Technical Report 686

Interactivity Theory: Analyzing Human Environments Using Linear Prediction Filters

Roland J. Hart and Stephen C. Bradshaw
Army Research Institute

ARI Field Unit at Presidio of Monterey
Training Research Laboratory

U. S. Army
Research Institute for the Behavioral and Social Sciences

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EDGAR M. JOHNSON
Technical Director

WM. DARRYL HENDERSON
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Analyses of complex interactive human environments pose analytic difficulties for commonly used methods. Linear prediction filters were selected as a methodology that could realistically reflect the characteristics of an interactive environment and conform to appropriate scientific criteria: empiricism, replication, prediction, and parsimony. Filters were compared to related linear models (e.g., ANOVA, path analysis). Scientific criteria were used to identify weaknesses in interactive applications of traditional linear methods.

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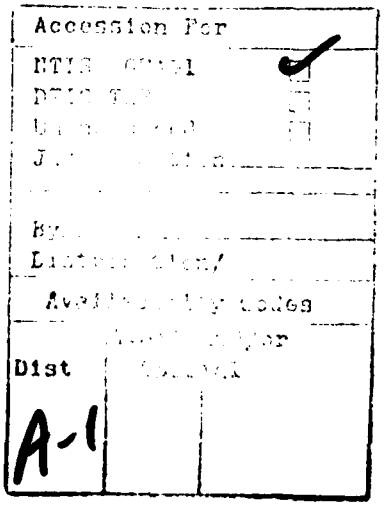
A multichannel linear prediction filter was derived and integrated with a measurement model, producing "time-series" factor analysis. The model was applied to data showing a long-term cyclical relationship between promotion rates in the U.S. Army and survey measures of company effectiveness.



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Roland J. Hart and Stephen C. Bradshaw
Army Research Institute

ARI Field Unit at Presidio of Monterey
Jack Hiller, Chief

Training Research Laboratory
Harold F. O'Neil, Jr., Director

U.S. ARMY RESEARCH INSTITUTE FOR THE BEHAVIORAL AND SOCIAL SCIENCES
5001 Eisenhower Avenue, Alexandria, Virginia 22333-5600

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FOREWORD

The Army Research Institute (ARI) Presidio of Monterey Field Unit is concerned with research related to improving unit collective training. Research in this area is conducted under the sponsorship of the U.S. Army Training Board (ATB) and the U.S. Army Combined Arms Center (CAC). The National Training Center (NTC) uses an instrumentation system to provide valuable detailed information about unit collective training. To capitalize on this valuable information, the Presidio of Monterey Field Unit has acquired the capability of replaying and analyzing NTC data. The collection and analysis of this dynamic and complex data breaks new ground. The development and adaption of methodological tools for the analysis of such data are required. This report is an in-house effort that reviews, evaluates, and develops times-series methodologies that can be applied to NTC data and other complex Army environments.

EDGAR M. JOHNSON
Technical Director

INTERACTIVITY THEORY: ANALYZING HUMAN ENVIRONMENTS USING LINEAR PREDICTION FILTERS

EXECUTIVE SUMMARY

Requirement:

The ability to analyze the details of Army unit performance quantitatively is important for many scientific and practical applications. Performance is used in a general sense to refer both to tactical behavior during field training exercises, and to day-to-day operations in garrison. The analysis of performance in such realistic Army environments is complex. It involves the analysis of dynamic forms of interaction over time between individuals and groups, and between groups. In addition, Army environments are not tightly controlled, and unexpected random events are commonplace. Approximate methods are needed to analyze performance quantitatively in these circumstances.

Procedure:

Analyses of complex interactive human environments pose analytic difficulties for commonly used methods. Linear prediction filters are selected as a methodology that can realistically reflect the characteristics of an interactive environment and conform to appropriate scientific criteria: empiricism, replication, prediction, and parsimony. Filters are compared to related linear models (e.g., ANOVA, path analysis). Scientific criteria are used to identify weaknesses in interactive applications of traditional linear methods. A multichannel linear prediction filter is derived and integrated with a measurement model, producing "time-series" factor analysis. The model is applied to data showing a long-term cyclical relationship between promotion rates in the U.S. Army and survey measures of company effectiveness.

Findings:

Multichannel linear prediction filters provide a useful set of methods that can be applied to the analysis of Army unit performance in dynamic environments. These filters are useful for predicting the dynamics of garrison performance as is shown in the Army example that is provided.

Utilization of Findings:

The National Training Center (NTC) uses a sophisticated automated data acquisition system to record events over time for NTC exercises. Another possible application of linear prediction filters is for the analyses of such data. In addition, the filters can be used usefully to simulate the dynamics of battle in computer-driven "battle" simulations.

INTERACTIVITY THEORY: ANALYZING HUMAN ENVIRONMENTS USING LINEAR PREDICTION FILTERS

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The application of the scientific method in psychological research has been the subject of continuing debate (see Elms, 1975; Gergen, 1973, 1976; Greenwald, 1976b; Schlinker, 1976; Secord, Note 1). Some critics have asserted that the application of the scientific method has been inadequate. As a consequence, informational content in the field becomes ideologically based, since theories remain untested by the scientific method. Far from being upset by ideological content, some psychologists have urged the creation of informational content "liberated both from the press of immediate fact and the necessity for verification." The application of emprirical methods to psychological problems is seen as inappropriate. The psychologist thus becomes liberated from the necessity for "discompassionate comportment in scientific affairs" (Gergen, 1978, p. 1344, 1982).

SCIENTIFIC METHOD

By contrast, we argue that ideologically based content in psychology is neither necessary or desirable when a more vigorous application of the scientific method can move informational content beyond ideology. Interactivity theory is directed toward the goal of improving the application of the scientific method in psychological research. The meaning of "scientific method" as used here is not synonymous with "randomization" or "controls" in a typical application of the experimental paradigm. In

fact, we argue (see Section 1.3.) that some applications of the traditional experimental paradigm in psychology can actually be poor applications of the scientific method. A simplified diagram of some essentials of the scientific method appears in Figure 1. To briefly summarize, the epistemology of the scientific method is empiricism, i.e., testing theory using systematic observation. The objective of the scientific method is to modify theory to match data. As noted in Figure 1, correspondence is required between the predictions that stem from theory and the predictions that stem from the application of the methodology. If this correspondence does not exist, "something" may be tested but it is not the theory in question. In addition, the methodology must be capable of making predictions in the first place so that theory becomes subject to testing and modification. Given equally precise prediction, concise representations are always preferred over inconcise ones (law of parsimony, or Occam's razor). Predictions that are confirmed should be replicable. These well known principles delineate the areas where

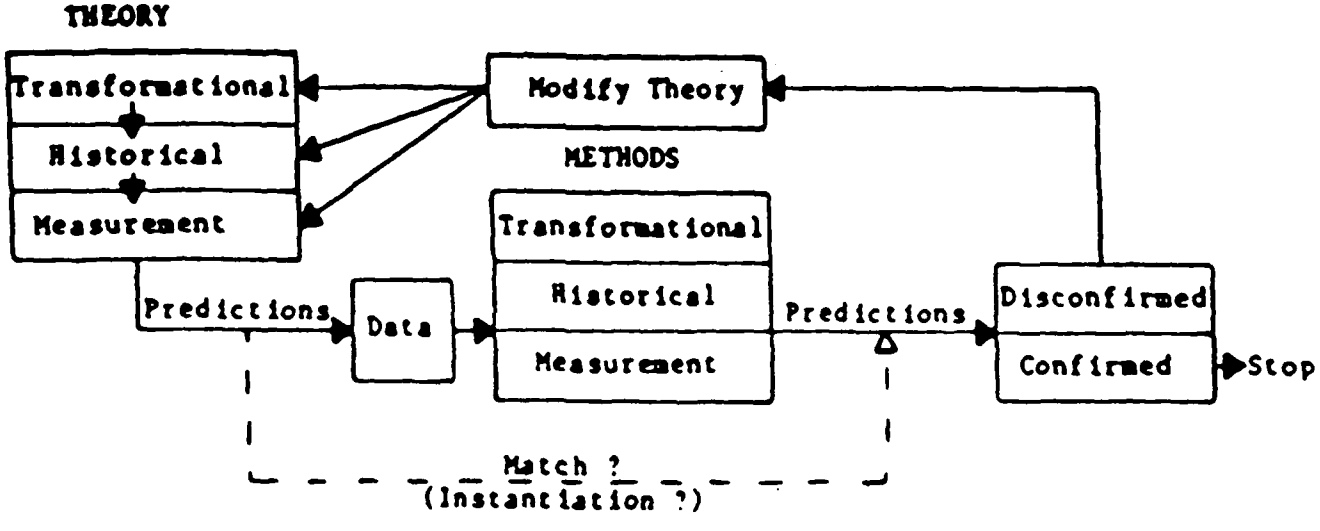

Figure 1. Some essentials of the scientific method.

problems can arise in the application of the scientific method (see Sections 1.3., 2.1.3., 3.3.4.). The goal of interactivity theory is to adhere to the principles of the scientific method in these potential problem areas, i.e., require (a) empiricism, (b) replication, (c) prediction, and (d) parsimony. These four scientific criteria were used as guidelines for methodological development and evaluation. Subsequent discussion is organized in these terms.

The relationships between these criteria are important since conclusions drawn from them can be in conflict. When conflict exists among objectives, the relationship between objectives is defined by the concept of preemptive prioritization--a concept borrowed from goal programming. In goal programming, the analyst must satisfy multiple mutually contradictory objectives by satisfying some ahead of others in order of a preestablished priority structure. An implicit priority structure exists among the scientific criteria: in order of priority, (a) empiricism, (b) replication, prediction (placed together as related criteria) and (c) parsimony. When conflict exists among criteria, empiricism must be ordered ahead of prediction so that theory can be tested by prediction. Empiricism requries observations to be made according to theory even if forecasts are not optimal under these conditions, because the lack of optimality provides a criterion for modifying theory. Parsimony involves refusing to multiply complexity beyond necessity. However, empiricism, prediction, and replication constitute necessities. Simplicity cannot be obtained at the expense of

empricism, prediction or replication, without engaging in over simplification or false parsimony. For these reasons parsimony is ordered last.

1. Empiricism

1.1. Bootstrapping Levels of Theory

Interactivity theory is partitioned into three levels of theory with three corresponding levels of methodology in Figure 1. The hierarchical organization of levels indicates a dependence of upper levels of theory upon lower levels. These same levels of theory exist in the physical sciences. However, in the physical sciences, the lower levels can be so straightforward that they become transparent to observers. Transformational theory is theory about change, i.e., what it takes to transform units of analysis from their present state to a new state as defined by predetermined goals. Such transformational questions are logically dependent upon historical theory, which describes present states in terms of past histories of the units of analysis. Historical theory, in turn, is dependent on a measurement theory that describes needed measurements and how to combine them in order to track appropriate "historical" variables. To recap, transformational theory cannot be tested without identifying current states first. Current states cannot be identified outside the context if their own histories, and histories cannot be tracked without measuring relevant variables first.

Empiricism requires that theory be tested at all levels. The only way to accomplish this is to test theory from the bottom up, from measurement theory through historical to transformational theory, in a bootstrap

fashion. Transformational theory cannot be tested prior to testing historical, and measurement theory; likewise, historical theory cannot be tested prior to measurement theory. Violating this order violates the principle of empiricism, and produces information that relies on a poor foundation. In psychology, it is generally much more difficult to bootstrap through lower levels of theory than in the physical sciences. Moreover, psychological theory (see Neel, 1977, for examples of theories) is frequently most elaborate at the transformational level and most sparce at the measurement levels--a situation incompatible with empiricism. The goal of interactivity theory is to adhere to empiricism by providing tools that allow observations to be made at all levels of theory. Almost by definition, this approch is data intensive, requiring appropriate observations at all levels. Initial emphasis is in measurement and historical models.

1.2. Correspondence of Theory and Method

Besides ignoring the necessity for bootstrapping, a second common way to violate the principle of empiricism is to mismatch theory and method as depicted in Figure 1. If the predictions provided by theory don't correspond directly to their realization through the method, "something" may be tested but it is not the theory in question. If the theory in question is not tested, the principle of empiricism has been violated. Instantiation is a descriptive term, borrowed from computer science terminology, that is used here to refer to this process of matching method to theory. Theory is like an inactive generic template.

Methodology must correctly activate specific instances of the theory. The process of correctly activating instances is called instantiation. Methodology must correctly instantiate theory.

Interactivity theory approaches the issue of instantiating theory by first defining generic characteristics of theory to which the methods can be applied. Interactivity theory was designed to apply to interactive human environments:

> The human environment can be characterized by complex, recursive, nonlinear, time-dependent exchanges between interacting people or groups. Interaction between participants is selective and goal oriented. The course of interaction adapts to fit circumstances; which can include random events. Interaction occurs in a context of social groups, organizations or institutions.

This generic characterization of the human environment is the starting point for interactivity theory. This starting point appears to be realistic and compatible with many psychological theories. The problem next becomes one of matching methods to the definition of the environment that will not violate the integrity of the initial definition. Linear prediction filtering (digital filtering) was selected as a technique that provides an appropriate match to the problem as defined

above. These filters are used in many high technology applications. To a large extent, the literature related to this approach is found outside psychology, e.g., in the economics, electronics engineering, and statistics literature (see Chow, 1975, 1981; Friedlander, 1982a, 1982b, 1983; Isaksson, Wennberg & Zetterberg, 1981; Jain, 1981; Kay & Marple, 1981). Many concepts differ from traditional deterministic concepts commonly found in psychology. Some familiarity with such models is necessary as a point of departure for further discussion. An example of a multichannel linear prediction filter is provided next.

1.2.1 Multichannel linear prediction filter. The model is a multivariate generalization of the autoregressive time-series model (see Chow, 1975, chap. 3). The definition of the model starts with model statements which define the linear relations between each variable at time t, and its own history and the histories of all other variables, as measured at equidistant and discrete points in past time. For example, model statements for a two-variable two-wave panel design can be written,

$$\begin{aligned} x_t &= a_{11}\, x_{t-1} + a_{12}\, y_{t-1} + e_{xt} \\ y_t &= a_{21}\, x_{t-1} + a_{22}\, y_{t-1} + e_{yt} \end{aligned} \qquad (2)$$

where x, y are measured variables, subscripts t, $t-1$ are the times associated with measurement intervals, a_{ij} are real linear coefficients showing relationships from past to present, and e_{xt}, e_{yt} represent

random residuals, or white noise processes. Likewise, the model statement for a two variable, three-wave design can be written,

$$\begin{aligned} \underline{x}_t &= \underline{a}_{11}\,\underline{x}_{t-1} + \underline{a}_{12}\,\underline{y}_{t-1} + \underline{a}_{13}\,\underline{x}_{t-2} + \underline{a}_{14}\,\underline{y}_{t-2} + \underline{e}_{xt} \\ \underline{y}_t &= \underline{a}_{21}\,\underline{x}_{t-1} + \underline{a}_{22}\,\underline{y}_{t-1} + \underline{a}_{23}\,\underline{x}_{t-2} + \underline{a}_{24}\,\underline{y}_{t-2} + \underline{e}_{yt}. \end{aligned} \tag{3}$$

The model statement can be written in the same way, as illustrated above, for both time-series and panel designs. With time-series many observations on a single case vary by time, while in panel designs, many cases are observed at just a few time intervals. The algebra is greatly simplified, if the model statement is rewritten in matrix notation as a first-order difference equation. Only a single time lag appears in a first-order equation. Since Equation 2 already has just one time lag, it translates directly into first order form as,

$$\begin{bmatrix} \underline{x}_{At} \\ \underline{x}_{Bt} \end{bmatrix} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_{At-1} \\ \underline{x}_{Bt-1} \end{bmatrix} + \begin{bmatrix} \underline{e}_{At} \\ \underline{e}_{Bt} \end{bmatrix}. \tag{4}$$

Since more than one time lag appears in the second model statement, it must be rewritten as,

$$\begin{bmatrix} \underline{x}_{At} \\ \underline{x}_{Bt} \\ \underline{x}_{At-1} \\ \underline{x}_{Bt-1} \end{bmatrix} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} & \underline{a}_{13} & \underline{a}_{14} \\ \underline{a}_{21} & \underline{a}_{22} & \underline{a}_{23} & \underline{a}_{24} \\ 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_{At-1} \\ \underline{x}_{Bt-1} \\ \underline{x}_{At-2} \\ \underline{x}_{Bt-2} \end{bmatrix} + \begin{bmatrix} \underline{e}_{At} \\ \underline{e}_{Bt} \\ 0 \\ 0 \end{bmatrix}. \tag{5}$$

Equations 4 and 5 can be written more compactly in first order form as,

$$\underset{\sim}{X}_t = \underset{\sim}{A}\underset{\sim}{X}_{t-1} + \underset{\sim}{v}_t, \text{ where} \qquad (6)$$

X_t is a n x m matrix of observed variables (starting at time t), with $\underset{\sim}{0}$ means. The term m represents the number of observations. The term $n = p \times \ell$. The term p is defined as the number of variables in the system at time t, and ℓ is the number of lagged observations on these variables, e.g., $\ell = 1$ in Equation 2 and $\ell = 2$ in Equation 3. The value ℓ represents the order of the equation. It should be noted that, in time series designs, the data matrix is transposed from the position typically given in a linear regression set-up. In addition, in order to simplify the notation, it is assumed that the observed variables have been scaled by a factor of $1/m^{\frac{1}{2}}$, so that covariance matrixes can be represented in the form $X_t X_t'$, without requiring division by m. $\underset{\sim}{A}$ is an n x n matrix of real linear coefficients (e.g., as shown in Equations 4 and 5). X_{t-1} is a n x m matrix of observed variables listed at time $t-1$ in relation to X_t. The vector v_t is a n x m matrix of random noise variables, uncorrelated with X_{t-1}, with $\underset{\sim}{0}$ means (see, e_{xt}, e_{yt} in Equations 4 and 5). When ℓ is greater than one, $\underset{\sim}{v}$ will contain some zero elements. It is clear that the pattern in Equations 5 and 6 readily generalizes to model statements with any number of variables and lags (see Chow, 1975, chap. 3).

Time is given by t, a counter that increases with time. Measurement lags are given by k, a counter measured in relation to t that increases backward in time from the starting value of 0 assigned at t. Synchronous relationships include only those measured at a common time, with $k = 0$. The expected

variance-covariance matrix between variables in synchronous time, can be written as a function of A and v in Equation 6 as follows:

$$\Gamma_{(t,0)} = E\,(X_{it}\, X'_{jt}) = V + AVA' + A^2VA'^2 + \ldots A^{t-1}VA'^{t-1}, \text{ where} \qquad (7)$$

the observed variables X_{it}, X_{jt-k} are expressed as deviations from their means, and $\Gamma_{(t,0}$ is a portion of the expected variance-covariance matrix at time lag $k = 0$. $\Gamma_{(t,0)}$ includes $n \times n$ covariances between those observed variables listed at the left in Equations 4, 5, and 6. A is the matrix of real coefficients from Equation 6. $V_{n \times n}$ is the variance-covariance matrix of the vector of residual noises v_t from Equation 6. Derivation of Equation 7 is provided in Chow (1975, chap. 3) and will not be repeated here. In the derivation of Equation 7, covariation between residuals is not permitted across time, anytime $k > 0$. However, covariation between residuals is permitted if it occurs in synchronous time ($k = 0$). The V matrix contains, then, variances of the residual noises for each variable. The pattern of covariances between residuals must be zero for lagged relationships ($k = 0$) but may differ from zero for synchronous relationships. Techniques for eliminating auto-correlated residuals are not discussed here (see Chow, 1975, pp. 61-63). Equation 7 is an infinite series. If the time-series is stationary (see discussion of stationarity in Sections 2.2.1.), the series reaches a steady state as time (denoted by t in Equation 7) goes to infinity, and additional terms on the right of Equation 7 become negligible.

The expected covariance matrix involving lagged relationships, $\underline{k} > 0$, is obtained by,

$$\Gamma_{(\underline{t},\underline{k}} = \underline{E}\,(\underset{\sim}{X}_{\underline{i}\underline{t}}\underset{\sim}{X}'_{\underline{j}\underline{t}-\underline{k}}) = \underset{\sim}{A}^{\underline{k}}\,\Gamma_{(\underline{t},0)}, \text{ where} \tag{8}$$

$\Gamma_{(\underline{t},1)}$, is the $\underline{n}$ x $\underline{n}$ section of the covariance matrix involving time at lag $\underline{k} \geq 1$. When $\underline{k} = 1$, $\Gamma_{(\underline{t},1)}$, involves the covariances between variables at $X_{\underline{t}}X'_{\underline{t}-1}$. Additional $\underline{n}$ x $\underline{n}$ sections of the covariance matrix can be projected backward in time as $\underline{k}$ increases, $\underline{k} > 1$.

It is clear from Equation 8 that $\underset{\sim}{A}$ can be calculated from the observed $\underline{n}$ x $\underline{n}$ sections of the covariance matrix,

$$\underset{\sim}{A} = \Gamma_{(1)}\,\Gamma_{(0)}^{-1}. \tag{9}$$

In order to make the notation more concise, $\underline{t}$ has been deleted from (t,k) in Equation (9) and subsequent equations. $\Gamma_{(1)}$ and $\Gamma_{(0)}$ must be stationary in this model.

$\underset{\sim}{V}$ can be calculated by eliminating the infinite series from Equation 7. This is accomplished by premultiplying Equation 7 by $\underset{\sim}{A}$ and post-multiplying it by $\underset{\sim}{A}'$ which yields,

$$\underset{\sim}{A}\,\Gamma_{(0)}\,\underset{\sim}{A}' = \underset{\sim}{A}\underset{\sim}{V}\underset{\sim}{A}' + \underset{\sim}{A}^2\underset{\sim}{V}\underset{\sim}{A}'^2 + \ldots\ldots \tag{10}$$

Subtracting Equation 10 from Equation 7 eliminates the infinite series, and provides an expression for $\underset{\sim}{V}$:

$$\underline{E}(\underset{\sim}{V}) = \Gamma_{(0)} - \underset{\sim}{A}\,\Gamma_{(0)}\,\underset{\sim}{A}'. \tag{11}$$

An alternate expression for Equation 11, in summation notation, is useful for purposes of compact calculation:

$$\underline{V}_{k\ell} = \sum_{\underline{\ell}=1}^{\underline{n}} \sum_{\underline{k}=1}^{\underline{n}} \left[\gamma_{(0)\,\underline{k\ell}} - \left(\sum_{\underline{i}=1}^{\underline{n}} \sum_{\underline{j}=1}^{\underline{n}} \underline{a}_{ki}\, \underline{a}_{\ell j}\, \gamma_{(0)\,\underline{ij}} \right) \right], \text{ where} \qquad (12)$$

$\underline{n}$ is defined as in Equation 6, and elements of $\underline{\underline{A}}$, $\Gamma_{(0)}$, and $\underline{\underline{V}}$ are represented by $\underline{a}$, $\gamma_{(0)}$, and $\underline{V}$, respectively.

Given an observed stationary covariance matrix, $\underline{\underline{A}}$ can be calculated first from Equation 9, and $\underline{\underline{V}}$ can be calculated next from Equations 11 or 12. To close the loop, $\underline{\underline{A}}$ and $\underline{\underline{V}}$ can be substituted into Equations 7 and 8, and the original covariance matrix can be regenerated exactly from $\underline{\underline{A}}$ and $\underline{\underline{V}}$.

Equation 7 includes infinite series to generate $\Gamma_{(0)}$. This is not a particularly efficient way to compute $\Gamma_{(0)}$. Appendix A provides alternate computational procedures.

1.2.2. Filter characteristics. The model just outlined represents a multichannel, finite, impulse-response (FIR) filter for zero-mean processes. Estimates are equivalent to estimates from simple multivariate regression computed on stationary, block Toeplitz matrices (see Section 2.1.1.). The regression is multivariate in a dual sense with multiple dependent as well as independent variables. The logic underlying a filtering model differs in a variety of ways from a deterministic regression model, however. In a deterministic regression equation, the regression coefficient represents the degree of deterministic input from "independent" variables, and "error" represents the lack of perfect

control. By contrast, in a filtering model, the stochastic white noise parameters (i.e., "error") represent the input that drives the system. Without the driver the system would die. Activation of, or driving energy for, a stochastic system must always come from sources that are at least partly stochastic (i.e., random). By contrast, in a deterministic model, activation or energy comes from entirely deterministic sources. In a stochastic system, the random driver feeds two types of processes: a colored (i.e., correlated) noise process that in turn feeds into an "autoregression" process. Together these processes produce filtered output--containing structure--from white noise as input.

This concept underlies the multivariate ARMA model (see Box & Jenkins, 1976; Granger & Newbold, 1977). In this model, white and colored noise processes drive filter coefficients (autoregression, AR, parameters) producing filtered output. Moving average (MA) parameters are associated with colored noise processes. A wide variety of physical and biological systems can be modeled in this way (Kay & Marple, 1981). The model in this section is a special case of an ARMA model. The model is a special case because colored noise is limited to synchronous time. The information is partitioned so that all information that is useful for forecasting is contained in $\underset{\sim}{A}$, the matrix of "autoregressive" coefficients, while all information about the noise processes that drive the system is contained in $\underset{\sim}{V}$. In a filtering model, if a particular process produced structure in data, the same process should be able to

decompose it, reducing residuals to white noise.

In the general case, iterative numerical techniques are required to estimate ARMA parameters (see Granger & Newbold, 1977). However, in the special case just noted, $\underset{\sim}{A}$ and $\underset{\sim}{V}$ parameters can be estimated in closed form using least squares estimators. In addition, the least squares estimators in the special case are also maximum likelihood estimators. A stationary process (see Section 2.1.1.) can be written in first order form as in Equation 6, and considered to follow a Markov process. According to the Gaus-Markov theorem, least squares estimators are also maximum likelihood (minimum variance) estimators under these circumstances (Wilks, 1962, pp. 283-286).

Filters as in Section 1.2.1. provide a number of useful features for modeling social interaction. In psychology, theoretical relationships are frequently stated in terms of discrete linear relationships. Parameters $\underset{\sim}{A}$ and $\underset{\sim}{V}$ provide a useful form for instantiating many theories. In addition, features can be modeled with a stochastic driver that are difficult to model with a deterministic one. As indicated by Definition 1, random events are continually introduced into the course of natural interaction, affecting its outcome. Filters with stochastic drivers can model and track interaction in the presence of such noise. In addition, stochastic drivers are useful for modeling selection and choice behavior. As indicated by Definition 1, goal-oriented behavior is an integral part of social interaction. Goals imply selection and choice. The range of possibilities from filtered output is narrower than that of the

stochastic input. Filters are selective by definition. A driver of white or colored noise can represent a "menu." Selections from the "menu" can be inferred from filtered output. By contrast, choice behavior is difficult to model with a totally deterministic model. Stochastic input is required in a stochastic model as in Section 1.2.1., but stochastic input does not preclude the addition of deterministic input to the system. In fact, much of modern controls theory involves calculation of deterministic inputs necessary to produce specified transformational changes in a stochastic system (Chow, 1981).

In spite of the possibilities for testing theory, most time-series models and engineering applications delete theory instantiation as a scientific criterion or goal. When this occurs, the model becomes a "black box" representation (*see Makridakis & Wheelwright, 1979*). Black box models are applied in practical situations where the goal is to predict accurately without concern for testing theory. ARMA models, "state-space" representations, and canonical parameter representations are generally black box models (see Akaike, 1976; Box & Jenkins, 1976; El-Sherief & Sinha, 1982). The objective is to provide the best forecasts possible with a minimum number of parameters.

1.2.3. Measurement model. In order to bootstrap through levels of theory, a measurement model needs to be added to the filter in Section 1.2.1. Measurement models are designed to reduce measurement error, and thus increase predictive accuracy. In addition, they increase parsimony by reducing the number of variables in the system.

Measure theory in psychology has traditionally been a latent variable, or "hypothetical construct" model (APA et al., 1954), that has been instantiated using factor analysis. Interactivity theory requires that a measurement model be able to distinguish between interactive historical effects and measurement effects produced by latent variables. Unfortunately, traditional synchronous-time measurement models, including factor analysis, are unable to make this distinction by themselves. Time must be added to these measurement models in order to validate that a true latent variable model exists, or is at least possible. Without validating that a true latent variable model exists, researchers run the risk of confusing interaction effects with measurement effects, with potentially disastrous results. In this section and subsequently whenever measurement models are discussed, it is assumed that the data has been standardized. Consequently, we will be dealing with correlation instead of covariance matrices. To simplify notation, correlation matrices are represented by $\underline{X}_t\underline{X}_t'$, without dividing by the product of sample size and variable standard deviations.

For example, a synchronous correlation matrix between four variables is provided in Equation 13,

$$\Gamma_{\underline{x}(0)} = \begin{bmatrix} 1.0 & .36 & 0 & 0 \\ .36 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & .51 \\ 0 & 0 & .51 & 1.0 \end{bmatrix}. \qquad (13)$$

$\Gamma_{\underline{x}(0)}$ in Equation 13 appears to be a perfectly appropriate candidate for a measurement model, so a common factor analysis was applied. The factor loading matrix is shown in Equation 14. A principal axes solution with iterations was used.

$$B = \begin{bmatrix} .60 & 0 \\ .60 & 0 \\ 0 & .71 \\ 0 & .71 \end{bmatrix} \tag{14}$$

From Equations 13 and 14, it appears appropriate to form two scales: Scale 1 is the average of items 1 and 2, and Scale 2 is the average of 3 and 4. Applying the Spearman-Brown prediction formula (Winer, 1971, p. 286), the reliabilities of these two scales are .53 and .68, respectively. From the classical relationship between reliability and validity ($r_{\underline{xy}} = \sqrt{r_{\underline{xx}} r_{\underline{yy}}}$), a researcher knows Scales 1 and 2 can never correlate higher than .60.

Unfortunately, history has been excluded from Equation 13. It is assumed that $\Gamma_{\underline{x}(0)}$ in Equation 13 is stationary. $\Gamma_{\underline{xa}(1)}$ in Equation 15 represents a stationary first-order historical process added to $\Gamma_{\underline{x}(0)}$ in Equation 13,

$$\Gamma_{\underline{xa}(1)} = \begin{bmatrix} .3 & .3 & 0 & 0 \\ .3 & .3 & 0 & 0 \\ -.7 & -.7 & .4 & .4 \\ -.7 & -.7 & .4 & .4 \end{bmatrix}. \tag{15}$$

Given the historical process in Equation 15, the lagged correlations for Scales 1 and 2 were computed,

$$\Gamma_{\underline{sa}(1)} = \begin{bmatrix} .30 & 0.0 \\ -.70 & .40 \end{bmatrix} \tag{16}$$

The lagged correlation between scales, $\underline{r}_{12} = -.70$, violates the upper boundary determined by reliability-validity theory, $.70 > .60$. Reliabilities are stationary in this example, so the upper boundary applies to lagged as well as synchronous correlations.

Assume a different history in Equation 17 were added to $\Gamma_{\underline{x}(0)}$ in Equation 13,

$$\Gamma_{\underline{xb}(1)} = \begin{bmatrix} .3 & .3 & .5 & .5 \\ .3 & .3 & -.5 & -.5 \\ -.5 & .5 & .4 & .4 \\ -.5 & .5 & .4 & .4 \end{bmatrix} \tag{17}$$

According to measurement theory, the creation of scales reduces measurement error. As a consequence, correlations between Scales 1 and 2 should be higher than corresponding correlations between individual items from different scales. Given the historical process in Equation 17, the lagged correlations were computed for Scales 1 and 2,

$$\Gamma_{\underline{sb}(1)} = \begin{bmatrix} .30 & 0 \\ 0 & .40 \end{bmatrix}. \tag{18}$$

The correlations in Equations 16 and 18 would be seriously misinterpreted if a true measurement model as depicted in Equations 13 and 14 were accepted. A falsely indentified "measurement" model can cover over significant interaction effects, and reduce the correlation between scales rather than increase it (e.g., see the 0 cross-lagged correlations in Equation 18). The time lagged relationships in Equations 15 and 17 represent relationships between separate variables rather than true latent variables. Synchronous-time measurement theory accepts a latent variable by definition. Time must be added to verify the existance of true latent variables. If true latent variables don't exist, then none of the traditional measurement theory follows. Tests for the existence of true latent variables can be made using the time-series factor analysis model described next.

The traditional factor analytic model can be written as follows:

$$\underset{\sim}{X}_{\underline{t}} = \underset{\sim}{B}\underset{\sim}{Z}_{\underline{t}} + \underset{\sim}{m}_{\underline{xt}}. \text{ where} \qquad (19)$$

$\underset{\sim}{X}_{\underline{t}}$ is a $\underline{p}$ x $\underline{1}$ column vector of observed variables, with $\underset{\sim}{0}$ means, measured at time $\underline{t}$; $\underset{\sim}{B}$ is a $\underline{p}$ x $\underline{f}$ matrix of factor loadings; $\underset{\sim}{Z}_{\underline{t}}$ is a $\underline{f}$ x 1 column of latent, unobserved variables; $\underset{\sim}{m}_{\underline{xt}}$ is a $\underline{p}$ x 1 column of independent measurement errors, uncorrelated with other variables, with $\underset{\sim}{0}$ means; and $\underline{p}$ and $\underline{f}$ are values identifying the number of observed and latent variables, respectively, at time $\underline{t}$. For the sake of concise notation, sample size $\underline{m}$ has been deleted from $\underset{\sim}{X}_{\underline{t}}$, $\underset{\sim}{Z}_{\underline{t}}$ and $\underset{\sim}{m}_{\underline{xt}}$ and these terms are represented as column vectors rather than matrices. Sample size is implicit in the vectors. Assuming $\underset{\sim}{X}_{\underline{t}}$ and $\underset{\sim}{Z}_{\underline{t}}$ to be standardized, the expected structure of the correlations at time $\underline{t}$ follows

Equation 19,

$$\underset{\sim}{E}(\underset{\sim}{X}_t \underset{\sim}{X}_t') = \underset{\sim}{B}\underset{\sim}{P}_{(0)}\underset{\sim}{B}' + \underset{\sim}{M}, \text{ where} \tag{20}$$

$\underset{\sim}{E}(\underset{\sim}{X}_t \underset{\sim}{X}_t')$ is the observed correlation matrix; $\underset{\sim}{P}_{(0)}$ is the correlation matrix of latent variables $\underset{\sim}{Z}_t \underset{\sim}{Z}_t'$ at time t; and $\underset{\sim}{M}$ is a diagonal variance-covariance matrix of measurement errors $\underset{\sim}{m}_{xt}$. Equation 20 defines the traditional factor analytic problem (Jöreskog, 1970; Mulaik, 1972).

Time is added to the traditional factor analytic model by assuming the latent variable to be the product of a stationary historical process (see Section 2.1.1.),

$$\underset{\sim}{Z}_t = \underset{\sim}{A}_{zt-1} + \underset{\sim}{e}_{zt}, \text{ with} \tag{21}$$

$\underset{\sim}{Z}_t$ written in first order form as in Equation 6. When time is added to the measurement model, replicability and stationarity become issues. Equation 21 applies the simplest assumption of first order stationarity. By substituting $\underset{\sim}{Z}_t$ from Equation 21--at time t only--into Equation 19, the model becomes:

$$\begin{bmatrix} \underset{\sim}{X}_{1t} \\ \underset{\sim}{X}_{2t} \\ \vdots \\ \underset{\sim}{X}_{pt} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{b}_{11} \cdots \underset{\sim}{b}_{1f} \\ \underset{\sim}{b}_{21} \cdots \underset{\sim}{b}_{2f} \\ \vdots \\ \underset{\sim}{b}_{p1} \cdots \underset{\sim}{b}_{pf} \end{bmatrix} \left[\begin{bmatrix} \underset{\sim}{a}_{111} \cdots \underset{\sim}{a}_{1f1} \\ \vdots \\ \underset{\sim}{a}_{f11} \cdots \underset{\sim}{a}_{ff1} \end{bmatrix} \begin{bmatrix} \underset{\sim}{Z}_{1t-1} \\ \vdots \\ \underset{\sim}{Z}_{ft-1} \end{bmatrix} + \begin{bmatrix} \underset{\sim}{a}_{11\ell} \cdots \underset{\sim}{a}_{1f\ell} \\ \vdots \\ \underset{\sim}{a}_{f1\ell} \cdots \underset{\sim}{a}_{ff\ell} \end{bmatrix} \begin{bmatrix} \underset{\sim}{Z}_{1t-\ell} \\ \vdots \\ \underset{\sim}{Z}_{ft-\ell} \end{bmatrix} + \begin{bmatrix} \underset{\sim}{e}_{z1t} \\ \vdots \\ \underset{\sim}{e}_{zft} \end{bmatrix} \right] + \begin{bmatrix} \underset{\sim}{m}_{x1t} \\ \underset{\sim}{m}_{x2t} \\ \vdots \\ \underset{\sim}{m}_{xpt} \end{bmatrix}, \text{ or}$$

$$\underset{\sim}{X}_t = \underset{\sim}{B}\left[\underset{\sim}{A}(1)\underset{\sim}{Z}_{t-1} + \cdots \underset{\sim}{A}(\ell)\underset{\sim}{Z}_{t-\ell} + \underset{\sim}{e}_{zt}\right] + \underset{\sim}{m}_{xt}.$$

The subscript $\underline{\ell}$ denotes the order of the latent variable process, or number of latent lags, which is assumed to be equal to the number of lags in observed variables. When $\underline{\ell} = 1$, the model does not continue beyond $\underset{\sim}{A}(1)$. In Equation 22, measurement error ($\underset{\sim}{m}_{\underline{xt}}$) is different from the error that drives the filter ($\underset{\sim}{e}_{\underline{zt}}$), in the sense that measurement error does not drive filter coefficients. It only occurs in synchronous time without history ever being added to it.

The overall model in Equation 22 can be considered nonlinear. There are a variety of approaches to estimating the parameters in such a model. An analytic solution was selected here for the sake of simplicity. The solution, while not necessarily optimal, is sufficiently accurate to illustrate the influence of time on the traditional factor analytic model. Observing Equation 20 it is clear that $\underset{\sim}{B}$ and $\underset{\sim}{M}$ can be estimated using traditional factor analytic techniques, $\underset{\sim}{M}_{(pxp)} = I - \text{diag}\ \underset{\sim}{C}^2$, where $\underset{\sim}{C}^2$ represents the communality and diag places the communalities as diagonal entries in a matrix. This leaves $\underset{\sim}{A}_{\underline{z}}$, $\underset{\sim}{V}_{\underline{z}} = \underset{\sim}{e}_{\underline{zt}}\underset{\sim}{e}'_{\underline{zt}}$, and the latent correlation structure to be estimated.

Time is added to the observed variables through backsubstitution:

$$\begin{aligned} \underset{\sim}{X}_{\underline{t}} &= \underset{\sim}{B}\underset{\sim}{Z}_{\underline{t}} + \underset{\sim}{m}_{\underline{xt}}, \\ &\vdots \\ \underset{\sim}{X}_{\underline{t-\ell}} &= \underset{\sim}{B}\underset{\sim}{Z}_{\underline{t-\ell}} + \underset{\sim}{m}_{\underline{xt-\ell}}. \end{aligned} \tag{23}$$

Next, we solve Equation 23 for $\underset{\sim}{Z}_{\underline{t}} \ldots \underset{\sim}{Z}_{\underline{t-\ell}}$, by letting $\underset{\sim}{B}^{+} = (\underset{\sim}{B}'\underset{\sim}{B})^{-1}\underset{\sim}{B}'$, the Moore-Penrose generalized inverse of $\underset{\sim}{B}$ (see Green, 1976, Appendix B):

$$\begin{aligned} \underset{\sim}{z}_t &= \underset{\sim}{B}^+ \underset{\sim}{x}_t - \underset{\sim}{B}^+ \underset{\sim}{m}_{xt}, \\ &\vdots \\ \underset{\sim}{z}_{t-\ell} &= \underset{\sim}{B}^+ \underset{\sim}{x}_{t-\ell} - \underset{\sim}{B}^+ \underset{\sim}{m}_{xt-\ell}. \end{aligned} \tag{24}$$

The expected latent correlation structure can be estimated from Equation 24,

$$\begin{aligned} \underline{E}\,(\underset{\sim}{z}_t \underset{\sim}{z}_t') &= \underset{\sim}{P}_{(0)} = \underset{\sim}{B}^+ (\underset{\sim}{x}_t \underset{\sim}{x}_t' - \underset{\sim}{M}) \underset{\sim}{B}^{+\prime}, \\ \underline{E}\,(\underset{\sim}{z}_t \underset{\sim}{z}_{t-1}') &= \underset{\sim}{P}_{(1)} = \underset{\sim}{B}^+ (\underset{\sim}{x}_t \underset{\sim}{x}_{t-1}') \underset{\sim}{B}^{+\prime}, \\ &\vdots \\ \underline{E}\,(\underset{\sim}{z}_t \underset{\sim}{z}_{t-\ell}') &= \underset{\sim}{P}_{(\ell)} = \underset{\sim}{B}^+ (\underset{\sim}{x}_t \underset{\sim}{x}_{t-\ell}') \underset{\sim}{B}^{+\prime}. \end{aligned} \tag{25}$$

It should be noted here that when B is estimated using a factoring technique in which latent variables are orthogonal, (e.g., Principal-axes method, Mulaik, 1972) $\underset{\sim}{P}_{(0)}$ will be in identity matrix.
The latent correlation structure is packed in first order form in the same way that $\Gamma_{(0)}$ and $\Gamma_{(1)}$ was packed in Section 1.2.1.:

$$\Gamma_{\underset{\sim}{z}(0)} = \begin{array}{c} \\ \underset{\sim}{z}_t \\ \vdots \\ \underset{\sim}{z}_{t-\ell+1} \end{array} \begin{array}{c} \begin{array}{cc} \underset{\sim}{z}_t & \cdots \underset{\sim}{z}_{t-\ell+1} \end{array} \\ \begin{bmatrix} \underset{\sim}{P}_{(0)} & \cdots \underset{\sim}{P}_{(\ell-1)} \\ \vdots & \vdots \\ \underset{\sim}{P}_{(\ell-1)} & \cdots \underset{\sim}{P}_{(0)} \end{bmatrix} \end{array},$$

$$\Gamma_{\underset{\sim}{z}(1)} = \begin{array}{c} \\ \underset{\sim}{z}_t \\ \vdots \\ \underset{\sim}{z}_{t-\ell+1} \end{array} \begin{array}{c} \begin{array}{cc} \underset{\sim}{z}_{t-1} & \cdots \underset{\sim}{z}_{t-\ell} \end{array} \\ \begin{bmatrix} \underset{\sim}{P}_{(1)} & \cdots \underset{\sim}{P}_{(\ell)} \\ \vdots & \vdots \\ \underset{\sim}{P}_{(\ell-2)} & \cdots \underset{\sim}{P}_{(1)} \end{bmatrix} \end{array}. \tag{26}$$

In Equation 26, the lag $\underline{k}$ from $\underset{\sim}{P}_{(k)}$ is $|\underline{k}_{row} - \underline{k}_{column}|$. When $\underline{\ell} = 1$, $\Gamma_{\underline{z}(0)}$ and $\Gamma_{\underline{z}(1)}$ are limited to the upper left block, $\underset{\sim}{P}_{(0)}$ and $\underset{\sim}{P}_{(1)}$ respectively. It is now possible to solve for $\underset{\sim}{A}_{\underline{z}}$ and $\underset{\sim}{V}_{\underline{z}}$ as in Section 1.2.1.:

$$\underset{\sim}{A}_{\underline{z}} = \Gamma_{\underline{z}(1)} \Gamma_{\underline{z}(0)}^{-1}$$

$$\underline{E}\ (\underset{\sim}{V}_{\underline{z}}) = \Gamma_{\underline{z}(0)} - \underset{\sim}{A}_{\underline{z}} \Gamma_{\underline{z}(0)} \underset{\sim}{A}'_{\underline{z}}. \qquad (27)$$

These equations represent a "time-series" factor analytic model, that was applied to data in the Example Section.

1.2.4. Interaction between groups. The definition of the human environment in Definition 1 refers to interaction between groups as well as individuals. When theory does refer to interacting groups, the interacting groups become the unit of analysis that must be used by the methodology to instantiate theory. Generally, interacting groups are found within a larger social organization. The structure of this larger social organization must be taken into account by methodology. In order to correctly instantiate theory involving group interaction, then, appropriate groups as opposed to individuals must be designated as the units of analysis, and the structure of these groups in the larger social organization must be taken into account. Research results are sensitive to these requirements.

The reasons for the conclusions are illustrated by example. When groups are the unit of analysis, group aggregated means provide scores to be used in an analysis. Generalization theory can be used to clarify the structure of these group aggregated scores. Generalization theory applies analysis of variance to complex data structures in order to estimate variance components, and generalized analogies of reliability and correlation coefficients (see Shavelson & Webb, 1981). A model

statement is provided for the U. S. Army data in the Example Section. The structure of this survey data at time $\underline{t}$ is described by:

$$Y = \mu + \underline{A} + \underline{B}(\underline{A}) + \underline{C}(\underline{AB}) + \underline{R} + \underline{AR} + \underline{BR}(\underline{A}) + \underline{CR}(\underline{AB}) + \underline{S}(\underline{ABCR}) + \underline{Q} + \underline{AQ} + \underline{BQ}(\underline{A}) + \underline{CQ}(\underline{AB}) + \underline{RQ} + \underline{ARQ} + \underline{BRQ}(\underline{A}) + \underline{CRQ}(\underline{AB}) + \underline{SQ}(\underline{ABCR}). \quad (28)$$

Nesting relationships are indicated by parentheses. Potential units of analysis are defined by nested levels of hierarchy; $\underline{A}$, $\underline{B}(\underline{A})$, $\underline{C}(\underline{AB})$, and $\underline{S}(\underline{ABCR})$; defined as brigade, battalion, company, and subject, in this example. $\underline{R}$ refers to subgroups that cross levels of hierarchy above subjects; in this example, rank and race, among others. $\underline{Q}$ refers to questionnaire items. From the complexity of Equation 28, it becomes clear that there is an increase in the number of ways correlations/covariances can be computed using group as opposed to individual level data. A rich variety of hypotheses can be tested with group level data. For example, interaction between subgroups within companies (e.g., superiors and subordinates, blacks and whites) is possible with such data.

Associated with each term that can serve as a unit of analysis, $\underline{A}$, $\underline{B}$, $\underline{C}$ and $\underline{S}$, is a variance component, $\sigma_{\underline{A}}^2$, $\sigma_{\underline{B}}^2$, $\sigma_{\underline{C}}^2$, and $\sigma_{\underline{S}}^2$. These variance components are the ones of interest in the analysis, even though the expected mean squares for $\underline{MS}_{\underline{A}}$, $\underline{MS}_{\underline{B}}$, $\underline{MS}_{\underline{C}}$, and $\underline{MS}_{\underline{S}}$ contain additional variance components, depending upon the sampling plan in the design (see Hart & Bradshaw, Note 2). In reliability or correlational

analyses, variance components are assigned the status of either "true" or "error" variance. Ultimately, the status of "true" variance ($\sigma^2_{\underline{true}}$) is assigned to $\sigma^2_{\underline{A}}$, $\sigma^2_{\underline{B}}$, $\sigma^2_{\underline{C}}$ or $\sigma^2_{\underline{S}}$ by theory. Theory pinpoints the unit of analysis term from among $\underline{MS}_{\underline{A}}$, $\underline{MS}_{\underline{B}}$, $\underline{MS}_{\underline{C}}$ and $\underline{MS}_{\underline{S}}$. The expectation of this term $\underline{E}(\underline{MS})$ must contain the true variance component $\sigma^2_{\underline{true}}$. For reasons to be discussed, "error" variance ($\sigma^2_{\underline{error}}$) status is assigned all the other components ($\sigma^2_{\underline{A}}$, $\sigma^2_{\underline{B}}$, $\sigma^2_{\underline{C}}$, or $\sigma^2_{\underline{S}}$) besides the one selected as the unit of analysis ($\sigma^2_{\underline{true}}$). With these assignments, the status of the variance components ($\sigma^2_{\underline{A}}$, $\sigma^2_{\underline{B}}$, $\sigma^2_{\underline{C}}$, $\sigma^2_{\underline{S}}$) changes as the corresponding unit of analysis ($\underline{A}$, $\underline{B}$, $\underline{C}$, or $\underline{S}$) changes. What is "true" variance at one level becomes "error" at the next. For example, if subjects represent the unit of analysis, $\sigma^2_{\underline{true}} = \sigma^2_{\underline{S}}$ and $\sigma^2_{\underline{error}} = \sigma^2_{\underline{A}}$, $\sigma^2_{\underline{B}}$ and $\sigma^2_{\underline{C}}$. $\underline{MS}_{\underline{total}}$ contains both true plus error variance, $\underline{MS}_{\underline{total}} = \underline{MS}_{\underline{S}}$. If companies represent the unit of analysis, $\underline{MS}_{\underline{total}} = \underline{MS}_{\underline{C}}$, $\sigma^2_{\underline{true}} = \sigma^2_{\underline{C}}$, and $\sigma^2_{\underline{error}} = \sigma^2_{\underline{A}}$, $\sigma^2_{\underline{B}}$, and $\sigma^2_{\underline{S}}$. The status of $\sigma^2_{\underline{C}}$ and $\sigma^2_{\underline{S}}$ changed as the unit changed.

Variance components can be divided into two classes by ordering them hierarchically, $\sigma^2_{\underline{A}}$, $\sigma^2_{\underline{B}}$, $\sigma^2_{\underline{C}}$, $\sigma^2_{\underline{S}}$, and separating the components above the designated unit of analysis from those below. The reason for assigning "error" status to these two classes differs. Correlational analyses like the one in Section 1.2.1. require independent observations on units of analysis. Components in the upper class are designated as error because they represent dependence between observations that are supposed to be independent. For example, if subjects represent the unit of analysis, the $\sigma^2_{\underline{A}}$, $\sigma^2_{\underline{B}}$, $\sigma^2_{\underline{C}}$ components represent "improper" dependencies

between individual level observations, analogous to "autoregression" errors in a time-series design (Hibbs, 1974). These dependencies need to be filtered out prior to entering the observations into correlational analyses. Components beneath the unit of analysis term are designed as error when they are sampled. For example, in the Example Section, companies are designated as the unit of analysis. Individual soldiers within companies were sampled in order to estimate company means. By sampling subjects, σ_S^2 becomes a component of "sampling error" that shows up in the expected mean square term for companies $E(MS_C)$.

The results of correlational analyses are determined by the relative size of "true" and "error" variance components. Total variance is a sum of true plus total error variance. Generalization theory defines correlation and reliability coefficients (positive sign) in terms of a ratio of true to total variance (see Kane & Brennan, 1977; Winer, 1971, pp. 283-296; Hart & Bradshaw, Note 2). Estimates for correlations and reliabilities are obtained by solving for this ratio definition using appropriate mean square terms. In this context, correlations can be estimated from a generalization of the intraclass correlation coefficient:

$$[MS_{total} - MS_{error}]/[MS_{total} + (n-1)MS_{error}]. \quad (29)$$

Likewise, reliability coefficients for mean scores can be estimated by:

$$(MS_{total} - MS_{error})/MS_{total}. \quad (30)$$

It is evident from Equations 29 and 30 that correlations and reliabilities are large when $\sigma^2_{\underline{true}}$, $\underline{MS}_{\underline{total}}$ are large, and $\sigma^2_{\underline{error}}$, $\underline{MS}_{\underline{error}}$ are small. Furthermore, the results will be sensitive to selection of the unit of analysis. The terms that can be substituted into these formulas vary as the unit of analysis changes. The term designated as the unit of analysis is always $\underline{MS}_{\underline{total}}$. $\underline{MS}_{\underline{error}}$ is a "sampling error" term that also varies with the unit of analysis. This term must either (a) belong to the nested hierarchy below the unit of analysis term, or (b) represent an interaction between the unit of analysis and the variable being correlated (see Hart & Bradshaw, Note 2). The value $\underline{n}$ represents the number of observations sampled (e.g., questionnaire items, subjects within companies). The use of Equations 29 and 30 is illustrated by comparing the following substitutions: for individuals, $\underline{MS}_{\underline{total}} = \underline{MS}_{\underline{S}}$, $\sigma^2_{\underline{true}} = \sigma^2_{\underline{S}}$, $\underline{MS}_{\underline{error}} = \underline{MS}_{\underline{SQ}}$, $\sigma^2_{\underline{error}} = \sigma^2_{\underline{SQ}}$, $\underline{n}$ = number of questionnaire items; for companies, $\underline{MS}_{\underline{total}} = \underline{MS}_{\underline{C}}$, $\sigma^2_{\underline{true}} = \sigma^2_{\underline{C}}$, $\underline{MS}_{\underline{error}} = \underline{MS}_{\underline{S}}$, $\sigma^2_{\underline{error}} = \sigma^2_{\underline{S}}$, $\underline{n}$ = number of subjects within companies. The first set of substitutions represent correlations between items for individuals, while the second set, correlations between subjects within companies at the company level. In this example, true variance in $\underline{MS}_{\underline{S}}$ at the individual level, became error variance at the company level.

Given that correlations are a function of the sizes of "true" and "error" variance components, and that true and error designations change as the unit of analysis changes, then it follows that conclusions reached at one level may be quite different than conclusions reached at another.

These results demonstrate why individual level data cannot be used to instantiate theory about group interaction, and why group data from one level of hierarchy cannot be used to instantiate theory about another level, without violating the principle of empiricism. Unfortunately, it is common to see group level theory "tested" using individual level data, or using an inconsistent mix of group and individual level data (see Bowers, 1973; Passmore, 1976; Taylor & Bowers, 1972, p. 54; Torbert, 1973).

1.3. Comparison

In order to clarify the nature of interactivity theory, it is instructive to contrast it with other commonly used methodologies; in this case with analysis of variance. Comparisons are made using relevant *scientific criteria as guidelines*, i.e., empiricism, prediction, replication and parsimony. In this case, theory instantiation provides the basis for comparison.

Analysis of variance, by itself, does not do a good job of instantiating an interactive theory of the environment, defined generically in Definition 1. Design requirements in experiments are dictated by the method of analysis, in this case analysis of variance. The use of analysis of variance on randomized experiments is justified by the Gaus-Markov theorem (Bock, 1975, pp. 175, 235-236). In order to create a system that approximates a Markov process, history must be removed by the random assignment of units (people or groups) to treatment conditions. In addition, analysis of variance assumes treatment conditions

affect only means, not variance-covariance structures. Since history affects variance-covariance structures (Chow, 1975), history must be removed by random assignment. With past history removed from a given situation, subjects may often have a short-term view of the future. In fact most experiments examine single one-step responses of subjects to treatment conditions. One-step responses are consistent with a Markov process. In addition to removing history, treatment conditions are deterministic, which means that all subjects within the same treatment condition must receive exactly the same predetermined treatment. Causation is unidirectional, from independent to dependent variables.

These analysis of variance design requirements create difficulties for instantiating an interactive theory of the human environment. For example, interaction as defined in Definition 1 requires time, with past history and future goals. This is difficult to achieve in most ANOVA designs, since history and variance-covariance effects must be removed through random assignments to new situations. By definition, interaction involves exchange. The forms of exchange in most ANOVA designs are highly restricted, limited to one-way, treatment-to-dependent-variable exchanges; in which the treatment party must remain nonresponsive to whatever the subject does, since treatment effects have to remain deterministic. Given such "unnatural" restrictions, it is not surprising that criticisms have been leveled at experimentalism. The meaning and interpretation of treatment effects, and the "external validity" of dependent variable responses, becomes a problem when time

(past and future) in the experimental setting is artificially truncated (see Alexander & Knight, 1971; Bem & Funder, 1978). "Spurious" forms of interaction, e.g., "social desirability" responses and demand characteristics, (Brehm, 1966; Orne, 1962; Rosenthal & Rosnow, 1969) may pop up in many experimental situations precisely because normal forms of interaction are so severely restricted. Human experiments are often designed like agricultural experiments. Analysis of variance has had a long and productive history in agricultural experimentation (see Cochran & Cox, 1957), but it should be recognized that human environments are often interactive in ways that plant environments are not.

These comparisons should not be interpreted as criticism of the experimental paradigm per se, but as a criticism of how analysis of variance has been uncritically applied to basically interactive problems, when other more appropriate forms of design and analysis are possible. Experiments can be designed in ways that take advantage of the interpretive benefits of random assignment while still accurately instantiating interactive situations. Linear prediction filters can be adapted for the analysis of interactive experiments. History could be reintroduced after random assignment, by allowing relatively free forms of interaction between participants in the situation. Deterministic input in the creation of treatment conditions, could be handled using one of two distinctly different formats: using open- or closed-loop filters. With open-loop filters, deterministic input is <u>not</u> contingent upon subjects' responses, as it is <u>not</u> in traditional experimental designs. With

"closed-loop" filters, input is contingent upon subjects' responses, as it is in operant conditioning. Open-loop input changes variance-covariance structures (Chow, 1975, pp. 150-152). The object of the design, using an open-loop format, would be to examine the effects of treatment conditions on covariance structures, as they reflect different patterns of change during interaction. By contrast, the object of the design using closed-loop filters could be to examine transformational changes in response to contingent input, to see if preestablished goals can be achieved in this way. Controls theory problems in engineering and economics are often described in this way (Chow, 1981; Elgerd, 1967). It should be possible to design closed-loop filters so that changes in means can be examined while changes in variance-covariance structures are controlled. The use of filters in the analysis of experimental data is an open area that requires further research.

2. Replication

Time is an inherent attribute of any interactive environment. The ability to replicate involves the ability to repeat observed relationships at new times. Relationships must be time-invariant for replication to occur. Parameters that are used to describe relationships are unreplicable if "population" parameters wander in some unknown way as a function of time. When they do, repeated samplings will produce different results. Patterns in relationships will be unreplicable. In order to replicate, then, complex relationships must be reduced to a time-invariant representation. This becomes more difficult as interaction

becomes more adaptive, since adaptivity frequently implies time-variability. The simplest case with little adaption is discussed first, followed by more difficult adaptive concepts.

2.1. Without Adaption

2.1.1. Stationarity. Time-invariant representations of relationships exist at different levels of complexity depending on the degree of adaptivity in the relationship: with low adaptivity, the simplest level of time-invariance for linear prediction filters is applicable. Time-invariance at this level is labeled covariance stationarity (see Granger & Newbold, 1977, pp. 3-5). The means and variance of variables following covariance stationary processes do not change through time. The covariances between variables remain constant at given distances or lags, producing a pattern of equalities in a covariance matrix. In addition, the linear parameters that generate a stationary covariance matrix are considered time-invariant too.

The model in Section 1.2.1. is based on the assumption of covariance stationarity. A stationary covariance matrix is one that follows a block Toeplitz pattern. A Toeplitz matrix has the property that all entries along diagonals--parallel to the principle diagonal--are the same. A block Toeplitz matrix is one that is partitioned into blocks with block entries down the diagonals. The corresponding elements within blocks are the same down the diagonals. In Section 1.2.1., the covariance matrix remains constant at a given lag, e.g., at lag $\underline{k} = 1$, $\underline{X}_{\underline{t}}\underline{X}'_{\underline{t}-1} = \underline{X}_{\underline{t}-1}\underline{X}'_{\underline{t}-2}$. This pattern of equalities produces a block Toeplitz

matrix, i.e., $\Gamma_{(0)}$ repeats itself down the principle diagonal. With this repetition, $\Gamma_{(0)}$ appears twice in Equation 11. The parameters in $\underset{\sim}{A}$ and $\underset{\sim}{V}$ generate this stationary covariance structure. Entries within the $\underset{\sim}{A}$ and $\underset{\sim}{V}$ matrices can differ but each entry is invariant as the system iterates through time, $\underline{t} = 1, 2 \ldots \infty$. The concept of a stationary representation is one factor that separates time-series models from structural equation models commonly used in psychology (Jöreskog & Sorbom, 1979).

2.1.2. Preemptive prioritization. Although first-order time invariance requires stationary (block Toeplitz) covariance matrices, sample covariance matrices frequently are not stationary exactly. In a panel design, for example, separate estimates are available for each element in a covariance matrix. In a stochastic system, sample covariances will differ from expected values in finite samples. For these reasons, sample covariances will not be exactly stationary in a panel design, even when the underlying data follow a model in which expected values of covariances are exactly stationary. In a time-series design, sample covariances may or may not have stationary block Toeplitz structure depending on the sampling window used to compute the sample covariance matrix. It is clear that there are four possible sampling windows that can be used to compute sample covariances (see Friedlander, 1982a, p. 838; Kay & Marple, 1981, p. 1391).

$$\begin{bmatrix} x_{\underline{a}1} & \cdots & x_{\underline{a}\ell+1} & \cdots & x_{\underline{a}t} \\ 0 & & x_{\underline{z}1} & \cdots & x_{\underline{z}t-\underline{\ell}} \cdots x_{\underline{z}t} \end{bmatrix} \qquad (31)$$

Covariance
Pre-Windowed
Post-Windowed
Autocorrelation

In Equation 31 $\underline{\ell}$ is defined as in Equation 6. Only the autocorrelation window produces exactly stationary block Toeplitz matrices. This is true because covariance estimates that share a common lag $\underline{k}$, are computed across data elements that are all identical in this instance. The data elements are not all identical for the other windows; consequently, the matrices will not be exactly Toeplitz, although they will generally be close to Toeplitz form.

A chi-square test is available to see if deviation from stationarity block Toeplitz form is statistically significant (see Browne, 1977; Burg, Luenberger & Wenger, 1982; Jöreskog & Sorbom, 1979; Steiger, 1980). Tests can be computed directly on sample covariance matrices from panel designs. In time-series designs, more than one sample from sequential windows of the time-series is needed.

Two mutually incompatible goals arise when covariance matrices are not exactly stationary. Stationarity requires a block Toeplitz structure, e.g., duplication of $\Gamma_{(0)}$ along the principle diagonal in Section 1.2.1. On the other hand, estimating linear parameters, e.g., $\underset{mm}{A}$, requires optimizing an achievement function, e.g., minimizing the trace of $\underset{mm}{V}$. For a stationary covariance matrix, the achievement function $\underset{mm}{V}$ is written as a function of $\Gamma_{(0)}$ (see Equation 11). In an unstationary representation, two different matrices appear in Equation 11 in the place of $\Gamma_{(0)}$. The goal of providing a stationary representation of an unstationary covariance matrix conflicts with the goal of minimizing the achievement function $\underset{mm}{V}$, which becomes apparent from the conflicting ways

$\Gamma_{(0)}$ must be handled to achieve each goal.

The concept of preemptive prioritization is borrowed from goal programming (Ignizio, 1976, 1982) to handle these conflicting goals. One goal is satisfied ahead of the other. The achievement function is an ordered variable that is lexicographically minimized (see Ignizio, 1982, chap. 17). Stationarity is given preemptive priority due to the centrality of the replication concept. A stationary block Toeplitz structure is imposed first on the sample covariance matrix using least squares or maximum likelihood estimators, and then linear parameters (e.g., $\underset{\sim}{A}$) are estimated next based on the imposed, stationary structure (see Burg et al., 1982). The objective of this procedure is to find the "best" time-invariant estimators. The estimators are time-invariant in a first-order sense. If the sample covariance matrix deviates significantly from stationary block Toeplitz structure, then the underlying process can no longer be considered time-invariant in a first order sense and more complex adaptive models are needed. The necessity for time-invariant representations becomes clear from a discussion of the consequences of ignoring them (see Section 2.3.).

2.2. Adaption

Interaction as defined by Definition 1 requires adaption. The attempt to model adaptive processes with linear parameters may produce estimates that are time-variable. The concept of higher-order time invariance is required to reduce representations to time-invariance. The concept of higher order time-invariance involves representing an

adaptive process in terms of a more complex model statement. In the model statement, parameters are written as products or sums of stationary and unstationary processes. For example, if the parameters $\underset{\sim}{A}$ from Equation 6 were time-variable, then the ways in which the $\underset{\sim}{A}$ parameters adapt can be modeled by treating $\underset{\sim}{A}$ like "data" in an equation of the following form:

$$\underset{\sim}{A}_t = \underset{\sim}{G}\underset{\sim}{A}_{t-1} + \underset{\sim}{e}_{At}, \tag{32}$$

Where $\underset{\sim}{G}$ represents a matrix of stationary time-invariant linear parameters, $\underset{\sim}{A}_t$ represents a matrix of unstationary time-variable parameters, and $\underset{\sim}{e}_{At}$ represents white noise, uncorrelated with $\underset{\sim}{A}_t$ or $\underset{\sim}{X}_t$. Substituting $\underset{\sim}{A}_t$ *in Equation 32 for* $\underset{\sim}{A}$ *in Equation 6 produces a model statement for* $\underset{\sim}{X}_t$ that is stationary or time-invariant in a second-order sense:

$$\underset{\sim}{X}_t = \underset{\sim}{G}\underset{\sim}{A}_{t-1}\underset{\sim}{X}_{t-1} + \underset{\sim}{v}_t. \tag{33}$$

While the $\underset{\sim}{A}$ parameters change as function of time, the way in which they change is modeled by the time-invariant $\underset{\sim}{G}$ matrix. If the way parameters adapt can be modeled, then time-invariance occurs at a higher level and replication of the processes is possible.

A wide variety of models are possible that are stationary in a higher order sense. Models analogous to Equation 33 have been applied to physical, biological, and economic systems (see Kalman Filtering,

Chow, 1981, chap. 6; Kashyap & Rao, 1976). In psychology, Kenny (1973, 1975b) introduced the concept of quasi-stationarity: Correlation matrices are permitted to be nonstationary so long as they become stationary after adjusting for measurement error. Extending this concept, measurement parameters can be allowed to vary as a function of time, while nonmeasurement "latent" parameters remain stationary. The measurement model in Section 1.2.3. can readily be adapted in this way. Models are possible in which stationary processes are analyzed after unstationary components are removed. Some adaptive models may simply track adaptive changes in linear parameters without modeling the form that this change takes (see Friedlander, 1982a; Kay & Marple, 1981). Even in this case, a stationary representation is imposed over a short sampling interval, and changes in stationary representations are tracked. It should be noted that reasonably large data bases may be required to support adaptive models. The number of parameters required to model adaptive systems will often increase over nonadaptive systems, requiring more data to estimate parameters. An extensive literature on adaptive systems exists in the electronics engineering literature (see Chow, 1981; Friedlander, 1982a). However, this discussion is limited to introducing the concepts of adaption and higher order time-invariance just discussed.

2.3. Comparison

Replication is a frequent problem in psychological research, particularly for interactive environments (see Epstein, 1980; Greenwald, 1975, 1976a). This problem is often due to parameters wandering in

unknown ways as a function of time. When this occurs it becomes impossible to replicate results, no matter how random the samples taken or how "normal" the population distribution. The problem is not that the parameters wander, but thay they wander in unknown ways. Change in wandering parameters can be accounted for by another stationary set that determines how the wandering occurs, producing time-invarance at a higher level. At a minimum, replication of results requires paying attention to the time-invariance issue. With the exception of cross-lagged correlation (Kenny, 1973, 1975b) and single case time-series designs, this issue has been largely ignored in psychology. Under these circumstances it is not surprising that replication has been difficult.

The issue of time invariance applies to transformational as well as historical change. For example, the issue applies to quasi-experimental studies designed to evaluate the effects of social intervention. Unstationary change in means or covariance are expected as a consequence of the intervention (see Kenny, 1975a). However, these changes are only unstationary in a first order sense. The intervention and only the intervention is supposed to account for this unstationary change. This means the same form of unstationary change is expected every time the intervention occurs and only when it occurs. As a consequence, expected change is stationary or time-invariant in a second order sense. Quasi-experimental intervention designs should be able to demonstrate this sort of second order time invariance. This could be accomplished by showing experimental groups undergo unstationary change of specified form

after an intervention, but not before; while change in a control group remains stationary. Again, this issue has not received direct attention in this context (see Cambell & Stanley, 1966; Simonton, 1977).

3. Prediction

Forecasts about the future based on past hisotry can be made for individual cases using linear prediction filters. These forecasts are made one step at a time (see Chow, 1975; Gilchrist, 1976; Granger & Newbold, 1977). The concept of prediction and replication overlaps in many ways. Prediction requires that linear coefficients (e.g., $\underset{mm}{A}$) replicate themselves from one step to the next, i.e., require a stationary representation. Replication itself is a prediction about past structure replicating itself in new samples.

3.1. *Prediction of Nonlinear Change*

In Section 1.2.1., information useful for prediction is contained in $\underset{mm}{A}$. The $\underset{mm}{A}$ matrix is composed of linear coefficients. This means that forecasts between adjacent sampling intervals must be linear. However, the definition of the environment in Definition 1 requires that nonlinear change be tracked. Change over long intervals seems inherently nonlinear. Linear projections over extended periods will always blow up toward infinity as time increases, which does not reflect the nature of interactive phenomena.

Linear prediction filters do, in fact, predict nonlinear change over multiple sampling intervals. When forecasts are made using $\underset{mm}{A}$, sequential one-step forecasts are made into the future, deleting unknown

future error terms (e.g., $\underline{v}_t$ in Equation 6). Since the coefficients in $\underset{\sim}{A}$ do not change as a function of time (see Section 2.1.1.), the equation becomes a homogeneous linear difference equation with constant coefficients (see Chow, 1975, chap. 2; Dahlquist & Björck, 1974, pp. 368-370). A polynomial equation (labeled the "characteristic" polynomial) describes the form of nonlinear chance projected by homongeneous linear difference equations. It is possible to vary the parameters of this polynomial to describe many forms of nonlinear change. The parameters of the characteristic polynomial are derived from the eigenvalues (roots) and eigenvectors of $\underset{\sim}{A}$. When a pair of roots is complex conjugate, the form of change described by these roots is a cosine wave. Positive real roots, less than one, produce nonoscillating exponential decay. The form of change is explosive and grows as a function of time, if the absolute value of the roots is greater than one, and is dampened and decays to zero if the absolute value is less than one. Negative real roots produce rapid oscillation. When different types of roots (i.e., positive real, negative real, complex conjugate) appear in the same stationary system, the final form of change is a composite defined by the sum of dampened cosine waves, and decaying exponentials. The relative contributions of each component to the final form of change is determined by the size of the absolute value of the roots. The largest roots dominate the composite, particularly as the system iterates forward in time away from the initial conditions.

It should be clear from the preceding discussion that linear prediction filters can track a large class of nonlinear changes. These changes

are described in terms of dampened cosine, and decaying exponential components. The linear forecast that occurs at each step is a discrete linear approximation to nonlinear change. Conditions must be imposed on the way the process is sampled in order for linear prediction to adequately approximate nonlinear change. First, it is much easier to identify the process if it is sampled on an equal interval basis. If the process is not sampled in this way, additional assumptions and methods to approximate the equal interval criterion are possible. In addition, samples must be taken frequently enough to ensure that the linear projection between adjacent intervals is an adequate approximation to the nonlinear change that actually occurred. As the sampling interval is reduced, resolution of the form of nonlinear change that actually occurred becomes better. When change involves oscillation, the size of the sampling interval, Δt, must be within the Nyquist frequency (Bloomfield, 1976, pp. 26-27):

$$2\Delta t \leq P, \text{ where} \tag{34}$$

P is the period of the highest frequency oscillation in the system. If sampling does not remain within the Nyquist frequency, aliasing occurs (see Bloomfield, 1976, pp. 204-208) and information is irretrievably lost. The most efficient sampling rate is at the Nyquist frequency, without exceeding it.

The covariance and correlation functions contain information about

both the stochastic and deterministic components of a model. The deterministic components are contained in $\underset{\sim}{A}$. These components are used to track and forecast nonlinear patterns of change. Since the covariance matrix also contains deterministic components, nonlinear change must be identifiable from the covariance matrix. The covariance coefficient reflects the degree of linear association between variables. However, the degree of linear association varies as a function of the measurement lag throughout the matrix, producing a pattern in the covariance matrix that identifies the type of nonlinear change that occurred. For example, when the roots of $\underset{\sim}{A}$ are complex conjugate, the pattern in the covariance matrix follows a dampened cosine wave which oscillates at the same frequency as the deterministic forecasts provided by $\underset{\sim}{A}$. When the roots of $\underset{\sim}{A}$ are positive and real, the covariances follow a decaying exponential pattern as in $\underset{\sim}{A}$ (see Chow, 1975, chap. 1-4; Granger & Newbold, 1977, chap. 1-3). However, in covariance matrices, coefficients vary backward in time as a function of k, instead of forward in time as a function of t, as do forecasts from $\underset{\sim}{A}$. The nonlinear patterns in correlation matrices are used to help identify time-series models (see Box & Jenkins, 1976; McCleary & Hay, 1980).

3.2. Covariances as Prefilters

There are two approaches to the analysis of time-series data: the direct approach via Fast Fourier Transform and the indirect approach via the autocorrelation/autocovariance function (Kay & Marple, 1981, p. 1383). Linear prediction filters follow the indirect approach. In Section 1.2.1.,

$\underset{\sim}{A}$ is a function of the covariance function as shown in Equation 9. The covariance/correlation function serves as a prefilter of the data prior to estimation of $\underset{\sim}{A}$. It follows that other measures besides covariances and correlations might be used in certain instances as prefilters. Section 1.2.4. suggests the possibility of using variance components as prefilters. These variance components can be used to reflect structure of group aggregated scores and to represent theoretically relevant aspects of group interaction. In addition, polychoric correlations could be examined as prefilters when there is some reason to suspect the adequacy of standard linear approximations between sampling intervals (see Martinson & Hamden, 1975).

3.3. Comparison

3.3.1. Nonlinear change. Unfortunately, the use of discrete linear models to approximate long-term nonlinear change, has been ignored in psychology. This seems to be due to an assumption that nonlinear information is unavailable; e.g., "... panel data used in psychological and educational research are often limited to two or three waves and thus a straight line model for growth is as complex as the data can support" (Rogosa, Brandt & Zimowski, 1982, p. 728). As noted in Section 3.1. and the Example Section, linear prediction filters can detect a variety of forms of nonlinear change from two or three waves of panel data. Applying the assumption that nonlinear information is unavailable to filters can lead to serious errors in selecting sampling intervals. Appropriate sampling intervals are necessary to enable

discrete linear statistics to model nonlinear change. Serious misconceptions can result from selecting sampling intervals that violate the Nyquist frequency (Equation 34). To illustrate, suppose the following $\underset{\sim}{A}$ and $\underset{\sim}{V}$ (see Equations 6 and 11) were to generate the correlation matrix in $\Gamma_{(0)}$ and $\Gamma_{(1)}$ below.

$$\underset{\sim}{A} = \begin{bmatrix} .2 & 55 \\ -.55 & .2 \end{bmatrix}, \underset{\sim}{V} = \begin{bmatrix} .658 & 0 \\ 0 & .658 \end{bmatrix}; \Gamma_{(0)} = \underset{\sim}{I}_{(2x2)}\Gamma_{(1)} = \begin{bmatrix} .2 & .55 \\ -.55 & .2 \end{bmatrix}. \quad (35)$$

Now suppose the researcher failed to sample this system at the correct interval $\underline{k}$ that generated the system, accidently violating Nyquist by sampling at twice the interval $2\underline{k}$. The researcher would identify the system in Equation 35 as (see Equation 8):

$$\underset{\sim}{A} = \begin{bmatrix} -.26 & -.22 \\ .22 & -.26 \end{bmatrix}; \Gamma_{(0)} = \underset{\sim}{I}; \Gamma_{(1)} = \begin{bmatrix} -.26 & -.22 \\ .22 & -.26 \end{bmatrix}. \quad (3$$

The cross-lagged as well as autoregression parameters and correlation coefficients are reversed in sign in Equation 36 compared to Equation 35. In other words, if the researcher misjudged the sampling interval by sampling at times 1 and 3 instead of 1 and 2, in this oscillating system, conclusions would be reversed from those made at the shorter interval (see Heise, 1970; Pelz & Lew, 1970).

The importance of sampling intervals is not limited to

quasi-experimental designs. If an experimental manipulation induces an oscillation in a dependent variable, the mean response in the experimental group can oscillate between being higher and lower than the control group mean, depending on when the dependent variable is sampled. Unfortunately, the dependent variable is often sampled only once. Problems with replication can stem from such sampling interval issues in addition to time-invariance issues discussed in Section 2. These issues require further research.

3.3.2. Prediction criterion. In general, linear simultaneous equation models admit a wide variety of possible specifications. However, not all of these models constitute linear prediction filters. Linear prediction filters are a subset of possible specifications that conform to prediction criteria. As a scientific guideline, the prediction criterion is similar to the principle of parsimony. Parsimony suggests that when faced with alternate explanations, the simplest one is accepted. Alternate specifications constitute alternate explanations. The principle of prediction says, that when faced with alternate explanations, the explanation that can predict and track the phenomenon is accepted. The rationale for this criterion is tied to theory. Phenomena are created through a historical time-related process, both in interactivity theory, and, perhaps, in all psychological theory. Specifications that can track and predict a phenomenon can represent the process that created it, or be related to it. By contrast, a specification that can't predict or track can't represent any phenomenon that was

created through time. A process created through time requires time-lagged coefficients to predict and track the process. The prediction criterion can be restated in terms of a maxim in modeling: "If a discrete linear system can't be simulated on the computer, it can't exist in nature." Application of the prediction criterion is illustrated next in Sections 3.3.3. and 3.3.4.

3.3.3. Single-time path-analytic models. In Table 1, a comparison is made between the filter defined in Section 1.2.1., and linear path-analytic models applied to single sample data. The correlations at the left in the table represent the output generated by a stationary historical process shown at the right. The coefficients in $\underset{\sim}{A}$ and $\underset{\sim}{V}$ represent the generative process. The coefficients in β and $\underline{e}$ represent three of the possible synchronous-time specifications. These specifications correspond in interpretation to $\underset{\sim}{A}$, with the exception that time has been left out of the path analytic models. The corresponding elements of β and $\underset{\sim}{A}$, within examples, should be compared to see how well a model without the benefit of time can represent a process generated through time.

The coefficients in β and $\underset{\sim}{A}$ in Table 1 are obviously very different. Furthermore, it is apparent that the synchoronous parameters (β) can't be tracking the dynamics of change in these examples. Examples 2, 3, and 4 provide cases where very different histories produced the same

Table 1
The Effects of Excluding History on Parameter Estimates

Example		Correlation Matrices[a]						Parameter Estimates[d]								
		History Excluded[b] $\Gamma_{(0)}$			History Included[c] $\Gamma_{(1)}$			History Excluded[b] β			$\underline{e}$	History Included[c] $\underline{\underline{A}}$			$\underline{\underline{V}}$		
		$\underline{x}_t$	$\underline{y}_t$	$\underline{z}_t$	$\underline{x}_{t-1}$	$\underline{y}_{t-1}$	$\underline{z}_{t-1}$	$\underline{x}_t$	$\underline{y}_t$	$\underline{z}_t$		$\underline{x}_{t-1}$	$\underline{y}_{t-1}$	$\underline{z}_{t-1}$	$\underline{x}_t$	$\underline{y}_t$	$\underline{z}_t$
1	$\underline{x}_t$	1.00	.28	.63	.71	.59	.56	--	.00	.63	.60	.59	.43	.00	.33	.00	.00
	$\underline{y}_t$	.28	1.00	.45	.14	.48	.21	.00	--	.45	.80	.00	.48	.00	.00	.77	.26
	$\underline{z}_t$	.63	.45	1.00	.79	.38	.85	.55	.30	--	.52	.42	.00	.59	.00	.26	.17
2	$\underline{x}_t$	1.00	.00	-.40	.20	-.63*	-.08	--	.00	-.40	.84	.20	.60*	.00	.60	.00	.00
	$\underline{y}_t$	.00	1.00	.00	.60*	.20	-.64*	.00	--	.00	1.00	-.46*	.20	.43*	.00	.41	.00
	$\underline{z}_t$	-.40	.00	1.00	-.08	.61*	.20	-.40	.00	--	.84	.00	-.64*	.20	.00	.00	.55
3	$\underline{x}_t$	1.00	.42	.33	.66	.21	.66	--	.42	.33	.71	.50	.00	.49	.34	.00	.00
	$\underline{y}_t$	.42	1.00	.00	.70	.78	.15	.48	--	-.16	.80	.45	.59	.00	.00	.23	.00
	$\underline{z}_t$	.33	.00	1.00	.44	-.33	.23	.41	-.17	--	.86	.70	-.63	.00	.00	.00	.48
4	$\underline{x}_t$	1.00	.42	.33	.21	.30	.21	--	.42	.33	.71	.00	.49	.00	.76	.32	.33
	$\underline{y}_t$	.42	1.00	.00	.49	.21	.00	.48	--	-.16	.80	.50	.00	-.61	.32	.58	.28
	$\underline{z}_t$	.33	.00	1.00	.00	-.44	.64	.41	-.17	--	.86	.00	.00	.64	.33	.28	.59
5	$\underline{x}_t$	1.00	.51	-.41	.60	.13	-.46	--	.90	-.83	.20	.50	.00	-.26	.58	.44	.00
	$\underline{y}_t$	.51	1.00	.47	.25	.50	.24	.83	--	.81	.20	.00	.50	.00	.44	.75	.43
	$\underline{z}_t$	-.41	.47	1.00	-.49	.08	.61	-.86	.91	--	.22	-.29	.00	.50	.00	.43	.55

* Changing the signs of a^*_{ij} in $\underset{\sim}{A}$ changes the signs of lagged correlations γ in $\Gamma_{(1)}$, but does not change parameters in β or synchronous correlations in $\Gamma_{(0)}$.

[a] The correlation matrices are partitioned by time into the $\Gamma_{(0)}$ block containing synchronous time correlations, and the $\Gamma_{(1)}$ block containing lagged correlations (see Section 1.2.1.). The correlation matrices are stationary, i.e., $\Gamma_{(0)}$ repeats itself down the principle diagonal.

[b] History Excluded refers to the parameter estimates (β) based solely on synchronous time correlations (lag $\underline{k}$ = 0 only).

[c] History Included involves the entire correlation matrix ($\underline{k}$ = 0, 1).

[d] Standardized regression coefficients (β) were computed treating each variable at the left as dependent. The residual variance from β is at $\underline{e}$. The matrices $\underset{\sim}{A}$ and $\underset{\sim}{V}$ were taken from Equations 9 and 11. For each example, the corresponding elements under β and $\underset{\sim}{A}$ should be compared.

synchronous correlations and parameters (β). In example 2, the signs of the coefficients a^*_{ij} in $\underset{\sim}{A}$ were changed without changing the synchronous correlations or parameters. In examples 3 and 4, very different histories produced the same synchronous results. In example 5, the coefficients in β could not have generated this stationary system. Due to a "dampening" variable, the coefficients in β are large; so large, in fact, that these coefficients would produce explosive unstationary change if they were treated as filter coefficients in $\underset{\sim}{A}$. Synchronous time specifications cannot predict or track because of missing data--time has been left out. Because of this, the two methods will never match, except by chance, even when they use the same estimation technique (e.g., regression, as in Table 1). Since the synchronous-time models cannot predict or track, they cannot represent the generative processs or increase our understanding about it. These conclusions are consistent with information theory (Shannon & Weaver, 1949). In information theory, information is produced by change, and if no change is measured, no information is generated. Synchronous-time models cannot measure change, and lose information as a consequence.

3.3.4. Econometric model. As just noted, synchronous-time linear models can't represent histroical processes. Even linear models that include time do not necessarily represent historical processes either, unless they conform to prediction criteria. The filter from Section 1.2.1. is compared next to specifications in Equation 37. Prediction again forms the basis for comparison. To simplify exposition, the model is

given for a two-variable, two-wave (one-lag) system, but conclusions readily generalize to any number of variables and lags:

$$\begin{bmatrix} \underline{x}_t \\ \underline{y}_t \end{bmatrix} = \begin{bmatrix} 0 & \underline{f}_{12} \\ \underline{f}_{21} & 0 \end{bmatrix} \begin{bmatrix} \underline{x}_t \\ \underline{y}_t \end{bmatrix} + \begin{bmatrix} \underline{a}^{\dagger}_{11} & \underline{a}^{\dagger}_{12} \\ \underline{a}^{\dagger}_{21} & \underline{a}^{\dagger}_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_{t-1} \\ \underline{y}_{t-1} \end{bmatrix} + \begin{bmatrix} \underline{e}_{xt} \\ \underline{e}_{yt} \end{bmatrix}, \text{ or} \tag{37}$$

$$\underset{m}{\underline{X}}_t = \underset{mm}{F}\underline{X}_t + \underset{m}{A^{\dagger}}\underline{X}_{t-1} + \underline{e}_{wt}.$$

$\underset{mm}{F}$ is subdiagonal; and the covariance of $\underline{e}_{wt}$ is $\underset{mm}{W}$. The specifications in Equation 37 are frequently applied in econometric modeling (Granger & Newbold, 1977, chap. 6), and in panel designs (Duncan, 1969; Jöreskog & Sorbom, 1979), and are called here the "econometric" model. A stationary correlation matrix generated by $\underset{mm}{A}$ and $\underset{mm}{V}$ is shown in Equation 38. The system includes two variables and one time lag. The generating $\underset{mm}{A}$ and $\underset{mm}{V}$ are shown in Table 2, specification 1. The econometric model is illustrated by specifications 2-8. All estimates are based on the correlation matrix in Equation 38:

$$\Gamma_{(0)} = \begin{bmatrix} 1.00 & .701 \\ .701 & 1.00 \end{bmatrix}; \; \Gamma_{(1)} = \begin{bmatrix} .521 & .562 \\ .706 & .774 \end{bmatrix}. \tag{38}$$

From Equation 37 and Table 2, it is clear that the econometric model includes time-lagged relationships, as do filters. However, the econometric model also includes synchronous-time "causal" relationships not

found in filters. This same kind of synchronous "causal" effect was unable to track change in the last section when history was excluded. How well, then, can synchronous-time "causal" relations conform to prediction criteria, when time has been added to the model?

Specifications 2, 5, and 6 cannot be simulated on the computer under any conditions, which violates the prediction criterion. Simulation in a discrete linear system requires iteration over time (see Pelz & Lew, 1970). Iteration is impossible with recursive, synchronous-time "causal" effects, e.g., specifications 2, 5, and 6. Future projections become impossible: in order to forecast $\underline{x}_{t+1}$, the value of $\underline{y}_{t+1}$ is required, but the value of $\underline{y}_{t+1}$ can't be estimated without $\underline{x}_{t+1}$, which is the value desired in the first place.

The econometric model specifications include forecasts between observed variables in synchronous time. These "forecasts" are not meaningful in a literal sense, because there is nothing to predict at time the prediction is made: by the time $\underline{y}_t$ is used to predict $\underline{x}_t$ in specification 7, $\underline{x}_t$ has already been observed, so there is no need to predict $\underline{x}_t$ at that time. Apparently, the effects in $\underset{\sim}{F}$ were not meant to literally represent synchronous effects but to represent short-term effects instead.

One approach to simulating the "short-term" effects in

Table 2

Comparing Specifications: Filter Versus Econometric Model

Coefficient	Model Specification[a] 1	2	3	4	5	6	7	8
$a_{11}^{\dagger}$	.250	.025	.025	.250	.025	3.48	.043	.250
$a_{12}^{\dagger}$	.387	0	0	.387	0	5.94	.032	.387
$a_{21}^{\dagger}$	.320	0	.320	0	.330	0	.320	.234
$a_{22}^{\dagger}$	.550	.054	.550	.054	.566	.055	.550	.417
f_{12}	0	.704	.704	0	.703	-10.1	.645	0
f_{21}	0	1.28	0	1.28	-.041	1.28	0	.344
w_{11}[b]	.652	.508	.508	.652	.508	40.6	.507	.652
w_{22}	.348	.842	.348	.842	.367	.842	.348	.271
w_{12}	.225	-.653	-.020	-.610	0	0	0	0

[a] Specification 1 is from the filter in Section 1.2.1. Specifications 2-8 are from the econometric model, Section 3.3.4.

[b] w_{ij} are elements of W, the covariance of $e_{wt} e_{wt}'$ in Equation 37.

specifications 3, 4, 7 and 8, Table 2, involves moving one variable ahead of the other a "slight amount," by computing this variable, with its error added to it, ahead of the other one. This approach violates the assumption of "synchronicity" (i.e., common sampling intervals), both for the model and data. Violating synchronicity does not seem to be a serious problem at first glance. However, violating this one assumption creates serious if not insoluable problems. It requires that synchronous coefficients in $\underset{\sim}{F}$ estimate time-lagged effects. As was shown in Table 1, this will never be true in general without making a series of highly restrictive assumptions. The size of the "short-term" interval must be assumed (i.e., the number of short intervals per long intervals). In addition the covariances and linear parameters must be assumed for all missing short-term intervals. In addition, assumptions about interpolating between shychronous effects and lagged effects are needed. The assumptions are so restrictive as to be almost certainly untrue. As shown by Table 1, results are sensitive to these assumptions. It is inappropriate to violate synchronicity for these reasons.

In order to simulate specifications 3, 4, 7 and 8 without violating synchronicity requires that forecasts be biased by dropping error terms. The off-diagonal elements of $\underset{\sim}{F}$ in Equation 37 create bias for forecasting. These elements create a linear combination of errors that are needed for forecasting (see the last two lines of Equations 1B and 2B). However, these errors have not occurred yet at the time the forecast is made so they cannot be used. As a consequence the forecast is biased by

dropping these errors. Specification 7 provides an example. At first glance, this specification looks optimal for forecasting, even better than the filter in specification 1, since the trace of $\underset{\sim}{W}$, the residual, is smallest here. To forecast $\underset{\sim}{x}_{t+1}$ in specification 7, $\underset{\sim}{y}_{t+1}$ must be forecast first, to enable $\underset{\sim}{x}_{t+1}$ to be forecast via $\underset{\sim}{f}_{12}\underset{\sim}{y}_{t+1}$. Unfortunately, the error for $\underset{\sim}{y}_{t+1}$ ($\underset{\sim}{e}_{yt+1}$) has not occurred yet at time t, when the forecast is made, so it must be dropped in making the forecast to $\underset{\sim}{x}_{t+1}$. As a consequence, the forecast to $\underset{\sim}{x}_{t+1}$ is biased by $\underset{\sim}{f}_{12}\underset{\sim}{e}_{yt}$, which biases the residual, $\underset{\sim}{f}_{12}^{2}\underset{\sim}{w}_{22} = .145$. Since this is precisely the amount by which the residual in specification 7 is smaller than in 1, "improved" prediction in this specification is due totally to prediction bias. The error $\underset{\sim}{w}_{11}$ in specification 7 is biased in the sense that it does not represent the total amount of prediction error in the system as it purports to do. Since the parameters in $\underset{\sim\sim}{F}$ and $\underset{\sim\sim}{A}\dagger$ are all estimated in relation to biased errors, the latter parameters are biased for forecasting too. The transformation equations between the filter in specification 1 (which can be used to forecast), and the econometric model in specifications 2-8 are found in Appendix B. The nature of forecasting biases is illustrated by the transformation equations. For example, from the last line of Equation 1B and from Equation 5B, the residual $\underset{\sim}{w}_{11}$ in specification 7 can be written as,

$$\underset{\sim}{w}_{11} = \underset{\sim}{v}_{11} - \underset{\sim}{f}_{12}^{2}\underset{\sim}{w}_{22} = \underset{\sim}{v}_{11} - (\underset{\sim}{v}_{12}/\underset{\sim}{v}_{22})^{2}\underset{\sim}{w}_{22}. \qquad (39)$$

In Equation 39 $\underline{w}$ and $\underline{f}$ are from specification 7 and $\underline{\underline{V}}$ is from specification 1 and Equation 11. Forecast bias is apparent in Equation 39. In attempting to simulate specification 7 over time, values of $\underline{w}_{22}$ in Equation 39 must be set to zero at time $\underline{t}+1$ when tracking or forecasting occurs, which means $\underline{w}_{11}$ will become equal to $\underline{V}_{11}$ over time. When a residual changes, the whole system changes, i.e., parameters in $\underline{\underline{F}}$, $\underline{\underline{A}}\dagger$, $\Gamma_{(0)}\Gamma_{(1)}$ also change. This means that the forecast bias that accrues from dropping future errors, makes it impossible to simulate econometric model specifications 3, 4, 7 and 8, and still maintain the same correlation matrix in Equation 38 and parameters. Furthermore, it is impossible to eliminate this forecast bias without violating the assumption of synchronicity with its attendant serious problems. Since none of the econometric specifications can be properly simulated, none of them conform to the prediction criterion.

An econometric model may instantiate theory one way via specifications 2-8, and predict another, via specification 1; however, this dual system is not consistent with the principle of prediction. Applying the prediction criterion to Table 2, specifications 1 alone can predict and track. As a consequence, this specification alone has the potential to represent, and increase our understanding of, processes created through time. Specifications 2-8 suffer from the same defect as do the synchronous-time path-analytic models--missing data. In order to measure short term effects, the system should have been sampled at appropriate short-term intervals.

Without the prediction principle, econometric models have fared poorly. Very simple autoregressive filters have been able to forecast as well or better than very complex econometric models (Granger & Newbold, 1977, chap. 8).

4. Parsimony

4.1. Model Identification

The principle of parsimony is defined by refusing to multiply complexity beyond necessity. The most common application of this principle to linear models involves its use as a selection criterion among alternate specifications. The specification that "fits best" with fewest parameters is selected from among the alternatives, other factors being equal (see Jöreskog & Sorbom, 1979). According to Akaike (1974), there is a relationship between the number of parameters in a model, and the predictive power of the model. An information criterion (AIC) is used for purposes of model identification. A curvilinear relationship between forecasting accuracy and simplicity is identified in this criterion. Forecasts improve as parameters increase, up to a point, beyond which limited samples can no longer support accurate parameter estimates, and then additional parameters decrease forecasting accuracy.

A related application of parsimony involves refusing to multiply possible alternative specifications beyond necessity. Model identification is a difficult problem, however (Leamer, 1978). Model identification is a current topic of intense research (El-Sherief & Sinha, 1982; Lee, 1983). A goal is to identify models in ways that simultaneously

conform to more than one scientific criterion, like instantiating theory where it exists, as well as maximizing prediction and parsimony.

4.2. Comparison

4.2.1. Synchronous-time models. Synchronous-time path-analytic models admit a great variety of possible specifications, due to the loss of ordering information that time provides. The number of possible specifications in these models is given by,

$$S = \sum_{\underline{r}=0}^{\underline{m}} \binom{\underline{m}}{\underline{r}}, \tag{40}$$

where $\underline{m} = \underline{p}(\underline{p}-1)$, $\underline{p}$ referring to the number of synchronous variables. The number of identifiable, and therefore estimable specifications, in Equation 40 can be found by substituting $(\underline{p}-1)$ for $\underline{m}$ in Equation 40. Using Equation 40, it is clear there are 64 possible synchronous specifications between the three synchronous variables in Table 1, 42 of which are identifiable. With four synchronous variables, there are 4096 possible specifications, 2510 of which are identifiable. All of these possible specifications can be ruled out using the prediction criterion (see Section 3.3.3.). Furthermore, there is no loss in information in doing so, since the possible interpretations of synchronous specifications all have an analogous time-related interpretation that can be taken directly from $\underset{\sim}{A}$ in Equation 6. To illustrate, $\underset{\sim}{A}$ from example 1, Table 1, shows an "intervening variable" relationship between $\underline{y}$ and $\underline{z}$

via $\underline{x}$. The alternate interpretation due to synchronous parameters in example 1 of "spurious" correlation, was ruled out by the prediction criterion.

Similarly, there is an exponentially expanding number of possible specifications in the econometric model that can be ruled out by the prediction criterion. No information is lost beyond what is available in $\underline{A}$ (see Section 3.3.4.). In this latter model, short-term effects must be measured if required by theory.

In spite of the ability to rule out large classes of possible specifications using the prediction criterion, the identification of filter models is by no means unique. The order of an "autoregression" model, or number of lags, must be identified. Furthermore, there is a large class of "latent" or "unobserved" variable models that remain as possible specifications.

4.2.2. Cross-lagged correlation model of spuriousness. Correlations and covariances are underidentified with respect to the possible structures that could have created them (Mulaik, 1972, chap. 13). It is possible to create a given correlation structure through simulation many different ways, including the use of unobserved or latent variables. The attempt to rule out latent or unobserved variable specifications has been defined in terms of "ruling out spuriousness" (Kenny, 1973, 1975b). The issue of "causal predominance" in the cross-lag model is not the issue here (see Rogosa, 1980). Currently, it is not possible to rule out spuriousness in the general sense of ruling out all possible latent variable specifications. However, Kenny (1973, 1975b) defined ruling

out spuriousness in terms of ruling out multiple, orthogonal, synchronous latent variables as the sole cause of an observed correlation matrix. Cross-lagged correlational differences do operate to rule out spuriousness in this narrowly defined sense; but not in a more general sense. To illustrate, the cross-lagged model of spuriousness can be written as,

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} + \begin{bmatrix} e_{xt} \\ e_{yt} \end{bmatrix},$$

$$X_t = BZ_t + e_{xt};$$

$$\begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix} \begin{bmatrix} z_{1t-1} \\ z_{2t-1} \end{bmatrix} + \begin{bmatrix} e_{z1t} \\ e_{z2t} \end{bmatrix} \qquad (41)$$

$$Z_t = DZ_{t-1} + e_{zt}.$$

In Equation 41 x_t and y_t are observed variables; z_{1t} and z_{2t} are two latent unobserved variables; B is a matrix of loadings from latent to observed variables; d_{11} and d_{22} are autoregression coefficients for the latent variables; e_{zt} are white noise processes that drive the system, and e_{xt} is a form of "measurement" error (see Section 1.2.3.). The representation in Equation 41 is the simplest one in which elements of B are "perfectly" stationary and do not adapt. The latent variables are uncorrelated, i.e., $E(e_{zt}e'_{zt}) = U = 0$.

The canonical variable transformation of the filter in Section 1.2.1. is shown in Appendix C. This transformation is identical to the model of spuriousness in Equation 41, with two exceptions. The canonical variable transformation is a special case of Equation 41 in which "measurement" error $\underset{\sim}{e}_{xt} = \underset{\sim}{0}$--which does not affect the spuriousness argument. However, the second exception is important. The canonical variables are allowed to be correlated as they almost certainly would be in nature: $E(\underset{\sim}{e}_{zt}\underset{\sim}{e}'_{zt}) = \underset{\sim}{U} \neq \underset{\sim}{0}$. When the latent variables are allowed to be correlated, then a number of variables in synchronous time can be the sole cause of an observed correlation matrix (see Appendix C). Furthermore, in contrast to the econometric model, the canonical variable transformation can be used to track and forecast, so it conforms to the prediction criterion. Therefore, cross-lagged correlation cannot rule out spuriousness in this slightly more general sense.

The correlation matrix in Equation 38 provides an example. Equation 2 and specification 1, Table 2, provide the customary explanation for this correlation matrix in terms of observed variables. The canonical variable transformation in Appendix C provides an alternative explanation. Applying Equations 4C, 5C and 8C to the example in Equation 38,

$$\underset{\sim}{D} = \begin{bmatrix} .78 & .0 \\ .0 & .02 \end{bmatrix}, \underset{\sim}{U}_{\underline{s}} = \begin{bmatrix} .39 & -.25 \\ -.25 & 1.00 \end{bmatrix}, \underset{\sim}{B}_{\underline{s}} = \begin{bmatrix} .75 & -.50 \\ 1.03 & .30 \end{bmatrix}. \quad (42)$$

$\underset{\sim}{A}$ and $\underset{\sim}{V}$ from Equations 6 and 11 can be partitioned to include latent

variables,

$$\underset{\sim}{A} = \begin{bmatrix} 0 & B_s \\ 0 & D \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Z_{t-1} \end{bmatrix}, \quad \underset{\sim}{V} = \begin{bmatrix} 0 & 0 \\ 0 & U \end{bmatrix}.$$

The matrices from Equation 43 can be used in Equations 7 and 8 to generate the observed and latent correlation matrices. In spite of a cross-lagged correlation difference in Equation 38, the correlation matrix in Equation 38 can be produced perfectly by two correlated, unobserved, latent variables in synchronous time (see Equations 42, 43, 9C, 10C).

The canonical variable transformation becomes complex when the eigenstructure of $\underset{\sim}{A}$ is complex. The canonical variable transformation cannot be considered an alternate explanation if latent variables and parameters are limited to real numbers, but the eigenstructure of $\underset{\sim}{A}$ is complex. It is possible to rule out spuriousness in the sense of ruling out multiple, correlated, latent variables in synchronous time, when the roots of $\underset{\sim}{A}$ are complex (as they were in data from the Example Section). However, it is still not possible to rule out spuriousness in a general sense, since multiple latent variables at multiple time lags could theoretically reproduce any correlation or covariance matrix.

Example

A sample composed of 59 combat companies in the U. S. Army was used. Companies are organizational units of approximately 125 soldiers

nested within the larger organizational hierarchy shown in Equation 28. Companies constitute the unit of analysis here. Approximately 18 enlisted men, grades E1 through E4, were randomly sampled from each company on three occasions at 10-week intervals. Sampling took place without replacement. Administrative record data were also collected for the same periods, including frequency of promotion to grades E3, E4, and E5 for lower enlisted soldiers. Frequency of promotion was unrelated to the numbers eligible for promotion within each company, so frequency counts were used. Frequency of promotion (Variable 1) plus three survey scales were selected to illustrate application of the filtering methodology. Scale 1 was labeled Goal Orientation (Variables $\underline{x}_2$ and $\underline{x}_3$); Scale 2, Intergroup Harmony (Variables $\underline{x}_4$, $\underline{x}_5$ and $\underline{x}_6$); and Scale 3, Enlisted Unity (variables $\underline{x}_7$ and $\underline{x}_8$).

These data were roughly stationary, with the possible exception of time-varying measurement error for Variable 1 and Scales 2 and 3. Means and standard deviations for the eight variables at three time intervals are given in Table 3. According to the model in Section 1.2., means and standard deviations do not vary as a function of time. Synchronization or "reliability" estimates using Equation 30 ($\underline{MS}_{total} = \underline{MS}_C$, $\underline{MS}_{error} = \underline{MS}_S$) were computed to represent homogeneity of responses within companies. These measures are expected to be stationary as a function of time. They are related to reliability in the sense of sampling soldiers to estimate total company means, but underestimate true reliability because (a) response sets have not been filtered out yet, and (b) sampling

fractions for small finite populations were not used. For lags $\underline{k}$ = 0, 1 and 2, synchronization measures were .33, .34 and .35 for Scale 1; .39, .21 and .50 for Scale 2; and .36, .30 and .57 for Scale 3. The covariance matrix between eight variables at three intervals is roughly stationary. Least squares estimators of a block Toeplitz covariance matrix were made by averaging the three synchronous blocks ($\underline{k}$ = 0) and two, lag-one ($\underline{k}$ = 1) blocks. The resulting block Toeplitz matrix was compared to the original using a chi-square goodness of fit test (see Browne, 1977; Jöreskog & Sorbom, 1979), χ^2 (300) = 326.7, $p \approx .5$.[1] Maximum likelihood estimates of block Toeplitz matrices are possible (see Section 2.1.), but least squares estimators are sufficiently accurate for illustrative purposes (Steiger, 1980).

Table 3

Means and Standard Deviations for Three Waves

lag $\underline{k}$	$\underline{x}_1$	$\underline{x}_2$	$\underline{x}_3$	$\underline{x}_4$	$\underline{x}_5$	$\underline{x}_6$	$\underline{x}_7$	$\underline{x}_8$
0	12.6	5.7	6.4	5.1	4.3	4.2	5.5	5.4
1	9.6	5.6	6.2	4.9	4.0	4.1	5.4	5.4
2	7.3	5.6	6.3	5.0	4.1	4.1	5.4	5.3
				Standard Deviations				
0	5.36	.54	.46	.81	.64	.62	.57	.48
1	4.86	.55	.48	.73	.68	.60	.52	.45
2	3.92	.54	.44	.90	.67	.67	.61	.58

Note. $\underline{x}_1$ = promotion frequency. Survey scales $\underline{x}_2$ - $\underline{x}_8$ were rated on 8-point scales and coded so that larger numbers were associated with positive responses.

[1] Covariance and correlation goodness-of-fit tests are sensitive to an assumption of multivariate normality.

The time-series factor analysis model from Section 1.2.3. was applied to these data. The block Toeplitz covariance matrix was standardized, since factor analysis is customarily applied to correlation matrices. All parameter estimates were based on the standardized block Toeplitz matrix. The original correlations between variables at three intervals are given, along with the expected correlations based on the time-series factor analysis model, in Table 4. The expected correlations were obtained by solving again for the "observed" correlations in terms of the latent correlations in Equation 25. The expected correlations were compared to the observed correlations. The chi-square goodness-of-fit test was again approximately equal to the degrees of freedom, $p \approx .5$. There was a close fit between the standardized block Toeplitz matrix and the expected correlations in Table 4, $p < .01$. Estimates of $\underset{\sim}{B}$ and $\underset{\sim}{M}$ were obtained by subjecting the synchronous section of the block Toeplitz matrix ($k = 0$) to common factor analysis, using a principal axes solution with iterations and varimax rotation. The first variable, promotion frequency, was not treated as latent. Scales 1, 2, and 3 were defined by the latent variables taken from $\underset{\sim}{B}$:

$$\underset{\sim}{B} = \begin{bmatrix} 1.0 & .0 & .0 & .0 \\ .0 & .70 & .10 & .12 \\ .0 & .64 & .14 & .23 \\ .0 & .05 & .81 & .36 \\ .0 & .20 & .81 & .07 \\ .0 & .10 & .59 & .13 \\ .0 & .27 & .17 & .85 \\ .0 & .17 & .23 & .72 \end{bmatrix}.$$

Estimates of $\underset{\sim}{M}$ were obtained from $\underset{\sim}{B}$ in the customary manner. Finally, latent correlations and parameters $\underset{\sim}{A}_z$ and $\underset{\sim}{V}_z$ are provided in Table 5.

The results in Table 5 reflect a highly dynamic oscillating system. The period of oscillation can be determined from the angle of the complex conjugate eigenvalues (roots) of $\underset{\sim}{A}_z$ (see Chow, 1975, pp. 27-30, 57-58). The relative amplitude of effects is a function of the absolute value of the roots. Four different periods of oscillation were represented in the roots of $\underset{\sim}{A}_z$, plus exponential decay stemming from one large positive real root. Listed in order of increasing amplitude, oscillation was found for periods of 5, 11, 27, and 23 months. The latter period corresponds roughly to the average tenure of enlisted soliders in a company. Promotions appear to be part of the latter cycle. A cycle of 24 months appears when comparing promotions with a composite scale (created by averaging within-scale items first and then averaging scales). A full multivariate frequence domain transformation, including phase lag relationships, is needed (see Friedlander, 1982b). Such transformations are the topic of current research.

In order to pick up the dynamics of change, it was important to include the third sampling time in this system. The intermediate sampling time helps to rule out aliasing effects due to violating the Nyquist frequency, at least for the longer period oscillations. The number of sampling intervals needs to be increased to eliminate concern over aliasing for the shorter period oscillations (e.g., five months).

The complex conjugate roots of $\underset{\sim}{A}_z$ mean that spuriousness can be ruled out in a limited sense: i.e., multiple correlated latent variables could not have produced the observed correlation matrix entirely (see

Table 4

Observed and Expected Correlations for Three Waves[a]

	x_{1t}	x_{2t}	x_{3t}	x_{4t}	x_{5t}	x_{6t}	x_{7t}	x_{8t}	x_{1t-1}	x_{2t-1}	x_{3t-1}	x_{4t-1}	x_{5t-1}	x_{6t-1}	x_{7t-1}	x_{8t-1}	x_{1t-2}	x_{2t-2}	x_{3t-2}	x_{4t-2}	x_{5t-2}	x_{6t-2}	x_{7t-2}	x_{8t-2}
x_{1t}	--	-08	-22	-02	-14	-02	-12	-21	19	30	25	14	26	06	16	18	21	12	25	51	40	40	35	26
x_{2t}	00	--	49	12	16	09	20	23	03	-01	08	02	10	18	-03	06	-35	24	10	-26	-11	-11	06	02
x_{3t}	01	49	--	16	25	11	36	20	-14	23	30	19	07	07	13	16	-22	05	13	-02	-02	03	02	02
x_{4t}	13	16	23	--	76	51	49	42	02	01	13	16	10	29	31	27	-26	07	-08	29	10	04	29	29
x_{5t}	12	23	26	69	--	44	45	36	14	06	12	21	24	33	24	06	-25	20	-01	20	01	-08	17	28
x_{6t}	09	15	18	53	50	--	23	42	-11	18	03	24	18	39	15	08	-32	25	15	15	03	14	18	16
x_{7t}	04	31	39	46	25	24	--	61	-12	03	04	01	-01	-05	13	14	-28	02	04	10	-01	-03	11	-02
x_{8t}	04	23	30	45	27	24	69	--	-22	05	-03	-06	-06	10	16	24	-34	27	08	03	-05	00	13	06
x_{1t-1}	23	02	01	-04	00	00	-09	-08	--	-01	15	34	24	12	24	01	29	19	30	34	17	-06	26	32
x_{2t-1}	27	16	15	10	14	09	06	05	00	--	41	09	20	00	23	19	16	22	12	27	27	27	06	16
x_{3t-1}	28	17	16	14	17	11	09	08	01	49	--	26	14	19	36	29	02	22	23	38	26	25	33	38
x_{4t-1}	20	22	22	32	36	25	13	12	13	16	23	--	67	63	23	19	02	23	24	46	29	22	46	39
x_{5t-1}	20	23	21	24	32	21	03	04	12	23	26	69	--	59	01	-06	-02	21	05	39	39	25	22	27
x_{6t-1}	15	16	16	19	24	16	05	05	09	15	18	53	50	--	04	02	-25	21	06	27	26	16	32	41
x_{7t-1}	28	15	18	32	27	21	29	26	04	31	39	46	25	24	--	71	04	15	25	30	14	05	39	35
x_{8t-1}	23	13	16	29	25	19	24	22	04	23	30	45	27	24	69	--	-05	-04	07	25	07	07	30	24
x_{1t-2}	19	-23	-24	-25	-23	-17	-26	-22	23	02	01	-04	00	-01	-09	-08	--	10	23	22	19	23	16	15
x_{2t-2}	17	16	15	06	08	05	08	06	27	16	15	10	14	09	06	05	00	--	58	24	35	22	43	34
x_{3t-2}	22	13	13	18	12	07	08	07	28	17	16	14	17	11	09	08	01	49	--	29	28	40	52	31
x_{4t-2}	61	-14	-11	28	15	12	02	05	28	22	22	32	36	25	13	12	13	16	23	--	65	45	60	63
x_{5t-2}	53	-10	-08	10	07	06	01	03	20	23	21	24	32	21	03	04	12	23	26	69	--	49	35	41
x_{6t-2}	41	-08	-06	10	07	06	01	02	15	16	16	19	24	16	05	05	09	15	18	53	50	--	32	34
x_{7t-2}	40	03	05	31	29	22	09	11	28	15	18	32	27	21	29	26	04	31	39	46	25	24	--	75
x_{8t-2}	38	-01	01	27	25	19	07	09	23	13	16	29	25	19	24	22	04	23	30	45	27	24	69	--

Note. Observed correlations appear in the upper triangle, and expected correlations generated by the time-series factor analysis model appear in the lower triangle. All entries should be multiplied by .01.

Table 5

System Parameters: Latent Correlations, $A_{\underline{z}}$, $V_{\underline{z}}$

latent variable	$\underline{z}_{1t}$	$\underline{z}_{2t}$	$\underline{z}_{3t}$	$\underline{z}_{4t}$	$\underline{z}_{1t-1}$	$\underline{z}_{2t-1}$	$\underline{z}_{3t-1}$	$\underline{z}_{4t-1}$	$\underline{z}_{1t-2}$	$\underline{z}_{2t-2}$	$\underline{z}_{3t-2}$	$\underline{z}_{4t-2}$
						Latent Correlations						
$\underline{z}_{1t}$	100	-02	15	02	23	33	15	20	19	10	60	32
$\underline{z}_{2t}$	-02	100	00	00	05	27	31	01	-27	36	-29	-08
$\underline{z}_{3t}$	15	00	100	00	00	07	37	24	-21	-02	12	41
$\underline{z}_{4t}$	02	00	00	100	-11	-07	-17	34	-18	01	04	04
			$V_{\underline{z}}$					$A_{\underline{z}}$				
$\underline{z}_{1t}$	43	21	-01	-10	06	10	-17	23	10	08	65	27
$\underline{z}_{2t}$	21	39	-10	14	10	43	56	-03	-22	17	-61	-22
$\underline{z}_{3t}$	-01	-10	69	-04	-06	07	28	11	-20	-04	05	32
$\underline{z}_{4t}$	-10	14	-04	76	-15	-19	-27	38	-13	15	30	01

Note. $\underline{z}_1$ = promotion frequency; $\underline{z}_2$ = Goal Orientation; $\underline{z}_3$ Intergroup Harmony; $\underline{z}_4$ = Enlisted Unity. All entries should be multiplied by .01. The eigenvalues of $A_{\underline{z}}$ are: (a) .65 $\pm$ .42$\underline{i}$, (b) .72, (c) .54 $\pm$ .32$\underline{i}$, (d) .22 $\pm$ .48$\underline{i}$, (e) -.22.

Section 4.2.2.). Similarly, lack of stationarity in measurement error at each time point could not account entirely for the observed correlation pattern, although it could have had an influence. Allowing measurement error to adapt would improve the fit to real data (see Section 2.2.).

The data in Table 5 suggest that good interpersonal relations led to high promotion rates, while high promotion rates produced a negative "sour grapes" reaction on a variety of scales. The expectation of replicating these results is improved by the fact that relationships are roughly stationary in a first order sense. Results are not inconsistent with evidence that promotions are rewarding. Promotions are rewarding, but rewarding at an individual level to the persons promoted. At the group level, many eligible persons are not promoted for each one who is, creating disappointment and conflict. The principle in Section 1.2.4. is illustrated here: Results from one level of analysis often will not be the same as results from another level. In another noticeable relationship, Intergroup Harmony is driving an oscillation in Goal Orientation (effort toward achieving company effectiveness).

These results also illustrate the principle of bootstrapping through levels of theory (see Section 1.1.). A measurement model was needed prior to estimating a historical model. The historical model was needed to understand the dynamics of intergroup or interpersonal interaction. Finally, understanding the dynamics of interaction are natural prerequisites to planning effective transformational change. The probability of increasing group productivity is improved by understanding system dynamics as shown in linear prediction filters (e.g., $\underset{\sim}{A}_{\underset{\sim}{z}}$).

Discussion

We have argued that failure to appropriately test theory leads to the predominance of ideologically-based content in the field. Ideological content is defined as untested theory. This problem can arise from an inability to appropriately test theory. A brief review of some commonly used methods in the field showed a number of serious problems in the application of the scientific method, and a more exhaustive review could undoubtedly uncover additional problems. Ideological content is, then, a problem. Interactivity theory attacks this problem by attempting to improve the application of the scientific method. An approach that seems particularly promising is linear prediction filter methodology. These filters have been successfully applied to many high technology problems, and are appropriate for application to interactive human environments. There is a large technically sophisticated tradition and literature in this area. Unfortunately, this tradition has been frequently ignored, perhaps because it appears irrelevant to psychological problems. Although many methods need to be adapted to meet the requirement of psychological theory, the relevance of this tradition to psychological problems has been demonstrated here.

Many important topics have not been covered here. What is covered, however, provides a tutorial for approaching the electronics engineering and economics literature on statistical aspects of filtering. Additional research and study is needed on such important topics as transformational change (see Chow, 1981), adaptive systems (see Friedlander, 1982a), and

higher order stationarity. In addition, estimation theory and statistical tests were not discussed in the context of filters. A variety of research issues exist here, e.g.: (a) behavior tests of overall system output, versus significance tests for single equations (Mass & Senge, 1978); (b) sensitivity of goodness-of-fit chi-square tests to the multivariate normality assumption (see Jöreskog & Sorbom, 1979); (c) distribution-free tests using "bootstrap" methods (Diaconis & Efron, 1983); and (d) conflict among root-based tests (Berndt & Savin, 1977; Kohler, Note 3).

Placing factor analysis in the context of filters means the resulting filter must forecast individual cases. Since the common factor analyses model is underidentified, factor scores must be estimated rather than identified exactly. Traditional methods for estimating factor scores (see Mulaik, 1982, chap. 13) do not seem satisfactory in the context of filters. Again, further research is required.

APPENDIX A

$\Gamma_{(0)}$ in Equation 7 can be written in closed form that excludes infinite series. This can be accomplished by using the following general identity (see Browne, 1977; Equation 6):

$$\underset{mmmm}{\text{vec}(RST')} = (\underset{mm}{T} \bullet \underset{mm}{R})\underset{mm}{\text{vec}S}, \text{ where} \tag{1A}$$

$\underset{mm}{R}$ and $\underset{mm}{T}$ are of order $\underline{m}$ x $\underline{n}$, $\underset{mm}{S}$ is of order $\underline{n}$ x $\underline{n}$; and $\bullet$ refers to the Direct or Kronecker product. Equation 1A can be used to rewrite Equation 11 as,

$$\underset{mm}{\text{vec}V} = \text{vec}\,\Gamma_{(0)} - (\underset{mm}{A} \bullet \underset{mm}{A})\text{vec}\,\Gamma_{(0)}, \text{ where} \tag{2A}$$

vec refers to an $\underline{n}^2$ x 1 column vector created by stacking, in order from 1 to $\underline{n}$, the $\underline{n}$ columns of an $\underline{n}$ x $\underline{n}$ matrix. The value $\underline{n}$ is defined in Equation 6. Factoring, and solving for $\Gamma_{(0)}$,

$$\text{vec}\,\Gamma_{(0)} = (\underset{mm}{I} - \underset{mm}{A} \bullet \underset{mm}{A})^{-1}\underset{mm}{\text{vec}V}. \tag{3A}$$

Unfortunately, this solution requires inverting a large matrix. However, an alternate expression for $\Gamma_{(0)}$ in closed form is provided by the eigenstructure of $\underset{mm}{A}$, as shown by Equations 7C and 9C, Appendix C, and Chow (1975, chap. 3):

$$\Gamma_{(0)} = \underset{\sim}{B}[(1-\text{conjg}(\lambda_j)\lambda_i)^{-1}u_{ij}]\underset{\sim}{B}' \tag{4A}$$

$\underset{\sim}{B}$ is the matrix of eigenvectors of $\underset{\sim}{A}$, λ_i and λ_j are elements of $\underset{\sim}{D}$, the diagonal matrix of eigenvalues of $\underset{\sim}{A}$ (see Equation 6), u_{ij} are elements of the residual covariance matrix $\underset{\sim}{U}$ (see Equation 5C) in canonical variable form, and conjg refers to taking the complex conjugate when λ_j is complex. In addition to Equation 4A, the lattice filter provides an efficient formula for the regeneration of covariance matrices in closed form using linear parameters and residuals (see Friedlander, 1982a).

APPENDIX B

The transformations between the filter in Section 1.2.1. and the econometric model in Section 3.3.4. are defined by the equations in this appendix. All the econometric model specifications can be transformed into the filter using the following equations. Let $\underset{\sim}{G} = \underset{\sim}{I} - \underset{\sim}{F}$. Then,

$$\begin{aligned} \underset{\sim}{X}_t &= \underset{\sim}{G}^{-1}\underset{\sim}{A}\dagger\underset{\sim}{X}_{t-1} + \underset{\sim}{G}^{-1}\underset{\sim}{e}_{wt}, \\ \underset{\sim}{A} &= \underset{\sim}{G}^{-1}\underset{\sim}{A}\dagger, \\ \underset{\sim}{v}_t &= \underset{\sim}{G}^{-1}\underset{\sim}{e}_{wt}, \\ \underset{\sim}{V} &= \underset{\sim}{G}^{-1}\underset{\sim}{W}\underset{\sim}{G}'^{-1}. \end{aligned} \tag{1B}$$

In Equation 1B $\underset{\sim}{W}$ is the covariance of residuals, $\underset{\sim}{e}_{wt}\underset{\sim}{e}'_{wt}$. Other terms are defined in Sections 1.2.1. and 3.3.4.

Conversely, the transformation from the filter to all econometric specifications involve the following equations,

$$\begin{aligned} \underset{\sim}{A}\dagger &= \underset{\sim}{G}\underset{\sim}{A} \\ \underset{\sim}{e}_{wt} &= \underset{\sim}{G}\underset{\sim}{v}_t \\ \underset{\sim}{W} &= \underset{\sim}{G}\underset{\sim}{V}\underset{\sim}{G}'. \end{aligned} \tag{2B}$$

It is possible to solve for the elements of $\underset{\sim}{G}$ (or $\underset{\sim}{F}$) from the elements of $\underset{\sim}{A}$ as follows. Solve for $\underset{\sim}{y}_{t-1}$ in the second line of Equation 2,

$$\underline{y}_{\underline{t}-1} = (\underline{y}_{\underline{t}} - \underline{a}_{21}\underline{x}_{\underline{t}-1} - \underline{e}_{\underline{yt}})/\underline{a}_{22}. \tag{3B}$$

Substitute the result into Equation 2, line 1, and collect terms:

$$\underline{x}_{\underline{t}} = (\underline{a}_{12}/\underline{a}_{22})\underline{y}_{\underline{t}} + (a_{11} - a_{12}a_{21}/a_{22})\underline{x}_{\underline{t}-1} - (\underline{a}_{12}/\underline{a}_{22})\underline{e}_{\underline{yt}} + \underline{e}_{\underline{xt}}. \tag{4B}$$

In Equation 4B $\underline{y}_{\underline{t}-1}$ becomes zero, and $f_{12} = -g_{12} = a_{12}/a_{22}$. Other elements of $\underset{\sim}{F}$ and $\underset{\sim}{G}$ can be solved analogously. Elements of $\underset{\sim}{G}$ can also be obtained from $\underset{\sim}{V}$, by rewriting the last line of Equation 2B as (see Equation 1A),

$$\text{vec}\underset{\sim}{W} = (\underset{\sim}{G} \bullet \underset{\sim}{G})\text{vec}\underset{\sim}{V}. \tag{5B}$$

Off-diagonal elements of $\underset{\sim}{W}$ can be set to zero. Particular elements of $\underset{\sim}{G}$ can then be solved in terms of elements of $\underset{\sim}{V}$. Using Equations 2B, 4B, and 5B, any of the parameters from the econometric specifications in Table 2 can be written in terms of filter coefficients (specification 1). For example, $a_{22}^{\dagger}$ from specification 5 is,

$$\underline{a}_{22}^{\dagger} = (\underline{a}_{12}^{2}\underline{v}_{22} - \underline{a}_{12}\underline{a}_{22}\underline{v}_{12})/(\underline{a}_{22}\underline{v}_{11} - \underline{a}_{12}\underline{v}_{12}) + \underline{a}_{22}. \tag{6B}$$

APPENDIX C

The transformation between the filter in Section 1.2.1. and its latent canonical variable representation is given here (see Chow, 1975, chap. 3). The canonical variable representation is similar to the cross-lagged correlation model of spuriousness discussed in Section 4.2.2.

A canonical variable is defined by:

$$\underset{\sim}{Z} = \underset{\sim}{B}^{-1}\underset{\sim}{X}_t. \tag{1C}$$

$\underset{\sim}{B}$ is the eigenvector matrix from the eigenstructure of $\underset{\sim}{A}$. $\underset{\sim}{A}$ and $\underset{\sim}{X}_t$ are defined in Equation 6. The following relationships follow from this definition:

$$\underset{\sim}{X}_t = \underset{\sim}{B}\underset{\sim}{Z}_t, \tag{2C}$$

$$\underset{\sim}{e}_{zt} = \underset{\sim}{B}^{-1}\underset{\sim}{v}_t, \tag{3C}$$

$$\underset{\sim}{Z}_t = \underset{\sim}{D}\underset{\sim}{Z}_{t-1} + \underset{\sim}{e}_{zt}. \tag{4C}$$

The term $\underset{\sim}{v}_t$ is defined in Equation 6, and $\underset{\sim}{e}_{zt}$ in Equation 41. $\underset{\sim}{D}$ is the diagonal matrix of eigenvalues from $\underset{\sim}{A}$. Using Equation 3C the covariance of latent canonical residuals becomes:

$$\underset{\sim}{U} = \underset{\sim}{e}_{zt}\underset{\sim}{e}'_{zt} = \underset{\sim}{B}^{-1}\underset{\sim}{V}\underset{\sim}{B}^{-1}. \tag{5C}$$

Using Equation 4C the synchronous covariance of latent canonical variables becomes,

$$\Gamma_{\underline{z}(0)} = \underset{\sim}{Z}_{\underline{t}} \underset{\sim}{Z}'_{\underline{t}} = \underset{\sim}{D} \Gamma_{\underline{z}(0)} \underset{\sim}{D}' + \underset{\sim}{U}. \tag{6C}$$

Using Equation 1A, $\Gamma_{z(0)}$ can be rewritten as,

$$\text{vec}\Gamma_{\underline{z}(0)} = [I - \text{conjg}\ (\underset{\sim}{D}) \otimes \underset{\sim}{D}]^{-1} \text{vec}\underset{\sim}{U}. \tag{7C}$$

In Equation (7C) conjg refers to the complex conjugate of the matrix $\underset{\sim}{D}$. This operation affects complex, but not real entries in $\underset{\sim}{D}$.

The indeterminacy in the eigenvector matrix $\underset{\sim}{B}$ can be eliminated by standardizing the latent covariance matrix $\Gamma_{z(0)}$. This can be accomplished by,

$$\underset{\sim}{B}_{\underline{s}} = \text{diag}[\Gamma_{\underline{x}(0)}]^{-\frac{1}{2}} \underset{\sim}{B} \text{diag}[\Gamma_{\underline{z}(0)}]^{\frac{1}{2}}, \tag{8C}$$

where $\underset{\sim}{B}_{\underline{s}}$ represent standardized coefficients, and diag represents diagonal elements of the respective matrices. In order to standardize $\underset{\sim}{U}$, $\underset{\sim}{U}_{\underline{s}}$ must be computed by substituting $\underset{\sim}{B}_{\underline{s}}$ for $\underset{\sim}{B}$ in Equation 5C. In order to standardize $\Gamma_{z(0)}$, $\Gamma_{\underline{zs}(0)}$ must be computed by substituting vec $\underset{\sim}{U}_{\underline{s}}$ for vec $\underset{\sim}{U}$ in Equation 7C. Finally, the observed correlation matrix can be written as,

$$\Gamma_{\underline{x}(0)} = \underset{\sim}{X}_{\underline{t}} \underset{\sim}{X}'_{\underline{t}} = \underset{\sim}{B}_{\underline{s}} \Gamma_{\underline{zs}(0)} \underset{\sim}{B}'_{\underline{s}} \tag{9C}$$

$$\Gamma_{\underline{x}(k)} = \underset{\sim}{X}_{\underline{t}} \underset{\sim}{X}'_{\underline{t-k}} = \underset{\sim}{B}_{\underline{s}} \underset{\sim}{D}^{k} \Gamma_{\underline{zs}(0)} \underset{\sim}{B}'_{\underline{s}}. \tag{10C}$$

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