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AIRCRAFT TRAJECTORY GENERATION
A LITERATURE REVIEW

by

M.C. WALLER

and

D.R. Blackman and T.F. Berreen
Department of Mechanical Engineering,
Monash University, Melbourne, Australia

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SUMMARY

There is a need in the current economic environment to minimize costs and maximize efficiency in aircraft operations. Optimal flight trajectory generation can reduce operating costs, increase passenger and aircrew comfort and in the case of military operations reduce the loss of aircrew and aircraft. This memorandum presents a review of the field of optimal flight path generation for both civil and military operations and gives some recommendations for future research.

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NOTATION

CAS = Calibrated air speed.
D = Drag.
E = Energy.
ff = Fuel flow.
g = Acceleration due to gravity.
h = Altitude.
L = Lift.
m = Mass.
T = Thrust.
rpm = Revolutions per minute.
V = Velocity.
x, y = Distance.

α = Angle of attack.
β = Heading angle.
γ = Flight path angle.
ε = Engine setting angle.
μ = Bank angle.
χ = Yaw angle.

1 INTRODUCTION

This memorandum reviews current areas of research in the area of aircraft trajectory generation. It extends the work published by Hill (1987) which presented a review of aircraft integrated control technology including the role of trajectory generation.

The generation of aircraft flight trajectories has applications in both civil and military operations. The main areas of application are:

- Determination of a near optimal strategy in the approach to flight path planning of civil passenger and transport aircraft within the restraints of air regulations.

- Determination of a flight path for military transport aircraft, depending on the mission task at hand.

- Flight path planning for military tactical fighter aircraft in two different applications: for the mission as a whole or the mission considered in segments, particularly during the ingress and egress at low level.

In this review a comprehensive set of papers and reports that outline current areas of research in the field was consulted. The list of references presented draws on some of these to direct the reader to further information on the topic.

Section 2 reviews the need for aircraft flight path optimization. Section 3 discusses aspects of the different aircraft models that have been used by researchers in trajectory generation research.

Sections 4 and 5 consider the work and research conducted to date. Two main areas are discussed: firstly the generation techniques that apply to the flight path as a whole, which are used mainly for civil aircraft applications, although they may also be applied to military transport and fighter aircraft; secondly the flight components (take off, climb, cruise, ingress, etc.), separated and treated as individual segments.

Section 6 contains concluding remarks which suggest areas for further research.

2 THE NEED FOR AIRCRAFT FLIGHT PATH OPTIMIZATION

The requirements to improve an aircraft's flight path stem from the need to increase safety-of-flight by increasing probability of survival and the need

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to make it more cost effective by reducing fuel usage, direct operating costs and maintenance costs. Ashley (1982) highlights the fact that aircraft flight path optimization has been underway since before World War 2, when efforts were centred on the minimization of fuel consumption or dollar expenditure for the cruise segment of the flight.

Increased survivability of military aircraft is achieved by selecting a flight path to decrease exposure to enemy's defences. Bise and Luhrs (1986) consider automatic generation of aircraft trajectories when devising their definition of safety-of-flight. They also lay down the requirements in controllability and aircraft operations to meet the standard that their definition of safety-of-flight implies. They propose the following additions to the MIL-SPECS to satisfy their safety-of-flight definition:

> "If the automatic trajectory control system (ATCS) is given authority to go into flight regimes where the pilot cannot recover the aircraft, upon any failure (which is not extremely remote) affecting path control, the ATCS must safely exit from that flight condition and permit reversion to manual control."

> "Any failure resulting in the loss of automatic trajectory control shall cause the pilot to be warned of the automatic disengagement of the ATCS. The amount of time necessary for the pilot to safely and smoothly affect control after notification of the ATCS shutdown shall be determined by the type of mission and the current aircraft position within the local environment, and shall comply with the failure transients requirements (Section 3.4.8 of MIL-F-8785C)."

Civil operators can increase flight economy by reducing operating costs and increasing passenger satisfaction (arriving on time so that people can catch connecting flights).

Aircraft flight path optimization addresses these areas for military and civil aircraft operations.

3 AIRCRAFT MODEL REPRESENTATION

The aircraft model is an approximation to the aircraft's motion. An approximation to the motion is needed in the analysis because an aircraft's motion is so complex that the analysis would not be possible without it. The aircraft

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model is one basic parameter that can affect the outcome of the analysis. There is however a trade off between reducing the complexity of the analysis and adequately representing the aircraft's motion and dynamics. Bryson, Desai and Hoffman (1969) state that the amount of difficulty and expense experienced in undertaking aircraft performance analysis is dependent primarily upon the complexity of the dynamic model used to represent the aircraft.

In general the models that have been used in trajectory generation research are different from those of aircraft stability and control analysis, where wind and body axes models represent the aircraft as a rigid body. Poi..t mass models[1] and energy state approximation models[2] tend to be favoured in trajectory generation analysis. Some researchers have however included rigid body dynamics into their aircraft models when representing them as a state space model[3]. Schultz and Zagalsky (1972) considered five sets of aircraft models, including the point mass model and energy state approximation. They addressed each of these sets giving properties of the control variables and characteristics of the velocity set for optimization by the maximum principle. The models in Schultz and Zagalsky (1972) are for flight in the vertical plane; the point mass approximation has been extended to flight in three dimensions and this is shown in the Appendix. Not only are the point mass equations considered in the vertical plane and in full three dimensional space but there is also reason to represent them in the horizontal plane. That representation is shown in Kreindler and Neuman (1982). Some of the analysis conducted by Schultz and Zagalsky is attacked in a paper by Speyer (1973) on the basis that they did not take into account higher order conditions to test for the existence of the optimum to be produced by the calculus of variations approach to the problem.

Calise and Moerder (1982) gave a brief account of how the field of singular perturbation theory has played an important role in aircraft trajectory optimization. Singular perturbation analysis has the ability to reduce the order of complexity of a problem to the degree where an approximate solution can be obtained for the problem. As a result the need to solve a two point boundary value problem, as is the case for the optimal control problem formulation, is removed. This is achieved by separating the variables into fast and slow varying groups of state derivatives and then solving the overall problem by considering the fast and slow problems. Hence the overall prob-

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[1] For example see Denton, Jones and Froeberg (1985).

[2] For example see Bryson, Desai and Hoffman (1969).

[3] For example see Simmons, Breza and Funk (1978).

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lem has been approximated by a number of problems of reduced complexity. Price, Calise and Moerder (1985) show how rapidly the first derivatives of the state variables vary for the point mass approximation. The order of this variation is shown below:

$$
\begin{array}{c c c}
& & \dot{x}, \dot{y} \\
\Uparrow & \text{Slower} & \dot{E} \\
& & \dot{\beta} \\
\Downarrow & \text{Faster} & \dot{h} \\
& & \dot{\gamma}
\end{array}
$$

Calise (1984) applied singular perturbation methods to the simplified point mass equations to optimize the altitude and flight path angle dynamics. Calise (1977) used singular perturbation theory to extend energy management methods of aircraft performance. Calise (1981) again used singular perturbation theory and generated optimum aircraft trajectories and showed that the method has the capability to reduce the order of complexity of the problem and in some cases even produce a solution that was not obtainable before.

4   THE GENERATION OF WHOLE AIRCRAFT FLIGHT PATHS

This section is concerned with the formation of an aircraft flight path for the whole flight. The term "whole flight" here means the climb, cruise and descent; it excludes the takeoff and landing segments.

The method of solution that has been addressed by researchers is to determine a control sequence that minimizes some performance index, such as fuel consumption, cost of fuel used or both cost of fuel and cost of time, and in special cases the duration of the flight. These performance measures reflect the more commercial nature of this type of problem.

In most cases the flight path is generated in the vertical plane. This is a reasonable approximation because with long range flights, the flight is conducted directly between waypoints and as a result the lateral movement is considered to be minimal. The changes in heading that occur at the waypoints are not considered.

Erzberger and Lee (1980) considered the formation of optimum trajectories over a specified range, modelling the flight in the vertical plane with

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a climb, steady cruise and descent using optimal control theory optimizing direct operating costs (the sum of fuel and time costs). They formulated the problem with the state variable being range–to–go and the independent variable being energy. The aircraft's energy increases monotonically during the climb and decreases monotonically during the descent. They neglected the loss of weight due to burning of fuel and highlighted some basic relevant aspects that are of concern to researchers, described as follows. The cruise segment is considered as steady state though it is known that this is non-optimal as shown by Speyer (1976). The steady cruise is used in practice because it is thought unacceptable to expose passengers to changing accelerations and have engines undergoing cyclic change. They state that the steady cruise is non–optimum. However, the penalty on performance is unknown because the non–steady optimum cruise is undetermined. In some examples Speyer (1976) has shown that the cyclic cruise condition, which is better than the steady cruise condition, does not give a marked improvement in the value of the performance index. The form of the cruise will be discussed further in the Section 5.

Erzberger and Lee (1980) stated that the existence of the cruise segment is dependent on the model boundary conditions imposed and the length of the flight. They found that for long flights (greater than 500 n.miles) an optimum cruise condition did occur. However for short flights the cruise could occur at a point below the optimum cruising energy, this being dependent on the thrust conditions imposed. If the thrust had limits imposed to some maximum in the climb and minimum in the descent they found that this situation of the cruise being conducted at a point below optimum did generally occur. However, if no thrust limits were applied this situation did not generally occur. It is shown in an example that the difference between the constrained and unconstrained thrust is only minor, of the order of 1%. Even though the actual trajectories followed by the aircraft for the two cases were different the variation in the performance index was only slight. This suggests the presence of a weak optimum.

Barman and Erzberger (1976) considered the case of a subsonic aircraft undergoing a short haul flight using energy state methods. The aim was to determine a flight path that takes advantage of the prevailing weather conditions, mainly winds but also temperature variations, to minimize the performance index. They restricted their analysis to climb, cruise and descent legs. These legs were represented by increasing, constant and decreasing energy states. This has implications in that it simplifies the calculus of variations approach to the problem. The paper by Barman and Erzberger highlights

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some important points of this problem as it becomes more complex with the consideration of the effects of winds. They state that there exist multiple minima in the cruise efficiencies when the wind cases are considered. They also state that in the presence of winds of the jet stream type, two cruise energy levels are possible. This highlights the complex nature of aircraft trajectory generation in that there are clearly many optimal solutions in some instances.

Simos and Jenkinson (1985) considered the problem of identifying flight paths for propeller aircraft operating over a short range. Their approach was different from that of other researchers in that they did take into account that propeller-driven aircraft will probably not have a sophisticated autopilot. As a result the parameters that are identified as control variables are those that can be controlled manually. For example they used indicated airspeed, throttle setting, propeller rpm and distance travelled as control variables during the climb. Because they chose pilot controlled parameters as control variables they labelled their trajectories as suboptimum. They used multivariate optimization techniques to arrive at the solution. The profiles generated are stated to have improved the fuel consumption figures by approximately 1.5% over the technique of formulating a flight profile from the flight manual.

Sorensen and Waters (1981) and others have considered the generation of trajectories that take into account, along with the other considerations discussed, time of arrival constraints. This is referred to as 4-D control. These works considered the situation of the pilot being informed as soon as possible by air traffic control at the destination of the expected delay to be encountered. Once this information is known the strategy is to slow down to absorb the expected delay during the flight to arrive in time to avoid being placed in a holding pattern. Sorensen and Waters considered both the cost of fuel and the cost of time in their formulation of the problem. They presented graphs showing the fuel saving over the other procedure of flying the computed optimum trajectory for the whole flight and then when reaching the destination going into a holding pattern, at the maximum endurance condition, to absorb the delay.

Chakravarty (1985) approached the 4-D control problem by taking into account winds and altitude variation with the reduction in speed to absorb the delay. The section of the flight under consideration was not the whole flight but the section from a point on the cruise to a point and time in space further down the track at a lower altitude. He used a singular perturbation approach to simplify the equations before applying Pontryagian's

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minimum principle to extract the required result. The wind representation was one that varies linearly with altitude. He used the cases of no wind, head wind and tail wind for the fuel optimal free terminal time condition. This form of descent was then compared with the more conventional type of "constant Mach number/CAS" using idle thrust, and the advantage in fuel consumption of the proposed technique over the "constant Mach number/CAS" descent was shown to be up to 2.4%.

5 THE GENERATION OF AIRCRAFT FLIGHT PATH SEGMENTS

The generation of aircraft flight path segments is now discussed.

The emphasis of the work carried out to date concerned with takeoff has been takeoff trajectories in the presence of windshear. Windshear, a variation in wind speed and/or direction within a short distance, can pose problems to the pilot during both takeoff and landing. The Bureau of Meteorology (1981) describes the forms and causes of windshear that are relevant to aviation and also describes characteristics of the weather that affect aviation. Anderson and Clark (1978) also address the problems of windshear flight giving results of a survey undertaken to determine the understanding of pilots and air traffic controllers of the different aspects of windshear. The survey by Anderson and Clark found that pilots and air traffic controllers have a varied understanding of windshear.

The generation of flight paths through a windshear at takeoff has been addressed by Psiaki and Stengel (1986), and Miele, Wang and Melvin (1985a, 1985b, 1985c and 1985d). Due to the variability of windshear conditions that could be encountered, these types of studies serve as a guide to highlight the most appropriate methods of control of the aircraft through the windshear condition and to determine the operational limits of the aircraft for the type of flight condition.

The best way to undertake a climb depends on the application. In the case where cost is the most relevant factor, the best way is to execute the climb so that total cost of the flight is minimized, or approximate the flight by the climb and then cruise. There have been cases where the flight path for the fuel minimization of the climb has been determined. Where the operator wants to achieve a certain altitude in the shortest possible time the climb can be considered in isolation, minimizing time-to-altitude to obtain the desired performance. Breakwell (1977) considered the problem of minimum

time aircraft climbs; he used a singular perturbation approach to formulate optimum flight-path angle transitions. Bryson, Desai and Hoffman (1969) used the energy state approximation to look at the minimum fuel and minimum time-to-climb problems; the energy state approximation was then used to address the question of maximum range for a supersonic aircraft. They drew the conclusion that the energy state approximation was suitable for performance analysis of supersonic aircraft, if correctly interpreted, because it allowed trading of potential and kinetic energy in zero time; an approximate representation of the zoom climb and dive was obtained by the model. They gave some numerical results for a supersonic aircraft.

In the cruise segment some of the areas that have come under investigation are trajectory generation for minimum fuel consumption, minimization of direct operating costs and arriving at the destination at a fixed time (this is closely related to the airport scheduling problem of arriving just in time and the minimum time intercept problem).

Calise (1981 and 1984) considered the problem of the minimum time intercept. He used singular perturbation techniques to reduce the complexity of the problem in optimizing the aircraft's altitude and flight path angle. It has been shown by Speyer (1976) that the steady cruise condition is not fuel optimal, better results being obtained with a type of cyclic cruise condition. Bilimoria, Cliff and Kelly (1985) used singular perturbation techniques in their study of the cruise–dash type of flight segment that is typical of a tactical fighter aircraft. The final result was one of a "chattering cruise" (an operation between two altitude/Mach number points).

Grimm, Well and Oberle (1986) considered the minimum fuel problem for an aircraft with constant and varying weight. It was formulated as an optimal control problem; the results were non-steady cruise conditions, where for an F-4 type aircraft the fuel saving over the steady state condition was approximately 2%. They stated that:

> "The difference between constant and variable weight is primarily of theoretical nature. The practical effects of decreasing weight was found to be small."

Their analysis and the concluding statement shown above display the fact that one of the most common assumptions used, that of constant mass, does not affect the results to any great extent. Consideration should be given though to the length of the flight where it is possible that the change in mass, due to fuel burned and perhaps to dropping of cargo or stores, may become a problem.

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Houlihan, Cliff and Kelley (1982) also addressed the question of cruise conditions. They discussed the chattering cruise using a singular perturbation type approach to undertake the analysis. The operation between two altitudes and Mach numbers attempts to overcome the nonconvexity of the solution. They highlighted some difficulties that can be encountered with the type of model adopted. For the analysis they conducted, it was found that, for the simplest model, the fuel saving for the chattering cruise was dependent on the ratio of maximum thrust to minimum drag.

Sorensen and Waters (1981) showed the amount of fuel that can be saved on the cruise for the minimization of direct operating costs (linear combination of cost of fuel and cost of time), with a delay expected at the arrival point.

The work carried out in the area of aircraft approach scheduling has been concerned with operations in air traffic controlled air space. Kreindler and Neuman (1982) formulate a minimum fuel flight trajectory at constant altitude, via the minimum principle. They give valuable conclusions about flight operations in the airport terminal area. The paper by Grepper and Huguenin (1981) describes a computing approach to simulate the generation of four-dimensional flight paths under air traffic control. They give most attention to the operation of transport type aircraft.

The tactical aircraft's ingress and egress at low altitude is one area that has had a fair research effort. The aim with terrain following (TF), terrain following/terrain avoidance (TF/TA) and terrain following/terrain avoidance and threat avoidance (TF/TA^2) is to minimize exposure to radar and the enemy's defence systems.

Dynamic programming has been used by researchers such as Denton and Marsh (1982) who, considered the problem of terrain/obstacle avoidance. The presence of obstacles is represented by the inclusion of a probability of hitting them or running into the ground. This implies that the location of the obstacles is known in advance or that they rely on information from forward-looking radar. Denton and Marsh undertook a study to compare their automatic trajectory generator with the trajectory formed by a human planner. The automatic generation technique produces trajectories that make better use of terrain masking and for trajectories that have similar values of the performance index, the automatic generated ones were shorter.

Denton, Jones and Froeberg (1985) use dynamic programming to formulate TF/TA trajectories: the performance index being the minimization of the weighted combination of lateral deviation from a reference trajectory (squared) and altitude above a reference altitude (squared). The resulting

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optimal trajectory is one that seeks out low altitude areas in the region of the reference trajectory (a straight line between way points). They describe how the dynamic programming algorithm can be implemented into a trajectory generation package. They apply dynamic programming over regions of the solution space and these regions overlap. As the aircraft is flying through one region the algorithm is generating the route for the next region. Before this region is reached the calculations have been completed and have been stored in a buffer and the calculations for the next region have commenced. The actual routes followed are not a summation of a number of independent calculations; as the calculations for the next region are underway the aircraft has predicted the route to be flown with estimated initial conditions and so no discontinuities are encountered. Chan and Foddy (1985) have used dynamic programming to form a massive data base of intermediate results. These are stored onboard and then they are recalled and used as required. The approach here, when a trajectory has to be recomputed, is to do a coarse two dimensional dynamic programming search to form an initial path. Once this is formed a full three dimensional dynamic programming search is performed in a corridor placed around the initial path.

Kupferer and Halski (1984) do the trajectory generation for a TF/TA task in a two step process. Firstly a path or corridor is generated, using dynamic programming, which minimizes exposure to ground defences. Then the TF/TA flight path is generated, taking into account aircraft constraints, to produce a flight path as low as possible around the calculated reference path which is within the corridor. For their terrain information they use digital maps to provide a lookahead facility and to allow efficient use of terrain masking.

Optimal control theory has been used to obtain the control sequence needed to undertake TF or TF/TA. Funk (1977) and Simmons, Breza and Funk (1978) are among the investigators who use the optimal control theory approach to the problem of TF.

Wendl, Katt and Young (1982) give an account of work conducted on generating TF/TA trajectories in three dimensions. They formulate the problem to be a nonlinear optimization problem and use the feasible directions method to optimize their performance index taking into account the imposed constraints. They conclude that TF/TA can significantly improve the aircraft's operation at low level in a threat environment.

Woodward and Hoover (1981) describe the use of NAVSTAR Global Positioning System (GPS) in the formation of a passive terrain following system. GPS is used to give the location of the aircraft and a stored map

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is used to determine details of the terrain that is ahead, thus almost eliminating the need for forward looking radar. Woodward and Hoover describe simulation and flight test results for a helicopter installed with the system.

Dissanayake and Perras (1984) describe how dynamic programming and artificial intelligence have been used for the real time modification of pre-mission generated optimum paths as new information becomes available. Here artificial intelligence is used to match precomputed missions with the current mission. Dynamic programming is used to do the pre-mission optimization of these trajectories.

Asseo (1988) uses the method of steepest descents in his generation of TF/TA trajectories. He states that in general the gradient methods such as steepest descents have poor convergence characteristics, however the method of steepest descents is limited to the horizontal plane, hence avoiding the problems with convergence associated with constraints in the vertical plane. Here the trajectory generation is separated into two components: the ground track is optimized using information on the location of ground based defences and then the vertical profile to be flown is formed using a parabolic flight segment.

6 CONCLUDING REMARKS

This memorandum has reviewed the main areas of research within the field of aircraft trajectory optimization, for civil and military aircraft.

With all of the methods mentioned throughout this memorandum there remains a great amount of research to be done into the techniques themselves and the algorithms by which they are implemented to achieve robust trajectory generating systems for real life applications.

The classic techniques of optimization which have been used for trajectory generation in the past (for example calculus of variations, Pontryagin's Principle etc.) do not guarantee that the global optimum is achieved. Also the necessary and sufficient conditions for an optimum which need to be applied are difficult to calculate in practice.

With the numerical techniques used up to present, the convergence to an optimum point may be slow and numerical instabilities can often be encountered. With the dynamic programming numerical technique, where these factors do not appear to be a problem, user introduced instabilities can occur and time of calculations and storage requirements are great.

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Future research into trajectory generation should emphasize:

- Calculation of real time optimum trajectories.

- Real life aspects of the trajectory generation question, such as:

    - Wind effects at high and low altitude.
    - Trajectory generation with multiple aircraft.
    - Determining the consequences and subsequent regeneration strategy if the aircraft is perturbed off its optimal flight path.
    - Determination of the required level of accuracy of the inputs to the optimization, so that no effort is wasted on calculating or sensing inputs beyond what is needed, hence producing an economical system.

- Determining the trajectory's sensitivity and uniqueness.

- Determining the optimum implementation of a single optimizing technique or a number of techniques.

- Determining the size of the solution space and grid spacing to achieve an optimum result with minimum effort.

Some areas mentioned have been researched but not completely, for example the analysis of wind effects. This area requires further research and analysis, particularly for takeoff and landing, considering the types of winds that are to be encountered in practice.

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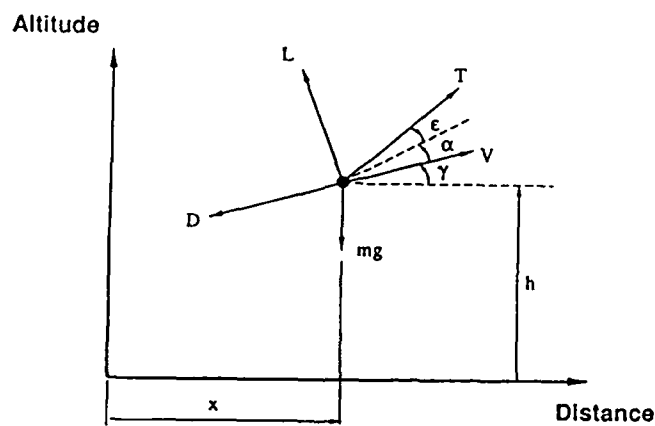
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APPENDIX — AIRCRAFT POINT MASS
EQUATIONS OF MOTION

1. Equations of Motion in the Vertical Plane

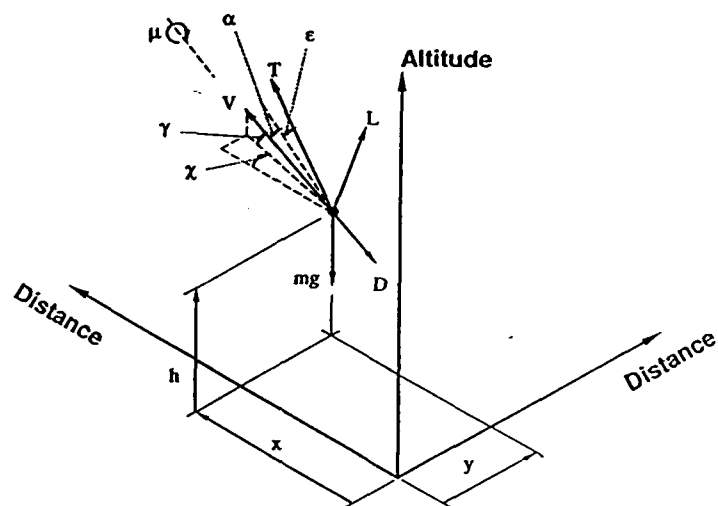$$\dot{V} = \frac{T\cos(\alpha + \varepsilon) - D - mg\sin\gamma}{m}$$

$$\dot{\gamma} = \frac{T\sin(\alpha + \varepsilon) + L - mg\cos\gamma}{mV}$$

$$\dot{h} = V\sin\gamma$$

$$\dot{x} = V\cos\gamma$$

$$\dot{m} = -ff$$

2. Equations of Motion in Three Dimensions

$$\dot{V} = \frac{T\cos(\alpha + \varepsilon_i) - D - mg\sin\gamma}{m}$$

$$\dot{\gamma} = \frac{(T\sin(\alpha + \varepsilon) + L)\cos\mu - mg\cos\gamma}{mV}$$

$$\dot{\chi} = \frac{(T\sin(\alpha + \varepsilon) + L)\sin\mu}{mV\cos\gamma}$$

$$\dot{h} = V\sin\gamma$$

$$\dot{x} = V\cos\gamma\cos\chi$$

$$\dot{y} = V\cos\gamma\sin\chi$$

$$\dot{m} = -ff$$

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16. ABSTRACT

There is a need in the current economic environment to minimize costs and maximize efficiency in aircraft operations. Optimal flight trajectory generation can reduce operating costs, increase passenger and aircrew comfort and in the case of military operations, reduce the loss of aircrew and aircraft. This memorandum presents a review of the field of optimal flight path generation for both civil and military operations and gives some recommendations for future research.

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