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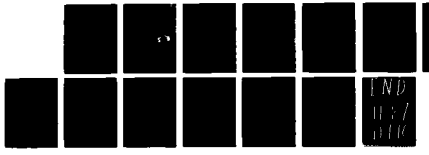

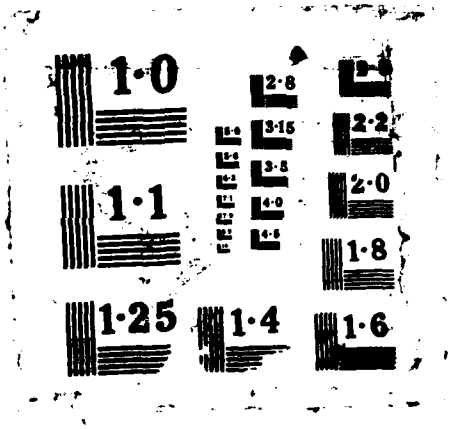
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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Coupled Systems Nonlinear PDE's, Operator Splitting, Adaptive Gridding

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Complex physical phenomena involving transport of heat or fluids are often modeled by coupled systems of nonlinear, time-dependent partial differential equations. Research has addressed each phase of the modeling process, utilizing physical, mathematical, numerical, and computational concepts. Techniques for treating systems of transport equations via modified method of characteristics, front tracking and mixed finite-element spatial discretizations have been developed. Adaptive grid refinement methods are capable of resolving important dynamic local phenomena. Stability of the partial differential equations (OVER)

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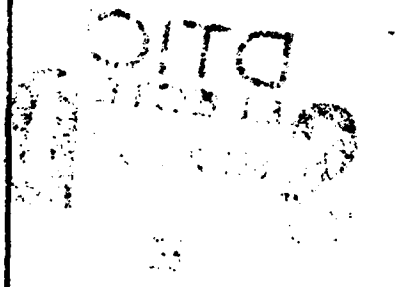
and their numerical counterparts have been studied. Computational procedures involving efficient data structures, preconditioning, domain decomposition, and vectorization and parallelization have been developed.



NUMERICAL AND ANALYTICAL METHODS IN NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

FINAL TECHNICAL REPORT

RICHARD E. EWING

04/30/87

U.S. ARMY RESEARCH OFFICE

CONTRACT NUMBER DAAG29-84-K-0002

UNIVERSITY OF WYOMING

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TABLE OF CONTENTS

I. STATEMENT OF THE PROBLEMS STUDIED

Complex physical phenomena involving chemically reacting systems or the transport of heat or fluids are often modeled by coupled systems of time-dependent, nonlinear partial differential equations. The difficulties in understanding the stability of the differential equation systems and in designing efficient, accurate numerical methods for their solution are widely recognized and were the focus of this research. We have worked on four general aspects of the analysis and numerical approximation of systems of partial differential equations. These areas of research are: (1) modeling aspects and stability analysis for nonlinear time-dependent partial differential equations; (2) use and analysis of finite element or finite difference methods to discretize coupled systems of nonlinear differential equations; (3) development of adaptive or local grid refinement capabilities to resolve local phenomena in large-scale applications; and (4) development of data structures, preconditioners, and efficient solution algorithms for large-scale problems on new computer architectures. Emphasis has been placed upon multiphase or multicomponent, transport-dominated flow processes with dynamic local phenomena. The research also involved a mix of analysis, algorithm development, and large-scale computation using newer computer architectures.

II. SUMMARY OF KEY RESULTS

The advent of orders-of-magnitude better computing capabilities has allowed the modeling of more complicated physical phenomena. The incorporation of more detailed physics in our models has necessitated the use of more sophisticated mathematics and numerics in the modeling process. In this way, a broader range of mathematics is needed for applications. The mathematical techniques which are used to model the transport of multicomponent or multiphase flows are representative of those needed for many other applications, such as chemically reacting or thermally driven flows, and have been studied as typical models to develop our understanding and capabilities. The equations which have been considered are strongly convection-dominated equations that have transport aspects of hyperbolic partial differential equations but which have thin moving regions with viscous effects where the modeling of diffusion is critical.

In order to treat the transport-dominated aspect of convection-diffusion type equations, we have developed [2,6,29,30,31,37] a "modified method of characteristics" temporal technique which can be combined with either finite difference or finite element spatial discretization. The computational aspects of this technique are much simpler than a true method of characteristics and can be extended to higher space dimensions with relative ease. In [30,37], we extended these ideas to transport problems with nonlinear and nonconvex flux functions. There are strong correlations with these and front tracking methods, but these techniques can incorporate the important diffusion phenomena much more readily. Ideas derived from characteristics-based methods have also been studied [35] to stabilize the transport-dominated problems, replacing the standard upstream weighting techniques which can cause serious problems for coupled nonlinear systems [2,3].

Ewing and co-workers have been successful in applying finite element tech-

niques to fluid flow simulation problems. Mixed finite element methods for accurate fluid velocities are described in [1,2,5-7,11,13,18,24,25,29,31,42]. Asymptotic error estimates for these procedures are described in [6,14,39-43]. Petrov-Galerkin variational methods and operator-splitting techniques have also been analyzed in [30,37] to treat the transport-dominated model problems. Applications to various engineering applications are described in [3,16,18,21,24,29,35]. Codes have been developed by Ewing, Russell, and graduate student Joe Koebbe, and with coworkers in Norway [35,37].

A major effort has also been concentrated upon adaptive and local grid refinement techniques for resolution of local behavior. References [4,8,10,12,17,18,20, 22,23,34,35] survey various aspects of local and adaptive grid refinement. A truly local refinement requiring a complex data structure for efficient implementation is described in [4,8,12,17,22]. Work is under way to implement these algorithms on a new Alliant parallel architecture computer recently acquired by Ewing's research group at the University of Wyoming using, in part, a DoD equipment grant. Patch types of local refinement which can be done more easily incorporated in large existing codes are described in [10,28,30,34,36-38]. Codes for implementing these concepts are being developed by graduate student Paul Jacobs and are almost complete.

Codes for modeling applications require the input of physical parameters that often cannot be measured directly. Various new methods for parameter determination are presented in [15,32,33,34]. Graduate students James Sochacki and Tao Lin have worked with Ewing on this analysis and corresponding code development.

Efficiency of computation is critical for the large-scale modeling applications. Applications of preconditioned conjugate-gradient methods were described in [26]. A variety of preconditioners and conjugate-gradient-like methods have

been coded and are currently being vectorized by graduate student Mark Oliver. Research is beginning on parallelization of these algorithms. Other preconditioners which allow a type of local refinement via domain decomposition are described in [28,30]. We are working on incorporation of these methods in large existing codes. Graduate students Joe Koebbe and Peng Lu and postdoc Uma Prasad [38] have been working on multiphase, multicomponent fluid flow codes with Ewing.

Finally, we have been looking at stability analyses for certain nonlinear, time-dependent partial differential equations [9,27,C1,C2]. Professor Brian Straughan from the University of Glasgow visited the University with partial support from this grant and to work with Professors Ewing and George on these problems. A joint paper [27] numerically considered a form of Burger's equation with various powers of the nonlinearity to understand the effect of initial conditions on these nonlinearities. Professor Straughan has extended this work to other convection-diffusion applications in [C1,C2].

III. LIST OF PUBLICATIONS

A. EWING'S PUBLICATIONS

1. Mixed finite element methods for miscible displacement problems in porous media (with B.L. Darlow and M.F. Wheeler), *Soc. Pet. Eng. J.* **4** (1984), pp. 391–398.
2. Simulation of miscible displacement using mixed methods and a modified method of characteristics (with T.F. Russell and M.F. Wheeler), SPE Reprint Series No. 20, Numerical Simulation II, Society of Petroleum Engineers, Dallas, and Soc. Pet. Eng. J. (to appear).
3. Problems arising in the modeling of processes for hydrocarbon recovery, Vol. I, *Mathematics of Reservoir Simulation*, R.E. Ewing, ed., Research Frontiers in Applied Mathematics, SIAM, Philadelphia, 1984, pp. 3–34.
4. Self-adaptive local grid refinement in enhanced oil recovery (with J.C. Diaz, R.W. Jones, A.E. McDonald, L.M. Uhler and D.U. von Rosenberg), *Proceedings Fifth Intl. Symp. on Finite Elements and Flow Problems*, Austin, Texas, January 23–26, 1984, pp. 479–484.
5. Computing accurate velocities for fluid flow in porous media (with J.V. Koebbe, R. Gonzalez and M.F. Wheeler) *Proc. Fifth Intl. Symp. on Finite Elements and Flow Problems*, Austin, Texas, January 23–26, 1984, pp. 131–136.
6. Convergence analysis of an approximation of miscible displacement in porous media by mixed finite elements and a modified method of characteristics (with T.F. Russell and M.F. Wheeler), *Computer Meth. Appl. Mech. Eng.*, R.E. Ewing, ed., **47** (1984), pp. 73–92.
7. Mixed finite element approximation of phase velocities in compositional reservoir simulation (with R.F. Heinemann), *Computer Meth. Appl. Mech. Eng.*, R.E. Ewing, ed., **47** (1984), pp. 161–176.
8. Adaptive mesh refinements in reservoir simulation applications, *Proc. Intl. Conference on Accuracy Est. and Adaptive Refinements in Finite Element Computations*, Lisbon, Portugal, June 19–22, 1984, pp. 31–40.
9. Viscous fingering in hydrocarbon recovery processes (with J.H. George), *Mathematical Methods in Energy Research*, K.I. Gross, ed., SIAM, Philadelphia, 1984, pp. 194–213.
10. The fast adaptive composite grid method for solving differential boundary-value problems (with S. McCormick and J. Thomas), *Proc. Fifth ASCE Specialty Conference "Engineering Mechanics in Civil Engineering,"* Laramie, Wyoming, August 1–3, 1984, pp. 1453–1456.
11. Mixed finite element methods for groundwater flow and contaminant transport (with J.V. Koebbe), *Proc. Fifth IMACS Intl. Symp. on Computer*

Meth. for Partial Differential Equations, Bethlehem, Pennsylvania, June 19–21, 1984, pp. 106–113.

12. Self-adaptive local grid refinement for time-dependent two-dimensional simulation (with J.C. Diaz, R.W. Jones, A.E. McDonald, L.M. Uhler and D.U. von Rosenberg), *Finite Elements in Fluids*, Vol. VI, John Wiley and Sons, Ltd., 1985, pp. 279-290.

13. Mixed finite element methods for accurate fluid velocities (with J.V. Koebbe, R. Gonzalez and M.F. Wheeler), *Finite Elements in Fluids*, Vol. VI, John Wiley and Sons, Ltd., 1985, pp. 233–249.

14. Galerkin methods for miscible displacement problems with point sources and sinks—unit mobility ratio case (with M.F. Wheeler), *Mathematical Methods in Energy Research*, K.I. Gross, ed., SIAM, Philadelphia, 1984, pp. 40–58.

15. Identification and control of distributed parameters in porous media flow (with J.H. George), *Distributed Parameter Systems*, Lecture Notes in Control and Information Sciences (M. Thoma, ed.) Springer-Verlag, **70**, May 1985, pp. 145-161.

16. Finite element methods for nonlinear flows in porous media, *Computer Meth. Appl. Mech. Eng.* **51** (1985), pp. 421–439.

17. Adaptive mesh refinement in large-scale fluid flow simulation, *Accuracy Estimates and Adaptivity for Finite Elements*, Chapter 16, I. Babuska, O.C. Zienkiewicz, and E. Oliveira, eds., John Wiley and Sons, New York, 1986, pp. 299–314.

18. Mixed finite-element methods for computing groundwater velocities (with M.B. Allen and J.V. Koebbe), *NUMETA 85, Numerical Methods in Engineering: Theory and Applications*, Volume II, J. Middleton and G.N. Pande, eds., A.A. Balkema Publishers, Rotterdam, Netherlands, 1985, pp. 609–614; and *Numerical Methods for Partial Differential Equations* **3** (1985), pp. 195–207.

19. Potential of HEP-like MIMD architecture in self adaptive local grid refinement for accurate simulation of physical processes (with J.C. Diaz), *Proc. Workshop on Parallel Processing Using the HEP*, Norman, Oklahoma, March 20–21, 1985, pp. 209–226.

20. Efficient adaptive procedures for fluid flow applications, *Computer Meth. Appl. Mech. Eng.* **55** (1986) pp. 89-103.

21. Applied mathematics in groundwater hydrology and contaminant transport (with M.B. Allen), *SIAM News* **18**(4) (July 1985), pp. 3, 14.

22. Local grid refinement for oil recovery simulation (with J.C. Diaz, R.W. Jones, A.E. McDonald, D.U. von Rosenberg, and L.M. Uhler), *Proceedings of SEG/SIAM/SPE Conference on Mathematical and Computational Methods in Seismic Exploration and Reservoir Modeling*, Houston, Texas, January 21–24, 1985, W.E. Fitzgibbon, ed., SIAM Publications (1986) pp. 238-240.

23. Adaptive local grid refinement, *Proc. SEG/SIAM/SPE Conference on Mathematical and Computational Methods in Seismic Exploration and Reservoir Modeling*, Houston, Texas, January 21–24, 1985, W.E. Fitzgibbon, ed., SIAM Publications (1986) pp. 235-247.

24. Finite element techniques for reservoir simulation (with J.V. Koebbe), *Innovative Numerical Methods in Engineering*, (R.P. Shaw, J. Periaux, A. Chaudouet, J. Wu CiMerino, and C.A. Brebbia, eds.) Springer-Verlag, Berlin, 1986, pp. 173-186.

25. Variational methods for fluid flow in porous media, *Variational Methods in Geosciences*, (Y. Sasaki, ed.), Elsevier, Amsterdam, 1986, pp. 251-263.

26. Application of conjugate gradient-like methods to a hyperbolic problem in porous media flow (with U. Obeysekare, M. Allen and J.H. George), *International Journal for Numerical Methods in Fluids* (to appear).

27. Nonlinear instability for a modified form of Burger's equation (with B. Straughan, P.G. Jacobs, and M.J. Djomehri), *Numerical Methods for Partial Differential Equations*, 3 (1987) pp. 51-64.

28. A preconditioning technique for the efficient solution of problems with local grid refinement (with J. Bramble, J. Pasciak, and A. Schatz), *Computer Methods in Applied Mechanics and Engineering* (to appear).

29. Mathematical modeling and large-scale computing in energy and environmental research, in New Directions in Applied and Computational Mathematics (R.E. Ewing, K.I. Gross and C.F. Martin, eds.) Springer-Verlag, Berlin, 1986, pp. 45-60.

30. Characteristic Petrov-Galerkin subdomain methods for two-phase immiscible flow (with Magne Espedal), *Computer Methods in Applied Mechanics and Engineering* (to appear).

31. Numerical solution of systems of partial differential equations *Transactions of Fourth Army Conference on Applied Mathematics and Computing*, ARO Report 87-1, 1987, pp. 583-595.

32. Parameter estimation for distributed systems arising in fluid flow problems via time series methods (with T. Lin), *Inverse Problems International Series of Numerical Mathematics*, Birkhauser Verlag, Basel, West Germany, Vol. 77, 1986, pp. 117-126.

33. Inverse and ill-posed problems in reservoir simulation (with T. Lin and R.S. Falk), *Notes and Reports on Mathematics in Science and Engineering*, Academic Press (to appear).

34. Adaptive grid refinement methods for time-dependent flow problems, *Communications in Applied Numerical Methods* (to appear).

35. Velocity weighting techniques for fluid displacement problems (with R.F. Heinemann, J.V. Koebbe, and U.S. Prasad), *Computer Methods in Applied Mechanics and Engineering* (to appear).

36. Adaptive grid-refinement techniques for treating singularities, heterogeneities and dispersion, *Proceedings of Symposium on Numerical Simulation in Oil Recovery*, Minneapolis, Minnesota, Springer-Verlag, Berlin (to appear).
37. Characteristic Petrov-Galerkin subdomain methods for convection diffusion problems (with H. Dahle and M. Espedal), *Proceedings of Symposium on Numerical Simulation in Oil Recovery*, Minneapolis, Minnesota, Springer-Verlag, Berlin (to appear).
38. Simulation techniques for multiphase and multicomponent flows (with M.S. Espedal, J.A. Puckett, and R.S. Schmidt), *Proceedings of Workshop on Special Topics in Computational Mechanics*, Dallas, Texas, April 13-14, 1987.
39. Alternating direction multistep methods for parabolic problems – iterative stabilization (with J.H. Bramble and Gang Li), (in preparation).
40. Numerical method for a model for the incompressible nuclear waste-disposal contamination in porous media (with Yirang Yuan and Gang Li), (in preparation).
41. The finite element method with moving mesh for local interpolation for incompressible miscible displacement (with Yirang Yuan and Gang Li), (in preparation).
42. A time-discretization procedure for a mixed finite element approximation of the incompressible nuclear waste-disposal contamination in porous media (with Yirang Yuan and Gang Li), (in preparation).
43. Finite difference methods for a model for compressible flow for the nuclear waste-disposal contamination in porous media (with Yirang Yuan and Gang Li), (in preparation).
44. Hyperbolic perturbation methods for parameter estimation of the numerical model for distributed systems (with Tao Lin), (in preparation).

B. Ewing's Books

1. *The Merging of Disciplines: New Directions in Pure, Applied, and Computational Mathematics*, Springer-Verlag, New York, 1986 (R.E. Ewing, K.I. Gross, and C.F. Martin, editors).
2. *Mathematical Modeling in the Energy and Environmental Sciences*, CBMS-NSF Regional Conference Series in Applied Mathematics, Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania (in preparation).

C. Other Supported Publications

1. Stability criteria for convection with large viscosity variations (B. Straughan), *Acta Mechanica* **61** (1986), pp. 59-72.
2. Finite amplitude instability thresholds in penetrative convection (B. Straughan), *Geophys. Astrophys., Fluid Dynamics* **34** (1985), pp. 227-242.

IV. INVITED PRESENTATIONS

A. INVITED ADDRESSES

1. "Mathematics of large-scale simulation," Symposium of Rocky Mountain Mathematics Consortium, Joint AMS-MAA Meetings, Louisville, Kentucky, January 26–28, 1984.
2. "Mathematical aspects of reservoir simulation," Visiting Professor, Institut National de Recherche en Informatique et en Automatique, Paris, France, June 9–16, 1984.
3. "Adaptive mesh refinement in reservoir simulation applications," Keynote Address, International Conference on Accuracy Estimates and Adaptive Refinements in Finite Element Computations (ARFEC), Lisbon, Portugal, June 19–22, 1984.
4. "Mixed finite element methods in fluid flow problems," Visiting Professor, Istituto per le Applicazioni del Calcolo (IAC), Mauro Picone, del Consiglio Nazionale delle Ricerche, Rome, Italy, July 1–7, 1984.
5. "Some applications to determination of distributed parameters in porous media flow," Second International Conference on Control Theory for Distributed Parameter Systems and Applications, Vorau, Austria, July 8–15, 1984.
6. "Finite element discretization for simulation of porous media flow," Gordon Research Conference on Modeling of Flow in Permeable Media, Andover, New Hampshire, July 30–August 3, 1984.
7. "Finite element methods for nonlinear flows in porous media," International Conference on Finite Element Methods in Nonlinear Mechanics (FENOMECH '84), Stuttgart, West Germany, September 10–14, 1984.
8. "The potential of finite element methods in reservoir simulation," Visiting Professor, Chevron Oil Field Research Company, La Habra, California, September 24, 1984.
9. "Mixed finite-element methods for computing groundwater velocities," International Conference Series on Advances in Numerical Methods in Engineering: Theory and Applications (NUMETA 85), Swansea, United Kingdom, January 7–11, 1985.
10. "Status of Enhanced Oil Recovery Institute," First Wyoming Enhanced Oil Recovery Symposium, Casper, Wyoming, May 21–22, 1985.
11. "Parameter estimation in large-scale simulation," Institute for Computation in Science and Engineering, NASA Langley, Hampton, Virginia, June 10–14, 1985.
12. "Efficient adaptive procedures for fluid flow applications," Symposium on Computational Mechanics, Second Joint ASCE/ASME Mechanics Conference, Albuquerque, New Mexico, June 23–26, 1985.

13. "Status of the oil and gas industry," Visiting Professor, Pacific Power and Light Company, Portland, Oregon, August 7, 1985.
14. "Mathematical modeling and large-scale computing in energy and environmental research," Conference on New Directions in Applied and Computational Mathematics, Laramie, Wyoming, August 8–10, 1985.
15. "Mathematics and engineering in large-scale computational science," National Science Foundation, Washington, D.C., October 11, 1985.
16. "Project type and multi-agency grants," 1985 National Chairmen's Research Colloquium for the Mathematical Sciences, Washington, D.C., October 12, 1985.
17. "Variational methods for petroleum reservoir simulation," Keynote Address, International Symposium on Variational Methods in Geosciences, Cooperative Institute for Mesoscale Meteorological Studies, Norman, Oklahoma, October 14–17, 1985.
18. "Large-scale simulation in enhanced oil recovery research," Annual Meeting Wyoming Colleges and Universities, Cheyenne, Wyoming, February 21, 1986.
19. "Finite element techniques for reservoir simulation" Fourth International Symposium on Numerical Methods for Engineers, Atlanta, Georgia, March 24-28, 1986.
20. "Adaptive local grid refinement" Third Stanford Reservoir Simulation Workshop Program, Stanford, California, March 31 - April 1, 1986.
21. "Introduction and overview of the Enhanced Oil Recovery Institute," Second Wyoming Enhanced Oil Recovery Symposium, Casper, Wyoming, May 15-16, 1986.
22. "Parameter estimation for fluid flow problems," Inverse Problems, Oberwolfach, West Germany, May 18-24, 1986.
23. "Numerical solution of partial differential equations," Fourth Army Conference on Applied Mathematics and Computing, Ithaca, New York, May 27-30, 1986.
24. "Mathematical modeling in the energy and environmental sciences." (Series of 10 invited lectures) Principal Lecturer at NSF-CBMS Conference, Morgantown, West Virginia, June 2-6, 1986.
25. "Estimation of spatially dependent parameters in parabolic partial differential equations," Alpine-U.S. Seminar on Inverse and Ill-posed Problems, St. Wolfgang, Austria, June 8-13, 1986.
26. "Adaptive local grid refinement," 3rd Mexican - American Conference on Computational Modeling in Science and Engineering, Avandaro, Mexico, July 22, 1986.
27. "Modeling of multiphase contaminant flows," Forum on NSF Research Activities in Subsurface Systems, Ann Arbor, Michigan, July 24, 1986.

28. "Domain decomposition method for adaptive local grid refinement," Workshop on Preconditioned Iterative Methods and Domain Decomposition, Mathematical Sciences Institute, Ithaca, New York, August 12, 1986.

29. "Adaptive grid refinement methods for time dependent flow problems," First World Congress on Computational Mechanics, University of Texas, Austin, Texas, September 25, 1986.

30. "Velocity weighting techniques for fluid displacement problems," First World Congress on Computational Mechanics, University of Texas, Austin, Texas, September 25, 1986.

31. "Optimization techniques in reservoir simulation," Special Session on Optimization and Inverse Problems in Reservoir Aquifer Modeling, Operations Research Society of America ORSA/TIMS National Meeting, Miami Beach, Florida, October 28, 1986.

32. "Large-scale computing in fluid flow problems," Special Session on Mathematics for Large-Scale Computing, 830th Meeting of the American Mathematical Society, Denton, Texas, October 31, 1986.

33. "Techniques for treating heterogeneities and dispersion in reservoir simulation," Symposium on Numerical Simulation in Oil Recovery, Institute for Mathematics and Its Applications, University of Minnesota, Minneapolis, Minnesota, December 1-12, 1986.

34. "The use of mixed finite element methods for accurate fluid velocities," Workshop on Recent Developments in Leaky Aquifer Mechanics, Instituto de Geofisica, Universidad Nacional de Mexico, Mexico City, Mexico, January 20-21, 1987.

35. "Simulation techniques for multiphase and multicomponent flows," Invited Lecture Series, IBM Bergen Scientific Center, Bergen, Norway, March 23-24, 1987.

36. "Parallel computation in operator splitting and self adaptive local grid refinement," Workshop on Special Topics in Computational Mechanics, Dallas, Texas, April 13-14, 1987.

37. "Status of the Enhanced Oil Recovery Institute," Third Wyoming Enhanced Oil Recovery Symposium, Casper, Wyoming, May 13-14, 1987.

38. "Reservoir Simulation at the University of Wyoming," Third Wyoming Enhanced Oil Recovery Symposium, Casper, Wyoming, May 13-14, 1987.

39. "Mathematics and Mathematical Modeling in Interdisciplinary Research and Large Scale Computation," Address to National Science Foundation, Washington, D.C., May 27, 1987.

40. "Analysis and Computation for a Model for Possible Contamination by Nuclear Waste-Disposal in Porous Media," 12th Biennial Conference on Numerical Analysis, University of Dundee, Scotland, June 23-24, 1987.

41. "Parameter Estimation Problems for Parabolic Problems," Workshop on Applications and Algorithms for Optimal Control and Parameter Identification, Universität Trier, Trier, West Germany, June 25-26, 1987.
42. "A Survey of Reservoir Simulation," Minisymposium on Simulation of Petroleum Reservoirs, First International Conference on Industrial and Applied Mathematics, Paris, France, June 29 – July 3, 1987.

B. Invited Colloquia and Special Seminars

1. Katholieke University, Nijmegen, The Netherlands, June 7, 1984.
2. Institut National des Sciences Appliquees de Lyon, Lyon, France, June 12, 1984.
3. INRIA, Paris, France, June 15, 1984.
4. Albert-Ludwegs Universitat, Freiburg, Germany, June 26, 1984.
5. Istituto di Analisi Numerica, Pavia, Italy, June 29, 1984.
6. Istituto per le Applicazioni de Calcolo (IAC), Rome, Italy, July 6, 1984.
7. Chevron Oil Field Research Co., La Habra, California, February 1, 1985.
8. University of Colorado, Boulder, February 8, 1985.
9. Denelcor Corporation, Denver, Colorado, February 27, 1985.
10. University of Houston, April 16, 1985.
11. Colorado State University, April 18, 1985.
12. University of Colorado, Boulder, May 2, 1985.
13. Pacific Power and Light Corp., Portland, Oregon, September 10, 1985.
14. Chevron Oil Field Research Co., La Habra, California, December 17, 1985.
15. University of Minnesota, Minneapolis, Minnesota, December 19, 1985.
16. University of Wyoming Foundation, Houston, Texas, February 11, 1986.
17. University of Wyoming, Laramie, Wyoming, February 17, 1986.
18. Chevron Oil Field Research Co., La Habra, California, March 27, 1986.
19. INRIA, Le Chesnay, France, June 24, 1986.
20. University of Colorado, Denver, Colorado, September 10, 1986.
21. National University of Mexico, Mexico City, Mexico, September 29, 1986.
22. Tulsa University, Tulsa, Oklahoma, October 29, 1986.
23. University of Minnesota, Minneapolis, Minnesota, December 4, 1986.
24. University of Wyoming, Laramie, Wyoming, February 10, 1987.
25. Utah State University, Logan, Utah, February 12, 1987.
26. University of Wyoming, Laramie, Wyoming, February 17, 1987.
27. Koninklijke/Shell Exploratie en Produktie Laboratorium, Rijswijk, The Netherlands, March 9, 1987.
28. Norsk Hydro Petroleum Research Centre, Bergen, Norway, March 20, 1987.
29. Universitetet Bergen, Bergen, Norway, March 23, 1987.
30. IBM, Bergen Scientific Centre, Bergen, Norway, March 24, 1987.
31. Christian Michelson Institute, Bergen, Norway, March 27, 1987.
32. Uppsala University, Uppsala, Sweden, March 31, 1987.
33. Chalmers University of Technology and The University of Göteborg, Göteborg, Sweden, April 1, 1987.

34. University of California at Berkeley, Berkeley, California, April 29, 1987.
35. Arizona State University, Tempe, Arizona, April 30, 1987.
36. Montana State University, Bozeman, Montana, May 21, 1987.
37. Montana State University, Bozeman, Montana, May 22, 1987.

V. PARTICIPATING SCIENTIFIC PERSONNEL

A. STUDENTS

1. James S. Sochacki – Ph.D., 1985
2. Upal R.B. Obeysekare – M.S., 1985
3. Joseph V. Koebbe
4. D. Lowell Smylie
5. Paul Jacobs
6. Mark Oliver
7. Tao Lin
8. Peng Lu

B. FACULTY

1. Brian Straughan
2. John H. George
3. Robert Sharpley
4. Uma Prasad

END

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