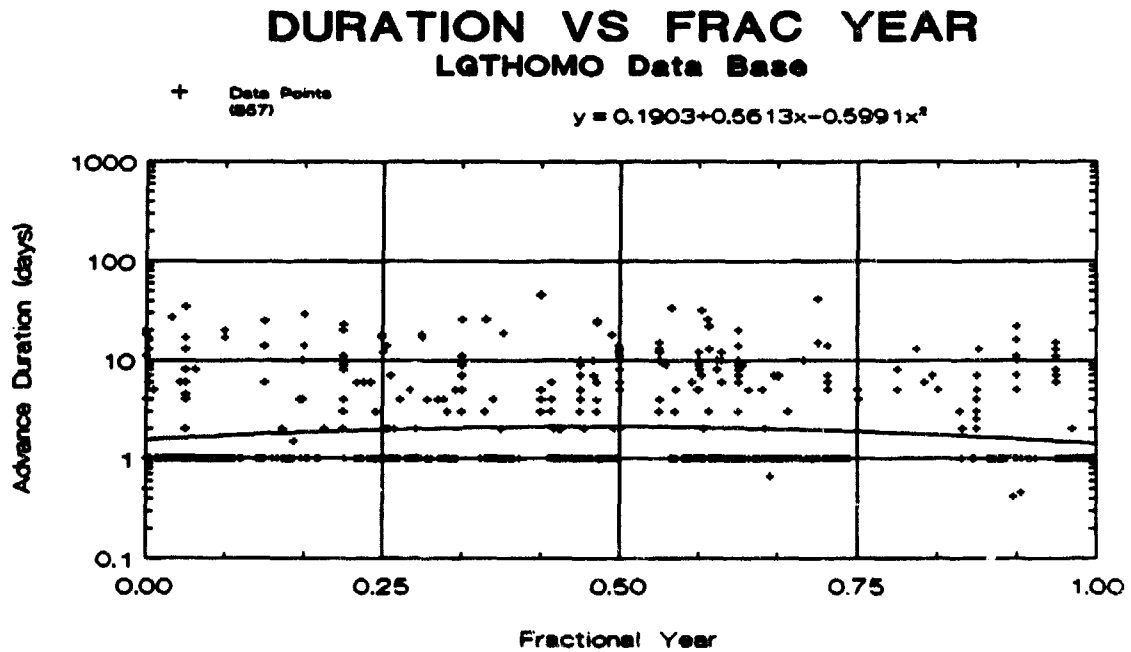


Figure 4-17d. Total Advance Duration Versus Fractional Year for LGTHOMO Data

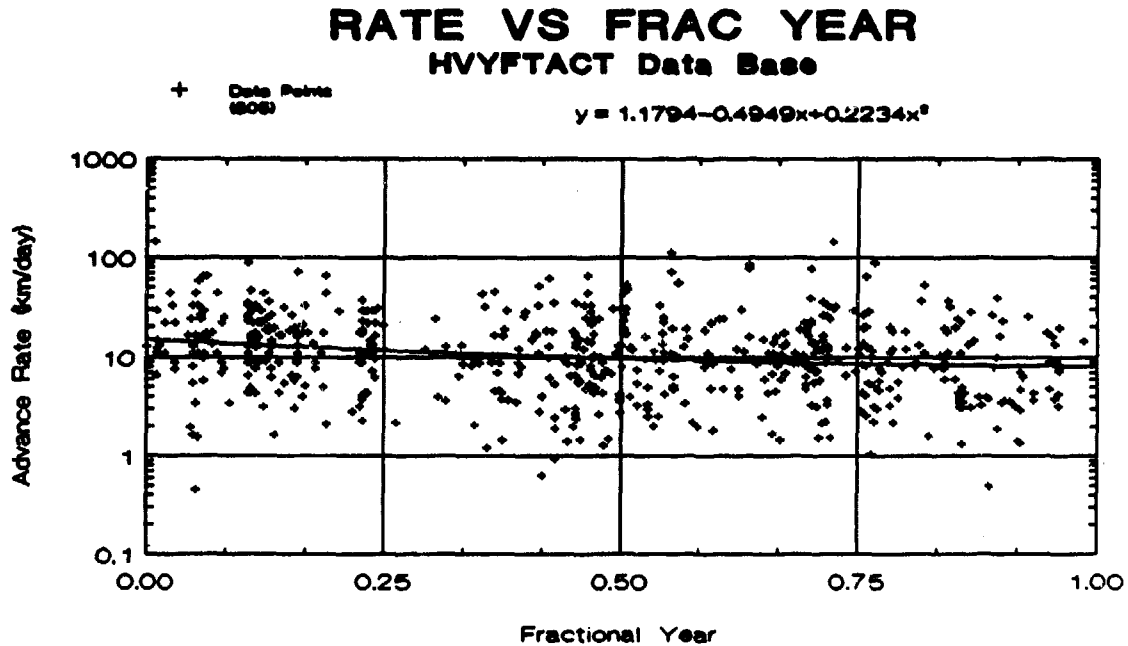


Figure 4-18a. Active Advance Rate Versus Fractional Year for HVYFOOT Data

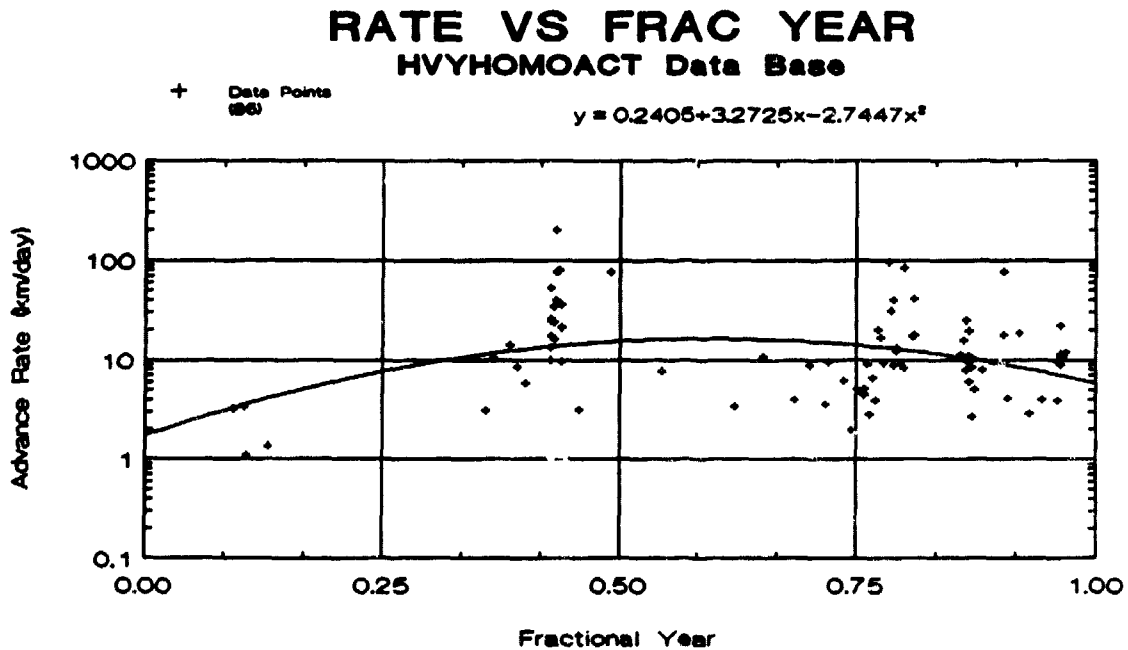


Figure 4-18b. Active Advance Rate Versus Fractional Year for HVYHOMO Data

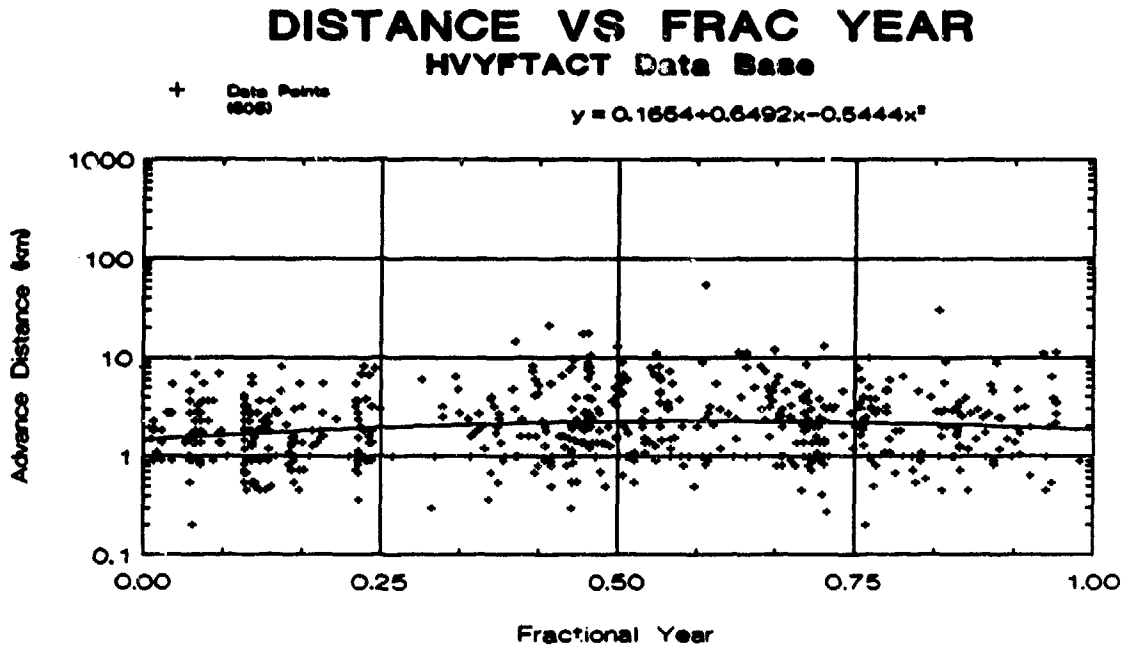


Figure 4-19a. Active Advance Distance Versus Fractional Year for HVYFOOT Data

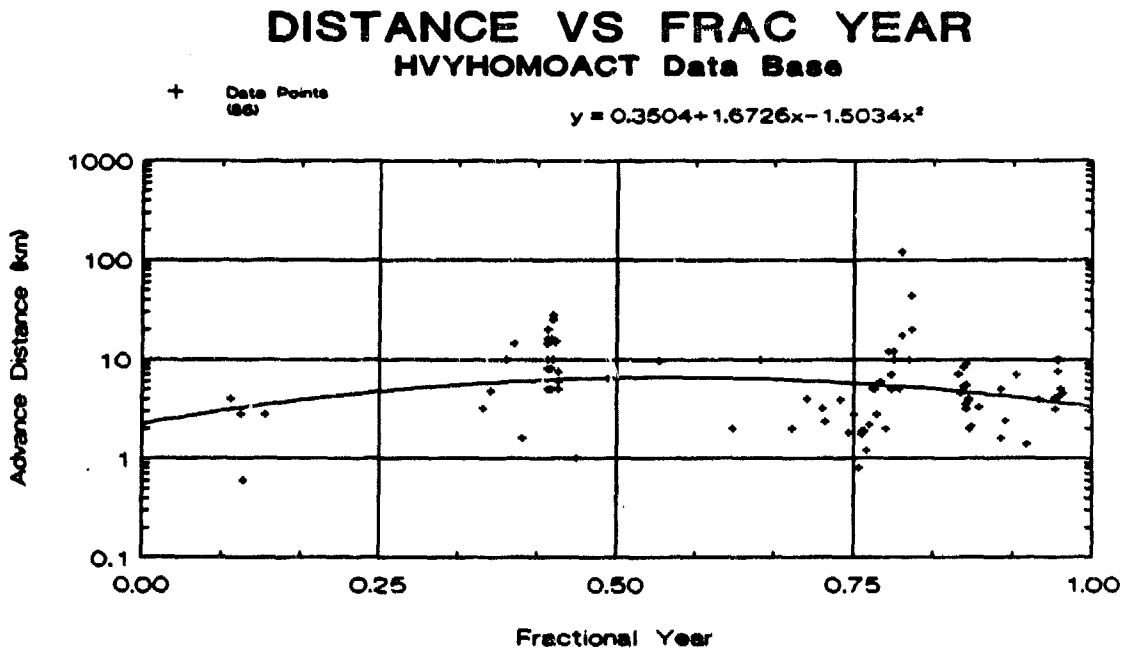


Figure 4-19b. Active Advance Distance Versus Fractional Year for HVYHOMO Data

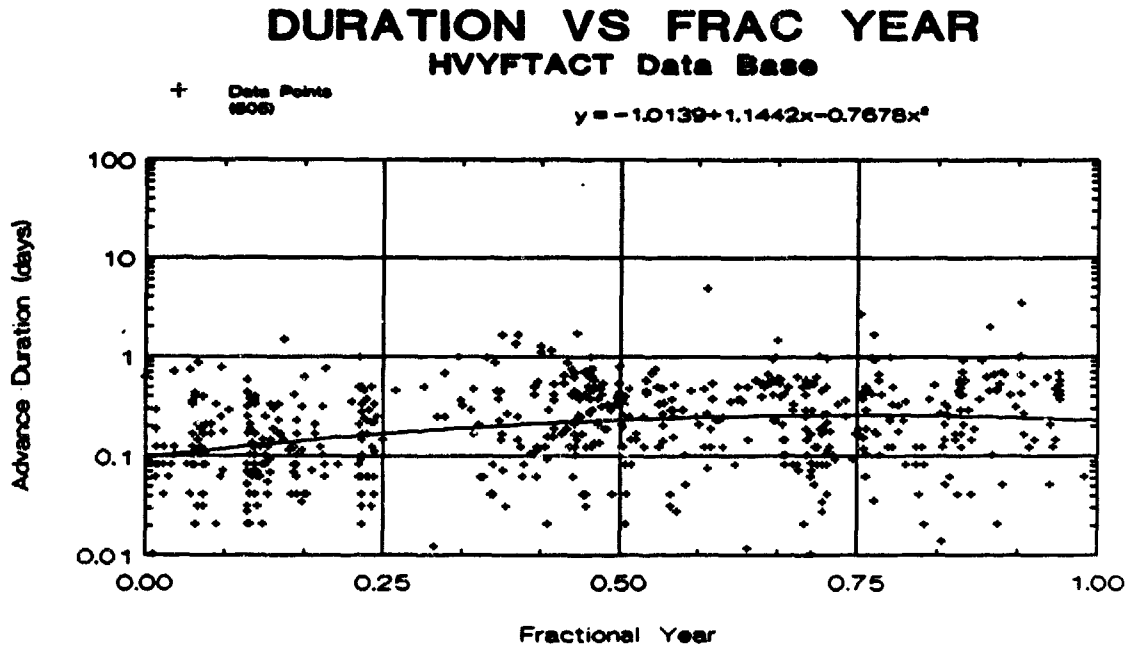


Figure 4-20a. Active Advance Duration Versus Fractional Year for HVVYFOOT Data

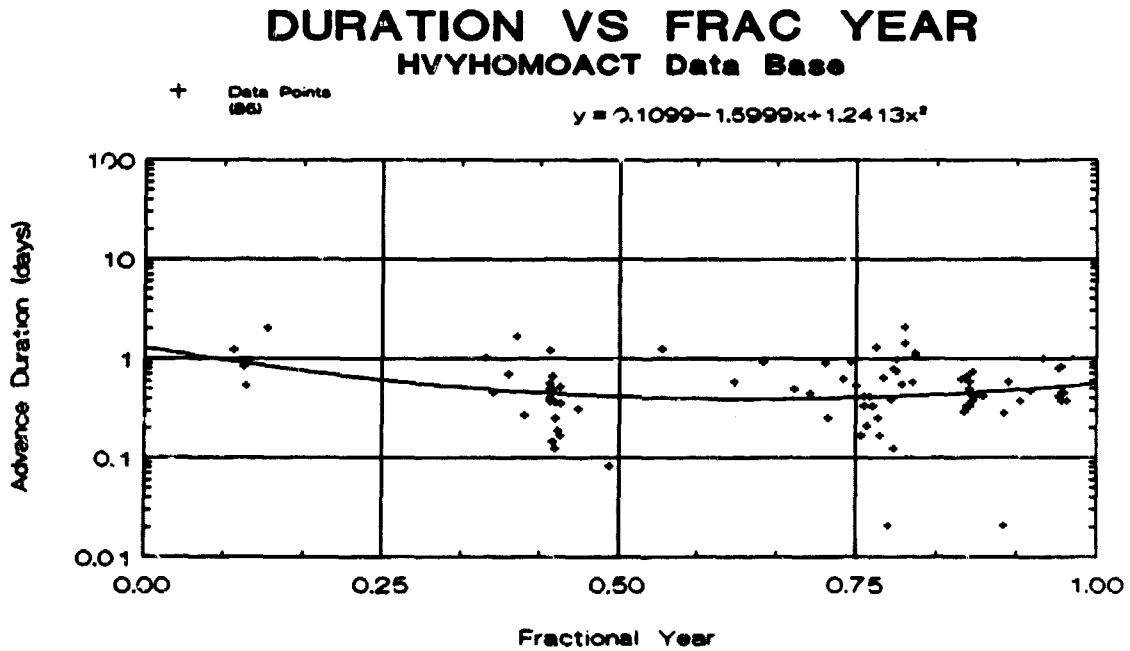


Figure 4-20b. Active Advance Duration Versus Fractional Year for HVVYHOMO Data



#### 4-10. RATES OF ADVANCE IN LONGER OPERATIONS

a. The proposition to be examined is that advance rates are lower for longer operations. That is, the advance rate declines the longer an operation lasts. Hence, the longer an operation lasts, the slower is its average advance rate.

b. The issue is whether advance rates are *consistently* lower for longer operations. If they are, then we are faced with what economists call diminishing returns to scale. The mental image is that of a factory producing advance distance as an output, using certain inputs such as labor, capital goods, expendables, and (most relevant for our purposes) time. Thus, the output (advance distance) may show diminishing returns to scale as any *one* input (such as time) is increased *while all others are held fixed*. Somewhat opposed to this view is Clausewitz's, to the effect that there is a sustainable advance rate which—if exceeded for too long—will damage the force (*cf.* paragraph 3-4c(2) of this report and the paragraph on Clausewitz in Appendix A of CAA-RP-90-3). The obvious generalization of Clausewitz's point of view is that there is a stabilizing feedback effect that tends to accelerate a force moving below its sustainable rate, and to decelerate one above it.

c. For this analysis we extracted from the primary data bases several derived data bases, as described in paragraph B-6 of Appendix B. Each of the them provided several cumulative values for a particular force's total advance distance and duration within a particular campaign. Figure 4-21 is an example of how we plotted cumulative total advance distance against cumulative total advance duration, using log-log axes. Figure 21-b is for operations in North Africa during WW II, as reported in the ANDREWS primary data base. It shows the campaigns led by Wavell, Rommel-I, Auchinleck, Rommel-II, and Montgomery. We prepared several similar figures, to see if a general trend could be discerned. They are presented as Figures 4-21 through 4-26.

d. The least squares straight line fit to each of these data bases (on a log-log scale) is also shown. The cumulative distances were generally linearly related to the cumulative times (on a log-log scale). That is, if  $D$  is the cumulative distance and  $T$  the corresponding cumulative time, then

$$\log(D) = a + b \log(T),$$

where logarithms to the base 10 are used, so that

$$D(T) = 10^a T^b. \quad (4-10a)$$

Note that the slope of the least squares line indicates whether the advance generally decelerated, accelerated, or proceeded at a uniform pace. If  $b < 1$ , then the advance decelerates with time (i.e., diminishing returns of cumulative distance as cumulative time increases). If  $b > 1$ , then the advance accelerates with time (i.e., increasing returns). If  $b = 1$ , then the advance proceeds at a steady rate with respect to cumulative time  $T$ . So the key question now is which of these cases is most typical of the reported data. The graphs of  $D$  versus  $T$  show a mixed and confusing picture, with some having a slope  $b < 1$ , and some having  $b > 1$ .

e. The simplest way to see the average trend is to (i) make vu-graph transparencies of Figures 4-21 to 4-26a, (ii) stack them atop one another, and (iii) look through the stack to find the darkest part. It will be found that the darkest part is a band paralleling a uniform speed line ( $b = 1$ ). To further investigate this, Figure 4-26b was prepared. It shows the slopes  $b$  versus the intercepts  $a$ , and the trend of decreasing slopes with increasing intercepts. The trend line shown in Figure 4-26b has the equation

$$b = 1.71 - 0.57a. \quad (4-10b)$$

The critical value of the intercept  $a$  that makes the slope  $b = 1$  is  $a = 1.25$ . For these

values of  $a$  and  $b$ , equation (4-10a) above implies that the first day's advance  $D(1) = 17.8 \text{ km} = 11 \text{ mi}$ . Hence, the trend shown by these figures implies that a force can maintain a steady advance of  $17.8 \text{ km/day} = 11 \text{ mi/day}$  indefinitely. If it starts faster, it will tend to decelerate. If it starts slower it will tend to accelerate. This phenomenon was noticed by Clausewitz and remarked on by Andrews (although he gave a different and *ad hoc* interpretation of it). The data are so variable that a particular advance will rarely follow this general trend with any exactitude.

f. Figure 4-27a shows the distribution of slopes  $b$ . The vertical scale is in units of standard deviation above and below the mean, and the horizontal scale is logarithmic. If the vertical scale were converted to the equivalent probability value, Figure 4-27a would look exactly like a conventional normal probability versus logarithmic axis chart. On such charts, lognormal distributions plot as straight lines. (See Appendix C for some basic facts about lognormal distributions.) As can be seen, the distribution of the slopes is approximately lognormal. Moreover, the median value of the slope is approximately 1, so the advances cluster around a uniform advance rate, with about half of them speeding up and half slowing down. This confirms the visual impression, given by sighting through the stacked transparencies, that the general trend is toward a uniform advance pace. Nobody has any good explanation for this phenomenon. We don't know how to predict which ones will speed up and which will slow down. The acceleration of advances in some cases may represent a kind of "learning effect," in which past experience teaches the force how to move a little faster. However, that's just a breezy hypothesis rather than a hard finding that can be adequately justified by what has been done so far. The fitted lognormal distribution has parameters  $\mu = -0.0283$  (which no doubt is not statistically significantly different from zero), and  $\sigma = 0.46448$ . This means that the sample average and standard error of the (natural) logarithms of the slopes is  $\mu$  and  $\sigma$ , respectively. The P-value for a lognormal distribution (using the Kolmogorov-Smirnov test) is 96.2 percent. The other distributions tried were the normal, exponential, Weibull, and gamma. None of them fit the observed values as well as the lognormal distribution.

g. Figure 4-27b shows typical cumulative distance versus cumulative time trends obtained from Equation (4-10a) for slopes and intercepts governed by Equation (4-10b). For this we took slopes  $b$  of  $1/2$ ,  $1$  or  $2$  (the corresponding intercepts  $a$  from Equation (4-10b) are  $2.12$ ,  $1.25$ , and  $-0.51$ ). These values of  $b$  were selected with an eye to Figure 4-27a, which shows that they include almost all of the empirically observed slope values. Note that all lines satisfying both Equation (4-10a) and (4-10b) go through a specific point. Such a family of lines forms what mathematicians sometimes call a "pencil" (the tip of the pencil is their common intersection point). We can find this common intersection point for our particular family of lines as follows. Let the coordinates of the common intersection point be  $(T_0, D_0)$ . To find them, use Equations (4-10a) and (4-10b) to set

$$a_1 + (1.71 - 0.57a_1)\log T = a_2 + (1.71 - 0.57a_2)\log T.$$

Solving this for  $\log(T)$ , we find it is independent of  $a_1$  and  $a_2$ , and is in fact given by

$$\log(T_0) = \frac{1}{0.57},$$

so that  $T_0 = 56.8$  days. Hence

$$\log(D_0) = a + (1.71 - 0.57a)\log(T_0) = \frac{1.71}{0.57} = 3.00,$$

and  $D_0 = 1000 \text{ km}$ . The average rate of advance is

$$R_0 = D_0/T_0 = 1000 \text{ km}/56.8 \text{ days} = 17.6 \text{ km/day} = 10.9 \text{ mi/day}.$$

This may be interpreted as saying that, on the average, no matter how fast or slow the force may start out, after about 56.8 days it will have covered about 1,000 km and hence averaged about 17.6 km/day. But the data are widely scattered and highly variable.

### CUMULATIVE DISTANCES AND TIMES

ANDREWS Data Base (Series 1-9)

+ Sherman    Δ Kluck    ○ Casperette    + Kleist    ▲ 4th Army  
 ● Vigen/Greer    ▽ Barbarossa    ◊ Malaya    □ No. Korea

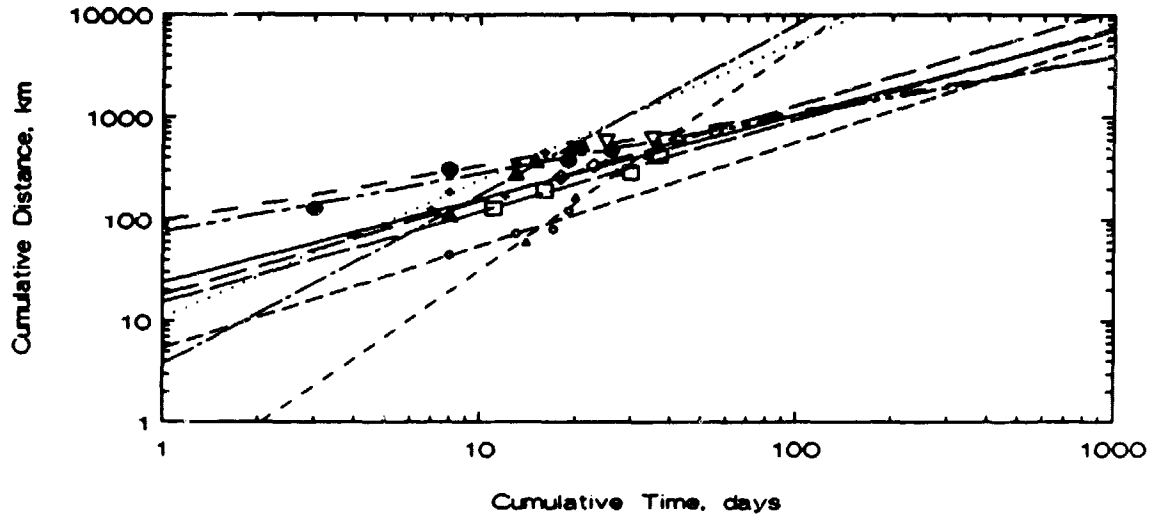


Figure 4-21a. Cumulative Distances and Times from ANDREWS (Series 1-9)

### CUMULATIVE DISTANCES AND TIMES

ANDREWS Data Base (Series 10-14)

+ Wevell    Δ Rommel-I    ○ Auchinleck    + Rommel-II    ▲ Montgomery

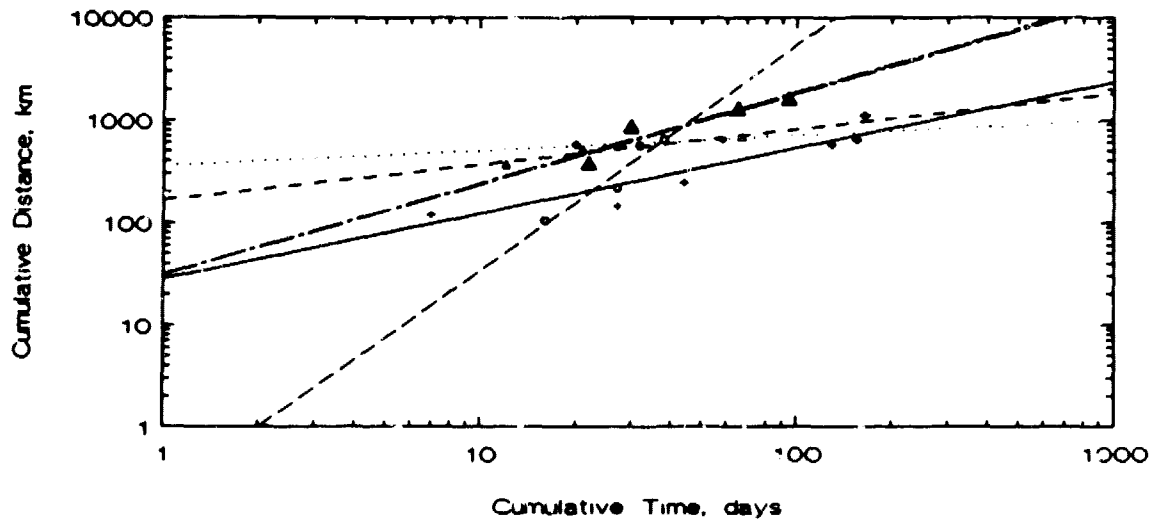


Figure 4-21b. Cumulative Distances and Times from ANDREWS (Series 10-14)

### CUMULATIVE DISTANCES AND TIMES

EASTFRON Data Base

+ Kurak      Δ Byeler      ○ Luv      + Odr

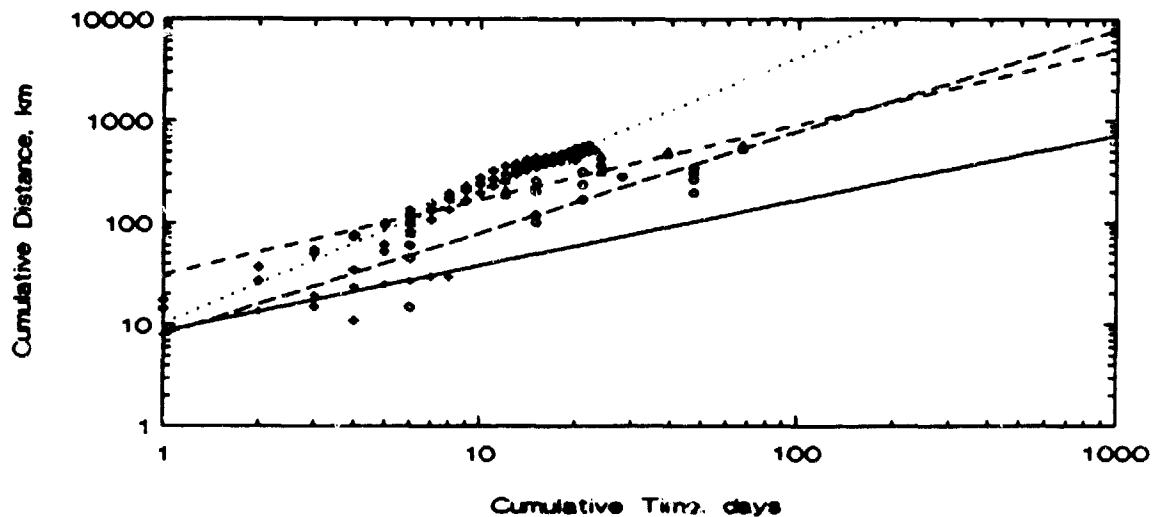


Figure 4-22a. Cumulative Distances and Times from EASTFRON

### CUMULATIVE DISTANCE VERSUS TIME

GIBBON Database

+ Data Points (187)

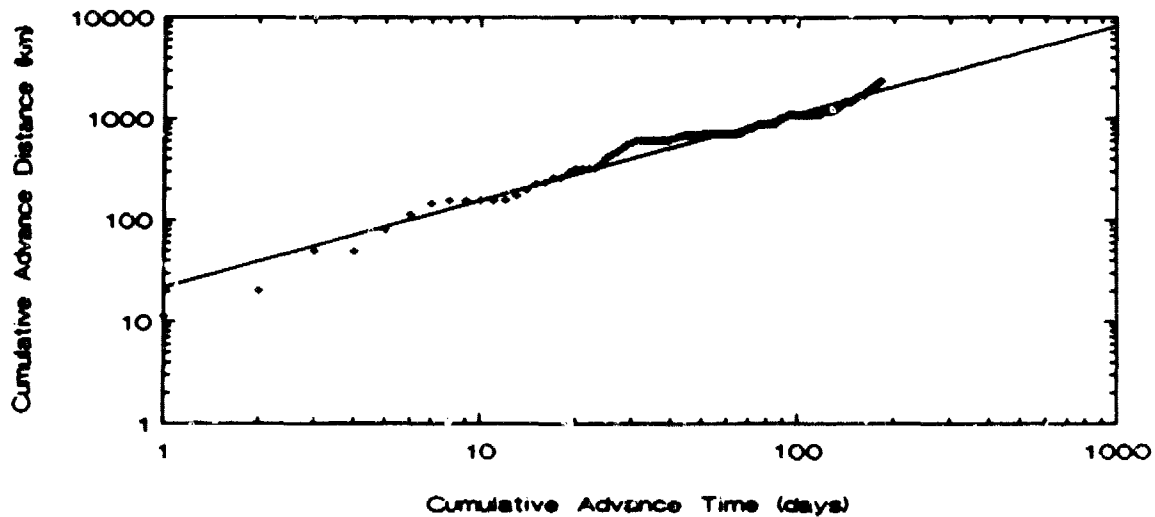


Figure 4-22b. Cumulative Distances and Times from GIBBON



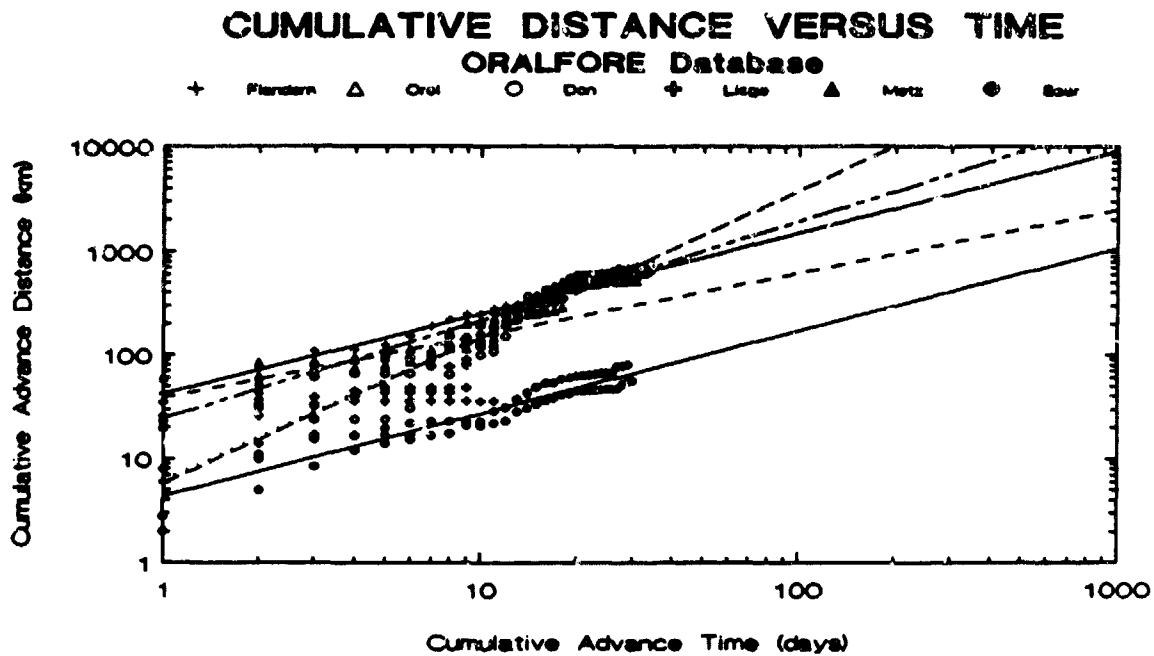


Figure 4-24a. Cumulative Distances and Times from ORALFORE

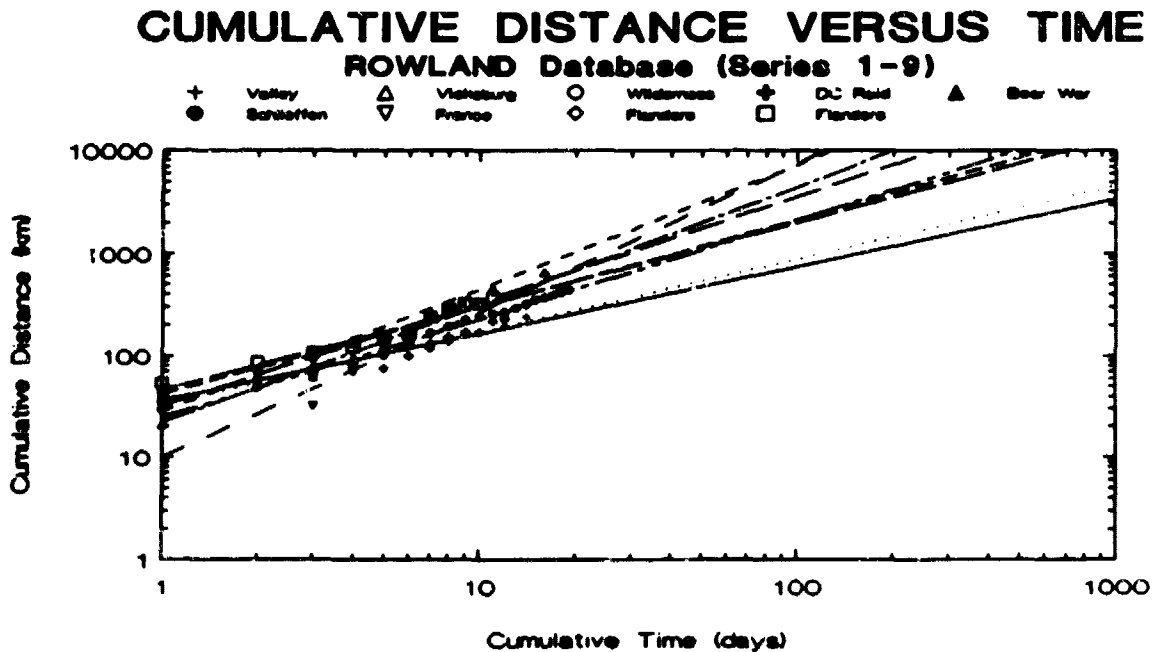


Figure 4-24b. Cumulative Distances and Times from ROWLAND (Series 1-9)

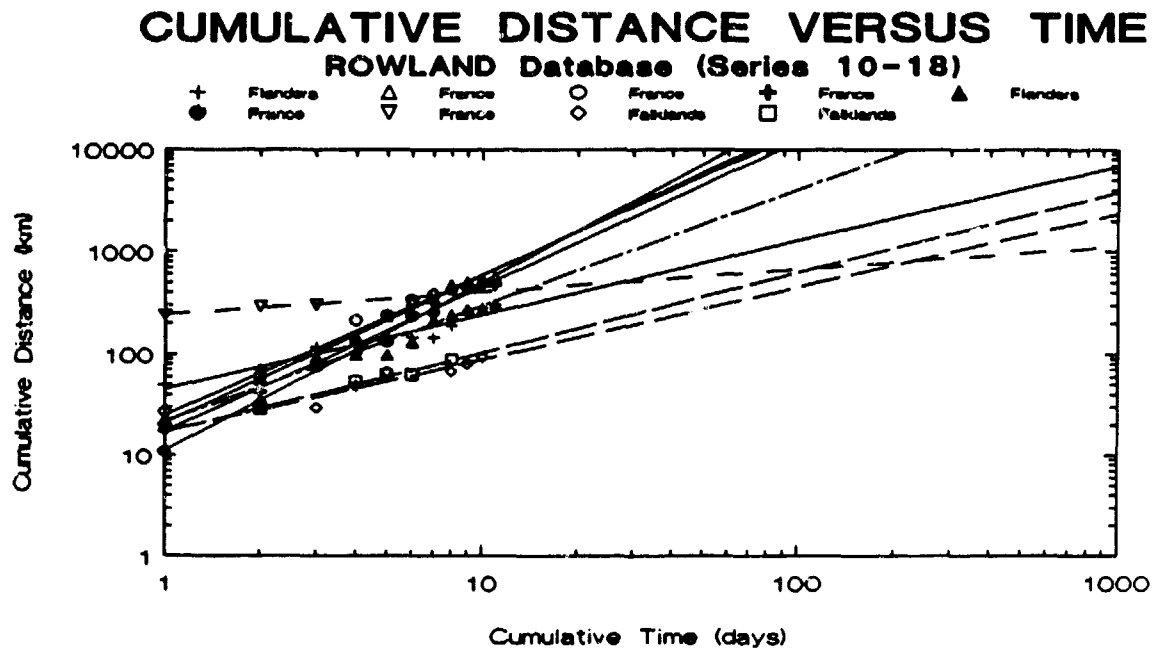


Figure 4-25a. Cumulative Distances and Times from ROWLAND (Series 10-18)

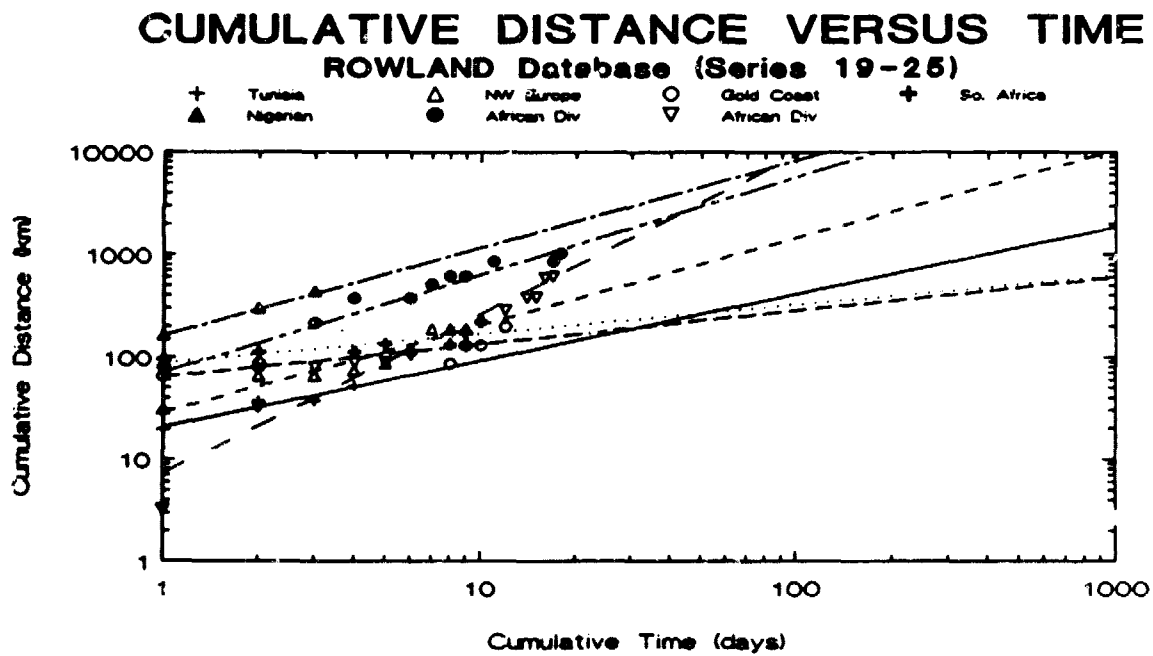


Figure 4-25b. Cumulative Distances and Times from ROWLAND (Series 19-25)

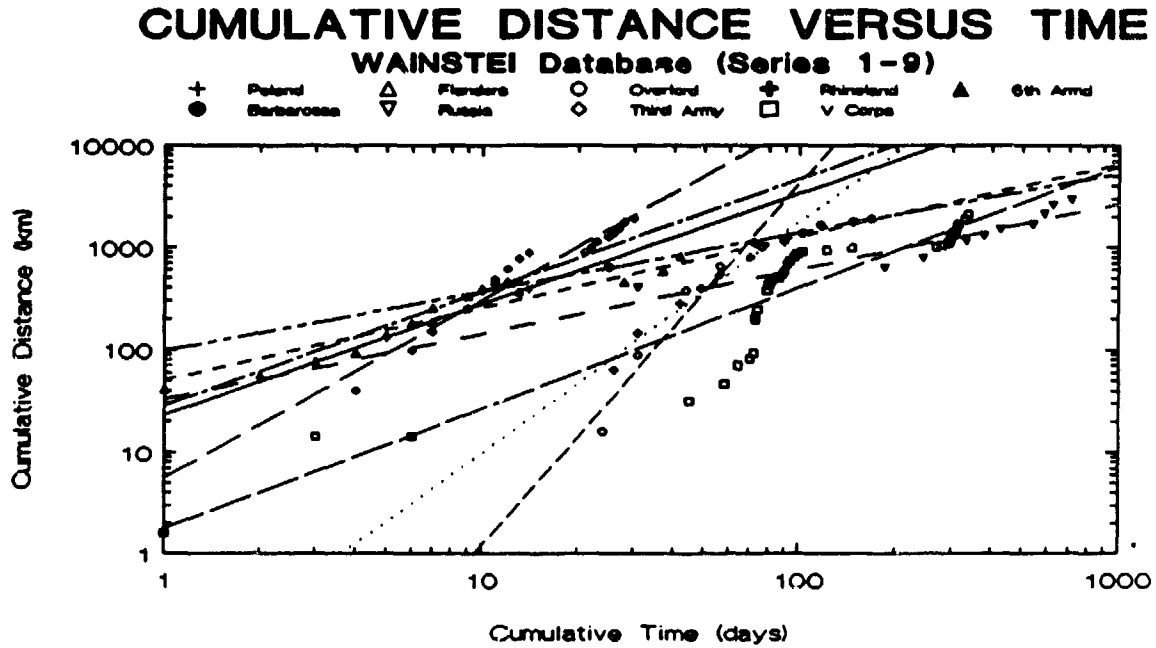


Figure 4-26a. Cumulative Distances and Times from WAINSTEI (Series 1-9)

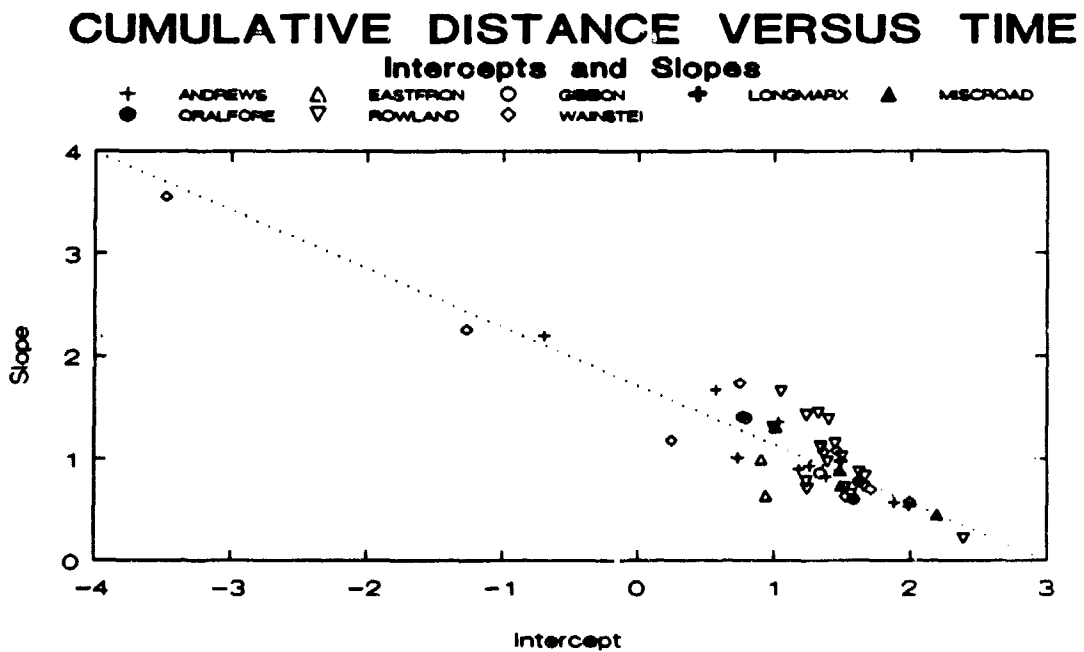


Figure 4-26b. Slope versus Intercept for Cumulative Distances and Times



### DISTRIBUTION OF VALUES

#### Slopes of Cumulative Advances

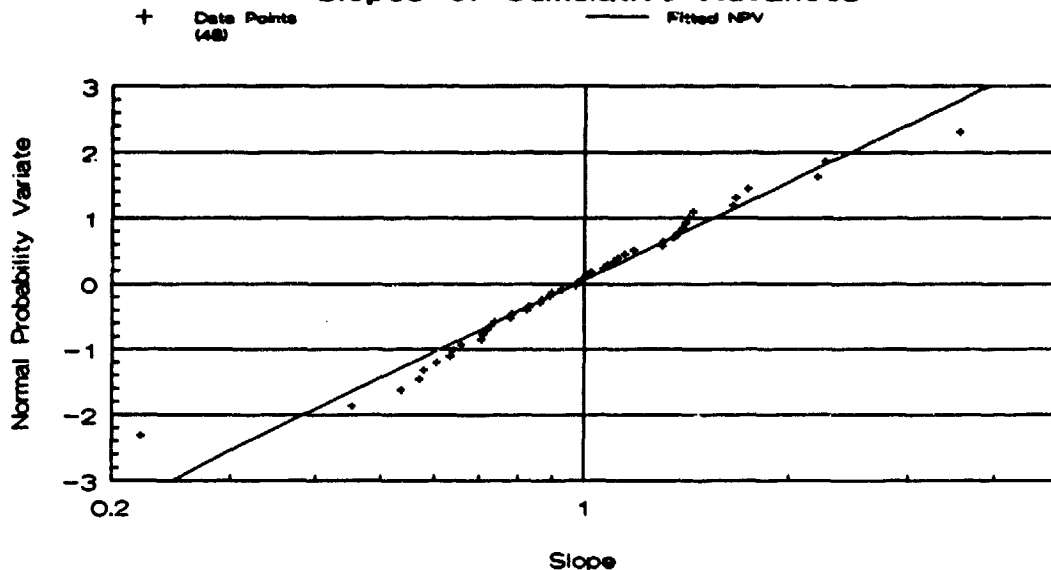


Figure 4-27a. Distribution of Slopes of Cumulative Advances

### CUMULATIVE DISTANCES AND TIMES

#### Typical Relationships

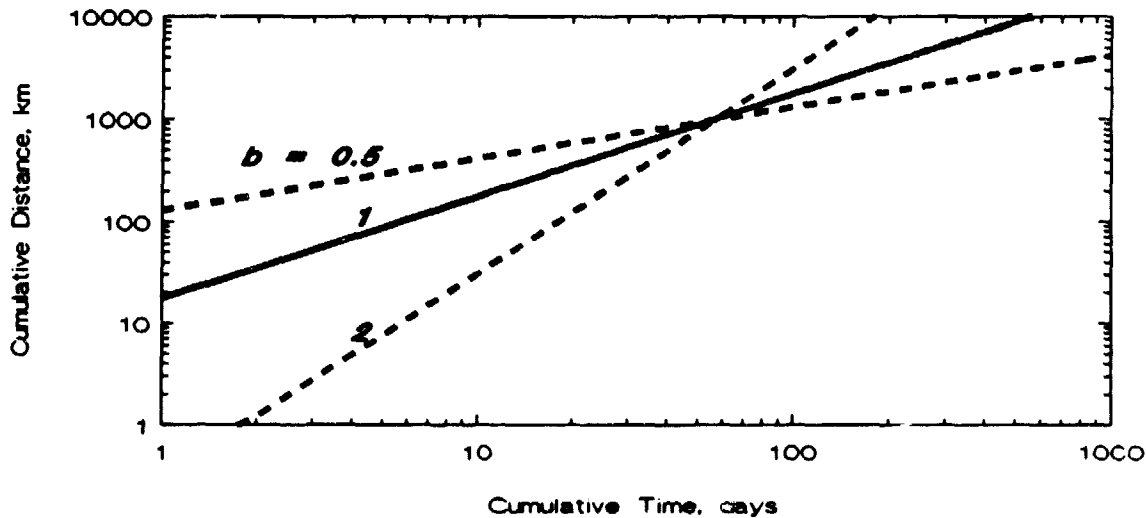


Figure 4-27b. Typical Cumulative Distance Versus Cumulative Time Relationships

## 4-11. DISTRIBUTION OF RATES OF ADVANCE

a. The proposition to be considered is that advance rates are normally distributed.

b. We have repeatedly remarked that the data are highly variable. Here we consider how best to characterize that variability. Our investigation of how the data are distributed used graphical methods, supported where applicable by the Kolmogorov-Smirnov test for goodness of fit. Several alternatives were evaluated, including the normal, exponential, lognormal, Weibull, and gamma distributions. Occasionally, because the data are so grainy, none of the distributions gave a theoretically acceptable fit. In such cases, the choice devolved upon the distribution that gave the best fit compared to the other alternatives.

c. Since we omitted cases with zero or negative advance distances, all of the following applies only to "successful" advances, i.e. to the distributions of advance distances, durations, and rates *conditional* upon achieving a measurable advance (it may be small, but has to be large enough to be measured).

d. Preliminary analysis of the lightly engaged data bases revealed that their lower reported advance rates (those in the 1 to 5 km/day range) were spurious. Many of them were actually heavily engaged cases accidentally included in the lightly engaged category, or cases where the unit was merely shifting its location slightly for administrative convenience, security, or other reasons not related to operational advances. So for this paragraph we dropped all of the lightly engaged cases that had reported advance rates less or equal to 5 km/day. This gave rise to the LGTFTO5L and LGTHMO5L derived data bases. (The first 3 letters indicate that they are for lightly engaged forces, the next two indicate the mode of movement, and the last two indicate that we omitted the lower 5 km/day rate bracket.) In doing this we omitted about 7 percent and 11 percent (respectively) from the LGTFOOT and LGTHOMO data bases. These data bases were then used unchanged for the distance and duration distributions. Considering the relatively large number of cases we have to work with, and the difficulty in obtaining accurate and valid data, I think this is acceptable. (If the reader objects to this, let him think of our results as applying to cases where lightly engaged forces are known to have an advance rate of at least 5 km/day.)

e. Figure 4-28a shows the distribution of daily advance rates for the HVYFOOT data base. The vertical scale is in units of standard deviation above and below the mean, and the horizontal scale is logarithmic. If the vertical scale were converted to the equivalent probability value, Figure 4-28a would look exactly like a conventional normal probability versus logarithmic axis chart. On such charts, lognormal distributions plot as straight lines. And we see that HVYFOOT daily advance rates are very close to a lognormal distribution. The occasional "jumps" so noticeable in Figure 4-28 are due to graininess in the data—obviously arising from a habit of reporting advance rates in round numbers of km/day or miles/day. Figures 4-28b and 4-29 show that daily advance rates for the other major derived data bases are also lognormally distributed. Note that the horizontal scale in Figures 28c and 28d is an order of magnitude higher than in Figures 28a and 28b (daily advance rates of lightly engaged forces average about an order of magnitude higher than for heavily engaged ones). By superimposing the charts, we see that heavily engaged forces have shallower slopes than lightly engaged ones. Shallower slopes correspond to a higher coefficient of variation (more variability in terms of percent deviation from the median). We also looked to see how well these data followed a normal, exponential, Weibull, or gamma distribution. The lognormal distribution was found to fit daily advance rate data consistently better than any of these alternatives.

f. Comparison of Figures 4-28a and 4-28b shows that daily advance rates reported in the HVYHOMO data base are about 1.5 times higher than those for the HVYFOOT data base, but have about the same slope (coefficient of variation). Similar

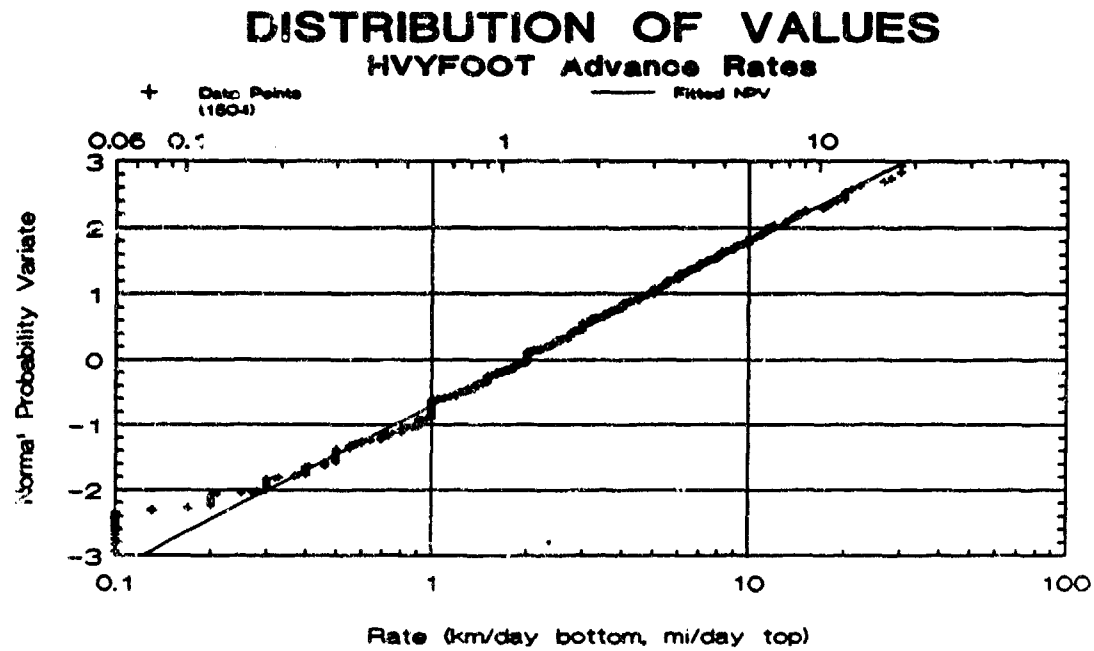


Figure 4-28a. Distribution of Daily Advance Rate for HVVYFOOT Data

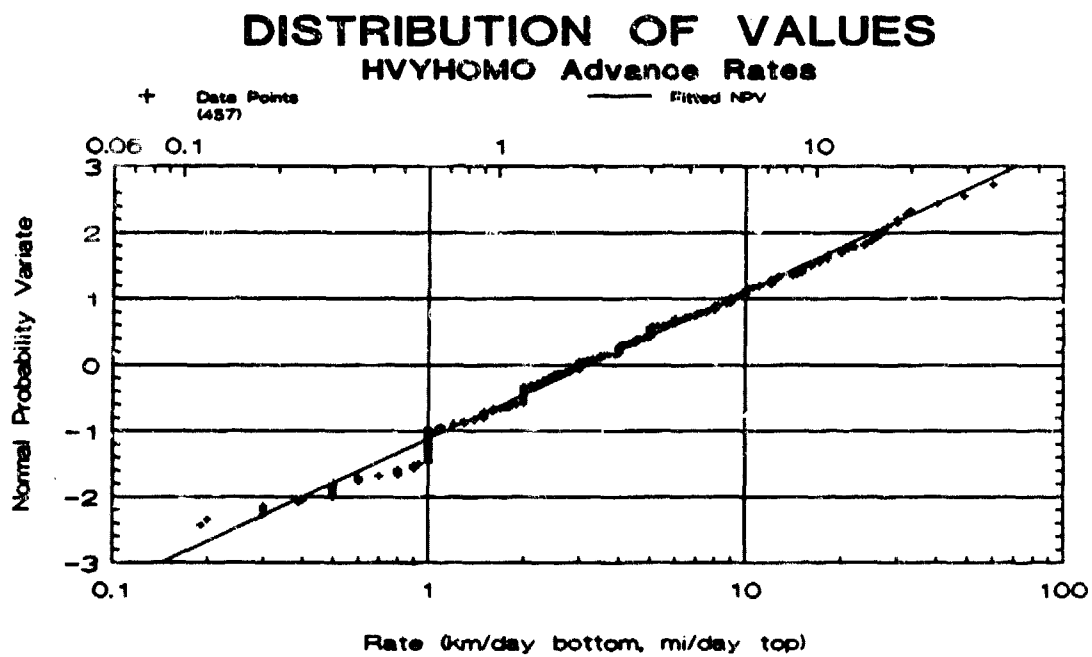


Figure 4-28b. Distribution of Daily Advance Rate for HVVYHOMO Data

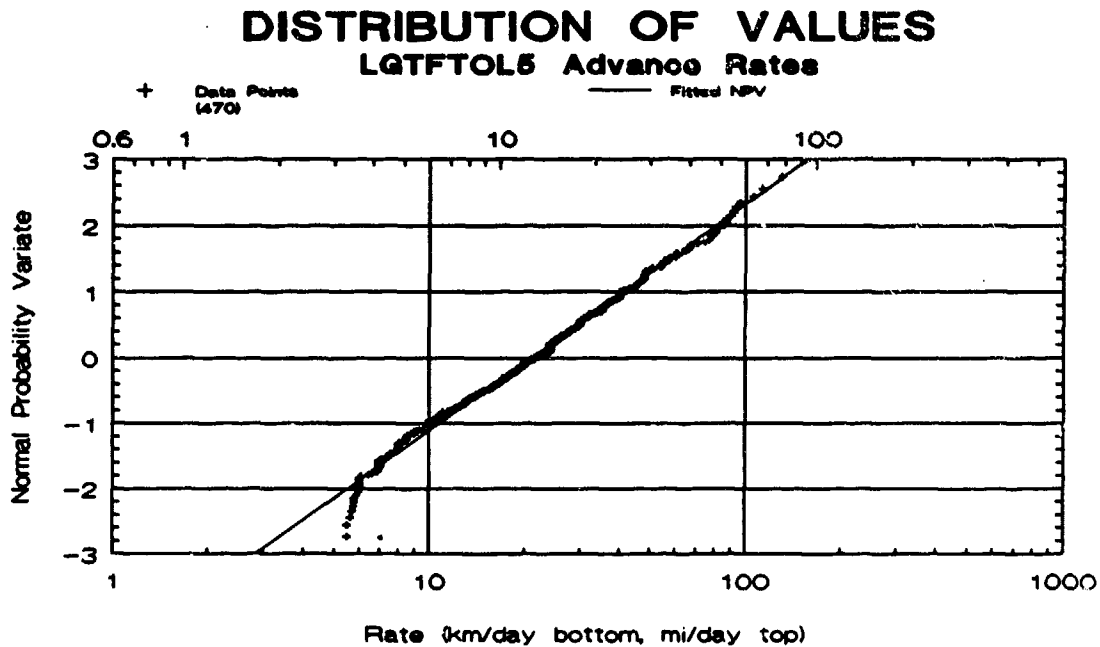


Figure 4-28c. Distribution of Daily Advance Rate for LGTFOOT Data

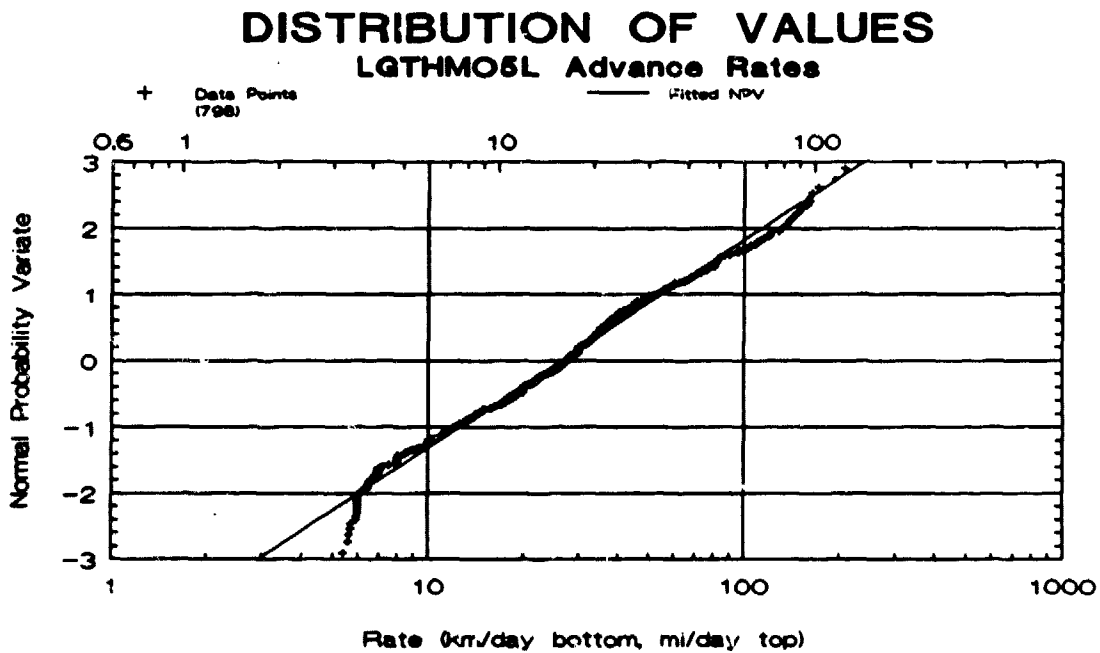


Figure 4-28d. Distribution of Daily Advance Rate for LGTHOMO Data

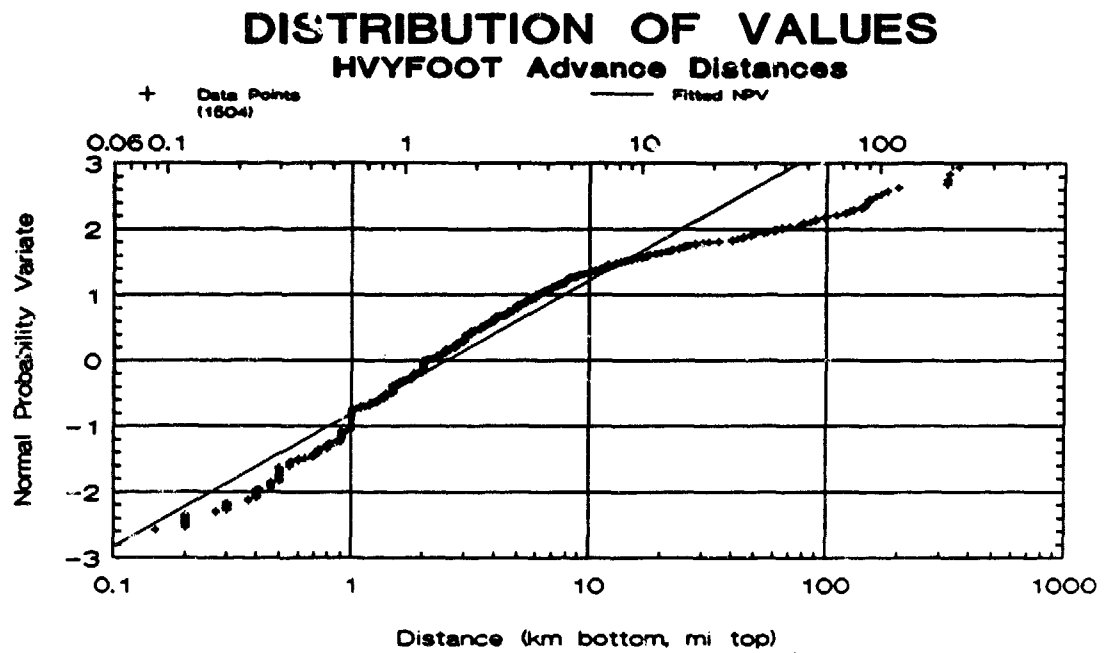


Figure 4-29a. Distribution of Total Advance Distance for HVYFOOT Data

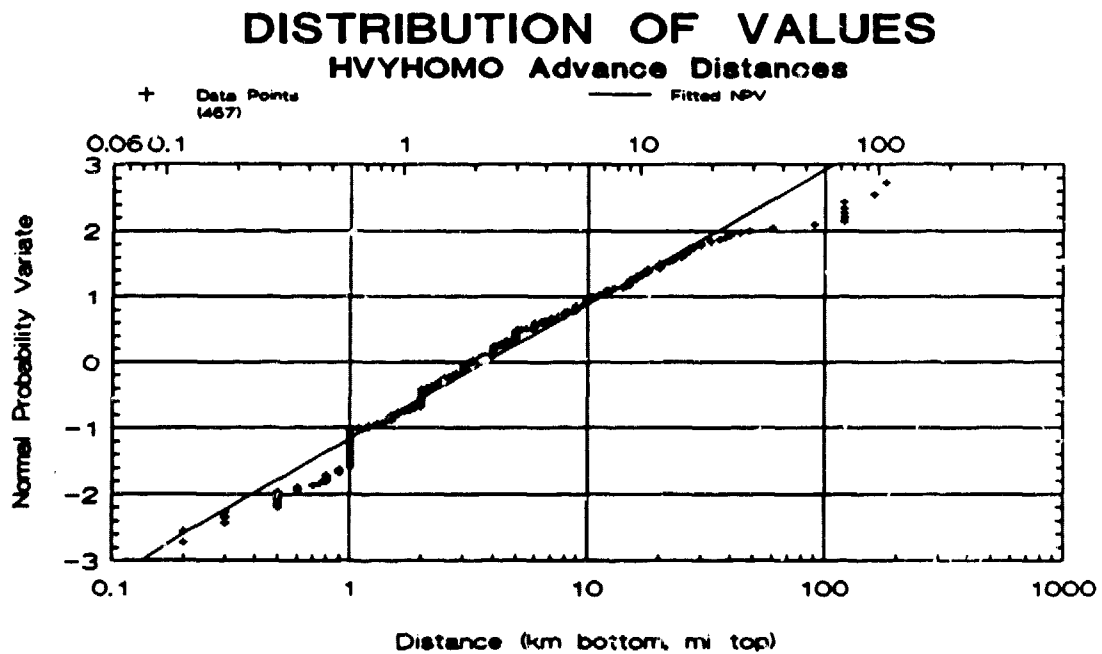


Figure 4-29b. Distribution of Total Advance Distance for HVYHOMO Data

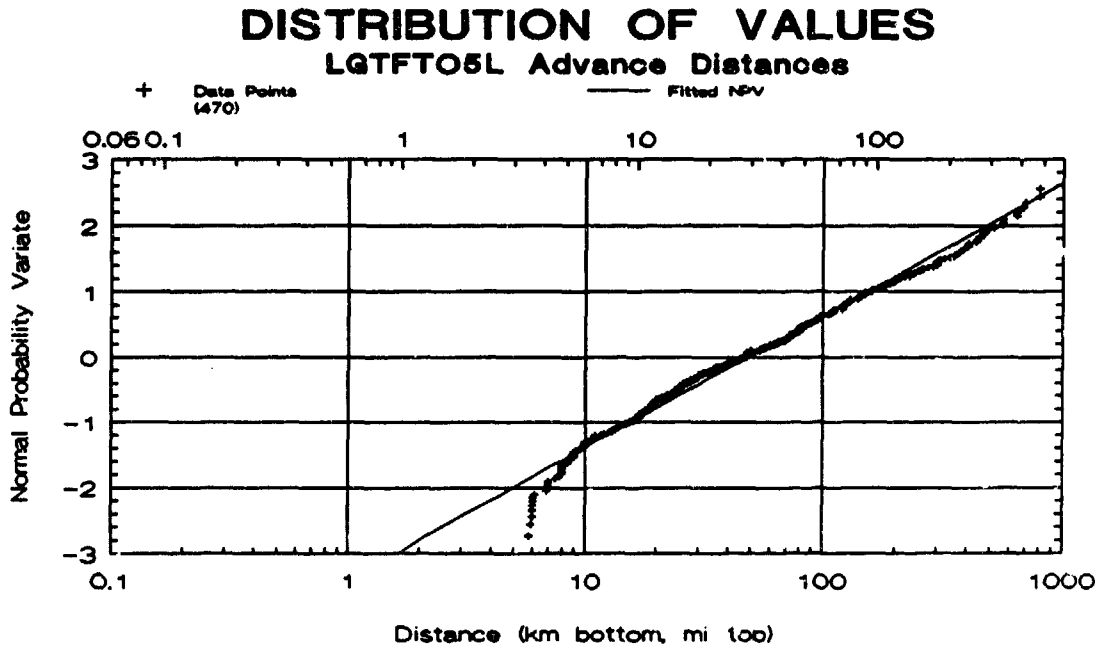


Figure 4-29c. Distribution of Total Advance Distance for LGTF05L Data

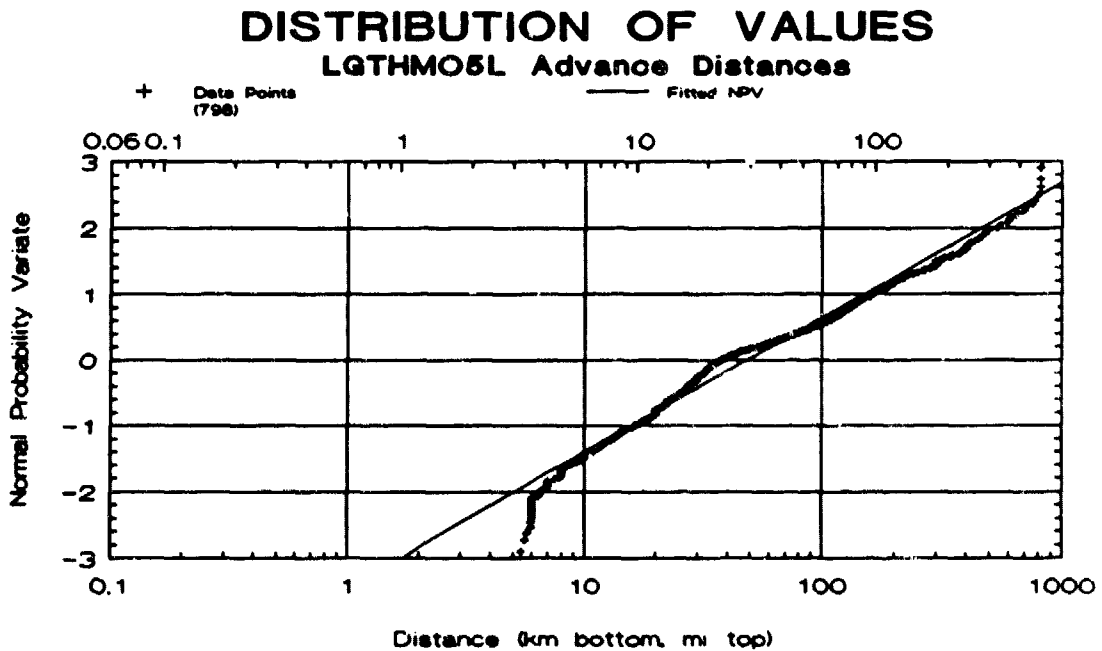


Figure 4-29d. Distribution of Total Advance Distance for LGTH05L Data

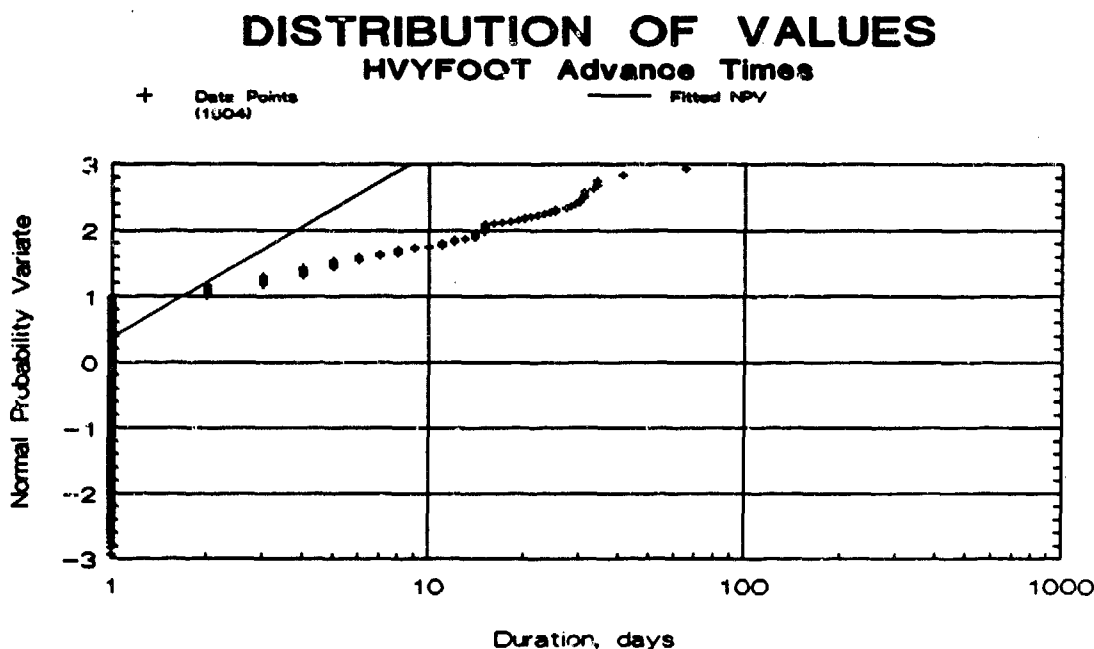


Figure 4-30a. Distribution of Total Advance Duration for HVYFOOT Data

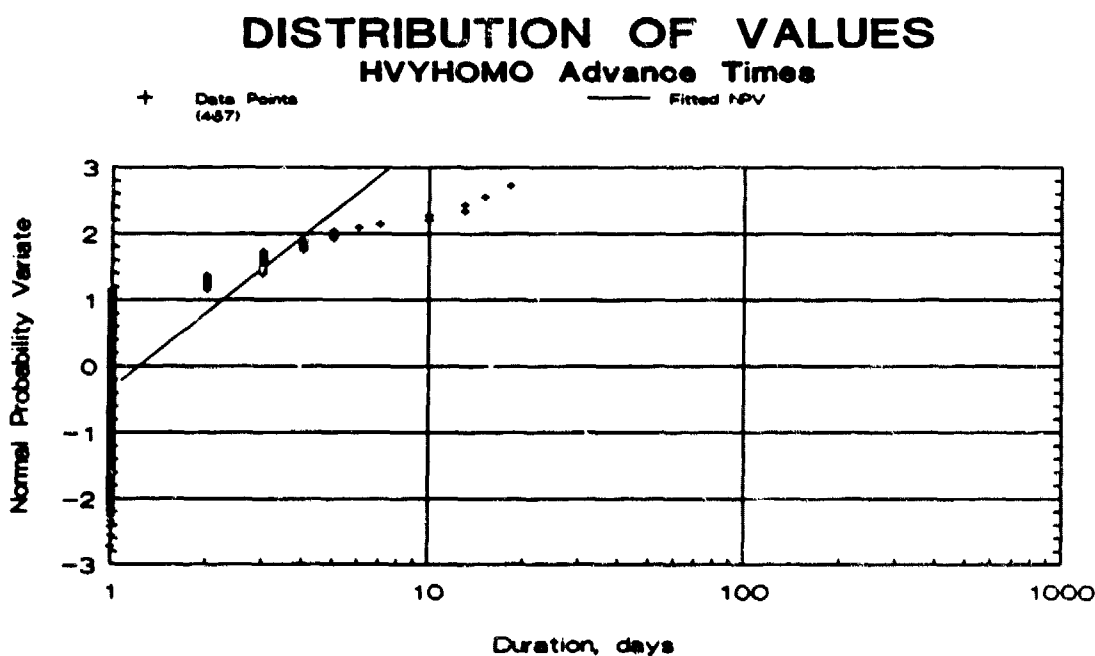


Figure 4-30b. Distribution of Total Advance Duration for HVYHOMO Data

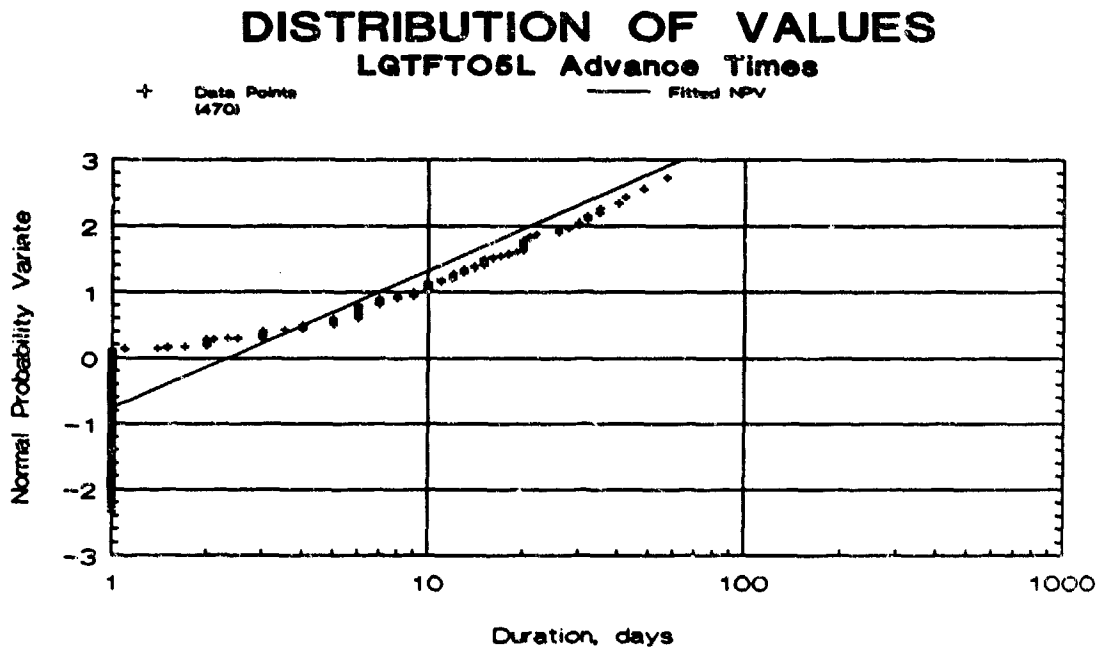


Figure 4-30c. Distribution of Total Advance Duration for LGTFOOT Data

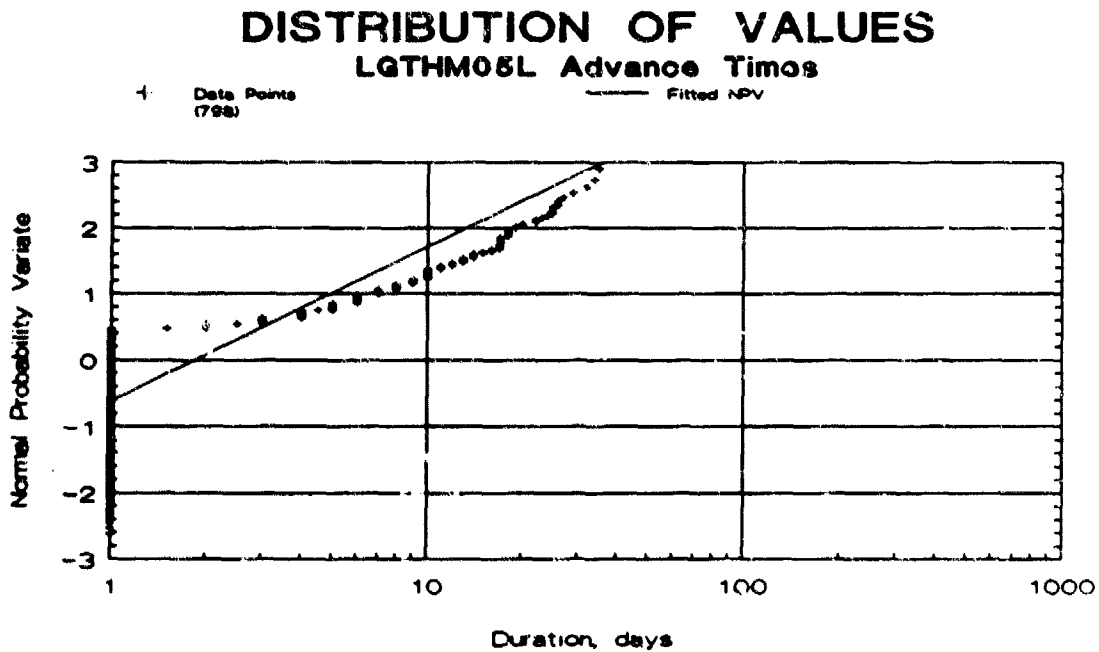


Figure 4-30d. Distribution of Total Advance Duration for LGTHOMO Data



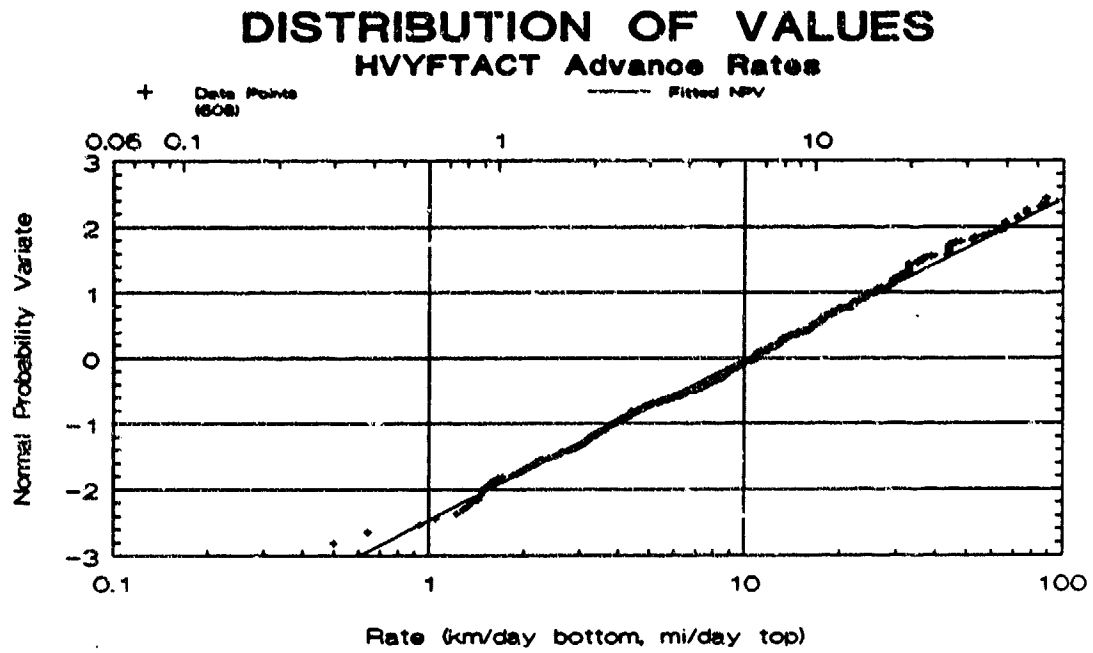


Figure 4-31a. Distribution of Active Advance Rate for HVVYFOOT Data

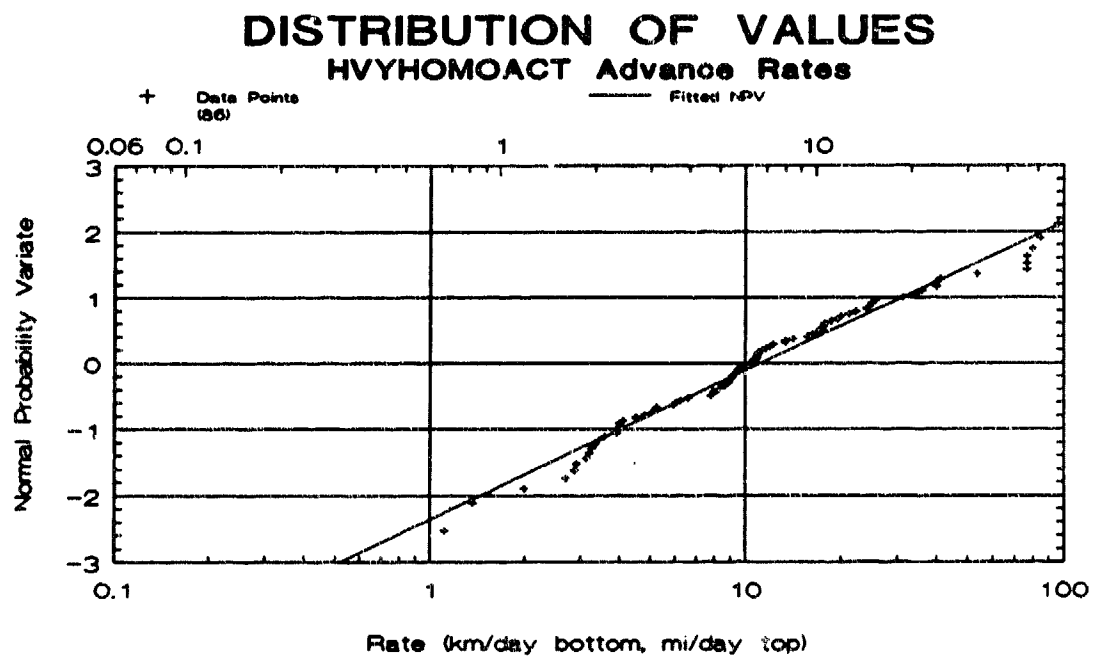


Figure 4-31b. Distribution of Active Advance Rate for HVVYHOMO Data

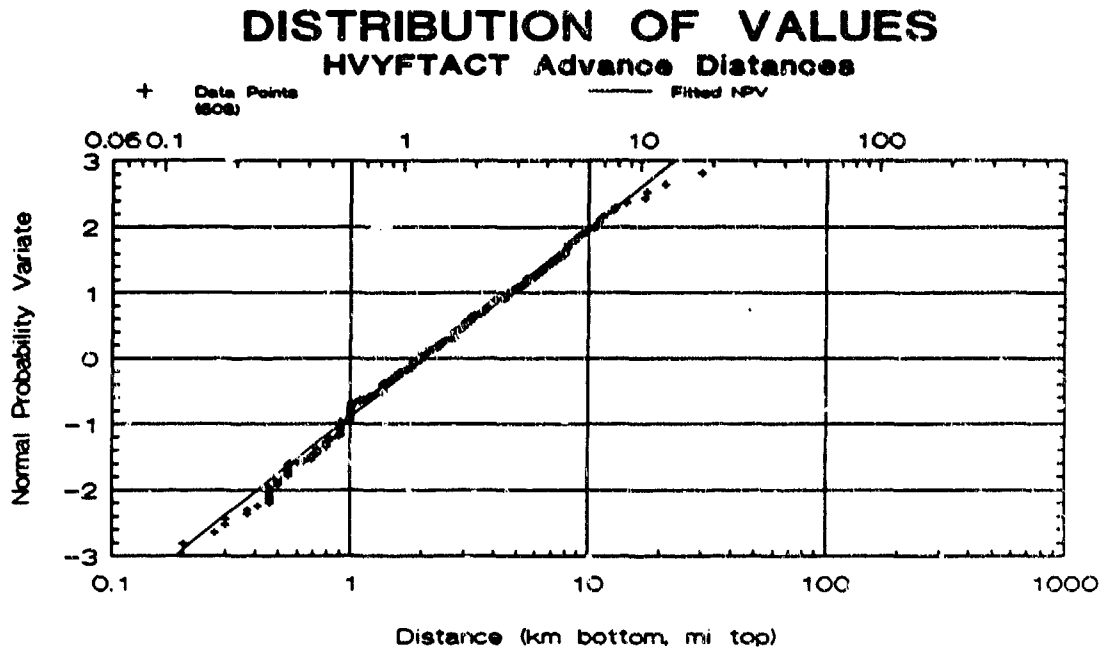


Figure 4-32a. Distribution of Active Advance Distance for HVFYFOOT Data

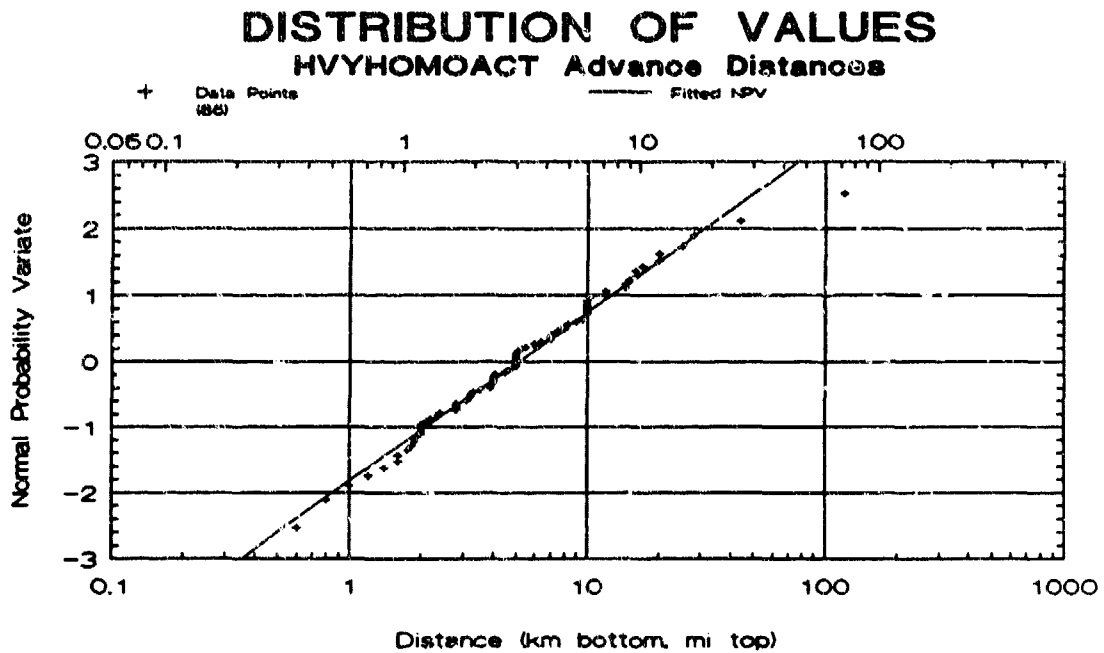


Figure 4-32b. Distribution of Active Advance Distance for HVYHOMO Data

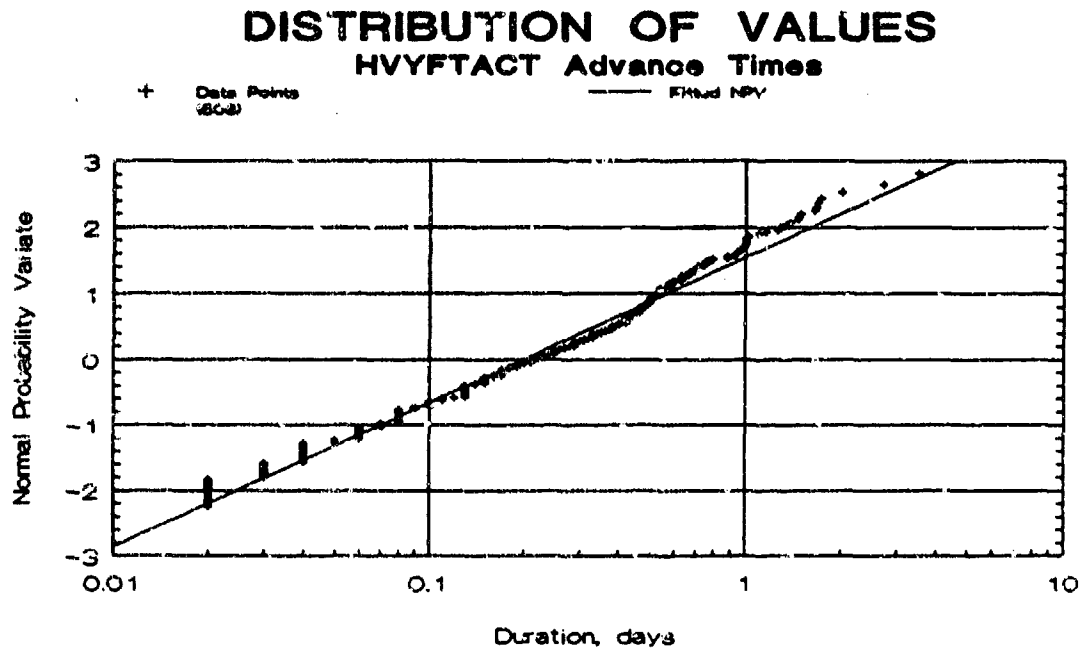


Figure 4-33a. Distribution of Active Advance Duration for HVYFOOT Data

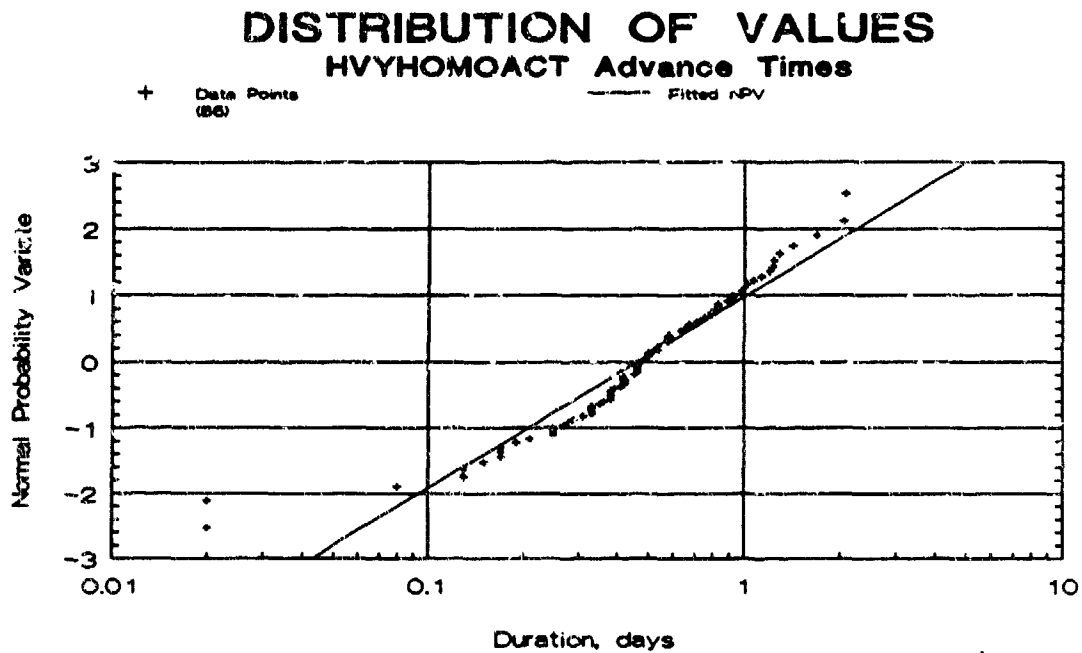


Figure 4-33b. Distribution of Active Advance Duration for HVYHOMO Data

remarks apply to Figures 4-28c and 4-28d, except that here the ratio of median values may be a little smaller. Figure 4-29 shows the distribution of total advance distances, and Figure 4-30 shows the distribution of total advance durations.

g. Figure 4-31 shows the distribution of active advance rates reported in the HVYFTACT and HVYHOMOACT derived data bases. Except for graininess, a lognormal distribution fits these data very well. We also looked to see how well these data followed the normal, exponential, Weibull, or gamma distributions. The lognormal distribution was found to fit active advance rate data consistently better than any of these alternatives. By comparing Figures 4-31a and 4-31b we see that the active advance rate distribution for the HVYHOMOACT data base practically coincides with that for the HVYFTACT data base, both in median value and in slope. Here again we see that the active advance rates of heavily engaged forces moving by horse or motor are very similar to those of heavily engaged forces moving by foot. However, inspection of Figures 4-32 and 4-33 shows that both active advance *distances* and *durations* are greater for the HVYHOMOACT than for the HVYFTACT data base. So only their ratios (active advance rates) are similar. Since both the active advance distances and durations are lognormally distributed, it is perhaps no surprise that the active advance rate is also lognormally distributed (*cf.* paragraph C-5c of Appendix C).

h. Comparison of Figure 4-31 with 4-28 shows that active advance rates of heavily engaged forces are about 4 or 5 times higher than their daily advance rates, although the slopes of their distributions are practically identical (so the coefficient of variation is the same). The inference seems inescapable that heavily engaged forces are standing still at least 75 percent to 80 percent of the time (relative to lightly engaged forces). This inference is supported by the observation that the distribution of daily and active advance *distance* is about the same for heavily engaged forces (compare Figures 4-29a to 4-32a and 4-29b to 4-32b—the “flare” in Figure 4-29’s distributions at higher values of total advance distance is probably associated with the longer duration advances, hence the closer agreement of active advance distance with the lower range of total advance distance). Comparison of Figures 4-30 and 4-33 clearly shows that active advance times are much shorter than total advance times.

i. The upshot of our analysis is that advance rates are *not* normally distributed. Their distribution is highly skewed, and much more closely fit by lognormal distributions than by any of the others tried (normal, exponential, Weibull, and gamma).

#### 4-12. RATES OF ADVANCE AND FORCE RATIOS

a. The proposition to be considered is that advance rates are determined by force ratios. That is, advance rates are very sensitive to changes in force ratio, and force ratio is the most important single factor governing advance rates.

b. Many wargames and simulations base their movement algorithms on this proposition. The MEFORD report<sup>17</sup> is the earliest I know of that explicitly and formally challenged the validity of this proposition. The debate over its range of validity has raged ever since. Trevor Dupuy has been a particularly vocal advocate of the view that this proposition is not supported by the empirical evidence.

c. In our investigations of this proposition, we used the CDB90 data base, because it is the only one that gives numerical forces in a convenient form. The ACSDB data base also gives numerical forces, but those data became available too late to be used in this study. The RMC data base occasionally gives forces in terms of the number of “platoons” involved, but this measure is too inaccurate to support analysis. Besides, it is available on both sides for only a handful of the RMC cases. So let us consider Figure 4-34a, which we constructed by grouping the CDB90 data by force ratio and advance rate categories (by the nature of the CDB90 data base, all these data are for

<sup>17</sup> RAC, Methodology for Force Requirements Determination (MEFORD), RAC-R-121, Research Analysis Corporation, McLean, VA, May 1971. AD 515-716L

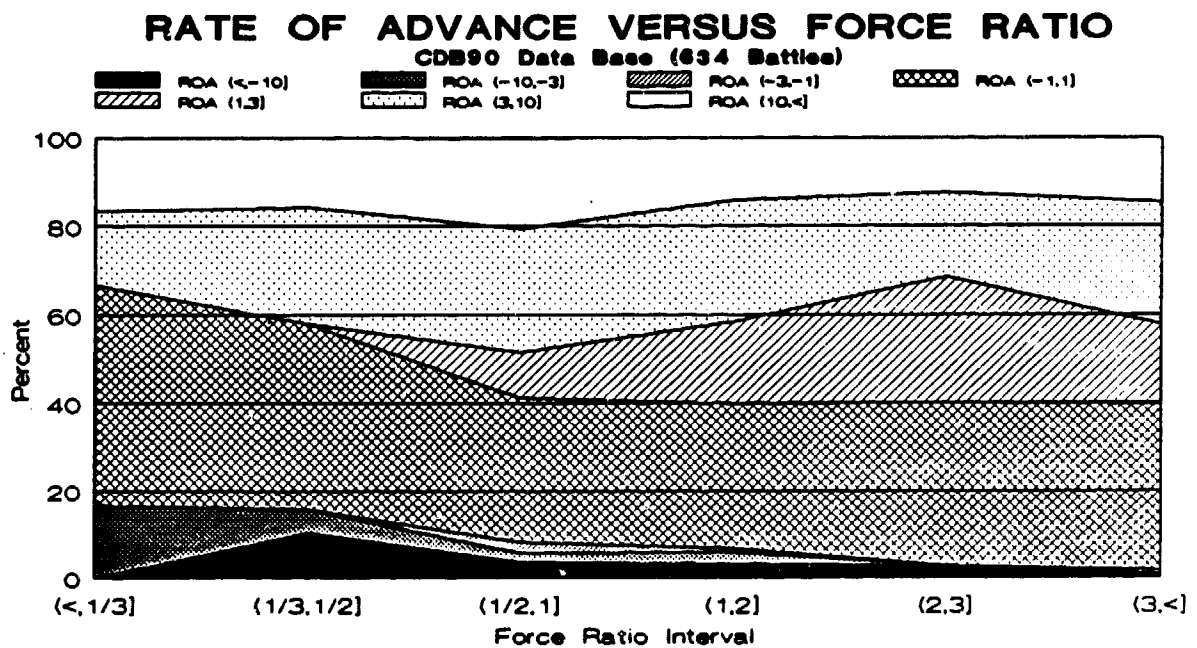


Figure 4-34a. Distribution of Advance Rate Versus Force Ratio

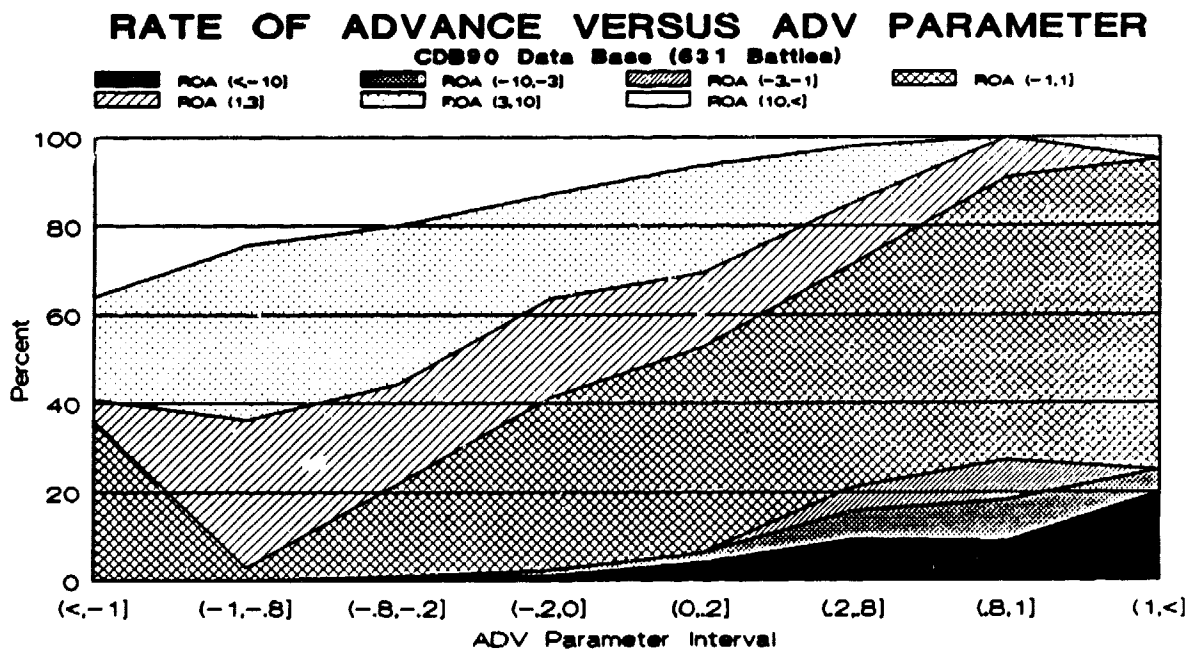


Figure 4-34b. Distribution of Advance Rate Versus ADV Parameter

active advance rates of heavily engaged forces.) Along the horizontal axis we have force ratios (of attacker to defender) of less than 1:3, between 1:3 and 1:2, etc. The solid black band along the bottom is for advance rates of less than  $-10$  km/day (i.e., the *defender* is advancing at better than 10 km/day). The clear band along the top is for advance rates of more than  $+10$  km/day (i.e., the *attacker* is advancing at better than 10 km/day). As can be seen, there is at best only a slight dependence of advance rate on force ratio. Figure 4-34b is a similar display, except that in place of the force ratio it uses the (*defender's*) advantage parameter, ADV. It shows a strong dependence of advance rate on the ADV parameter. Figure 4-35a explains how the ADV parameter is computed and shows how its mathematical form was inspired by a reparameterization of the Lanchester square law of combat attrition.<sup>18</sup>

d. It may be quite difficult to find a single parameter that is much more closely associated with advance rate than ADV. If there *were* a better, it presumably could be no better than knowing which side won (or will win). So consider the results when the reported winning side is used in place of the ADV parameter, as shown in Figure 4-35b. The qualitative and quantitative similarity of Figures 4-35b and 4-34b shows that ADV is about as good at predicting the advance rate as knowing which side won! In fact, Figure 4-36 shows that the ADV parameter is much more closely associated with the winning side than is the force ratio. However, there is still a lot of scatter in the data, and no single parameter is a consistently accurate predictor of advance rate—even if we know which side won, we still don't know as much as we'd like to know about advance rate. We also tried the following alternative single parameters, but none of them had as strong a relationship to advance rate as either the ADV parameter or the winning side. Here the notation is as defined in Figure 4-35a.

- (1) Fractional exchange ratio (i.e.,  $FER = f_x/f_y$ , where  $f_x$  is the attacker's casualty fraction and  $f_y$  is the defender's casualty fraction).
- (2) Casualty exchange ratio (i.e.,  $CER = C_x/C_y$ , where  $C_x$  is the number of attacker casualties and  $C_y$  is the number of defender casualties).
- (3) Attacker's casualty fraction ( $f_x$ ).
- (4) Defender's casualty fraction ( $f_y$ ).

What we found was that the ADV parameter was as good as any, and better than most. We also found that force ratio was definitely the worst of the bunch! As for the attacker's and the defender's casualty fractions, the combination of both of them worked much better than either alone. In fact, when we fed them both into a stepwise regression computation, it elected to use the logarithms of both, but with opposite signs. That means that it wanted to use the fractional exchange ratio ( $FER$ ), which is very closely related to the ADV parameter both conceptually and numerically, as indicated in Figure 4-35a. There is still a lot of scatter in the data, however, and none of these

<sup>18</sup> For more information on the ADV parameter and its uses see, among others, the following papers by Dr. Robert L. Helmbold: Historical Data and Lanchester's Theory of Combat, CORG-SP-128, 1961, AD-480 975L; Historical Data and Lanchester's Theory of Combat: Part II, CORG-SP-190, 1964, AD-480 109L; Lanchester's Equations, Historical Battles, and War Games, Proceedings of the Eighth Military Operations Research Symposium, Pt. Mugu, CA, 1961; Some Observations on the Use of Lanchester's Theory for Prediction, Opns Res, 12(1964), pp 778-781; Air Battles and Ground Battles—A Common Pattern?, RAND P-4548, 1971, AD-718 975; Integrated Warfare—Representing the Decision Processes Within Computer Simulations of Combat, co-author, US Army Concepts Analysis Technical Paper CAA-TP-85-1, Jan 85; Combat History Analysis Study Effort (CHASE): Progress Report for the Period August 1984-June 1985, US Army Concepts Analysis Agency Technical Paper CAA-TP-86-2, August 1986, AD F-860-122; Do Battles and Wars Have a Common Relationship Between Casualties and Victory?, CAA-TP-87-16, Nov 87, AD-A 196 126.

## HELMBOLD'S REPARAMETRIZATION

$$dx/dt = -Dy, \quad dy/dt = -Ax, \quad x(0) = x_0, \quad y(0) = y_0 \quad (1)$$

$$a = x/x_0, \quad d = y/y_0, \quad 1 - a = f_x, \quad d = 1 - f_y \quad (2)$$

$$\mu = (y_0/x_0)\sqrt{D/A}, \quad \lambda = \sqrt{AD} \quad (3)$$

$$da/dt = -\lambda\mu d, \quad dd/dt = -\lambda\mu^{-1}a, \quad a(0) = d(0) = 1 \quad (4)$$

$$\mu^2 = (1-a^2)/(1-d^2) = [(1-a)(1+a)]/[(1-d)(1+d)] \quad (5)$$

$$\mu^2 = [f_x(2-f_x)]/[(f_y(2-f_y))] \doteq f_x/f_y = \text{FER} \quad (6)$$

$$\text{ADV} = \ln(\mu) \doteq (1/2)\ln(\text{FER}) \quad (7)$$

$$a = \cosh(\lambda t) - \mu \sinh(\lambda t) \quad (8)$$

$$d = \cosh(\lambda t) - \mu^{-1} \sinh(\lambda t) \quad (9)$$

Figure 4-35a. Derivation of the ADV Parameter

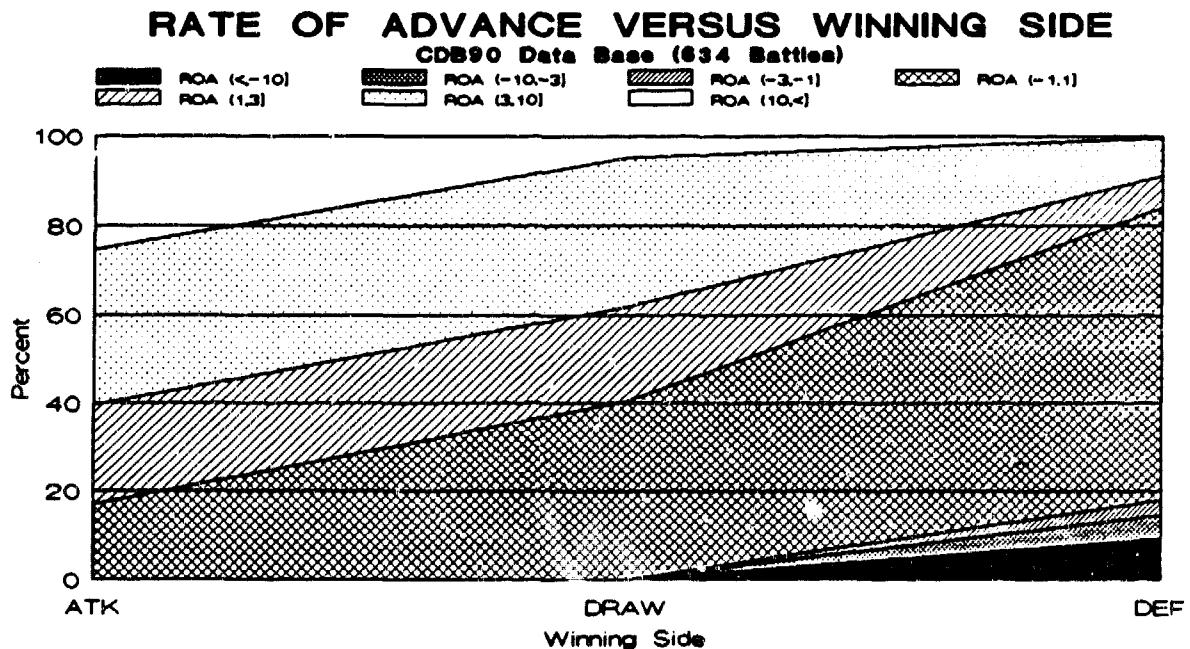


Figure 4-35b. Distribution of Advance Rate Versus Winning Side

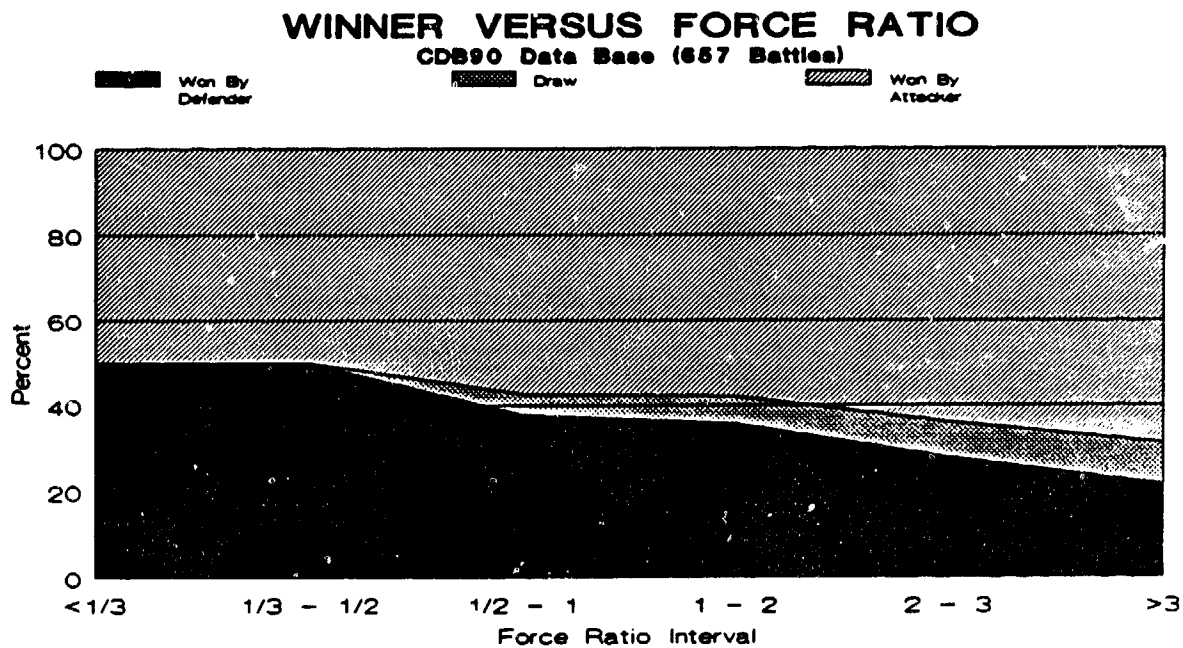


Figure 4-36a. Distribution of Winning Side Versus Force Ratio

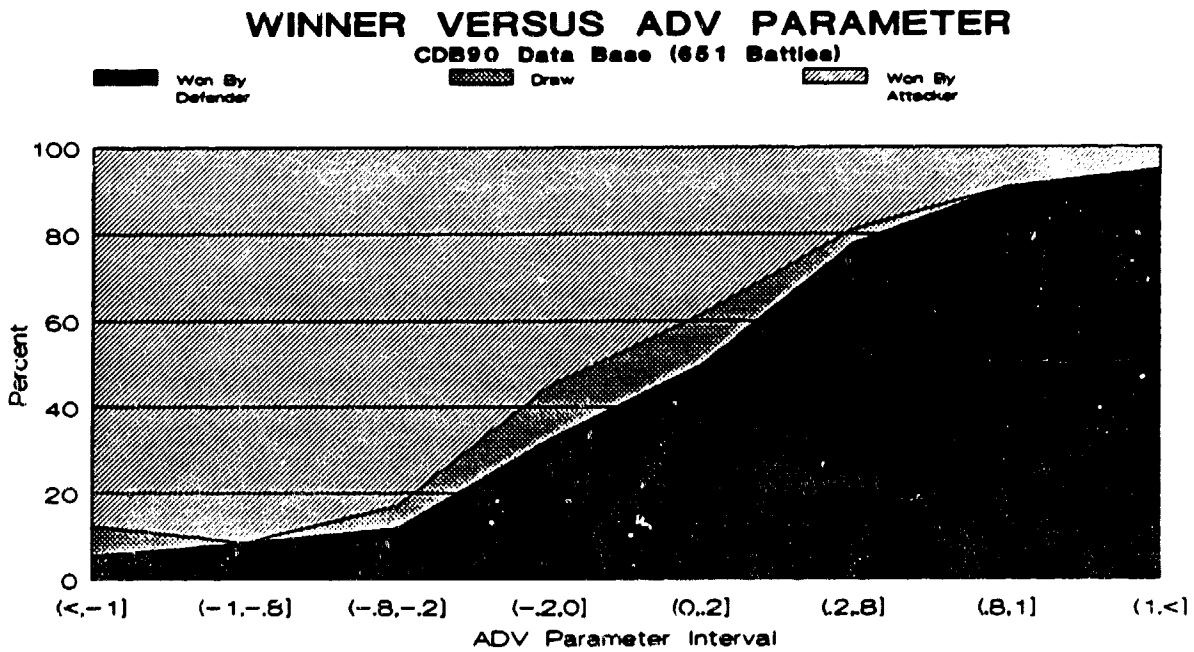


Figure 4-36b. Distribution of Winning Side Versus ADV Parameter



predictors is consistently accurate. Yet it looks as though the ADV parameter is at least a step in the right direction. If so, then perhaps a clever modification or augmentation of it might get measurably closer to the truth.

e. We also tried various plots of the advance distance or duration versus force ratio, casualty exchange ratio, attacker's casualty fraction, defender's casualty fraction, fractional exchange ratio, and the ADV parameter. These did not reveal anything of importance beyond what is already provided by Figures 4-34 and 4-35.

f. The upshot of our investigation is that force ratio is among the poorest and ADV among the best single-parameter predictors of advance rates. While neither is a consistently accurate and dependable predictor of advance rate, the ADV parameter is a far better choice than force ratio.

#### 4-13. PREDICTING RATES OF ADVANCE

a. The proposition to be considered is that advance rates are predictable and generalizable. By this we mean that advance rates are sufficiently well understood that we can rather accurately and reliably estimate what the advance rate will be, both in future times (prediction) and in new tactical and operational situations (generalization).

b. In earlier paragraphs we examined the dependency of advance rates on such factors as the speed of the governing mode of transportation, calendar date, force size, whether the governing mode of transportation was foot as opposed to horse or motor, whether the force was lightly or heavily engaged, season of the year, length of the operation, and single parameters such as the force ratio or ADV parameter. In none of these investigations did we find any single factor that would satisfactorily predict advance rate. Instead, each investigation found that the reported data are highly variable and widely scattered. However, those investigations used only a single factor at a time. They did not consider that some combination of factors might provide a better predictor of advance rate. This paragraph addresses that issue.

c. To do that we performed a great many ordinary and stepwise regression computations, using as the candidate independent variables various subsets of those mentioned in paragraph 4-13b, and several others. In many of these analyses, we transformed some of the original values to a logarithmic form, as seems desirable in view of the results of paragraph 4-12. In some analyses, we had to transform both positive and negative values logarithmically. To do that we employed the signed-logarithm (*sl*) transform defined by

$$sl(x) = \text{sgn}(x)\ln(1 + |x|),$$

where *sgn*(*x*) is the signum function (equal to  $-1$  if  $x < 0$ ,  $+1$  if  $x > 0$ , and  $0$  if  $x = 0$ ). The signed logarithm function brings logarithmically distributed values closer to normal, as shown in Figure 4-37. The CDB90 data base was the only data base used in these analyses because it is the only one that has enough data to compute all of the key parameters (specifically including the ADV parameter). Also, it is largely free of the successful advance bias. *Note that this means all the discussion in this paragraph applies only to the active periods of heavily engaged forces.*

d. In one such analysis we stepwise-regressed the dependent variable *sl*(*KMD.A*) against the independent variables *ln*(*DURN*), *ln*(*f<sub>x</sub>*), *ln*(*f<sub>y</sub>*), *ln*(*SIZE*), *FRY*, *ln*(*EPS*), *ln*(*λ*), and *ln*(*DENRO*). Here *KMDA* is the kilometer distance advanced by the attacker (and may be either positive or negative, depending on whether the attacker moved forward or backward), *DURN* is the duration (in days) of the active period, *f<sub>x</sub>* and *f<sub>y</sub>* are the attacker's and defender's casualty fractions as defined in Figure 4-35a, *SIZE* is the geometric mean of the number of personnel in the attacking and defending force, *FRY* is the fractional year at which the battle started, *EPS* is the bitterness of the

## SIGNED LOGARITHM FUNCTION

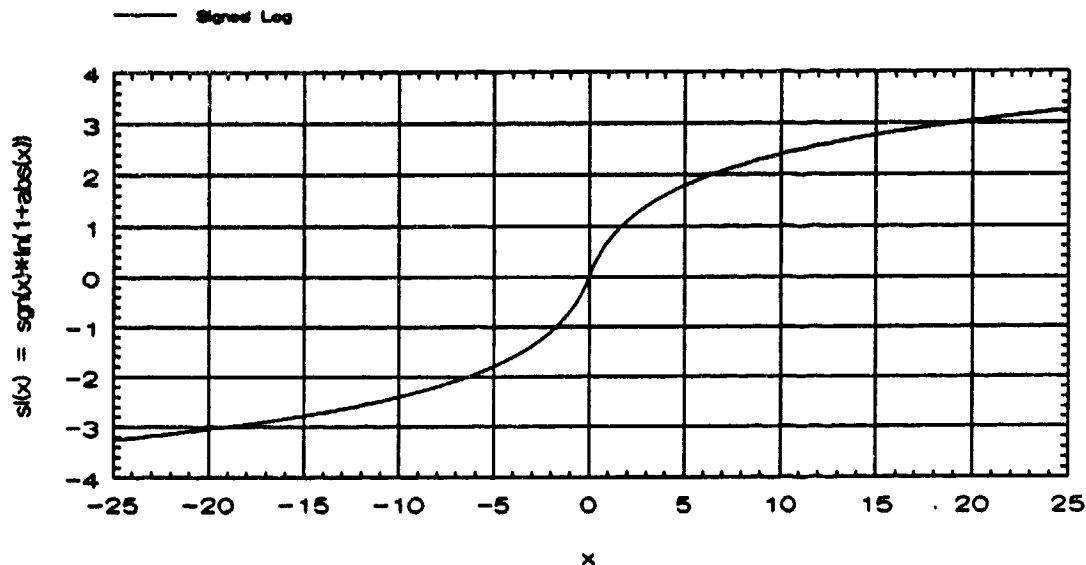


Figure 4-37. Signed Logarithm Transformation

battle and is equal to the product  $\lambda t$  (where  $t$  is the battle duration  $DURN$ ,  $\lambda$  is the battle intensity defined in Figure 4-35a), and  $DENRO$  is the ratio of densities ( $SIZE$  divided by the initial width of front in kilometers). This stepwise regression selected the variables  $\ln(DURN)$ ,  $\ln(f_x)$ , and  $\ln(f_y)$ . These gave an  $R^2$  value of 0.34, and so account for about 34 percent of the variance in  $sl(KMDA)$ . The resulting estimated regression equation may be expressed as

$$sl(KMDA) = 0.34\ln(DURN) - 0.36\ln(f_x) + 0.35\ln(f_y) + N(0, 0.89).$$

Here  $N(\mu, \sigma)$  stands for a normal random variable with mean  $\mu$  and standard deviation  $\sigma$ . It represents the "error term" in the regression equation. The F-ratio is 80 on 2 and 311 degrees of freedom, which corresponds to a P-value of practically zero.

e. These results are typical of our linear regression results. We also tried using either  $sl(KMDA)$  or  $sl(ROA)$  as the dependent variable (where  $ROA$  is the rate of advance in km/day), and various combinations of the above independent variables with  $\ln(SIZEA)$ ,  $\ln(CER)$ , and  $\ln(FER)$ . Here  $SIZEA$  is the number of personnel in the advancing force (which may be either the force that initially was attacking or its opponent),  $CER$  is the casualty exchange ratio (attacker's casualties divided by defender's casualties), and  $FER$  is the fractional exchange ratio ( $f_x/f_y$ ). The upshot of these analyses seems to be that:

(1) If  $\ln(f_x)$  and  $\ln(f_y)$  both appear in the independent variable list, then stepwise or multiple regression selects coefficients for them that are opposite in sign and approximately equal in magnitude. Thus, the regression program tends to combine  $\ln(f_x)$  and  $\ln(f_y)$  into a single factor that approximates either  $\ln(FER)$  or  $ADV$ .

(2) Given several additional independent variables to choose from, stepwise regression still selects  $\ln(DURN)$  and  $ADV$  as the main regressors on which  $sl(KMDA)$  depends.

(3) Using a nonlinear combination of the ADV and  $\ln(DURN)$  regressors may improve the quality of the regression slightly, but not enough to be very helpful. In particular, no appreciable improvement in  $R^2$  was obtained by going from  $sl(KMDA)$  regressed on ADV and  $\ln(DURN)$  to  $sl(KMDA)$  stepwise-regressed on ADV,  $ADV^2$ ,  $\ln(DURN)$ ,  $[\ln(DURN)]^2$ , and  $ADV \times \ln(DURN)$ ; or to  $sl(KMDA)$  stepwise-regressed on ADV,  $ADV^3$ ,  $\ln(DURN)$ ,  $[\ln(DURN)]^3$ , and  $ADV \times \ln(DURN)$ .

(4) So it appears that  $sl(KMDA)$  depends primarily on  $\ln(DURN)$  and ADV, and that other regressors express their effect on  $sl(KMDA)$  via  $\ln(DURN)$  and ADV. Unfortunately, none of the linear regressions give very dependable (consistently accurate) predictions. Even the best of them account for only about 33 percent of the variance in  $sl(ROA)$  or  $sl(KMDA)$ , and have coefficients of variation around 1.1 or so. So the actual  $KMDA$  values often differ from their "predicted" values by factors of about 2 or 3.

f. We then tried some other nonlinear regressions. Some preliminary analyses suggested using the "explanatory" variables ADV,  $FRY$ ,  $FRY^2$ ,  $TERINDEX$ ,  $\ln(SIZE)$  or  $\ln(SIZEA)$ ,  $[\ln(SIZE)]^2$  or  $[\ln(SIZEA)]^2$ ,  $DATE$ , and  $HOMO$ . This analysis was an attempt to find out which combination of them (as opposed to using them singly) best fits/predicts the value of  $sl(ROA)$  (i.e., signed log of the rate of advance). Here  $TERINDEX$  is a composite terrain index that corresponds to the difficulty of the terrain as reflected in the terrain codes of the CDB90 data base,  $DATE$  is the date the battle started, and  $HOMO$  is a categorical variable distinguishing cases of movement by horse or motor ( $HOMO = 1$ ) from movement by foot ( $HOMO = 0$ ).

(1) Multiple regression of  $sl(ROA)$  versus ADV,  $FRY$ ,  $FRY^2$ ,  $TERINDEX$ ,  $\ln(SIZE)$ ,  $[\ln(SIZE)]^2$ ,  $HOMO$ , and  $DATE$  gave an  $R^2 = 0.33$ . The variables ADV,  $FRY$ ,  $FRY^2$ , and  $DATE$  had regression coefficients significantly different from zero at the 6 or 7 percent level. However, ADV alone would give an  $R^2 = 0.31$ , so adding the other variables doesn't really improve the fit very much. And, indeed, the corresponding stepwise regression picked out ADV and no other explanatory variables.

(2) Multiple regression of  $sl(ROA)$  versus ADV,  $FRY$ ,  $FRY^2$ ,  $TERINDEX$ ,  $\ln(SIZEA)$ ,  $[\ln(SIZEA)]^2$ , and  $HOMO$  gave an  $R^2 = 0.32$ . Only the coefficient of ADV (and marginally  $HOMO$ ) was significant at the 10 percent level. Again, the regression using ADV alone gave an  $R^2 = 0.31$ , so the other variables didn't really add much to the fit. This was confirmed by a stepwise regression which picked out only the ADV variable.

(3) A stepwise regression of ADV versus  $HOMO$ ,  $FRY$ ,  $FRY^2$ ,  $TERINDEX$ ,  $\ln(SIZEA)$ , and  $[\ln(SIZEA)]^2$  was done to see what could explain the ADV values. This picked out the  $HOMO$  variable as statistically the best, but since its  $R^2$  is only 0.0303 ( $R = -0.174$ ), it is hardly a very good predictor. However, although its sign is "right," i.e., horse and motor give the attacker an advantage (defender advantage is negative for  $HOMO = 1$  versus  $HOMO = 0$ ).

g. We also tried an analysis to follow up on the suggestion that advance rates may be largely determined by the time spent "waiting around." If that is true, then the eight qualitative factors in the CDB90 data base for leadership, planning, intelligence, and surprise ( $LEADA$ ,  $LEADAA$ ,  $SURPA$ ,  $SURPAA$ ,  $PLANA$ ,  $INTELLA$ ,  $LOGSA$ , and  $LOGSAA$ ) ought to have a substantial impact on reducing the waiting around time. So they ought to have an impact on advance rate. To examine this hypothesis, we constructed  $BIGSUM$  and  $SUM$  as indices of these qualitative factors, defined as follows:

(1) *BIGSUM* = sum of the eight qualitative ratings for *LEADA*, *LEADAA*, *SURPA*, *SURPAA*, *PLANA*, *INTELA*, *LOGSA*, and *LOGSAA*,

(2) *SUM* = sum of the four qualitative ratings for *LEADA*, *LEADAA*, *PLANA*, and *INTELA*.

We then discarded all cases with  $BIGSUM \leq -30$  or  $SUM \leq -17$ , since they correspond to instances where the qualitative ratings are not given in the CDB90 data base. This left us with 614 cases. We then tried multiple regressions of  $sl(ROA)$  on ADV, on ADV and *BIGSUM*, and on ADV and *SUM*. We found that *BIGSUM* and ADV are correlated with each other about as strongly as either is with  $sl(ROA)$ . Technically, *BIGSUM* does add somewhat to ADV as a predictor of  $sl(ROA)$  ( $R^2 = 0.34$  using both ADV and *BIGSUM*, while  $R^2 = 0.31$  using just ADV). However, neither is a very satisfactory predictor (coefficient of variation is 1.1 using both ADV and *BIGSUM*, and 1.2 using just ADV). So the upshot of this exercise was that the contribution of *BIGSUM* to  $sl(ROA)$ , as compared to ADV, does not contradict the hypothesis that much of the advance rate for heavily engaged forces is due to  $C^3$  delays. However, it does not prove that any such effect exists, either! The correlation between *SUM* and *BIGSUM* is  $0.896 \approx 0.90$ . Accordingly, though *SUM* could have been used in place of *BIGSUM*, the results would have been about the same, since they are nearly statistically equivalent.

h. Another analysis was done to clarify the results described in paragraph 4-13g. To do this, we stepwise regressed  $sl(ROA)$ ,  $sl(KMDA)$ , or ADV on various independent variables, but chiefly  $\ln(f_x)$ ,  $\ln(f_y)$ ,  $\ln(FER)$ ,  $\ln(FR)$ ,  $sl(DURN)$ , ADV, *SUM*, *BIGSUM*, and  $ADVBIGSUM = ADV \times BIGSUM$ .

(1) Stepwise regression of  $sl(KMDA)$  versus  $\ln(f_x)$ ,  $\ln(f_y)$ ,  $\ln(FER)$ ,  $\ln(FR)$ , ADV,  $sl(DURN)$ , *SUM*, *BIGSUM*, and  $ADVBIGSUM$  selected  $\ln(f_x)$ ,  $sl(DURN)$ , and *SUM* as the independent variables, yielding an  $R^2 = 0.45$ , with an F-ratio of 163 on 3 and 612 degrees of freedom, for a P-value practically 0. However, the coefficient of variation is still 0.86, so that the fitted values are only good to within a factor of  $2.4^{\pm 1}$ , which, of course, is not really terribly impressive! Stepwise regression of  $sl(KMDA)$  on *SUM* and *BIGSUM* selected *SUM*, with an  $R^2 = 0.23$ .

(2) Stepwise regression of  $sl(ROA)$  versus  $\ln(f_x)$ ,  $\ln(f_y)$ ,  $\ln(FER)$ ,  $\ln(FR)$ , ADV,  $sl(DURN)$ , *SUM*, *BIGSUM*, and  $ADVBIGSUM$  selected ADV,  $sl(DURN)$ , and *SUM* as the independent variables, yielding an  $R^2 = 0.39$ , with an F-ratio of 130.25 on 3 and 612 degrees of freedom, for a P-value practically 0. However, the coefficient of variation is still 1.06, so that fitted values are only good to within a factor of  $2.9^{\pm 1}$ , which is not impressive. Using Andrew's sine function, robust regression of  $sl(ROA)$  on ADV and *SUM* gave an  $R^2 = 0.52$ , with an F-ratio of 331 on 2 and 610 degrees of freedom, for a P-value practically 0. However, the coefficient of variation is still 0.59, so that fitted values are only good to within a factor of  $1.8^{\pm 1}$ , which is still not very helpful. Stepwise regression of  $sl(ROA)$  versus ADV, *SUM*, and *BIGSUM* selected *SUM* as the independent variable, yielding an  $R^2 = 0.38$ , with an F-ratio of 185 on 2 and 612 degrees of freedom, for a P-value practically 0, and a coefficient of variation of 1.07. A stepwise regression of  $sl(ROA)$  versus *SUM* and *BIGSUM* selected *SUM* as the independent variable, with an  $R^2 = 0.33$ .

(3) Although the foregoing results do not contradict the hypothesis that  $C^3$  delays (and the subjective factors that affect them) have a major impact on advance rates, they cannot be interpreted as proving the hypothesis is correct. One of the more serious objections to such an inference is the possibility of vicious circles like those mentioned in paragraph 3-4a(3) in which the subjective factors are based in part on the outcome (i.e., advance rate or distance). Obviously, using the subjective factors *SUM* and *BIGSUM* leaves the door wide open to such vicious circles, so the (relatively) high  $R^2$  values found by including these factors may well be more artifact than fact.

i. We did many additional analyses, but they didn't substantially change the results described above. None of the analyses found any consistently accurate way to predict advance rates. The variability and scatter in the data are very large, and even the best predictors account for only a modest amount of that variability. Those past studies that at first sight seem to have found excellent "predictors" of advance rates have done so, as we discussed in Chapter 3, only by committing serious epistemological errors and various kinds of statistical mayhem on the data! When interpreting the foregoing multiple regression analyses, the following facts must be considered (here *WINA* is a categorical variable describing which side wins,  $WINA = -1$  if the defender wins,  $+1$  if the attacker wins, and  $0$  if the battle is a draw).

(1) Regression of *SLROA* on *ADV* alone yields an  $R^2 = 0.31$ , and a coefficient of variation of 1.21.

(2) Regression of *SLROA* on *WINA* alone yields an  $R^2 = 0.39$ , and a coefficient of variation of 1.03.

(3) Regression of *SLROA* on *ADV* and *WINA* combined yields an  $R^2 = 0.50$ , and a coefficient of variation of 1.03.

Accordingly, only the multiple correlations in which subjective factors were used give  $R^2$  values much above that achievable using *ADV* alone.

j. The upshot of our analysis seems to be that both I and several careful and conscientious investigators who used sound methods have tried to find consistently accurate predictors. But none of the epistemologically sound studies have had much success. In fact, they have managed to account for only about 1/3 the variance in the data. So 2/3 of the variance remains a mystery.<sup>19</sup> How many failures to build a perpetual motion machine do we have to have before we decide it is impossible? I think it is time we seriously entertain the possibility that advance rates are inherently random and highly variable, and begin to explore systematically the implications when the amount of unaccounted for and inherently unexplainable variance in advance rates is substantial.

#### 4-14. ANALYSIS FINDINGS

a. Proposition 1. Are advance rates about equal to the transport speed? No. Reported advance rates average 1 or two orders of magnitude *below* the speed of their governing mode of transportation. This suggests that land combat operations stand still about 90 percent or 99 percent of the time. The data are widely scattered and highly variable.

b. Proposition 2. Are advance rates much higher now than in the past? No. Reported advance rates haven't changed much over the last 400 years, or even longer. But the data are widely scattered and highly variable.

c. Proposition 3. Are advance rates much higher for small units? Can't answer simply yes or no. Reported *daily* advance distances for heavily engaged forces are higher for *large* units! But reported *active* advance rates do seem to be higher for small units, by factors of about 2 or 3. However, in this context a unit with more than about 900 or 1000 people in it is not longer "small." The data are widely scattered and highly variable, and the reported versus actual bias confuses the pattern.

<sup>19</sup> Perhaps we should not be *too* disappointed at that. Recently a psychologist has said that "Nevertheless, even if the heritability of IQ scores is at the bottom of this range [from 30 percent to 70 percent], it is a remarkable finding. To account for 30 percent of the variance of anything as complex as IQ scores is a remarkable achievement." (See Robert Plomin, 'The Role of Inheritance in Behavior, *Science*, 248(13 April 1990, pp 183-188.)

d. Proposition 4. Are advance rates much higher for motor than for foot? Can't answer simply yes or no. Reported heavily engaged daily advance rates are somewhat higher for horse and motor. But reported heavily engaged active advance rates are the *same* for horse and motor as for foot. But reported lightly engaged advance rates *do* seem to be somewhat higher for horse and motor than for foot, by factors of about 1.5 to 2. The data are widely scattered and highly variable.

e. Proposition 5. Are advance rates higher for lightly engaged forces? Yes. Reported daily advance rates for lightly engaged forces are about an order of magnitude higher than for heavily engaged ones. But reported *daily* advance rates for *lightly* engaged forces are higher than reported *active* advance rates for *heavily* engaged ones only by factors of 2 to 5. The data are widely scattered and highly variable.

f. Proposition 6. Are advance rates higher in summer than in winter? Yes. More so for horse and motor than for foot, by a factor of 2 or 3. The data are widely scattered and highly variable.

g. Proposition 7. Are advance rates lower for longer operations? Not consistently. While it is true for many operations, in about as many others reported advance rates actually *increase* with the passage of time. And we don't know how to predict which ones will slow down and which will speed up. Although the general trend is for a steady uniform speed throughout the operation, the data are widely scattered and highly variable.

h. Proposition 8. Are advance rates normally distributed? No. Reported advance rate distributions are highly skewed and are consistently much closer to the lognormal than to either the normal, exponential, Weibull, or gamma distributions.

i. Proposition 9. Are advance rates determined by force ratios? No. Reported advance rates are practically independent of force ratios. They are *much* more strongly associated with CER, FER, and ADV than with force ratios. Force ratios are among the worst of the single ratios on which to base advance rates. But even the best are of little help in practical situations because the data are so widely scattered and highly variable.

j. Proposition 10. Are advance rates (consistently and accurately) predictable? Not by current knowledge. The highest proportion of the variance accounted for by the epistemologically acceptable studies typically seems to cluster somewhere around 1/3. So despite great efforts by many distinguished analysts and historians, only weak general trends and broad tendencies have been found. How many attempts to build a perpetual motion machine have to fail before we decide it is impossible? I think it is time we seriously consider the possibility that there is a large, irreducible random component in advance rates. If there is, we just have to face up to that and behave accordingly.

#### 4-15. OTHER OBSERVATIONS

a. **Burning Questions.** The "burning questions" regarding advance rates in land combat operations were raised in paragraph 2-8a, and are repeated below. If we could give good answers to these questions, then we would know just about all that was worth knowing about advance rates in land combat operations. So far as I know, this is the first time they have been stated so clearly and concisely. Unfortunately, no one has any good answers for them. And, in view of the remarks in paragraphs 4-13i and j, it seems that adequate answers to them are not in the presently available data bases. I can say it no better than Tukey<sup>20</sup> (see Figure 4-38). Here are those "burning questions:"

<sup>20</sup> John W. Tukey, *Sunset Salvo*, *The American Statistician*, 40(1986), 1(Feb), pp72-76.

- (1) What starts a force in motion?
- (2) Once started, what governs its speed and direction?
- (3) What eventually stops or reverses its motion?
- (4) Where and when does it stop or reverse direction?

b. **Successful Advance Bias.** As mentioned earlier, nearly all of our conclusions are conditional on there *being* a successful advance. That's because most of the data bases describe only cases of successful advances. I haven't yet found a good way of correcting for or dealing with this successful advance bias. Since it has been widely overlooked in the past, no one has studied it enough to properly quantify its effects.

c. **Reported versus Actual Bias.** Past studies completely missed this particular bias. Now that we know about it, we can start figuring out how best to deal with it. My investigations to date show that it can bias advance rates downward by factors of at least 3 to 5.

d. **Generalization and Prediction.** These are epistemological issues. It is time to consider seriously that usefully accurate generalization and prediction may not be possible. We've been looking for the key, but what if there is no key? If that's the way it is, then we should implement Taguchi's advice—see Figure 4-39—in a practical way, so that our operations will be robust against unavoidable variability. I suggest we have a lot fewer studies looking for high fidelity predictors of advance rates, and a lot more on how to cope rationally with extreme variations in advance rates. In particular, how do we design our forces and operations to be robust in the face of such variations?

e. **The C<sup>3</sup> Delay Hypothesis.** I think it is very important to study the *non*-movement periods. No one has thought of that in the past. But we need to understand what's causing them and governing their durations. How long are they? How frequently do they occur? What causes them to start and end? What's going on while they last? More work on these issues could be very helpful.

f. **Correlations.** How best to deal with correlations in time, space, and space-time?

## JOHN W. TUKEY

- THE DATA MAY NOT CONTAIN THE ANSWER.
- THE DATA MAY NOT EVEN CONTAIN THE APPEARANCE OF AN ANSWER.
- EXPLORATION (SEEKING APPEARANCES) MAY BE AIDED BY CALCULATIONS.

Figure 4-38. Tukey's Advice

## GENICHI TAGUCHI

"The most important quality of process or product design is its robustness against variation."

Figure 4-39. Taguchi's Advice



## CHAPTER 5

## CONCLUSIONS AND OBSERVATIONS

5-1. **OBJECTIVE.** This chapter presents the conclusions and observations of our work to date.

## 5-2. DATA BASE FINDINGS

a. A lot of statistical data on advance rates in land combat operations is available, but for a given purpose only a properly chosen part of it is useful. Hence we will use the primary data bases as a resource, and draw from them such derived data bases as are most suitable for various purposes. Sometimes only a tiny fraction of the data is suitable. Sometimes none of it applies.

b. Past work used a bewildering variety of *ad hoc* descriptors and terminology. Most of the sources use rather brief descriptors to indicate the tactical situation's environmental and operational conditions. But no one descriptor is used by all of the sources, nor is there any source that uses all of the descriptors. For example, one study reported natural obstacles such as rivers and canals. None of the other studies did that. Usually, it is hard to tell just what the descriptors actually mean in objectively measurable terms. Nor is it likely that a given descriptor is defined the same way by all sources. This lack of standardization makes comparisons always difficult, and sometimes impossible. In addition, appreciable difficulties for future quantitative work have been created by the use of subjective or qualitative descriptors not meaningfully defined in terms of objectively measurable quantities (e.g., the use of such subjective/qualitative descriptors as "intensity of enemy opposition," "degree of difficulty of the terrain," and the like).

c. Some past work made mistakes in figuring elapsed times from calendar dates and/or times of day; some even made mistakes in computing rates from distances and elapsed times. Also, a variety of units were used, including miles, kiloyards, hectometers, Persian parasangs (~ 3.5 miles), Chinese lis (~ 1/3 mile), Greek or Roman stadia (~ 607 feet), and French leagues (~ 3 miles). In all cases we recomputed elapsed times from starting and ending times and converted all units to days and to kilometers. (Of course, a rate of 100 km/day does not necessarily mean that the distance was 100 km or that the time was 1 day.)

d. Several sources, some intentionally and others unintentionally, selected cases of a successful advance by the attacker. This biases the data against successful defensive efforts and in favor of advances by attackers. Where this bias is present, study results on advance rates are conditional on there having been a successful advance.

e. Reported advance rates are systematically biased toward lower values than are actually achieved. As explained in Chapter 4, this bias can cause reported rates to be too low by factors of around 3 to 5, and seriously distorts the apparent influence of size upon rate of advance.

## 5-3. SURVEY FINDINGS

a. The literature is full of loud but wildly conflicting claims. Many of these "findings" are merely *post hoc* rationalizations or hasty overgeneralizations with little objective basis in fact.

b. Appreciable difficulties for this and for future quantitative work have been created by the use of subjective/qualitative descriptors not defined in terms of objectively measurable quantities (e.g., the use of such subjective/qualitative descriptors as "intensity of enemy opposition," "degree of difficulty of the terrain," and the like).

c. Several epistemological weaknesses affect past work. Among the more important are:

(1) Inadequately caveating hasty and premature overgeneralizations based on only a small number of cases, or on a narrow sample of cases representing only a particular time and operational context.

(2) Theory and observation are seldom compared directly, quantitatively, and in detail.

(3) Despite their effectiveness in other contexts, powerful multivariate statistical methods have been singularly unsuccessful when dealing with advance rates. Some possible reasons for this are:

(a) There has been a habit of using far too many variables. This, in turn, has led to excessive overfitting. The use of formal model selection criteria would help to control these excesses.

(b) Excessive focus on purely statistical "significance" at the expense of the practical importance of the results. There has also been too much focus on "trends" and too little attention to the variability of the data above and below the trend line.

(c) Normality has been assumed, sometimes tacitly, without adequate justification. Yet the data are highly skewed and poorly represented by normal distributions.

(d) The data have tacitly been assumed to follow a smooth distribution. Yet the reported data are very "grainy" because of the tendency to report certain simple values (e.g., 1 km/day).

(e) No allowance has been made for the successful advance bias and the reported versus actual bias mentioned in points 5-2d and 5-2e above.

(f) Correlations between events that are close together in time or space have been ignored.

#### 5-4. ANALYSIS FINDINGS

a. **Proposition 1.** Are advance rates about equal to the transport speed? No. Reported advance rates average 1 or two orders of magnitude *below* the speed of their governing mode of transportation. This suggests that land combat operations stand still about 90 percent or 99 percent of the time. The data are widely scattered and highly variable.

b. **Proposition 2.** Are advance rates much higher now than in the past? No. Reported advance rates haven't changed much over the last 400 years, or even longer. But the data are widely scattered and highly variable.

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## APPENDIX A

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<u>Volume No.</u>	<u>AD Number</u>
1	B-086-797L
2	B-087-718L
3	B-087-719L
4	B-087-720L
5	B-087-721L
6	B-087-722L

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## APPENDIX B

## DESCRIPTIONS OF THE DERIVED DATA BASES

**B-1. PURPOSE AND SCOPE.** This appendix describes the new and derived data bases extracted from the primary data bases for analysis purposes. The principal source used was A Compilation of Data on Rates of Advance in Land Combat Operations, CAA-RP-90-4. Our analyses also used some material from recent issues of the Guinness Book of World Records<sup>tm</sup>, and from publications of Jane's Information Group.

**B-2. MANNER OF PRESENTATION.** Each derived data base is given a short name for reference purposes. These short names are listed in alphabetical order in para B-3b below. Their descriptions start in para B-4. Each description identifies the primary data sources used, describes how the derived data were extracted from them, and explains any special notations or codes used. Since the actual data are the same as in the primary data bases, they are not repeated here.

**B-3. LIST OF SHORT NAMES FOR THE DATA BASES.** The following short names for the data bases are used for ease of reference. Some of these short names may look a bit odd. This is because we shortened them to comply with the MS-DOS<sup>tm</sup> limitation of eight characters for file names.

a. List of Short Names for the Primary or Original Data Bases. (NOTE: This list differs from that in CAA-PR-90-4 by inclusion of the ARTYWWII, GIBBON and GUINNESS data bases. The ARTYWWII data base consists of some artillery unit movement data from J. Duncan Love's Artillery Usage in World War II, Vol II, Technical Memorandum ORO-T-375, Operations Research Office, Bethesda, MD, April 1959, AD-208 021. The GIBBON data base consists of the horse-mounted movements of COL Gibbon's command as described under the heading "Custer" in Appendix B of CAA-RP-90-4. The GUINNESS data base is an extract of running, walking, hurdling, steeplechase, and endurance accomplishments afoot from recent issues of the Guinness Book of World Records<sup>tm</sup>.)

<u>Number</u>	<u>Short name</u>	<u>No. of data items</u>
1	ACSDB	3401
2	ALEXANDE	124
3	ANDREWS	55
4	ARTYWWII	NA
5	BAORG	320
6	CAESAR	27
7	CDB90	660
8	DESANTIS	84
9	EASTFRON	212
10	ENGELS	19
11	GIBBON	182
12	GLANTZ	114
13	GUINNESS	205
14	GUSTAVUS	8
15	HANNIBAL	4
16	HULSE	19
17	LONGMARX	371
18	MISCROAD	173
19	NAPOLEON	31
20	NORMANDY	50

21	ORALFORE	360
22	OVERHOLT	0
23	PARSONS	33
24	QUICKWIN	30
25	RADZIEV	40
26	RAND	0
27	RECORD	22
28	RMC	250
29	ROWLAND	288
30	SAVKIN	84
31	SIEGFRIE	40
32	WAINSTEI	237
33	XENOPHON	38

b. List of Short Names for the Derived Data Bases

(1) Numerical. 1BAL TIC, 1BYELOR, 2BYELOR, 3BYELOR, 3GUARDS, 4THARMY, 6TH ARMORED, 13CAA, 38CAA, 60CAA

(2) A. AFRICAN DIVI, AFRICAN DIVII, ANCIENT, ARGENTAN-LIEGE, AUCHINLECK

(3) B. BAORACT, BARBAROSSA (Andrews), BARBAROSSA (Wainstein), BOER WAR

(4) C. CAPORETTO, CDB90, CDB90FT, CDB90ACT, CDB90NEW, CDB90RAT

(5) D. DAVIS, DC RAID, DESANACT, DON, DON R.-CAUCASUS

(6) F. FALKLANDSI, FALKLANDSII, FLANDERS (ORALFORE), FLANDERS (WAINSTEI), FLANDERSI, FLANDERSII, FLANDERSIII, FLANDERSIV, FRANCEI, FRANCEII, FRANCEIII, FRANCEIV, FRANCEV, FRANCEVI

(7) G. GOLD COAST, GUSTANO

(8) H. HVYFOOT, HVYHOMO, HVYFTACT, HVYHOMOACT

(9) K. KLEIST, KLUCK, KURSK

(10) L. LEMANS-METZ, LGTFOOT, LGTFTO5L, LGTHOMO, LGTMOO5L, LGTHMO5L, LIEGE, LONGMARX

(11) M. MALAYA, METZ, MONTGOMERY

(12) N. NO. KOREA, NIGERIAN23D, NW EUROPE

(13) O. ODER1, ODER2, ODER3, ODER4, OREL, OVERLORD

(14) P. PARSOACT, POLAND

(15) R. RECROW, RHINELAND, ROMMEL I, ROMMEL II, RUSSIA

(16) S. SAAR, SAAR (LORRAINE), SCHLIEFFEN, SHERMAN, SO. AFRICA

(17) T. THIRD ARMY, TUNISIA

(18) V. V CORPS, VALLEY, VICKSBURG, VISTULA-ODER

(19) W. WAVELL, WILDERNESS, WILSON

(20) Y. YUGO/GREECE

#### B-4. DERIVATION OF THE DAILY ADVANCE DATA BASES

a. As noted in Chapter 4, six major derived data bases were constructed. Additional derived data bases were also constructed, and are described elsewhere, but the six major ones were LGTFOOT, LGTHORSE, LGTMOTOR, HVYFOOT, HVYHORSE, and HVYMOTOR. All of these data bases are for daily advance distances, times, and rates. The HVYHOMO data base is simply a combination of the HVYHORSE and the HVYMOTOR data bases. Similarly, the LGTHOMO data base combines the LGTHORSE and LGTMOTOR data bases. The first three letters indicate the degree of engagement and the last 4 or 5 letters indicate the primary mode of movement (i.e., the mode of movement that seemed likely to dominate the distance and times, and particularly the rates, of the reported movements). They contain the following information:

- (1) No (sequence or index number in the derived data base).
- (2) Source Name (where did this data come from?—needed for trace-backs).
- (3) Source No (index number in the source data base).
- (4) Start date of the advance (year and fractional year).
- (5) Size of force.
- (6) Distance advanced (km).
- (7) Duration of the advance (days).
- (8) Rate of advance (km/day).
- (9) Notes (information needed for trace-back, e.g., unit identification).

b. If the source gave a designation rather than a specific number for force size we used the following nominal values:

Army Group = 500,000  
 Front = 750,000  
 Army = 100,000  
 Corps = 50,000  
 Division = 18,000  
 Bde = 6,000 (old style); 4,500 (new style)  
 Rgt = 3,000  
 Bn = 800-1,000  
 Co = 150-250

c. Data from the primary data bases was extracted and incorporated into the six major derived data bases according to the procedures given below.

(1) ACSDB. The primary data are already daily advance distances, so all are candidates. Use just the division and independent brigade data (discard the corps and army data as duplicative). Use both opposed advances and opposed withdrawals, i.e., take absolute values of distances, and discard all primary data base cases having a zero DIST\_OP\_ADV coupled with either a zero or an unknown DISPlacement. Treat all opposed advances and withdrawals as heavily engaged. Treat all other "displacements" as lightly engaged. Use principal mode of movement as motor if division/brigade is armored, panzer, or mecz; and as foot otherwise.

(2) ALEXANDE. Treat all values as lightly engaged, movement by foot or horse as indicated by the mode of movement column.

(3) ANDREWS. Use only positive advances. Treat all values as lightly engaged, movement by foot or motor as indicated by the mode of movement column.

(4) BAORG. Treat the data as daily advance distances (*i.e., ignore the tabulated actual durations of the advance, and take the durations to all be one day*). [The justification for this is to keep the data from BAORG on the same footing as that from the other data bases of daily advances. However, a separate analysis of the BAORG data, using the actual advance durations, is in order, and must be done to see what differences that would make.] Delete all cases with zero advance distances. Delete all cases having "URBAN" topography. Treat all values as heavily engaged, and as movement by foot.

(5) CAESAR. Treat all values as lightly engaged, movement by foot or horse as indicated by the mode of movement column.

(6) CDB90. Use both positive and negative advances (*i.e., delete zero and unknown advance distances, use force size as the size of the advancing force, take absolute values of the advance distance so that it corresponds to the advancing force*). Treat all values as heavily engaged. Base decision on mode of movement on unit designation, size of cavalry, and number of tanks. Keep actual advance times and rates in file CDB90ACT.WQ!. But for this analysis, use the times rounded up to the next largest integral day, and keep rates based on these "rounded up" times in file CDB90RND.WQ!. [As for the BAORG analysis, this is to maintain maximum comparability with the other data on daily advances. *But a separate analysis must be done later to determine the amount of the bias introduced by this procedure.*]

(7) DESANTIS. Use only positive advances by attacker. Treat all values as heavily engaged. Mode of movement as indicated in the data base. Keep actual times and rates in file DESANACT.WQ!, but use in this analysis the times rounded up to the next nearest integral day and corresponding rates, which are in file DESANRND.WQ!. As usual, this was done for consistency with other data on daily advances, *but a separate analysis must be done later to determine the amount of the bias introduced by this procedure.*

(8) EASTFRON. Treat line numbers 1-50 [*i.e., prior to Nov 43, including Defense of Moscow, Soviet Counteroffensive, Citadel (Kursk), and Melitopol Breakthrough*] as heavily engaged, and line numbers 51-212 [*i.e., Nov 43 to Aug 45*] as lightly engaged. Use only positive advances by attacker. Treat mode of movement as by foot, unless the unit is a Panzer, Mecz, Mtzd, Tank, or similar type of unit—in which case treat the mode of movement as by motor. Also, treat the following operations as movements by motor, since their rates of advance were largely dictated by motor transport: Lvov-Sandomierz, Yassy-Kishiniev, Vistula-Oder, East Prussia, and Manchuria.

(9) ENGELS. NOT USED. Gives rates only, without either times or distances.

(10) GIBBON. The data are daily advance distances only. Discard zero advances. Treat all values as lightly engaged. Mode of movement is by horse.

(11) GLANTZ. Use positive known advances only. Delete cases with zero or unknown advance distances, or with unknown time durations. Treat all values as lightly engaged, since they fall right in line with other lightly engaged values, such as ANDREWS. Treat mode of movement as by motor for all GLANTZ data points.

(12) GUSTAVUS. Use positive advances only. Treat all values as lightly engaged. Mode of movement is indicated in the data base as "All arms" or as "NA." We elected to use the "Foot" mode of movement for all, because the movements were largely dictated by foot mobility.

(13) HANNIBAL. Treat all values as lightly engaged, movement by foot or horse as indicated by the mode of movement column.

(14) HULSE. Treat all as lightly engaged. Treat all as motor mode of movement.

(15) LONGMARX. **NOT USED.** The data are daily advance distances only. Because of the uncertainty regarding the validity of these data, they were not used in this analysis.

(16) MISCROAD. Use only the Wilson's Raid data. Discard zero advances. Treat as lightly engaged. Treat as horse mode of movement.

(17) NAPOLEON. Use positive advances only. Treat all values as lightly engaged. Mode of movement is indicated in the data base as "All arms" or as "NA." We elected to use the "Foot" mode of movement for all, because the movements were largely dictated by foot mobility.

(18) NORMANDY. Use positive advances only. Treat all entries as heavily engaged.. Treat mode of movement as foot (all of the cases are for infantry divisions).

(19) ORALFORE. The data are daily advance distances only. Discard zero advances. Treat as heavily engaged if OPP'N INT'Y is Intense or Moderate. Treat as lightly engaged if OPP'N INT'Y is Slight or Negligible. Treat movement as by foot, unless the unit is an armored or mecz division.

(20) OVERHOLT. **NOT USED.** Graphical data only.

(21) PARSONS. Treat all as heavily engaged. Treat all as movements by foot. Actual times and rates are in file PARSOACT.WQ!. Rounded up times and corresponding rates are in file PARSORND.WQ!, which was used for this analysis. As usual, this was done to maintain consistency with other data on daily advances, *but a separate analysis must be done later to determine the extent of the bias.*

(22) QUICKWIN. Treat all as lightly engaged. Treat movement mode as indicated in column on MODE OF MOVEMENT.

(23) RADZIEV. **NOT USED.** The data are for distances and *maximum* rates of advance only. They are not suitable for use in this analysis.

(24) RAND. **NOT USED.** Graphical data only.

(25) RECORD. Treat all as lightly engaged. Treat all as movement by motor.

(26) RMC. Delete all cases of zero or unknown advance distance. Treat all of the remainder as heavily engaged. Treat all as movement by foot if MECZ CODE of advancing side is  $\leq 5$ , else as movement by motor.

(27) ROWLAND. Delete all cases of zero advance distance. Delete all total and summary values. Treat all as lightly engaged. Treat mode of movement as indicated in the MODE OF MOVEMENT column of this data base.

(28) SAVKIN. Treat all as lightly engaged, because they fall in line with other lightly engaged data, such as ANDREWS. Treat mode of movement as indicated in the MODE OF MOVEMENT column of this data base.

(29) SIEGFRIE. Treat all as heavily engaged. Treat mode of movement as indicated in the MODE OF MOVEMENT column of the data base (all are by foot).

(30) WAINSTEI. NOT USED. Data base does not give adequate information on degree of engagement, or on mode of movement.

(31) XENOPHON. NOT USED. This data base seems to be an outlier, and the data it contains do not seem to be reliable.

#### B-5. DERIVATION OF THE ACTIVE ADVANCE DATA BASES

a. We created the HVYFTACT data base by combining the data from BAORACT, PARSOACT, CDB90ACT(foot mode only), and DESANACT(foot mode only). We also created the HVYMOACT by combining the data from CDB90ACT(motor mode only) and DESANACT(motor mode only). We also created the HVYHOACT data base from CDB90ACT(horse mode only). In doing this, we filtered the unknown times out of CDB90ACT and then split it up by mode of movement, and we also split up DESANACT by mode of movement. The HVYHOMOACT data base was created by combining the HVYHOACT and HVYMOACT data.

b. Start with CDB90ACT.WQ!. Sort it on ATPBHR1. Delete all cases with  $ATPBHR1 \geq 9999$ . Delete cases where  $DIST = 0$  or unknown. This leaves 261 cases. Sort them by foot/horse/motor mode of movement, and then by rate. Then we find that cases number 1-210 are foot, cases 211-218 are horse, and cases 219-261 are motor. Save the result as CDB9AACT.WQ!.

c. Start with DESANACT.WQ!. Sort it by mode of movement.

d. Copy to HVYFTACT the foot mode of movement cases from CDB9AACT and DESANACT. Add the entirety of PARSOACT and BAORACT. This gives 608 cases.

e. Copy to HVYMOACT the motor mode of movements from CDB9AACT and DESANACT. This gives 77 cases for HVYMOACT.

f. Copy to HVYHOACT the horse mode of movements from CDB9AACT. There are only 9 cases in the HVYHOACT data base.

#### B-6. DERIVATION OF THE CUMULATIVE ANALYSIS DATA BASES

a. To examine the hypothesis that daily advance distances diminish as time into a particular advance increases, special data bases were extracted from the ANDREWS, EASTFRON, GIBBON, LONGMARX, MISCROAD, ORALFORE, ROWLAND, and WAINSTEI data bases. Notice that the are all unopposed or lightly opposed advances (except for the ORALFORE and possibly the EASTFRON cases). The criterion for selecting these data bases was that they gave data on advances in disjoint, contiguous, time intervals, so that cumulative distance and the corresponding cumulative times could be found. The data bases thus created are as described below. [The ones used to construct Figure 3-2 are indicated in para B-7.]

b. We extracted 14 data bases from the ANDREWS data base, one for each of the 14 major operations it describes. However, there are only 4 (or in one case only 3) data points in each of these subsets, or series.

- (1) SHERMAN: Atlanta to Savannah, 1864.
- (2) KLUCK: Belgium and France, 1914.
- (3) CAPORETTO: Italy, 1917.
- (4) KLEIST: Holland, Belgium, and France, May 1940.



- (5) 4THARMY: Somme to Spain, Jun 40.
- (6) YUGO/GREECE: German invasion of Yugoslavia and Greece, Apr 41.
- (7) BARBAROSSA: German invasion of Russia, Jun-Dec 41.
- (8) MALAYA: Malaya to Singapore, 1941-42.
- (9) NO. KOREA: North Korean invasion of South Korea, Jun-Aug 50.
- (10) WAVELL: Egypt, Libya and Cyrenaica, Dec 40-Feb 41.
- (11) ROMMEL I: North Africa, Mar-May 41.
- (12) AUCHINLECK: North Africa, Nov-Dec 41.
- (13) ROMMELII: North Africa, Jan-Jul 42.
- (14) MONTGOMERY: North Africa, Nov 42-Feb 43.

c. There were 13 data bases extracted from the EASTFRON data base, as described below.

- (1) KURSK: Voronezh Army Group Sector, 5-12 Jul 43.
- (2) 1BAL TIC: Byelorussian Offensive, 1st Baltic Army Gp,  
23 Jun-28 Aug 44.
- (3) 3BYELOR: Byelorussian Offensive, 3d Byelorussian Army Gp,  
23 Jun-28 Aug 44.
- (4) 2BYELOR: Byelorussian Offensive, 2d Byelorussian Army Gp,  
23 Jun-28 Aug 44.
- (5) 1BYELOR: Byelorussian Offensive, 1st Byelorussian Army Gp,  
23 Jun-28 Aug 44.
- (6) 3GUARDS: Lvov-Sandomierz Offensive, 3d Guards Combined Arms  
Army, 13 Jul-29 Aug 44.
- (7) 13CAA: Lvov-Sandomierz Offensive, 13th Combined Arms Army,  
13 Jul-29 Aug 44.
- (8) 60CAA: Lvov-Sandomierz Offensive, 50th Combined Arms Army,  
13 Jul-29 Aug 44.
- (9) 38CAA: Lvov-Sandomierz Offensive, 38th Combined Arms Army,  
13 Jul-29 Aug 44.
- (10) ODER1: Vistula-Oder Offensive, 1st Byelorussian Army Gp, Combined  
Arms Armies, 14 Jan-3 Feb 45.
- (11) ODER2: Vistula-Oder Offensive, 1st Byelorussian Army Gp, Tank  
Armies, 15 Jan-2 Feb 45.
- (12) ODER3: Vistula-Oder Offensive, 1st Ukrainian Army Gp, Combined  
Arms Armies, 12-31 Jan 45.
- (13) ODER4: Vistula-Oder Offensive, 1st Ukrainian Army Gp, Tank Armies,  
12-31 Jan 45.

For analysis purposes, these were grouped into four operations, namely KURSK (1), BYELOR (2-5), LVOV (6-9), and ODER (10-13).

d. One data base was used from the GIBBON data. It was the 182-day record of daily advance distances for cavalry units (Table B-1 of the data base paper<sup>1</sup>).

e. One data base was used from the LONGMARX data. It was the 371-day record of daily advance distances.

- f. Two data bases were extracted from the MISCROAD data base. They were
- (1) WILSON: The 18 values of distances and times for Wilson's Raid, and
  - (2) DAVIS: The 18 values of distances and times for Jefferson Davis' flight.

g. Six major data bases were extracted from the ORALFORE data base. They are listed below. For some analyses, they were further divided or combined.

(1) FLANDERS: Ardennes-Flanders operation of 10-24 May 40 (Ger 7th Pz Div).

(2) OREL: Orel to Moscow operation of 13-30 Nov 41 (Ger XLVII Corps, Ger 18th Pz Div, and Ger 29th Mtzd Div).

(3) DON: Don River to Caucasus operation of 21 Jul-23 Aug 42 (Ger XL Pz Corps and Ger 3d Pz Div).

(4) LIEGE: Argentan to Liege operation of 13 Aug-12 Sep 44 (US VII Corps, US 3d Armd Div, US 1st Inf Div, and US 9th Inf Div).

(5) METZ: LeMans to Metz operation of 14 Aug-13 Sep 44 (US XX Corps and US 7th Armd Div).

(6) SAAR: Saar (Lorraine) operation of 8 Nov-7 Dec 44 (US XII Corps and US 4th Armd Div).

h. The 25 data bases or series extracted from the ROWLAND data base. are as follows.

- (1) VALLEY: Winchester Campaign, May-June 1862.
- (2) VICKSBURG: Grierson's Raid of 17 April-2 May 1863.
- (3) WILDERNESS: Wilderness Campaign of May 1864.
- (4) DC RAID: Early's Raid of July 1864.
- (5) BOER WAR: Operations of 12-17 February 1900.
- (6) SCHLIEFFEN: Schlieffen Plan operations of August 1914.
- (7) FRANCEI: Battle of France, Ger XXXVIII Korps, June 1940.
- (8) FLANDERSI: Battle of Flanders, Ger 2d Pz Div, May 1940.
- (9) FLANDERSII: Battle of Flanders, Ger 1st Pz Div, May 1940.
- (10) FLANDERSIII: Battle of Flanders, Ger 10th Pz Div, May 1940.
- (11) FRANCEII: Battle of France, Ger 1st Pz Div, June 1940.
- (12) FRANCEIII: Battle of France, Ger 29th Mtzd Div, June 1940.
- (13) FRANCEIV: Battle of France, Ger 2d Panzer Div, June 1940.
- (14) FLANDERS IV: Battle of Flanders, Ger 7th Panzer Div, May 1940.
- (15) FRANCEV: Battle of France, Ger 7th Panzer Div, June 1940.

<sup>1</sup> A Compilation of Data on Rates of Advance in Land Combat Operations, CAA-RP-90-4, US Army Concepts Analysis Agency, February 1990.

- (16) FRANCEVI: Battle of France, Ger 7th Panzer Div, June 1940.
- (17) FALKLANDSI: British 45th Commando, May-June 82.
- (18) FALKLANDSII: British 3d Para, May 82.
- (19) TUNISIA: British 2d Para, Nov-Dec 42.
- (20) NW EUROPE: British 4th Grenadier Guards, Mar-Apr 45.
- (21) GOLD COAST: 24th Gold Coast Bde, Feb 41.
- (22) SO. AFRICA: 1st South Africa Bde, Feb 41.
- (23) NIGERIAN: 23d Nigerian Bde, Feb 41.
- (24) AFRICAN DIVI: 11th African Div, Feb-Mar 41.
- (25) AFRICAN DIVII: 11th African Div, Mar-Apr 41.

i. The 9 series extracted from the WAINSTEI data base are as follows.

- (1) POLAND: German invasion of Poland, Sep 39.
- (2) FLANDERS: German invasion of Flanders and France, May-Jun 40.
- (3) OVERLORD: Allied invasion of France, Jul-Sep 44.
- (4) RHINELAND: Allied campaign in Rhineland, Feb-May 45.
- (5) 6TH ARMORED: Operations of US 6th Armd Div in France, Jul-Aug 44.
- (6) BARBAROSSA: German offensive of Jun-Dec 41.
- (7) RUSSIA: Russian offensive, Jan 43-Dec 44.
- (8) THIRD ARMY: US Third Army Headquarters moves (Forward Echelon), Aug 44-May 45.
- (9) V CORPS: US Vth Corps Command Post locations, Jun 44-May 45.

#### B-7. DATA BASES USED FOR FIGURE 3-2

a. Figure 3-2 of para 3-4e(6) used some of the derived cumulative data bases mentioned in para B-6.

b. The correspondence between the names used in Figure 3-2 and those in para B-6 is as follows:

<u>Used in Figure 3-2</u>	<u>Used in para B-6</u>
GIBBON	GIBBON (B-6d)
LONGMARX	LONGMARX (B-6e)
DON	DON (B-6g(3))
LIEGE	LIEGE (B-6g(4))
METZ	METZ (B-6g(5))
SAAR	SAAR (B-6g(6))
VISTULA-ODER	ODER3 (B-6c(12))

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## APPENDIX C

SOME BASIC FACTS ABOUT LOGNORMALLY DISTRIBUTED  
RANDOM VARIABLES

**C-1. PURPOSE AND SCOPE.** This appendix presents for ready reference some standard facts about lognormally distributed random variables. Only those facts that are especially pertinent to this paper are presented. Their proofs are either sketched briefly or omitted entirely. However, the detailed proofs are very easy and are available from many sources. Some useful references are:

- a. J. Atchison and J. A. C. Brown, The Lognormal Distribution, Cambridge University Press, Cambridge, UK, 1957
- b. Harald Cramer, Mathematical Methods of Statistics, Princeton University Press, Princeton, NJ, 1946
- c. Edwin L. Crow and Kunio Shimizu (eds.), Lognormal Distributions: Theory and Applications, Marcel Dekker, New York, 1988
- d. N. A. J. Hastings and J. B. Peacock, Statistical Distributions, Butterworths, London, 1975
- e. N. L. Johnson and S. Kotz, Distributions in Statistics: Continuous Univariate Distributions—1, Houghton-Mifflin, Boston, 1970
- f. Maurice G. Kendall and Alan Stuart, The Advanced Theory of Statistics, Hafner Publishing Co., New York, 1967-1968
- g. R. J. Lawrence, The Lognormal Distribution of the Duration of Strikes, J. R. Statist. Soc. A (1984), v. 147, Part 3, pp 464-483
- h. Alexander McFarlane Mood, Introduction to the Theory of Statistics, McGraw-Hill Book Co, New York, 1950
- i. C. Radhakrishna Rao, Linear Statistical Inference and Its Applications, John Wiley & Sons, New York, 1965
- j. Samuel S. Wilks, Mathematical Statistics, John Wiley & Sons, New York, 1962

**C-2. DEFINITIONS.** The quantity  $x$  is distributed as a lognormal random variable with parameters  $\mu$  and  $\sigma$  if, and only if, the quantity  $\ln(x)$ —the natural logarithm of  $x$ —is distributed as a normal random variable with mean  $\mu$  and standard deviation  $\sigma$ . Here  $x$  ranges from zero to infinity. An equivalent alternative definition is that  $x$  is a lognormal random variable with parameters  $\mu$  and  $\sigma$  if, and only if,

$$x = e^y$$

and  $y$  is distributed as a normal random variable with mean  $\mu$  and standard deviation  $\sigma$ .

**C-3. DISTRIBUTION FUNCTION.** If  $x$  is a lognormal random variable with parameters  $\mu$  and  $\sigma$ , then the cumulative distribution of  $x$  is

$$F(x) = CUNO[(\ln(x) - \mu)/\sigma]$$

where  $x$  is nonnegative and  $CUNO(z)$  is the standard cumulative normal distribution function

$$CUNO(z) = (2\pi)^{-1/2} \int_{-\infty}^z e^{-(u^2/2)} du.$$

$F(x)$  is zero for all  $x$  less than zero. The median value of  $x$  (i.e., the value of  $x$  that makes  $F(x) = 1/2$ ) is  $x_{50} = e^{\mu}$ .

**C-4. MOMENTS.** If  $x$  is a lognormal random variable with parameters  $\mu$  and  $\sigma$ , then the moment of  $x^m$  about the origin is

$$E(x^m) = e^{m\mu + m^2\sigma^2/2}.$$

In particular, the average value of  $x$  is

$$E(x) = e^{\mu + \sigma^2/2},$$

and the variance of  $x$  is

$$\begin{aligned} VAR(x) &= E(x^2) - [E(x)]^2 \\ &= e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2}. \end{aligned}$$

The standard deviation of  $x$  is

$$SD(x) = \sqrt{VAR(x)}.$$

### C-5. DISTRIBUTIONS OF CERTAIN FUNCTIONS OF LOGNORMAL RANDOM VARIABLES

a. If  $x$  is a lognormal random variable with parameters  $\mu$  and  $\sigma$  then, for any real value  $p$ ,  $x^p$  is a lognormal random variable with parameters  $p\mu$  and  $|p|\sigma$ . This is proved by observing that, since  $x = e^y$  where  $y$  is normal with mean  $\mu$  and standard deviation  $\sigma$ , it follows that  $x^p = e^{py}$ , where  $py$  is normal with mean  $p\mu$  and standard deviation  $|p|\sigma$ . In particular, if  $x$  is lognormal with parameters  $\mu$  and  $\sigma$ , then  $x^{-1}$  is lognormal with parameters  $-\mu$  and  $+\sigma$ .

b. Suppose that  $x_1$  and  $x_2$  are lognormal random variables with parameters  $\mu_1, \sigma_1$  and  $\mu_2, \sigma_2$ , respectively. Then their product  $x = x_1x_2$  is a lognormal random variable with parameters given by  $\mu = \mu_1 + \mu_2$  and  $\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\rho_{12}$ , where  $\rho_{12}$  is the correlation between  $\ln(x_1)$  and  $\ln(x_2)$ . These facts follow immediately from well-known facts regarding normal random variables by noting that, by the definition of  $x$ ,  $\ln(x) = \ln(x_1) + \ln(x_2)$ , where both  $\ln(x_1)$  and  $\ln(x_2)$  are normal random variables.

c. Suppose that  $x = x_1/x_2$ , where  $x_1$  and  $x_2$  are as in paragraph C-5b above. Then  $x$  is lognormal with parameters given by  $\mu = \mu_1 - \mu_2$  and  $\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho_{12}$ . This can be seen by paralleling the argument of paragraph C-5b.

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**RATES OF ADVANCE IN HISTORICAL  
LAND COMBAT OPERATIONS**

**STUDY  
SUMMARY  
CAA-RP-90-1**

**THE REASON FOR PERFORMING THIS STUDY** was that original analyses of the statistical data on rates of advance are called for. Past analyses have used more limited data bases and often a narrower set of alternative hypotheses. We are not aware of any other work that covers this area as thoroughly as this Research Paper does.

**THE STUDY SPONSOR** was the Secretary of the Army. This is the third and last paper to be prepared by Dr. Robert L. Helmbold under his Secretary of the Army Research and Study Fellowship.

**THE STUDY OBJECTIVE** was to provide the Army with original analyses of the available data on rates of advance, using a larger set of data than has been used in the past, and a range of alternative hypotheses. As such, it furnishes a valuable resource for further work in this important field.

**THE SCOPE OF THE STUDY** was intended to be broad, in the sense of using all of the available data to examine a wide range of worthy hypotheses. No doubt it is too much to hope that the paper used literally all of the available data or included all of the important hypotheses. Nevertheless, its analyses should be very helpful to military historians and operations researchers.

**THE MAIN ASSUMPTION** of this paper is that no data or hypothesis that would substantially alter its principal findings has been overlooked.

**THE BASIC APPROACHES** used in this study were to:

- a. Obtain through extensive personal visits, correspondence, and phone calls all of the noteworthy documents with statistical data on rates of advance,
- b. Compile, computerize, describe, critique, and comparatively review them, and then to
- c. Use these data to examine a wide range of alternative hypotheses about rates of advance.

**THE PRINCIPAL FINDINGS** of the work reported herein are:

- a. A lot of statistical data on rates of advance in land combat operations is available, but for a given purpose only a properly chosen part of it is useful. Sometimes none of it applies.
- b. Past work used a variety of subjective descriptors and *ad hoc* terminology. This subjectivity and lack of standardization makes systematic comparisons difficult, and sometimes impossible.
- c. Several sources, some intentionally and others unintentionally, tended to select cases of a successful advance by the attacker. This biases the data against successful defensive efforts and in favor of advances by attackers.
- d. Reported advance rates tend to be systematically biased toward lower values than are actually achieved. This bias can cause reported rates to be too low by factors around 3 to 5, and seriously distorts the apparent influence of size upon rate of advance.

e. Several epistemological weaknesses affect past work. Among the more important are:

(1) Inadequately caveating hasty and premature overgeneralizations based on only a small number of cases, or on a narrow sample of cases representing only a particular time and operational context.

(2) Theory and observation are seldom compared directly, quantitatively, and in detail.

(3) Despite their effectiveness in other contexts, advanced multivariate statistical methods have been singularly unsuccessful and often misused when dealing with advance rates.

f. Reported advance rates do not seem to have changed much over the last 400 years or so. But the data are widely scattered and highly variable.

g. Reported advance rates may be somewhat higher for battalion-sized units than for larger ones. But the data are widely scattered and highly variable.

h. For heavily engaged forces, reported advance rates of mechanized and armored units are about the same as for infantry units. But for lightly engaged forces reported advance rates of mechanized and armored units are somewhat higher than for infantry units. But again the data are widely scattered and highly variable.

i. Reported advance rates for lightly engaged forces are substantially higher than for heavily engaged forces. However, the evidence indicates that both lightly and heavily engaged forces stand still about 90 to 99 percent of the time. This observation suggests that the key to understanding advances by land combat forces may lie not with their periods of movement, but instead with their periods of standing still. As in other cases, the data are widely scattered and highly variable.

j. Reported advance rates are somewhat higher in summer than in winter—more so for mechanized and armored units than for infantry, but the data are widely scattered and highly variable.

k. Reported advance rates are not consistently lower for longer operations. In fact, on the average, extended operational advances proceed at a steady uniform pace. But the data are widely scattered and highly variable.

l. Reported advance rates are not normally distributed. They are highly skewed and follow a lognormal distribution much more closely than they do either a normal, exponential, Weibull, or gamma distribution.

m. Reported advance rates are practically independent of force ratios. They are much more strongly associated with other indexes of combat capability. But the data are widely scattered and highly variable.

n. Both our and past efforts to devise consistently accurate schemes for predicting advance rates have been unsuccessful. Accordingly, the hypothesis that advance rates are governed primarily by chance should receive serious consideration in future work. Also, the nonmovement phases should be studied in conjunction with the movement phases of land combat operations.

**THE STUDY EFFORT** was directed by Dr. Robert L. Helmbold, Office, Special Assistant for Model Validation.

**COMMENTS AND SUGGESTIONS** may be sent to the Director, US Army Concepts Analysis Agency, ATTN: CSCA-MV, 8120 Woodmont Avenue, Bethesda, Maryland, 20814-2797.