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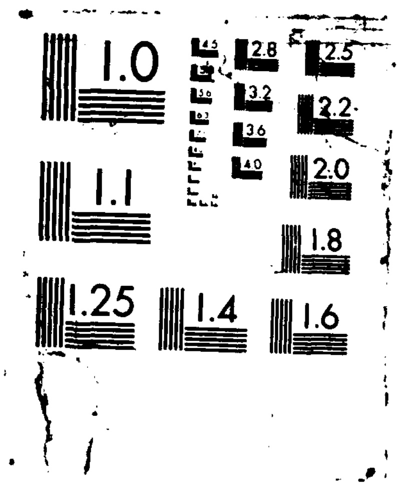
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Survey of European Studies on the Far Field Characteristics of a Source Oscillating and Translating Near a Free Surface

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The report presents a comprehensive survey of the leading papers written in French, German, and Russian on the formulation, construction, analysis of wave pattern, and evaluation of wave amplitude of the Green's function for a pulsating three-dimensional source which is translating in a straight line below the free surface of an infinite fluid. This important Green's function is of fundamental importance in analyses of ship motions and contains as two special limiting cases the widely used functions for zero frequency (Kelvin wake case) and zero speed (ring wave case). For each topic, the report describes in detail the major approaches used in these papers. The survey covers the period from the late 1940's to the early 1980's.

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SURVEY OF EUROPEAN STUDIES ON THE FAR FIELD CHARACTERISTICS OF A SOURCE OSCILLATING AND TRANSLATING NEAR A FREE SURFACE

1. INTRODUCTION

Starting from the late 1940's and continuing to the present day, a number of significant papers have been written in the European languages (French, German, and Russian) investigating various aspects of the Green's function for a pulsating three-dimensional source which is translating in a straight line below the free surface of an infinite fluid. This Green's function constitutes the basic building block for many modern day computer programs which analyze the motions, forces, and wave patterns of ships in a seaway. In addition, this Green's function contains as two special limiting cases the widely used functions for zero frequency (Kelvin wake case) and zero forward speed (circular ring wave case).

This report summarizes the leading papers written in French, German, and Russian on the formulation, construction, analysis of wave pattern, and approximate evaluation of wave amplitude of this important Green's function. Since many of the papers use similar methods in treating the above topics, the approach taken here is not to present a detailed summary of each paper. Instead, for each topic, the report describes in some detail the major approaches used in these papers.

On the whole, the notation used in the equations follows that of the original papers. However, for the sake of uniformity, the original equations have been rewritten (when necessary) to conform to the coordinate system shown in Fig. 1 and the consistent notation for principal variables, shown below. Figure 1 shows that the x axis is in the direction of motion of the source. In those studies where the source is taken to be fixed, the x axis then coincides with the direction of the current. The y axis is the horizontal axis perpendicular to x, so that the xy plane represents the undisturbed free surface. The z axis is directed vertically upwards.

Principal Notation

k	wavenumber
r	$= \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$
r_1, r'	$= \sqrt{(x-x')^2+(y-y')^2+(z+z')^2}$
R	$= \sqrt{(x-x')^2+(y-y')^2}$
U	speed of source or current
x,y,z	field point location
x',y',z'	source location

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β,γ	$= \tan^{-1}(y - y')/(x - x')$, spatial direction
θ,u	wave propagation direction
ν	$= U\omega/g$, ratio of source speed to phase speed
ω	circular frequency

2. CONSTRUCTION OF GREEN'S FUNCTION

2.1 Basic Formulation

It is well known that the Green's function $G(x,y,z,t)$ for an oscillating translating source located at (x',y',z') should satisfy the following conditions in a fixed coordinate system

$$\nabla^2 G = 0;\ z < 0;\ (x,y,z) \neq (x',y',z') \tag{2.1a}$$

$$\frac{\partial^2 G}{\partial t^2}(x,y,z,t) + g\,\frac{\partial G(x,y,z,t)}{\partial z} = 0 \quad \text{on} \quad z = 0 \tag{2.1b}$$

$$\lim_{z \to -\infty} \frac{\partial G}{\partial z} = 0 \tag{2.1c}$$

$$\lim_{R \to \infty} \nabla G = 0. \tag{2.1d}$$

In addition, a radiation condition of no incoming waves at $R \to \infty$ is needed to ensure uniqueness of the solution.

In a coordinate system moving with the velocity U in the x direction, the only explicit change is that Eq. (2.1b) is replaced by

$$\frac{\partial^2 G}{\partial t_1^2} - 2U\,\frac{\partial^2 G}{\partial x_1 \partial t} + U^2\,\frac{\partial^2 G}{\partial x_1^2} + g\,\frac{\partial G}{\partial z_1} = 0 \ \text{on}\ z_1 = 0 \tag{2.2}$$

where the subscript 1 denotes independent variables for a moving coordinate system. This follows from the well known kinematic relationship between time derivatives in fixed and moving coordinate systems

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_1} - U\,\frac{\partial}{\partial x_1} \tag{2.3a}$$

$$\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial t_1^2} - 2U\,\frac{\partial^2}{\partial t_1 \partial x_1} + U^2\,\frac{\partial^2}{\partial x_1^2}\,. \tag{2.3b}$$

Grekas [2] shows that the free surface equation (2.2) also applies in a fixed coordinate system for a fixed source in the presence of a uniform stream U in the $-x$ direction.

The construction of the Green's function in most studies uses the Fourier transform approach. Here, the description largely follows the lucid and detailed approach given by Bougis [1]. In particular, he gives the key differences between approaches using moving and stationary coordinate systems. Others who have used similar approaches include Grekas [2,3], Haskind [4], and Sretenskii [5]. The approach by Sretenskii, which gives a Fourier transform for the wave elevation itself, is also briefly described. The convolution time domain approach used by Brard [6,7] is described in the next section. The approach of Simmgen [8], which obtains G as a distribution of dipoles and is given in single integral form, is described in the last section.

2.2 Fourier Transform Approach

As is usual in this type of approach, Bougis expresses G in the fixed coordinate system as the sum of two terms

$$G = \frac{\cos \omega t}{r} + G_1(x,y,z,x',y',z',t) \tag{2.4}$$

where the first term is a Rankine source. Noting that the elemental wave solution

$$W(x,y,z) = e^{kz} e^{ik(x \cos\theta + y \sin\theta)} \tag{2.5}$$

satisfies Eqs. (2.1a) and (2.1c), G is written in the following Fourier transform form over wavenumber k and wave direction θ space

$$G_1 = \frac{1}{\pi} \operatorname{Re} \left\{ \int_{-\pi/2}^{\pi/2} d\theta \int_0^\infty \hat{G}_1(\theta,k;t) e^{kz} e^{ik(x \cos\theta + y \sin\theta)} k dk \right\}. \tag{2.6}$$

For the sake of brevity, the use of a fictitious viscosity coefficient by Bougis, which serves to determine the proper sense of the contour of integration around the poles to satisfy the radiation condition but which is ultimately set equal to zero, is omitted in the present description. The use of the fictitious viscosity is described in greater detail in Chapter 4, in connection with Haskind's work [4]. Using the key identity

$$\frac{1}{r} = \frac{1}{\pi} \operatorname{Re} \left\{ \int_{-\pi/2}^{\pi/2} d\theta \int_0^\infty e^{-k(z - z')} e^{ikw} dk \right\} \tag{2.7a}$$

where

$$w = (x - x') \cos\theta + (y - y') \sin\theta \tag{2.7b}$$

and noting that in a fixed coordinate system

$$\frac{dx'}{dt} = U \tag{2.8}$$

the free surface condition (2.1b) leads to the following differential equation for the transform $\hat{G}_1(\theta,k;t)$

$$\left[\frac{\partial^2}{\partial t^2} + gk \right] \hat{G}_1(\theta,k;t) = \Big[(\omega^2 + U^2 k^2 \cos^2\theta + gk) \cos \omega t \; -$$

$$2iUk\cos\theta\,\omega\sin\omega t\Big]\; e^{kz'}e^{ik(x'\cos\theta+y'\sin\theta)}\,\frac{1}{k}\,. \tag{2.9}$$

Bougis points out that it is not convenient to solve this equation since x' is a function of time. By expressing $\cos\omega t$ and $\sin\omega t$ in terms of complex exponentials and the following transformation between moving (x_1,y_1,z_1) and fixed (x,y,z) coordinate systems

$$x = x_1 + Ut \tag{2.10a}$$

$$y = y_1 \tag{2.10b}$$

$$z = z_1 \tag{2.10c}$$

Eq. (2.9) becomes

$$\left(\frac{\partial^2}{\partial t^2} + gk\right)\hat{G}_1(\theta,k;t) = \frac{1}{2k}\,e^{kz_1'}e^{-ik(x_1'\cos\theta+y_1'\sin\theta)}$$

$$\Big\{e^{i(\omega-Uk\cos\theta)t}\,[\omega^2 + U^2k^2\cos^2\theta + gk - 2Uk\cos\theta\,\omega] +$$

$$e^{-i(\omega+Uk\cos\theta)t}\,[\omega^2 + U^2k^2\cos^2\theta + gk + 2Uk\cos\theta\,\omega]\Big\}. \tag{2.11}$$

Solving this equation for $\hat{G}_1(\theta,k;t)$, discarding the transient solutions, again using the key identify (2.7), and returning to the fixed coordinate system by using Eqs. (2.10), the following form for G is obtained

$$G = \sum\left\{\frac{\pm\cos\omega t}{[(x-x')^2+(y-y')^2+(z\mp z')^2]^{1/2}}\right.$$

$$\left.-\frac{1}{\pi}\,\mathrm{Re}\left[e^{\pm i\omega t}\int_{-\pi/2}^{\pi/2}d\theta\int_0^{\infty}\frac{e^{k(z+z'+iw)}gk}{(\omega\mp Uk\cos\theta)^2-gk}\,dk\right]\right\} \tag{2.12}$$

where the terms are summed for the upper and lower signs.

Sretenskii [5] uses a similar approach to obtain G and proceeds further to obtain an expression for the wave elevation ζ in Fourier integral form. He considers a fixed coordinate system with flow U in the $+x$ direction past a fixed pulsating source located at $x' = 0$, $y' = 0$, with depth of submergence z'. Here, the sign of the term involving $2U$ in the free surface condition (2.2) is changed from minus to plus, resulting in

$$\frac{\partial^2 G}{\partial t^2} + 2U\frac{\partial^2 G}{\partial x\,\partial t} + U^2\frac{\partial^2 G}{\partial x^2} + g\frac{\partial G}{\partial z} = 0 \text{ on } z = 0. \tag{2.13}$$

As in the preceding formulation, G is taken to be the sum of a Rankine source, its mirror image about the free surface, and a harmonic function, as follows

$$G = \frac{Q}{4\pi}\left(\frac{1}{r} - \frac{1}{r'}\right) \cos \omega t + G' \cos \omega t + G'' \sin \omega t \tag{2.14}$$

where Q is the strength of the pulsating source. By defining G_1 as the complex sum of G' and G''

$$G_1 = G' + iG'' \tag{2.15}$$

substituting Eq. (2.14) into the free surface condition (2.13), and using the key identity (2.7a), G_1 is obtained in the following double integral form similar to the last term of Eq. (2.12)

$$G_1 = \frac{gQ}{4\pi^3} \int_0^\infty ke^{k(z-z')}dk \int_{-\pi}^{\pi} \frac{e^{ik(x \cos \theta + y \sin \theta)}}{gk - (\omega + Uk \cos \theta)^2}\, d\theta$$

$$= \frac{gQ}{4\pi^3} \int_0^\infty ke^{k(z-z')}dk \int_{-\pi}^{\pi} \frac{e^{ikR \cos (\theta - \gamma)}}{gk - (\omega + Uk \cos \theta)^2}\, d\theta \tag{2.16}$$

where use has been made of the polar coordinate relations

$$x = R \cos \gamma, \quad y = R \sin \gamma. \tag{2.17}$$

The wave elevation ζ is given by

$$\zeta = \text{Re} \left(i\omega G_1 + U \frac{\partial G_1}{\partial x} \right) e^{i\omega t} \quad \text{at } z = 0. \tag{2.18}$$

Substituting G_1 given by the second equality in Eq. (2.16) gives ζ in the following double integral form

$$\zeta = \text{Re} \left\{ \frac{igQ}{4\pi^2} \int_{-\pi}^{\pi} d\theta \int_0^\infty \frac{k(\omega + Uk \cos \theta)\, e^{-kz'} e^{ikR \cos (\theta - \gamma)}}{gk - (\omega + Uk \cos \theta)^2}\, dk \right\} e^{i\omega t}. \tag{2.19}$$

2.3 Convolution Time Domain Approach

Brard [6] starts with the two-dimensional case of a doublet at (x', z') moving in an arbitrary path parallel to the vertical xz plane and with strength M varying as a function of t. Letting $\mathrm{v} = x + iz$, $\mathrm{v}' = x' + iz'$, the potential of a doublet in an infinite fluid is

$$F_0 (\mathrm{v},t) = -\frac{\partial}{\partial z'} \log (\mathrm{v} - \mathrm{v}')M(t) = \int_0^\infty e^{ik(\mathrm{v}-\mathrm{v}')} M(t)dk. \tag{2.20}$$

Expressing G as the sum of F_0 and F

$$G = F_0 + F \tag{2.21}$$

and further, F in terms of its Fourier transform P

$$F(\mathrm{v},t) = \int_0^\infty e^{-ik\mathrm{v}} P(k,t)dk \tag{2.22}$$

the free surface condition (2.1b) leads to the following differential equation for P

$$\left[\frac{\partial^2}{\partial t^2} + gk\right] P = -\left[\frac{\partial^2}{\partial t^2} - gk\right] e^{ik\bar{v}'}M \tag{2.23}$$

where a bar denotes the complex conjugate. The solution of this equation for P and its substitution into Eq. (2.22) results in the following final expression for F (v,t) in double integral form

$$F(\mathrm{v},t) = \int_0^\infty e^{ik(\mathrm{v}-\bar{\mathrm{v}}')}M(t)dk$$

$$-2\int_0^\infty dk\, e^{-ik\mathrm{v}} \int_{t_0}^{t} \frac{\partial}{\partial \tau}(e^{ik\bar{\mathrm{v}}'}M)\cos\sqrt{gk}\ (t-\tau)d\tau \tag{2.24}$$

where the first term on the right hand side corresponds to an image doublet which is symmetrically located above the free surface.

For the generalization to the three-dimensional fixed coordinate system (x,y,z), the doublet term F_0 is replaced by the source with strength $S(t)$

$$\phi_0 = \frac{-S(t)}{r} = -\int_0^\infty e^{k(z-z')}J_0(kR)S(t)dk$$

$$= \frac{-1}{2\pi}\int_0^\infty dkS(t)\int_0^{2\pi} e^{ik[w+i(z+z')]}d\theta \tag{2.25}$$

where w is defined in Eq. (2.7b). Comparing Eqs. (2.20) and (2.25) shows that the generalization from two to three dimensions involves an additional integration of the wave propagation direction θ from 0 to 2π, and the following two related substitutions:

$$\mathrm{v} = x + iz \Longrightarrow x\cos\theta + y\sin\theta + iz \tag{2.26a}$$

$$\mathrm{v}' = x' + iz' \Longrightarrow x'\cos\theta + y'\sin\theta + iz'. \tag{2.26b}$$

The resulting expression for Φ, the three-dimensional equivalent of F, the regular part of the Green's function, then becomes

$$\Phi(x,y,z,t) = -\frac{S(t)}{r'} + \frac{1}{\pi}\int_0^\infty dk\int_0^{2\pi} d\theta e^{-ik(x\cos\theta+y\sin\theta+iz)} \times$$

$$\int_0^t \frac{\partial}{\partial \tau}\left[e^{ik[x'(\tau)\cos\theta+y'(\tau)\sin\theta-iz'(\tau)]}S(\tau)\right] \times$$

$$\left\{\cos[\sqrt{gk}\ (t-\tau)]\right\}d\tau \tag{2.27}$$

where, analogous to the two-dimensional case, the first term represents an image source symmetrically placed above the free surface.

The above expression represents the Green's function for a source with strength $S(t)$ and location $x'(t)$, $y'(t)$, $z'(t)$ varying arbitrarily with time. To specialize to the case considered in the present report of a sinusoidally pulsating source moving rectilinearly, such that

$$x'(t) = Ut,\ y'(t) = 0,\ z'(t) = \ const\ < 0,\ S(t) = S \cos \omega t \tag{2.28}$$

Φ becomes

$$\Phi(x,y,z,t) = \frac{S}{r'} + \frac{S}{4\pi} \int_0^{2\pi} dk \int_0^{2\pi} e^{-ik[x \cos \theta + y \sin \theta + i(z+z')]} d\theta \ \times$$

$$\int_{-\infty}^{t} \left[i(kU \cos \theta + \omega)\, e^{i(kU\cos \theta + \omega)\tau} + i\ (kU\cos \theta - \omega)\, e^{i(kU\cos \theta - \omega)\tau} \right] \times$$

$$\left[e^{-i\sqrt{gk}\,\tau}\, e^{i\sqrt{gk}\,t} + e^{i\sqrt{gk}\,\tau}\, e^{-i\sqrt{gk}\,t} \right] d\tau. \tag{2.29}$$

2.4 Superposition of Dipoles

Simmgen [8] carries out his formulation in a fixed (x,y,z) coordinate system, with a uniform flow U in the x direction. The double integral key identity in Eq. (2.7a) is rewritten in single integral form, as follows

$$\frac{1}{r} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \mathrm{Re}\, \frac{i}{w}\, d\alpha \tag{2.30}$$

where

$$w = (x - x') \cos \alpha + (y - y') \sin \alpha + i(z - z') = p + i(z - z'). \tag{2.31}$$

Since Re (i/w) represents a dipole in the complex pz plane, Eq. (2.30) shows that a Rankine source may be represented by the integral over the inclination α of a distribution of dipoles. Similar to the case of a source, the presence of a free surface for a pulsating dipole g_α requires the addition of a harmonic function k_α, as follows

$$g_\alpha = \mathrm{Re}\, \frac{i}{w} + k_\alpha \tag{2.32}$$

where the time-dependent multiplicative factor $e^{i\omega t}$ has been omitted. The Green's function G_2 is then obtained as the integral over α of g_α

$$G_2 = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} g_\alpha d\alpha \tag{2.33}$$

The harmonic function k_α is determined by the free surface condition in the form

$$\left[\left(i\, \frac{k_0}{\cos^2\alpha}\, \nu \cos \alpha + \frac{\partial}{\partial p} \right)^2 + \frac{k_0}{\cos^2\alpha}\, \frac{\partial}{\partial z} \right] g_\alpha = 0 \ \text{ on } z = 0 \tag{2.34}$$

where $\nu = U\omega/g$ and $k_0 = g/U^2$. Adopting an approach used by Haskind [10] to study the Green's function for the two-dimensional case of a pulsating, translating source, g_α is expressed in terms of the functions $F^\pm$

$$g_\alpha = \mathrm{Re} \left(\frac{i}{w} + \frac{i}{w_1} \right) + F^+(w_1) + F^-(w_1) \tag{2.35}$$

where

$$w_1 = w + 2iz' = p + i(z + z')$$

$$F^{\pm} = \frac{k_0}{\cos^2\alpha} \frac{1}{k_1 - k_2} \left[k_1 e^{-ik_1 w_1} \int_{-\infty}^{w_1} \frac{e^{ik_1 t}}{t} \, dt - k_2 e^{-ik_2 w_1} \int_{\mp\infty}^{w_1} \frac{e^{ik_2 t}}{t} \, dt \right] \tag{2.36}$$

$$k_{1,2}^{+} = \left[\frac{1}{2} + \nu \cos\alpha \pm \sqrt{\frac{1}{4} + \nu \cos\alpha} \right] \frac{k_0}{\cos^2\alpha}$$

$$k_{1,2}^{-} = \begin{cases} \left[\frac{1}{2} - \nu \cos\alpha \pm \sqrt{\frac{1}{4} - \nu \cos\alpha} \right] \frac{k_0}{\cos^2\alpha}, & \nu \cos\alpha < \frac{1}{4} \\ \left[\frac{1}{2} - \nu \cos\alpha \mp i \sqrt{\nu \cos\alpha - \frac{1}{4}} \right] \frac{k_0}{\cos^2\alpha}, & \nu \cos\alpha > \frac{1}{4} \end{cases}$$

Upon integrating Eq. (2.35) with respect to α, the following expression for G_2 is obtained

$$G_2 = \frac{1}{r} - \frac{1}{r'} + \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} [F^{+}(w_1) + \overline{F^{-}(w_1)}] \, d\alpha . \tag{2.37}$$

In order to reduce G_2 to single integral form, Simmgen defines the following changes of variables

$$\tau = k_0 \sigma = k(w_1 - t) \tag{2.38a}$$

$$\cosh\gamma = \frac{1}{\cos\alpha} . \tag{2.38b}$$

The purpose of the first change is to convert the integral expression for $F^{\pm}$ in Eq. (2.36) so that use can be made of the following relation

$$\int_{-\infty}^{\infty} \frac{e^{ikt}}{t} \, dt = 2\pi i . \tag{2.39}$$

Eggers [9] points out that the complex variable γ in the second change of variables is related to the direction of wave propagation, θ, by means of the intermediate real variable μ, as follows

$$\sin\theta = \tanh\mu \tag{2.40a}$$

$$\mu = \gamma - \pi i . \tag{2.40b}$$

The resultant final single integral form for G_2 is of the form

$$G_2 = \frac{1}{r} - \frac{1}{r_1} + k_0 \, i \int_{L_1 + L_2} \frac{\lambda(\gamma)}{\rho(\gamma)} \, e^{-ik_0\lambda(\gamma) w_1(\gamma)} d\gamma \tag{2.41}$$

where

$$\lambda(\gamma) = \left[\nu + \frac{1}{2} \cosh \gamma + \rho(\gamma) \right] \cosh \gamma$$

$$\rho(\gamma) = \sqrt{\frac{1}{4} \cosh^2\gamma + \nu \cosh \gamma} \, .$$

The paths L_1 and L_2 lie in two sheets of the γ-plane, as shown in Fig. 2, which is adapted from Fig. 3 of Simmgen. Eggers [9] points out that γ_0 in this figure is missing the term $i\pi/2$

$$\gamma_0 = x + i\epsilon + i\pi/2. \tag{2.42}$$

Otherwise, the symmetry between $+(y - y')$ and $-(y - y')$ would be lost.

3. FAR FIELD WAVE PATTERNS

Bougis [1], Grekas [2,3], Haskind [4], Sretenskii [5], Brard [6,7], Becker [11], and Eggers [12] study the far field wave patterns due to the Green's functions derived in the preceding chapter. With the exception of [2,3,4], which are discussed in the following chapter, the investigations are limited to obtaining the far field curves of constant phase, for example, the wave crests. Becker [11] points out that such a presentation may be considered to be an instantaneous photograph of the free surface with neighboring wave crests differing by a phase of 2π.

There are two principal methods for determining the curves of constant phase. The most common approach is to use the well known method of stationary phase [1,2,3,5,6,7]. For the Green's functions obtained by using the Fourier transform approach the description here will largely follow the concise and direct approach used by Sretenskii [5]. His unfortunate error in choosing the integration paths, which leads to an incorrect placement of his wave patterns, is clearly pointed out. The wave patterns shown are those recently obtained by Bougis [1]. A description is also given of the application of this method by Brard [6,7] to his convolution time domain Green's function, Eq. (2.29). The other method is the approach taken in [11,12], where the wave patterns are constructed from basic considerations concerning the frequencies of the waves observed in moving and stationary coordinate systems. This may be considered a more physical alternate approach as compared to the more mathematical method of stationary phase. The description here largely follows the formulation given by Becker [11]. The principal differences between the approach taken by Eggers [12] from that of Becker [11] are also pointed out.

3.1 Method of Stationary Phase Applied to Fourier Transform Approach

The usual procedure is to consider the double integral in the various expressions for Green's functions (Eqs. (2.12), (2.16), or (2.19)) as a single integral of the form

$$I = \int h(\theta) e^{iRf(\theta)} d\theta. \tag{3.1}$$

For large R, the method of stationary phase states that I may be approximated by

$$I = h(\theta_0) \left(\frac{2\pi}{|Rf''|} \right)^{1/2} e^{i(Rf \pm \pi/4)} \tag{3.2}$$

where θ_0 is given by

$$f'(\theta_0) = 0. \tag{3.3}$$

If there is more than one root, i.e., point of stationary phase, then Eq. (3.2) must be summed over all these roots. The entire Eq. (3.2) must be considered if one is interested in the wave amplitudes. This would involve an evaluation of the integral over k in order to obtain $h(\theta_0)$. However, if only the curves of constant phase are of interest, then only Eq. (3.3) need be considered. From either Eq. (2.16) for the Green's function G_1 or Eq. (2.19) for the wave elevation ζ, $f(\theta)$ is given by

$$f(\theta) = k \cos(\theta - \gamma) \tag{3.4}$$

where $k_{1,2}$ are the roots of the denominator of the integrand in Eqs. (2.16) or (2.19)

$$gk - (\sigma + Uk \cos\theta)^2 = 0 \tag{3.5a}$$

and are given by

$$k_1, k_2 = \frac{(g - 2\,U\sigma\cos\theta) \pm \sqrt{g(g - 4U\sigma\cos\theta)}}{2U^2\cos^2\theta}. \tag{3.5b}$$

Eqs. (3.5) arise due to the fact that the residues of the inner integral in Eqs. (2.16) or (2.19) are evaluated at the poles given by Eq. (3.5b). Taking the derivative of $f(\theta)$ in Eq. (3.4) and setting it equal to zero results in

$$\frac{\sin(\theta - \gamma)}{\sin\gamma} = \frac{-2U}{g}(\sigma + Uk\cos\theta). \tag{3.6}$$

Substituting the values of k_1,k_2 into Eq. (3.6) gives two equations relating the wave propagation direction θ to the spatial direction γ

$$-\text{ctn}\,\gamma = \tan\theta \pm \frac{2}{\sin 2\theta}\sqrt{1 - 4\nu\cos\theta} \tag{3.7}$$

where the upper sign is for k_1, the lower sign for k_2, and $\nu = U\omega/g$. The term $-4\,\nu\cos\theta$ under the square root sign suggests that the characteristics of the wave patterns may be significantly different for $\nu \lesseqgtr 1/4$.

Since Eq. (3.7) holds for either G_1 or ζ, this means that the far field curves of constant phase are identical for both G_1 and ζ. The actual amplitudes are, of course, different since the integrands are different in Eqs. (2.16) and (2.19).

The proper ranges of θ to be used in Eq. (3.7) are determined by placing the contours of integration in the inner integral with respect to k above or below the poles k_1 and k_2 to satisfy the radiation condition that the energy of all the wave systems move away from the source [1]. Sretenskii chooses paths of integration which surround all the poles with upper semicircles. Unfortunately, this leads to systems of divergent and transverse waves, which arise from the pole at k_1, lying upstream of the source with the resultant propagation of energy toward the source. Haskind [4] (as does Bougis [1]) uses a fictitious viscosity μ, which is ultimately set equal to zero, which serves to give the proper placement of the contours around the poles. He shows that the contour should always pass above k_2 but the contour around k_1 depends on θ. In particular, for $\cos\theta > 0$, i.e., $\theta < \pi/2$, the contour should pass below the pole k_1. With this change, the wave patterns obtained by Sretenskii would be similar to those of Becker [11], who shows two sets of wave patterns for $\nu \lesseqgtr 1/4$.

Bougis, who carries out a systematic and detailed implementation of the above procedure, shows that there are actually three regions: $0 < \nu < 1/4$, $1/4 < \nu < 1/\sqrt{2}$, and $1/\sqrt{2} < \nu$. These patterns are reproduced in Fig. 3. This figure shows that for $\nu < 1/4$ there are five waves, a set of ring waves and two complete sets of Kelvin like waves, each composed of transverse and divergent waves. For $\nu > 1/4$, there are four waves with a region in front of the source which is now disturbance free. The ring waves and the inner transverse and divergent waves remain, but the outer set of transverse waves has disappeared, leaving only the divergent waves. The only difference between the cases $1/4 < \nu < 1/\sqrt{2}$ and $1/\sqrt{2} < \nu$ is that in the former case, the forward part of the ring waves, near the wedge lines defining the boundaries of the disturbance free region, has θ-values which correspond to forward propagating waves. This fact had previously been noted by Eggers [12].

3.2 Method of Stationary Phase Applied to Convolution Time Domain Approach

Brard [6,7] also uses the method of stationary phase. However, instead of the single application noted above, this method must be applied three times to the triple integral time domain Green's function given by Eq. (2.29): first with respect to θ, then with respect to k, and finally with respect to τ. This last application leads to a solution of

$$d\Psi(\tau)/d\tau = 0 \tag{3.8a}$$

with $\Psi(\tau)$ given by

$$\Psi(\tau) = \frac{g(t-\tau)^2}{4R_\tau} \mp \omega\tau \tag{3.8b}$$

where R_τ is defined in Fig. 4. This figure shows a sketch of a triangle whose vertices are the field point m, and s_τ and s_t, the projections of the positions of the source at times τ and t onto the undisturbed free surface. The lengths of the sides are R_τ, R, and $U(t-\tau)$, and the base angles of the triangle may be interpreted as being γ and θ, respectively the field point angle and wave propagation direction, as above. Upon noting that

$$\frac{dR_\tau}{d\tau} = -U\cos\theta \tag{3.9}$$

$d\Psi(\tau)/d\tau = 0$ leads to

$$\cos\theta = \frac{2R_\tau}{U(t-\tau)} \pm \nu\frac{4R_\tau^2}{U^2(t-\tau)^2}\,. \tag{3.10}$$

Using the relation

$$\frac{R_\tau}{U(t-\tau)} = \frac{\sin\gamma}{\sin(\theta+\gamma)} \tag{3.11}$$

converts Eq. (3.10) to an equation between γ and θ, similar to Eq. (3.7). Physically, Eq. (3.10) corresponds to the finding of the positions of the source at times τ, s_τ, which give waves which reach field point m at current time t.

3.3 Geometrical Construction Technique

Becker considers the case of a fixed pulsating source in the presence of a uniform stream U in the $-x$ direction. He starts by noting that for an observer fixed in space, all wave processes occur at the source oscillation frequency f. Also, relative to a fluid at rest, the wave crests propagate in the direction θ at phase velocity V, which is a function of wavelength λ. In the presence of the current U, which retards the wave motion by the velocity $U \cos\theta$, the resultant velocity is therefore $V - U\cos\theta$. The frequency f of waves propagating past a fixed point is then related to this velocity by

$$\frac{V(\lambda) - U\cos\theta}{\lambda} = f. \tag{3.12}$$

Since f must always be positive, the signs in the numerator must be switched if $V(\lambda) < U\cos\theta$. With this understanding, Eq. (3.12) may be solved for $V(\lambda)$, as follows

$$U\cos\theta \pm f\,\lambda = V(\lambda) \tag{3.13}$$

where the upper or lower sign is taken according to whether $U(\lambda)$ is greater or less than $V\cos\theta$. The intersections of the two straight lines

$$f_1(\lambda) = U\cos\theta + f\,\lambda \tag{3.14a}$$

$$f_2(\lambda) = U\cos\theta - f\,\lambda \tag{3.14b}$$

with the $V(\lambda)$ curve gives the wavelengths as a function of θ, $\lambda(\theta)$.

In order to obtain a $\theta - \gamma$ relation, similar to the preceding Eqs. (3.7) and (3.10), Becker uses the following two properties of the nth curve of constant phase

$$\phi\,(x,y;\theta) \equiv n\lambda(\theta) - x\cos\theta - y\sin\theta = 0 \tag{3.15}$$

$$\frac{\partial\phi}{\partial\theta} = n\,\frac{d\lambda}{d\theta} + x\sin\theta - y\cos\theta = 0. \tag{3.16}$$

An expression for $d\lambda/d\theta$ is obtained by using Eq. (3.13). It turns out that the same $\theta - \gamma$ relation is obtained by using either the upper or lower sign. Differentiating Eq. (3.13) (with the upper sign) with respect to θ results in

$$-U\sin\theta = \frac{dV}{d\lambda}\,\frac{d\lambda}{d\theta} - f\,\frac{d\lambda}{d\theta}. \tag{3.17}$$

Using the following relationship between the phase velocity V and the group velocity V_g to obtain an expression for $dV/d\lambda$

$$V_g = V(\lambda) - \lambda\,\frac{dV}{d\lambda} \tag{3.18}$$

converts Eq. (3.17) to

$$\frac{d\lambda}{d\theta} = \frac{U\sin\theta}{f - \dfrac{V - V_g}{\lambda}}. \tag{3.19}$$

Using Eqs. (3.12) and (3.15) to eliminate f and λ from the right hand side of the above equation results in

$$\frac{d\lambda}{d\theta} = \frac{1}{n} \frac{U \sin\theta}{V_g - U\cos\theta} (x\cos\theta + y\sin\theta). \tag{3.20}$$

Substituting this equation into Eq. (3.16) and expressing x and y in terms of the polar coordinates R and γ results in the desired $\gamma - \theta$ relationship

$$\tan(\gamma - \theta) = \frac{U\sin\theta}{V_g - U\cos\theta} \tag{3.21}$$

which may also be transformed to the more direct form

$$\tan\gamma = \frac{V_g \sin\theta}{V_g\cos\theta - U}. \tag{3.22}$$

For the deep water case considered in this report, V_g is given by

$$V_g = \frac{1}{2} V = \frac{1}{2}\sqrt{\frac{g\lambda}{2\pi}}. \tag{3.23}$$

Becker then considers a velocity triangle whose sides are $\vec{U}$ directed along the x axis, $\vec{V}_g$ at angle θ, and the resultant energy transport $\vec{V}_E$ given by

$$\vec{V}_E = \vec{U} + \vec{V}_g. \tag{3.24}$$

He shows that the angle which $\vec{V}_E$ makes with the x-axis, γ^*, is given by

$$\tan(\gamma^* - \theta) = \frac{U\sin\theta}{V_g - U\cos\theta} \tag{3.25}$$

i.e., comparing with Eq. (3.21)

$$\gamma^* = \gamma. \tag{3.26}$$

That is, at a given point (x,y) in space, a wave normal is directed at angle θ, but the resultant energy is transported at the polar angle γ of the point.

Eggers [12] starts with the same frequency condition as Becker, Eq. (3.12). However, he carries out his analysis in terms of k instead of λ, and explicitly introduces the deep water relation (Eq. (3.23) expressed in terms of $k = 2\pi/\lambda$) from the start instead of at the end. As a result, his analysis is of a more mathematical nature and to a large extent resembles the method of stationary phase described in section (3.1) of this chapter.

4. ESTIMATES OF FAR FIELD WAVE AMPLITUDE

Only Grekas [2,3] and Haskind [4] give explicit formulas for the amplitudes of the far field waves. As shown below, the formulations are for small values of the parameter $\nu = U\omega/g$ or the velocity U.

4.1 Grekas's Approximate Formula

Since Reference [3] is basically a summary of Part 1 of the doctoral thesis [2], the description here summarizes the derivation given in Chapter 4 of [2]. The analysis is carried out in a fixed coordinate system for a pulsating source fixed in the presence of a horizontal stream. Grekas actually considers the stream to have components in both horizontal directions x and y. To conform to the convention used in this report, with no actual loss of generality, his analysis will be given for flow U in only the $+x$ direction. The crucial simplification of his approach is to consider U small such that U^2 can be neglected everywhere. In the case of the free surface condition, Eq. (2.13) is simplified to

$$\frac{\partial^2 G}{\partial t^2} + 2U \frac{\partial^2 G}{\partial x \partial t} + g \frac{\partial G}{\partial z} = 0 \text{ on } z = 0. \tag{4.1}$$

Taking the strength of the pulsating source to be given by

$$Q = Q^* \cos \omega t \tag{4.2}$$

G is then decomposed into the usual sum of a Rankine source and a function which is harmonic everywhere in the lower half space

$$G = G_1 + G_2 = \frac{-Q^* \cos \omega t}{4\pi r} + G_2. \tag{4.3}$$

Putting G_2 in the usual Fourier transform form, Eq. (2.6), and again noting the key identity (2.7), the use of the free surface condition (4.1) leads to G in the form

$$G = \frac{-Q^* \cos \omega t}{4\pi} \left(\frac{1}{r} - \frac{1}{r_1} \right)$$

$$- \frac{Q^*}{4\pi^2} \operatorname{Re} \left\{ e^{-i\omega t} \int_{-\pi}^{\pi} d\theta \int_0^{\infty} \frac{e^{k(z+z'+iw)}\, gk\, dk}{-(\omega^2 - 2k\omega U \cos\theta - gk)} \right\} \tag{4.4}$$

where w is defined in Eq. (2.7b). The neglect of U^2 in the free surface condition simplifies the denominator of the integrand to $gk + 2k\omega U \cos\theta - \omega^2$, which has a simple pole in k, whereas the more general free surface condition (2.2) leads to the more complex denominator in Eq. (2.12), $gk - (\omega \mp Uk \cos\theta)^2$, which has a double pole in k.

The equation for the "meridian" lines (or wave crest contours) of the deformed free surface is taken to be

$$h = \frac{-1}{g} \frac{\partial \phi}{\partial t} = h_1 + h_2 \tag{4.5}$$

where h_1 is a near field solution which goes to zero as $1/R^n$, $n \geq 1$, and h_2 is the far field solution. It should be emphasized h is not the total wave elevation since this would include the convective term $(U/g)\partial\phi/\partial x$. This term can, however, be easily obtained from the final form for h_2, given below in Eq. (4.9). The far field solution is put in the usual form

$$h_2 = -\frac{Q^* \omega}{2\pi} \operatorname{Re} \left\{ e^{-i\omega t} \int_{\theta_1}^{\theta_2} f(\theta)\, e^{iRg(\theta)}\, d\theta \right\} \tag{4.6}$$

where

$$g(\theta) = k_0 \cos(\theta - \beta) = \frac{\omega^2 \cos(\theta - \beta)}{g + 2\omega U \cos\theta}$$

$$f(\theta) = \frac{k_0\, e^{k_0 z'}}{g + 2\omega U \cos\theta}$$

$\beta = \tan^{-1} y/x$ is the spatial angle.

The quantity k_0 is the wavenumber in terms of the intrinsic frequency ω_0

$$k_0 = \frac{\omega_0^2}{g} = \frac{(\omega - U \cos\theta\, k_0)^2}{g} \tag{4.7a}$$

$$\approx \frac{\omega^2 - 2\omega\, U \cos\theta\, k_0}{g} \tag{4.7b}$$

where Eq. (4.7b) results by again neglecting the U^2 term. Solving Eq. (4.7b) gives k_0 in terms of ω as follows

$$k_0 = \frac{\omega^2}{g + 2\,\omega U \cos\theta}\,. \tag{4.8}$$

Applying the method of stationary phase to Eq. (4.6), the following final form for h_2 is obtained

$$h_2 = \frac{Q^* \omega}{2\pi\sqrt{R}}\ \mathrm{Re}\left\{ f(\theta_0) \sqrt{\frac{2\pi}{|g''(\theta_0)|}}\ e^{i[k_0(\theta_0)R \cos(\theta_0 - \beta) - \pi/4 - \omega t]} \right\} \tag{4.9}$$

where

$$g''(\theta_0) = \frac{-g \cos(\theta_0 - \beta)\, \omega^2}{(g + 2\omega U \cos\theta_0)^2} \tag{4.10a}$$

and θ_0 is the root of the $\theta - \beta$ relationship given by

$$g'(\theta_0) = g \sin(\beta - \theta_0) + 2\omega U \sin\beta = 0\,. \tag{4.10b}$$

Grekas's expression for $g''(\theta_0)$ erroneously has the denominator raised to a power of 3 instead of 2. Figure 5 shows that there is only a set of ring waves, as opposed to the ring waves plus two sets of ship transverse and divergent waves shown in Fig. 3 for $\nu < 1/4$.

4.2 Haskind's Approximate Formula

Haskind [4] uses a moving coordinate system, for which the free surface condition is given by Eq. (2.2). The Fourier transform technique is also used to obtain G_2 in the expression

$$Ge^{i\omega t} = \left[\frac{1}{r} - \frac{1}{r_1} + G_2 \right] e^{i\omega t}\,. \tag{4.11}$$

A fictitious viscosity μ is included in his formulation. This viscosity, although ultimately set equal to zero, serves to indicate the proper way of surrounding the poles in the integration with respect to k. Basically, the semi-circles surrounding the poles must be chosen such that their imaginary parts must have the same sign as the imaginary parts of the poles k'_1, k'_2 when $\mu \neq 0$

$$k'_{1,2} = k_{1,2} - if(\mu) \tag{4.12}$$

where k_1, k_2 are given in Eq. (3.5b). He shows that

$$\text{sign Im } k'_1 = - \text{ sign } \cos\theta \tag{4.13a}$$

$$\text{sign Im } k'_2 = - \text{ sign } \nu = - \text{ sign } \frac{U\omega}{g}. \tag{4.13b}$$

For $\nu > 0$, he gives the following definitive integral representation for G_2, including the paths of integration for the integral with respect to k

$$G_2 = F_1(x,y,z) + F_2(x,y,z) + F_3(x,y,z) \tag{4.14}$$

where

$$F_1 = -\frac{1}{\pi}\int_{-\pi/2}^{\pi/2}\int_{0(L_1)}^{\infty} \frac{k\, e^{[k(z+z'+iw)]}}{(k-k_1)\sqrt{1+4\nu\cos\theta}}\, dk\, d\theta$$

$$-\frac{1}{\pi}\int'^{\,\pi/2}_{-\pi/2}\int_{0(\bar{L}_1)}^{\infty} \frac{k\, e^{[k(z+z'-iw)]}}{(k-\bar{k}_1)\sqrt{1-4\nu\cos\theta}}\, dk\, d\theta$$

$$F_2 = \frac{1}{\pi}\int_{-\pi+\theta_0}^{+\pi-\theta_0}\int_{0(L_2)}^{\infty} \frac{k e^{[k(z+z'+iw)]}}{(k-k_2)\sqrt{1+4\nu\cos\theta}}\, dk\, d\theta$$

$$F_3 = -\frac{K}{\pi\nu^2}\int_{\pi-\theta_0}^{\pi+\theta_0}\int_0^{\infty} \frac{k e^{[k(z+z'+iw)]}}{(k-k_1)(k-k_2)\cos^2\theta}\, dk\, d\theta$$

$\bar{k}_{1,2}(\theta) = k_{1,2}(\theta+\pi)$

$'$ signifies that the integration is to exclude the interval $(-\theta_0, +\theta_0)$

$K = \omega^2/g$

$\theta_0 = 0$ for $\nu < \frac{1}{4}$

$= \cos^{-1}\frac{1}{4\nu}$ for $\nu > \frac{1}{4}$.

The contour L_1 connects the points $k = 0$ and $k = \infty$ by passing above $k = k_1$, the contour $\bar{L}_1$ connects these points by passing below $k = \bar{k}_1$, and the contour L_2 passes above $k = k_2$.

For values of $|\nu| < \frac{1}{4}$, Haskind gives the following asymptotic far field formula for G_2 in terms of the polar coordinates R, β

$$G_2 = 2(1 \pm 2\nu \cos\beta) \frac{\sqrt{2\pi k}}{R} e^{\{k(1+2\nu\cos\beta)[z + z' \mp i(R - x'\cos\beta - y'\sin\beta)] \mp i\pi/4\}} \tag{4.15}$$

where the upper sign is to be used for positive values of ν and the lower sign for negative values of ν. Unfortunately, Haskind does not provide any details of the derivation. However, similar to the formulation by Grekas, Eq. (4.9), the above equation gives rise to a single system of ring waves which propagate in all directions from the source. This again suggests that the free surface condition (2.2) gives rise to a simple pole in k. Since the term $U^2\, \partial^2/\partial x^2$ gives rise to a square term in k, it must have been neglected in the derivation.

5. ACKNOWLEDGEMENT

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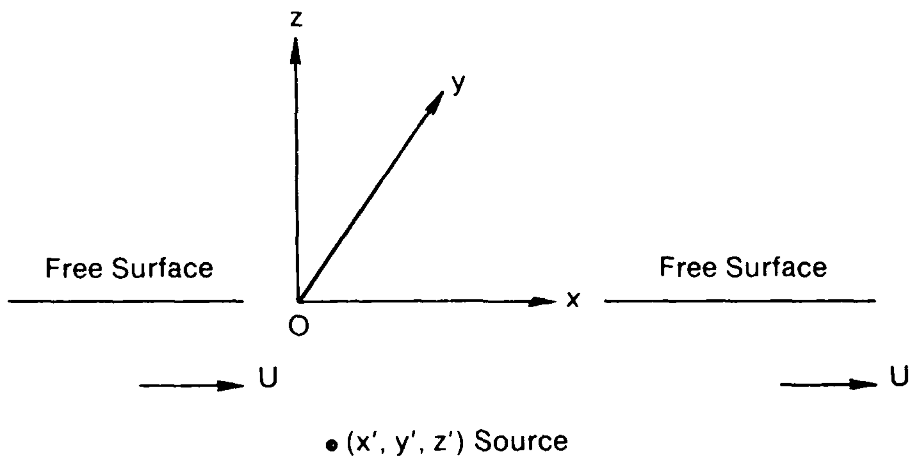
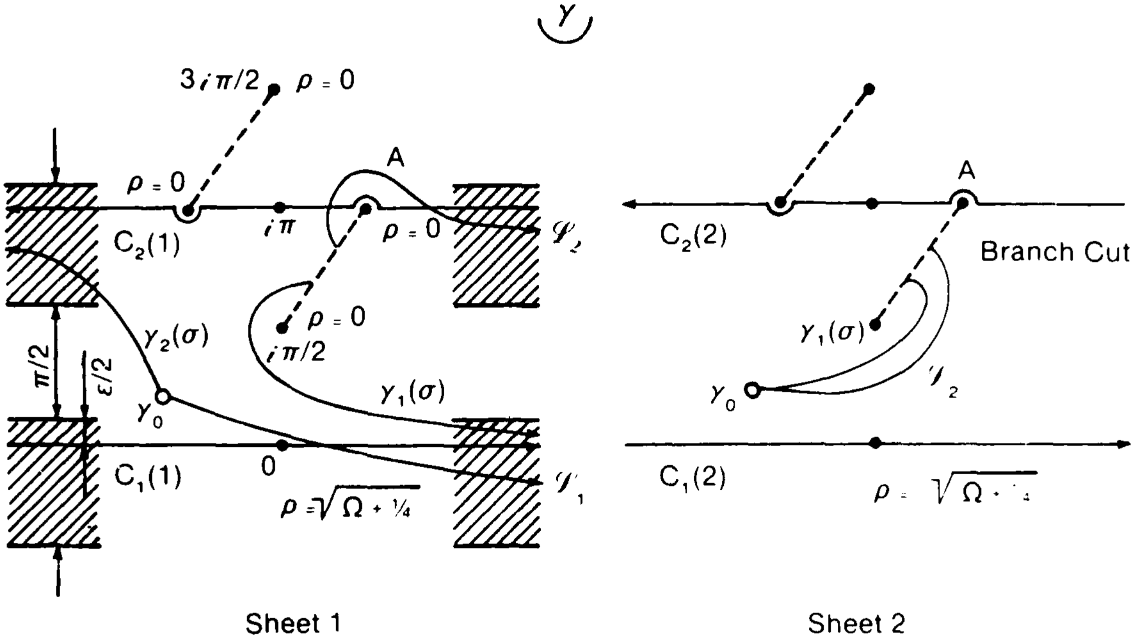


Fig. 1 — Definition of coordinate system

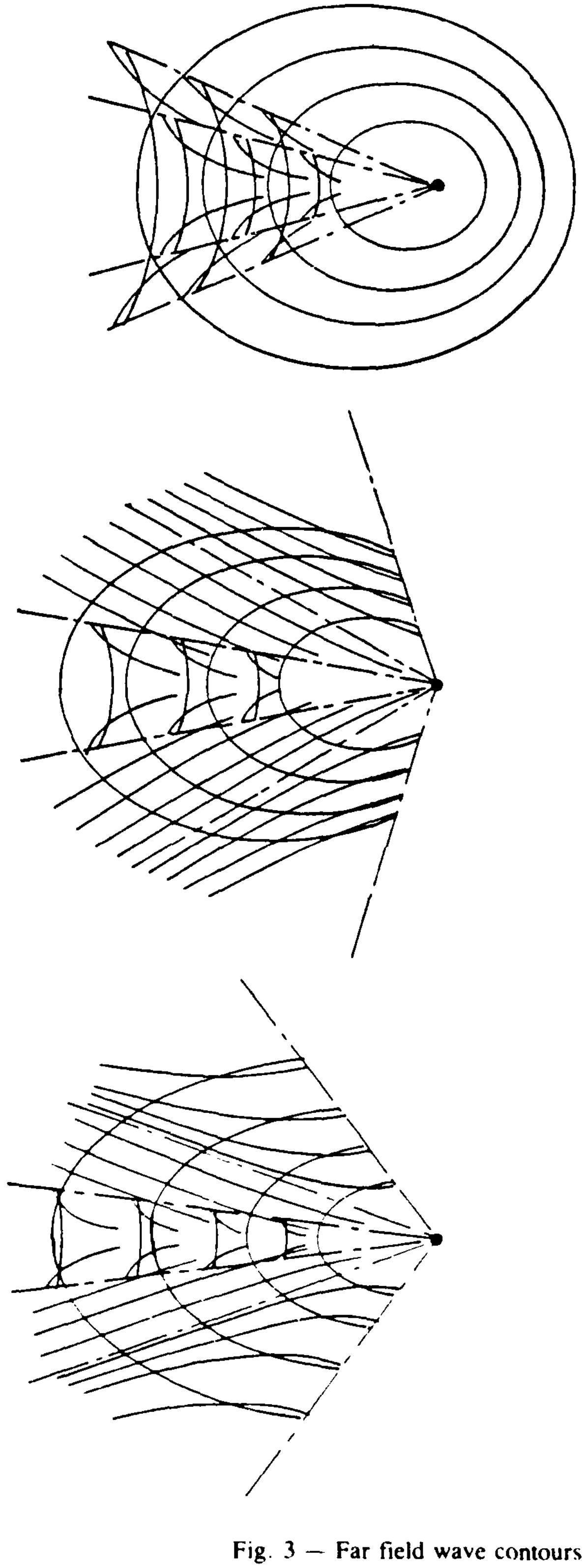


Inside of the shaded strips, the paths go to infinity

$$\gamma_0 = \chi + i\varepsilon, \ \chi = -\sinh^{-1}(x-x')/\sqrt{(y-y')^2 + (z+z')^2}$$

$$\varepsilon = -\tan^{-1}(y-y')/(z+z')$$

Fig. 2 — Sketch of integration paths L_1 and L_2 in the complex γ-plane (from Fig. 3 of Ref. [8])



$0 < \nu < 1/4$

$1/4 < \nu < 1/\sqrt{2}$

$1/\sqrt{2} < \nu$

Fig. 3 — Far field wave contours of equal phase
(from Fig. 7t of Ref. [1])

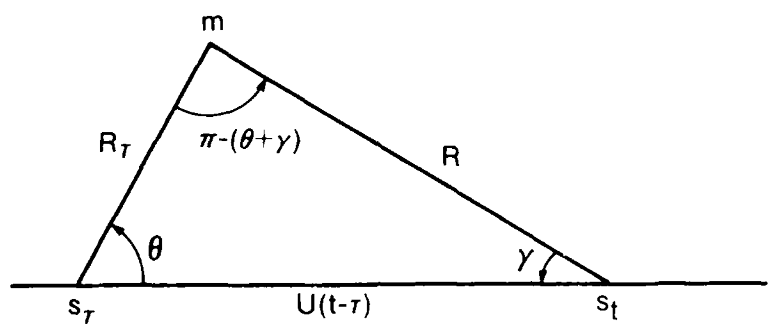


Fig. 4 — Sketch of locations of source and field point (from Fig. 1 of Ref. [6])

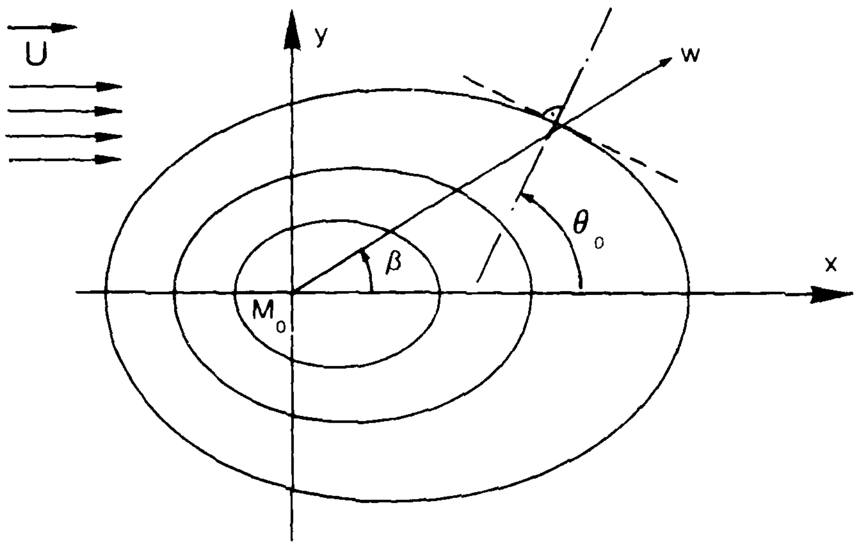


Fig. 5 — Far field wave crest contours for low speed case
(from Fig. 4 of Ref. [2])

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