Anisotropy and Stress Path Effects in Clays With Applications to the Pressuremeter Test

S. E. Chegaraei, A. Skardarjan and J. L. Chambon

The research study undertaken at Purdue focuses towards increasing our understanding of fundamental issues related to the behavior of clays, especially with regard to modelling their anisotropy. There is ample evidence that in situ soils are anisotropic, elasto-plastic, stress path and rate dependent, unlike what is generally assumed in test interpretations. Therefore, the study of mechanisms of deformation of in situ tests and their interpretation must take into account these features, and especially the anisotropic nature of the soil deposit.

Expressions were derived for the normalized shear strength \( \tau_s/C' \), using the Cam clay model, the modified Cam clay model, and the extended Cam clay and modified Cam clay models with spacing ratio. Both extended models with spacing ratio predict values that compare well with experimental data. Unlike the excellent agreement with experimental results obtained for \( \tau_s/C' \), predictions made for Skempton's A parameter at failure \( A_s \) were marginal for all models. \( A_s \) is very sensitive to \( A \), which is a function of \( C' \) and \( C_0 \). This severely

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ANISOTROPY AND STRESS PATH EFFECTS IN CLAYS WITH APPLICATIONS TO THE PRESSUREMETER TEST

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S. Thevanayagam
A. Skandarajah
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influences $A$ and thus affects the predictions of $A_{p}$. Using critical state concepts a simplified procedure to predict the overconsolidation ratio of clays is presented. Since the OCR estimate is not sensitive to the $C/C_{o}$ ratio, a good prediction is possible from the knowledge of $e/e_{L}$ and $C_{o}$.

The limitations of existing theories to model the anisotropy of clays were identified. A novel approach is presented that incorporates the essential features of anisotropic behavior of clays, and the theory is validated using experimental data. The model captures the essential features of initial and induced anisotropy, yet yielding close form solutions for all the failure parameters of clay. The proposed theory is applicable to a number of geotechnical problems; in this report, it is used to investigate several important issues related to the determination of in situ properties using the self-boring pressuremeter (SBPM):

(i) The stress state and mode of deformation in pressuremeter tests are identified. The generality of plane strain condition during expansion of an SBPM cavity is shown and conditions where it is far from plane strain are indicated. Experimental data are used to substantiate the theoretical findings. All indications are that these findings can settle the questions regarding the mode of deformation and the role of the vertical stress in expansion of cylindrical cavity.

(ii) The possibility of radial cracking (negative circumferential stress) was analyzed in detail and shown to be unlikely.

(iii) Using the theory developed in this work, the SBPM failure parameters can be used to determine the same parameters for many different modes of failure other than that of the SBPM.

(iv) A procedure has been proposed to evaluate the error in strain induced by initial movement of the pressuremeter cavity. After evaluation of the error in strain, actual strength parameters can be calculated.
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CHAPTER 1
INTRODUCTION

The research study undertaken at Purdue focuses towards increasing our understanding of fundamental issues related to the behavior of clays, especially with regard to modelling their anisotropy. The information being developed is applicable to most geotechnical problems, however, in illustrating their usefulness, emphasis is placed on interpretation of in situ tests, and in particular pressuremeter and self-boring pressuremeter (SBPM) tests. There is ample evidence that in situ soils are anisotropic, elasto-plastic, stress path and rate dependent, unlike what is generally assumed in test interpretations. Therefore, the study of mechanisms of deformation of in situ tests and their interpretation must take into account these features, and especially the anisotropic nature of the soil deposit. A simple and reliable anisotropic theory will be most useful in the study of several important factors related to pressuremeter testing such as possibility of radial cracking, role of vertical stress, stress conditions at failure, and the effects of initial disturbance which cannot be measured. It will also be possible to estimate undrained strength of the clay in other modes of failure using SBPM data only.

In Chapter 2 of this report, the viability of modelling the anisotropic behavior of clays within the context of the modified cam clay (MCC) model (Roscoe and Burland 1968, Wroth 1984) is stu-
died. Some useful relationships to predict the behavior of clays in triaxial space are presented, and the limitations of the MCC model are indicated. A cuboidal shear device is used to illustrate the influence of stress path and fabric anisotropy on the behavior of clays. In addition, using critical state concepts (Schofield and Wroth 1968, Wroth 1979) a simplified procedure to predict the overconsolidation ratio of clays is presented.

In Chapter 3, the limitations of existing theories to model the anisotropy of clays are identified. A novel approach is presented that incorporates the essential features of anisotropic behavior of clays, and the theory is validated using experimental data. The model captures the essential features of initial and induced anisotropy, yet yielding close form solutions for all the failure parameters of clay. All the facets of anisotropy known to date are incorporated. It can be applied to a number of geotechnical problems, such as stability of embankments, retaining systems, etc. Its potential is demonstrated by studying the following issues in pressuremeter testing of soils. (i) The stress state and mode of deformation in pressuremeter tests are identified. The generality of plane strain condition during expansion of an SBPM cavity is shown and conditions where it is far from plane strain are indicated. Experimental data are used to substantiate the theoretical findings. All indications are that these findings can settle the questions regarding the mode of deformation and the role of the vertical stress in expansion of cylindrical cavity (SBPM).
(ii) Possibility of radial cracking is analyzed in detail, showing little chance for its occurrence. (iii) A method to obtain anisotropic failure parameters using SBPM test data is outlined. (iv) An analytical technique to make allowance for the disturbance in test data is presented.
2.1 Experimental Illustration of Anisotropy Effects

Strength anisotropy consists of two major components. One is inherent anisotropy, which occurs due to preferred particle arrangement during sedimentation, resulting in a fabric that is not identical in all directions. The other component is stress induced anisotropy which is caused by an anisotropic state of stress at the end of consolidation. Inherent anisotropy implies that the soil behaves anisotropically even if the initial stress state is isotropic. Stress induced anisotropy means that the soil behaves anisotropically depending on the direction of loading due to its initial anisotropic stress state even if the soil properties such as $c'$ and $\phi'$ are isotropic. The ratio of triaxial compression strength to that of triaxial extension is often referred to as a measure of anisotropy. Using the cuboidal shear device, anisotropy was illustrated by comparing the strength derived from strain controlled tests on sedimented samples loaded vertically and horizontally under plane strain conditions.

Three tests were carried out on WP Georgia silty clay under essentially plane strain conditions: the first two tests on $K_0$ consolidated samples and the following one on an isotropically consolidated sample. In the first test (CK$_0$ UC-PSV), major and minor principal stress increments were applied horizontally, while the intermediate principal stress increment was applied vertically,
ensuring plane strain conditions, with the plane of deformation being horizontal (this situation is typical of a horizontally loaded pile foundation). In the second test (CK\textsubscript{UC}-PSH) intermediate and minor principal stress increments were applied horizontally, while the major principal stress increment was applied vertically, ensuring plane strain conditions, with the plane of deformation being vertical (this is a typical field loading condition, e.g., long embankment strip footing, etc.). The same test was repeated on an isotropically consolidated sample (CIUC-PSH). The results of these plane strain tests are given in Table 2.1.

To quantify strength anisotropy, several techniques have been proposed by previous researchers (e.g. Aas, 1965, Duncan and Seed, 1966, Lo and Morin, 1972, Berre and Bjerrum, 1973, Krishnamurthy, et al., 1980, Nakase and Kamei, 1983). In the experimental program described herein, the specimen is never rotated. The axis remains vertical during consolidation and shear. Anisotropy is measured by the ratio of the shear strength of a horizontally loaded specimen to the strength of a vertically loaded replicate specimen.

From Table 2.1, the three different tests that followed different stress paths and had undergone different consolidation (K\textsubscript{o} and isotropic consolidation) yielded different strength values. The first sample (CK\textsubscript{UC}-PSV), K\textsubscript{o} consolidated and loaded horizontally, gave the lowest strength. The reason is that the extension test stress path meets the failure envelope before the compression test stress path does. In the other two tests where the plane
### Table 2.1 Plane Strain Test Results
(WP Georgia Silty Clay)

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Test Type</th>
<th>$(t_f/\sigma_{cv})_{max}$</th>
<th>$[t_f/\sigma_{cv}]_H$</th>
<th>$[t_f/\sigma_{cv}]_v$</th>
<th>$[t_f/\sigma_{cv}]_{K_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CK UC-PSV</td>
<td>0.47</td>
<td></td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>CK UC-PSH</td>
<td>0.60</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>CIUC-PSH</td>
<td>0.70</td>
<td></td>
<td>-</td>
<td>0.86</td>
</tr>
</tbody>
</table>
strain condition was maintained in horizontal direction and the loading was applied vertically (conventional plane strain tests), the samples showed higher strengths than the first one, with the isotropically consolidated sample being stronger than the $K_o$ consolidated one.

The three different stress path tests yielded different strengths which clearly illustrate the stress dependent anisotropic nature of the soil. The anisotropic ratio $[\frac{\tau_f}{\sigma_{vc}}]_H/[\frac{\tau_f}{\sigma_{vc}}]_V$ of the $K_o$ consolidated sample is 0.78, which shows that the stress path has a significant influence on the anisotropic strength. The strength ratio between the $K_o$ and isotropically consolidated samples is not unity ($= 0.86$) which indicates that induced anisotropy effects exist.

2.2 Theoretical Studies of Stress Path and Anisotropy

2.2.1 Predicting Undrained Shear Strengths of $CK_o\, UC$ from CIUC Tests

Isotropically consolidated undrained compression (CIUC) triaxial tests are the common laboratory tests to determine the undrained shear strength of clays primarily because of the convenience and simplicity of the experimental procedures. Anisotropically consolidated (CAUC) tests are more complicated and take much longer to run. Natural soil deposits, however, are rarely isotropic nor do they often have an in situ $K_o = 1$. Therefore, development of a theoretical expression to predict the actual behavior/strength (i.e. $CK_o\, UC$ strength) from CIUC test results
would be very beneficial.

Although Ladd (1965) concluded that it is not possible to predict CK<sub>0</sub>UC strength from CIUC tests because the tests follow two different stress paths, Sivakugan, et al. (1988b) found that a reasonably good estimate of the CK<sub>0</sub>UC strengths may be obtained from a single CIUC test on the same soil. The ratio of \( \frac{\tau_f}{C_{vo}} \) to \( \frac{\tau_f}{C_{vo}} \) CIUC was shown to be a function of the coefficient of earth pressure at rest \( K_0 \) and Skempton's pore pressure parameters at failure \( A_f \) for both tests. Thus, \( \frac{\tau_f}{C_{vo}} \) from a CIUC test may be used with estimates of \( K_0 \) and the \( A \) parameters to estimate the normalized shear strength of \( K_0 \)-consolidated specimens. Although only limited data are available, the procedure appears to be sufficiently accurate for practical purposes. It is briefly summarized here, as details are given in Sivakugan (1987) and Sivakugan, et al. (1988b).

It was shown that for normally consolidated clays, the strength ratio can be given by:

\[
\frac{\tau_f}{C_{vo}} \text{CK}_0 \text{UC} \quad \frac{\tau_f}{C_{vo}} \text{CIUC} = \frac{K_0 + 2(1 - K_0) A_f, I}{K_0 + 2(1 - K_0) A_f, K_0} \frac{A_f, K_0 (1 - K_0) + K_0}{K_0 + 2(1 - K_0) A_f, K_0} (2.1)
\]

where \( \frac{\tau_f}{C_{vo}} \) CK<sub>0</sub>UC = normalized undrained shear strength for CK<sub>0</sub>UC test

\( \frac{\tau_f}{C_{vo}} \) CIUC = normalized undrained shear strength for CIUC test
$A_f, i$ - Skempton's $A$ parameter at failure for the CIUC test;

$A_f, K_o$ - Skempton's $A$ parameter at failure for the CK$_o$UC test

$K_o$ - coefficient of at rest earth pressure.

Typical representations of this strength ratio equation are given in Fig. 2.1 for $K_o = 0.4$. From this figure and others at different $K_o$ values, it can be seen that $A_f, i$ has a much greater influence on the strength ratio than $A_f, K_o$, and even a crude estimate of $A_f, K_o$ is usually sufficient for a reasonably good estimate of the strength ratio.

The parameters required for the computation are $K_o$, $A_f, i$ and $A_f, K_o$. It is admittedly not easy to obtain accurate estimates of $K_o$ in practice, e.g. see Massarsch et al. (1975) and Tavares et al. (1975). In the interim, $K_o$ may be obtained from published correlations (Holtz and Kovacs, 1981) or estimated from Jaky's (1948) relationship. The isotropic $A$ parameter $A_f, i$ can be obtained from the CIUC test results, while $A_f, K_o$ requires $K_o$-triaxial tests (just what we want to avoid). Fortunately, as indicated previously, the strength ratio is not very sensitive to $A_f, K_o$, and only crude estimates are required for good estimates of the strength ratio. In the absence of any other information, the mere knowledge of whether $A_f, i$ is greater or less than $A_f, K_o$ is sufficient for a reasonably good estimate of this parameter. Numerical examples to predict the strength ratio are given in Sivakugan et al. (1988b).
Figure 2.1 Variation of $A_{f,i}$ with $A_{f,Ko}$ for Strength Ratios
(Eq. 2.1) of 0.8-1.2 for $K_o = 0.4$
To check the applicability of the proposed technique, calculated values of the strength ratio were compared to experimental data (Table 2.2) for 24 different soils, and the results are plotted in Fig. 2.2. This figure shows that for carefully performed tests on replicate specimens, the assumptions made and equations used to derive Eq. 2.1 are reasonably valid. From Table 2.2, it is seen that $A_f, K_0$ can be less or greater than $A_f, i$ by as much as 100%. The slight differences in $\phi'$ between isotropic and $K_0$ tests can be neglected in the procedure. This is justified by a statistical analysis of experimental data from CIUC and CAUC tests by Mayne (1985), which showed that $\phi'_{CAUC} = 0.97 \phi'_{CIUC}$.

### 2.2.2 Spacing Ratio

The Cam clay (Roscoe et al., 1963) and the modified Cam clay models (Roscoe and Burland, 1968) are the most widely accepted critical state soil models. They have undergone several developments over the past two decades (e.g., Egan, 1977, Pender 1978, and van Echelen and Potts, 1978). Sivakugan (1987) and Sivakugan et al. (1988a) give a review of the main characteristics of these models. The Cambridge soil models were developed essentially for isotropically consolidated clays, although most soil deposits encountered in nature are one dimensionally consolidated with no lateral deformation.

In the Cambridge soil models, it was hypothesized that the $K_0$ consolidation line ($K_0$ CL) in the $p' - q - v$ space lies on the state
Table 2.2 Experimental and Predicted Strength Ratios

<table>
<thead>
<tr>
<th>No.</th>
<th>Soil</th>
<th>$K_0$</th>
<th>$A_{f,1}$</th>
<th>$A_{f,K_0}$</th>
<th>$\phi^*$</th>
<th>CIUC</th>
<th>$K_{0,UC}$</th>
<th>Predicted</th>
<th>Expt.</th>
<th>Ref.</th>
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<tr>
<td>1</td>
<td>Boston Blue Clay</td>
<td>0.54</td>
<td>1.10</td>
<td>0.60</td>
<td>27.5</td>
<td>26.5</td>
<td>1.16</td>
<td>1.10</td>
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<tr>
<td>2</td>
<td>Weald Clay</td>
<td>0.61</td>
<td>0.92</td>
<td>1.80</td>
<td>26.0</td>
<td>26.0</td>
<td>0.87</td>
<td>0.84</td>
<td>36</td>
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<td>3</td>
<td>Vicksburg Buckshot Clay</td>
<td>0.54</td>
<td>1.05</td>
<td>1.05</td>
<td>24.0</td>
<td>23.5</td>
<td>1.02</td>
<td>1.00</td>
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<td>4</td>
<td>Undist. Kawasaki Clay</td>
<td>0.52</td>
<td>0.80</td>
<td>0.50</td>
<td>37.0</td>
<td>33.0</td>
<td>0.98</td>
<td>0.93</td>
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<td>Undist. Brobekkveien Oslo   Clay</td>
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<td>30.0</td>
<td>26.5</td>
<td>1.09</td>
<td>1.00</td>
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<td>0.45</td>
<td>0.84</td>
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<td>37.2</td>
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<td>0.94</td>
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<td>10</td>
<td>Illite</td>
<td>0.50</td>
<td>0.43</td>
<td>0.10</td>
<td>22.3(^a)</td>
<td>24.9(^a)</td>
<td>0.85</td>
<td>0.88</td>
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<td>Spestone Kaolinite</td>
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<td>Nagoya Clay</td>
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<td>0.50</td>
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<td>0.89</td>
<td>0.86</td>
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Table 2.2, continued

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<thead>
<tr>
<th>No.</th>
<th>Soil</th>
<th>$K_o$</th>
<th>$A_{f, i}$</th>
<th>$A_{f, K_o}$</th>
<th>$\phi'$</th>
<th>CIUC</th>
<th>$C_{K, UC}$</th>
<th>Predicted</th>
<th>Expt'1</th>
<th>Ref.</th>
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<td>0.42</td>
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<td>0.68</td>
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<td>0.40</td>
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<td>0.74</td>
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<td>0.78</td>
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<td>Kawasaki Clay</td>
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<td>41.0</td>
<td>0.89</td>
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<td>Whitefish Falls</td>
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<td>1.03</td>
<td>0.68</td>
<td>27.0</td>
<td>26.0</td>
<td>1.09</td>
<td>1.06</td>
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<tr>
<td>18</td>
<td>Wallaceburg</td>
<td>0.51</td>
<td>0.70</td>
<td>0.44</td>
<td>23.0</td>
<td>25.0</td>
<td>0.92</td>
<td>0.97</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Boston Blue Clay</td>
<td>0.50</td>
<td>1.0</td>
<td>0.5</td>
<td>25.4$^a$</td>
<td>26.1$^a$</td>
<td>1.13</td>
<td>1.07</td>
<td>14</td>
<td></td>
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<tr>
<td>20</td>
<td>Marine Clay</td>
<td>0.51</td>
<td>1.08</td>
<td>1.25</td>
<td>34.0</td>
<td>29.0</td>
<td>1.01</td>
<td>0.94</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Vicksburg Buckshot Clay</td>
<td>0.55</td>
<td>0.88</td>
<td>0.41</td>
<td>26.7</td>
<td>24.9</td>
<td>1.07</td>
<td>0.96</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>EABPL clay</td>
<td>0.64</td>
<td>0.92</td>
<td>0.60</td>
<td>21.7</td>
<td>20.2</td>
<td>1.04</td>
<td>0.98</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>K50</td>
<td>0.47</td>
<td>1.06</td>
<td>1.05</td>
<td>32.0</td>
<td>29.1</td>
<td>1.03</td>
<td>0.94</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Kaolinite</td>
<td>0.49</td>
<td>1.15</td>
<td>1.03</td>
<td>26.9</td>
<td>24.2</td>
<td>1.09</td>
<td>0.93</td>
<td>77</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Computed from Eq. 1 of Bjerrum and Simmons (1960).
Figure 2.2 Experimental versus Predicted Strength Ratio

\[
\text{Strength Ratio} = \frac{(\tau_f/\sigma_{vo})_{Ck0UC}}{(\tau_f/\sigma_{vo})_{ClUC}}
\]
boundary surface between the critical state line (CSL) and the isotropic consolidation line (ICL) (Atkinson and Bransby, 1978, and Roscoe and Poorooshasb, 1963). Furthermore, ICL, \( K_o \) CL, and CSL are assumed to be three parallel lines with a negative slope of \( \lambda \) in the \( v - \ln p' \) plane, as shown in Fig. 2.3 (Atkinson and Bransby, 1978), resulting in the following equations, respectively:

\[
\begin{align*}
\nu &= N - \lambda \ln p' \\
\nu &= N_o - \lambda \ln p' \\
\nu &= \Gamma - \lambda \ln p'
\end{align*}
\]

where \( N \) = the specific volume \( (\nu = 1 + e) \) of a specimen isotropically (hydrostatically) consolidated under \( p' = 1 \text{kN/m}^2 \). \( N_o \) is the specific volume of a specimen one dimensionally consolidated with \( p' = 1 \text{kN/m}^2 \), and \( \Gamma \) is the specific volume at the critical state when \( p' = 1 \text{kN/m}^2 \).

A new state parameter, called spacing ratio and denoted by \( r \), is introduced herein (Sivakugan et al., 1988a) to represent the relative position of \( K_o \) CL with respect to ICL and CSL. The stress path of a \( K_o \) UC test is AF (Fig. 2.3), and the spacing ratio can be defined as:

\[
r = \frac{N_o - \Gamma}{N - \Gamma} \quad (0 < r < 1)
\]

When this ratio takes its extreme values of 0 or 1, \( K_o \) CL coincides with the CSL or ICL, respectively.

Expressions were derived for spacing ratios for extended Cam clay model and for extended modified Cam clay model in order to
Figure 2.3 ICL, K₀CL, CSL, and CK₀UC Test Path on v - ln p' Plane
consider $K_o$ consolidated soils instead of isotropically consolidated soils. The spacing ratio for both extended models are given in Table 2.3. Sivakugan et al. (1988a) have derived expressions for the normalized shear strength $\tau_f/\sigma_{vo}^*$ using the Cam clay model, modified Cam clay model and the extended models with spacing ratio. The derived expressions are summarized in Table 2.3. For detailed derivations the reader is referred to Sivakugan (1987). The normalized shear strength is a function of the friction angle and the consolidation characteristics of the soil. The expressions are derived assuming that the normally consolidated clay is idealized as an elasto-plastic material exhibiting isotropic strain hardening. Based on constitutive equations proposed by Sekigushi and Ohta (1977), Ohta and Nishihara (1985) developed similar equations for the normalized shear strength but using rheological and dilatancy characteristics of soils.

A typical variation of normalized undrained shear strength with $\phi^*$ predicted by the extended models is shown in Fig. 2.4. The extended modified Cam clay model gives slightly higher values than the extended Cam clay model for all values of $K_o$ and $\Lambda$, however, the differences are small. The normalized undrained shear strength varies approximately linearly with $\phi_{CIUC}^-$. It varies between 0.25 and 0.45 for typical values of $K_o$, $\Lambda$ and $\phi^*$ (Sivakugan et al., 1988a).

For the limited data available in the literature, predictions of $\tau_f/\sigma_{vo}^*$ for $K_o$ consolidated clays were made from the extension of
Table 2.3 Expressions for Spacing Ratio, Normalized Shear Strength and Skempton's A-Parameter at Failure

<table>
<thead>
<tr>
<th>Cam Clay Model</th>
<th>Modified Cam Clay Model</th>
<th>Cam Clay</th>
<th>Modified Cam Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio, $r$</td>
<td></td>
<td>1 - $\frac{\eta_o}{M}$</td>
<td>$\frac{\ln \left( \frac{2M^2}{M^2 + \eta_o^2} \right)}{\ln 2}$</td>
</tr>
<tr>
<td>Normalized Shear Strength $\tau_f/\sigma_{vo}$</td>
<td>$\frac{1}{2} M e^{-\Lambda}$</td>
<td>$\frac{1}{2} M 2^{-\Lambda}$</td>
<td>$\frac{M}{6} (1 + 2 K_o) e^{-r\Lambda}$</td>
</tr>
<tr>
<td>Skempton's Pore-Pressure Parameter $A_f$</td>
<td>$\frac{1}{3} - \frac{1}{M} + \frac{1}{M} e^{\Lambda}$</td>
<td>$\frac{1}{3} - \frac{1}{M} + \frac{1}{M} 2^{\Lambda}$</td>
<td>$\frac{1}{3} + \frac{e^{r\Lambda} - 1}{M - \eta_o e^{r\Lambda}}$</td>
</tr>
</tbody>
</table>

$M = \text{stress ratio } q/p^\prime \text{ at critical state } = \frac{6 \sin \phi}{3 - \sin \phi}$

$\eta_o = \text{stress ratio } q/p^\prime \text{ immediately after } K_o \text{ consolidation } = \frac{q_o}{p_o} = \frac{3(1 - K_o)}{(1 + 2 K_o)}$

$\Lambda = 1 - \frac{C_t}{C_c} = 1 - \frac{k}{\lambda}$
Figure 2.4 Variation of $\frac{\tau_f}{\sigma'_{vo}}$ with $\varphi'$ for $K_0 = 0.6$
the Cam clay and modified Cam clay models. The data and the predicted values are given in Table 2.4. Both extended models show excellent agreement with the experimental results as illustrated in Fig. 2.5.

The Cam clay models as well as the extended models can also be used to predict the pore pressures at failure by evaluating Skempton's A parameter at failure, \( A_f \). This parameter is given by:

\[
A_f = \frac{\Delta u_f}{q_f} = \frac{p_0 + \frac{1}{3} q_f - p_f}{q_f} \tag{2.4}
\]

At failure \( q_f = M p_f \). Therefore,

\[
A_f = \frac{1}{3} - \frac{1}{M} + \frac{1}{M} \frac{p_o}{p_f} \tag{2.5}
\]

The resulting relationships between \( A_f \) and \( M, r_o, r, \) and \( \lambda \), are given in Table 2.3 (see Sivakugan, 1987, for a detailed derivation). A typical variation of \( \phi^\prime, K_o \), the A parameter at failure for \( K_o \) consolidated clays, with \( \phi^\prime \) predicted from the extended models is shown in Fig. 2.6. For low values of \( \lambda \) both models predict about the same \( A_f, K_o \). For higher \( \lambda \), the extended modified Cam clay model gives lower values than the extended Cam clay model. \( A_f, K_o \) decreases with increasing \( \phi^\prime \), decreasing \( \lambda \), and increasing \( K_o \). For typical values of \( \phi^\prime, \lambda \) and \( K_o \), \( A_f, K_o \) varies between 0.9
Table 2.4 Experimental and Predicted Values of $\tau_f/\sigma_{vo}$ and $A_f$

<table>
<thead>
<tr>
<th>No.</th>
<th>Soil</th>
<th>$K_o$</th>
<th>$\phi_{CIUC}$ deg.</th>
<th>$\Lambda$</th>
<th>Expt.1</th>
<th>Predicted</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\tau_f/\sigma_{vo}$</td>
<td>$A_f$</td>
<td>$\tau_f/\sigma_{vo}$</td>
<td>$A_f$</td>
<td>$\tau_f/\sigma_{vo}$</td>
<td>$A_f$</td>
<td>$\tau_f/\sigma_{vo}$</td>
<td>$A_f$</td>
<td>Ref.</td>
</tr>
<tr>
<td>1</td>
<td>Hokkaido Silt 1</td>
<td>0.45</td>
<td>37.2</td>
<td>0.80</td>
<td>0.40</td>
<td>0.58</td>
<td>0.34</td>
<td>1.71</td>
<td>0.35</td>
<td>1.57</td>
<td>49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Hokkaido Silt 2</td>
<td>0.45</td>
<td>35.1</td>
<td>0.84</td>
<td>0.34</td>
<td>2.00</td>
<td>0.32</td>
<td>2.11</td>
<td>0.33</td>
<td>1.94</td>
<td>49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Hokkaido Clay</td>
<td>0.47</td>
<td>36.1</td>
<td>0.89</td>
<td>0.36</td>
<td>1.03</td>
<td>0.30</td>
<td>2.60</td>
<td>0.32</td>
<td>1.89</td>
<td>49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Spestone Kaolinite</td>
<td>0.64</td>
<td>22.6</td>
<td>0.89</td>
<td>0.21</td>
<td>3.60</td>
<td>0.22</td>
<td>3.39</td>
<td>0.23</td>
<td>2.97</td>
<td>62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Kaolinite</td>
<td>0.50</td>
<td>26.9</td>
<td>0.85</td>
<td>0.27</td>
<td>1.03</td>
<td>0.28</td>
<td>3.14</td>
<td>0.28</td>
<td>2.97</td>
<td>77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>K50</td>
<td>0.50</td>
<td>32.0</td>
<td>0.90</td>
<td>0.32</td>
<td>1.05</td>
<td>0.29</td>
<td>2.66</td>
<td>0.30</td>
<td>2.40</td>
<td>77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Weald Clay</td>
<td>0.59</td>
<td>25.9</td>
<td>0.66</td>
<td>0.27</td>
<td>1.80</td>
<td>0.28</td>
<td>1.63</td>
<td>0.28</td>
<td>1.52</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Sapporo Clay</td>
<td>0.52</td>
<td>1.6</td>
<td>0.49</td>
<td>0.30</td>
<td>1.24</td>
<td>0.34</td>
<td>0.98</td>
<td>0.34</td>
<td>0.94</td>
<td>50,51</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 2.5 Predicted vs. Measured Normalized Strength Ratio
Figure 2.6 Variation of $A_{f,k_0}$ with $\phi'$ for $k_0 = 0.6$
and 4.0 (Sivakugan et al., 1988a).

Predictions of $A_f$ for $K_o$ consolidated clays were made for the eight soils given in Table 2.4. The data and results are given in Table 2.4 for both the extended Cam clay model and extended modified Cam clay models. Unlike the excellent agreement obtained for the $\gamma_f/\sigma_{vo}$ ratio, the $A_f$ prediction from both models are inconsistent, being excellent for some tests and poor for others (Fig. 2.7). It seems that $A_f$ is very sensitive to $\Lambda$, which is a function of both $C_c$ and $C_r$, the compression index and the swelling index, respectively. Measurement of $C_c$ is rather straightforward; however, $C_r$ varies with OCR and no standard procedure is generally adopted in measuring $C_r$. This severely influences $\Lambda$ and thus significantly affects the predictions of $A_f$. To avoid the problems in determining $\Lambda$ from $C_r$, Mayne (1980) has proposed to redefine $\Lambda$ in terms of overconsolidated strength data as:

$$\Lambda = \frac{\log \left[ \left( \frac{\tau_f}{\sigma_{vo}} \right)_{OC} \right] - \log \left[ \left( \frac{\tau_f}{\sigma_{vo}} \right)_{NC} \right]}{\log OCR}$$

where $(\tau_f/\sigma_{vo})_{OC}$ and $(\tau_f/\sigma_{vo})_{NC}$ are the normalized shear strength of overconsolidated and normally consolidated clays, respectively. The normalized undrained strength values at two different OCR's are required for this determination. For the data given in Table 2.4, such information was not available, and thus it was not possible to compute $\Lambda$ by this method.

It has also been suggested to determine $\Lambda$ from correlations.
Figure 2.7 Predicted vs. Measured A-parameter at Failure
Few empirical correlations have been cited in the literature relating $M$ with $A$. Schofield and Wroth (1968) proposed that $M/A = 1.5$; Karube (1975) suggested that it is 1.75. However, based on experimental data from literature, Sivakugan and Holtz (1986) showed that there appears to be no correlation between $M$ and $A$. As shown in Fig. 2.8, the rather large scatter makes a linear correlation questionable.

2.2.3 Conclusions

Although $CK_{UC}$ and $CIUC$ tests follow different stress paths, it has been shown that a good estimate of the $CK_{UC}$ strength can be obtained from a single $CIUC$ test. The normalized strength ratio, $\left( \tau_f/\sigma_{vo} \right)_{CK_{UC}}/\left( \tau_f/\sigma_{vo} \right)_{CIUC}$, was found to be a function of earth pressure at rest $K_o$ and Skempton's pore pressure parameters at failure $A_f$ for both tests. $K_o$ may be obtained from published correlations or estimated from Jaky's relationship. The isotropic $A$ parameter $A_{f_i}$ can be obtained from the $CIUC$ test results. Since the strength ratio is not very sensitive to $A_{f,K_o}$, an estimate of $A_{f,K_o}$ is sufficient for the calculation.

Expressions were derived for the normalized shear strength $\tau_f/\sigma_{vo}$ using the Cam clay model, the modified Cam clay model, and the extended Cam clay and modified Cam clay models with spacing ratio. The normalized shear strength is a function of the friction angle and the consolidation characteristics of the soil. Both extended models with spacing ratio predict values that compare well
with experimental data.

Expressions were also derived for estimating Skempton's $A_f$ parameter at failure for isotropically consolidated soils and $K_0$ consolidated soils. Unlike the excellent agreement with experimental results obtained for the strength ratio, the $A_f$ prediction is marginal for all models. $A_f$ is very sensitive to $A$, which is a function of $C_c$ and $C_r$. This severely influences $A$ and thus affects the predictions of $A_f$.

2.3 Normalization of Consolidation Lines

2.3.1 Introduction

A good estimate of compression index, $C_c$, recompression index, $C_r$, and preconsolidation pressure, $p_c$, are prerequisites for settlement analysis of embankments or buildings on cohesive soils. For this, it is common practice to perform consolidation tests on specimens prepared from undisturbed samples taken at various locations within the clay stratum. From the consolidation test results, e-log $
u$ plots are generated, and $C_c$, $C_r$, and $p_c$ are obtained. These steps involve effort and expense, and, for preliminary designs, empirical correlations are often used to estimate these parameters (e.g. Holtz and Kovacs, 1981). $C_c$ has been correlated with natural water content, liquid limit and initial void ratio (e.g. Skempton, 1944, Nishida, 1956, Cozzolino, 1961, Azzouz et al., 1976, and Koppula, 1981). $C_r$ is often assumed to be $0.1$ to $0.2$ times $C_c$. Estimating $p_c$ from correlations remains a problem
yet.

The slope and position of virgin consolidation lines in the $e$-$\log \sigma_v$ space depend primarily on liquid limit (or initial void ratio) and sensitivity of the soil. Therefore, for clays of low to medium sensitivity, the slope and the intercept on the $e$-axis depend mainly on the liquid limit. When $e$-$\log \sigma_v$ lines for several normally consolidated clays are plotted together, a wide range of void ratios may be observed for a given $\sigma_v$, depending upon their respective liquid limits. Based on the Gouy-Chapman diffuse double layer theory, Nagaraj and Srinivasamurthy (1983 and 1986) showed that when $e$ is normalized with respect to the void ratio at the liquid limit, $e_L$, all the lines tend to fall into a narrow band irrespective of the clay mineral type or pore fluid characteristics.

This is confirmed from published consolidation test data for 16 different clays and experimental data from three artificially sedimented clays. The existence of such a normalized relationship facilitates predictions of the overconsolidation ratio (OCR). A simple procedure to predict OCR is proposed. A summary of this study and its conclusions are given below.

2.3.2 Critical State Model

The virgin consolidation line is often assumed to be linear (i.e. constant $C_c$) for settlement calculations. An idealized virgin consolidation line in $e$-$\log \sigma_v$ space is shown in Fig. 2.9. In
Figure 2.9 Idealized Virgin Consolidation Line in $e$-$\log \sigma'_v$ Space
reality, it may be slightly curved as the plastic limit is approached. The equation of the virgin consolidation line is given by:

\[ e = -C_c \log \sigma_v + e_1 \]  

(2.7)

Normalizing Eq. 2.7 by \( e_L - e_p \):

\[ \frac{e}{e_L - e_p} = -\frac{C_c}{e_L - e_p} \log \sigma_v + \frac{e_1}{e_L - e_p} \]  

(2.8)

where \( e_p \) is the void ratio at plastic limit. The liquidity index \( LI \) is defined by:

\[ LI = \frac{w - PL}{LL - PL} \]

\[ = \frac{e - e_p}{e_L - e_p} \]  

(2.9)

From Eqs. 2.8 and 2.9:

\[ LI = -\frac{C_c}{e_L - e_p} \log \sigma_v + \frac{e_1 - e_p}{e_L - e_p} \]  

(2.10)

Thus, if \( e \) versus \( \log \sigma_v \) is a straight line, so is \( LI \) versus \( \log \sigma_v \) and vice versa.

There is experimental evidence that when the liquidity index is plotted against logarithm of undrained shear strength, all the points fall within a narrow band (e.g. Skempton and Northey, 1953, Houston and Mitchell, 1969, Mitchell, 1976). Based on the large number of experimental results given by Skempton and Northey (1953) and Youssef et al. (1965), Schofield and Wroth (1968) adopted a value of 1.7 kN/m\(^2\) as the best estimate of the undrained shear
strength at the liquid limit, and estimated the shear strength at
the plastic limit to be 100 times that at the liquid limit, i.e.,
cu = 170 kN/m² at the plastic limit. The idealized relationship
often used in critical state models (Atkinson and Bransby, 1978,
Wroth and Wood, 1978, and Wroth, 1979) is shown in Fig. 2.10(a).
Using the critical state model and the above LI versus cu relation-
ship, Wroth (1979) developed an idealized relationship between
vertical effective stress and liquidity index which is shown in
Fig. 2.10(b). Based on the experimental evidence Wroth estimated
that σv' = 6.3 kPa at the liquid limit (or LI = 1) and σv' = 630 kPa
at the plastic limit (LI = 0).

2.3.3 Normalized Virgin Consolidation Lines

Normalizing Eq. 2.7 by eL:

\[ \frac{e}{e_L} = \frac{-C}{e_L} \log \sigma_v' + \frac{e_L}{e_L} \]  \hspace{1cm} (2.11)

where e_L, the void ratio at liquid limit, is obtained from:

\[ e_L = \frac{LL}{100} G_s \]  \hspace{1cm} (2.12)

and G_s is the specific gravity of the soil grains.

With the diffuse double layer theory, Nagaraj and
Srinivasamurthy (1953) analytically developed and supported with
experimental data that the half-space distance, d, is linearly
correlated with e/e_L, and linearly correlated with log\sigma_v'. This
aspect was examined further using the e-log\sigma_v' curves of six natural
Figure 2.10 Idealized Relationship Between Liquidity Index and (a) Remolded Shear Strength
(b) Effective Vertical Stress.
soils, the liquid limit water contents of which varied over a four fold range. All six curves fell into a narrow range, and, with a correlation coefficient of 0.953, the best fit straight line was functionally expressed as:

\[ \frac{e}{e_L} = 1.099 - 0.2237 \log_{10} \sigma_v \]  

(2.13)

where \( \sigma_v \) is in kN/m². With additional data for eleven soils, Nagaraj and Srinivasamurthy (1986) further refined the above equation (correlation coefficient of 0.962) and expressed it as:

\[ \frac{e}{e_L} = 1.122 - 0.2343 \log_{10} \sigma_v \]  

(2.14)

This was further confirmed in this study, using the critical state model and the data obtained from Skempton (1944) for 16 clays of different geological origin (Table 2.5). When the wide scattered consolidation lines were normalized by their corresponding \( e_L \)'s, they fell into a narrow range as shown in Fig. 2.11. The equation of the best fit straight line was found as:

\[ \frac{e}{e_L} = 1.084 - 0.2154 \log_{10} \sigma_v \]  

(2.15)

where \( \sigma_v \) is in kN/m².

In addition, conventional consolidation tests were performed on three different artificially sedimented soils, kaolinite, "K50" containing 50% kaolinite and 50% silt, and grundite, an illitic clay. The soil slurries were made to a water content of approximately 2 1/2 times the liquid limit, and were consolidated in a
### Table 2.5 Properties of Skempton's (1944) Clays

<table>
<thead>
<tr>
<th>No.</th>
<th>Soil</th>
<th>LL</th>
<th>PI</th>
<th>$G_s$</th>
<th>$e_1$</th>
<th>$C_c$</th>
<th>$e_L$</th>
<th>$e_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Boulder Clay</td>
<td>28</td>
<td>14</td>
<td>2.69</td>
<td>0.76</td>
<td>0.12</td>
<td>0.75</td>
<td>0.38</td>
</tr>
<tr>
<td>2</td>
<td>Wealden Clay</td>
<td>39</td>
<td>20</td>
<td>2.73</td>
<td>1.25</td>
<td>0.24</td>
<td>1.06</td>
<td>0.52</td>
</tr>
<tr>
<td>3</td>
<td>R. Severn Alluvium</td>
<td>46</td>
<td>21</td>
<td>2.59</td>
<td>1.22</td>
<td>0.21</td>
<td>1.19</td>
<td>0.65</td>
</tr>
<tr>
<td>4</td>
<td>Kaolin</td>
<td>50</td>
<td>18</td>
<td>2.64</td>
<td>1.51</td>
<td>0.23</td>
<td>1.32</td>
<td>0.84</td>
</tr>
<tr>
<td>5</td>
<td>Oxford Clay</td>
<td>53</td>
<td>26</td>
<td>2.57</td>
<td>1.56</td>
<td>0.30</td>
<td>1.36</td>
<td>0.69</td>
</tr>
<tr>
<td>6</td>
<td>Belfast Clay</td>
<td>67</td>
<td>37</td>
<td>2.66</td>
<td>1.64</td>
<td>0.32</td>
<td>1.78</td>
<td>0.80</td>
</tr>
<tr>
<td>7</td>
<td>Ganges Clay</td>
<td>69</td>
<td>41</td>
<td>2.77</td>
<td>2.06</td>
<td>0.42</td>
<td>1.91</td>
<td>0.78</td>
</tr>
<tr>
<td>8</td>
<td>Gosport Clay</td>
<td>75</td>
<td>46</td>
<td>2.67</td>
<td>2.03</td>
<td>0.46</td>
<td>2.00</td>
<td>0.77</td>
</tr>
<tr>
<td>9</td>
<td>London Blue Clay</td>
<td>77</td>
<td>49</td>
<td>2.71</td>
<td>2.26</td>
<td>0.49</td>
<td>2.09</td>
<td>0.76</td>
</tr>
<tr>
<td>10</td>
<td>London Brown Clay</td>
<td>88</td>
<td>56</td>
<td>2.65</td>
<td>2.44</td>
<td>0.56</td>
<td>2.27</td>
<td>0.85</td>
</tr>
<tr>
<td>11</td>
<td>Argille Plastique</td>
<td>128</td>
<td>97</td>
<td>2.58</td>
<td>3.44</td>
<td>0.81</td>
<td>3.30</td>
<td>0.80</td>
</tr>
<tr>
<td>12</td>
<td>Bosporous</td>
<td>36</td>
<td>18</td>
<td>2.71</td>
<td>1.24</td>
<td>0.25</td>
<td>0.98</td>
<td>0.49</td>
</tr>
<tr>
<td>13</td>
<td>Boston Blue Clay</td>
<td>39</td>
<td>16</td>
<td>2.78</td>
<td>1.22</td>
<td>0.21</td>
<td>1.08</td>
<td>0.64</td>
</tr>
<tr>
<td>14</td>
<td>Vienna Miocene</td>
<td>47</td>
<td>25</td>
<td>2.76</td>
<td>1.47</td>
<td>0.30</td>
<td>1.30</td>
<td>0.61</td>
</tr>
<tr>
<td>15</td>
<td>Bosporous Blue Clay</td>
<td>58</td>
<td>32</td>
<td>2.85</td>
<td>1.61</td>
<td>0.32</td>
<td>1.65</td>
<td>0.74</td>
</tr>
<tr>
<td>16</td>
<td>Denmark Marine Clay</td>
<td>127</td>
<td>91</td>
<td>2.77</td>
<td>4.00</td>
<td>0.91</td>
<td>3.52</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Figure 2.11 e versus log $\sigma'_v$ and Normalized (e/e_L) Consolidation Lines (After Skempton, 1944)
consolidation chamber. Details of the slurry preparation method, consolidation chamber etc. are available in Sivakugan (1987) and Sivakugan et al. (1988b). The slurry was consolidated to a vertical pressure of 10 to 20 psi under $K_0$ conditions. At the end of consolidation, a specimen of 63.5 mm in diameter and 25.5 mm in height was trimmed from the consolidated cake for the oedometer test. To ensure full saturation all oedometer tests were carried out under a back pressure of 30 psi. Since pore pressure measurements were not required, drainage was allowed from both top and bottom of the specimen. The index properties and the parameters obtained from the consolidation tests are given in Table 2.6. The $e/e_L$ vs. $\log \sigma_v$ lines resulting from these experiments fall within the range of the other data (Figure 2.12). Eq. 2.14 proposed by Nagaraj and Srinivasamurthy (1986) was essentially the same as the average of the normalized lines, as shown in Fig. 2.12.

2.3.4 Estimation of OCR

Wroth (1979) used the critical state model shown in Fig. 2.10 to predict OCR in offshore deposits, assuming $C_r/C_c = 0.2$. Similar predictions are possible with the $e/e_L - \log \sigma_v$ model. From Eq. 2.15, the "unique" (i.e. average) consolidation line can be plotted in the $e/e_L$ versus $\log \sigma_v$ plot. This is the normally consolidated-virgin compression line with the slope of $C_c$. By estimating the ratio $C_r/C_c$ ratio, the slope of the swelling line is known and it can be drawn through any point (such as $\sigma_v = 1000$ kPa in Fig. 2.13). Then, for OCR values of 2, 5, 10, etc., lines can be drawn
### Table 2.6 Properties of Experimental Clays

<table>
<thead>
<tr>
<th>Soil</th>
<th>LL</th>
<th>PL</th>
<th>$G_s$</th>
<th>$C_c$</th>
<th>$C_r$</th>
<th>$e_1$</th>
<th>$e_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaolinite</td>
<td>63</td>
<td>36</td>
<td>2.65</td>
<td>0.35</td>
<td>0.106</td>
<td>1.93</td>
<td>1.67</td>
</tr>
<tr>
<td>K50</td>
<td>37</td>
<td>23</td>
<td>2.69</td>
<td>0.22</td>
<td>0.022</td>
<td>1.17</td>
<td>1.00</td>
</tr>
<tr>
<td>Grundite</td>
<td>51</td>
<td>29</td>
<td>2.75</td>
<td>0.36</td>
<td>0.060</td>
<td>1.76</td>
<td>1.40</td>
</tr>
</tbody>
</table>
Figure 2.12 Experimental Results on the Normalized Consolidation Plot.

\[
\frac{e}{e_L} = 1.084 - 0.2154 \log_{10} \sigma' \quad \text{(Sivakugan et al., 1988)}
\]

\[
\frac{e}{e_L} = 1.122 - 0.2343 \log_{10} \sigma' \quad \text{(Nagaraj and Srinivasamoorthy, 1986)}
\]
parallel to the NC line, through points with $\sigma_v^*$ of 500, 200, 100 kPa, respectively, on the swelling line. The rationale for this is provided by the critical state model, i.e. $K_0$ consolidated and other over consolidation lines are parallel to the isotropic consolidation line. Prediction charts are given in Figs. 2.13 and 2.14, where the OCR contours, which are straight lines parallel to the virgin consolidation line, are shown for $C_r/C_c$ of 0.1 and 0.2, respectively.

When taking samples from saturated clay deposits, disturbance should not cause a significant change in water content. Therefore, a good prediction of in situ void ratio at any depth is possible. $e_L$ can be obtained from liquid limit test on the remolded soil, and $\sigma_v^*$ can be estimated if the soil profile and the densities of each strata are known. Thus, knowing $e/e^*_L$ and $\sigma_v^*$ at a given depth, the corresponding point can be located in Figures 2.13 or 2.14 to provide an estimate of the OCR. The estimate is not sensitive to the assumed value of the ratio $C_r/C_c$, at least in the range 0.0 - 0.2.

2.3.5 Summary

This review and evaluation of test data confirmed that, when normalized with respect to the void ratio at liquid limit, consolidation lines fall in a narrow band. Based on this, a simple procedure was described to predict OCR of saturated clays using an average normalized virgin consolidation line. Since the OCR estimate is not sensitive to the $C_r/C_c$ ratio, a good OCR prediction is possible from the knowledge of $e/e^*_L$ and $\sigma_v^*$. 
Figure 3.18 OCR Prediction Chart for $\frac{c_r}{c_f} = 0.1$
CHAPTER 3

MODELLING ANISOTROPY OF CLAYS - A NOVEL APPROACH

3.1 Introduction

Experimental studies conducted in the past have demonstrated the significance of anisotropy on shear behavior of natural clays (e.g. Duncan and Seed, 1966, Lewin, 1973, Mitchell, 1972, Saada, 1976, Saada and Bianchini, 1975, Tavenas and Leroueil, 1977, Ting, 1968). Lewin's stress probe experiments (1973) showed differences between lateral and axial strains during isotropic consolidation of an initially anisotropic specimen of clay. Anisotropy can also be induced; Ting (1968) showed that an anisotropically consolidated sample reaches an asymptotic isotropic state at an isotropic stress about 3 times higher than its initial vertical stress. Experimental data (e.g., Tavenas and Leroueil, 1977, Graham et al., 1983) show that the yield loci of natural deposits of clays are centered around the $K_o$ line in triaxial stress space.

These fundamental aspects of soil anisotropy must be taken into account by geotechnical researchers and engineers if major improvements in the interpretation and use of in situ measurements are to be made. A simple model that captures the basic facets of anisotropy and offers closed form solutions for strength and other parameters at failure along various stress paths would be a valuable tool to accomplish this. In addition, the fundamental issues involved in in situ testing can be best investigated, if this model
allows for sophistication without loss of the physical basis of the model.

3.2 Limitations of the Current Thinking.

Since the pioneering work of Roscoe and his co-workers (e.g., Roscoe, Schofield and Thurairajah 1963, Roscoe and Burland 1968), significant improvements have been made in modelling soil behavior. Yet, none of the models existing to date can be used routinely in the interpretation of pressuremeter and especially self-boring pressuremeter (SBPM) tests. Simple models such as the MCC model (as discussed in Chapter 2) suffer from their inability to capture the above features of anisotropy. More sophisticated soil models (e.g., Prevost, 1978, Dafalias and Herrman, 1980, and Kavvadas, 1982) require extensive calibration parameters that would prevent routine use or mask the outcome of the investigations of in situ devices. Some of these models (e.g., Prevost, 1978 and 1979) are limited to the deviatoric stress space and thus cannot be used in situations where pore pressures are involved.

Perhaps, more importantly, a common shortcoming of all these models is that they do not provide a better understanding of the physical basis of the model, such as the work assumption used in the isotropic MCC model. With these shortcomings, one is left with the dissatisfaction that the important aspects such as mode of failure, possibility of radial cracking, effects of disturbance, etc., in SBPM testing can not be studied at a best rational level.
A research program has been undertaken with the aim of addressing the above shortcomings, i.e. of creating a model that captures the anisotropic features of clays and is applicable to SBPM interpretation. Four stages are contemplated for this work:

a. Develop a simple anisotropic theory built on a sound physical basis.

b. Build upon this model to a sophisticated level without loss of generality.

c. Use (a) and/or (b) to interpret SBPM data.

d. Use (b) to study the fundamental issues of SBPM.

Stages (a) and (c) are well underway. An outline of the essential features of the theory and its application to pressuremeter problems are reported in the following; details are given in Thevanayagam (1988a,b).

3.3 New Theoretical Developments

The purpose of this section is to provide the essential features of a plasticity model that captures the behavior of anisotropic clays at the critical state. The emphasis is upon: i) The anisotropic nature of the yield surface; ii) The effect of induced anisotropy; iii) The prediction of the failure parameters of clays, using simple parameters ($\phi_c$ and OCR); and iv) The ability to interpret any measured in situ strength, and transform it to any other
stress path applicable to a given design problem.

Test results published in the literature are used for validation. These includes many natural soils that have been tested in triaxial compression/extension (TC/TE), plane strain compression/extension (PSC/PSE), model pressuremeter (PM) and simple shear (SS).

3.3.1 Background Material: The Axisymmetric Model

The notations are that of critical state soil mechanics (Schofield and Wroth 1968); they are appended to this chapter (Section 3.7). Elastic isotropy is assumed, and elastic shear strains are assumed to exist. The parameters \( \kappa \) and \( \lambda \) refer to the slope of the swelling and virgin \( K_0 \) lines in \( e \) vs. \( \ln p \) plane. All stresses are effective stresses. Compressive stresses and strains are taken positive and tensor notation is used. Due to the mode of deposition, natural clays are generally cross anisotropic; the vertical axis is assumed to be the axis of cross anisotropy.

The original Cam Clay theory (Roscoe, Schofield and Thurairajah, 1963) began with the work assumption:

\[
p \varepsilon^p_v + q \varepsilon^p_q = M p \varepsilon^p_p
\]  

(1)

Considering the continuity requirements of work relation, Eq.1 was modified to the form of the modified cam clay model, MCC, (Roscoe and Burland, 1968):
\[
\frac{p \dot{\varepsilon}_p + q \dot{\varepsilon}_q}{\sqrt{(\dot{\varepsilon}_p)^2 + (M \dot{\varepsilon}_p)^2}} = 1/2
\]

which, assuming the associative flow rule, resulted in an isotropic yield surface of the form:

\[
f_{iso} = p^2 - p\rho_0 + \frac{q^2}{M^2} = 0
\]

Dafalias (1987) introduced a modification to the work assumption:

\[
p \dot{\varepsilon}_p + q \dot{\varepsilon}_q = \sqrt{(\dot{\varepsilon}_p)^2 + (M \dot{\varepsilon}_p)^2 + 2 \alpha \dot{\varepsilon}_p \dot{\varepsilon}_q}
\]

where \(\alpha\) is a nondimensional parameter with an absolute value less than that of \(M\). Dafalias (1987) considered the term \(\alpha\) to account for anisotropic internal residual stresses and coupling of \(\dot{\varepsilon}_p\) and \(\dot{\varepsilon}_q\). However, it can be shown that irrespective of the nature of the soil (whether it is isotropic or not), Eq. 4 is strictly true and has a definite meaning if the following relationship is used for \(\alpha\) (Thevanayagam, 1988a):

\[
2 \alpha = 2 \tau + \left(\tau^2 - M^2\right) \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_q}
\]

The selection of the parameter \(\alpha\) dictates the importance of the coupling term in Eq. 4, and also the shape of the yield locus as deduced from the work equation. If the normality condition is assumed, then \(\alpha\) is related to the shape of the yield locus, \(f\), by (from Eq. 5):
Thus, in general, \( a \) must be a function of \( M, p, q \) and \( r_n \), i.e. \( a = g_1(M,p,q,r_n) \). The assumptions \( a = g_1(r_n) \) (Dafalias, 1987) or \( a = 0 \) (Roscoe and Burland, 1968) are obviously special cases of this more general formulation for \( a \).

When assuming a constant value of \( a \), it is appropriate to relate it to the end of \( K_o \) virgin consolidation of clay from a slurry. At that stage, the following conditions must be satisfied:

\[
\begin{align*}
\frac{\dot{\epsilon}_v}{\epsilon_v} &= 3/2; & \frac{\dot{\epsilon}_p}{\epsilon_p} &= \frac{\dot{\epsilon}_v - \dot{\epsilon}_v}{\epsilon_v}; & \text{and } \frac{\dot{r}_o}{p} &= \frac{3 (1 - K_o)}{(1 + 2 K_o)} \\
\frac{\dot{\epsilon}_q}{\epsilon_q} &= \frac{\dot{\epsilon}_p}{\epsilon_p} - \frac{\dot{\epsilon}_q}{\epsilon_q} \\
\end{align*}
\]

where \( r_o \) refers to the slope of the virgin \( K_o \) line in \( p-q \) plane.

Using the assumption of elastic isotropy and \( e \) vs. \( \ln p \) relationships for the elastic strains, the following relationships exist:

\[
\begin{align*}
\dot{K} &= \frac{\dot{P}}{\kappa} = \frac{(1 + e)p}{\kappa}; & \dot{e}_e &= \dot{e}_p = q/(3G), & \frac{G}{K} = \frac{3 (1 - 2 \nu)}{2 (1 + \nu)} \\
\end{align*}
\]

where \( K, G \) and \( \nu \) are the elastic bulk modulus, shear modulus and Poisson's ratio of the clay, respectively. Using Eqs. 5 and 7-8, \( a \) at the end of \( K_o \) virgin consolidation, \( a_o \), is given by:

\[
a_o = \frac{3 a_1 r_o + \eta_o^2 - M^2}{3 a_1}; \text{with } a_1 = \frac{1 - \kappa/\lambda}{1 - \left[ \frac{\kappa \eta_o (1+\nu)}{3 \lambda (1 - 2\nu)} \right]} (9a)
\]

If the elastic shear strain is neglected in Eq.7, \( a_1 = 1 - \kappa/\lambda \), and \( a_o \)
reduces to:

\[ \alpha_0 = \frac{3(1-\kappa/\lambda)\eta_o + \eta_i^2 - M^2}{3(1-\kappa/\lambda)} \]  

(9b)

If the clay is isotropically consolidated (\(\varepsilon = \varepsilon^P = 0\)), \(\alpha_0\) becomes zero. It may be noted that no assumption on the shape of the yield surface or on the flow rule is necessary to derive \(\alpha_0\).

Assuming a constant value of \(\alpha\), i.e. \(\alpha = \alpha_0\), the shape of the yield locus function is deduced from the work equation as:

\[ \frac{p}{\eta_i} = \frac{M^2 + \eta_i^2 - 2\alpha \eta_i}{M^2 + \eta_i^2 - 2\alpha \eta_i} \]  

(10a)

where \(p\) and \(\eta_i\) correspond to any two positions on the yield surface, \(f = 0\) (Fig. 1a). However, \(M\) takes the value \(M_c\) (compression side) for points located above point A in Figure 1a, and \(M_e\) (extension side) for points below A. Point A corresponds to the condition of zero increment in plastic shear strain, i.e. \(\alpha f/\alpha q = 0\). At that point, the value of \(\eta_i\) is equal to \(\alpha\) (from Eq. 6 with \(\alpha f/\alpha q = 0\)). The equations for the state boundary surface can be derived from Eq. 10a as:

\[ (\eta_i - \alpha)^2 = -(M - \alpha)^2 + \frac{(M^2 - \alpha^2)}{(M_c^2 - \alpha^2)} \left| \eta_i - \alpha \right|^2 + \frac{(M^2 - \alpha^2)}{(M_c^2 - \alpha^2)} \left| \eta_i - \alpha \right|^2 \exp \left| \frac{N_o - \nu \lambda}{\lambda - \kappa} \right| \]  

(10b)

\[ \left( \frac{\eta_i - \alpha}{p_i} \right)^2 = -(M - \alpha)^2 \left| \frac{p}{p_i} \right|^2 + \frac{(M^2 - \alpha^2)}{(M_c^2 - \alpha^2)} \left| \eta_i - \alpha \right|^2 + \frac{(M^2 - \alpha^2)}{(M_c^2 - \alpha^2)} \left| \eta_i - \alpha \right|^2 \exp \left| \frac{p}{p_i} \right| \frac{\lambda - \kappa}{\lambda - \kappa} \]  

(10c)

The parameters \(p_i\), \(N_o\), and \(\nu \lambda\) are defined as shown in Fig. 1b and in
Fig. 1a. A Typical Anisotropic Yield Surface in the Triaxial p-q Space
Fig. 1b. Critical State Parameters
Section 3.7.

Equations 10b and 10c are plotted in Figs. 2 and 3 for kaolin using the following parameters: $N_o=3.23$, $\lambda=0.19$, $\kappa=0.06$, $M_C=0.85$, $M_e=-0.85$, and $r_o=0.43$. The state boundary surface predicted from these equations are compared to experimental data on stress paths obtained by Atkinson et al. (1987). This comparison shows that the model with a constant $\alpha$ captures the main features of the state boundary surface with some deviations. The first deviation occurs in the initial part of the triaxial extension (TE) stress path, as this part of the TE stress path lies inside the state boundary surface; however, this is not completely unexpected, at least in the initial part of the stress path, as the initial part of the TE stress path remains within the yield locus and thus must be elastic (Thevanayagam and Prapaharan, 1988). Therefore, this initial part of the TE stress path is expected to lie inside the state boundary surface. The stress path in triaxial compression (TC) closely follows the state boundary surface up to the peak deviatoric stress and then the most significant deviation occurs beyond this point (clearly illustrated in Figs. 2 and 3). These deviations at large strains are attributed to the effects of induced anisotropy.

The comparison between predictions and experimental data in Figs. 2 and 3 demonstrates that the proposed model provides a good representation of the material behavior; however, if the post peak behavior in TC and large strain behavior in TE are of interest then the induced anisotropy should be accounted for in the model. In
Fig. 3. Normalized Stress Paths and State Boundary Surface (Compression: Constant \( p' \Delta \); Extension: Undrained Triaxial \( O \); Constant \( p' + \); Measured - after Atkinson et al., 1947)
Fig. 3 Normalized Stress Paths and State Boundary Surface (Compression: Constant l' Δ; Extension: Undrained Triaxial (○; Constant p' +)): Measured after Atkinson et al., 1987)
what follows, the effects of induced anisotropy are introduced by considering that $\alpha$ does not remain constant, but varies during shearing proportionally to the initial value $\alpha_0$. The other assumptions of the model, related to the other evolution characteristics of the yield surface $f$, are: (i) the projection of the yield surface on the $e$ vs. $\ln p$ plane has a constant slope $\kappa$, and (ii) the hardening is such that point $E$ (i.e. $p_E$ in Fig. 1a) projected on the $e$ vs. $\ln p$ plane remains on the $K_{o NC}$ line, independent of plastic loading.

The expressions for undrained strength ratio in TC and TE are developed first. Assuming that the value of $\alpha$ is given by $\alpha = \gamma_0$ (i.e. proportional to $\alpha_0$) at failure, the values of $\gamma$ in TC and TE are obtained by calibrating the theoretical expressions for strength ratio using experimental data. This is followed by the development of a 3-D model. The results of the axisymmetric and 3-D models are used to obtain closed form solutions for failure parameters of clays. Predictability is demonstrated by comparison with experimental data. Subsequently the application of this model to the pressuremeter will be illustrated.

3.3.2 Undrained Shear in Triaxial Compression

Referring to Table 1, the undrained strength in TC is given by:
<table>
<thead>
<tr>
<th>Use Eqns. Eq.</th>
<th>TC, Fig. 20a</th>
<th>TE, Fig. 20b</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (10) )</td>
<td>( A1 )</td>
<td>( A1-A4 )</td>
</tr>
<tr>
<td>1- ( \frac{p_o}{p_a} = \frac{M^2 - \alpha^2}{M^2 + \eta_o^2 - 2\eta_0} )</td>
<td>( \frac{p_f}{p_a} = \frac{M^2 - \alpha^2}{M^2 - 2M_e} )</td>
<td>( \frac{p_o}{p_a} = \frac{M^2 - \alpha^2}{M^2 + \eta_o^2 - 2\eta_0} )</td>
</tr>
<tr>
<td>( \frac{p_f}{p_a} = \frac{M^2 - \alpha^2}{M^2 - 2M_e} )</td>
<td>( \frac{p_f}{p_a} = \frac{M^2 - \alpha^2}{M^2 - 2M_e} )</td>
<td></td>
</tr>
<tr>
<td>( q_f = Np_f )</td>
<td>( A2 )</td>
<td>( \frac{q_f}{2} = \frac{Mc^2}{2} )</td>
</tr>
<tr>
<td>( \frac{\tau_f}{2} = \frac{Mc^2}{2} )</td>
<td>( \frac{\tau_f}{2} = \frac{Me^2}{2} )</td>
<td></td>
</tr>
<tr>
<td>( e ) vs. ( \ln(p) )</td>
<td>( \dot{e}_o = e_f - \kappa \ln \left( \frac{p_o}{p_f} \right) )</td>
<td>( \dot{e}_o = e_f - \kappa \ln \left( \frac{p_o}{p_f} \right) )</td>
</tr>
<tr>
<td>( \dot{e}<em>o = e_f - \kappa \ln \left( \frac{p</em>{\text{max}}}{p_{oc}} \right) - \lambda \ln \left( \frac{p_o}{p_{\text{max}}} \right) )</td>
<td>( \dot{e}<em>o = e_f - \kappa \ln \left( \frac{p</em>{\text{max}}}{p_{oc}} \right) - \lambda \ln \left( \frac{p_o}{p_{\text{max}}} \right) )</td>
<td></td>
</tr>
<tr>
<td>( A1-A4 )</td>
<td>( A5 )</td>
<td>( A2, A5 )</td>
</tr>
<tr>
<td>( \frac{(\tau_f)}{\sigma_{vo}} )_{TC}^A = \frac{M_c}{6} \cdot \left( 1 + 2K_o, nc \right) \cdot \frac{M^2 + \eta_o^2 - 2\eta_0}{2M^2 - 2M_e} )</td>
<td>( \frac{\tau_f}{\sigma_{vo}} )_{TE}^A = \frac{M_e}{6} \cdot \left( 1 + 2K_o, nc \right) \cdot \frac{M^2 + \eta_o^2 - 2\eta_0}{2M^2 - 2M_e} )</td>
<td>( \frac{1 - \frac{6}{M_c \left( 1 + 2K_{oc} \right)}(\frac{\tau_f}{\sigma_{vo}})<em>{TC}}{\eta</em>{oc} - \left( 1 + 2K_{oc} \right)(\frac{\tau_f}{\sigma_{vo}})_{TC}} )</td>
</tr>
<tr>
<td>( \frac{1 - \frac{6}{M_e \left( 1 + 2K_{oc} \right)}(\frac{\tau_f}{\sigma_{vo}})<em>{TE}}{\eta</em>{oc} - \left( 1 + 2K_{oc} \right)(\frac{\tau_f}{\sigma_{vo}})_{TE}} )</td>
<td>( \frac{1 - \frac{6}{M_e \left( 1 + 2K_{oc} \right)}(\frac{\tau_f}{\sigma_{vo}})<em>{TE}}{\eta</em>{oc} - \left( 1 + 2K_{oc} \right)(\frac{\tau_f}{\sigma_{vo}})_{TE}} )</td>
<td></td>
</tr>
</tbody>
</table>

Note: \( (\frac{\tau_f}{\sigma})_{TE} \) is negative.
\[
\left( \frac{\tau_f}{\sigma_{vo}} \right)_{TC} = \left( \frac{\tau_f}{\sigma_{vo}} \right)_{nc} \frac{1+2K_o, nc}{1+2K_o, nc} \frac{K}{\lambda} \left| \frac{\sigma_{OCR}}{\sigma_{vo}} \right|^\lambda; \lambda = 1 - \frac{\kappa}{\lambda}
\]

(11a)

where
\[
\left( \frac{\tau_f}{\sigma_{vo}} \right)_{nc} = \frac{1+2K_o, nc}{6} M_c D_1; D_1 = \frac{M_c^2 + r_o^2 - 2a r_o}{2M_c^2 - 2a M_c}
\]

(11b)

\(\sigma_{vo}\) is the initial vertical stress before shearing, \(\alpha\) represents the anisotropic state at failure, and \(nc\) and \(oc\) stands for normally consolidated clay and overconsolidated clay, respectively. The shear strength \(\tau_f\) is as defined in Table 1. If \(\alpha\) is set to zero, Eq.11b reduces to the strength expressions for \(CK_o\) and \(CIUC\) (when \(r_o = 0, for K = 1\)) strength of the MCC model (Wroth 1984, also see Chapter 2 of this report). If desired, simplifications to Eq.11 can be made as follows.

The relationship between the slope of the virgin consolidation line and swelling line in \(e\) vs \(\ln p\) plane (denoted by \(\lambda\) and \(k\)) and those in \(e\) vs \(\ln \sigma_v\) plane (denoted by \(\lambda_v\) and \(k_v\)) are given by (Thevanayagam, 1988a):

\[
\lambda = \lambda_v \quad \text{and} \quad \kappa = \frac{\ln(OCR)}{\left( \frac{1+2K_o, nc}{OCR} \right) \left| \frac{\sigma_{OCR}}{\sigma_{vo}} \right|^\lambda} \frac{K_v}{\lambda}
\]

(12)

Using typical \(K_o\)-OCR relationships (e.g. Brooker and Ireland, 1965) for clays, Eq.12 reduces to:
\[
\kappa = 1.3 \kappa_v 
\]

Thus, \( \Lambda \) in Eq. 11.b can be written as:

\[
\Lambda = 1 - \frac{1.3 \kappa_v}{\lambda} 
\]

Furthermore, substituting Eq. 13 in Eq. 11, \( \frac{\tau_f}{\sigma_{vo}} \) can be expressed as:

\[
\frac{\tau_f}{\sigma_{vo}}_{TC} = D_2 \left( \frac{\tau_f}{\sigma_{vo}} \right)_{nc} \Lambda_v \text{OCR} 
\]

with

\[
D_2 = \frac{1 + 2 K_{o,oc}}{1 + 2 K_{o,nc}} \frac{1.3 \kappa_v}{\lambda} 
\]

and

\[
\Lambda_v = 1 - \frac{\kappa_v}{\lambda} 
\]

For typical values of soil parameters, \( \kappa_v/\lambda \approx 0.1 - 0.3 \) and typical \( K_o \)-OCR relationships (e.g., Brooker and Ireland, 1965), \( D_2 \) is close to 1.0, reducing Eq. 15 to:

\[
\frac{\tau_f}{\sigma_{vo}}_{TC} = \left( \frac{\tau_f}{\sigma_{vo}} \right)_{nc} \text{OCR} \Lambda_v 
\]

The form of Eq. 16 is similar to the empirical relationship proposed by Ladd, et al. (1977) and the theoretical relationship for isotropic clays (Wroth 1984).
3.3.3 Undrained Shear in Triaxial Extension

Similarly, the undrained shear strength in TE is given by (see Table 1):

\[
\frac{\tau_f}{\sigma_{vo}}\text{TE} = \frac{M_c}{M_c - \alpha} \left( \frac{\tau_f}{\sigma_{vo}}\text{TC} \right) \frac{M_c - \alpha}{M_c + \alpha} (1 - \frac{1.3 \kappa \nu}{\lambda})
\]

where \(\frac{\tau_f}{\sigma_{vo}}\text{TC}\) is given by Eqs. 11 or 16; in this case, \(\alpha\) represents the anisotropic state at the critical state after shearing in TE. Note that the value of \(M_e\) is negative. Classically, critical state parameters \(M_c\) and \(M_e\) are obtained using the expressions (Roscoe and Burland 1968, Dafalias 1987): \(M_c = \frac{6\sin\phi}{3 - \sin\phi}\) and \(M_e = -\frac{6\sin\phi}{3 + \sin\phi}\), where the \(\phi\) angle is the angle of friction at failure obtained from TC (\(\sin \phi = (\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)\) at \(\sigma_1 - \sigma_3\) max). However, this expression is valid only for truly isotropic clays. Existing data (e.g. Saada et al., 1975) show that the friction angle in TE is generally greater than that in TC. In this report, using the failure criteria developed later in Eq. 23b, a better relationship was obtained for \(\phi_c\) and \(\phi_e\) (demonstrated in Eq. 23c), and comparison with experimental data showed very good agreement. Using this information, Eq. 17 reduces to a simple expression (absolute value of \(\tau_f\) in TE is taken):

\[
\frac{\tau_f}{\sigma_{vo}}\text{TE} = \frac{M_c - \alpha}{M_c + \alpha} (1 - \frac{1.3 \kappa \nu}{\lambda}) \frac{\tau_f}{\sigma_{vo}}\text{TC}
\]
3.3.4 Effect of Induced Anisotropy and Variations in Soil Parameters

The effects of variability on soil parameters \( v \) and \( (k_v/\lambda) \) on the expressions for strength ratio in TC and TE were studied. Using the experimental data on many clays the values of \( \gamma \) in TC and TE were calibrated. The following three cases were considered:

a. \( \alpha = 0 \) (i.e. isotropic MCC model);

b. Initial value of \( \alpha, \alpha_0 \), given by Eq. 9b and \( \alpha \) at critical state given by \( \alpha = \gamma \alpha_0 \);

c. Case (b), but using Eq. 9a for \( \alpha_0 \).

Case (a) when compared to cases (b) and (c) is used to study whether the initial anisotropy is an important factor in the undrained strength ratio of clays. Cases (b) and (c) were chosen to study the influence of Poisson's ratio on the strength ratio. Using these cases, the values of \( \gamma \) for TC and TE were calibrated from experimental data on peak strength.

A reasonable variation of the ratio \( k_v/\lambda \) in the range 0.1-0.3 showed that the above relationships were fairly insensitive to this ratio, and most calculations were thus made with \( k_v/\lambda = 0.20 \). For the TC strength (Eqs. 11-16), case (c) was virtually unaffected by any variation in \( v \), and the results were very similar to that of case (b). Based on regression analysis, case (b) was found to be the best, thus indicating that Eq. 9b is appropriate for \( \alpha_0 \). The single most influential parameter was \( \phi_c \). The comparison of the
model prediction with experimental data at peak strength in TC showed that \( a \) is not a very sensitive parameter in TC, and thus, as a first approximation, \( \gamma = 1.0 \) is a reasonable assumption in TC. The predictability of residual strength of clays was not studied at this stage; it is expected that \( \gamma = 1.0 \) will not be a satisfactory assumption if the strength at very large strains is of interest. Also the rate effects were not considered in the process of calibration.

For TE, the influence of induced anisotropy was significant. A value of \( \gamma = 0.6 \) was found to be most appropriate for the data evaluated, resulting in Fig.4. However, since the term \( \left( \frac{\tau_f}{\sigma_{vo}} \right)_{TC} \) in Eq.18 is not very sensitive to \( a \), for simplicity in expressing the strength ratio in terms of that in TC, it can be sufficient to substitute \( a = \gamma a_o \) only in the first term of Eq.18. For highly overconsolidated clays, since most of the stress path lies inside the yield locus, the induced anisotropy may be expected to be small, i.e. \( \gamma \) may tend towards 1.0. Further simplifications can be made to Eq.18 as follows; Using a typical value of \( k_v/\lambda = 0.2 \), taking a Taylor series expansion, and neglecting higher order terms Eq.18 reduces to:

\[
\frac{\tau_f}{\sigma_{vo}}_{TE} \approx \left( \frac{c}{M + \alpha} \right) \frac{\tau_f}{\sigma_{vo}}_{TC} \tag{19}
\]

where the relationships:

\[
a = \gamma a_o \quad \text{and} \quad \gamma = 0.6 \tag{20}
\]
Fig. 4 Predicted Versus Experimental Undrained Strength

\[
\left( \frac{T_F}{\sigma_i} \right)_{\text{Predicted}} \quad \left( \frac{T_E}{\sigma_i} \right)_{\text{Measured}}
\]

Ratio \( \left( \frac{T_F}{\sigma_i} \right)_{\text{TE}} \) (Numbers Refer to Cases in Table 8)
appear applicable to many clays.

Using Eqs. 11 to 19 and \( q_f = M_{pf} \) at failure, the stresses and shear induced pore pressure parameters at failure can be derived for both TC and TE (Table 2). Table 1 contains the expression for Skempton's A-parameter as well.

The determination of failure parameters for modes of failure other than TC and TE is presented in the next sections. Two important aspects are discussed: (i) development of the necessary theoretical relationships required to obtain failure parameters in any mode of failure; and (ii) rationale for failure criteria of an anisotropic material, and physical explanation for the forms chosen in this development.

3.3.5 General 3-D Behavior of Clays at Critical State

Let the failure criterion for a clay, given by \( f^f = 0 \), satisfy the critical state condition, and be dependent on the initial state of the clay. For any arbitrary point \( A \) on this failure criterion, the corresponding state of the yield surface is given by \( f^y_A \). No point can lie outside the failure criterion, and no intersection of yield and failure criteria is possible except tangentially. Using the associative flow rule, the gradient of \( f^y_A \) at \( A \) is perpendicular to the hydrostatic stress axis, as no plastic volumetric strain takes place at critical state. Since \( f^y_A \) and \( f^f \) are in contact tangentially at \( A \), it follows that the gradient of \( f^f_d = 0 \) is parallel to the gradient of \( f^y_A \) at \( A \), where \( f^f_d = 0 \) is the
<table>
<thead>
<tr>
<th>MODE AT FAILURE</th>
<th>IC</th>
<th>T</th>
<th>ETO</th>
<th>EM</th>
<th>S-X</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td>STATE OF STRAIN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
| $i_j = 0; i 
eq j$ | $\left( \sigma_{\max} - \sigma_{\min} \right)$ | $\left( \frac{\sigma_{\max} - \sigma_{\min}}{2} \right)$ | $\left( \frac{\sigma_{\max} - \sigma_{\min}}{2} \right)$ | $\left( \frac{\sigma_{\max} - \sigma_{\min}}{2} \right)$ | $\left( \frac{\sigma_{\max} - \sigma_{\min}}{2} \right)$ | $\left( \frac{\sigma_{\max} - \sigma_{\min}}{2} \right)$ |
| $\frac{\tau_{f}}{\tau_{f c}}$ | 1.0 | $(\frac{M_c - 0.5a}{M_c + a})$ | $(\frac{1.15M_c + 1.04a}{M_c + a})$ | $(\frac{1.15M_c - 0.46a}{M_c + a})$ | $\frac{1.5a}{M_c + a}$ | $(\frac{2M_c + 0.5a}{\sqrt{3}(M_c + a)})$ |
| $\frac{\sigma_{1}}{\sigma_{0}}$ | $\frac{5M_c + 6}{3M_c}$ | $(\frac{\tau_f}{\sigma_{0}c})$ | $(\frac{6-6M_c}{3M_c})$ | $\left[ \frac{\frac{X_{c}}{M_{c}^2}}{M_{c}^2} + 1.15M_{c} \right]$ | $\left[ \frac{\frac{X_{c}}{M_{c}^2}}{M_{c}^2} + 0.71a - 1.15M_{c} \right]$ | $\frac{\sqrt{3x} + 2.25}{M}$ | $\frac{x}{0.5}$ $\frac{a}{M_c + a} \left( \frac{\tau_f}{\sigma_{0}c} \right)$ |
| $\frac{\sigma_{2}}{\sigma_{0}}$ | $\frac{6-2M_c}{M_c}$ | $\left( \frac{\tau_f}{\sigma_{0}c} \right)$ | $\left( \frac{6+2M_c}{M_c} \right)$ | $\left( \frac{X_{c}}{M_{c}^2} - 0.5a \right)$ | $\left( \frac{X_{c}}{M_{c}^2} - 0.5a \right)$ | $\frac{\sqrt{3x} + 2.25}{M} + 1$ | $\frac{a}{M_c + a} \left( \frac{\tau_f}{\sigma_{0}c} \right)$ |
| \( \frac{3}{c} \) |
| \( \frac{2}{c} \) |
| \( \frac{\tau}{v_0} \) |
| \( \frac{1}{M + \alpha} \) |
| \( \frac{\alpha}{M + \alpha} \) |
| \( \frac{C}{3M^2 + 2.25} \) |

| \( \sin(\theta) \) |
| \( \frac{2M_c}{6+M_c} \) |
| \( \frac{2.31M + 2.08\alpha}{\frac{\tau}{M_c^2} + 0.2} \) |
| \( \frac{2.31M - 0.92\alpha}{\frac{\tau}{M_c^2} + 0.5} \) |
| \( \frac{2x}{\frac{2}{3}M^2 + 2.25} \) |

| \( \frac{\Delta l_{sh}}{v_0} \) |
| \( \frac{\tau f}{\tau f} \) |
| \( \frac{1+2K_{o,ocr}}{3} \) |
| \( \frac{1+2K_{o,ocr}}{3} \) |
| \( \frac{1+2K_{o,ocr}}{3} \) |
| \( \frac{1+2K_{o,ocr}}{3} \) |

| \( \frac{x}{\sqrt{3}} \) |
| \( \frac{(7.9M_c^2 + 14.35M_c a + 7.61a^2)^{1/2}}{2} \) |
| \( \frac{(7.9M_c^2 - 6.35M_c a + 2.39a^2)^{1/2}}{2} \) |
| \( \frac{2M_c + 0.5a}{\sqrt{3}a} \) |
deviatoric component of the failure criterion, obtained by projecting \( f^f \) at a given initial state onto the deviatoric plane in stress space. Since \( f^f \) is dependent on the initial state of the clay, the function \( f^f_d \) is also dependent on the initial state. The general form of \( f^f \) in 3-D stress space may be obtained by the evolution of the intersection of \( f^f_d \) and another, non-deviatoric surface, \( f^f_p = 0 \). In this development \( f^f_p \) is assumed to be independent of the initial state of the clay. The resulting failure surface \( f^f \) is given by:

\[
f^f = f^f_d + f^f_p = 0, \quad \text{with,} \quad f^f_d = 0, \quad \text{and} \quad f^f_p = 0 \quad \text{at failure.} \quad (21)
\]

The strain increment upon loading at failure (i.e. at critical state) is given by:

\[
\varepsilon_{ij} = \beta \frac{\partial f^f_d}{\partial \sigma_{ij}} \quad (22)
\]

where \( \beta \) is a proportionality constant. In undrained shearing only 5 of the 6 components of strain are independent. Therefore, Eq. 22 yields only five independent equations. The functions \( f^f_d = 0 \) and \( f^f_p = 0 \) offer two more independent equations. The unknowns are the failure stresses (6 components) and the proportionality constant \( \beta \). Therefore with the knowledge of the strain increment at failure \( \varepsilon_{ij} \), and Eqs. 21 and 22, the failure stresses \( \sigma_{ij} \) can be solved for in closed form, i.e. the strain path and Eqs. 21 and 22 lead to the solution for failure stresses.
The choices made for $f_d^f$ and $f_p^f$ and their physical meanings are as follows. The form of $f_d^f$ is given by a general Mises criteria type:

$$f_d^f = \frac{3}{2} (S_{ij} - a_{ij}) (S_{ij} - a_{ij}) - k^2 = 0 \quad (23a)$$

with $S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$

where $\delta_{ij}$ is the Kronecker delta, $\sigma_{ij}$ the stress tensor, $S_{ij}$ the deviatoric stress tensor, and $a_{ij}$ represents the anisotropic state of the material at failure, with $k$ and $a_{ij}$ dependent on the initial state of the material only. If the clay is initially cross-anisotropic (e.g. natural 1-D consolidated clays) the axes of stresses and strains are chosen to coincide with the axes of anisotropy (Fig.5). If a different coordinate system is chosen, the results of this study should be used with appropriate direction cosines (as later indicated by Eq. 25). For cross anisotropic clays, $a_{33} = a_{22} = -a_{11}/2$, and $a_{ij} = 0$ for $i \neq j$.

The form for $f_p^f$ is given by an extended von Mises type:

$$f_p^f = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 - \frac{2H^2}{9} (\sigma_1 + \sigma_2 + \sigma_3)^2 = 0 \quad (23b)$$

where $\sigma_1, \sigma_2$ and $\sigma_3$ are principal stresses and $H$ is a constant. The failure criteria given in Eqs. 23a and 23b are interpretable within the context of mechanics. The terms $a_{ij}$ can be considered to represent the deviatoric component of residual stresses in the clay, that is locked-in stresses indicative of the anisotropic
Fig. 5 Axis System for Different Tests

- Triaxial Comp. (c)
- Triaxial Ext. (E)
- Conventional Simple Shear
nature of the soil. The deviatoric stresses internally available in the soil are given by $(S_{ij} - a_{ij})$. From an energy standpoint, Eq. 23a can be interpreted in terms of the shear strain energy internally stored in the soil, while Eq. 23b shows that the strain energy externally available to do work is dependent on the mean stress. Taken together, those equations imply that the mean and deviatoric stresses at failure are dependent on the orientation of the failure plane.

The general 3-D failure criteria, Eqs. 23a-b, should satisfy some specific conditions: (i) $q_f = M\, p_f$ in TC and TE; and (ii) the results obtained using the axisymmetric model given earlier. These conditions can be used to solve for the model parameters $a_{ij}$, $k$, and $H$. The first condition implies that $H^2 = M^2$, and it can be used to obtain the relationship between $\phi_c$ and $\phi_e$:

$$
\sin\phi_e = \frac{\sin\phi_c}{1 - \frac{2}{3} \sin\phi_c} \quad (23c)
$$

The predictions made with Eq. 23c are given in Fig. 6 (dashed line) for the experimental data used earlier. This agreement between predicted and experimental values is very good. For comparison, the prediction made using the work by Lade and Mussante (1977 and 1978), and Matsuoka and Nakai (1982) are also given in Fig. 6. For a given $\phi_c$, these relationships tend to underpredict the value of $\phi_e$. The results in Fig. 6 indicate that, based upon the TC and TE failure modes, the proposed criteria are satisfac-
Fig. 6 Proposed Relationship Between $\phi_e$ and $\phi_c$ for 1-D Consolidated Clays (Numbers Refer to Cases in Table 8)
tory. However, alternate forms could be explored based upon data for other modes of failure.

Using the second condition that the failure criteria must satisfy, i.e. the axisymmetric results in TC and TE, the two unknowns $a_{11}$ and $k$ in Eq. 23a can be solved for (Thevanayagam, 1988a):

$$k = \frac{2M + 0.5\alpha}{M + \alpha} \frac{\tau_f}{\sigma_{vo}}_{TC}$$  \hspace{1cm} (24a)

and

$$\frac{a_{11}}{\sigma_{vo}} = \frac{\alpha}{M + \alpha} \frac{\tau_f}{\sigma_{vo}}_{TC}$$ \hspace{1cm} (24b)

where $(\tau_f/\sigma_{vo})_{TC}$ is given by Eqs. 11 or 16.

3.4 Summary of the New Model.

As indicated earlier, with the knowledge of strain increments at failure, Eqs. 22 to 23a-b gives seven independent equations. There are 7 unknowns ($\sigma_{ij}$'s and $\beta$). Using OCR, $M_c = \frac{6 \sin \phi_c}{3 - \sin \phi_c}$, Eqs. 9b, 11 or 16, 24, $a = \gamma \alpha$, and $\sigma_{vo}$, all the model parameters can be determined. Consequently, using Eq. 22 and 23a-b, the failure stresses in any mode of failure can be solved for in closed form. As noted earlier, if a coordinate system different from that shown in Fig. 5 is chosen, appropriate direction cosines must be applied to Eqs. 22 and 23a-b. For example, if the strain increments at failure are known in a new coordinate system different
from that in Fig. 5, Eq. 22 should be modified as follows:

\[ \varepsilon_{mn} = \beta l_{im} l_{jn} \frac{\partial f}{\partial \sigma_{ij}} \]  

(25)

where \( \sigma_{ij} \) are stresses in the coordinate system shown in Fig. 5, \( l_{ij} \) are the direction cosines, i.e., cosine of the angle between axis \( i \) in the system shown in Fig. 5 and axis \( j \) in the new system, and \( \varepsilon_{ij} \) is the strain increment at failure in the new coordinate system.

Using Eqs. 15, and 21-24, the strength relationships for some of the most commonly encountered modes of failure are given by (see Table 2):

\[
\left( \frac{\tau_f}{\sigma_{vo}} \right)_{SS} = \frac{1.5 \alpha}{M_c + \alpha} \left( \frac{\tau_f}{\sigma_{vo}} \right)_{TC} 
\]  

(26a)

\[
\left( \frac{\tau_h}{\sigma_{vo}} \right)_{SS} = 0.9 \frac{2}{\sqrt[3]{3}} \frac{M_c + 0.5 \alpha}{(M_c + \alpha)} \left( \frac{\tau_f}{\sigma_{vo}} \right)_{TC} 
\]  

(26b)

\[
\left( \frac{\tau_f}{\sigma_{vo}} \right)_{PSC} = \frac{2}{\sqrt[3]{3}} \frac{M_c + 1.04 \alpha}{M_c + \alpha} \left( \frac{\tau_f}{\sigma_{vo}} \right)_{TC} 
\]  

(27)

\[
\left( \frac{\tau_f}{\sigma_{vo}} \right)_{PSE} = \frac{2}{\sqrt[3]{3}} \frac{M_c - 0.46 \alpha}{M_c + \alpha} \left( \frac{\tau_f}{\sigma_{vo}} \right)_{TC} 
\]  

(28)

with \( \alpha = \gamma \sigma_o \).
The factor 0.9 in the above (Eq. 26b) was introduced to take into account the non-uniformity in stress distribution in simple shear (SS) tests conducted in the laboratory, which was found to reduce the strength of the clay by about 10% (Prevost and Hoeg 1976).

Additional information on stresses, friction angles, and shear induced pore pressure parameters at failure are given in Table 2 for various stress paths. Eqs. 21-23 can be used to generate the same information for any other mode of failure that is of interest. Invoking Jaky's relationship (1948) for $K_o\left(K_o = 1 - \sin \phi_c \right)$, the relationships for friction angle and undrained shear strength in various modes of failure can be further simplified. Figs. 7 and 8 show these simplified relationships.

From simple shear test data on natural clays, Ladd et al. (1977) showed that the strength ratio of overconsolidated clays can be related to that in NC clays and OCR by:

$$
\frac{\tau_f}{\sigma_{vo}}_{oc} = \left(\frac{\tau_f}{\sigma_{vo}}\right)_{nc} \cdot OCR^m
$$

(29)

where $m$ is a constant, approximately equal to 0.80. For isotropic clays, Wroth (1984) showed a similar theoretical expression for strength ratio in TC. Using the theoretical development reported herein, it is possible to show a general theoretical relationship for many modes of failure for 1-D consolidated clays. Recalling Eqs. 16, 20, 26-28, the following holds true for any particular type of undrained shear in TC/TE, PSC/PSE, PM and SS:
Fig. 7  Simplified Relationship for Undrained Strength Ratio $\left(\frac{\tau}{\tau_0}\right)$ for NC Clays
Fig. 6  Simplified Relationships Between Friction Angles in Many Modes of Failure
Figs. 4, 9a-b and 10 show the measured and predicted strength of several natural clays, for many different stress paths, geologic origins and OCR. The close agreement observed in these figures as well as in Fig. 6 substantiates the proposed theory and the choice of the failure criteria. The general concept of the approach and its advantages are summarized as follows:

i. The notion that mean stress at failure is dependent on the mode of undrained failure is included;

ii. One axisymmetric yield surface and a general form for the failure surface $f$ are introduced, which allow the complete determination of failure parameters for any mode of failure.

iii. The initial anisotropy is properly taken into account in the yield surface and failure criteria.

iv. The effect of induced anisotropy is captured and incorporated by a calibration procedure. Given the initial conditions of the soil, the complete failure behavior at critical state (strength, pore pressures, friction angles, etc.) along any mode of failure is given by the relationships in Table 2 or Eqs. 21-23.

v. Limited laboratory testing of soil is required to calibrate the model.
Fig. 4a: Predicted versus Measured Undrained Strength Ratio in Simple Shear. Numbers refer to cases in Table 1.
Fig. 9. Predicted Versus Measured Undrained Strength
Ratio in P1. Strain Compression and Extension
(Numbers refer to Cases in Table B)
Fig. 13 Predicted Versus Measured Undrained Strength Ratio in Triaxial Compression. Numbers Refer to Cases in Table F1.
vi. Any relationship derived in this work can be related using the two most commonly used soil parameters, $\phi_c$ and OCR (with a reasonable value of $k_v / \lambda$ such as 0.2).

vii. The model implementation is simple and could be used during field tests as experiments are being performed.

To the authors' knowledge, it is the first time that a model which captures the essential features of initial and induced anisotropy, yet yielding closed form solutions for all the failure parameters of clays, is presented. Each of the steps involved in the development is physically explained and complemented using experimental data. The model strictly adheres to all the facets of anisotropy known to date. Sophistication of the model thereof is possible and is being developed.

The proposed theory can certainly be applied to a number of geotechnical engineering problems (Thevanayagam, 1988a and 1988b), such as:

i. stability analysis of embankments, retaining walls, vertical cuts, etc.

ii. interpretation of vane shear test

iii. study of SBPM, etc.

The potential that the theory has for SBPM interpretation is introduced in the next section.
Table 3: Some Selected Forms of Failure Criteria

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f^f_p$</td>
<td>$f^f_p + f^f_d = 0$</td>
<td>$f^f_p + f^f_d = 0$</td>
<td>$f^f_p + f^f_d = 0$</td>
</tr>
<tr>
<td>$f^f_p$</td>
<td>$1/2 \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 \right]$</td>
<td>Max of $(\sigma_i - \sigma_j)$</td>
<td>Max of $S_{ij}$</td>
</tr>
<tr>
<td></td>
<td>$+ (\sigma_1 - \sigma_3)^2 \right] = M^2 p^2$</td>
<td>$= M_p$</td>
<td>$= 2/3 M_p, i=j$</td>
</tr>
<tr>
<td>$f^f_d$</td>
<td>$3/2 (s_{ij} - a_{ij})$</td>
<td>$3/2 (s_{ij} - a_{ij})$</td>
<td>$3/2 (s_{ij} - a_{ij})$</td>
</tr>
<tr>
<td></td>
<td>$(s_{ij} - a_{ij}) - k^2 = 0$</td>
<td>$(s_{ij} - a_{ij}) - k^2 = 0$</td>
<td>$(s_{ij} - a_{ij}) - k^2 = 0$</td>
</tr>
</tbody>
</table>

Note: Case I corresponds to Eqs. 23a-b in Sec. 3.3.5.
Since the induced anisotropy factor $\gamma$ was calibrated using data on peak strength in TC and TE, the model is not applicable to obtain the residual strength of clays. Also, calibrated values of $\gamma = 1.0$ for TC and $\gamma = 0.6$ for TE were deduced from experimental data for many clays. For individual clays these values may be slightly different. If data is available for a clay of interest, then using the theoretical development of this report the value of $\gamma$ may be calibrated and used in Eqs. 21-24 to obtain failure parameters in any mode of failure that is of interest to the analyst for that particular clay.

In this development two other forms were also considered for $f^p$. They are shown in Table 3. These forms also give the relationship between $\psi_c$ and $\psi_e$ given by Eq. 23c. Without further data on the failure stresses of clays, the relative merits of these different criteria could not be studied. Eq. 23b was selected because of its interpretability in terms of shear strain energy and mathematical simplicity. Based on existing data, it is shown (e.g. see predictions in Table 5a-b) that $f^p$ in Eq. 23b predicts failure stresses reasonably well, and appears to perform better than the other two possibilities shown in Table 3. An experimental program to study the relative merits of these criteria will certainly be highly revealing.

3.5 Some Novel Applications to Pressuremeter Problem

Applications of this theory are focusing on a study of the
following aspects of self boring pressuremeter measurements.

1. State of stresses at failure and mode of failure;
2. Possibility of radial cracking;
3. Methods to determine $\phi_c$ and OCR using SBPM data;
4. Development of a solution technique to interpret disturbed SBPM data;
5. Possible effects of rate of testing, creep and relaxation time on SBPM data, and
6. Possible effects of partial drainage.

Promising results were obtained to date with regard to aspects 1, 2, 3, and 4. They are summarized in the following; Details can be found in Thevanayagam (1988b).

3.5.1 State of Stress

The stress path during the pressuremeter test is not fully agreed upon and much speculation has been offered (e.g. Prevost, 1979, Ladd et al., 1980, Wroth, 1984, Ladanyi, 1977, Wood and Wroth, 1977). A clear understanding of the mode of deformation during testing is instrumental to the development of a superior interpretation method. Using the theory reported herein and a "bending mechanism", the mode of failure in pressuremeter testing of clays was identified. The axisymmetric nature of loading in
pressuremeter tests requires that the soil elements around the probe be within two extreme modes of deformation, plane strain and plane stress. The state of stress and behavior of clays at failure for these two extreme cases i) plane strain, \( \varepsilon_1 = 0 \), and (ii) plane stress, \( \sigma_v = \text{constant} \), will be analyzed first. This information and available experimental data (Wood and Wroth 1977, Huang 1986) will then be used to develop plausible arguments to show that the assumption of plane strain \( \varepsilon_1 = 0 \) is usually satisfactory in the interpretation of pressuremeter data. Conditions for which the plane strain state may not prevail are identified.

For undrained plane strain shearing with \( \varepsilon_z = 0 \) and \( \varepsilon_2 = -\varepsilon_3 \) (subsequently denoted by the subscript pm), the methodology of this work, Eqs. 21-23, yields the following relationships for stresses at failure (also see Table 2):

\[
\begin{align*}
\frac{\sigma_r}{\sigma_v} \text{ pm} &= \frac{\tau_f}{\sigma_v \text{ pm}} = \frac{2 M_c + 0.5 a}{\sqrt{3} (M_c + a)} \frac{\tau_f}{\sigma_v} \text{ TC} \\
\frac{\sigma_r}{\sigma_v} \text{ pm} &= \frac{M_c - 1.05 a}{\sqrt{3} (M_c + a)} \frac{\tau_f}{\sigma_v} \text{ TC} \\
\frac{\sigma_v}{\sigma_v} \text{ pm} &= \frac{M_c + 1.55 a}{\sqrt{3} (M_c + a)} \frac{\tau_f}{\sigma_v} \text{ TC}
\end{align*}
\]

where \( \sigma_v = \sigma_{11}, \sigma_r = \sigma_{22}, \sigma_\theta = \sigma_{33} \)

The expression for \( \frac{\sigma_r}{\tau_f} \text{ pm} \) is given by:
\[ \frac{\sigma_r}{\sigma_{vo}} = \sqrt{\frac{(3X^2 + 2.25)^{1/2}}{M_c}} + X - 0.5 \]  \[ \frac{\tau_f}{\sigma_{vo}} \text{ pm} = \frac{M_c}{X} \]  \[ \text{with } X = \frac{2M_c + 0.5 \alpha}{\sqrt{3} \alpha} \]

and the relationships for \( \sigma_r, \sigma_\theta \) and \( \sigma_v \) are:

\[ \frac{\sigma_r}{\sigma_{vo}} \text{ pm} = \sqrt{\frac{3X^2 + 2.25}{M_c}} + X - 0.5 \left( \frac{a}{M_c + a} \right) \left( \frac{\tau_f}{\sigma_{vo}} \text{ TC} \right) \]  \[ \frac{\sigma_v}{\sigma_{vo}} \text{ pm} = \sqrt{\frac{3X^3 + 2.25}{M_c}} + 1 \left( \frac{a}{M_c + a} \right) \left( \frac{\tau_f}{\sigma_{vo}} \text{ TC} \right) \]  \[ \frac{\sigma_\theta}{\sigma_{vo}} \text{ pm} = \sqrt{\frac{3X^2 + 2.25}{M_c}} - X - 0.5 \left( \frac{a}{M_c + a} \right) \left( \frac{\tau_f}{\sigma_{vo}} \text{ TC} \right) \]

The stresses at failure can also be obtained using the other choices for \( f_f \) given earlier in Table 3; they are shown in Table 4. Only results obtained with the failure criterion in Eqs. 23a-b are reported in the following discussion.

Eqs. 31a-c imply that, independent of OCR and initial magnitudes of principal stresses in horizontal and vertical directions, the following inequalities hold true:

\[ \sigma_r > \sigma_v > \sigma_\theta \text{ at failure.} \]  \[ (32) \]

and that the r-\( \theta \) plane is always the failure plane. Figs. 11a and
Table 4: Stress Parameters at Failure in Plane Strain $\varepsilon_z = 0$ Test in Clays

<table>
<thead>
<tr>
<th>Eqn</th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>$f^f_p$</td>
<td>$f^f_p + f^f_d = 0$</td>
<td>$f^f_p + f^f_d = 0$</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$f^p_f$</td>
<td>$1/2 \left[ \left( \sigma_1 - \sigma_2 \right)^2 + \left( \sigma_2 - \sigma_3 \right)^2 \right] + \left( \sigma_1 - \sigma_3 \right)^2 = M^2 p^2$</td>
<td>$\text{Max of } (\sigma_1 - \sigma_j) = M_p$</td>
</tr>
<tr>
<td>$B_3$</td>
<td>$f^f_d$</td>
<td>$3/2(s_{ij} - a_{ij})$</td>
<td>$3/2(s_{ij} - a_{ij})$</td>
</tr>
<tr>
<td>$B_4$</td>
<td>$\left( \frac{\tau_f}{\sigma_{vo}} \right)_{\text{PM}}$</td>
<td>$\frac{M_c + 0.5a}{\sqrt{3}(M_c + \alpha)} \cdot \left( \frac{\tau_f}{\sigma_{vo}} \right)_{\text{TC}}$</td>
<td>$\frac{M_c + 0.5a}{\sqrt{3}(M_c + \alpha)} \cdot \left( \frac{\tau_f}{\sigma_{vo}} \right)_{\text{TC}}$</td>
</tr>
<tr>
<td>$B_5$</td>
<td>$\left( \frac{\sigma_z}{\sigma_{vo}} \right)_{\text{PM}}$</td>
<td>$\left[ \frac{3}{2M_c} \left( x + 0.5 \alpha \right) + 1 \right] \cdot \frac{\alpha}{M_c + \alpha} \cdot \left( \frac{\tau_f}{\sigma_{vo}} \right)_{\text{TC}}$</td>
<td>$\left[ \frac{3}{2M_c} \left( x + 0.5 \alpha \right) + 1 \right] \cdot \frac{\alpha}{M_c + \alpha} \cdot \left( \frac{\tau_f}{\sigma_{vo}} \right)_{\text{TC}}$</td>
</tr>
<tr>
<td>$B_6$</td>
<td>$\left( \frac{\sigma_\tau}{\sigma_{vo}} \right)_{\text{PM}}$</td>
<td>$\left[ \frac{\sqrt{3} x^2 + 2.25}{M_c} \right] \cdot \frac{\alpha}{M_c + \alpha} \cdot \left( \frac{\tau_f}{\sigma_{vo}} \right)_{\text{TC}}$</td>
<td>$\left[ \frac{\sqrt{3} x^2 + 2.25}{M_c} \right] \cdot \frac{\alpha}{M_c + \alpha} \cdot \left( \frac{\tau_f}{\sigma_{vo}} \right)_{\text{TC}}$</td>
</tr>
</tbody>
</table>
Table 4: (cont'd)

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>B7</td>
<td>( \frac{\sigma_v}{\sigma_{vo}}_{PM} )</td>
<td>[ \frac{\sqrt{3x^2 + 2.25}}{M_c} - x - 0.5 ]</td>
<td>[ \frac{3}{2M_c} (x + 0.5) - x - 0.5 ]</td>
<td>[ \frac{2x}{M_c} - x - 0.5 ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B8</td>
<td>( \sin(\phi_{pm}) )</td>
<td>\frac{2X}{2 \sqrt{3x^2 + 2.25} - 1}</td>
<td>\frac{3}{M_c} (x + 0.5) - 11</td>
<td>\frac{4x}{M_c} - 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B9</td>
<td>( \frac{\Delta U_{sh}}{\sigma_{vo}} )</td>
<td>\frac{1 + 2K_{ocr}}{3} \left( \frac{\sqrt{3x^2 + 2.25}}{M_c} \right) ]</td>
<td>\frac{1 + 2K_{ocr}}{3} \left( \frac{\sqrt{3x^2 + 2.25}}{M_c} \right) ]</td>
<td>\frac{1 + 2K_{ocr}}{3} \left( \frac{\sqrt{3x^2 + 2.25}}{M_c} \right) ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B10</td>
<td>( \frac{\sigma_r}{\tau_{fpm}} )</td>
<td>\left( \frac{\sqrt{3x^2 + 2.25}}{M_c} + x - 0.5 \right)</td>
<td>\left( \frac{3}{2M_c} (x + 0.5) + x - 0.5 \right)</td>
<td>\left( \frac{2x}{M_c} + x - 0.5 \right)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B11</td>
<td>( X )</td>
<td>\frac{2M_c + 0.5\alpha}{\sqrt{3\alpha}} ]</td>
<td>\frac{2M_c + 0.5\alpha}{\sqrt{3\alpha}} ]</td>
<td>\frac{2M_c + 0.5\alpha}{\sqrt{3\alpha}} ]</td>
<td></td>
</tr>
</tbody>
</table>

Note: Case I corresponds to Eqs. 23a-b in Sec. 3.3.5.
11b show the variation of $\sigma_r/\sigma_{vo}$, $\sigma_v/\sigma_{vo}$ and $\sigma_\theta/\sigma_{vo}$ for NC and OC clays (OCR = 10) as a function of $\phi_c$. Fig. 11c gives the variation of $\sigma_\theta/\sigma_{vo}$ for various values of OCR and $\phi_c$, and for $\gamma$ values of 0.6 and 1.0. Table 5a-b, Figs. 12a and 12b compare measured values of failure stresses for Spestone kaolin (Wood and Wroth, 1977) to those predicted by Eqs. 31f-h. Evaluation of these results lead to the following implications:

1. As long as the plane strain condition, $\varepsilon_z = 0$, is maintained, the stresses $\sigma_r$ and $\sigma_\theta$ become the major and minor principal stresses respectively, irrespective of the initial state of stress;

2. There exists a critical OCR, called $OCR_c$, at which $(\sigma_v/\sigma_{vo})$ at failure remains equal to 1.0. For most of the clays, this OCR lies in a narrow range of 1.5-2.5 (Fig.13). For a clay with $OCR < OCR_c$, $\sigma_v$ decreases sharply and becomes the intermediate principal stress at failure. The stress $\sigma_r$ may increase or decrease depending on the characteristics of the clay (for very weak clays $\sigma_r$ decreases), and $\sigma_\theta$ decreases becoming the minor principal stress at failure (Figs.11a-c). If $OCR = OCR_c$, then $\sigma_r$ increases and $\sigma_\theta$ decreases, while $\sigma_v$ remains constant and becomes the intermediate principal stress. If $OCR > OCR_c$, $\sigma_v$ and $\sigma_r$ increase while $\sigma_\theta$ decreases. The increase in $\sigma_v$ depends on the value of OCR and the clay. However, the changes in $\sigma_v$ and $\sigma_r$ are such that $\sigma_v$ always becomes the intermediate principal stress. For any
Fig. 1a: \( \frac{\partial x}{\partial y} \) vs. \( z \) relationship for Mr. F's eyes.
Fig. 11b $\frac{\sigma_{xx}}{\sigma_{vo}}$ vs. $\kappa_c$ Relationship for OC Clays.
<table>
<thead>
<tr>
<th>1.0</th>
<th>2.8</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>2.2</td>
<td>2.0</td>
</tr>
<tr>
<td>1.25</td>
<td>1.8</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Fig. 11c Simplified $\frac{\sigma_0}{\sigma_v}$ vs $\phi_c$ relationship
Fig. 11d State of Stress and Orientation of Failure Planes (Proposed Theory)
Table 5a: Numerical Prediction of Stress Parameters \( \epsilon_0 = 0 \) Path (Test Data - Wood et al., 1977).

<table>
<thead>
<tr>
<th>Case</th>
<th>Test</th>
<th>OCR</th>
<th>Prd`td</th>
<th>Meas`d</th>
<th>Prd`td</th>
<th>Meas`d</th>
<th>Prd`td</th>
<th>Meas`d</th>
<th>Prd`td</th>
<th>Meas`d</th>
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<th>Prd`td</th>
<th>Meas`d</th>
</tr>
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<tbody>
<tr>
<td>No</td>
<td>( \frac{\sigma_v}{\sigma_{vo}} )</td>
<td>( \frac{\sigma_v}{\sigma_{vo}} )</td>
<td>( \frac{\sigma_r}{\sigma_{vo}} )</td>
<td>( \frac{\sigma_r}{\sigma_{vo}} )</td>
<td>( \frac{\sigma_\theta}{\sigma_{vo}} )</td>
<td>( \frac{\sigma_\theta}{\sigma_{vo}} )</td>
<td>( \phi_{pm} )</td>
<td>( \phi_{pm} )</td>
<td>( 2\frac{\sigma_v}{r+\sigma_\theta} )</td>
<td>( 2\frac{\sigma_v}{r+\sigma_\theta} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K4</td>
<td>1.0</td>
<td>0.576</td>
<td>0.51</td>
<td>0.727</td>
<td>0.674</td>
<td>0.324</td>
<td>0.316</td>
<td>22.5</td>
<td>22.5-26.0</td>
<td>1.10</td>
<td>1.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>K2</td>
<td>2.5</td>
<td>1.19</td>
<td>1.02</td>
<td>1.513</td>
<td>1.19</td>
<td>0.674</td>
<td>0.516</td>
<td>22.5</td>
<td>22.5-26.0</td>
<td>1.10</td>
<td>1.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K3</td>
<td>5.46 (2.14)</td>
<td>1.44</td>
<td>(2.4)</td>
<td>1.91</td>
<td>(1.07)</td>
<td>0.77</td>
<td>22.5</td>
<td>22.5-26.0</td>
<td>1.10</td>
<td>1.1</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>K3</td>
<td>5.46</td>
<td>2.24</td>
<td>1.44</td>
<td>2.8</td>
<td>1.91</td>
<td>1.26</td>
<td>0.77</td>
<td>22.5</td>
<td>22.5-26.0</td>
<td>1.10</td>
<td>1.1</td>
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<tr>
<td>K4</td>
<td>1.0</td>
<td>0.617</td>
<td>0.51</td>
<td>0.767</td>
<td>0.674</td>
<td>0.364</td>
<td>0.316</td>
<td>20.8</td>
<td>22.5-26.0</td>
<td>1.09</td>
<td>1.11</td>
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<td></td>
</tr>
<tr>
<td>II</td>
<td>K2</td>
<td>2.5</td>
<td>1.28</td>
<td>1.02</td>
<td>1.59</td>
<td>1.19</td>
<td>0.757</td>
<td>0.516</td>
<td>20.8</td>
<td>22.5-26.0</td>
<td>1.09</td>
<td>1.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K3</td>
<td>5.46</td>
<td>2.4</td>
<td>1.44</td>
<td>2.98</td>
<td>1.91</td>
<td>1.415</td>
<td>0.77</td>
<td>20.8</td>
<td>22.5-26.0</td>
<td>1.09</td>
<td>1.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K4</td>
<td>1.0</td>
<td>0.654</td>
<td>0.51</td>
<td>0.804</td>
<td>0.674</td>
<td>0.4</td>
<td>0.316</td>
<td>19.5</td>
<td>22.5-26.0</td>
<td>1.09</td>
<td>1.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>K2</td>
<td>2.5</td>
<td>1.36</td>
<td>1.02</td>
<td>1.67</td>
<td>1.19</td>
<td>0.83</td>
<td>0.516</td>
<td>19.5</td>
<td>22.5-26.0</td>
<td>1.09</td>
<td>1.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K3</td>
<td>5.46</td>
<td>2.54</td>
<td>1.44</td>
<td>3.12</td>
<td>1.91</td>
<td>1.55</td>
<td>0.77</td>
<td>19.5</td>
<td>22.5-26.0</td>
<td>1.09</td>
<td>1.1</td>
<td></td>
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<td></td>
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</tbody>
</table>

Note a: ( ) denotes that \( \sigma = \sigma_0 \) was assumed.

Note b: Case I corresponds to Eqs. 23a-b in Sec. 3.3.5.
Table 5b: Predicted and Measured Critical OCR.

<table>
<thead>
<tr>
<th>Predicted</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>OCR_c</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Note: Measured value was interpreted by the writers using the experimental data from Wood et al. (1977).
Figure 12a Theoretical Versus Measured Stress Ratio at Failure in Undrained Pl. Strain $\varepsilon_z = 0$

Shearing of Clays (Measured after Wood and Wroth 1977)
Figure 12b  Theoretical Versus Measured Stress Ratio at Failure in Undrained Pl. Strain $\varepsilon_2 = 0$

Shearing of Clays (Measured after Wood and Wroth, 1977)
Fig. 13 OCRC vs C Relationship
value of OCR, the failure takes place in the r-θ plane. This is summarized in Fig. 11d. Experimental data obtained by Wood and Wroth (1977) (Figs. 12a-b) support these trends.

3. The mean principal stress, in general, does not remain constant. For lightly OC clays the mean principal stress decreases whereas it increases for highly OC clays;

4. The angle φ_p is always greater than the corresponding angle φ_c obtained in TC (Fig. 8).

5. Fig. 11c indicates that, for φ_c > 37°, σ_θ becomes negative. Also, the chances for negative value of σ_θ increases with γ approaching 1.0. Since γ is expected to be close to 1.0 for highly OC clays, the likelihood for negative σ_θ is slightly higher for OC clays than for NC clays.

The solution for failure stresses σ_r/σ_v, σ_θ/σ_v and ε_z/ε_r in undrained plane stress shear of clays (σ_v = constant, ε_r > 0) can be obtained by solving the following set of equations:

\[(2σ_v - σ_r - σ_θ - 3σ_{11})^2 + (2σ_r - σ_v - σ_θ + 1.5 σ_{11})^2 + (2σ_θ - σ_r - σ_v + 1.5 σ_{11})^2 - 6 k^2 = 0 \quad (33a)\]

\[(σ_v - σ_θ)^2 + (σ_v - σ_r)^2 + (σ_r - σ_θ)^2 - \]
\[
\frac{2Mc^2}{9} (\sigma_r + \sigma_\theta + \sigma_v)^2 = 0
\]  \hspace{1cm} (33b)

\[
\frac{\varepsilon_z}{\varepsilon_r} = \frac{2\sigma_v - \sigma_r - \sigma_\theta - 3\sigma_{11}}{2\sigma_r - \sigma_v - \sigma_\theta + 1.5 \sigma_{11}}
\]  \hspace{1cm} (33c)

where \(\sigma_{11} = \sigma_v\), \(\sigma_{22} = \sigma_r\), \(\sigma_{33} = \sigma_\theta\), \(\varepsilon_{11} = \varepsilon_z\) (vertical strain increment), and \(\varepsilon_{22} = \varepsilon_r > 0\). These equations can be solved numerically. Fig. 14 shows the variation of \(\varepsilon_z/\varepsilon_r\) as a function of \(\phi_c\) for various values of OCR. This figure shows that, in general, the vertical strain increment is compressive for lightly OCR clays, whereas it is tensile for highly OCR clays. For each value of \(\phi_c\), there exists an OCR at which the incremental vertical strain is zero (i.e. plane strain). This value of OCR is the value of OCR deduced earlier using the results for plane strain shearing of clays.

Let us consider the application of radial stresses in a cylindrical cavity wall as shown in Fig. 15. For the purpose of discussion, the element \(A_1\) is chosen as a reference point if end effects are neglected, or any other element \(A_i\) below \(A_1\) can be selected if end effects are assumed to exist. In deep tests, the soil above the level \(A_1\) can be considered as a thick plate clamped at some distance away from the probe. Using the above descriptions of clay behavior in plane stress and plane strain shearing, the mode of failure and the mechanism causing it in pressuremeter tests can be deduced from the following reasoning.
Fig. 14: Strain Ratio Vs. Friction Angle at Critical State

in Plane Stress $\sigma_y = \sigma_{yo}$ Shearing (PSS)
Figure 15 A Schematic Diagram of Pressuremeter Testing and Vertical Stress Distribution Around the Cavity.
Assuming that plane stress conditions prevail, the results in Fig. 14 indicates that depending on the value of OCR, $\varepsilon_z$ can be tensile (highly OC), zero (at OCR$_c$), or compressive (lightly OC). Considering the soil layer above the level A$_1$ as a thick plate, a compressive strain increment, $\varepsilon_z$, requires that the plate with a central hole deflect downwards. Since the flexural rigidity of the plate is large, a small amount of deflection requires a substantial reduction in the stress distribution $\sigma_v$ under the plate, i.e. in the horizontal plane at the level of A$_1$. This contradicts the original assumption that plane stress in vertical direction prevails. The same argument but using a hogging type of movement can be used for the case of highly OC clays. It should be remembered that this argument is valid only if the plate is very thick. However, in the case of OCR = OCR$_c$, this bending mechanism is absent ($\varepsilon_z = 0$), and the soil elements satisfy both plane stress and plane strain conditions, independently of the depth at which the test is performed. Therefore, the soil elements around the probe in pressuremeter testing at large depths cannot be expected to experience the plane stress mode of failure except when OCR = OCR$_c$.

Assuming that plane strain conditions prevail, for OCR = OCR$_c$, $\sigma_v$ remains equal to $\sigma_{vo}$, and the mode of failure is both plane strain and plane stress shearing, as already discussed above. If OCR > OCR$_c$, the theory reported herein (see Figs. 11a-c) and limited experimental data (Wood and Wroth, 1977) imply that $\sigma_v$ increases. To achieve this, an upward movement of the thick plate
of soil is necessary. Therefore, the soil elements around the probe would apparently deviate from the plane strain condition. However, considering the high flexural rigidity of the plate, this increase in $\sigma_v$ is possible with a very small upward deflection of the thick plate at level $A_1$; thus, this upward deflection is in fact negligible and the element at level $A_1$ can be considered to be in plane strain. Similar argument (with downward deflection, and decrease in $\sigma_v$) is plausible for a case of $OCR < OCR_c$. Therefore, it can be concluded that for pressuremeter testing of clays at relatively large depths, the soil elements closely experience a state of plane strain shearing. The "bending mechanism" of the thick plate of soil above the pressuremeter probe is responsible for this. However, the argument will not hold true for testing at shallow depths except when $OCR = OCR_c$. With further development of the model reported herein, a complete numerical analysis of this problem is possible and is proposed for further study. Nevertheless, based on the plausible arguments stated herein the following conclusions can be drawn without loss of generality. For a clay with $OCR = OCR_c$, the pressuremeter data can be interpreted at any depth using the assumption of plane strain shearing. For deep tests the mode of failure is very close to that in plane strain and the theoretical results reported earlier are applicable. For shallow depths, the mode of failure is far from that in plane strain.
3.5.2 Interpretation of \( \phi_c \) and OCR from SBPM Data

It is noted that Eq. 31e depends only on the value of \( \phi_c \), while Eq. 31a depends on both \( \phi_c \) and OCR of the clay. Hence, using these relationships, a method to obtain \( \phi_c \) and OCR of the insitu clay using SBPM data can be developed. A step by step example of this method is given in the following:

1. Using pressuremeter expansion and pore pressure measurements, obtain \( (\tau_f)_pm \) and corresponding \( (c_r)_pm \) at failure.

2. Assume a value for \( \phi_c \); Estimate \( M_c = 6 \sin \phi_c / (3 - \sin \phi_c) \) and \( a_0 \) (from Eq. 9b).

3. Use steps 1 and 2 to check that Eq. 31e is satisfied. Repeat steps 2 and 3 with a different estimate of \( \phi_c \) until convergence is reached.

The value of OCR may be deduced from the value of \( \phi_c \) at convergence and Eq. 31a. Using data from model pressuremeter tests simulated in a calibration chamber (Chameau et al., 1987), the parameters \( \phi_c \) and OCR were obtained with this procedure for two clays. The predicted values are compared to those measured from TC tests in Table 6. While the predictions for normally consolidated clays is excellent, the OCR is slightly under predicted for highly OC clays (tests CP23 and CP16). This deviation is attributed to the presence of shear stresses on the pressuremeter membrane due to the consolidation around it and subsequent swelling to prepare
Table 6 Prediction of Soil Parameters Using Anisotropic Theory

<table>
<thead>
<tr>
<th>TEST # SOIL</th>
<th>Predicted</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi_{tc}$</td>
<td>OCR</td>
</tr>
<tr>
<td>CP 6 [K100]</td>
<td>19.5</td>
<td>1.02</td>
</tr>
<tr>
<td>CP 8 [K50]</td>
<td>20.2</td>
<td>0.99</td>
</tr>
<tr>
<td>CP 23 [K50]</td>
<td>20.8</td>
<td>7.5</td>
</tr>
<tr>
<td>CP 16 [K100]</td>
<td>23.5</td>
<td>7.3</td>
</tr>
</tbody>
</table>

$\phi_{tc} =$ Friction Angle in Triaxial Compression Test

Measured: Based on data from Huang, 1986.
highly OC clays in the calibration chamber tests.

3.5.3 Interpretation of Disturbed SBPM Data

The self boring technique does produce changes in the initial loading condition and the initial position of the cavity wall. Wroth and Hughes (1973) reported an outward radial movement of the cavity wall corresponding to about 0.5% of radial strain. Denby (1978) also reported similar movements of the cavity wall well before the beginning of the expansion test. Later studies using a larger size cutting shoe (Benoit and Clough, 1986) also pointed out this possibility. A recent integrated study of the problems involved in the pressuremeter tests (Prapaharan, 1987 and Prapaharan et al., 1988), showed that this initial movement is the most influential when compared to any other problems identified with the test.

This section deals with developing an interpretation technique that may be used to improve the current difficulties in interpreting pressuremeter data with initial movement (i.e. initial disturbance). An outline of this new interpretation method is given and illustrated using model pressuremeter test data. The solution procedure requires the expansion curve and pore pressure measurements at the cavity wall (for overconsolidated clays OCR is also required).

Let \( \hat{C}_r \) refer to the strain obtained from measurements in the disturbed test (i.e. \( \hat{C}_r \) = measured strain), and \( \Delta C_0 \) be the error.
The error $\Delta \varepsilon_o$ in the measured strain may be estimated in the following manner [For illustration purposes, the figures used herein were obtained by applying this technique to model pressuremeter test data (Test No. CP6) of Huang, 1986]:

1. Assume that the measured strain $\hat{\varepsilon}_r$ includes some unknown error $\Delta \varepsilon_o$:

$$\varepsilon_r = \hat{\varepsilon}_r + \Delta \varepsilon_o$$

(34)

2. For several values of $\Delta \varepsilon_o$, using Eq. 34 and $\hat{\varepsilon}_r$ vs. probe pressure measurements, obtain $\varepsilon_r$ vs probe pressure data. This data can be fitted using a spline function (i.e. one spline function for each $\Delta \varepsilon_o$).

3. For each spline fit in step 2, using the method of derivatives (e.g. Palmer, 1972, Ladanyi, 1972, Baugelin, et al., 1972) obtain the peak strength $\tau_{fpm}$, the corresponding $\sigma_r$ and strain $\varepsilon_r$ at the cavity wall. These are shown in Table 7 and Figs. 16 and 17 for the data set considered.

4. For each value of $\Delta \varepsilon_o$, using the values in step 3, and Eqs. 31a and 31e, obtain the values of $\phi_c$ and OCR.

5. Establish the relationship between OCR and $\Delta \varepsilon_o$ (see example in Fig. 18)

6. If the value of OCR of the clay is estimated by other means (e.g. dilatometer or piezocone data), using this OCR and Fig. 18 (for the data considered), the correct value of $\Delta \varepsilon_o$
Fig. 1: Model Pressuremeter Expansion Curve (Huang 1981)
Fig. 17  Derived Stress-Strain Curves for Each $\Delta \sigma_0$
### Table 7: Resulting OCR, \( \phi_c \) for Assumed Values of Error \( \Delta \varepsilon_o \)

<table>
<thead>
<tr>
<th>( \Delta \varepsilon_o )</th>
<th>( \tau_{fpm} )</th>
<th>( \varepsilon_r )</th>
<th>( \varepsilon_r )</th>
<th>( \sigma_r )</th>
<th>( \phi_c )</th>
<th>OCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>kPa</td>
<td>%</td>
<td>%</td>
<td>kPa</td>
<td>(deg.)</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>117.0</td>
<td>0.5</td>
<td>0.25</td>
<td>185.0</td>
<td>1.58</td>
<td>62.</td>
</tr>
<tr>
<td>0.15</td>
<td>91.0</td>
<td>0.43</td>
<td>0.28</td>
<td>185.5</td>
<td>2.04</td>
<td>38.</td>
</tr>
<tr>
<td>0.05</td>
<td>71.0</td>
<td>0.36</td>
<td>0.31</td>
<td>191.0</td>
<td>2.70</td>
<td>25.</td>
</tr>
<tr>
<td>0.0</td>
<td>64.4</td>
<td>0.50</td>
<td>0.50</td>
<td>213.3</td>
<td>3.31</td>
<td>19.</td>
</tr>
<tr>
<td>-0.05</td>
<td>60.0</td>
<td>0.7</td>
<td>0.75</td>
<td>241.9</td>
<td>4.03</td>
<td>15.</td>
</tr>
<tr>
<td>-0.15</td>
<td>53.3</td>
<td>0.8</td>
<td>0.95</td>
<td>260.</td>
<td>4.88</td>
<td>12.</td>
</tr>
</tbody>
</table>
Fig. 18 OCR vs $\Delta \varepsilon_0$ (Refer to Table 7)
can be determined.

For the data in this example problem, OCR is equal to 1.0, the error is found to be about $\Delta \varepsilon_0 = \pm 0.02\%$ from Fig. 18. Using this value of $\Delta \varepsilon_0$, the measured strain can be corrected, and true pressuremeter strength and material parameters deduced. For this example, they are given by:

$$\tau_{fpm} = 64.4 \text{kPa}, \quad \frac{T_{fpm}}{C_{vo}} = 0.236, \quad \phi_c = 19.5 \text{ deg.}$$

The value of $\phi_c$ is in excellent agreement with that measured in TC, about 20.7°.

3.5.4 Existence of Tensile Stress $\sigma_0$

Ladanyi (1977) indicated the possibility of radial cracking for frozen soils. Wood and Wroth (1977) observed negative value of interpreted circumferential stress $\sigma_0$ for some pressuremeter tests in clays. This was thought to lead to radial cracking. If radial cracking is present, then the mode of failure may become that of unconfined compression (Ladanyi 1977). This is a serious problem, because all the current interpretation methods and the necessary relationships to obtain the stress-strain curve of the soil are based on the assumptions that the soil around the cavity remains intact and compatibility of strain holds. If radial cracking is present, then those interpretation methods are no longer valid. This subject has not been studied analytically in detail except some model pressuremeter tests (Huang, 1986 and Chameau et al.,
1987) and simulated plane strain $\varepsilon_z = 0$ tests on clays in true triaxial tests (Wood and Wroth, 1977). None of the simulated tests or model pressuremeter tests had any evidence of negative circumferential stress. The only theoretical study on mode of deformation in SBPM was done assuming soil as an isotropic-elastic material that is governed by a Mohr-Coulomb criterion with $\sigma_{vo}$ remaining constant (Wood and Wroth, 1977) (Fig. 19). Clearly this is not a valid assumption for natural clays.

The theoretical results reported herein, Fig. 11c, indicates that radial cracking is possible only when $\phi_c$ is greater than about $37^\circ$, with slightly higher susceptibility for highly overconsolidated clays. Therefore, it appears that the likelihood of tensile stresses during the expansion of the probe is very small, possibly with the exception of frozen soils, however, in this case, the theory may not be applicable. This suggests that tensile stresses that have been reported based on current interpretation techniques may not be real, but only a reflection of the interpretation method. A plausible reason for these observations is proposed next.

It can be noted that, in general, whenever a case of tensile $\sigma_\theta$ is reported, it is always combined with an "interpreted" strength much higher than expected for that particular deposit. It is likely that, in many of these reported cases, the interpreted strength is high because the measured expansion curve contains an error in the measurements of radial strain due to initial movement.
Fig. 19 Nonr-Coaxial Failure with Different Orientations of Failure Plane (after Wood et al., 1977)

- Initial State of Stress
- State of Stress at Failure

Condition for non-coaxial failure:
\[ \frac{1}{2 + \sin \phi} \leq K_1 \leq \frac{1}{1 - \sin \phi} \]
of the probe. This can lead to tensile stresses being computed erroneously as shown in the following.

Let the true material stress-strain of the material be given by:

\[ \sigma_r - \sigma_\theta = q(\varepsilon_r) \]  \hspace{1cm} (35a)

Then, the "true" \( \sigma_\theta \) is:

\[ \sigma_\theta = \sigma_r - q(\varepsilon_r) \]  \hspace{1cm} (35b)

Let the interpreted stress-strain of the material be denoted by:

\[ (\sigma_r - \sigma_\theta)^i = q^i(\hat{\varepsilon}_r) \]  \hspace{1cm} (36)

where \( \hat{\varepsilon}_r \) is the measured strain, and the superscript "i" refers to "interpreted" parameters. The interpreted circumferential stress is:

\[ \sigma_\theta^i = \sigma_r - q^i(\hat{\varepsilon}_r) \]  \hspace{1cm} (37)

If the interpreted relation \( q^i \) is greater than the actual relation \( q \), at least for some range of strain, then:

\[ \sigma_\theta^i < \sigma_\theta \]  \hspace{1cm} (38)

Thus, if \( q^i \) is interpreted high, then it is possible that \( \sigma_\theta^i \) will reach negative values while \( \sigma_\theta \) will in fact be positive. This condition can happen only when the interpreted strength is higher than expected and, thus, any observation related to negative stresses are most probably due to erroneous interpretation. How-
ever, since negative $\sigma_0$ is a theoretical possibility (though small) for soils with high $\phi_c$, model pressuremeter tests on such clays are needed for clarification of this issue.

3.6 Conclusion

A general 3-D anisotropic model has been developed to study the behavior of clays at the critical state. The model is simple, and holds the promise of being very successful, as shown by comparisons with available data. With the developed relationships one can predict the strength and other failure parameters in any particular mode of failure. In addition, given the initial conditions, one can transform the data to predict the behavior in a different mode of deformation. Using the theoretical development of this report and the data from SBPM, the strength, friction angle, and other failure parameters, along many different modes of failure other than that of SBPM can be predicted. Also, the in situ OCR can be predicted using SBPM data and this theory. A novel approach has been presented to solve the problems caused by initial disturbance in SBPM testing. Possibility of radial cracking is shown to be a rare case. The theory holds the promise of being developed further. It offers a valuable model to study the fundamental aspects of many insitu testing devices in a new perspective.

3.7 Notations

The following symbols are used in this paper:
\( A_f \) = Skempton's A-parameter at failure;

CIUC = isotropically consolidated undrained shear test in TC;

\( C_{K_{UC}} \) = 1-D consolidated consolidated undrained shear test in TC;

CSL = critical state line;

\( e \) = void ratio;

\( f \) = \( f(p,q,r_n) \) - yield surface;

\( G \) = shear modulus;

\( K \) = bulk modulus;

\( K_0 \) = coefficient of earth pressure at rest;

\( K_{o_{NCL}} \) = 1-D consolidated normal compression line;

\( K_{o_v} \) = specific volume intercept of the normal compression line - \( v \) vs. \( \ln p \) at \( p=1 \text{ kPa} \);

\( M \) = \( q/p \), stress ratio at critical state;

PM = pressuremeter test;

PSC = plane strain compression;

PSE = plane strain extension;

PSS = plane stress shearing;

\( p \) = mean effective stress;

\( P_{eq} \) = \( \exp (K_{o_v}-v/\lambda) \), equivalent pressure;

\( q \) = deviatoric effective stress;

\( r_n \) = memory variable, hardening parameter;

SS = simple shear;

TC = triaxial compression;

TE = triaxial extension;

\( v \) = \( 1+e \), specific volume;

\( v_{\lambda} \) = \( (v + \lambda \ln p) \), equivalent specific volume;

\( \alpha \) = anisotropic parameter;

\( \alpha_0 \) = anisotropic parameter at virgin consolidated state;

\( \alpha_{ij} \) = anisotropic tensor;

\( \varepsilon_{ij} \) = strain tensor;

\( \varepsilon_{v} \) = \( (\varepsilon_1 + 2\varepsilon_3) \) volumetric strain;

\( \varepsilon_{q} \) = \( \frac{2}{3} (\varepsilon_1 - \varepsilon_3) \) deviatoric strain;

\( \kappa \) = slope of the swelling line in \( e-\ln p \) plane;

\( \kappa_v \) = slope of the swelling line in \( e-\ln(\sigma_v) \) plane;

\( \lambda \) = slope of the normal compression and critical state lines in \( e-\ln p \) or \( e-\ln(\sigma_v) \) plane;
\( \nu = \text{Poisson's ratio; } \)
\( \eta = q/p, \text{ stress ratio; } \)
\( \eta_0 = (q/p)_0, \text{ stress ratio during virgin consolidation; } \)
\( \phi = \text{effective angle of shearing resistance; } \)
\( \psi = \frac{\partial f}{\partial p} \frac{\partial p}{\partial q} \)
\( a_{ij} = \text{stress tensor; } \)

Subscripts:

- \( c \) = compression;
- \( e \) = extension;
- \( nc \) = normally consolidated clay;
- \( oc \) = over consolidated clay;
- \( vo \) = initial state in vertical direction;
- \( z \) = vertical direction

Superscripts:

- \( e \) = elastic part of ... 
- \( p \) = plastic part of ... 
- \( t \) = increment of ...
<table>
<thead>
<tr>
<th>NOTATION</th>
<th>SOIL</th>
<th>REFERENCE</th>
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<tr>
<td>1</td>
<td>Florida clay - 1</td>
<td>Saada, et al. (1975)</td>
</tr>
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<td>2</td>
<td>Florida clay - 2</td>
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<td>3</td>
<td>Florida clay - 3</td>
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<td>4</td>
<td>Hydrite - 10-2</td>
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<td>Hydrite - 10-3</td>
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<td>6</td>
<td>Girundite - 2</td>
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<td>7</td>
<td>Atchafalay clay - 1</td>
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<td>8</td>
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<tr>
<td>11</td>
<td>Speckled kaolinite</td>
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<tr>
<td>12</td>
<td>Boston Blue clay</td>
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</tr>
<tr>
<td>13</td>
<td>Haney clay</td>
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<tr>
<td>14</td>
<td>Remolded Sapporo</td>
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</tr>
<tr>
<td>15</td>
<td>Kawasaki - M-30 clay</td>
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<td>Vicksburg Buck Shot clay</td>
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<td>21</td>
<td>Undisturbed Kawasaki clay</td>
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<td>22</td>
<td>Undisturbed Brobekkein clay</td>
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<td>23</td>
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<td>29</td>
<td>Kaolinite (K50)</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Portsmouth Sensitive Marine clay</td>
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<tr>
<td>31</td>
<td>Bangor clay</td>
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<tr>
<td>32</td>
<td>San Francisco Bay mud</td>
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<tr>
<td>33</td>
<td>Weald clay</td>
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</tr>
<tr>
<td>34</td>
<td>Portland Marine clay</td>
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</tr>
<tr>
<td>35</td>
<td>Maine Organic clay</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>Resedimented FEC</td>
<td></td>
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<tr>
<td>37</td>
<td>Connecticut-Northampton Varved clay</td>
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<tr>
<td>38</td>
<td>Atchafalay clay</td>
<td></td>
</tr>
<tr>
<td>CP6</td>
<td>Kaolin, 100%</td>
<td></td>
</tr>
<tr>
<td>CP8</td>
<td>Kaolin, 50%; silt, 50%</td>
<td></td>
</tr>
<tr>
<td>CP16</td>
<td>Kaolin, 100%</td>
<td></td>
</tr>
<tr>
<td>CP23</td>
<td>Kaolin, 50%; silt, 50%</td>
<td></td>
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Fig. 20 Effective Stress Path Diagram in Undrained TC/TE and Yield Surfaces at Failure (Initial state: I, Effective stress path: IF; Failure state: F)
CHAPTER 4
CONCLUSIONS

This chapter summarizes the main conclusions drawn from this research program.

1. Although CK\textsubscript{0} UC test and CIUC test follow different stress paths, it has been shown that a good estimate of the CK\textsubscript{0} UC strength can be obtained from a single CIUC test on the same soil. It is a useful relation since the conventional CIUC tests are much easier to run than CK\textsubscript{0} UC tests.

2. Expressions were derived for the normalized shear strength $\tau_f / \sigma_{vo}$ using the Cam clay model, the modified Cam clay model, and the extended Cam clay and modified Cam clay models with spacing ratio. The normalized shear strength is a function of the friction angle and the consolidation characteristics of the soil. Both extended models with spacing ratio predict values that compare well with experimental data. Unlike the excellent agreement with experimental results obtained for $\tau_f / \sigma_{vo}$, predictions made for Skempton's A parameter at failure $A_f$ were marginal for all models. $A_f$ is very sensitive to $\Lambda$, which is a function of $C_c$ and $C_r$. This severely influences $\Lambda$ and thus affects the predictions of $A_f$.

3. A simple procedure is described to predict OCR of saturated clays using normalized virgin consolidation lines. Since the OCR estimate is not sensitive to the $C_r / C_c$ ratio, a good OCR prediction is possible from the knowledge of $e/e_L$ and $\sigma_v'$. 
4. Emphasis of this work was placed on anisotropy in clays, leading to the development of a general 3-D anisotropic model to study the behavior of clays at the critical state. To the authors' knowledge, it is the first model that captures the essential features of both initial and induced anisotropy yet yielding closed form solutions for all the failure parameters of clays. The main characteristics of the model are as follows:

i. The notion that mean stress at failure is dependent on mode of undrained failure is included;

ii. One axisymmetric yield surface and a general form for the failure surface $f^f$ are introduced, which allow the complete determination of failure parameters for any mode of failure.

iii. The initial anisotropy is properly taken into account in the yield surface and failure criteria.

iv. The effect of induced anisotropy is captured and incorporated by a calibration procedure. Given the initial conditions of the soil, the complete failure behavior at critical state (strength, pore pressures, friction angles, etc.) along any mode of failure can be determined.

v. Limited laboratory testing is required to calibrate the model.
5. The proposed theory is applicable to a number of geotechnical problems; in this report, it was used to investigate several important issues related to the determination of in situ soil properties using the self-boring pressuremeter (SBPM):

(i) The states of stresses at failure in plane strain and plane stress were studied with the anisotropic model. Using these results it was shown that the assumption of the plane strain mode of failure is satisfactory for relatively deep tests. $\sigma_T$ and $\sigma_6$ become the major and minor principal stresses respectively, irrespective of the initial state of stresses. It was also shown that the vertical stress and mean stress do not remain constant in general, in contrast to the predictions made by isotropic-elastic analysis. Currently available experimental data support these findings. All indications are that these findings settle the questions regarding the mode of deformation and the role of vertical stress during the expansion of the cylindrical cavity.

(ii) The possibility of radial cracking (negative circumferential stress) was analyzed in detail and shown to be unlikely; According to this theory, radial cracking is possible only for large friction angles, $\phi_c > 37^\circ$.

(iii) A method to obtain $\phi_c$ and OCR using the SBPM (ideal) data was outlined, and illustrated through a step by step solution. Using the theory developed in this work, the SBPM failure parameters can be used to determine the same parameters for many different modes of failure other than that of the SBPM.
(iv) A procedure has been proposed to evaluate the error in strain induced by initial movement of the pressuremeter cavity. It is based upon the determination of OCR using the anisotropic model and its independent estimation from other means. After evaluation of the error in strain, actual strength parameters can be calculated.
REFERENCES


[22] Henkel, D. J. and Sowa, V. A. (1963), "The Influence of Stress History on Stress Paths in Undrained Triaxial Tests on


APPENDIX I
WRITTEN PUBLICATIONS

Three Ph.D. students were partly supported by the research project. One Ph.D. dissertation has been completed:


The work reported herein will represent an important part of the Ph.D. theses of Mr. S. Thevanayagam and A. Skandarajah.

Two technical papers have already been published or accepted for publication:


There are several papers submitted for publication or in preparation which will provide adequate coverage and diffusion of the work described in this report. The following papers will acknowledge the support received from the U.S. AFOSR for this research (for the papers in preparation, tentative authors are given in parentheses in alphabetical order):

"Spacing Ratio - A New State Parameter for Anisotropically Consolidated Clays" (Chameau, Holtz, Sivakugan).


Prapaharan, S., Chameau, J. L. and Holtz, R. D., "Effect of Strain Rate on Undrained Strength Derived from Pressuremeter Test", submitted for publication in Geotechnique, April 1988.

"Anisotropy of Clays, a Novel Approach" (Chameau and Thevanayagam); this paper will be composed of two parts: Part I - theory, and Part II - application to pressuremeter testing.

We also expect to diffuse the research results through presentations at technical meetings, and interaction with our colleagues in the geotechnical community.