

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

SECRET

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release;
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AD-A211 068

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R(S)

5. MONITORING ORGANIZATION REPORT NUMBER(S)

AFOSR-TR- 89-1064

6a. NAME OF PERFORMING ORGANIZATION

6b. OFFICE SYMBOL
(if applicable)

7a. NAME OF MONITORING ORGANIZATION

Air Force Office of Scientific Research

6c. ADDRESS (City, State, and ZIP Code)

7b. ADDRESS (City, State, and ZIP Code)

302 Administration Bldg.
East Lansing, MI 48824-1226

Building 410
Bolling AFB, DC 20332-6448

8a. NAME OF FUNDING/SPONSORING
ORGANIZATION

8b. OFFICE SYMBOL
(if applicable)

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

AFOSR

NM

AFOSR-87-0267

8c. ADDRESS (City, State, and ZIP Code)

10. SOURCE OF FUNDING NUMBERS

Building 410
Bolling AFB, DC 20332-6448

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	A9	

11. TITLE (Include Security Classification)

Solidation Front/Viscous Phase Transitions, Forwards-Backward Heat Equations

12. PERSONAL AUTHOR(S)

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13a. TYPE OF REPORT

Final

13b. TIME COVERED

FROM 1 Jun 87 TO 28 Feb 89

14. DATE OF REPORT (Year, Month, Day)

15. PAGE COUNT

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Three important papers were written during the grant period. These were 1. Instability of periodic states for the Sivashinsky equation. 2. On Cahn-Hilliard type equation. 3. Stable patterns in a viscous diffusion equation all the above dealt with solidification and phase change/separation.

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT

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21. ABSTRACT SECURITY CLASSIFICATION

UNCLASSIFIED

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22c. OFFICE SYMBOL

NM

AFOSR-TR. 89-1064

Final Technical Report
AFOSR Grant # 87 0267

June 26, 1989

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Final Report
AFOSR Grant #87-0267

1. Directional Solidification

In the paper, "Instability of periodic steady states for the Sivashinsky equation", it is demonstrated that periodic one dimensional steady states of the equation

$$\begin{aligned}u_t &= (-u + \frac{1}{2}u^2 - ku_{xx})_{xx} - \alpha u \\u_x(0) &= u_{xxx}(0) = 0 \\u_x(L) &= u_{xxx}(L) = 0\end{aligned}\tag{1}$$

if they exist, must be unstable in the sense that the associated eigenvalue problem possesses unstable eigenvalues. Symmetric steady states are likewise shown to be unstable. These results cast serious doubt on the ability of equation (1) to sensibly model the persistence of stable cellular structures in directional solidification.

2. Directional Solidification and Phase Separation

"On Cahn-Hilliard type equations", studies equations in R^n , $n < 4$ of the form

$$\begin{aligned}u_t &= \Delta(f(u) - K\Delta u) & x \in \Omega \\n \cdot \nabla u &= n \cdot \nabla \Delta u = 0 & x \in \partial\Omega\end{aligned}\tag{2}$$

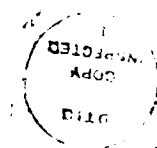
where $|f(x)| \rightarrow \infty$ as $x \rightarrow \pm\infty$. Equation (2) generalizes both the Cahn-Hilliard equation and the Sivashinsky equation in the small segregation coefficient limit. In terms of nonlinear stability of the planar state, two theorems are given which prescribe conditions under which solutions decay exponentially fast to their mean. In one-dimension it is shown that all nonmonotone steady states are dynamically unstable.

3. Phase Separation

The paper, "Stable patterns in a viscous diffusion equation" (with R.L. Pego) treats the equation

$$\begin{aligned}u_t &= \Delta(f(u) + \nu u_t) & x \in \Omega \\n \cdot \nabla(f(u) + \nu u_t) & & x \in \partial\Omega\end{aligned}\tag{3}$$

where Ω is a bounded domain and where f is assumed to be nonmonotone. Equation (3) arises in the context of phase separation of viscous binary mixtures. Global existence for initial data in L_∞ is proven using invariant regions. Discontinuities in the initial data persist and no new discontinuities may appear in finite time. If f satisfies a certain "nondegeneracy" condition, then solutions converge to steady state. For equation (3) steady states consist of all bounded measurable functions $u_e(x)$ which satisfy $f(u_e(x)) = \text{constant}$. If $f'(u) > 0$ a.e., then $u_e(x)$ is dynamically stable. Smooth solutions may converge asymptotically to discontinuous solutions, and solutions may converge to steady states which are not absolute minimizers of the free energy.



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