AD-A179 126   EXTREME VALUE THEORY FOR SUPREMA OF RANDOM VARIABLES   1/1
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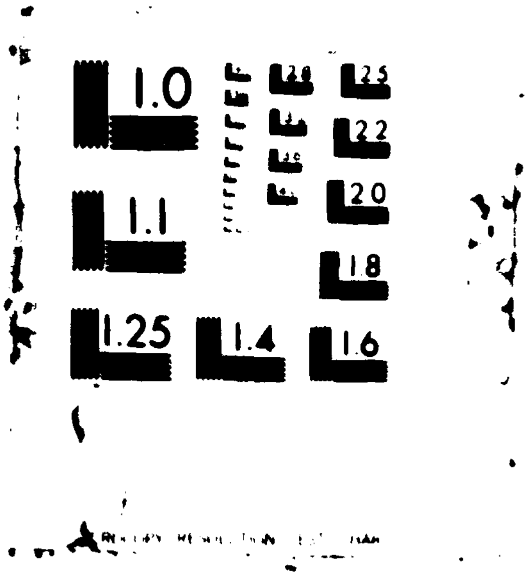
HILL CENTER FOR STOCHASTIC PROC.. T HSING JUL 86
UNCLASSIFIED   TR-140 AFOSR-TR-87-0319 F49620-82-C-0009   F/G 12/1   NL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AD-A179 126

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REPORT DOCUMENTATION PAGE

UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY
NA

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE
NA

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for Public Release; Distribution Unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
Technical Report No. 140

5. MONITORING ORGANIZATION REPORT NUMBER(S)
AFOSR-TR- 87-0319

6a. NAME OF PERFORMING ORGANIZATION
University of North Carolina

6b. OFFICE SYMBOL (If applicable)

7a. NAME OF MONITORING ORGANIZATION
AFOSR/NM

6c. ADDRESS (City, State and ZIP Code)
Center for Stochastic Processes, Statistics
Department, Phillips Hall 039-A,
Chapel Hill, NC 27514

7b. ADDRESS (City, State and ZIP Code)
Bldg. 410
Bolling AFB, DC 20332-6448

8a. NAME OF FUNDING/SPONSORING ORGANIZATION
AFOSR

8b. OFFICE SYMBOL (If applicable)

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
~~F49620 85 C 0144~~ F49620 82 C 0009

8c. ADDRESS (City, State and ZIP Code)
Bldg. 410
Bolling AFB, DC

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
6.1102F	2304		

11. TITLE (Include Security Classification)
"Extreme value theory for suprema of random variables with regularly varying tail probabilities.

12. PERSONAL AUTHOR(S)
Hsing, T.

13a. TYPE OF REPORT
technical

13b. TIME COVERED
FROM 9/81 TO 8/85

14. DATE OF REPORT (Yr., Mo., Day)
July 1986

15. PAGE COUNT
10

16. SUPPLEMENTARY NOTATION
Short title: The extremes of suprema of random variables.

17. COSATI CODES

FIELD	GROUP	SUB. GR.
xxxxx	xxxxxxxxxxx	xxxx

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Keywords: extreme values, point processes, regular variation, weak limits.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL
Peggy Ravitch

22b. TELEPHONE NUMBER (Include Area Code)
919-962-2307

22c. OFFICE SYMBOL
AFOSR/NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE. UNCLASSIFIED

CENTER FOR STOCHASTIC PROCESSES

AFOSR-TR- 87-0319

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EXTREME VALUE THEORY FOR SUPREMA OF RANDOM VARIABLES WITH REGULARLY VARYING TAIL PROBABILITIES

by

T. Hsing

Technical Report No. 140

July 1986

EXTREME VALUE THEORY FOR SUPREMA OF RANDOM VARIABLES WITH REGULARLY VARYING TAIL PROBABILITIES

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Consider a stationary sequence $X_j = \sup_i c_i Z_{j-i}$, $j \in I$, where $\{c_i\}$ is a sequence of constants, and $\{Z_i\}$ a sequence of i.i.d. random variables with regularly varying tail probabilities. For suitable normalizing functions $v_1, v_2, \ldots$, the limit form of the two dimensional point process with points $(j/n, v_n^{-1}(X_j))$, $j \in I$, is derived. The implications of the convergence are briefly discussed, while the distribution of the joint exceedances of high levels by $\{X_j\}$ is explicitly obtained as a corollary.

Short title: The extremes of suprema of random variables.

AMS 1980 subject classification: Primary 60F05, Secondary: 60F17, 60G55.

Key words: extreme values, point processes, regular variation, weak limits.

Research supported by the Air Force Office of Scientific Research Grant No. ~~AFOSR~~F49620 82 C 0009

1. Introduction

Extreme value theory concerns the joint tail behavior and related problems of random variables (r.v.'s). Recent emphasis has been the extension of the classical theory, which considers independent and identically distributed (i.i.d.) r.v.'s to the more general setting of stationarity. Progress has been made on topics such as notions of asymptotic independence, general extremal types theorems, studies of related point processes, etc. See [13] for a comprehensive account of the subject.

We are interested in the extremal properties of stationary sequences whose members are certain functions of i.i.d. r.v.'s. In this direction, [1, 4, 15] investigated moving average sequences under various assumptions. Through the particular structure of the sequences, these studies provided invaluable insights into the theory in general. In this paper, we consider a stationary sequence $\{X_j\}$ consisting of the weighted suprema -- instead of sums as in the case of moving averages -- of certain i.i.d. r.v.'s whose tail probabilities are regularly varying. A sequence with this structure may be used to model random exchanges (cf. [7, 8]), and is a useful tool in studying multivariate extreme value theory (cf. [5]).

In Section 2 we introduce some general results concerning the asymptotic tail behavior of the supremum of independent r.v.'s, and consider the marginal of $\{X_j\}$ as a special case. Section 3 contains a main result Theorem 3.2, which is a limit theorem of certain point processes defined for $\{X_j\}$. Section 4 discusses the application of Theorem 3.2, and its connection with some related results. The distribution of the joint exceedances of high levels by $\{X_j\}$ is also derived.

2. Framework

We first summarize some relevant facts concerning the tail behavior of the supremum of independent r.v.'s. Unless otherwise stated, assume that each sequence mentioned, random or nonrandom, is indexed by the set of integers I.

Theorem 2.1. Let $\{Y_i\}$ be a sequence of independent r.v.'s. Then $\sup Y_i < \infty$ a.s., or $= \infty$ a.s. Furthermore, $\sup Y_i < \infty$ a.s. if and only if $\sum_i P[Y_i > x] < \infty$ for some $x < \infty$.

Proof. As is shown in [3], Theorem 1, the claims follow readily from the zero-one law, and the Borel-Cantelli Lemma. □

Lemma 2.2. Let $\{Y_i\}$ be a sequence of independent r.v.'s. Suppose that $\sup Y_i$ converges to X a.s., and that $P[X < x_0] = 1$ where $x_0 = \sup\{u: P[X \le u] < 1\}$. Then $P[X > u] \sim \sum_i P[Y_i > u]$ as $u \uparrow x_0$.

Proof. Write $f(y) = -\log(1-y)-y$, $y \in [0,1)$. It is simply seen that $f(y) \ge 0$, and $f(y) \sim y^2/2$ as $y \to 0$. The assumption $P[X < x_0] = 1$ implies that there exists an x such that $0 < P[X > u] < 1$, $u \in [x,x_0)$, and therefore that $P[Y_i > u] < 1$, $u \in [x,x_0)$, $i \in I$. Hence

$$\sum_i P[Y_i > u] \le -\sum_i \log P[Y_i \le u] = -\log P[X \le u]$$

$$= P[X > u] + f(P[X > u]),\ u \in [x,x_0).$$

By this and Boole's inequality,

$$0 \le \sum_i P[Y_i > u] - P[X > u] \le f(P[X > u]),\ u \in [x,x_0).$$

Since x_0 is not an atom, $P[X > u] \to 0$ as $u \uparrow x_0$. This concludes the proof. □

Let $\{Z_i\}$ be a sequence of i.i.d. r.v.'s whose tail probabilities are regularly varying at ∞ with index $-\alpha$, $\alpha > 0$; i.e. $P[Z_1 > z] = z^{-\alpha}L(z)$, $z > 0$, where L is slowly varying (cf. [6]). To avoid trivialities, assume that the Z_i are positive and unbounded above. The following result is similar to [2], Lemma 2.2 (ii).

Theorem 2.3. Let $\{c_i\}$ be a sequence of nonnegative constants with $\sup c_i > 0$. Then $\sup c_i Z_i < \infty$ if and only if $\sum_i c_i^{\alpha}L(c_i^{-1}) < \infty$, where $c^{\alpha}L(c^{-1})$ denotes zero if $c = 0$. Moreover,

$$P[\sup c_i Z_i > x] \sim x^{-\alpha}L(x) \sum_i c_i^{\alpha}, \quad \text{as} \quad x \to \infty, \tag{2.1}$$

if there exist a constant $\delta > 0$, and a sequence of constants $\{a_i\}$ such that $\sum a_i < \infty$, and $c_i^{\alpha}L(c_i^{-1}x)/L(x) \le a_i$ for all $x > \delta$, $i \in I$. In particular, (2.1) holds if either of the following holds:

(a) $\sum_i c_i^{\varepsilon} < \infty$ for some $\varepsilon \in (0,\alpha)$;

(b) $\sum_i c_i^{\alpha} < \infty$ and $L(tx)/L(x)$ is uniformly bounded for all $t > \rho$, $x > \delta$, where ρ and δ are positive constants.

Proof. We first show that $\sup c_i Z_i < \infty$ a.s. if and only if $\sum c_i^{\alpha}L(c_i^{-1}) < \infty$. It is obvious that in either case $c_i \to 0$ as $|i| \to \infty$. Thus for each $x > 0$, $\sum P[c_i Z_i > x] = \sum c_i^{\alpha}L(c_i^{-1}x) < \infty$ if and only if $\sum c_i^{\alpha}L(c_i^{-1}) < \infty$ by the limit comparison test for series. The claim now follows from Theorem 2.1. Next assume the existence of δ and $\{a_i\}$ as described. Then by Lemma 2.2 and dominated convergence,

$$\lim_{x\to\infty} \frac{P[\sup c_i Z_i > x]}{x^{-\alpha}L(x) \sum c_i^{\alpha}} = \lim_{x\to\infty} \frac{\sum c_i^{\alpha}L(c_i^{-1}x)}{L(x) \sum c_i^{\alpha}} = 1,$$

proving (2.1). Suppose now (a) holds. Then it is obvious that c_i^{-1} is bounded away from zero, and thus there exist positive constants δ and k such that $L(c_i^{-1}x)/L(x) \le kc_i^{\varepsilon-\alpha}$ for each $x \ge \delta$ and $i \in I$. The conclusion follows since one can take a_i to be kc_i^{ε}. (b) can be shown similarly, concluding the theorem. □

For $\{Z_i\}$ and a sequence of nonnegative constants $\{c_i\}$ satisfying either (a) or (b) in Theorem 2.3, define a ststionary sequence $\{X_j\}$ by $X_j = \sup_i c_i Z_{j-i}$, $j \in I$. $\{X_j\}$ is similar in appearance to a moving average sequence, and we shall see that the parallels in extremal properties between the two are also interesting. It is worth noting that in some cases it may be profitable to represent $\{X_j\}$ in an "autoregressive form" (much as in the case of regular moving average). For example, if $c_i = \rho^i$, $i \geq 1$, where $\rho \in (0,1)$ is a constant, then $\{X_j\}$ can be defined recursively: $X_j = \max(Z_j, \rho X_{j-1})$.

3. Point Process Convergence

In this and the following section, some theory of point processes is required. The reader is referred to [12] for details.

It follows from [13], Theorem 1.6.2 that there exist constants $a_n > 0$ such that $P^n[Z_1 \leq a_n^{-1}x] \to \exp(-x^{-\alpha})$, $x > 0$. Write $v_n(\tau) = a_n^{-1}\tau^{-1/\alpha}$, $\tau > 0$, $n \geq 1$, and denote by v_n^{-1} the inverse of v_n. It is simply seen that for each $\tau > 0$, $P[Z > v_n(\tau)] \sim \tau/n$ as $n \to \infty$.

For each $n \geq 1$, define a point process N_n on $\mathbb{R} \times \mathbb{R}_+' = (-\infty,\infty) \times (0,\infty)$ by $N_n = \sum_j \delta_{(j/n, v_n^{-1}(X_i))}$, where $\delta_{(x,y)}$ is the measure with a single unit mass at (x,y). For simplicity of presentation, the normalization v_n is used instead of the more traditional linear normalization so that (as we shall see) N_n converges weakly to a homogeneous limit.

Closely related to N_n are the point processes N, $N^{(k)}$, $k \geq 1$, defined by $N = \sum_i \sum_j \delta_{(S_i, c_j^{-\alpha} T_i)}$, $N^{(k)} = \sum_i \sum_{|j| \leq k} \delta_{(S_i, c_j^{-\alpha} T_i)}$ where the (S_i, T_i) are the points of a homogeneous Poisson process on $\mathbb{R} \times \mathbb{R}_+'$ with mean one, and, as a convention, the inner summations extend over the set of j for which $c_j \neq 0$. It is clear that $N^{(k)}$ converges to N a.s., and hence in distribution.

Lemma 3.1. For $n, k \geq 1$, denote by $N_n^{(k)}$ the point process with points $(j/n, v_n^{-1}(\max_{|i| \leq k} c_i Z_{j-i}))$, $j \in I$. Then for each fixed k, $N_n^{(k)}$ converges in distribution to $N^{(k)}$ as n tends to infinity.

Proof. Let k be fixed. Write h for the mapping $h\mu = \sum_i \sum_{|j| \leq k} \delta_{(x_i, c_j^{-\alpha} y_i)}$ if $\mu = \sum_i \delta_{(x_i, y_i)}$ is a locally finite counting measure on $\mathbb{R} \times \mathbb{R}_+'$. h is a continuous mapping on the space of locally finite counting measures on $\mathbb{R} \times \mathbb{R}_+'$ to itself. For $n \geq 1$, denote by η_n the point process $\sum_i \delta_{(j/n, v_n^{-1}(Z_j))}$. It is well known (cf. [13], Theorem 5.7.1) that η_n converges in distribution to a homogeneous Poisson process on $\mathbb{R} \times \mathbb{R}_+'$ with mean one. By the continuous mapping theorem (cf. [12], 15.4.2), $h\eta_n \xrightarrow{d} N^{(k)}$. Therefore it suffices to show that $N_n^{(k)}$ and $h\eta_n$ have the same limit, or, by Theorem 4.2 of [12], to show that

$$\lim_{n \to \infty} \{P[N_n^{(k)} B_m = i_m,\ 1 \leq m \leq \ell] - P[(h\eta_n) B_m = i_m,\ 1 \leq m \leq \ell]\} = 0$$

for each choice of $\ell \geq 1$, $i_m \geq 0$, $B_m \in \mathcal{P}$ where $\mathcal{P}$ denotes the semiring of sets of the form $[a,b) \times [c,d)$ in $\mathbb{R} \times \mathbb{R}_+'$. Since

$$|P[N_n^{(k)} B_m = i_m,\ 1 \leq m \leq \ell] - P[(h\eta_n) B_m = i_m,\ 1 \leq m \leq \ell]|$$
$$\leq \sum_{m=1}^{\ell} P[N_n^{(k)} B_m \neq (h\eta_n) B_m],$$

it suffices to show that $\lim_{n \to \infty} P[N_n^{(k)} B \neq (h\eta_n) B] = 0$ for each B in $\mathcal{P}$. Let $B = [a,b) \times [c,d)$ be a set in $\mathcal{P}$. Since $v_n^{-1}(cx) = c^{-\alpha} v_n^{-1}(x)$ for $c, x > 0$, the event $[N_n^{(k)} B \neq (h\eta_n) B]$ occurs only if at least one of the following events $E_{n,1}$, $E_{n,2}$, $E_{n,3}$ occurs:

$E_{n,1} = [c(1)Z_j > v_n(d)$ for some j in $([na]-k, [na]-k+1, \ldots, [na]+k)]$,

$E_{n,2} = [c(1)Z_j > v_n(d)$ for some j in $([nb]-k, [nb]-k+1, \ldots, [nb]+k)]$,

$E_{n,3} = [c(1)Z_i > v_n(d)$ and $c(1)Z_j > v_n(d)$ for some pair i,j in $([na], \ldots, [nb])$ such that $|i-j| \leq 2k]$,

where $c(1) = \max c_j$, and $[x]$ denotes the integer part of x. By Boole's inequality and the fact that $P[c(1)Z_1 > v_n(d)] \sim (c(1))^{\alpha} d/n$, we have

$$\lim_{n\to\infty} \{P(E_{n,1}) + P(E_{n,2})\} \leq 2(2k+1) \lim_{n\to\infty} P[c(1)Z_1 > v_n(d)] = 0,$$

$$\lim_{n\to\infty} P(E_{n,3}) \leq \lim_{n\to\infty} \sum_{m=[na]}^{[nb]} P[c(1)Z_i > v_n(d),\ c(1)Z_j > v_n(d) \text{ for some pair } i \neq j \text{ in } (m, m+1, \ldots, m+2k-1)]$$

$$\leq \lim_{n\to\infty} ([nb]-[na]+1)\, k(2k-1)\, P^2[c(1)Z_1 > v_n(d)] = 0.$$

The conclusion follows. □

The main result of this section is the following.

Theorem 3.2. N_n converges in distribution to N as n tends to infinity.

Proof. Let $N_n^{(k)}$ and $\mathcal{P}$ be as in Lemma 3.1. We have shown earlier that $N_n^{(k)} \overset{d}{\to} N^{(k)}$ as $n \to \infty$ for $k = 1,2,\ldots$, and that $N^{(k)} \overset{d}{\to} N$ as $k \to \infty$. By [12], Theorem 4.2, it suffices to show that

$$\lim_{k\to\infty} \lim_{n\to\infty} \{P[N_n B_m = i_m,\ 1 \leq m \leq \ell] - P[N_n^{(k)} B_m = i_m,\ 1 \leq m \leq \ell]\} = 0$$

for each choice of $\ell \geq 1$, $i_m \geq 0$, $B_m \in \mathcal{P}$, or as in Lemma 3.1, that

$$\lim_{k\to\infty} \lim_{n\to\infty} P[N_n B \neq N_n^{(k)} B] = 0 \text{ for each } B \text{ in } \mathcal{P}. \qquad (3.1)$$

Suppose $B = [a,b) \times [c,d)$ is a set in $\mathcal{P}$. The event $[N_n B \neq N_n^{(k)} B]$ occurs only if the event $[c_i Z_{j-i} > v_n(d)$ for some i,j such that $|i| > k$, and $[na] \leq j \leq [nb]]$ occurs, the probability of the latter event being bounded by $([nb] - [na] + 1) \sum_{|i|>k} P[c_i Z_1 > v_n(d)]$. As $n \to \infty$, the expression tends to $(b-a)d \sum_{|i|>k} c_i^{\alpha}$, which tends to zero as k tends to infinity by the choice of $\{c_i\}$. This proves (3.1). □

4. Applications and Remarks

In general settings, the problems concerning weak convergence of point processes similar to N_n have been studied extensively. See, for example, [10,13,14].

Applying the continuous mapping theorem, a number of conclusions regarding the extremes of $\{X_j\}$ follow readily from Theorem 3.2. [4] demonstrates in detail the manner in which this is done. Since no new ideas are involved, the reader is referred there for details. However, the following is of some special interest to us.

It can be shown easily that the Laplace transform functional (cf. [12]) of N is $L_N(f) = \exp\{-\int_{\mathbb{R} \times \mathbb{R}'_+} [1 - \exp(-\sum_j f(s, c_j^{-\alpha} t))] dsdt\}$ where f is a nonnegative and compactly supported function on $\mathbb{R} \times \mathbb{R}'_+$. This is consistent with the representation in [10], Theorem 4.7. For $\tau > 0$, the exceedance point process $\Lambda_n^{(\tau)}$ on $\mathbb{R}$ studied in [11,15] consists of the set of points $\{j/n: j \in I, X_j > u_n(\tau)\}$. Note that $\Lambda_n^{(\tau)}(B) = N_n(B \times (0,\tau))$ for each Borel set B in $\mathbb{R}$. Using arguments similar to those in Section 3 of [4], it is straightforward to show that for any choice of $\tau_1 > \tau_2 > \ldots > \tau_k$, $(\Lambda_n^{(\tau_1)}, \ldots, \Lambda_n^{(\tau_k)})$ converges in distribution to $(N(\cdot \times (0,\tau_1)), \ldots, N(\cdot \times (0,\tau_k)))$, where the vectors are regarded as random elements in the product space of spaces of locally finite counting measures on $\mathbb{R}$. The distribution of $(N(\cdot \times (0,\tau_1)), \ldots, N(\cdot \times (0,\tau_k)))$ may be conveniently described (cf. [9]) by the functional

$$L(f_1, \ldots, f_k) = E \exp[-\int_{\mathbb{R}} \sum_{j=1}^{k} f_j dN(\cdot \times [0,\tau_j))]$$

$$= E \exp[-\int_{\mathbb{R}\times\mathbb{R}_+} \sum_{j=1}^{k} f_j(s) 1(t < \tau_j) dN]$$

where $f_1, \ldots, f_k$ are nonnegative compactly supported functions on $\mathbb{R}$. Using the Laplace transform of N obtained earlier, it is seen that

$$L(f_1, \ldots, f_k) = \exp\{-\int_{\mathbb{R}} \sum[1-\exp(-\sum_{j=1}^{k} f_j(s) i_j)]\pi(i_1, \ldots, i_k)ds\}$$

where the first summation extends over the set $\{(i_1,\ldots,i_k): i_1 \geq i_2 \geq \ldots \geq i_k \geq 0, i_1 \geq 1\}$, $\pi(i_1, \ldots, i_k) = \max[0, \min_{1\leq j\leq k} \tau_j c^\alpha(i_j) - \max_{1\leq j\leq k} \tau_j c^\alpha(i_j + 1)]$, and $\{c(i)\}$ is a rearrangement of $\{c_j\}$ with $c(1) \geq c(2) \geq \ldots$ If $k = 1$, L simply reduces to the Laplace transform of a compound Poisson process on $\mathbb{R}$.

The following comparison is interesting. Consider the moving average $Y_j = \sum_i c_i Z_{j-i}$, $j \in I$, where the Z_i are as before, and the c_i are now constrained by $\sum_i c_i^\varepsilon < \infty$ for some $\varepsilon < \min(1,\alpha)$ so that Y_1 is a.s. finite (cf. [4]). Then, as shown by [4], the point process $\tilde{N}_n \overset{def}{=} \sum_j \delta_{(j/n,\, v_n^{-1}(Y_j))}$ converges in distribution to the same limit as N_n does. It would be interesting to see whether this parallel extends to more general situations, for example, where the Z_i have subexponential distributions (cf. [16]).

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