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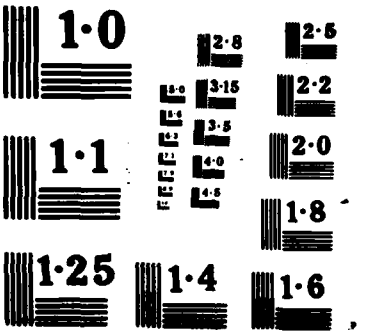
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MULTIVARIATE NONPARAMETRIC CLASSES IN RELIABILITY

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ABSTRACT

Recent research in multivariate nonparametric classes in reliability is surveyed.

1. Introduction. This paper is a sequel to the survey paper of Hollander and Proschan (1984) who examine univariate nonparametric classes and methods in reliability. In this paper we will examine multivariate nonparametric classes and methods in reliability.

Hollander and Proschan (1984) describe the various univariate nonparametric classes in reliability. The classes of adverse aging described include the IFR, IFRA, NBU, NBUE and DMRL classes. The dual classes of beneficial aging are also covered. Several new univariate classes have been introduced since that time. One that we will briefly mention is the HNBUE class, since we are aware of several multivariate generalizations of this class.

The univariate classes in reliability are important in applications concerning systems where the components can be assumed to be independent. In this case the components are often assumed to experience wearout or beneficial aging of a similar type. For example, it is often reasonable to assume that components have increasing failure rate (IFR). In making this IFR assumption it is implicit that each component separately experiences wear and no interactions among components can occur. However in many realistic situations, adverse wear on one component will promulgate adverse wear on other components. From another point of view a common environment will cause components to behave similarly. In either situation, it is clear that an assumption of independence on the components would not be valid. Consequently multivariate concepts of adverse or beneficial aging are required.

Multivariate nonparametric classes have been proposed as early as 1970. For background and references as well as some discussion of univariate classes with multivariate generalizations in mind see Block and Savits (1981). In the present paper ~~we shall only~~ only describe a few fundamental developments prior to 1981

and focus on developments since then. The coverage will not be exhaustive but will emphasize the topics which we feel are most important.

Section 2 deals with multivariate nonparametric classes. In section 2.1 multivariate IFRA is discussed with emphasis on the Block and Savits (1980) class. Multivariate NBU is covered in Section 2.2 and multivariate NBUE classes are mentioned in Section 2.3. New developments in multivariate IFR are considered in Section 2.4 and in Section 2.5 the topics of multivariate DMRL and HNBUE are touched on.

Familiarity with the univariate classes is assumed. The basic reference for the IFR, IFRA, NBU and NBUE classes is Barlow and Proschan (1981). See also Block and Savits (1981). For information on the DMRL class see Hollander and Proschan (1984). The HNBUE class is relatively recent and the best references are the original articles. See for example, Klejsjö (1982) and the references contained there.

2. Multivariate Nonparametric Classes. Many multivariate versions of the univariate classes were proposed using generalizations of various failure rate functions. These multivariate classes were extensively discussed in Block and Savits (1981). Other classes were proposed by attempting to imitate univariate definitions in a multivariate setting. (See also Block and Savits (1981).) One of the most important of these extensions was due to Block and Savits (1980) who generalized the IFRA class. This multivariate class was proposed to parallel the developments of the univariate case where the IFRA class possessed many important closure properties. As in the univariate case the following multivariate class of IFRA, designated the MIFRA class, satisfies important closure properties. First, as in the univariate case, monotone systems with MIFRA lifetimes have MIFRA lifetimes and independent sums of MIFRA lifetimes are MIFRA. From the multivariate point of view, subfamilies of MIFRA are MIFRA, conjunctions of independent MIFRA are

MIFRA, scaled MIFRA lifetimes are MIFRA, and various other properties are satisfied. We discuss this extension first since several other classes have been defined using similar techniques.

2.1 Multivariate IFRA. Using a characterization of the univariate IFRA class in Block and Savits (1976) the following definition can be made.

(2.1.1) Definition. Let $\underline{T}=(T_1,\ldots,T_n)$ be a nonnegative random lifetime. The random vector $\underline{T}$ is said to be MIFRA if

$$E^{\alpha}[h(\underline{T})] \leq E[h^{\alpha}(\underline{T}/\alpha)]$$

for all continuous nonnegative nondecreasing functions h and all $0 < \alpha \leq 1$.

This definition as mentioned above implies all of the properties one would desire for a multivariate analog of the univariate IFRA class. Part of the reason for this is that the definition is equivalent to many other properties which are both theoretically and intuitively appealing. The statement and proofs of these results are given below; the form in which these are presented is influenced by the paper of Marshall and Shaked (1982) who defined a similar MNBU class.

Notes. 1) Obviously in (2.1.1) we need only consider h defined on $\mathbb{R}_+^n = \{\underline{x} | \underline{x} \geq \underline{0}\}$. Hence all of the functions and sets mentioned below are assumed to be Borel measurable in $\mathbb{R}_+^n$.

2) We say a function g is homogeneous (subhomogeneous) on $\mathbb{R}_+^n$ if

$$\alpha g(\underline{t}) = (\leq)\ g(\alpha \underline{t}) \qquad \text{for all } 0 \leq \alpha \leq 1,\ \ \underline{0} \leq \underline{t}.$$

3) A is an upper set if $\underline{x} \in A$ and $\underline{x} \leq \underline{y}$ implies $\underline{y} \in A$.

(2.1.2) Theorem The following conditions are all equivalent to $\underline{T}$ being MIFRA.

i) $P^{\alpha}\{\underline{T} \in A\} \leq P\{\underline{T} \in \alpha A\}$ for all open upper sets in $\mathbb{R}^n_+$, all $0 < \alpha \leq 1$.

ii) $P^{\alpha}\{\underline{T} \in A\} \leq P\{\underline{T} \in \alpha A\}$ for all upper sets in $\mathbb{R}^n_+$, all $0 < \alpha \leq 1$.

(i.e. $E^{\alpha}(\phi(\underline{T})) \leq E(\phi^{\alpha}(\underline{T}/\alpha))$ for all nonnegative, binary, nondecreasing ϕ on $\mathbb{R}^n_+$).

iii) $E^{\alpha}(h(\underline{T})) \leq E(h(\underline{T}/\alpha))$ for all nonnegative, nondecreasing h on $\mathbb{R}^n_+$, all $0 < \alpha \leq 1$.

iv) For all nonnegative, nondecreasing, subhomogeneous h on $\mathbb{R}^n_+$, $h(\underline{T})$ is IFRA.

v) For all nonnegative, nondecreasing, homogeneous h on $\mathbb{R}^n_+$, $h(\underline{T})$ is IFRA.

Proof: i)=>ii): By Theorem 3.3 of Esary, Proschan and Walkup (1967) for an upper set A and any $\epsilon > 0$ there is an open upper set A_{ϵ} such that $A \subset A_{\epsilon}$ and $P\{\underline{T} \in \alpha A_{\epsilon}\} \leq P\{\underline{T} \in \alpha A\} + \epsilon$. Thus

$$P^{\alpha}\{\underline{T} \in A\} \leq P^{\alpha}\{\underline{T} \in A_{\epsilon}\} \leq P\{\underline{T} \in \alpha A_{\epsilon}\} \leq P\{\underline{T} \in \alpha A\} + \epsilon.$$

ii) => iii): Let h_k, $k = 1,2,\ldots$ be an increasing sequence of increasing step functions such that $\lim_{k\to\infty} h_k = h$. Specifically take

$$h_k(\underline{t}) = \begin{cases} \frac{i-1}{2^k} & \text{if } \frac{i-1}{2^k} \leq h(\underline{t}) < \frac{i}{2^k}, \quad i = 1,2,\ldots,k2^k, \\ k & \text{if } h(\underline{t}) \geq k, \end{cases}$$

i.e. $h_k(\underline{t}) = \sum_{i=1}^{k2^k} \frac{1}{2^k} I_{A_{i,k}}(\underline{t})$ where $I_{A_{i,k}}$ is the indicator function of the upper set $A_{i,k} = \{\underline{t} | h(\underline{t}) \geq \frac{i}{2^k}\}$. Thus we need only prove the result for functions of the form

$$h(\underline{t}) = \sum_{i=1}^{m} a_i I_{A_i}(\underline{t}), \qquad a_i \geq 0,$$

where $A_1, \ldots, A_m$ are upper sets, since the remainder follows by the monotone convergence theorem. We have

$$E^{\alpha}\left(\sum_{i=1}^{m} a_i I_{A_i}(\underline{T})\right) = \left[\sum_{i=1}^{m} a_i P\{\underline{T} \in A_i\}\right]^{\alpha} \leq \left[\sum_{i=1}^{m} a_i P^{\frac{1}{\alpha}}\{\underline{T} \in \alpha A_i\}\right]^{\alpha}$$

$$= \left[\sum_{i=1}^{m} \left\{\int a_i^{\alpha} I_A^{\alpha}(\underline{t}/\alpha)\, dF(\underline{t})\right\}^{\frac{1}{\alpha}}\right]^{\alpha} \leq \sum_{i=1}^{m} \int a_i I_A(\underline{t}/\alpha)\, dF(\underline{t})$$

$$= E\left(\left[\sum_{i=1}^{m} a_i I_{A_i}(\underline{T}/\alpha)\right]^{\alpha}\right)$$

where the last inequality is due to Minkowski.

iii) => Def. Obvious.

Def. => i). From Esary, Proschan and Walkup (1967) for any open upper set A there exist nonnegative, nondecreasing, continuous functions h_k such that $h_k \uparrow I_A$. Then apply the monotone convergence theorem.

iii) => iv). Let h be nonnegative, nondecreasing and subhomogeneous. Then

$$P^{\alpha}\{h(\underline{T}) > t\} = E^{\alpha}(I_{(t,\infty)}(h(\underline{T}))) \leq E(I_{(t,\infty)}(h(\underline{T}/\alpha)))$$

$$\leq E\left(I_{(t,\infty)}\left(\frac{1}{\alpha} h(\underline{T})\right)\right) = P\{h(\underline{T}) > \alpha\, t\}$$

where the first inequality follows from iii) and the second by the subhomogeniety.

iv) => v): Obvious.

v) => i). Let A be an open upper set and define

$$h(\underline{t}) = \begin{cases} \sup\{\theta > 0: \frac{1}{\theta}\,\underline{t} \in A\} & \text{if } \{\theta > 0: \frac{1}{\theta}\,\underline{t} \in A\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

Then h is nonnegative, nondecreasing and homogeneous. Thus

$$P^{\alpha}\{\underline{T} \in A\} = P^{\alpha}\{h(\underline{T}) > 1\} \leq P\{h(\underline{T}) > \alpha\} = P\{\underline{T} \in \alpha A\}.$$

(2.1.3) Note. The following two alternate conditions could also have been added to the above list of equivalent conditions (provided $\bar{F}(0) = 1$).

vi) $P^{\alpha}\{\underline{T} \in A\} \leq P\{\underline{T} \in \alpha A\}$ for each set A of the form $A = \bigcup_{i=1}^{n} A_i$ where $A_i = \{\underline{x} | \underline{x} > \underline{x}_i\}$, $\underline{x}_i \in \mathbb{R}_+^n$ and for all $0 < \alpha \leq 1$.

vii) For each $k = 1,2,\ldots$, for each a_{ij}, $i = 1,\ldots,k$, $j = 1,\ldots,n$, $0 \leq a_{ij} \leq \infty$, and for each coherent life function τ of order kn $\tau(a_{11}T_1, a_{12}T_1, \ldots, a_{1n}T_1, a_{21}T_2, \ldots, a_{kn}T_n)$ is IFRA. (See Block and Savits (1980) for a definition of coherent life function and for some details of the proof).

In conjunction with the preceding result the following lemma makes it easy to demonstrate that a host of different lifetimes are MIFRA.

(2.1.4) Lemma. Let $\underline{T}$ be MIFRA and $\psi_1,\ldots,\psi_m$ be any continuous, subhomogeneous functions of n variables. Then if $S_i = \psi_i(\underline{T})$ for $i = 1,\ldots,m$, $\underline{S} = (S_1,\ldots,S_m)$ is MIFRA.

Proof: This follows easily by considering a nonnegative, increasing, continuous function h of m variables and applying the MIFRA property of $\underline{T}$ and the monotonicity of the ψ_i.

(2.1.5) Corollary. Let $\tau_1,\ldots,\tau_m$ be coherent life functions and $\underline{T}$ be MIFRA. Then $(\tau_1(\underline{T}),\ldots,\tau_m(\underline{T}))$ is MIFRA.

Proof: Since coherent life functions are homogeneous this follows easily.

(2.1.6). Example. Let $X_1,\ldots,X_n$ be independent IFRA lifetimes and $\phi \neq S_i \subset \{1,2,\ldots,n\}$, $i = 1,\ldots,m$. Since it is not hard to show that independent IFRA lifetimes are MIFRA, it follows that $T_i = \min_{j \in S_i} X_j$ $i = 1,\ldots,m$ are MIFRA. Since many different types of multivariate IFRA can be generated in the above way, the example shows that any of these are MIFRA. See Esary and Marshall (1979) where various types of multivariate IFRA of the type in this example are defined. See Block and Savits (1982) for relationships among these various definitions.

Multivariate shock models with multivariate IFRA properties have been treated in Marshall and Shaked (1979) and in Savits and Shaked (1981).

2.2 Multivariate NBU. As with all of the multivariate classes, the need for each of them is evident becuase of the usefulness of the corresponding univariate class. The only difference is that in the multivariate case, the independence of the components is lacking. In particular the concept of NBU is fundamental in disucssing maintenance policies in a single component system. For a multicomponent system, where components are dependent, marginally components satisfy the univariate NBU property under various maintenance protocols. However, a joint concept describing the interaction of all the components is necessary. Hence multivariate NBU concepts are required.

Mose of the earliest definition of multivariate NBU (see for example Buchanan and Singpurwalla (1977)) consisted of various generalizations of the defining property of the univariate NBU class. For a survey of these see definitions (1)-(5) of Section 5 of Block and Savits (1981). For shock models which satisfy these definitions see Marshall and Shaked, (1979), Griffith (1982), Ebrahimi and Ghosh (1981) and Klefsjo (1982). Other definitions involving generalizations of properties of univariate NBU distributions are given by (7)-(9) of the same reference. These are similar to definitions used by Esary and Marshall (1979) to define multivariate IFRA distributions. Properties (7) and (8) of the Block and Savits (1981) reference represent a certain type of definition and bear repeating here. The vector $\underline{T}$ is said to be multivariate NBU if:

$$\tau(T_1,\ldots,T_n) \text{ is NBU for all } \tau \text{ in a certain class of life functions;} \tag{2.2.1}$$

$$\text{There exist independent NBU } X_1,\ldots,X_k \text{ and life functions } \tau_i,\ i=1,\ldots,n \text{ in a certain class such that } T_i = \tau_i(\underline{X}),\ i=1,\ldots,n. \tag{2.2.2}$$

El-Neweihi, Proschan and Sethuraman (1983) have considered a special case of (2.2.2) where the τ_i are minimums and have related this case to some other definitions including the special case of (2.2.1) where τ is any minimum.

As shown in Theorem 2.1, definitions involving increasing functions can be given equivalently in terms of upper (or open upper) sets. Two multivariate NBU definitions which were given in terms of upper sets were those of El-Neweihi (1981) and Marshall and Shaked (1982). These are respectively:

For every upper set $A \subset \mathbb{R}_+^n$ and for every $0 < \alpha < 1$

$$P\{\underline{T} \in A\} \leq P(\min(\frac{\underline{T}'}{\alpha}, \frac{\underline{T}''}{1-\alpha}) \in A) \tag{2.2.3}$$

where $\underline{T}, \underline{T}', \underline{T}''$ are independent and have the same distribution.

For every upper set $A \subset \mathbb{R}_+^n$ and for every $\alpha > 0$, $\beta > 0$

$$P\{\underline{T} \in (\alpha+\beta)A\} \leq P\{\underline{T} \in \alpha A\}\ P\{\underline{T} \in \beta A\}. \tag{2.2.4}$$

Relationships among these definitions are given in El-Neweihi (1981). A more restrictive definition than either of the above has been given in Berg and Kesten (1984):

For every upper $A, B \subset \mathbb{R}^n$,

$$P(\underline{T} \in A+B) \leq P(\underline{T} \in A)P(\underline{T} \in B) \tag{2.2.5}$$

This definition was shown to be useful in percolation theory as well as reliability theory.

A general framework involving generalizations of the concept (2.2.1) called taking the C-closure of $\mathcal{F}$ and of the concept (2.2.2) called C-generating from $\mathcal{F}$ (where $\mathcal{F}$ is the class of univariate NBU lifetimes in (2.2.1) and (2.2.2)) has been given by Marshall and Shaked (1984). Many of the previous NBU definitions are organized within this framework. A similar remark applies when the classes $\mathcal{F}$ are exponential, IFR, IFRA and NBUE. See Marshall and Shaked (1984).

2.3 Multivariate NBUE. Along with the multivariate NBU versions of Buchanan and Singpurwalla (1977) are integrated versions of these definitions. These authors give three versions of multivariate NBUE. The relations among these and closure properties are discussed in Ebrahimi and Ghosh (1981). Furthermore the latter authors relate these multivariate NBUE definitions to four definitions of multivariate NBU (i.e. definitions (1)-(4) of Section 5 of Block and Savits (1981)).

Some other multivariate NBUE classes are mentioned by Block and Savits (1981) and Marshall and Shaked (1984). One extension of a univariate characterization of the NBUE class mentioned in Block and Savits (1978) has been proposed by Savits (1983b).

2.4 Multivariate IFR. Perhaps the most important univariate concept in reliability is that of increasing failure rate. One reason for this is that in a very simple and compelling way this idea describes the wearout of a component. Many engineers, biologists and actuaries find this description fundamental. The monotonicity of the failure rate function is simple and intuitive and occurs in many physical situations. This also is crucial in the multicomponent case where the components are dependent.

Several authors have attempted to describe the action of the failure rates increasing for n components simultaneously. These cases were discussed in Block and Savits (1981) and in the references contained therein.

A recent definition of multivariate IFR was given by Savits (1983a) and is in the spirit of the classes defined by Block and Savits (1980) and Marshall and Shaked (1982). For shock models involving multivariate IFR concepts see Ghosh and Ebrahimi (1981).

It is shown in Savits (1983a) that a univariate lifetime T is IFR if and only if $E[h(\underline{x},T)]$ is log concave in $\underline{x}$ for all functions $h(\underline{x},t)$ which are log concave in $(\underline{x},t)$ and are nondecreasing in t for each fixed $\underline{x} \geq 0$. This leads to the following multivariate definition.

(2.4.1) Definition. Let $\underline{T}$ be a nonnegative random vector. Then $\underline{T}$ has an MIFR distribution if $E[h(\underline{x},\underline{T})]$ is log concave in $\underline{x}$ for all functions $h(\underline{x},\underline{t})$ which are log concave in $(\underline{x},\underline{t})$ and nondecreasing in $\underline{t} \geq 0$ for each fixed $\underline{x} \geq 0$.

This class enjoys many closure properties. Among these are that all marginals are MIFR, conjunction of independent MIFR are MIFR, convolutions of MIFR are MIFR, scaled MIFR are MIFR, nonnegative nondecreasing concave functions of MIFR are MIFR, and weak convergence preserves MIFR. See Savits (1983a) for details. From these results it follows that the multivariate exponential distribution of Marshall and Olkin (1967) is MIFR, as are all distributions with log concave densities. Since the multivariate folded normal has a log concave density, it also is MIFR.

The technique used in Definition 2.4.1 for the MIFR class extends to other multivariate classes. In particular, if we replace log concave with log subhomogeneous, we get the same multivariate IFRA class as in Definition 2.1.1; if we replace log concave with log subadditive, we get a new multivariate NBU class which is between that of (2.2.3) and (2.2.4). For more details see Savits (1983a, 1983b).

2.5 Multivariate DMRL and HNBUE. Few definitions of multivariate DMRL have been discussed in the literature, although E. El-Neweihi has privately communicated one to us. Since developments are premature with respect to this class we will not go into details.

Multivariate extensions of the HNBUE class have been proposed by Basu and Ebrahimi (1981) and Klefsjo (1980). The extensions of the former authors are similar in spirit to the multivariate NBUE classes of Ghosh and Ebrahimi (1981). The latter author's definition extend the univariate definition by replacing the univariate exponential distribution with the bivariate Marshall and Olkin (1967) distribution and considering various multivariate versions of the definition.

Basu and Ebrahimi (1981) show relationships among their definitions and Klefsjo's, give some closure properties and also point out relations with multivariate NBUE classes.

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