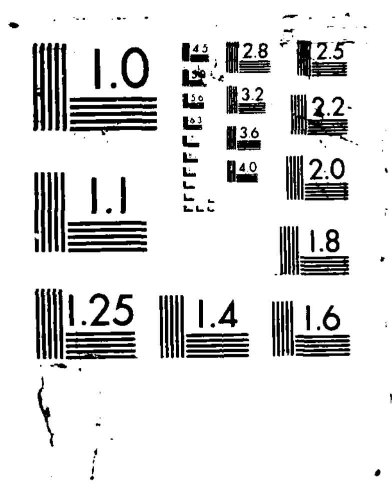
AD-A191 315 AN ALGORITHM FOR RANDOM ACCESS COMMUNICATION OVER A 1/1
NOISY CHANNEL..(U) VIRGINIA UNIV CHARLOTTESVILLE DEPT
OF ELECTRICAL ENGINEERING.. P PAPANTONI-KAZAKOS
UNCLASSIFIED 28 JAN 88 UVA/525415/EE88/110 F/G 25/5 NL


1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

(4)

DTIC FILE COPY

AD-A191 315

A Technical Report
Grant No. N00014-86-K-0742
September 1, 1986 - August 31, 1988

AN ALGORITHM FOR RANDOM ACCESS COMMUNICATION OVER A NOISY CHANNEL

Submitted to:

Office of Naval Research
Department of the Navy
800 N. Quincy Street
Arlington, VA 22217-5000

Submitted by:

P. Papantoni-Kazakos
Professor

M. Paterakis
Graduate Assistant

Report No. UVA/525415/EE88/110
January 1988

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901

A Technical Report
Grant No. N00014-86-K-0742
September 1, 1986 - August 31, 1988

AN ALGORITHM FOR RANDOM ACCESS COMMUNICATION OVER A NOISY CHANNEL

Submitted to:

Office of Naval Research
Department of the Navy
800 N. Quincy Street
Arlington, VA 22217-5000

Submitted by:

P. Papantoni-Kazakos
Professor

M. Paterakis
Graduate Assistant

Department of Electrical Engineering
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA

Report No. UVA/525415/EE88/110
January 1988

Copy No. _____

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS None
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release, distribution unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) UVA/525415/EE88/110	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION University of Virginia Dept. of Electrical Engineering 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Office of Naval Research Resident Representative
6c. ADDRESS (City, State, and ZIP Code) Thornton Hall Charlottesville, VA 22901	7b. ADDRESS (City, State, and ZIP Code) 818 Connecticut Avenue, N.W. Eighth Floor Washington, DC 20006
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Office of the Chief of Naval Research 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-86-K-0742
8c. ADDRESS (City, State, and ZIP Code) 800 N. Quincy Street Arlington, VA 22217-5000	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)

An Algorithm For Random Access Communication Over A Noisy Channel

12. PERSONAL AUTHOR(S)
P. Papantoni-Kazakos and M. Paterakis

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical	FROM 9/1/86 TO 8/31/88	1988, January 28	

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

We consider synchronous random access systems with noisy transmission channels. We model the channel noise as two types of erroneous feedbacks observed by the users. Specifically, with some probabilities, a successful channel slot and an empty channel slot can be seen as collision slots. For such systems, we analyze the stability conditions of a random access algorithm, both in its full sensing and limited sensing versions. We exhibit the superior resistance of the algorithm to the above feedback errors.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL R. N. Madan	22b. TELEPHONE (Include Area Code) (202) 696-4217 22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

Unclassified

I. Introduction

In data networks with bursty users whose population may be changing, the most appropriate multiple-access techniques are those belonging to the class of Random Access Algorithms (RAAs). The latter do not require knowledge of the user population in their operations, they can accommodate bursty independent users with relatively low delays, and they can induce satisfactory throughput.

The important performance characteristics of some RAA are: Stability, throughput, transmission delays, and insensitivity to channel errors. Stability ensures operation in the presence of changing user population, good throughput translates to satisfactory utilization of the channel bandwidth, and low delays are self explanatory. Insensitivity to channel errors means maintaining good performance characteristics when operating in noisy environments. The source of noise may be interferences from other transmissions, multi-path fading, etc., and its effects can be modelled by erroneously read feedbacks, [4].

In this paper, we consider the RAA in [1] and [3], and we present its analysis in the presence of two simultaneous types of feedback errors. The latter algorithm operates in both full sensing and limited sensing feedback environments, it is synchronous, it requires CNC binary (collision versus noncollision) feedback per slot, and in the absence of feedback errors and the presence of the limit Poisson user model, it attains throughput 0.43 (same as the algorithm in [2]). This algorithm also has simple operational properties which allow its analysis in the presence of strict delay limitations [1], and induces delays that are uniformly better than those induced by the algorithm in [2]. As first found in [3], and as will be further exhibited in this paper, it also has superior error resistance qualities.

The paper is organized as follows: In Section II, we present the system model and the description of the algorithm. In Section III, we include detailed stability analysis. In Section IV, we draw conclusions and discuss comparisons with other existing algorithms.

II. The Model and the Algorithm

We consider independent, identical, and packet transmitting users, with stationary and memoryless packet generating processes, who communicate with each other via a single common channel. We also consider a synchronous system, where the channel time is divided into slots, each of length identical to the length of a packet, and where each packet transmission can only start at the beginning of some slot. We call a slot empty (E), successful (S), or collision (C), if it is not occupied by some packet transmission(s), or is occupied by a single packet transmission, or at least two simultaneous packet transmissions have occured in it. respectively. We assume that in the event of a collision, all the involved packets are destroyed, and retransmission is then necessary. We consider the case where feedback per slot exists, and is received by the users at the end of each slot without propagation delays. We assume that the observed by the users feedback is CNC binary (collision versus noncollision), where an observed NC slot may be either empty (E) or successful (S). We also assume that due to noisy conditions, the following feedback errors exist: An E slot may be seen by the users as a C slot, with probability ε. An S slot may be seen by the users as a C slot, with probability δ. A C slot is always read correctly as a C slot by all the users.

For the system described above, we adopt the RAA in [3]. This RAA utilizes a window Δ on the arrival axis, it is implemented independently by each user in the system, and has a full sensing and a limited sensing versions. The full sensing version requires that each user know the total feedback history of the system, at all times. The limited sensing version only requires that each user observe the feedback continuously, from the time he generates a packet to the time this packet is successfully transmitted. For completeness, we describe here both versions of the algorithm. In our descriptions, time will be measured in slot units, where slot t occupies the time interval $[t, t+1)$. We will then denote by x_t the feedback of slot t, as seen by the users in the system, where x_t is either C or NC.

The Full Sensing Algorithm

The algorithm utilizes a window of length Δ. Let t be a time instant such that, for some $t_1<t$, all the packet arrivals in $(0, t_1]$ have been successfully transmitted by the algorithm and there is no information regarding the arrival interval $(t_1, t]$, and such that t corresponds to the beginning of some slot. The instant t is then called Collision Resolution Point, (CRP), the arrival interval $(0, t_1]$ is called "resolved interval", and the interval $(t_1, t]$ is called "the lag at t". In slot t, the packet arrivals in $(t_1, t_2 \overset{\Delta}{=} \min (t_1 + \Delta, t)]$ attempt transmission, and the arrival interval $(t_1, t_2]$ is then called the "examined interval". If $x_t = NC$, then $(t_1, t_2]$ contains at most one packet and is resolved at t. If $x_t = C$, instead, then $(t_1, t_2]$ either contains at least two packets or it contains at most one packet but erroneous feedback has been observed. In the latter case, a collision at t is perceived by the users, and its resolution starts with slot t+1. Until the perceived collision at t is resolved, no arrivals in (t_2, ∞) are allowed transmission. The time period required for the resolution of the latter perceived collision is called the Collision Resolution Interval, (CRI). During some CRI, each user acts independently via the utilization of a counter whose value at time t is denoted r_t. The counter values can be either 1 or 2, and they are updated and utilized according to the rules below.

1. The user transmits in slot t, if and only if $r_t = 1$. A packet is successfully in t, if and only if $r_t = 1$ and $x_t = NC$.
2. The counter values transition in time as follows:
   (a) If $x_{t-1} = NC$ and $r_{t-1} = 2$, then $r_t = 1$
   (b) If $x_{t-1} = C$ and $r_{t-1} = 2$, then $r_t = 2$
   (c) If $x_{t-1} = C$ and $r_{t-1} = 1$, then
$$r_t = \begin{cases} 1, \text{ with probability } 0.5 \\ 2, \text{ with probability } 0.5 \end{cases}$$

Remark 1. From the operations described above, it is not hard to see that a CRI which starts with a C slot ends the first time two consecutive NC slots occur. Even in the erroneous feedback environment considered here, upon such occurence, the users "know" with certainty that some CRI has ended.

The Limited Sensing Algorithm

Let some user generate a new packet within the time interval $[t_1, t_1 + 1)$. Then, he immediately starts observing the feedback sequence $\{x_t\}_{t\geq t_1}$, starting with the feedback x_{t_1}. Let us define the sequence $\{t_i\}_{i\geq 2}$, as follows: t_2 is the first time after t_1, such that $x_{t_2} = x_{t_2-1} = NC$. Then, as explained above, t_2 corresponds to the ending slot of some CRI, and from $t_2 + 1$ on, the user can identify the ending slots of CRIs induced by the algorithm (where some of the latter CRIs may have unity length). Each t_i corresponds to the ending slot of a CRI, and t_{i+1} is the first after t_i observed by the user such point. At t_i, the user updates his arrival instant to the value $t_1^{(i)} \stackrel{\Delta}{=} t_i + (i-2)\Delta$; we call the sequence $\{t_1^{(i)}\}_{i\geq 2}$, updates. Let t_k be such that: $t_k \varepsilon \{t_i\}_{i\geq 2}$, $t_1^{(i)} < t_i - 1 - \Delta$; $\forall i \leq k-1$, and $t_1^{(k)} > t_k - 1 - \Delta$. Then, in slot t_k+1, the user enters a CRI, and transmits his packet successfully during its process. The operations of the algorithm during the latter CRI are exactly as described under its full sensing version.

Remark 2. From the above description, it is clear that the following global operations are induced by the limited sensing version of the algorithm: Let T be a slot that corresponds to the end of some CRI. Then, in slot T+1, all the users with current updates in $(t-\Delta-1, T-1]$ transmit. If x_{T+1} = NC, then the CRI which started with slot T+1 lasts one slot, and a new CRI starts with slot T+2. If x_{T+1} = C, instead, then a collision is perceived by the users whose resolution starts with slot T+2. No arrivals that did not participate in the perceived collision at T+1 are transmitted, until the latter is resolved. During the collision resolution process, the users operate as in items 1 and 2, in the description of the full sensing version of the algorithm.

III. Stability Analysis

For the stability analysis, we will adopt the limit Poisson user model (infinitely many Bernoulli users). Indeed, as proven in [6], the throughput obtained under this user model, is a lower bound to the throughput in the presence of any number of independent and identical users whose packet generating processes are memoryless. We note that both the full sensing and the limited sensing versions of the algorithm attain identical throughputs, in the presence of the limit Poisson user model; the delays induced by the latter, however, are generally longer than those induced by the former. Thus, in our stability analysis, it suffices to consider the full sensing version of the algorithm only.

Consider the system model in Section II, and the full sensing version of the algorithm. Let the system start operating at time zero, and let then $\{C_i\}_{i\geq 1}$ denote the sequence of lags induced by the algorithm. The first lag corresponds to the empty slot zero; thus, $C_1 = 1$. In addition, $\{C_i\}_{i\geq 1}$ is a Markov chain whose state space is at most countable. To see that, let us define:

$l_{u,d}$: The number of slots needed to examine an interval of length u, given that the lag equals d.

$P(l \mid u,d)$: Given that the interval to be examined has length u and the lag equals d, the probability that the corresponding collision resolution interval has length l.

From the operations of the algorithm, we then have:

$$C_{i+1} = \begin{cases} l_{d,d}\ , \text{ if } C_i = d \leq \Delta\ ; \text{w.p } P(l \mid d,d) \\ C_i - \Delta + l_{\Delta,d}\ , \text{ if } C_i = d > \Delta\ ; \text{w.p. } P(l \mid \Delta,d) \end{cases} \tag{1}$$

We, therefore, conclude that $\{C_i\}_{i\geq 1}$ is a Markov chain with state space the countable set $E = \{x\geq 1 : x = k-m\Delta,\ k,\ m\ \varepsilon\ N\}$. It can be seen that any state can be reached from any other; therefore, $\{C_i\}_{i\geq 1}$ is an irreducible Markov chain. Since $P(C_{i+1} = 1 \mid C_i = 1)>0$, we conclude that $\{C_i\}_{i\geq 1}$ is also aperiodic. Thus, Pake's Lemma [5] applies, and gives that the following condition is sufficient for the ergodicity of the Markov chain:

$$E(l \mid \Delta, d) < \Delta \tag{2}$$

On the other hand, if $E(l \mid \Delta, d) > \Delta$, then the Markov chain is not ergodic, and the system is unstable. Let L_k denote the expected length of a collision resolution interval given that it starts with a collision of multiplicity k. We can then write:

$$E(l \mid \Delta, d) = \sum_{k=0}^{\infty} L_k \, e^{-\lambda\Delta} \frac{(\lambda\Delta)^k}{k!} \tag{3}$$

In Section III.A, we show that:

(i) L_k ; $k \geq 0$ can be computed recursively, and

(ii) L_k are quadratically upper bounded, $L_k \leq L_k^u \stackrel{\Delta}{=} \alpha\, k^2 + \beta\, k + \gamma$; $k \geq 2$.

Due to the upper bound on L_k, we conclude then that the following condition is sufficient for stability:

$$\sum_{k=0}^{30} L_k \, p(k \mid \Delta) + \sum_{k=31}^{\infty} L_k^u \, p(k \mid \Delta) < \Delta \tag{4}$$

where

$$p(k \mid \Delta) = e^{-\lambda\Delta} \frac{(\lambda\Delta)^k}{k!}$$

After some manipulations, we conclude that (4) is equivalent to:

$$f(\lambda\Delta) = \sum_{k=0}^{30} L_k \, p(k \mid \Delta) + \alpha \left\{ (\lambda\Delta)^2 + \lambda\Delta - \sum_{k=0}^{30} k^2 \, p(k \mid \Delta) \right\} +$$

$$+ \beta \left[\lambda\Delta - \sum_{k=0}^{30} k \;\, p(k \mid \Delta) \right] + \gamma \left[1 - \sum_{k=0}^{30} p(k \mid \Delta) \right] < \Delta \tag{5}$$

Let us now define:

$$x \stackrel{\Delta}{=} \lambda\Delta \tag{6}$$

Then, from (5) and (6), we conclude that, for the stability of the algorithm, it is sufficient that the input rate λ satisfies the following inequality.

$$\lambda < \sup_{x \geq 0} \frac{x}{f(x)} \tag{7}$$

The following condition specifies a region of λ values for which the algorithm is unstable.

$$\lambda > \sup_{x \geq 0} \frac{x}{g(x)} \tag{8}$$

where $g(x) = \sum_{k=0}^{30} L_k \, e^{-x} \, x^k / k!$

The maximization of expessions in (7) and (8) has been done numerically, and provides the throughput, as well as the optimal window size Δ.

III.A Computation of L_k

We define:

$G_{n,k-n}$: The expected number of slots needed by the algorithm, for the successful transmission of k packets, given that n of those packets have counter values equal to one, and k-n of the packets have counter values equal to two.

Notice that $L_k = G_{k,0}$, for $k \geq 2$, while $L_k \neq G_{k,0}$, for $k<2$. We first show how we compute L_0 and L_1.

(i) Computation of L_0:

From the operation of the algorithm we have

$$L_0 = \begin{cases} 1 \; ; \text{ w.p. } (1-\varepsilon) \\ 1 + G_{0,0} \; ; \text{ w.p. } \varepsilon \end{cases} \tag{9a}$$

where

$$G_{0,0} = \begin{cases} 1 + L_0 \; ; \text{ w.p. } (1-\varepsilon) \\ 1 + G_{0,0} \; ; \text{ w.p. } \varepsilon \end{cases} \tag{9b}$$

From (9a) and (9b) we find that

$$L_0 = \frac{1}{(1-\varepsilon)^2} \tag{9c}$$

(ii) Computation of L_1

It was found that L_1 satisfies the following

$$L_1 = \begin{cases} 1 \; ; \text{ w.p. } (1-\delta) \\ 1+G_{1,0} \; ; \text{ w.p. } 0.5\delta \\ 1+G_{0,1} \; ; \text{ w.p. } 0.5\delta \end{cases} \tag{10a}$$

where $G_{1,0}$ and $G_{0,1}$ satisfy the following

$$G_{1,0} = \begin{cases} 1+L_0 \; ; \text{ w.p. } (1-\delta) \\ 1+G_{1,0} \; ; \text{ w.p. } 0.5\delta \\ 1+G_{0,1} \; ; \text{ w.p. } 0.5\delta \end{cases} \tag{10b}$$

$$G_{0,1} = \begin{cases} 1+ L_1 \; ; \text{ w.p. } (1-\varepsilon) \\ 1+G_{0,1} \; ; \text{ w.p. } \varepsilon \end{cases} \tag{10c}$$

From (10a), (10b), and (10c) we find that

$$L_1 = \left[1- \frac{\delta}{2-\delta} \right]^{-1} \left[1+ \frac{\delta}{2(1-\varepsilon)} + \frac{\delta}{(2-\delta)} \left[1+ \frac{(1-\delta)}{(1-\varepsilon)^2} + \frac{\delta}{2(1-\varepsilon)} \right] \right] \tag{10d}$$

(iii) Computation of L_k, for $k\geq 2$.

From the operation of the algorithm we obtain:

$$G_{n,k-n} = 1 + \sum_{i=0}^{n} \binom{n}{i} 2^{-n} G_{i,k-i} \;\; ; \; n\geq 2 \, , \, k\geq n \tag{11a}$$

where

$$G_{0,k} = \frac{1}{(1-\varepsilon)} + L_k \tag{11b}$$

$$G_{1,k} = \frac{1+L_k(1-\delta) + 0.5\delta\left[\frac{1}{(1-\varepsilon)} + L_{k+1}\right]}{1-0.5\delta} \tag{11c}$$

It can be shown by induction that $G_{n,k-n}$ has the following form

$$G_{n,k-n} = A_n^{(1)} G_{k,0} + A_n^{(2)} G_{k-1,0} + A_n^{(3)} \;\; ; \;\; 2\leq n\leq k \tag{12}$$

; where $A_n^{(i)}$, i=1,2,3 are independent of k and can be computed recursively as follows:

$$A_2^{(1)} = \frac{2+\delta}{6-3\delta} \, , \, A_2^{(2)} = \frac{4(1-\delta)}{6-3\delta} \, , \, A_2^{(3)} = \frac{14-3\delta-12\varepsilon + 4\varepsilon\delta}{3(1-\varepsilon)\,(2-\delta)} \tag{13a}$$

$$A_n^{(1)} = [1-2^{-n}]^{-1} \, 2^{-n} \left\{ 1+\frac{\delta n}{(2-\delta)} + \sum_{i=2}^{n-1} \binom{n}{i} A_i^{(1)} \right\} \;\; , \;\; n\geq 3 \tag{13b}$$

$$A_n^{(2)} = [1-2^{-n}]^{-1} \, 2^{-n} \left\{ \frac{n(1-\delta)}{(1-0.5\delta)} + \sum_{i=2}^{n-1} \binom{n}{i} A_i^{(2)} \right\} \, , \; n\geq 3 \tag{13c}$$

$$A_n^{(3)} = [1-2^{-n}]^{-1} \left\{ 1+ \frac{n2^{-n}}{(1-0.5\delta)} + \frac{2^{-n}}{(1-\varepsilon)} + \frac{\delta n 2^{-n}}{(2-\delta)(1-\varepsilon)} + 2^{-n} \sum_{i=2}^{n-1} \binom{n}{i} A_i^{(3)} \right\}, \; n\geq 3 \tag{13d}$$

For n=k, expression (12) gives:

$$L_k = G_{k,0} = \frac{A_k^{(2)}}{1-A_k^{(1)}} L_{k-1} + \frac{A_k^{(3)}}{1-A_k^{(1)}} \;\; , \;\; k\geq 2 \tag{14}$$

Expression (14) together with the recursions in (13) provide a mean for the computation of L_k, $k\geq 2$.

Development of an upper bound on L_k

It can be seen by induction that:

$$A_k^{(1)} \leq \frac{1}{3} + \frac{\delta}{2-\delta} \;\; ; \, k\geq 2 \tag{15a}$$

$$\frac{A_k^{(2)}}{1-A_k^{(1)}} = 1 \; ; \, k\geq 2 \tag{15b}$$

$$A_k^{(3)} \leq \frac{(2-2\varepsilon+\delta)}{(2-\delta)(1-\varepsilon)} k + \frac{1}{(1-\varepsilon)} \tag{15c}$$

Here, we prove (15b) ; the proofs for (15a) and (15c) are similar. By summing (13b) and (13c) and after some simple manipulations we obtain:

$$A_n^{(1)} + A_n^{(2)} = \frac{1+n+\sum_{i=2}^{n-1} \binom{n}{i} (A_i^{(1)} + A_i^{(2)})}{2^n - 1} \tag{16}$$

We also have that $A_2^{(1)} + A_2^{(2)} = 1$. Next we assume that $A_i^{(1)} + A_i^{(2)} = 1$ for $i = 3,4,.., n\text{-}1$. Expression (16) then gives

$$A_n^{(1)} + A_n^{(2)} = \frac{1+n+\sum_{i=2}^{n-1} \binom{n}{i}}{2^n - 1} = 1 \tag{17}$$

which completes the proof.

From (14) and (15) we finally find:

$$L_k \leq L_{k-1} + \frac{3(2-2\varepsilon+\delta)}{(1-\varepsilon)(4-5\delta)} k + \frac{3(2-\delta)}{(1-\varepsilon)(4-5\delta)} \text{ , } k\geq 2 \tag{18}$$

from which we can show that

$$L_k \leq \frac{3(2-2\varepsilon+\delta)}{2(1-\varepsilon)(4-5\delta)} k^2 + \frac{3(6-2\varepsilon-\delta)}{2(1-\varepsilon)(4-5\delta)} k + \left[L_1 - \frac{6(2-\varepsilon)}{(1-\varepsilon)(4-5\delta)} \right] \stackrel{\Delta}{=} L_k^u = \alpha k^2 + \beta k + \gamma \tag{19}$$

III.B <u>Computation of L_k, for the Capetanakis Dynamic Algorithm</u>

For the operation of the algorithm the interested reader is referred to [2] and [5]. Here, the quantitites L_k, $k\geq 2$ can be computed recursively as follows:

$$L_k = [1-2\,2^{-k}]^{-1} \left\{ 1+2\,L_0\,2^{-k} + 2\sum_{i=1}^{k-1} L_i \binom{k}{i} 2^{-k} \right\} \text{ , } k\geq 2 \tag{20}$$

where

$$L_0 = \frac{1}{(1-2\varepsilon)} \text{ and } L_1 = \frac{1-2\varepsilon+\delta}{(1-2\varepsilon)(1-\delta)} \tag{21}$$

Moreover the following upper bound on $L_k, k\geq 4$, has been found.

$$L_k \leq \left[\frac{3(1-\varepsilon)}{(1-2\varepsilon)} + \frac{2(\delta-\varepsilon)}{(1-2\varepsilon)(1-\delta)} \right] k - 1 \stackrel{\Delta}{=} L_k^u = \mu k + v \tag{22}$$

From (2), and due to the upper bound in (22), we conclude that the following condition is sufficient for stability:

$$h(\lambda\Delta) = \sum_{k=0}^{30} L_k\, p(k\,|\,\Delta) + \mu \left[\lambda\Delta - \sum_{k=0}^{30} k\, p(k\,|\,\Delta) \right] + v \left[1 - \sum_{k=0}^{30} k\, p(k\,|\,\Delta) \right] < \Delta \tag{23}$$

or equivalently

$$\lambda < \sup_{x \geq 0} \frac{x}{h(x)} \tag{24}$$

,where $x = \lambda\Delta$.

The following condition specifies a region of the input rate values λ, for which the algorithm is unstable.

$$\lambda > \sup_{x \geq 0} \frac{x}{p(x)} \tag{25}$$

where $p(x) = \sum_{k=0}^{30} L_k \, p(k \mid \Delta)$

IV. Conclusions and Comparisons

We analyzed the proposed algorithm, as well as the Capetanakis's Dynamic Algorithm [2], (in [5], the Capetanakis's Nondynamic algorithm is considered). Binary CNC observed feedback was assumed. Both algorithms attain the same throughput ($\lambda^* = 0.429$) when $\varepsilon = \delta = 0$.

In Table 1, the throughputs for the proposed algorithm and the Capetanakis's Dynamic Algorithm are presented for various values of the error probabilities ε and δ. The values of ε and δ have been chosen to demonstrate that the proposed algorithm is remarkably insensitive to feedback channel errors. Even for the practically extreme values $\varepsilon = \delta = 0.1$, the throughput is almost 90% of its value in the error free case. From the results presented in Table 1, we also conclude that the proposed algorithm allows operation (positive throughput) as long as $\varepsilon < 1$ and $\delta < 1$, while if $\varepsilon \geq 0.5$ the throughput for the Capetanakis's algorithm is then zero. We notice for example that for $\varepsilon = 0.5$ and $\delta = 0$, the proposed algorithm attains throughput as high as $\lambda^* = 0.325$.

References

[1] M. Paterakis, L. Georgiadis, and P. Papantoni-Kazakos, "A Full Sensing Window Random-Access Algorithm for Messages with Strict Delay Constraints," Algorithmica, International Journal in Computer Science, Springer-Verlag, Special Issue on Algorithms for Computer Communications, to appear.

[2] J. J. Capetanakis, "Tree Algorithms for Packet Broadcast Channel," IEEE Trans. Inform. Theory, Vol. IT-25, pp. 505-515, Sept. 1979.

[3] M. Paterakis, and P. Papantoni-Kazakos, "A Simple Window Random Access Algorithm with Advantageous Properties," University of Virginia, Electrical Engrg. Dept. Technical Report UVA/525415/EE88/106, July 1987. Also submitted for publication.

[4] Ayal Bar-David, and Moshe Sidi, "Collision Resolution Algorithms in Multi-Station Packet-Radio Networks," submitted for publication.

[5] J. L. Massey, "Collision Resolution Algorithms and Random Access Communications," in Multi-User Communication, Ed. G. Longo, Springer-Verlag, CISM.

[6] M. Paterakis, L. Georgiadis and P. Papantoni-Kazakos, "On the Relation Between the Finite and the Infinite Population Models for a Class of RAA's," IEEE Transactions on Communications, Vol. COM-35, No. 11, pp. 1239-1240, November 1987.

Table 1

ε	δ	λ^* proposed alg.	λ^* (Cap. Dyn. alg.)
0.0	0.0	0.4295	0.4295
0.0	0.01	0.4248	0.4258
0.0	0.10	0.3873	0.3920
0.0	0.20	0.3463	0.3535
0.0	0.40	0.2655	0.2731
0.0	0.50	0.2251	0.2310
0.0	0.70	0.1422	0.1429
0.0	0.90	0.0526	0.0049
0.01	0.0	0.4274	0.4272
0.10	0.0	0.4117	0.4043
0.20	0.0	0.3930	0.3706
0.40	0.0	0.3503	0.2329
0.45	0.0	0.3382	0.1524
0.5	0.0	0.3250	0.0000
0.1	0.1	0.3706	0.3672
0.2	0.2	0.3139	0.2972
0.3	0.3	0.2589	0.2166
0.4	0.4	0.2064	0.1205
0.3	0.5	0.1885	0.1511
0.3	0.7	0.1183	0.0886
0.7	0.7	0.0699	0.0000
0.9	0.9	0.0105	0.0000

DISTRIBUTION LIST

Copy No.	
1 - 6	Director Naval Research Laboratory Washington, D.C. 20375 Attention: Code 2627
7	Dr. R. N. Madan Electronics Division, Code 1114SE Office of Naval Research 800 N. Quincy Street Arlington, VA 22217-5000
8	Mr. James G. Smith Office of Naval Research Code 1241 800 N. Quincy Street Arlington, VA 22217-5000
9	Mr. Michael Georgiopoulos EECS Department University of Central Florida Orlando, FL 32816
10	Professor Mike Athans MIT, Bldg. 35 Cambridge, MA 02139
11	Professor A. Ephremides Electrical Engineering Dept. University of Maryland College Park, MD 20742
12	Dr. Dave Castanon ALPHATECH, Inc. 2 Burlington Executive Center 111 Middlesex Turnpike Burlington, MA 01803-4901
13	Dr. John P. Lehoczky Dept. of Statistics Carnegie Mellon University Schenley Park Pittsburgh, PA 15213-3890
14	Professor Alex Levis MIT, Bldg. 35 Cambridge, MA 02139

(Continued on Next Page)

15	Dr. Harold L. Hawkins Manager, Perceptual Sciences Code 1142 Office of Naval Research 800 North Quincy Street Arlington, VA 22217-5000
16	Dr. G. Malecki Office of Naval Research Code 1142 800 N. Quincy Street Arlington, VA 22217-5000
17	Dr. David Kleinman ESE Dept., U-157 The University of Connecticut STORRS, CT 06268
18 - 29	Defense Technical Information Center, S47031 Bldg. 5, Cameron Station Alexandria, VA 22314
30 - 31	P. Kazakos, EE
32	M. Paterakis
33 - 34	E. H. Pancake, Clark Hall
35	SEAS Publications Files
*	Office of Naval Research Resident Representative 818 Connecticut Ave, N.W. Eighth Floor Washington, D.C. 20006 Attention: Mr. Michael McCracken, Administrative Contracting Officer

*Send Cover Letter Only

0935:ald:DW210R

END
DATE
FILMED
5-88
DTIC