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FINITE ELEMENT FORMULATIONS FOR COUPLED FLUID / STRUCTURE EIGENVALUE ANALYSIS

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Abstract

This report summarizes recent work directed toward the development of an effective finite element-based eigensolution capability for coupled fluid/structure interaction problems in which the fluid compressibility is included. Solution of these coupled problems is essential in the evaluation of structural acoustic transmission characteristics and enclosure acoustics. The methodologies are also of direct relevance to the acoustic radiation problem as well as other forms of time domain analysis. A comprehensive review of the four general methods of treating this problem is presented with the objective of establishing future directions most compatible with existing finite element analysis capability. Several example applications of the different methods are presented to highlight practical considerations. Of the formulations reviewed, a three-field system appears to have the most potential as a general purpose approach, although considerable effort is required in the development of a suitable eigensolver for the system of choice. The report also presents a discussion of a number of methods of incorporating concentrated damping in finite element models of acoustic domains.

Résumé

Le présent rapport résume les travaux effectués pour mettre au point une solution caractéristique aux éléments finis efficace aux problèmes d'interaction fluide/structure couplès, solution qui tient compte de la compressibilité du fluide. Il est essentiel de résoudre ces problèmes de systèmes couplés pour évaluer les caractéristiques de transmission de sons par les structures et l'acoustique d'une enceinte. Les méthodes sont en outre directement liées au problème de rayonnement acoustique ainsi qu'à d'autres formes d'analyse du domaine temporel. Une étude compréhensif des quatre méthodes générales de traitement de ce problème est présentée dans le but d'établir des orientations futures qui soient des plus compatibles avec les moyens actuels d'analyse par éléments finis. Plusieurs applications des différentes méthodes sont données en exemple pour faire ressortir des facteurs d'ordre pratique. Des formulations présentées, un système à trois champs semble le plus prometteur comme méthode de résolution générale, même si un effort considérable est nécessaire pour trouver une solution caractéristique appropriée au système choisi. Le rapport renferme aussi une analyse de plusieurs méthodes pour intégrer des amortissements concentrés dans des modèles de domaines acoustiques aux éléments finis.

Contents

Notation

In the presentation of many different formulations, consistency of notation is difficult. In this report, every endeavour has been made to retain the same symbol for the same type of discrete operation, although the terms involved may be quite different. Thus for example, L always refers to a coupling matrix generated from cross-products of structural and fluid variable representations, whether based on modal or interpolation functions. Vectors are printed in boldface type.

A	Admittance function, boundary area
A_1, A_2, A_3	Coupling matrices
B, B_1	Fluid coupling matrices
c, c_a	Sound speed in fluid medium, absorptive material
C	Absorptive boundary damping matrix
D	Elasticity matrix
D_s	Distributed structural damping matrix
D_f	Distributed fluid damping matrix
E	Young's modulus
F	Load vector
g	gravitational acceleration
H, H_1	Fluid stiffness type matrices
H_r	Rotational stiffness matrix
I	Identity matrix
j	$\sqrt{-1}$
K	Structural stiffness matrix
K_s	Structural factor of absorptive material
$\bar{K}$	Generalized stiffness $\varphi^T K \varphi$
L, L_1, L_2	Fluid/structure coupling matrix

M	Structural mass matrix
$\bar{M}$	Generalized mass matrix $\varphi^T M \varphi$
M_a	Added mass matrix
$\mathbf{n}$	Unit normal vector
N	Finite element interpolation function
p, p_0	Dynamic, hydrostatic pressure
P_n	Non-dimensional pressure coefficient
Q	Fluid mass type matrix
Q_a	Mass type matrix for absorptive material
R	Absorptive material flow resistivity
$\mathbf{u}$	Structural displacement vector
u_i	Displacement component
v	Fluid particle, boundary velocity
$\mathbf{X}$	Body force vector
Z	Impedance function
α	Penalty function coefficient
β	Fluid bulk modulus
$\Gamma_w, \Gamma_a, \Gamma_v$	Wetted, absorptive, specified velocity boundary
δ	Variational operator
ς	Generalized structural displacement coordinates
η	Generalized fluid displacement coordinates
λ	Lamé constant
μ	Lamé constant (shear modulus)
ν	Poisson's ratio

ρ, ρ_s	Dynamic, static, fluid density
ϕ	Velocity potential function
φ_s, φ_f	Structural, fluid displacement mode
ψ	Displacement potential function
ω	Coupled natural frequency
ω_a, ω_s	Rigid-walled fluid cavity, structural natural frequency
Ω	Absorptive material porosity
Ω_f, Ω_s	Fluid, structural domain
$*$	Cross-product operator
$\nabla\cdot, \nabla$	Divergence, gradient operator

1 Introduction

This report discusses finite element methodologies for the prediction of natural modes of vibration of coupled fluid/structure systems in which the fluid compressibility is included. These formulations find applicability in a wide variety of important problems, such as the evaluation of acoustic transmission characteristics, prediction of enclosure acoustics, and vibration prediction for submerged structures. The solution of the coupled eigenproblem is also often of direct relevance to other forms of steady state and transient response analysis.

The relatively straightforward application of finite element methods to the acoustic modal analysis of bounded domains and the extent to which this method has become the standard for structural analysis raise few suspicions that the treatment of coupled compressible fluid/structure systems would be problematic. That this is not the case is apparent from the continued discussion of this topic in the literature, a full twenty years after the initial approaches were undertaken. While the original intent of this investigation was simply to establish a viable approach to the coupled eigenvalue problem within the framework of an existing structural finite element analysis program suite, the large number of seemingly disparate approaches proposed in the literature has led to a much more in-depth review of this problem.

Finite element formulation of an initially undamped coupled eigenvalue problem leads to one or more of the following problems: the presence of unsymmetric coefficient matrices; the presence of a pseudo damping matrix; inefficiency arising from the inclusion of extra variables; highly singular and/or indefinite coefficient matrices or the presence of numerous zero strain energy modes; necessity of matrix inversion; large bandwidths and ill-conditioning. The occurrence of most of these characteristics is a function of the particular choice of the variable or variables used to describe the behaviour in the individual structural and fluid domains. To exploit conventional structural finite element solution systems to advantage, displacements are invariably chosen for the structural domain, although it could be argued that force variables would be a more logical choice. In the fluid region, one finds formulations based on one or more of the fundamental variables of displacement, displacement potential, pressure, and velocity potential. It is of note that formulations which appear to be quite different can often be derived from each other using the simple relationships which must exist between these variables in a steady state harmonic system.

By far the largest effort has been directed towards formulations which provide symmetric coefficient matrices, to again exploit the efficient algorithms of conventional solver systems. Since most of these solver systems operate in simple arithmetic, formulations which do not produce a damping matrix are preferable. Unfortunately, the mixing of primal (displacement) and dual (pressure) representations, which is attractive from the point of view of minimizing the number of variables required in the coupled problem, leads inevitably to unsymmetric forms. Various methods of symmetrizing such systems have been

proposed; these can be shown to be equivalent to reductions of three-field formulations, in which two scalar variables are used in the fluid model, to two-field formulations. These reduction methods require matrix inversion, thus restricting their applicability in almost all cases. These important limitations do not appear to have been properly discussed in many of the formulations which have been reviewed. Other forms providing symmetry result in a damping or damping-like matrix, which presents its own problems.

The review and development work discussed here forms part of an overall program of enhancement of the numerical analysis capability within DREA for solution of fluid/structure interaction problems. The merits of a number of the classical approaches are presented with the objective of establishing the formulation(s) most compatible with the existing structural finite element analysis system VAST.[47] The VAST system is fairly typical of conventional finite element solver systems; symmetry of the coefficient matrices is assumed, and simple arithmetic is used throughout. Two well-known eigenvalue solution techniques are available in VAST, the direct and subspace iteration algorithms. Although these two techniques are suitable in the context of structural analysis, they are a limitation in the application of VAST to certain formulations of the coupled problem. It is also not likely that complex arithmetic will be implemented in VAST in the near future, hence those formulations that result in damping matrices cannot currently be considered as potential candidates for development.

Several of the methods presented in this document for treating coupled systems have been evaluated using small test problems, and the results are reported here. These methods include the structural analogy and displacement-based approach, which require only the VAST system, and several of the symmetrized pressure-based systems, for which a separate program has been developed. The latter is typical of the reduction techniques, being relatively inefficient and having limited applicability. The theoretical basis of this program is presented in the section on scalar formulations, and the operation of the program is presented as an appendix. Irrespective of its current limitations, the pressure-based approach embodied in this formulation can form the basis of a three-field system which should offer a much more efficient and general purpose capability, although a specialized eigensolver is required in place of those currently available in the VAST program. Development of that solver system is ongoing; future work will report on its application to practical problems and performance.

While the pursuit of effective formulations of the undamped problem is worthwhile in other interaction problems, and makes for an interesting academic study in the present case, it is well-known that damping plays a major role in most systems containing highly compressible fluids. Damping can be incorporated into the finite element formulations in a number of ways, and a section of this report is devoted to discussion of these approaches. Frequency response analysis must then generally be utilized to determine the natural modes of the coupled system.

In the presentation of this work, it is assumed that the reader is familiar with finite element formulations of dynamics problems. A brief mathematical development of the continuum problem in the individual fluid and structural domains is first presented as a basis for discussion of the various numerical treatments.

2 Theoretical Review

A mathematical foundation for the coupled continuum problem is relatively easy to develop and is a necessary prerequisite to a systematic consideration of the various numerical treatments. Since the formulation of the governing dynamical equations of the structural system is invariant in all of the formulations considered, this section presents the development of only the fluid field equations in detail. These equations will be referenced extensively in the following sections.

The treatment of the fluid field embodies the classic assumptions of linear acoustics in which viscous and convective terms are neglected, and relations between temporal and spatial distributions of pressure, particle velocity and density are sought. Three equations are required, these typically comprising a continuity or conservation of mass relation in the form (no sources),

$$\frac{\partial \rho}{\partial t} + \rho_s(\nabla \cdot \mathbf{v}) = 0 \tag{1}$$

where ρ_s is invariant, a momentum equation,

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \rho \mathbf{g} \tag{2}$$

and a relation derived from an adiabatic equation of state

$$p = \beta \frac{d\rho}{\rho_s} = \rho_s c^2 \frac{d\rho}{\rho_s} \tag{3}$$

in which β is the adiabatic bulk modulus. This last equation can be interpreted as a scalar constitutive relation, since

$$\frac{d\rho}{\rho_s} = -\nabla \cdot \mathbf{u} \tag{4}$$

thus,

$$p = -\rho c^2 \nabla \cdot \mathbf{u} \tag{5}$$

Hence stress in the form of pressure is linearly related to strain through a modulus β.

Appropriate manipulation of these equations provides three different functions. In terms of density, we obtain a wave equation in the form

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho + \rho_s \nabla \cdot \mathbf{g} = 0 \tag{6}$$

Using equation (3) in the above, an equivalent pressure relation becomes

$$\frac{\partial^2 p}{\partial t^2} - c^2 \nabla^2 p + c^2 \rho_s \nabla \cdot \mathbf{g} = 0 \tag{7}$$

A vector relation of some importance in the current context can be derived for the equivalent function in terms of velocities. Taking the gradient and partial time derivative of equations (1) and (2) respectively, we have in terms of velocity

$$\frac{\partial^2 \mathbf{v}}{\partial t^2} - c^2 \nabla(\nabla \cdot \mathbf{v}) - \frac{\partial \mathbf{g}}{\partial t} = 0 \tag{8}$$

or displacements

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - c^2 \nabla(\nabla \cdot \mathbf{u}) - \frac{\partial \mathbf{g}}{\partial t} = 0 \tag{9}$$

The necessity of assuming an irrotational flow field becomes evident in the above vector relations, since

$$\nabla(\nabla \cdot \mathbf{v}) = \nabla^2 \mathbf{v} + \nabla * (\nabla * \mathbf{v}) \tag{10}$$

Irrotationality of the flow (or displacement) is required to eliminate the cross-product term. With such an assumption, equations (8) and (9) can be cast in scalar form via the introduction of velocity and displacement potential functions respectively.

It can be shown that the contributions of gravity effects, such as surface waves, are generally small in comparison to compressibility contributions in a coupled system and the gravitational terms in the above relations are often neglected. This term is obviously retained in the formulation of solutions of the sloshing problem, in static problems and in problems of large scale in which variation of pre-stresses may be important. The common forms of the scalar equations are the classical wave equations of potential or pressure in the fluid domain

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0 \tag{11}$$

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \tag{12}$$

In the remainder of this report, the symbol ρ will replace ρ_s, except where the distinction is required.

The boundary conditions for the fluid field are reasonably self-evident. For the velocity potential and pressure formulations respectively, they are appropriate combinations of

$$\rho \frac{\partial \phi}{\partial n} = \dot{\mathbf{u}}_s \cdot \mathbf{n} \qquad \frac{\partial p}{\partial n} = -\rho \ddot{\mathbf{u}}_s \cdot \mathbf{n} \tag{13}$$

on the elastic interface,

$$\rho\frac{\partial\dot{\phi}}{\partial n}=0 \qquad \frac{\partial p}{\partial n}=0 \tag{14}$$

on a rigid wall, and

$$\rho\dot{\phi}=0 \qquad p=0 \tag{15}$$

on a free surface of zero ambient pressure. Here, $\mathbf{u}_s$ is the structural displacement vector, and the overdot notation is used for convenience to represent time differentiation.

An equivalent scalar formulation using a displacement potential ψ in place of the velocity potential can also be developed, the appropriate boundary conditions becoming

$$\nabla\psi\cdot\mathbf{n}=\mathbf{u}_s\cdot\mathbf{n} \tag{16}$$

on the elastic interface,

$$\nabla\psi\cdot\mathbf{n}=0 \tag{17}$$

on a rigid wall, and

$$\ddot{\psi}=0 \tag{18}$$

on a free surface. Other impedance type boundary conditions are sometimes required, such as the radiation condition for unbounded domains, and treatments of partially transmitting boundaries. These will be discussed in more detail in following sections.

The preceding development provides the differential equations and boundary conditions relevant to the complete fluid domain. To utilize the finite element method, an integral relation is required, which when solved, provides an approximate solution to these equations and boundary conditions. Two related procedures are used to obtain the integral functional: direct application of the weighted residual method, and formulation of the system on the basis of a variational principle. The former is much more general, but there is always impetus to establish a variational principle, if one exists, since it imparts attractive features such as bounding to a well-posed numerical solution. In this case, variational principles based on energy can be developed for both the fluid and structural domain, provided that the individual systems considered are conservative. For such cases, the minimization of the functional corresponds exactly to the solution of the relevant differential equations with their proper boundary conditions. In general, if a variational principle exists, it can often be obtained using appropriate operations on the basic weighted residual statement, as is presented below. Here, as is often most logical, the Galerkin weighted residual method is utilized, in which the weighting functions and interpolation functions are the same.

For the scalar(p,ϕ) relations developed, the general form of the basic weighted residual statement is

$$\int_{\Omega_f}\delta p(\nabla^2 p-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2})\,d\Omega \;+\; \int_{\Gamma_w}\delta p(\nabla p+\rho\ddot{\mathbf{u}}_s)\cdot\mathbf{n})\,d\Gamma=0 \tag{19}$$

where Ω_f represents the entire fluid volume, and Γ_w represents the flexible portion of the fluid/structure interface. Assuming that the variation of the pressure function δp satisfies the boundary condition of equation (15), (the essential boundary condition), equation (19) can be integrated once by parts to obtain a weak form of the governing equations in the form of a variational statement

$$\delta \int_{\Omega_f} \frac{1}{2}(p^2 + \rho c^2 \nabla p \cdot \nabla p)\, d\Omega - \int_{\Gamma_w} \rho p \ddot{\mathbf{u}}_s \cdot \mathbf{n}\, d\Gamma \;\; = 0 \qquad (20)$$

The volume integral in this functional can be used as the basis for finite element solutions of rigid-walled acoustic cavity problems. In that case, the absence of a surface integral imposes the boundary condition of equation (14) automatically. Here, the surface integral enforces the coupling boundary condition at the elastic interface. Equation (20) is in fact a variational principle, in which the kernel of the volume integral is the acoustic Lagrangian. The surface integral accounts for the work done by the external loads as seen by the fluid.

While the formulation of the scalar relations in terms of a velocity potential is probably more familiar, the use of a displacement potential is usually more convenient in the present application, since a discrete system can be obtained without the presence of velocity dependent terms. This point will be discussed further in the next section.

Following the same process as described in some detail for the fluid, a weighted residual process can also be used to develop a variational form of the governing equations of the structural system. In this case, the functional contains terms representing the kinetic and potential energies, the latter comprising the strain energy, work of the body forces, and work done by external loads. The fluid pressure term is separated from other loads, and is the source of the fluid/structure coupling. A typical variational form in terms of the structural displacements is

$$\delta \int_{\Omega_s} (\frac{1}{2}\rho_s \dot{\mathbf{u}}_s \dot{\mathbf{u}}_s - \frac{1}{2}\nabla \mathbf{u}_s^T D \nabla \mathbf{u}_s - \mathbf{X}^T \mathbf{u}_s)\, d\Omega - \int_{\Gamma_s} \mathbf{F}^T \mathbf{u}_s\, d\Gamma - \int_{\Gamma_w} (\mathbf{u}_s \cdot \mathbf{n}) p\, d\Gamma = 0 \qquad (21)$$

where D is the elasticity matrix, X are the body forces, and F are the external loads. The solution of the coupled problem thus involves the simultaneous satisfaction of relations in the form of (20) and (21). The finite element solution proceeds in the standard manner of assuming local approximating functions for the $(\mathbf{u}_s, p, \phi, \psi)$ variables, as appropriate, numerically evaluating the integrals and assembling the elemental matrices into an overall system. The following section discusses the numerous types of discrete formulations of these relations which have been proposed.

Within the assumptions of linear acoustics and small displacement theory, there is no mathematical approximation necessary in the derivation of the above functionals. Of course, any numerical procedure can introduce errors into the solution and destroy the features of such variational-based solutions. For example, the use of reduced integration, which has

been found beneficial in certain formulations, may corrupt the underlying functional, and lead to a solution of an unknown set of governing equations.

3 Discrete Formulations of the Coupled Eigenproblem

The numerous discrete formulations of the coupled problem can be crudely divided into three groups based on the main objective of the approach. One group is concerned primarily with the response of the structure, another is concerned primarily with the acoustic response of the fluid, and a third group treats both domains. In the case of the former two groups, simplifying assumptions about the behaviour of the complementary domain can often be exploited in order to substantially decouple the problem. This section begins with a very brief discussion of these first two types of approaches, but the emphasis will be on formulations which consider the response of both structural and fluid domains. Further discussion of uncoupled analysis techniques can be found in several of the references quoted.

Much of the early finite element work in the area of fluid/structure coupling was directed toward the prediction of the effect of the fluid on the dynamic response of large structures such as gravity dams.[2,28,45] In these cases, only the structural response in the low frequency range (below, say, 20 Hz) is of interest. For these problems, (ka small), the fluid effect is almost purely reactive, and compressibility effects could in most cases be neglected. This leads to the well-known added mass concept, and if free surface gravity waves are included, to an added damping. Finite elements of various forms have been applied with much success to this problem, and although it is essentially an unbounded one, these developments have laid the groundwork for many of the finite element formulations of the bounded problem. Reference 45 remains essential reading for anyone working in this area.

Similar developments occurred during roughly the same time period in the use of finite elements for acoustic modal analysis and the prediction of acoustic response of coupled systems.[4,5,14,37] To a large extent, the structural response was of secondary importance in these studies, and approximate structural modes were often assumed representative of the boundary forcing functions in the acoustic domain. Here too, finite element formulations, usually pressure-based, have proven very effective for the prediction of acoustic response at low to moderate frequencies. The modal approximation approaches have a number of merits, and can be used effectively for complete solution of the coupled problem, as will be discussed.

Numerical treatments which utilized finite element modelling of both domains began to appear in the mid-1970's, although the mathematical basis for their development had probably been presented a decade previously in the work of Gladwell and Zimmerman,[20,21] and again by Zienkiewicz.[45] Since then, a continuous discussion of this problem can be found in the literature.

It is instructive to divide the various finite element formulations into subsections based on the particular variable or variables used to characterize the fluid field. In this way, the merits and drawbacks of each general method can be discussed, and the links to other approaches made in some consistent fashion. A brief summary of the relevant characteristics is also given at the end of the section. A pressure formulation is presented in some detail to define the basic terms that occur in some form in almost all of the approaches.

The choice of field variables can have different effects on solutions of transient and steady state problems. In this discussion, we consider only the ramifications for the solution of the eigenproblem. The application of some of the formulations as applied to time domain solutions can be found in certain of the references.[1,4,16,19,22,41]

3.1 Scalar Formulations

The use of scalar variables to describe the fluid field is obviously appealing since it results in a minimum number of fluid degrees of freedom, while automatically satisfying the irrotationality condition arising from the small displacement assumption in the fluid. Thus, these formulations are immediately applicable for solution of the scalar wave or Helmholtz equation. Numerous variations have been popular, most notably those utilizing fluid pressure, velocity potential and more recently displacement potential. The scalar analogy is, as its title suggests, analogous to these formulations, but it is discussed separately here because of its rather unique basis and requirements.

3.1.1 Pressure

The description of the fluid field in terms of a pressure variable is conceptually appealing as well as practical from the viewpoint of applying certain boundary conditions. This approach can be used very effectively in cavity acoustics problems and uncoupled systems such as the prediction of added mass, but poses some difficulties for compressible fluid/structure interaction.

To start, the coupled equations arising from the variational statements, equations (20) and (21), or more generally from the application of a Galerkin finite element approach if damping is to be included, can be cast in the form of two discrete differential equations. In keeping with standard structural finite element methodology, all of the approaches reviewed utilize the set of nodal structural displacements δ as one set of unknowns. In terms of the set of nodal fluid pressures p, these two equations are

$$[M]\ddot{\delta} + [D_s]\dot{\delta} + [K]\delta = [L]p + \{F_s\} \tag{22}$$

$$[Q]\ddot{p} + [D_f]\dot{p} + [H]p = -\rho[L]^T\ddot{\delta} \tag{23}$$

with ρ the fluid density, and F_s the vector of structural forces independent of the fluid. The structural equation contains the familiar mass, damping and stiffness matrices for a multi-degree of freedom system. The coupling terms occur as forcing functions in each equation; for the fluid, the acceleration of the structural interface produces fluid pressures, and for the structure, those induced pressures produce structural forces. In the standard approach, the fluid and structure have been discretized into finite elements within which the structural displacements and fluid variables, here pressure, are described by

$$\delta = \sum N_i^\delta \delta_i \tag{24}$$

$$p = \sum N_i^p p_i \tag{25}$$

and the N_i represent interpolation functions, which can be different between the fluid and the structure. The subscripted field variables are the nodal values, and the superscripts denote the variable with which the interpolation function is associated. With reference to equation (20), the fluid coefficient matrices in equation (2) are assembled forms of the inertial terms

$$q_{ij} = \frac{1}{c^2} \int_{\Omega_f} N_i^p N_j^p \, d\Omega + \frac{1}{g} \int_{\Omega_f} N_i^p N_j^p \, d\Omega \tag{26}$$

the fluid damping terms

$$(d_f)_{ij} = \frac{-1}{c} \int_{\Omega_f} N_i^p N_j^p \, d\Omega \tag{27}$$

and the acoustic 'stiffness' terms

$$h_{ij} = \int_{\Omega_f} \nabla N_i^p \cdot \nabla N_j^p \, d\Omega \tag{28}$$

The L matrix is the coefficient matrix relating pressure, acting normal to the interface, to global forces at the nodes

$$l_{ij} = \int_{\Gamma_w} \frac{\partial N_i^\delta}{\partial n} N_j^p \, d\Gamma \tag{29}$$

Neglecting the damping terms for the moment (radiation damping is usually assumed small, but concentrated acoustic damping may be quite significant), the eigenvalue problem associated with equations (1) and (2) becomes

$$\left[\begin{bmatrix} K & -L \\ 0 & H \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 \\ \rho L^T & Q \end{bmatrix} \right] \begin{Bmatrix} \delta \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{30}$$

Despite the inherent symmetry in the K, H and Q submatrices, the assembled form of this coupled system of this formulation is unsymmetric. Conventional solution systems such as

those available in the VAST program cannot be used to solve this problem. Equation (30) is the basis of a host of variations on the theme of scalar formulations.

The lack of symmetry in this system is a direct result of the mixed nature of this common formulation, which utilizes displacement variables in the structure and dual (force) variables in the fluid. On a mathematical basis, the finite element models in the two domains are approximating solutions of quite different quantities; a displacment-based solution generally satisfies compatibility, but approximates equilibrium, while complementary formulations satisfy equilibrium, but approximate compatibility. Thus, as pointed out by Tabarrok,[43] the only consistent solution is the exact solution, which is never obtained. The consequences of such mixed formulations are often difficult to assess *a priori* , but at the very least, the solutions cannot be used as bounds, as is often the case for consistent formulations.

The inconsistency can be avoided if a complementary formulation utilizing force variables is used for the structure. To this author's knowledge, no results of finite element formulations utilizing the consistent dual principle have appeared in the literature.

Because the undamped coupled problem can be developed from a variational principle in quadratic form, symmetric forms of equation (30) must exist. In fact, four independent modifications of that system can be made in order to symmetrize the system, each of which involves a substitution and requires an inverse of one of the four submatrices K,M,H or Q. For example, a useful modified relation is obtained if the substitution

$$p = \omega^2 H^{-1}(\rho L^T \delta + Qp) \tag{31}$$

is made in the structural partition of equation (30). Multiplication of equation (31) by Q and division by ρ gives a new second row, and a standardized system is obtained in the form

$$\left[\begin{bmatrix} K & 0 \\ 0 & \frac{1}{\rho}Q \end{bmatrix} - \omega^2 \begin{bmatrix} M+\rho LH^{-1}L^T & LH^{-1}Q \\ - & \frac{1}{\rho}QH^{-1}Q \end{bmatrix}\right] \begin{Bmatrix} \delta \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{32}$$

where a dash implies symmetry across the diagonal. Application of the three alternative substitutions which can be used to obtain symmetric forms of equation (30) provide the systems

$$\left[\begin{bmatrix} K+\rho LQ^{-1}L^T & -LQ^{-1}H \\ - & \frac{1}{\rho}HQ^{-1}H \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 \\ 0 & \frac{1}{\rho}H \end{bmatrix}\right] \begin{Bmatrix} \delta \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{33}$$

$$\left[\begin{bmatrix} M & 0 \\ 0 & \frac{1}{\rho}H \end{bmatrix} - \omega^2 \begin{bmatrix} MK^{-1}M & -MK^{-1}L \\ - & \frac{1}{\rho}Q+L^TK^{-1}L \end{bmatrix}\right] \begin{Bmatrix} \delta \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{34}$$

$$\left[\begin{bmatrix} KM^{-1}K & -KM^{-1}L \\ - & \frac{1}{\rho}H + L^TM^{-1}L \end{bmatrix} - \omega^2 \begin{bmatrix} K & 0 \\ 0 & \frac{1}{\rho}Q \end{bmatrix}\right] \begin{Bmatrix} \delta \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (35)$$

Each of the equations (32) to (35) is a symmetric system which may be soluable with conventional finite element solution techniques, provided that the system can in fact be formulated. Since several of these systems have received attention in the literature, it is worthwhile to consider the requirements and limitations in each approach, a point not well-considered in publications reviewed by this author. Taking each equation in turn, we note that system (32) requires a positive definite structural stiffness matrix K and fluid matrices H and Q. In most cases, the rigid body constraint of the structure is not problematic, and Q is non-singular; however, for closed cavities, the H matrix is singular, and inversion is possible only through the constraint of at least one pressure degree of freedom. For such problems, experience has shown that such a constraint always corrupts the solution. More will be said shortly about this particular system.

System (33) requires the inversion of the fluid mass matrix Q, which is always realizable, and which will not be particularily onerous if a diagonalized matrix can be utilized, or if the fluid domain is small. Unfortunately, the presence of a singular H matrix in the diagonal block of the assembled stiffness matrix renders that matrix singular. Shifting will not alleviate this problem, since the shift involves the use of the singular H matrix. Thus, equation (33) is not a useable formulation for the general problem.

System (34) is attractive in that it retains the banded nature of the individual systems in the assembled stiffness matrix, and that the use of shifting may allow the presence of singular H matrices. The approach requires a positive definite mass matrix, and the inversion of the structural stiffness matrix. Without considering the former, the approach must be dismissed on the premise that the such an inversion is not practical for anything other than purely academic problems. This approach has apparently been successfully utilized in References 10 and 11 for some very small systems.

The final system of equation (35) requires a structural mass matrix inversion, which is tractable for diagonalized forms. As well, the application of shifting should allow solutions of singular fluid systems. Aside from the poorly banded form of the assembled stiffness matrix, this approach presents several other numerical difficulties. First, the structural mass matrix may not be well-conditioned when elements utilizing rotational degrees of freedom are used in the structural model. This characteristic arises from the disparate values of translational and rotational mass components, and is not alleviated through the natural resort to a diagonal mass matrix. Since these element types are a necessary part of a realistic structural model, this is a non-trivial problem. Second, the squaring of terms in the structural stiffness matrix can lead to numerical overflows when structural constraints are

imposed through the use of large spring stiffnesses, such as are utilized in current versions of VAST. The use of smaller stiffness values may adversely affect the accuracy of the prediction of structural response. As well, there is still considerable matrix manipulation required even when a diagonal mass matrix is utilized. Presumably, the removal of the constrained degrees of freedom would eliminate this second problem, but the first would remain. Further work is required to investigate if rotational masses could be modified to provide better conditioning of the structural mass matrix.

Despite its inherent limitations, the reduced formulation in equation (32) is straightforward to produce, and can be used in the solution of constrained types of coupled problems. This system has been coded in a stand-alone computer program COUPLE, which creates the fluid submatrices Q,H, and L, carries out the required manipulations, and assembles the overall system with the structural submatrices K and M. This program incorporates the previously developed acoustic finite element generator program ACOUSTIC.[44] The program is also configured to construct the system of equation (35), although the practical application of this approach is still quite limited due to the problems discussed above. Further discussion of this program is contained in the appendix.

Of note in equation (32) is the conventional added mass effect provided by this symmetrized formulation in the limit of fluid incompressibility. As the sound speed tends to infinity, all terms involving Q vanish (aside from the small surface wave contributions, see equation (26)), and equation (32) reduces to

$$[[K] - \omega^2[M + M_A]]\{\delta\} = 0 \tag{36}$$

in which

$$[M_A] = \rho L H^{-1} L^T \tag{37}$$

This is identical to the form utilized in the current VAST fluid element formulation for added mass,[33] and shows how the limiting assumption of incompressibility uncouples the general system.

As first pointed out by Morand and Ohayon[30], the symmetrized systems of equations (32) to (35) can be interpreted as statically reduced forms of a three-field system. For example, equation (32) is the reduced form of an expanded symmetric system in the (δ, p, ψ) variables, in which ψ can be shown to be equivalent to a displacement potential function in the fluid. Three-field systems offer a number of advantages over their two-field counterparts, and will be discussed further in the subsection covering that topic. The relationships between the two and three-field systems are discussed in some detail by Fellipa,[17,18] who also considers at least to some degree the utility of the various reduced systems, which, as pointed out, have quite serious limitations.

3.1.2 Displacement Potential

A displacement potential formulation for the fluid domain can be developed from a direct application of the Galerkin weighted residual technique to the wave equation in terms of displacement potential. A weak form of the system becomes

$$\int_{\Omega_f} (\frac{1}{2}\ddot{\psi}^2 + c^2 \nabla\psi \cdot \nabla\psi)\, d\Omega - c^2 \int_{\Gamma_w} \psi(\mathbf{u}_s \cdot \mathbf{n})\, d\Gamma \tag{38}$$

with a discrete system obtained from a variation with respect to ψ,

$$[H]\{\psi\} + [Q]\{\ddot{\psi}\} + [L]^T\{\delta\} \;\; = 0 \tag{39}$$

The coupling term in the otherwise unchanged structural equation can be obtained through substitution of the constitutive relation of equation (3), and the displacement potential wave equation into the last term of equation (21),

$$-\int_{\Gamma_w} (\mathbf{u}_s \cdot \mathbf{n}) p\, d\Gamma \;\; = \;\; \rho \int_{\Gamma_w} (\mathbf{u}_s \cdot \mathbf{n}) \ddot{\psi}\, d\Gamma \tag{40}$$

The assembled system then becomes

$$\left[\begin{bmatrix} K & 0 \\ L^T & H \end{bmatrix} - \omega^2 \begin{bmatrix} M & \rho L \\ 0 & Q \end{bmatrix}\right] \begin{Bmatrix} \delta \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{41}$$

Evidently, the use of a displacement potential does not result in a symmetric formulation. The form of the assembled system is very similar to the pressure formulation of equation (30), and there is no apparent advantage in using such an approach on its own. The reversal of the coupling terms suggests that combinations of the two variables might yield some form of symmetric system. As was alluded to earlier, the utility of the displacement potential variable indeed becomes evident in the three-field formulations.

3.1.3 Velocity Potential

A reformulation of the basic scalar problem using a fluid velocity potential in place of the fluid pressure has been suggested by Everstine.[15] The relationship between the pressure and velocity potential is that commonly associated with potential flow theory

$$p \propto \dot{\phi} \tag{42}$$

Making this substitution in equation (30), dividing the fluid partition of that equation by $-\rho$ and integrating in time gives the system

$$\left[\begin{bmatrix} K & 0 \\ - & -\frac{1}{\rho}H \end{bmatrix} - \omega \begin{bmatrix} D_s & L \\ - & -\frac{1}{\rho}D_f \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 \\ - & -\frac{1}{\rho}Q \end{bmatrix}\right] \begin{Bmatrix} \delta \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{43}$$

Regardless of the presence of fluid or structural damping terms, the time integration required in this formulation moves the coupling terms in L to the damping matrix. For the eigenvalue problem, this transforms the undamped, unsymmetric problem into a complex symmetric one. For systems which operate in simple arithmetic, this is not a viable formulation. For other types of time domain analyses, the approach does have redeeming features.

3.1.4 Scalar Analogy

An alternative method of generating the required fluid submatrices H and Q in the coupled problem is to use standard structural solid elements with three displacement degrees of freedom per node, in what Everstine has termed a fluid-structural analogy.[16] In this quite elegant trick, the wave equation

$$\nabla^2 \phi = \frac{1}{c^2} \ddot{\phi} \tag{44}$$

can be solved with structural formulations based on the Navier equations of elasticity in the form

$$\left(\frac{\lambda + 2\mu}{\mu}\right) \nabla^2 u_1 + \left(\frac{\lambda + \mu}{\mu}\right) (u_{2,xy} + u_{3,xz}) = \frac{\rho}{\mu} \ddot{u} \tag{45}$$

when the u_2 and u_3 components of displacement are zero, and the conditions

$$\frac{\lambda + 2\mu}{\mu} = 1 \tag{46}$$

$$\frac{\rho}{\mu} = \frac{1}{c^2} \tag{47}$$

are satisfied. Everstine has related these requirements for the Lamé constants of elasticity, μ (the shear modulus) and λ, to the more conventional material property constants E and ν, which are more easily specified in finite element programs as input data. A typical approach is to arbitrarily assign μ a value of unity and specify Young's modulus E large, say 1.0E+15. The specification of a Poisson's ratio ν=$E/2$ and a density of $1/c^2$ then makes the solution of equations (45) and (44) equivalent. An uncoupled acoustic cavity analysis capability is then immediately available from an eigenvalue analysis using stiffness and mass matrices assembled from standard elements. An added fluid mass prediction capability is also easily defined by using standard constraints on the exterior surface of a fluid model.

The advantage of this approach is that no special elements need be developed, and standard pre/post processors can be used for interpretation of input and output data; however, for a program such as VAST, there are several distinct disadvantages. First, two degrees of freedom at each fluid node must be constrained to simulate a scalar field. These constrained freedoms are not eliminated in the VAST solution, hence the problem size can be significantly larger than necessary.

While quite elegant for uncoupled analysis, the major drawback in the present context is that the scalar formulation of the coupled problem is in exactly the unsymmetric form given previously for the pressure formulation. Since several well developed capabilities already exist for added mass prediction and the programs ACOUSTIC or COUPLE are available for efficiently generating the matrices required for acoustic cavity modal analysis, the scalar analogy does not offer any enhancement to the current capability. Finally, the Poisson's ratio specification required in this analogy is structurally unrealistic, and causes problems in programs such as VAST which check for such characteristics. The analogy has been tested with the VAST program, and a recent comparison with results from acoustic elements is given in the next section. Further discussion of this approach in the context of VAST operation can also be found in Reference 3.

3.2 Vector Formulations - Displacement Fluid Variables

Since virtually all structural analysis codes use displacements as primary variables, it is natural to formulate the coupled problem completely in terms of displacements. Such an approach should avoid the inconsistency inherent in mixed formulations and thus lead directly to symmetric forms. This is in fact the case, although the immediately apparent drawback is the larger number of fluid variables required. The disadvantage of larger problem size may be offset by the bandedness which can usually be conserved in consistent formulations.

A less obvious but serious drawback is that a straightforward application of the displacement variable formulation will not satisfy the irrotationality condition which is implicit in the use of the scalar formulations. The result is the presence of what have been called circulation modes in the fluid domain. These modes are displacement configurations which involve zero volumetric strain in the elements, their exact form and frequency often being dependent on the particular grid, element order and integration point location. In general, these modes will have non-zero frequencies, and for detailed grids, the number of circulation modes may make determination of the true compressible modes very difficult.[1,24,27]

Although a simple solution would appear to be the use of the form

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - c^2 \nabla^2 \mathbf{u} = 0 \tag{48}$$

as a starting point, a weak form of this system produces untractable surface integrals. Recalling that this equation is a reduced form of equation (9), in which the cross-product term of the expansion (10) was assumed to vanish, a more general starting point is

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - c^2 \nabla(\nabla \cdot \mathbf{u}) = 0 \tag{49}$$

Unfortunately, there is now no guarantee that the displacement field satisfies the irrotationality requirement. This is the form that an unmodified finite element program such as VAST will emulate. One solution to this problem is the addition of a penalty function to equation (49) based on the relation of equation (10). In this case, the starting point becomes

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - c^2 \nabla(\nabla \cdot \mathbf{u}) + \alpha c^2 \nabla * (\nabla * \mathbf{u}) = 0 \tag{50}$$

with α a non-zero penalty coefficient. A weak form of this equation can be obtained (two rules of vector differentiation are required) in the form

$$\int_{\Omega_f} \left(\delta \mathbf{u} \frac{\partial^2 \mathbf{u}}{\partial t^2} + (\nabla \cdot \delta \mathbf{u})(\nabla \cdot \mathbf{u}) + \alpha c^2 (\nabla * \delta \mathbf{u})(\nabla * \mathbf{u}) \right) d\Omega + \int_{\Gamma} \delta \mathbf{u} (\nabla \cdot \mathbf{u}) \, d\Gamma = 0 \tag{51}$$

The surface integral produces a load term in both domains which is a function of the strain at the interface. For rigid-walled cavities, and coupled systems in which consistent interpolation functions are utilized in the two domains, the integral vanishes. If mixed interpolants are used, a fictitious load term results.

When utilized, the discrete form of the penalty function takes the form of an added stiffness term H_r. The assembled form of the coupled system becomes

$$\left[\begin{bmatrix} K & 0 & 0 \\ & K+H+H_r & 0 \\ - & & H+H_r \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 & 0 \\ & M+Q & 0 \\ - & & Q \end{bmatrix} \right] \begin{Bmatrix} \delta_s \\ \delta_{sf} \\ \delta_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{52}$$

The use of large values of α, on the order of one to ten times the fluid bulk modulus, suppresses the circulation modes in most cases, and does not appear to adversely degrade the prediction of the true modes.[1,24] In current versions of the program ADINA[48], which offers displacement-based formulations with or without rotational stiffness terms, the penalty coefficient is taken as one thousand times β.

Certain displacement formulations[13] also include interface elements between the fluid and structure in order to decouple the tangential displacements, consistent with the inviscid assumption. This decoupling has been shown to provide better solutions than a completely coupled form in some cases, which appears to be too rigid. Another suggestion for tangential decoupling in cases where interface elements are not available, is the use of short stiff rod elements between fluid and structural nodes at the interface.[1]

To utilize this formulation with a conventional displacement-based finite element program such as VAST, a modified elasticity matrix is required for the fluid elements in order to enforce the zero shear stress model. For 3-dimensional problems, the standard D matrix

must take the form

$$\begin{bmatrix} \beta & \beta & \beta & 0 & 0 & 0 \\ \beta & \beta & \beta & 0 & 0 & 0 \\ \beta & \beta & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{53}$$

where β is the bulk modulus of the fluid. Appropriately reduced forms of this matrix are utilized for 2-dimensional or axisymmetric analyses. The VAST program currently allows direct input of the elasticity matrix for certain membrane, plate, shell and solid elements.

The consistency of the displacement variable approach is appealing, since any graphics and post-processing features developed for the general analysis program can readily be utilized. The method has been applied in the analysis of several simple problems, the results of which are presented in the next section. The formulation can be used to solve the coupled problem in which no constraints are applied in the fluid, for which the symmetrized scalar approaches were unworkable due to the presence of the singular fluid stiffness matrix. In both coupled and uncoupled solutions, circulation modes are evident, and it is suggested that these would be very problematic in large scale analysis. The inclusion of a penalty stiffness term for selected elements would not be a difficult modification in VAST, and the existing interface elements could readily be used to provide tangential decoupling. Additional capability for entering the elasticity matrix would be required; again this would require only minor modification.

The application of this method in the solution of general problems is not without difficulties, and the simple configurations considered in the next section highlight some of these. First, the highly singular nature of the elasticity matrix imposes stringent requirements on the boundary conditions which must be applied to the problem in order to avoid ill-conditioning of the overall stiffness matrix. For general geometries, the use of awkward skewed coordinates is also necessary in the application of correct boundary conditions. As well, even when a rotational stiffness is available, it is difficult to specify an appropriate value of the penalty coefficient a priori, and repeated evaluations and plotting of mode shapes may be required to determine true modes. In Reference 35, poor performance of the displacement-based method was also noted in the prediction of coupled eigenmodes for nearly-incompressible fluids. These immediately apparent drawbacks do not instill confidence that this method can be successfully applied to general fluid/structure interaction problems.

3.3 Three-field Approaches

Three-field systems, which utilize two scalar field variables in the fluid domain, lie midway between two-field and displacement-based approaches i the number of degrees of freedom present in the fluid model. As with the latter, the three-field formulations have a banded form, which will usually offset the disadvantages of having a larger overall dimension than a symmetrized two-field form. The first three-field system appears to have been proposed by Morand and Ohayon[30], who also noted its relationship to the condensed system. Their approach utilized the pressure and displacement potential as fluid field variables, and provided a sparse system in the form

$$\left[\begin{bmatrix} K & 0 & 0 \\ & \frac{1}{\rho}Q & 0 \\ - & & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 & L \\ & 0 & B \\ - & & -\frac{1}{\rho}H \end{bmatrix}\right] \begin{Bmatrix} \delta \\ p \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{54}$$

in which a fluid coupling term is introduced, in this case of the form

$$b_{ij} = \frac{1}{c^2} \int_{\Omega_f} N_i^p N_j^\psi \, d\Omega \tag{55}$$

The linear dependence of the three variables is apparent in the assembled mass matrix. Those authors reduced this system to a two-field (δ, p) relation in the form of equation (32). Solution difficulties associated with the singularity of the assembled stiffness matrix in the full system were not addressed.

Very similar approaches have more recently been taken,[19,39] in which the displacement potential and pressure variables can be described independently. This approach allows the use of lower order interpolation functions for ψ, which is justified in some sense since the displacement potential is an integral of the pressure. As a consequence of the possibility of independent discretization, the B and H submatrices need not be square. In practice, it is probably simpler to maintain consistency in the interpolation order, since several of the submatrices are then identical.

In Reference 19, the constant potential mode solution associated with the singularity in the H matrix, $\{e\} = c\{1, 1, ...\}$, with c a constant, is eliminated by forcing the solution eigenvectors to be orthogonal to this mode. This constraint can be imposed at the variational statement level, much like the inclusion of rotational stiffness terms in the displacement variable fluid formulation. The result is an added degree of freedom, λ, in the assembled system,

$$\left[\begin{bmatrix} K & 0 & -L_e & 0 \\ & \frac{1}{\rho}Q & B_e & 0 \\ & & 0 & 0 \\ - & & & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 & 0 & L \\ & 0 & 0 & B \\ & & 0 & 0 \\ - & & & -\frac{1}{\rho}H \end{bmatrix}\right] \begin{Bmatrix} \delta \\ p \\ \lambda \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{56}$$

in which

$$[B_e] = [B]\{e\}^T \qquad [L_e] = [L]\{e\}^T \tag{57}$$

This system can be partitioned into two variable sets (δ, p, λ) and (ψ). If at least one constraint in the ψ field is imposed, then reduced forms of the above system, reminiscent of the symmetrized system of equation (32) can be formulated in only the first variable set for systems which are actually singular. This is an approximation, but since the fluid field variables are assumed independently, the error introduced with such a constraint applied to one field may not significantly influence the solution in the second field. The reduction scheme suffers from the same type of computational overhead as the other symmetrization approaches, since an explicit inverse is required. Reference 19 also discusses a method for solution of the assembled system of equation (56), which would appear to be a more practical approach in most cases.

Another three-field system in which the (δ, p, ψ) variable set is utilized was given in Reference 39. The assembled system in this case is

$$\left[\begin{bmatrix} K & 0 & -L^T \\ & 0 & B_1^T \\ - & & -\frac{1}{\rho}Q \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 & 0 \\ & \rho H_1 & 0 \\ - & & 0 \end{bmatrix}\right] \begin{Bmatrix} \delta \\ \psi \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{58}$$

in which

$$(b_1)_{ij} = \int_{\Omega_f} \nabla N_i^p \cdot \nabla N_j^\phi \, d\Omega \tag{59}$$

and the interpolation functions may again be independent. The fluid stiffness submatrix retains the form

$$(h_1)_{ij} = \int_{\Omega_f} \nabla N_i^\phi \cdot \nabla N_j^\phi \, d\Omega \tag{60}$$

but may be of reduced order. The linear dependence inherent in this three-field system appears in the assembled stiffness matrix, rather than the mass matrix as in equation (54).

While these three-field approaches provide a symmetric form for the coupled problem, contain fewer fluid variables than displacement variable formulations, and partially maintain a bandedness in the assembled matrices, they present a number of difficulties in terms of solution algorithm requirements. All systems have assembled stiffness matrices which are singular. In the case of equation (58), the B matrix is semidefinite, and this combined with the negative diagonal terms leaves the assembled stiffness matrix negative definite. Also, as a result of utilizing dependent fluid variables, there are more zero energy modes in the stiffness matrix than can be removed through shifting.

Despite these adverse numerical characteristics, the three-field system is physically well-posed. Unfortunately, current eigensolver algorithms within VAST cannot treat these systems, and the development of non-standard techniques will be required. The generation of

the submatrices, based on isoparametric axisymmetric and three-dimensional elements, and assembly into the three-field form can currently be carried out with the COUPLE program.

Static condensation of equation (58), in this case of the pressure variable, yields the two-field system

$$\left[\begin{bmatrix} K+\rho LQ^{-1}L^T & -\rho LQ^{-1}B_1 \\ - & \rho B_1 Q^{-1} B_1^T \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 \\ - & \rho H_1 \end{bmatrix}\right] \begin{Bmatrix} \delta \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{61}$$

which is essentially in the form of the system of equation (33), with allowance for the use of mixed interpolation. The comments made previously on that system still apply. As suggested in Reference 22, it is almost always more efficient to solve the original highly banded system (58) with a specialized eigensolver than form and solve the system of equation (61).

Another interesting three-field approach has been suggested by Olson and Bathe[36], in which a single hydrostatic pressure variable p_0 and a set of nodal velocity potentials ϕ is used to describe the fluid. In this form, both static and dynamic problems can be solved, and many fewer fluid variables are required than is the case for standard three-field representations. This approach introduces three new matrices,

$$(a_1)_i = -\frac{-1}{\beta} \int_{\Omega_f} d\Omega \tag{62}$$

$$(a_2)_i = \int_{\Gamma_w} \frac{\partial N_i^\phi}{\partial n}\, d\Gamma \tag{63}$$

$$(a_3)_i = \frac{1}{c^2} \int_{\Omega_f} N_i^\phi \, d\Omega \tag{64}$$

where β is the fluid bulk modulus. The assembled system becomes

$$\left[\begin{bmatrix} K & 0 & A_2^T \\ & -H & 0 \\ - & & A_1 \end{bmatrix} - \omega \begin{bmatrix} 0 & L^T & 0 \\ & 0 & A_3^T \\ - & & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 & 0 \\ & -Q & 0 \\ - & & 0 \end{bmatrix}\right] \begin{Bmatrix} \delta \\ \phi \\ p_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{65}$$

As was the result in the scalar use of velocity potential, the effective time integration of the pressure causes the coupling terms to be velocity dependent. According to the authors, this formulation does not result in a true damping matrix, and the above system can be solved with algorithms operating in simple arithmetic. The assembled stiffness matrix is also negative definite. The approach has been implemented in a version of the finite element code ADINA,[48] in which the determinant search eigenvalue solver system has been modified to obtain solutions of this problem. The approach is appealing for its generality, despite the additional matrices required since these terms are easily calculated. Unfortunately,

VAST does not use a determinant search method, and further work would be required to investigate whether modified subspace iteration or direct iteration methods could be used on this problem. Alternatively, a specialized determinant search method could be developed for the VAST system.

3.4 Modal Coupling Methods

An alternative to the direct formulations requiring finite element discretization of both structural and fluid regions is to treat the coupled problem using modal representations of either or both of the domains. In the latter case, the development is quite analagous to that utilized in structural component modal synthesis, although the assembled system for a coupled fluid/structure problem is usually more cumbersome. The modal sets required in the formulation may also be more computationally demanding to obtain. Much of the modal-based work is aimed more towards the accurate prediction of the acoustic field response than the structural response. Thus, several authors have used quite simplified structural modal representations but retained a more detailed fluid model. Regardless of these simplying assumptions, even the simplest modal methods often can provide a good deal of insight into the behaviour of the coupled system.

A simple mixed approach which retains a detailed pressure-based finite element fluid model but utilizes boundary structural modes was proposed by Craggs.[5] In this case, the flexible boundary displacement is represented by a modal expansion in the form

$$\mathbf{u}_s = \varphi_s \varsigma \tag{66}$$

where φ_s has an associated natural frequency ω_s, and ς is a generalized displacement. For a harmonic forcing function F at frequency ω, the displacement response amplitude then becomes

$$\mathbf{u}_s = \frac{\varphi_s^T \varphi_s F}{\bar{M}(\omega_s^2 - \omega^2)} \tag{67}$$

and the boundary velocity amplitude due to F becomes

$$\dot{\mathbf{u}}_s = -j\omega \frac{\varphi_s^T \varphi_s F}{m(\omega_s^2 - \omega^2)} \tag{68}$$

These relations utilize generalized mass $\bar{M}$ and stiffness $\bar{K}$ terms defined by

$$\bar{M} = \varphi_s^T M \varphi_s \quad \bar{K} = \varphi_s^T K \varphi_s \tag{69}$$

If the force results from the acoustic backpressure p, then equation (68) defines a boundary admittance function A, where

$$\dot{\mathbf{u}}_s = -j\omega \frac{\varphi_s^T \varphi_s L}{\bar{M}(\omega_s^2 - \omega^2)} p = Ap \tag{70}$$

This relation can be used directly in the boundary term of the standard acoustic functional, equation (20), to yield the system

$$\left[\left[H - \frac{\rho\omega^2 L\varphi_s\varphi_s^T L}{\bar{M}(\omega_s^2 - \omega^2)}\right] - \omega^2[Q]\right]\{p\} = 0 \tag{71}$$

This relatively simple formulation provides a direct solution to the coupled problem in the limiting cases for which the boundary frequency ω_s is much smaller or much larger than the forcing frequency, or equivalently, the fundamental acoustic cavity mode. In the former, the boundary is mass controlled, and the admittance relation results in an added stiffness term, increasing the acoustic cavity modal frequencies. In the latter, the response is stiffness controlled, leading to an added mass effect, which decreases the cavity modal frequencies. That the controlling and response factors seem opposite is a consequence of using the pressure type of formulation. Solutions for near-resonant cases cannot be obtained directly with this formulation.

Resonant boundaries can be treated with a more versatile modal formulation, in which the generalized boundary displacements ς for the resonant modes are incorporated into the solution vector. A symmetric system can be obtained in the form

$$\left[\begin{bmatrix} H + H_2 & -\rho{\omega_s}^2 L\varphi_s \\ - & \rho\bar{K}{\omega_s}^2 \end{bmatrix} - \omega^2 \begin{bmatrix} Q & 0 \\ 0 & \rho\bar{K} \end{bmatrix}\right] \begin{Bmatrix} p \\ \varsigma \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{72}$$

in which $\bar{K}$ is the generalized resonant modal stiffness defined in equation (69), and

$$[H_2] = \rho \sum_{i=1}^{n} \frac{L(\varphi_s)_i(\varphi_s)_i^T L}{\bar{M}_i} \tag{73}$$

represents the added stiffnesses of all of the boundary modes considered. As discussed in Reference 5, this approach can treat multiple cavities and both coupling and flanking boundaries.

Following a similar approach, Dowell[9] presented a general form of coupled modal solution in which both the fluid and structural boundary motion were represented with modal sets (φ_f, φ_s) respectively. The coupled equations in this case utilize non-dimensional pressure expansion coefficients P_n and generalized structural displacements ς. The undamped system becomes

$$\left[\begin{bmatrix} \omega_a^2 & 0 \\ -L_1 & \omega_s^2 \end{bmatrix} - \omega^2 \begin{bmatrix} I & L_1 \\ 0 & I \end{bmatrix}\right] \begin{Bmatrix} P_n \\ \varsigma \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{74}$$

in which the L_1 matrix now contains coupling components defined in terms of integrated modal cross-products of the form

$$(l_1)_{ij} = \frac{1}{A_w} \int_{\Gamma_w} \varphi_s\varphi_f \, d\Gamma \tag{75}$$

The respective modal frequencies in each domain appear as diagonal spectral submatrices in the assembled stiffness matrix. The pressure field can be recovered once the P_n are available. Following the methods outlined earlier, this system can be symmetrized in several ways.

A form of this approach in which axisymmetric finite element analysis was used to define the structural and acoustic natural modes was utilized in Reference 38 to determine the coupled response of an air-filled cylinder. As can often occur in practice, a large number of structural modes were predicted in the vicinity of the acoustic cavity modes, which fortunately were widely separated. Although many of the structural modes may have little effect on the coupled response of the system, their individual contributions are unknown *a priori*, and they must be maintained in the problem formulation. Thus, the modal approach maintains its effectiveness in the general case when only a very limited frequency range is considered.

In the above formulations, *in-vacuo* modes of the structure have been utilized with at least some success in the analysis of coupled systems which involve low density fluids. In fact, neither the *in-vacuo* structural modes nor the acoustic cavity modes satisfy the true interface boundary condition of equation (13), and this can lead to significant error in problems which involve liquids. The work of Morand, Valid and Ohayon[30,34] in particular has addressed this issue, in which mathematically admissible modal basis functions have been defined for the general problem. For an arbitrary geometry, the modal sets are determined via finite element analysis. In their approach, two types of modal basis functions are defined: the first is a combination of the wet modes of the structure, where the fluid is treated as incompressible, coupled with quasi-static elastic fluid modes; the second set is the acoustic rigid-walled modes of the cavity. The first set of modes satisfies the required interface boundary condition, while the acoustic cavity modes have a vanishing pressure gradient on the boundary, and thus do not corrupt this interface condition. The price to be paid in this more mathematically correct approach is the increased computation required to define the modified wet modes. The assembled system ends up very much resembling that in equation (74)

$$\left[\begin{bmatrix} \omega_a^2 & 0 \\ - & \omega_s^2 + L_2 \end{bmatrix} - \omega^2 \begin{bmatrix} I & L_1 \\ - & I + L_3 \end{bmatrix}\right] \begin{Bmatrix} \eta \\ \varsigma \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{76}$$

in which the L_1, L_2 and L_3 terms again represent modal coupling contributions.

A related approach has more recently been taken by Daniels[34] in the context of applying a fixed-interface component mode technique to fluid/structure interaction problems. As in the work of Morand and Ohayon, the formulation utilizes modified wet modes and acoustic cavity modes to satisfy the interface boundary condition. This basis set is augmented with up to two other types of modes: a set of fixed interface modes ($p = 0$ on Γ_w) analagous to those utilized in certain formulations of component mode synthesis, and so-called constraint

modes in the fluid. The latter are incompressible modes which include the fluid nodal constraints, and are included in the analysis of fluid domains modified by the release of particular constraints. Again, for low density fluids, the *in-vacuo* structural modes could be used in place of the wet modes, since it is less critical that the correct boundary condition be satisfied. The wet modes should be used for cases involving liquids. The fixed-interface modes are redundant, but apparently improve the convergence rate of the eigensolution of the overall system.

With this seemingly extensive modal set, a non-symmetric modal formulation of the coupled problem can be developed, which can be symmetrized using any of a number of the standard substitutions. Because of its notational awkwardness, the reader is referred to Reference 34 for the actual form of the assembled system. While this approach appears cumbersome, the matrices involved are generally of quite small dimension, and as in the other modal approaches, several of the submatrices are diagonal. The only constraint in this method appears to be a requirement that the structural stiffness matrix be positive definite. The reference contains an investigation of the efficiency of the above form in the analysis of several small fluid/structure problems, comparing the performance to the equivalent symmetrized form of the pressure formulation, equation (34). It is concluded that the present modal formulation is more effective from a purely computational point of view, although as discussed, equation (34) is no doubt the most inefficient of the symmetrized systems. A similar modal approach is also suggested for predicting a compressibility correction on the pressure field in a nearly-incompressible fluid.

The use of modal methods has the potential advantage of reduced problem size, which can be critical in many problems of practical importance. As well, modal concepts often simplify the system when a specific frequency range can be considered. On the other hand, this can be a limitation when more general response characteristics are required. To maintain correct mathematical admissibility, the more demanding calculation of wet modes is required, which will itself require a coupled finite element model. The use of only the wet modes to represent the structural response assumes that the fluid compressibility does not affect the structural mode shape, which may not always be the case. Lastly, as with the basic scalar approaches, the symmetrization process destroys much of the banding available in the modal systems.

In light of these considerations, it would appear that the modal methods will have distinct advantages over direct solutions in problems for which wet modes are not required, for time domain solutions which span limited frequency ranges and perhaps for modification type problems. The direct methods, on the other hand, offer an increased generality. In the development of an effective analysis system for the coupled problem, it would be desirable to embody both approaches.

3.5 Summary

This section has presented four quite different numerical treatments of the coupled fluid/structure interaction problem, and a number of variations within each general form. To reiterate, two-field scalar formulations in pressure or displacement potential resulted in unsymmetric coefficient matrices, necessitating secondary symmetrizing algorithms. The symmetrization itself can be quite inefficient, and each of the symmetrized systems was shown to have serious limitations in practical application. Use of the velocity potential provided symmetry, but resulted in the presence of a damping matrix. This form is thus untractable for current finite element procedures, but may be effective in time domain or direct frequency response solutions. The scalar analogy, while extending the usefulness of conventional finite element codes to treat uncoupled acoustic cavity problems, suffers from the same non-symmetric limitation in its application to coupled problems. It is concluded that any system based on a two-field scalar formulations will have a limited applicability for small problems, but will not provide an effective solution capability for general problems.

Displacement-based formulations offer bandedness of the coefficient matrices, and ease in coupling of structural and fluid domains, but suffer from inefficiency, necessity of skew coordinates, presence of circulation modes or measures to eliminate them, and numerical instability. These drawbacks appear to preclude the use of this approach as a general purpose capability.

Three-field systems offer reduced problem size over displacement-based formulations and maintain to some degree a banded form in the coefficient matrices, at the expense of adverse numerical characteristics of the assembled system. These approaches are easily formulated from the basic acoustic and structural submatrices, and appear to offer an effective capability provided that the necessary use of more sophisticated eigensolution algorithms is relatively efficient. It has not been possible to address the latter point in this report.

The modal-based methods are appealing in certain circumstances, typically providing non-symmetric systems which can be symmetrized comparatively easily due to their smaller size and more diagonal structure. Their lack of generality could be a major drawback.

Considering the alternatives, if a single approach is developed for solution of coupled problems, one of the three-field systems would appear to be most appropriate, subject to the condition that a reasonably efficient solver system can be developed as well. The presence of the (δ, ϕ, p_0) method in current versions of ADINA is an indication that that approach is considered useful and reliable; however, the eigensolution system required in that approach is quite specialized. If significant effort need be directed at implementing a new solver system, it would be desirable that it be as general as possible. Based on these conclusions, in-house and contracted work is currently underway on an original eigensolution capability for the (δ, ψ, p) three-field formulation. The complete system will use VAST to generate

the structural submatrices, COUPLE for acoustic submatrix generation and assembly of the submatrices, and the eigensolver for solution of the system. At the time of writing, the eigensolver is operational for small systems, but work remains to enable large problems to be handled effectively. Effort is being made to keep file formats and details consistent with current VAST requirements, and, if appropriate, the completed eigensolver could be included in a special version of the VAST program.

4 Example Applications

This section presents several example applications of each of two fundamental approaches to the coupled problem: the scalar and displacement variable formulations. Unfortunately, only a limited number of the methods discussed in the previous section can be considered, since many of them would require extensive program development or modification. Despite the limited scope, these simple analyses can highlight difficulties inherent in the approaches considered, specifically in the adaptation or integration of some of the methods for use with the VAST program. Analyses using the VAST-COUPLE system illustrate a symmetrized two-field scalar formulation, and the VAST program itself is used to illustrate the scalar analogy and displacement-based formulation. The scalar analogy approach is utilized only for prediction of acoustic cavity modes since it results in unsymmetric forms of the assembled coefficient matrices in the interaction problem.

Although the COUPLE program can be used to construct the overall mass and stiffness matrices for three-field formulations, evaluations of that approach were not undertaken at this time since testing of the new eigensolver system is not complete. Details of the operation of the COUPLE program are presented in Appendix A.

4.1 Symmetrized Pressure Formulation

The VAST-COUPLE system has been used in the analysis of several small coupled problems, the results of one of which is presented here. This problem utilizes the bottle analysed in Reference 44, and exploits axisymmetric modelling both for the fluid and structural models. The structural model consists of 28 3-noded axisymmetric shell elements, giving 58 nodes, the fluid model comprises 24 8-noded axisymmetric acoustic elements, giving 123 nodes. In this case, all structural nodes are interface nodes. The models are shown in Figure 1. Pressure relief constraints were applied on two fluid nodes at the open end of the bottle, and rigid body modes of the bottle were eliminated by appropriate structural constraints.

Table 1 presents the results for the first five vibration modes of the three forms of analyses, with both air and water as the fluid. Modal displacement plots for four of the modes of the structure alone and the coupled structure/water system are presented in

Figures 2-5. The modes with air as the fluid are indistinguishable from the purely structural modes, and are not presented.

Mode	Structural	Cavity[1]	Coupled[1]	Cavity[2]	Coupled[2]
1	276	227	227	1036	275
2	1041	1327	276	6027	989
3	1359	2040	1041	9268	1034
4	1412	2901	1327	13186	1042
5	1460	3736	1356	16982	1326

[1] Air filled
[2] Water filled

Table 1: Comparison of modal frequency predictions (Hz) for the uncoupled and coupled bottle/fluid system.

Despite the matching in the natural frequencies of the two individual systems when the fluid is air, there no apparent interaction. The individual system modes are recovered virtually unchanged. This is not surprising, since for open-ended systems, the coupling effect shows predominantly as an added mass term, which is directly proportional to the fluid density.

When the air is replaced by water, only one of the acoustic cavity modes is in the range of the lowest structural modes (column 5), but due to the higher fluid density, there is significant mass-dominant modal coupling. While the fundamental structural mode is apparently unaffected by the presence of the fluid, it is suggested that the particular model used here is not well-suited for predicting the coupling effects for this mode. This is a result of the inconsistency in the fluid and structural models at the closed end. The introduction of triangular axisymmetric elements would be helpful in avoiding this problem. Aside from the fundamental mode, the coupled modes are now quite distinct from the purely structural modes, as shown in Figures 3-5.

Note that this particular problem is well suited for analysis with the standardization approach implemented in the COUPLE program, since the open bottle end allows the necessary constraints to be correctly applied. The VAST added mass model would provide a useful comparison solution for this problem; however, axisymmetric fluid elements are not currently available in VAST.

4.2 Scalar Analogy

The scalar analogy has been used in conjunction with the VAST program for the prediction of the acoustic cavity modes of a rectangular box. In this method, the solid elements available in the VAST program mimic single-degree-of-freedom acoustic elements. This simple analysis highlights some of the characteristics of the approach, although in fact this is not an interaction problem.

The finite element model of the box, which was also used as an evaluation problem in Reference 44, is shown in Figure 6. In accordance with the requirements of the scalar analogy, the Young's modulus E was taken as 1.0E+15, Poisson's ratio was taken as E/2.0, and a sound speed in air of 330 m/sec was assumed. The material density appropriate for the analogy method is thus $\rho = 1/c^2$. In fact, this density required scaling upwards to avoid starting vector problems in the VAST program during the eigenvalue analysis, and certain actions were required to avoid problems with the unrealistic value of Poisson's ratio. Two of the three degrees of freedom per node were constrained, this being a very ineffective requirement even for the small problem considered (102 constraints). The results of the analysis are shown in Table 2.

Mode	Scalar Analogy (Hz)	COUPLE+VAST (Hz)	Theoretical (Hz)
1	701.8	701.8	699.2
2	1506	1505	1500
3	1541	1541	1398
4	1664	1664	1654
CPU Time (Sec)	232	53	

Table 2: Comparison of modal frequency predictions for the rectangular cavity.

As is evident, the two approaches provide comparable accuracy, although the penalty of carrying extra degrees of freedom is reflected in the increased CPU time required for the scalar analogy. The poor prediction of the third mode in both cases is a result of the coarseness of the model.[44] It must be noted that the order of the modal predictions of both forms of analysis are sensitive to the shifting parameter used in the decomposition process.

4.3 Displacement-based Formulation

The results of two applications of the displacement-based elements in VAST in the solution of fluid problems are presented below. The first is the prediction of axial cavity

modes for the constrained system shown in Figure 7. To make the approach general, the problem was modelled with solid elements. The results, which are typical of this formulation, are shown in Table 3 and Figure 7. True modes are obtained as modes 2 and 8, the others being circulation modes, with frequencies the same as the true modal frequencies. A number of the true modes are not obtained from the analysis. Similar results were obtained in Reference 1.

Mode	Frequency (Hz)	Characteristic	Theoretical (Hz)
1	22.45	Circulation	22.36
2	22.45	True	44.72
3	22.45	Circulation	67.08
4	22.45	Circulation	89.44
5	45.46	Circulation	111.80
6	45.46	Circulation	-
7	45.46	Circulation	-
8	69.58	True	-
9	123.30	True	-

Table 3: VAST predictions of axial natural modes of the fixed boundary problem using a three-dimensional displacement-based element.

A second more interesting example, taken from a test problem used by Olson and Bathe[35] is a rigid moving boundary problem for which an analytical solution can be obtained. The problem schematic and finite element model for this problem are shown in Figure 8. The first two axial modes of this system are predicted well by the numerical solution, as shown in Table 4.

Mode	Frequency (Hz) Analytical	Frequency (Hz) VAST
1	99.3	99.3
2	1460	1467

In-vacuo frequency: 1.58 Hz

Table 4: VAST predictions of axial natural modes of the moving boundary problem using a three-dimensional displacement-based element (m=1.0).

The analytical values are solutions of the transcendental expression

$$2\pi f = \omega = \sqrt{\frac{k + \rho c \omega A \cot(\frac{\omega L}{c})}{m}} \tag{77}$$

where the relevant parameters are shown on Figure 8. This closed problem cannot be solved using the symmetrized pressure formulation since the fluid stiffness matrix is singular. As in Reference 35, rigid (no bending) vertical motion was enforced via nodal constraint equations in the VAST solution. Although in this case skewed coordinate systems were not required, it is easy to imagine problems in which the boundary conditions would be very difficult to specify.

A modification of this problem in which the axial constraints are removed at the end opposite the moving boundary can be used to test the capability of the displacement-based formulation for the prediction of added mass. This prediction in turn can be used for comparison with both the VAST added mass system,[33] and the symmetrized pressure formulation in the form of equation 32 available in the COUPLE program.

The spring/plate system, with the mass m decreased to 0.01 in place of 1.0, has an in vacuo natural frequency of 15.8 Hz. The pressure or displacement release at the opposite end knocks out the compressibility stiffening, and the effect will be of an added mass. Using exactly the same model in each case, the three approaches all produce the same result, as shown in the second column of Table 5.

Method	Fundamental[1] Frequency (Hz)	Fundamental[2] Frequency (Hz)
Couple + VAST	6.67	6.50
VAST added mass	6.68	6.50
VAST displacement	6.67	264

[1]Full end release
[2]Partial end release

Table 5: Comparison of predictions of the fundamental axial mode for the partially constrained moving boundary problem.

While the agreement in solutions in the fully released problem as described above is encouraging, similar results should be obtained when a partial end release is used. The results from the COUPLE system and the VAST added mass systems agree well and are in the order expected, as shown in column 3 of the table. The fundamental displacement-based solution is a complicated displacement mode of high frequency that does not model

the purely added mass effect that a pressure release condition provides. This points out the consideration that must be given to the boundary conditions in the various approaches; simple release conditions may not be equivalent, as is indicated here.

5 Damped systems

The neglect of damping is a legitimate approximation in many types of fluid/structure interaction, but a realistic numerical treatment of the acoustics of low density fluids such as air will necessitate a model capable of including the effects of concentrated damping. Since much of the work described in this document has been oriented toward the prediction of acoustic pressures in such systems, it is appropriate to consider in a very cursory way how damping forces can be introduced into finite element solutions of these problems. In all but a few special cases, the inclusion of damping precludes the use of the simple modal decomposition techniques which have been the subject of this report. In certain cases, complex eigensolution techniques can be utilized, but if damping is frequency dependent, then direct frequency response methods become a more effective means to define system characteristics.

Loss of acoustical energy in an acoustic system can occur in the form of reflection, radiation or dissipation in absorbent material. Systems which incur such losses can sometimes be treated by variational methods by coupling the lossy system to a second system which gains the energy, the overall system remaining conservative. Alternatively, a direct application of the weighted residual techniques can be utilized, these having particular application in cases where the mean flow in the system is non-zero. The variational approach was formally presented by Gladwell,[21] using what is commonly called an adjoint system. The problem variables are complex, but the resulting system can be assembled into a set of real linear equations, the size of which is double that of the original undamped problem. Alternatively, solution algorithms operating in complex arithmetic can be utilized. A slightly modified form of the adjoint system approach has been embodied in the work of Craggs,[5-8] and others[25,26,46], in the analysis of reactive and dissipative mufflers and vehicle interiors, and the calculation of acoustic impedances. Although the purely reactive system is not dissipative, the damping terms have a similar form to the dissipative systems, which are discussed in greater detail below.

Two general methods are commonly utilized in the finite element treatment of absorptive linings. In the first, the effect of the lining is modeled with a complex impedance boundary condition on the lined surfaces (an admittance in the pressure-based formulations). This approach is attractive in the sense that no new degrees of freedom are added to the existing system, but is limited in the thickness of lining which can be modeled accurately. For thick linings, the assumption of a local normal impedance implicit in this approach is

inappropriate.

5.1 Thin Absorptive Linings

In both variational or weighted residual formulations, the inclusion of non-homogeneous boundary conditions such as absorbing boundaries results in additional boundary integrals in the functional to be minimized. For absorptive linings over a region Γ_a, the integrals are similar to the fluid/structure coupling conditions discussed previously, and take the form

$$\int_{\Gamma_a} j\omega\rho p v \, d\Gamma \tag{78}$$

in which p and v are complex, (i.e. $p = p_r + jp_i$). As in the development presented for the modal methods, the pressure and velocity are related through a complex impedance (admittance) function

$$v = Ap = \frac{1}{Z}p \tag{79}$$

Substituting this relation into equation (78) yields damping terms in the form

$$c_{ij} = A\rho\omega \int_{\Gamma_a} N_i^p N_j^v \, d\Gamma \tag{80}$$

for the discretized system. For purely reactive boundaries, the absorptive linings are in effect replaced by openings with appropriate characteristic impedance values (typically ρc). An assembled form of the damped acoustic system then takes the form

$$\begin{bmatrix} [H] - \omega^2[Q] - \omega[C_i] & -\omega[C_r] \\ \omega[C_r] & [H] - \omega^2[Q] - \omega[C_i] \end{bmatrix} \begin{Bmatrix} p_r \\ p_i \end{Bmatrix} = \begin{Bmatrix} F_r \\ F_i \end{Bmatrix} \tag{81}$$

where the damping matrices are sparse according to the configuration of the lined boundary. Organization of the boundary terms into related groups can allow the damping matrix to be constructed in a very compact form. The standard acoustic stiffness and mass matrices appear in equation (81), which is formulated and solved for each value of frequency to determine the system response. This system is commonly used to evaluate transmission loss values for muffler systems.

The right-hand side of the assembled system is now non-zero, and although in principle a standard eigenvalue form of this system can be defined[26], this is not always practical since the impedance relation may be strongly dependent on frequency, necessitating an iterative approach. As well, specialized eigensolvers are required, since the modal solutions are complex. The load terms F arise from specified pressure or velocity forcing functions. For

piston-type problems, where a boundary nodal velocity vector v is specified over a region Γ_v, the load terms take the form

$$f_i = \rho\omega[C^*]\{V_i\} \tag{82}$$

where

$$c^*_{ij} = \int_{\Gamma_*} N_i^p N_j^p \, d\Gamma \tag{83}$$

Direct specification of pressure boundary forces is useful for modelling hydrodynamically induced loads, such as occur at openings of moving acoustic cavities.

An important requirement in this approach, and one which does not appear to be well-discussed, is the definition of an appropriate impedance function. Both empirical and experimental values have been utilized, the empirical techniques obviously having a greater generality, perhaps at the expense of some accuracy. Following Reference 12, Craggs[6,7,8] first defines a characteristic normal impedance as a function of the frequency, gas sound speed and the density and flow resistivity R of the damping material. This characteristic impedance is then used to calculate the impedance of the lining for a specific thickness. The validity of this approach is limited to certain ranges of frequency (typically 200-4000 Hz) and flow resistivity (1K-50K Rayls). These ranges are adequate for most applications amenable to finite element analysis.

5.2 Thick Absorptive Linings

A second general approach to the inclusion of concentrated acoustic damping is the direct finite element discretization of the absorptive material. This approach allows the treatment of linings of extended thicknesses, which can behave differently than thin linings. In this case, the bulk material properties (assumed isotropic) of the absorptive layer are imbedded in modified dynamic and constitutive relations governing the pressure and velocity distributions within the damping material. Within these elements, these relations take the form

$$(j\omega\rho K_s + R)\mathbf{u} = -\nabla p \tag{84}$$

$$-\frac{j\omega\Omega}{\rho c_a^2} p = \nabla \cdot \mathbf{u} \tag{85}$$

where Ω is the material porosity, ρK_s is an effective material density, and R is again the flow resistivity. The term K_s is the structural factor, intended to account for the inertia effect of the rigid portion of the damping material. For further discussion of the theoretical basis of these concepts, the reader is referred to References 12 and 29.

Applying the concept of an adjoint system and variational formulation, an assembled system based on the relations (84) and (85) can be developed for the dissipative elements,

in the form

$$\begin{bmatrix} [H]-\omega^2 K_s\Omega[Q_a] & -\frac{1}{\rho}\omega R\Omega[Q_a] \\ \frac{1}{\rho}\omega R\Omega[Q_a] & [H]-\omega^2 K_s\Omega[Q_a] \end{bmatrix} \begin{Bmatrix} p_r \\ p_i \end{Bmatrix} = \begin{Bmatrix} F_r \\ F_i \end{Bmatrix} \tag{86}$$

The Q_a matrix is formed from the usual functions, with the effective sound speed c_a used in place of the standard sound speed. The forcing functions are defined in the same manner as previously discussed.

For practical use, these elements require coupling to the acoustic elements at the interface. Craggs[8] has shown that equality of both pressure and normal velocity are required at the interface in order that these elements perform correctly. The latter condition necessitates a modification of the absorptive element matrices of equation (86) in the form of a multiplication of the matrices by the complex factor

$$\frac{j\omega\rho}{j\omega\rho + R} \tag{87}$$

before assembly into the complete system. This factoring and assembly must be repeated for each new value of frequency.

The absorptive elements can be either rigid-backed or allow structural interaction, and the other impedance boundaries presented can be added to the complete system. Thus, the finite element approach can be developed to handle the major requirements of a complete acoustical system without great difficulty. Certainly, specific features such as these damping models, which are not available in conventional finite element codes would be very useful. Several such special purpose codes appear to be used commercially.[22,29,46]

The development of a capability for modelling concentrated damping within the VAST program would be straightforward, although such a committment may not be justified at this time. Additional requirements are essentially limited to the modification of existing algorithms for surface integrations of interpolation cross-products, and the development of effective means of boundary definition and overall system assembly.

6 Concluding Remarks

This report has presented a systematic review of four different formulations of the coupled fluid/structure eigenvalue problem. The merits and drawbacks of each general approach have been discussed, and illustrated with examples in several cases. Based on the conclusions of this work, a three-field formulation utilizing structural displacement and fluid displacement potential and pressure has been chosen for further development. This work entails the generation and assembly of the submatrices required, and most importantly the development of an eigensolver capable of dealing with the difficult numerical characteristics

of the assembled system. At the time of writing, a capability for submatrix generation and assembly has been completed, in conjunction with a two-field symmetrization approach, in the program COUPLE. The symmetrization technique is applicable only to relatively small problems and requires the imposition of at least one constraint in the fluid domain. The development of the eigensolver for the three-field system is largely complete, but the system remains to be optimized to effectively handle larger problems. Future work will report on details of the eigensolver, and its application to some problems of practical interest. In the meantime, the standardization of the unsymmetric eigenvalue problem provides at least a nominal capability for the analysis of certain types of coupled problems, but the necessity of matrix inversion limits the usefulness of this approach.

The major portion of the current work has concerned itself with undamped systems. While the capability to solve the real eigenvalue problem is an important one, it is nevertheless a small subset of the general acoustic interaction problem. As discussed, a more comprehensive numerical treatment of the problem will necessarily include damping models, and several methods of including damping in finite element models have been presented. Solutions for transient response, frequency response, complex eigenvalue and nonlinear problems are also necessary. While VAST has a well-developed capability for the solution of some of these problems in the case of uncoupled systems, much work remains to be done in the development of effective methods of handling the general coupled problem.

As a first step towards further development, it is suggested that capabilities for including the effects of damping be included in the solution system. This is particularily relevant to the acoustic fluid/structural system in which significant concentrated damping can easily be introduced in real problems through the use of damping materials. In addition to the concentrated fluid damping, structural damping is important. This is often more of a hysteritic form (displacement proportional), than the viscous form of proportional or modal damping currently available.

The introduction of a more general purpose capability to include either fluid or structural damping will necessitate development of algorithms operating in complex arithematic. While this is more costly in terms of required computer resources, it is essential for the meaningful treatment of coupled acoustic fluid /structural interaction. Further effort should be directed towards this goal.

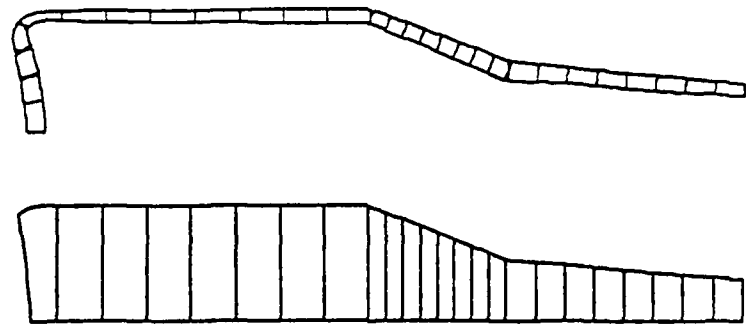

Figure 1: Axisymmetric finite element models of the bottle and fluid

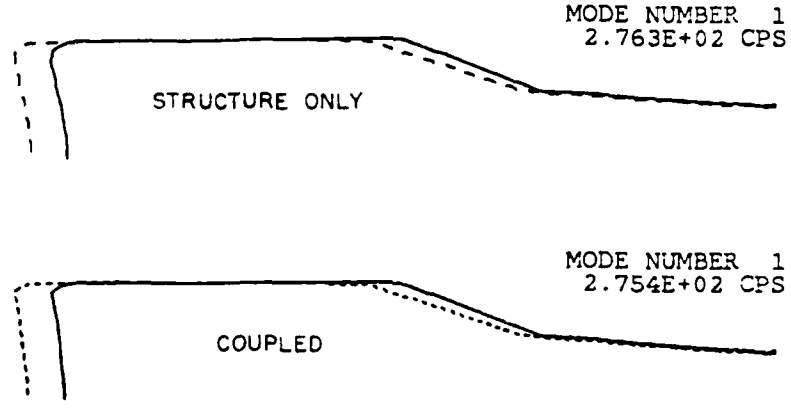

Figure 2: Mode 1 displacement plots for the bottle and coupled bottle/water system.

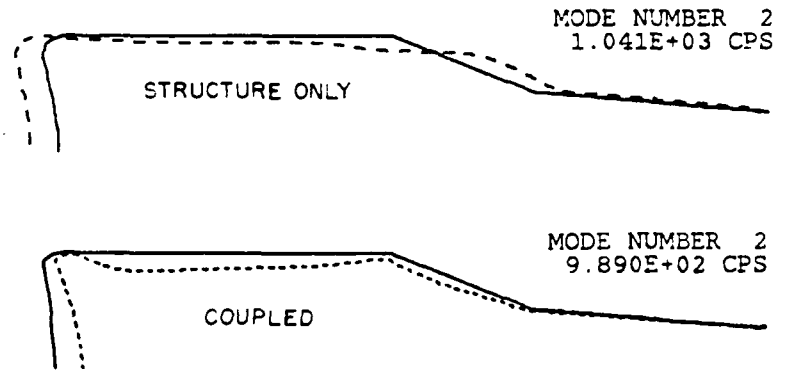

Figure 3: Mode 2 displacement plots for the bottle and coupled bottle/water system.

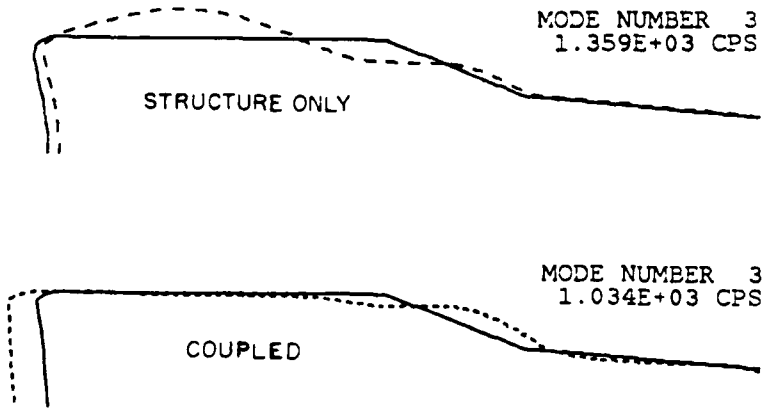

Figure 4: Mode 3 displacement plots for the bottle and coupled bottle/water system.

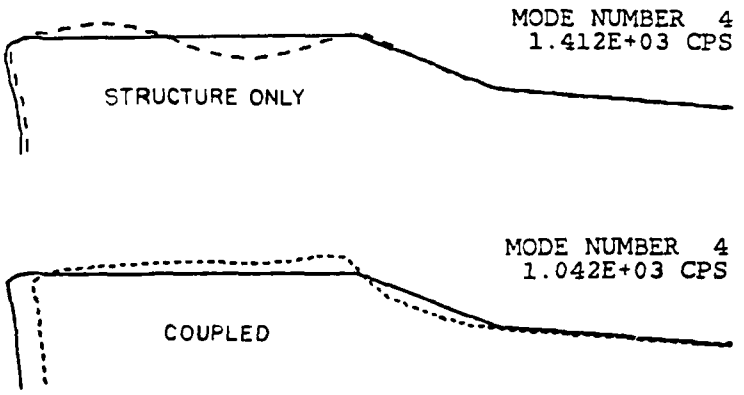

Figure 5: Mode 4 displacement plots for the bottle and coupled bottle/water system.

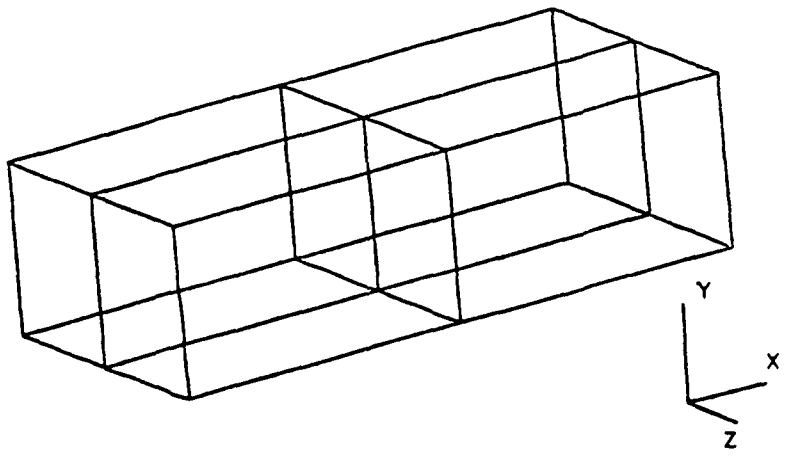

Figure 6: Finite element model for rectangular box of xyz dimension 0.236, 0.128, 0.110 m.

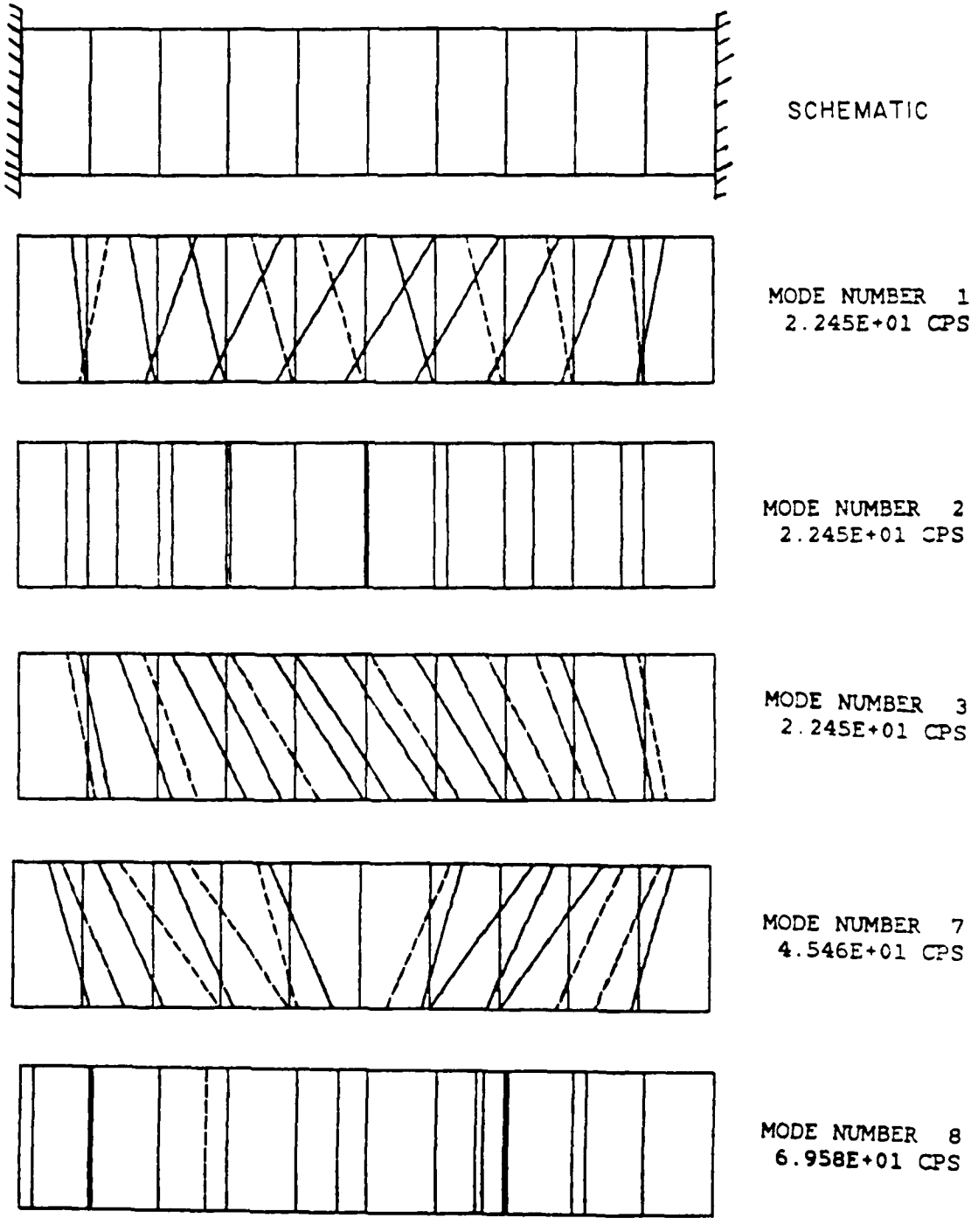

Figure 7: Finite element schematic and selected axial modes determined using a displacement-based method without rotational stiffness correction.

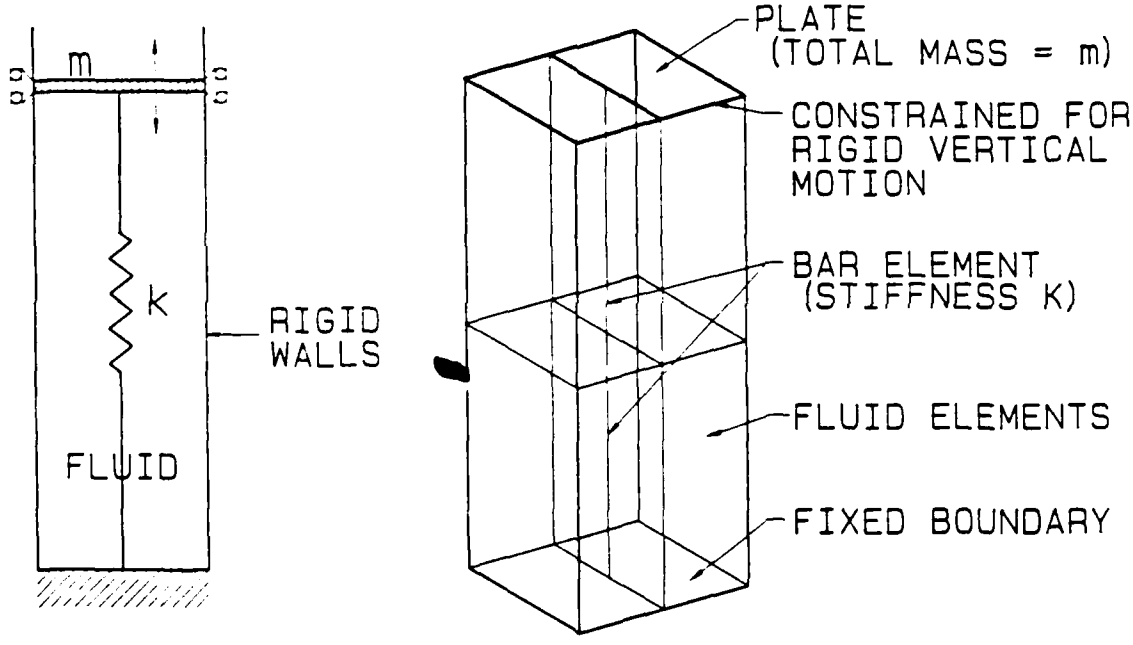

Figure 8: Piston-boundary interaction problem and finite element model.

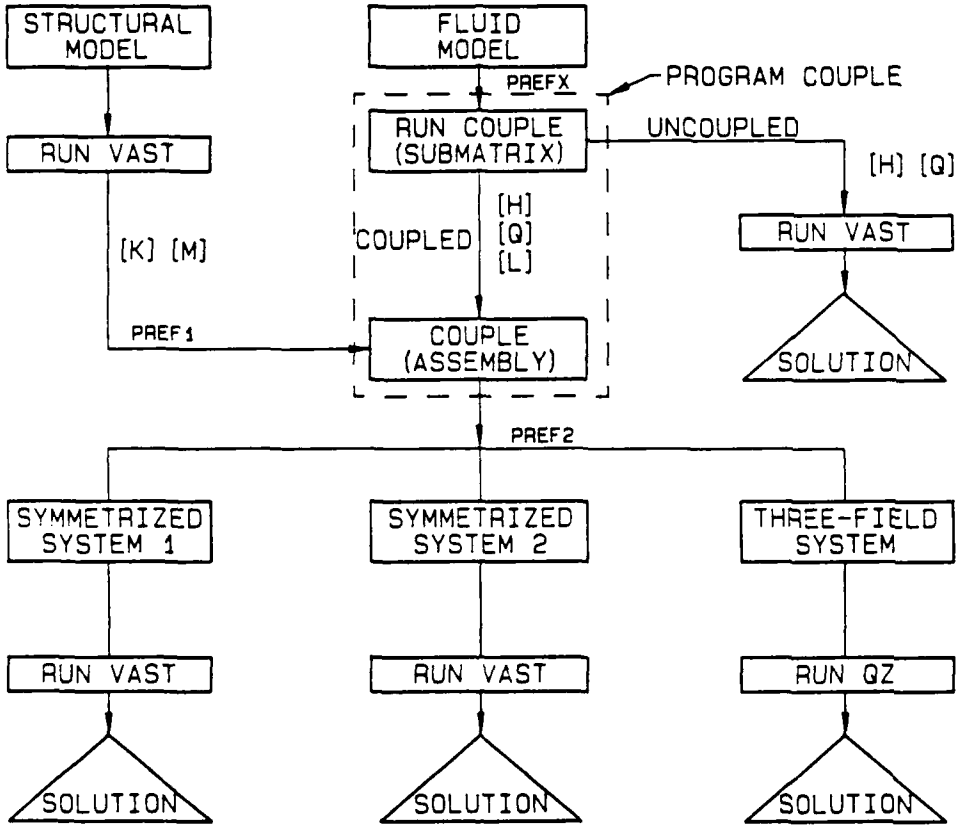

Figure 9: Flow chart detailing interaction of the COUPLE, VAST and QZ programs for eigenvalue analysis of acoustic cavity systems.

Appendix A: Program COUPLE

This section describes the program COUPLE, which can be used to generate the fluid submatrices required for a cavity modal analysis alone, and/or assemble the equivalent stiffness and mass matrices for one of two symmetrized two-field systems or a three-field formulation of the coupled eigenproblem. For reference, the uncoupled system is simply

$$[[H] - \omega^2[Q]]\{p\} = \{o\} \tag{88}$$

the two symmetrized systems available are

$$\left[\begin{bmatrix} K & 0 \\ - & \frac{1}{\rho}Q \end{bmatrix} - \omega^2 \begin{bmatrix} M + \rho L H^{-1} L^T & L H^{-1} Q \\ - & \frac{1}{\rho} Q H^{-1} Q \end{bmatrix}\right] \begin{Bmatrix} \delta \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{89}$$

and

$$\left[\begin{bmatrix} K M^{-1} K & -K M^{-1} L \\ - & \frac{1}{\rho} H + L^T M^{-1} L \end{bmatrix} - \omega^2 \begin{bmatrix} K & 0 \\ - & \frac{1}{\rho} Q \end{bmatrix}\right] \begin{Bmatrix} \delta \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{90}$$

and the three-field system is

$$\left[\begin{bmatrix} K & 0 & -L^T \\ & 0 & B_1^T \\ - & & -\frac{1}{\rho}Q \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 & 0 \\ & \rho H_1 & 0 \\ - & & 0 \end{bmatrix}\right] \begin{Bmatrix} \delta \\ \psi \\ p \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{91}$$

Since the actual application of the system (90) has not been particularily successful to date, the emphasis in the following discussion will be on the formulation of equations (89) and (91). The generalized eigenvalue systems of equations (88),(89) and (90) are solved using VAST, while the three-field system will be solved with an eigenvalue solution system (QZ) which is still under development. The characteristics of the system of equation (88) have been discussed in Reference 44, and will not be repeated here. A flow chart for the interaction of the various programs for the uncoupled and coupled eigenvalue problem is shown in Figure 9.

Since structural, fluid and coupled data files are all required at some stage during the formulation of the coupled system, the COUPLE program requires three distinct file prefix specifications. For the purposes of this discussion, the terms PREFX, PREF1, and PREF2 denote the user-specified file prefixes for the fluid model, structural model and assembled model respectively. For the uncoupled cavity analysis, only the fluid PREFX is required.

The overall operation of the program is controlled by the fluid model .USE file, i.e. PREFX.USE in the current notation. The format of this file is exactly as required for operation of the VAST program, up to and including stiffness matrix modification. The file format is detailed in Reference 47. The key features of this file are the line of master control

switches, which specify the modules of the program to be executed, and the additional data which control operations within each module. Where possible, analogous operations are performed by the COUPLE and VAST program, as indicated in the operation summary given in Table 6.

The two operations of fluid submatrix generation and assembly are quite distinct within the COUPLE program and will be discussed separately. The structural submatrix generation (for K and M) is assumed to have been performed with the VAST program, and will not be discussed further here.

Master contol switch	COUPLE operation controlled	Module data required
1	Element generation	yes
2	Bandwidth minimization (optional)	yes
3	Submatrix assembly	yes
4	Constraint imposition	yes
5	Inverse or three-field assembly	no
6	Creation of .T84 and .T85	no
7	Creation of .T86 and .T87	no
8	Two-field coupled assembly	no

Table 6: Master control switch operations for the COUPLE program.

Submatrix Generation

The fluid submatrix generation routine is essentially an extension of the ACOUSTIC program described in Reference 44. The operations performed by this section of the COUPLE program are thus:

1. Fluid element generation
2. Bandwidth minimization (optional)
3. Assembly of element matrices
4. Imposition of fluid constraints

The first four master control switches control execution of these four operations, and most of the module control data in a standard VAST structural .USE file is relevant. Restarts

are possible at bandwidth minimization, assembly and constraint application. Assembly in this case refers to element assembly.

For fluid submatrix generation, the input files required by the COUPLE program are:

1. PREFX.USE - control file for submatrix generation and assembly
2. PREFX.AMD - fluid geometry file

The format of these files can be found in Reference 47. The only difference occurs in the PREFX.AMD file, in which an additional record containing the element group sound speed must appear as the record following specification of the element code and number of elements, and the final record, which specifies fluid density, is not required. The fluid element codes are as specified in the VAST fluid added mass section, with the exception of the 8-noded axisymmetric element. The element codes are as follows:

- IEC=3 - 8-noded interface element
- IEC=4 - 20-noded fluid element
- IEC=5 - 8-noded axisymmetric element
- IEC=8 - 3-noded axisymmetric interface element

In all other aspects, the .AMD file format is exactly that required by the current VAST program. For the uncoupled cavity analysis, interface elements are obviously not required.

The output files created by the submatrix generation section of the COUPLE program depend on whether the cavity (uncoupled) or coupled analysis is specified during program execution. For cavity modal analysis alone, the COUPLE program produces only the assembled H and Q matrices. For the coupled problem, the coupling L matrix is also generated, using the interface elements implemented for added mass computation by finite element methods.[33] The following files may thus be created:

1. PREFX.LPA - formatted output data file
2. PREFX.T31 - element mass matrices
3. PREFX.T32 - element stiffness matrices
4. PREFX.T33 - interface element matrices, coupled analysis
5. PREFX.T34 - coupling submatrix L, coupled analysis
6. PREFX.T41 - partial geometry file, uncoupled analysis

7. PREFX.T45 - bandwidth reduction mapping file
8. PREFX.T46 - assembled stiffness matrix before boundary conditions are imposed
9. PREFX.T48 - fluid stiffness submatrix H
10. PREFX.T49 - fluid mass submatrix Q

Note that the file extensions are consistent with those utilized in the VAST program for files which contain similar data. All output files are unformatted binary with the exception of the .LPA file, which resembles the .LPT file produced by VAST. The print control parameters in the geometry and bandwidth control lines in the .USE file are active.

Because constraints are often required on the fluid stiffness matrix H, a stiffness matrix modification algorithm has been imbedded in the COUPLE program. The algorithm is a modified form of that used in the VAST program, and the input, read from the ISTIFM section of the .USE file, is exactly as required for VAST. A minimum of one constraint is required if the two-field symmetrization option of equation (89) is to be utilized; unconstrained systems are allowable in each of the other systems.

Submatrix Assembly

The submatrix assembly sections of the COUPLE program obviously function quite differently depending on the assembly option specified, but each option requires the following input data files:

1. PREF1.T48 - structural stiffness matrix K (from VAST)
2. PREF1.T49 - structural mass matrix M (from VAST)
3. PREFX.T48 - fluid stiffness matrix H (from COUPLE)
4. PREFX.T49 - fluid mass matrix Q (from COUPLE)
5. PREFX.T34 - coupling matrix L (from COUPLE)

and generates the output files:

1. PREF2.T41 - partial geometry control file for eigenvalue analysis
2. PREF2.T48 - assembled, coupled stiffness matrix
3. PREF2.T49 - assembled, coupled mass matrix

If the option specified is assembly of the three-field system of equation (91), the execution is controlled completely by the fifth master control switch in the PREFX.USE file. Although VAST is not being used for the eigenvalue analysis for the three-field system (see Figure 9), these files maintain the format currently required by the VAST program.

For assembly of the system of equation (89), at least one constraint must have been applied to the fluid stiffness matrix H. It is also recommended that sufficient constraints be applied to the structure to ensure that the structural stiffness submatrix K is also positive definite.

The computational aspects of the symmetrization options of the COUPLE program are somewhat more demanding than those for the three-field system, and a number of intermediate output files are utilized, to both reduce core memory requirements and facilitate restarts. With reference to equations (89) and (90), the following files additional to the output files noted above are created:

1. PREFX.T82 - H^{-1} or M^{-1}
2. PREFX.T84 - $H^{-1}Q$ or $M^{-1}L$
3. PREFX.T85 - $QH^{-1}Q$ or $L^TM^{-1}L$
4. PREFX.T86 - $LH^{-1}Q$ or $KM^{-1}L$
5. PREFX.T87 - $\rho LH^{-1}L^T$ or $KM^{-1}K$

The fifth through eighth master control switches in the PREFX.USE file control the execution of the symmetrization procedure in these options within the COUPLE program. In this case, there is no analogy with the operations controlled by these switches in VAST, and no module data is required for these operations. Restarts are possible from each of the control switches.

To complete the coupled eigenvalue analysis for the symmetrized systems, a PREF2.USE file will be required to control the decomposition and eigenvalue solution via VAST.

System constraints

The current version of the COUPLE program can assemble systems with up to 600 structural nodes and 400 fluid nodes. This limitation actually applies only to the symmetrization options, since much of that system has been developed on the basis of in-core matrix manipulation. As this approach is not well-suited for the solution of large problems, further expansion is probably unwarranted. For the three-field formulation, the current size limitations could be increased substantially. In both approaches, it is important to exploit

the bandwidth minimization options in the program, since the matrix manipulation algorithms and solvers can utilize the banded nature of the various submatrices and assembled system respectively.

In the symmetrization option of equation (89), the inverse matrix is calculated using an IMSL[49] algorithm LIN1PB specifically designed for banded symmetric matrices. If a diagonalized mass matrix is utilized, a simpler inversion algorithm is used. Implicit calculation of the inverse matrices is not practical for these formulations.

The Q matrix is naturally positive definite, and hence if the rigid body modes are constrained in the structural stiffness matrix K, the assembled, coupled stiffness matrix of equation (89) will be positive definite. Thus, frequency shifting will not be required in the decomposition section of VAST. If the structure is free or partially constrained, shifting should facilitate solution; however, the extremely large variation in the order of terms in the overall system can lead to excessive roundoff error, and the large band in the assembled mass matrix makes the shifting procedure inefficient. If possible, shifting should be avoided. Note that the presence of any unconnected fluid nodes will render the Q matrix singular.

It should be noted that the fluid mass matrix Q used in the cavity modal analysis should not be used directly in the coupled analysis, unless the reference sound speed used was unity. This in general is not possible in the uncoupled system, since the entries in the Q matrix become very small, and the current version of VAST will not generate a starting vector for eigenvalue analysis. In the coupled problem, the assembled mass matrix of equation (89) is divided by ρ^2, which generally circumvents problems with starting vectors. The eigenvalues calculated by VAST must then be divided by ρ to obtain the true frequencies.

Program input

In addition to the options specified in the .USE file, a small amount of interactive input is required by the program. The input is dependent on the option taken, either uncoupled or coupled forms (89), (90) or (91). For an uncoupled analysis, the following input is required:

1. User-specified PREFX
2. Uncoupled analysis specification
3. Reference sound speed

For a coupled analysis, the following input is required:

1. User-specified PREFX
2. Coupled analysis specification
3. User-specified PREF1

4. User-specified PREF2

5. Fluid density

6. Two or three-field option specification

7. H or M inverse option (if applicable)

8. Diagonal or consistent mass matrix M option, (if applicable)

For both reference sound speed and fluid density specification, the program provides a choice of some common values, or a user-specified value can be entered. For the coupled analysis specification, the reference sound speed is automatically set to unity, since that option cannot be utilized in the coupled problem. As previously mentioned, the calculated eigenvalues must be divided by the fluid density to obtain the true frequencies for the coupled analysis when the form of equation (89) is used.

Program Modules and Executable Image Formation

The current COUPLE program consists of eleven FORTRAN source code modules which contain a total of approximately 7500 lines of code. The modules perform operations generally consistent with a single master control switch in the .USE file. The COUPLE module controls the overall program operation. An executable image of the COUPLE program can be obtained by compiling and linking the following modules:

COUPLE, CELEMS, ABANRD, ASSEM, ASTIFM, MINVT, MKMUL, MLMUL, KMASSM, MKASS, KMASS3

In addition, the LINK command must currently include the location of the object library containing the IMSL routine LIN1PB, or a suitable equivalent. The submatrix generation and assembly is carried out in double precision, but the matrix manipulation required in the symmetrization approach utilizes single precision. The COUPLE program is currently compatible with version 5 of the VAST program. In future, the symmetrization algorithms may not be required. In that case, the modules

COUPLE, CELEMS, ABANRD, ASSEM, ASTIFM, KMASS3

will be all that are required. Minor changes to the COUPLE module will also be necessary.

References

1. Akkas, N., Akay, H.U., Yilmaz, C., 'Applicability of General-Purpose Finite Element Programs in Solid-Fluid Interaction Problems', Computers and Structures, Vol. 10, 1979, pp 773-783.

2. Chakrabarti, P., Chopra, A.K., 'Earthquake Analysis of Gravity Dams Including Hydrodynamic Interaction', Earthquake Engineering Structural Dynamics, Vol. 2, 1973, pp 143-160.

3. Chernuka, M.W., Nigh, G.L., 'Error Estimation and Reliability Assessments for Adaptive Finite Element Modelling - Phase II', DREA Contract Report DREA/8AE81-00092, 1984.

4. Craggs, A., 'The Transient Response of a Coupled Plate-Acoustic System Using Plate and Acoustic Finite Elements', J. Sound and Vibration, Vol. 15 (4), 1971, pp 509-528.

5. Craggs, A., 'An Acoustic Finite Element Approach for Studying Boundary Flexibility and Sound Transmission Between Irregular Enclosures', J. Sound and Vibration, Vol 30 (3), 1973, pp 343-357.

6. Craggs, A., 'A Finite Element Method for Damped Acoustic Systems: An Application to Evaluate the Performance of Reactive Mufflers', J. Sound and Vibration, Vol. 48 (3), 1976, pp 377-392.

7. Craggs, A., 'A Finite Element Method for Modelling Dissipative Mufflers with a Locally Reactive Lining', J. Sound and Vibration, Vol. 54 (2), 1977, pp285-296.

8. Craggs, A., 'A Finite Element Model for Rigid Porous Absorbing Materials', J. Sound and Vibration, Vol. 61 (1), 1978, pp 101-111.

9. Craggs, A., 'Coupling of Finite Element Acoustic Absorption Models', J. Sound and Vibration, Vol. 66 (4), 1979, pp 605-613.

10. Daniels, W.J.T., 'Performance of Reduction Methods for Fluid-Structure and Acoustic Eigenvalue Problems', Int. J. Numerical Methods Engineering, Vol. 15, 1980, pp 1585-1594.

11. Daniels, W.J.T., 'Modal Methods in Finite Element Fluid-Structure Eigenvalue Problems', Int. J. Numerical Methods Engineering, Vol. 15, 1980, pp 1161-1175.

12. Delany, M.E., Bazley, E.N., 'Acoustic Properties of Fibrous Absorbent Materials', Applied Acoustics, Vol. 3, 1970, pp 106-116.

13. Deshpande, S.S., Belkune, R.M., Ramesh, C.K., 'Dynamic Analysis of Coupled Fluid-Structure Interaction Problems', in Numerical Methods for Coupled Problems, Conference Proceedings, Swansea, Ed. Hinton, E, et al, Pineridge Press, 1981, pp 335-345.

14. Dowell, E.H., Gorman, G.F., Smith, D.A, 'Acoustoelasticity: General Theory, Acoustic Natural Modes and Forced Response to Simusoidal Excitation, Including Comparisons with Experiment', J. Sound and Vibration, Vol. 52 (4), 1977, pp 519-542.

15. Everstine, G.,C., 'A Symmetric Potential Formulation for Fluid-Structure Interaction', Letter, J. of Sound and Vibration, Vol. 79(1), 1981, pp 157-160.

16. Everstine, G.C., 'Structural Analogies for Scalar Field Problems', Int. J. for Numerical Methods in Engineering, Vol. 17, 1981, pp 471-476.

17. Felippa, C.A., 'Symmetrization of the Contained Compressible-Fluid Vibration Eigenproblem', Communications in Applied Numerical Methods, Vol. 1, 1985, pp 241-247.

18. Felippa, C.A., 'Some Aspects of the Symmetrization of the Contained Compressible-Fluid Vibration Eigenproblem', in Innovative Numerical Methods in Engineering, Proceedings, Fourth Int. Sym. on Numerical Methods Engineering, Georgia, Ed. Shaw, R.P., et al, 1986, pp 249-254.

19. Geradin, M., Robert, G., Huck, A., 'Eigenvalue Analysis and Transient Response of Fluid-Structure Interaction Problems', Engineering Computation, Vol. 1, 1984, pp 151-160.

20. Gladwell, G.M.L., Zimmermann, G., 'On Energy and Complementary Energy Formulations of Acoustic and Structural Vibration Problems', J. Sound and Vibration, Vol. 3 (3), 1966, pp 233-241.

21. Gladwell, G.M.L., 'A Variational Formulation of Damped Acousto-Structural Vibration Problems', J. Sound and Vibration, Vol. 4 (2), 1966, pp 172-186.

22. Göransson, P., 'Overview of Computational Vibro-Acoustics State of the Art and Future Development', Report FFA/TN 1988-17, Aeronautical Research Institute of Sweden, 1988.

23. Gupta, D.K., Ramakrishnan, C.V., Rao, J.S., 'Fluid-Structure Interaction in Turbine Blade Vibration', in Advances in Fluid-Structure Interaction, ASME Pressure Vessel Piping Series, Vol. 78, 1984, pp 89-96.

24. Hamdi, M.A., Ousset, Y., Verchery, G., 'A Displacement Method for the Analysis of Vibrations of Voupled Fluid-Structure Systems', Int. J. Numerical Methods Engineering, Vol. 13, 1978, pp 139-150.

25. Joppa, P.D., Fyfe, I.M., 'A Finite Element Analysis of the Impedance Properties of Irregular Shaped Cavities With Absorptive Boundaries', J. Sound and Vibration, Vol. 56 (1), 1978, pp 61-69.

26. Kagawa, Y., Yamabuchi, T., Mori, A., 'Finite Element Simulation of an Axisymmetric Acoustic Transmission System with a Sound Absorbing Wall', J. Sound and Vibration, Vol. 53 (3), 1977, pp 357-374.

27. Kiefling, L., Feng, G.C., 'Fluid-Structure Finite Element Vibrational Analysis', AIAA Journal, Vol. 14 (2), 1976, pp 199-203.

28. Liaw, C.Y., Chopra, A.K., 'Earthquake response of Axisymmetric Tower Structures Surrounded by Water', EERC Report No. 73-25, University of California, Berkely, Cal., 1973.

29. McCulloch, C., Coyette, J., 'Special Finite Element Developments for Acoustics', Finite Element News, Issue 6, 1988, pp 32-35.

30. Morand, H, Ohayon, R., 'Substructure Variational Analysis of the Vibrations of Coupled Fluid-Structure Systems. Finite Element Results', Int. J. Numerical Methods Engineering, Vol. 14, 1979, pp 741-755.

31. Morse, P.M., Ingard, K.U., 'Theoretical Acoustics', McGraw-Hill, New York, 1968.

32. Müller, W.C., 'Simplified Analysis of Linear Fluid-Structure Interaction', Int J. Numerical Methods Engineering, Vol. 17, 1981, pp 113-121.

33. Norwood, M., 'Application of the Finite Element Method for the Calculation of the Hydrodynamic Added Mass of Marine Propeller Blades', DREA Informal Communication, 1977.

34. Ohayon, R, Valid, R., 'True Symmetric Formulations of Free Vibrations of Fluid-Structure Interaction - Applications and Extensions', in Numerical Methods for Coupled Problems, Conference Proceedings, Swansea, Ed. Hinton, E, et al, Pineridge Press, 1981, pp 335-345.

35. Olson, L., Bathe, K.J., 'A Study of Displacement-Based Fluid Finite Elements for Calculating Frequencies of Fluid and Fluid-Structure Systems', Nuclear Engineering and Design, Vol. 76, 1983, pp 137-151.

36. Olson, L., Bathe, K.J., 'Analysis of Fluid-Structure Interactions. A Direct Symmetric Coupled Formulation Based on the Fluid Velocity Potential', Computers and Structures, Vol. 21, 1985, pp 21-32.

37. Petyt, M., Lea, J., Koopmann, G.H., 'A Finite Element Method for Determining the Acoustic Modes of Irregular Shaped Cavities', J. Sound and Vibration, Vol. 45 (4), 1976, pp 495-502.

38. Petyt, M., Lim, S.P., 'Finite Element Analysis of the Noise Inside a Mechanically Excited Cylinder', Int J. Numerical Methods Engineering, Vol. 13, 1978, pp 109-122.

39. Sandberg, G., Göransson, P., 'A Symmetric Finite Element Formulation For Acoustic Fluid-Structure Interaction Analysis', J. of Sound and Vibration, Vol. 123(3), 1988, pp 507-515.

40. Sandberg, G., 'Finite Element Modelling of Fluid-Structure Interaction', Lund Institute of Technology Report TVSM-1002, Lund, Sweden, 1986.

41. Sharan, K.S., Gladwell, G.M.L.,' A General Method for the Dynamic Response Analysis of Fluid-Structure Systems', Computers and Structures, Vol. 21, No. 5, 1985, pp 937-943.

42. Stanko, M.J., Seireg, A., 'Finite Element Simulation for the Acoustical Response of Closed and Open Spaces', Computers in Mechanical Engineering, October, 1982, pp 28-33.

43. Tabarrok, B., 'Dual Formulations for Acousto-Structural Vibrations', Int. J. Numerical Methods Engineering, Vol. 13, 1978, pp 197-201.

44. Vernon, T.A., Tang, S., 'Prediction of Acoustic Cavity Modes by Finite Element Methods', DREA Technical Communication 89/302, 1989.

45. Zienkiewicz, O.C., Newton, R.E., 'Coupled Vibrations of a Structure in a Compressible Fluid', Proceedings of International Symposium on Finite Element Techniques, Stuutgard, 1969, pp 359-378.

46. Finite Element Applications in Acoustics, Proceedings, ASME Vibration Conference, Hartford, Conn., Ed. Kamal, M. M., Wolf, J. A., 1981.

47. 'Vibration and Strength Analysis Program (VAST): User's Manual Version 04', DREA Contract Report CR/86/429, 1986.

48. 'ADINA - Automatic Dynamic Incremental Nonlinear Analysis Program User's Manual', Adina R and D Incorporated, Watertown , Maine, USA, 1987.

49. 'IMSL Users Manual', Edition 9.2, Vol. 2, IMSL Ltd., Houston, Tex., 1984.

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This report summarizes recent work directed toward the development of an effective finite element-based eigensolution capability for coupled fluid/structure interaction problems in which the fluid compressibility is included. Solution of these coupled problems is essential in the evaluation of structural acoustic transmission characteristics and enclosure acoustics. The methodologies are also of direct relevance to the acoustic radiation problem as well as other forms of time domain analysis. A comprehensive review of the four general methods most compatible with existing finite element analysis capability. Several example applications of the different methods are presented to highlight practical considerations. Of the formulations reviewed, a three-field system appears to have the most potential as a general purpose approach, although considerable effort is required inthe development of a suitable eigensolver for the system of choice. The report also presents a discussion of a number of methods of incorporating concentrated damping in finite element models of acoustic domains.

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