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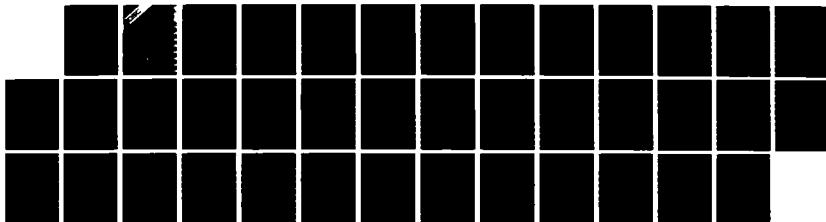
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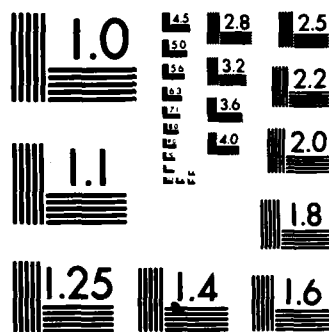
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Longwave Propagation — A Brief Review

F. P. Snyder



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19 ABSTRACT (Continue on reverse if necessary and identify by block number) This brief review of the state-of-knowledge of longwave electromagnetic propagation in the near-earth environment includes some discussions of commonly available computer models for this frequency regime. The frequency range considered in this report encompasses the ELF, the VLF, and the lower part of the LF band. Because longwave propagation is characterized by high stability in both amplitude and phase, and because these frequencies propagate to very great distances, longwave signals are of considerable practical importance to global communications, navigation systems, and to global frequency and time comparisons.					
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1.0 INTRODUCTION

This report provides an overview of the state-of-knowledge of long wavelength electromagnetic propagation (frequently termed longwave propagation) in the near-earth environment. Included in this review is a discussion of some commonly available computer models for this frequency regime. This report does not, however, dwell on analytical considerations of the models. Rather, it is concerned with the characteristics of longwave propagation and the ability of the models to predict these characteristics.

The frequency range considered encompasses what is conventionally termed the extremely low frequency (ELF), the very low frequency (VLF), and the lower part of the low frequency (LF) bands. The ELF band extends from 30 Hz to 3 kHz, the VLF from 3 kHz to 30 kHz and the LF band from 30 kHz to 300 kHz. The term longwave propagation is often used to exclude the ELF band, or to include all of the LF band although this is not universal. Makarov et al. (1970), for example, indicates that the Russian usage extends from 1 kHz to 60 kHz. It should be noted that the propagation within the LF band undergoes a transition from the longwave characteristics to the more traditional shortwave characteristics, such as is common to the medium frequency (MF), high frequency (HF), and higher frequencies. It is this transitioning that precludes discussion of the upper LF regime in this report. This report also does not consider the application of the LF regime to such systems as the Loran-C navigation system, as such applications employ propagation characteristics typical to ground wave propagation in the shortwave regime.

Although there are unique features characteristic to each of the frequency regimes, there are certain connective features which are common throughout the bands, at least up to the mid-LF band. This commonality stems from the confinement of the radio wave energy essentially within the space between the earth and the ionosphere. This region is often called the earth-ionosphere waveguide and an understanding of the propagation mechanisms for the longwave frequencies is aided by reference to the well-known microwave waveguide. The frequency-dependent properties of waveguide cutoff, dispersion, mode conversion due to discontinuities, signal attenuation due to lossy dielectric loading, etc. are all exhibited within the earth-ionosphere waveguide for the longwave regime.

Because longwave propagation is characterized by high stability in both amplitude and phase, and because these frequencies propagate to very great distances, the longwave frequencies are of considerable practical importance to communications, to navigation systems, and to global frequency and time comparisons. The longwave signals are not greatly affected by most naturally occurring ionospheric disturbances apart from polar cap events and radio communications and can usually be maintained under conditions that make communications very difficult at higher frequencies.

A review of longwave propagation characteristics is presented in the following sections. Some of the important computational methods that are available for predicting the phase and amplitude of longwaves are also discussed. Because computed results are very sensitive to the input waveguide boundary parameters, an

understanding of the electrical properties of the boundary regions of the earth-ionosphere waveguide is necessary to understanding the properties of longwave propagation. The boundary regions are also discussed in this report.

2.0 THE WAVEGUIDE BOUNDARIES

The lower boundary of the earth-ionosphere waveguide is, of course, the earth. It is generally characterized as homogeneous, isotropic and smooth, having an effective but variable conductivity and dielectric constant. The conductivity ranges from about 5 S/m, which is characteristic of sea water, to as little as 10^{-5} S/m, which is characteristic of large areas of the permafrost or ice, such as the Greenland ice cap. For conductivity this low, the skin depth can be so large that consideration must be given to the lower substrata (Westerlund, 1974).

The detailed geometrical characteristics of the surface are not very important for VLF waves because the principal electric vector is largely perpendicular to the surface. However, if the electric vector has an appreciable horizontal component, these characteristics can be quite important (Galejs, 1972b). Quite the opposite, however, is true concerning the conductivity (Westerlund, 1974).

The upper boundary of the VLF waveguide is considerably more complicated than the lower. The reflection of VLF waves from the upper boundary (the D region and lower E region) is generally accepted to be from about 70 km for daytime and about 90 km for nighttime. The reflecting medium is a slightly ionized plasma, rendered anisotropic by the geomagnetic field. Because of variable solar control, variations in electro-chemical processes and variations in thermal and mechanical influences, the medium is further rendered highly inhomogeneous.

The reflection process is quite a complicated process, of which all aspects have still not been fully explained. Both the density gradients and collision frequencies are known to play a major role (Budden, 1961b). Reflection at VLF does not occur at the altitude where the operating frequency is suitably related to the plasma frequency, as in HF ionospheric propagation, but is controlled to a significant extent by the ratio of the plasma to the collision frequency. It can be said, however, that the upper boundary of the earth-ionosphere waveguide is controlled by the ionization at heights below about 100 to 110 km.

The upper boundary is characterized by electron and ion density distributions and collision frequency height profiles. There are three procedures commonly used to determine ionospheric density profiles appropriate for longwave calculations. One method is to use profiles analytically defined with arbitrary coefficients to calculate signal strengths corresponding to observed signal strength data and to then use numerical methods to find the "best fit" coefficients for the profiles. In another method, a compiled list of observed (i.e., experimentally measured) profiles is searched for a profile that best corresponds to the requisite geophysical conditions. A third method uses an analytically specified profile wherein the coefficients defining the profile are determined parametric in geophysical conditions through an appropriate regression analysis from the measured profile data base. Note that both of the latter two methods are independent of longwave propagation characteristics, whereas the first method is designed to specifically consider the propagation characteristics.

2.1 The Lower Ionosphere

This section is concerned with the observed (measured) characteristics of the ionospheric boundary (the D and E regions). Because this region is not easily studied by ground-based probing due to poor height resolution or to insufficient sensitivity, our present knowledge of the lower ionosphere is based in large part on rocket-borne ion mass spectrometer observations coupled with improved measurements of electron concentrations carried out since 1963. The electron density-height gradient, which is particularly important, is not measured accurately by ground-based radiowave probes. Electron concentrations less than about 1000 cm^{-3} are particularly important within the longwave regime but are not very accurately measured. The chemically active minor neutral constituents, and the relatively high ambient pressures, which permit many-body interactions, provide for the unique character of the lower ionosphere.

The lower ionosphere consists mainly of ions of the gases nitrogen, oxygen, hydrogen, and their compounds (Bowhill, 1975). Ions in the D and E regions of the ionosphere are primarily molecular and it happens that the lifetime for molecular ionization within these regions is from a few seconds to a few minutes. Therefore, the solar control of the D and E layers is very strong, causing them to disappear almost completely at night. In fact, much of the residual ionization in the E region consists of atomic ions of metallic debris from meteors. In the E region, the chemistry of the metallic ions is poorly understood, particularly in regard to their disappearance below 90 km, perhaps by oxidation.

It would be nice to be able to report that the D and lower E regions are well understood. This is not, however, the case. It has been asserted, that many of the most interesting unsolved problems of the ionosphere are related to the D region (Bowhill, 1975). In a recent review of the D region, Thomas (1974) pointed out, that although there have been advances in theoretical models of the D region in recent years, there has not been a significant improvement in our understanding of the aeronomical processes operating.

Ionization below 100 km is produced by electromagnetic and corpuscular radiation that bombards the earth from a variety of sources. The principal ionization sources for the quiet-time lower ionosphere are considered to be galactic cosmic rays, solar x-rays, and solar Lyman- α , both direct and scattered. An additional source only recently recognized and not yet universally accepted involves precipitation of energetic electrons.

The photoionization of nitric oxide (NO) by solar Lyman- α constitutes the major source of ionization during the daytime between approximately 65- and 90-km altitude. Soft x-rays (31-100 Å) dominate above 90 km and galactic cosmic rays are most important below 65 km. At nighttime, ionization is usually assumed to be produced primarily by Lyman- α in the nightglow drizzle of energetic particles continually raining down onto the mid-latitude atmosphere. According to Potemra and Zmuda (1970), these electrons are the dominant nighttime ionization source above about 80 km. Additional sources of nighttime ionization have been proposed, including cosmic x-rays and solar ultraviolet ionization of metastable oxygen. These have, however, been discounted or demonstrated to be only of secondary importance.

The immediate effect of these ionization sources is the production of free electrons and positive ions of atomic or molecular size. The electrons can combine with neutral particles, forming negative ions. Positive ions and electrons, as well as positive and negative ions, can recombine forming neutrals. Balance equations describing these processes can be formulated and solved. However, the coefficients involved in such balance equations are usually quite uncertain. Additional chemical and physical processes, such as the presence of ions of larger sizes than single molecules, further complicate the balance, resulting in the frequent use of nominal working models of the ionosphere. Such models range from a homogeneous conductor sharply bounded at an assumed reflection height, to an electron only (positive ions assumed to be infinitely massive) exponentially varying ionosphere, to a lumped parameter model involving electrons together with positive and negative ions of a single mass.

Figure 1 shows a typical daytime profile and Figure 2 shows a nighttime profile. Such density profiles are typically characterized by a series of regions or layers:

1. A region above approximately 85 km, the base of the E layer, which is the simplest part of the region.
2. A region where the electron density increases rapidly, the so-called "ledge" region around 82-85 km.
3. A region between the ledge and 70 km, the D layer.
4. A region between 50-70 km, the so-called C layer, where negative ions become appreciable.
5. The region below 50 km where there are no electrons under normal conditions.

The presence of the C layer is most often inferred from ground-based radio wave measurements although its existence is well accepted today (Belrose, 1982).

Also illustrated in the figures are some exponential gradient profiles obtained via the first method discussed in the previous section, that is, by "best fit" determination of profile coefficients using radio propagation data. These profiles are discussed, along with the somewhat limited VLF/LF propagation data bases used to obtain them, by Ferguson (1980) and Morfitt (1977).

In general, there are no marked differences between profiles measured at equatorial and at middle latitudes. High latitude profiles, however, exhibit distinct characteristics which are attributable to the polar geomagnetic field effects on solar particles.

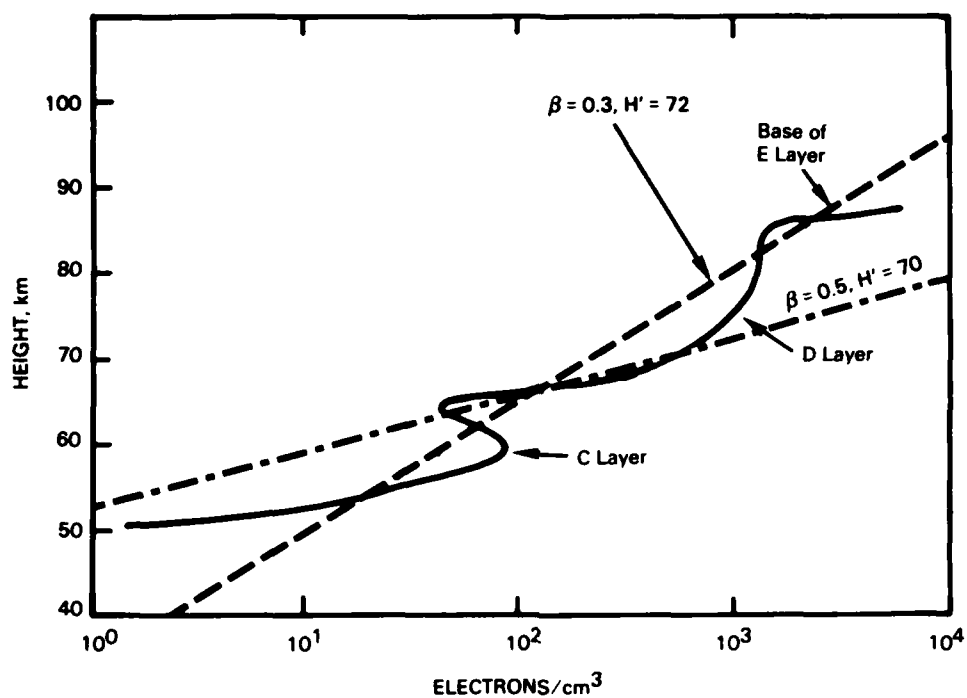


Figure 1. Daytime electron density profiles and collision frequency profiles.

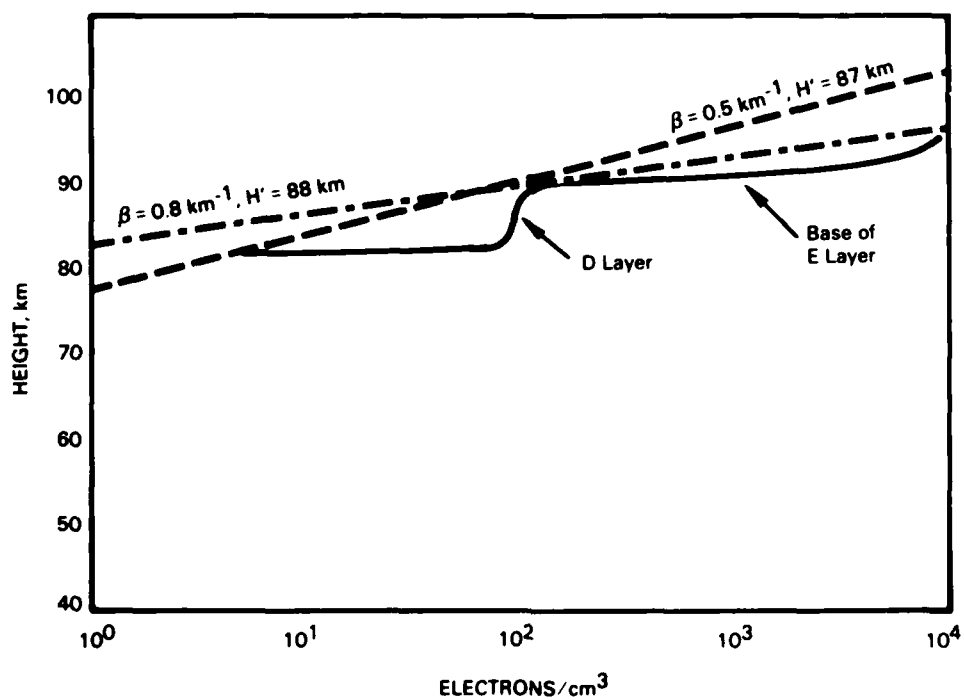


Figure 2. Nighttime electron density profiles and collision frequency profiles.

In addition to the normal ionosphere, there are also a number of lower ionospheric disturbances that can affect longwave propagation. There is no doubt that changes in lower ionosphere ionization follow geomagnetic storms (Belrose, 1982). There are also storm "after effects" which have a seasonal variation with the winter season more strongly affected. The most important ionospheric disturbances are listed below:

1. Auroral events.

These events are caused by precipitation of energetic electrons and cover regions of several hundred kilometers in extent and occur most frequently in the auroral zone during nighttime. Rapid variations in propagation result.

2. Polar Cap Absorption (PCA) events.

These events are caused by increased precipitation of solar protons and occur over both polar regions simultaneously and may extend over the polar caps down to 50-60 degrees latitude. The frequency of occurrence varies from a few events per year during sunspot minimum conditions to several per month near sunspot maximum. A single event may last from 1 to 10 days, with the greatest effects on radio waves during daytime.

3. Relativistic Electron Precipitation (REP) events.

These events are caused by electrons with energies above about 100 keV. They occur most frequently at subauroral latitudes in the early morning hours and cause excess ionization at very low heights in the D region, with rapid onset and a slow recovery during the course of a few hours.

4. Sudden Ionospheric Disturbances (SID).

These result from a sudden burst of solar x-rays and ultraviolet radiation that cause excess ionization and occur simultaneously over the entire sunlit hemisphere during solar "flares." Normal duration is from 30 minutes to several hours.

3.0 THE VLF/LF REGIME

We begin the discussion of longwave propagation with VLF and LF. This portion of the longwave frequency band richly exhibits, in one way or another, all of the characteristics of the total band. For convenience, we will use the term VLF to denote the entire VLF band as well as the lower portion of the LF band which exhibits the longwave propagation characteristics.

The propagation of VLF radio waves has been of interest since the beginning days of radio. Marconi's experiments at the beginning of the century were made using what is now called the LF band. There are many practical disadvantages to using this frequency band (Watt, 1967), which include the very high atmospheric noise level, necessitating high power transmissions, the limited bandwidth available, and the high cost and low efficiency of transmitting antennas. These disadvantages did not exist at shorter wavelengths and the development of shortwave radio in the 1920s led to the near demise of interest in the longer wavelengths.

For a variety of reasons, interest in VLF radiowave propagation was revived and has greatly increased during the last two decades. Near the earth's surface, the structure of the VLF electromagnetic field depends on the properties not only of the earth but also of the lower regions of the ionosphere between 50 to 100 km, the D region and lower E region. VLF signals are not greatly affected by most ionospheric disturbances, apart from polar cap events (Al'pert and Fligel, 1970). VLF signals exhibit low attenuation (on the order of a few dB per 1000 km), and this frequency band is widely used in low data rate, global communication systems. The phase of VLF transmission is highly stable and undergoes nearly reproducible diurnal variations. It has been found, for example (Belrose, 1968), that the stability of VLF transmissions is sufficient to permit frequency comparisons to within a few parts in 10^{12} which is a few powers of ten better than is possible at high frequencies. This phase stability leads to applications in navigational systems, frequency comparisons, and clock synchronization or timing systems.

For short distances, less than 1000 km, the so-called ground wave from a VLF antenna can usually exceed any energy received from ionospheric reflections. For greater distances, however, this is not true, and the influence of both the ground and the ionosphere on the total field must be considered. This section addresses the propagation of VLF radiowaves to great distances. VLF propagation to great distances has been considered in a large number of theoretical papers, using a variety of formulations. We will not attempt to review this massive literature but will concentrate on some general ideas that have been incorporated into the various formulations.

3.1 Theoretical Developments

One of the major formulations for consideration of VLF propagation to great distances, the VLF wave-hop theory, has been presented by Berry and Chrisman (1965), Berry et al. (1969), and Berry and Herman (1971). In this formulation, the solution for propagation of VLF waves between a spherical earth and a concentric ionosphere is developed in a rigorous complex integral representation and the integrand

is then expanded in terms of a geometrical-type series. With the order of summation and integration exchanged, evaluation of the complex integrals using saddle point approximations leads to the identification of the series as the ray-hop series of geometric optics. The complex integrals are thus called wave-hops. For distances near or beyond the caustic, numerical integration or application of residue theory must be employed to evaluate the integrals. The wave-hop formulation requires both azimuthal and longitudinal invariance of the earth and ionosphere, a situation which can exist only for short ranges.

Johler (1970) has presented a theory for radiowave propagation from VLF to MF in terms of spherical wave functions. As with the wave-hop formulation, the spherical wave formulation is not applicable to consideration of variations of the earth or ionosphere in either the longitudinal or azimuthal directions. The spherical wave function formulation has not been widely used because of its complexity and difficulty of numerical implementation.

VLF propagation to great distances can be conveniently represented in terms of waveguide mode propagation, where the finitely conducting curved earth and the anisotropic, imperfectly conducting curved ionosphere with dipping magnetic field form the boundaries of a waveguide. The basic idea that the earth and lower ionosphere form a waveguide actually goes back to the work of Watson (1918, 1919), who postulated that the Kennelly-Heaviside layer could be represented as a conducting shell concentric with the spherical conducting earth.

The theory of VLF waveguide mode propagation has been presented in the literature in two different formulations. One formulation, based on the pioneering work of Watson (1919) and more recently in the theory given by Schumann (1952, 1954) and developed extensively by Wait (1970), treats the problem in spherical coordinates wherein the fields are expanded in terms of azimuthal, longitudinal, and radial functions. In a sequence of simplifying approximations, including the assumption of azimuthal and longitudinal independence of both ground and ionospheric properties, Wait (1970) developed a modal (eigenvalue) equation. The solution of the eigenvalue equation yields the propagation characteristics of the VLF modes in the earth-ionosphere waveguide, and consideration of orthogonality properties of the radial functions yields excitation factors of the modes. The excitation factor is a quantity that gives the amplitude of the wave excited in a given mode by a given source.

Another simplifying approximation involves the replacement of the inhomogeneous ionosphere with an impedance boundary placed at the height where the bulk of the VLF energy is assumed to be reflected. A similar impedance boundary replacement is also made for the earth. The impedance boundary formulation permits specification of the ratio of an electric field component to a magnetic field component at the boundary without further consideration of the region beyond the boundary. The replacement of an inhomogeneous, isotropic ionosphere by an "equivalent" impedance boundary is a straightforward procedure. Such a replacement for an anisotropic ionosphere is greatly complicated by the coupling of transverse magnetic and transverse electric polarizations. Boundary conditions must then be formulated in terms of an impedance matrix. Unfortunately, the complication is sufficient to render many of Wait's results inapplicable to the highly anisotropic nighttime ionosphere and

to limit their application to the daytime ionosphere, for which the effect of anisotropy is slight. The theory has been significantly modified and extended to anisotropic ionospheres by Galejs (1972b).

Another formulation of the earth-ionosphere waveguide problem has been presented by Budden (1961a). In this formulation, a modal equation is developed in terms of reflection coefficients of the ionosphere and the earth. Formulation in terms of reflection coefficients permits determination of propagation characteristics of waveguide modes for essentially any ionospheric or ground conditions, limited only by the ability to compute reflection coefficients for the environment considered. Budden's formulation is essentially planar, although he does point out how to include earth curvature in the direction of propagation in an approximate way by modifying the refractive index in the space between the earth and the ionosphere (Budden, 1962). A more rigorous technique was introduced by Richter (1966) and has been employed in VLF propagation studies by Pappert (1968) and Abbas et al. (1971). This technique assumes cylindrical stratification and effects a conformal transformation from cylindrical to Cartesian coordinates. Modal excitation values are determined in Budden's formulation by applying the residue theory rather than by using the modal orthogonality properties of conventional waveguide theory employed by Wait.

Whichever formulation is used at the outset, the use of waveguide mode theory leads to specification of the VLF field as a summation of terms called modes. Because many VLF transmitters in common use radiate a vertically polarized field, only the radial or vertical component of the electric field is usually considered. For a time-harmonic source, the VLF modesum for the vertical electric field may be written as

$$E_v(d) = \frac{K(P,f)}{[\sin(d/a)]^{1/2}} \sum A_n G_n(T) G_n(R) e^{-ikS_n d} \quad (1)$$

where $K(P,f)$ is a complex constant dependent on transmitted power (P) and frequency (f), d is the distance from the transmitter on a homogeneous, smooth earth of radius a , A_n is the excitation factor for mode n , normalized to unity for flat earth, perfectly conducting boundaries, k is the free space wave number, S_n is the propagation factor, and $G_n(T)$, $G_n(d)$ represent altitude dependent height-gain functions for mode n , normalized to unity at the ground. One height-gain function is needed for the transmitter location (T) and one for the path point location (d). Generally, both A_n and S_n are complex. The real part of S_n determines the distance dependence of the phase for a mode while the imaginary part of S_n determines the attenuation rate. Thus

$$\begin{aligned} \alpha_n &= -k\text{Im}(S_n) \\ V_n/c &= 1/\text{Re}(S_n) \end{aligned} \quad (2)$$

where α_n is the modal attenuation rate, V_n is the phase velocity, and c is the speed of light in free space.

Numerous assumptions and approximations are implicit in equation (1). It is assumed that the propagation environment is homogeneous in all but the vertical direction. Thus, the propagation factor, S_n , is independent of position. The influence of the curvature of the earth transverse to the propagation direction is approximately accounted for in the spreading term, $\sin(d/a)$. The usual $1/r^{1/2}$ term for a cylindrical wave is replaced by the $1/[\sin(r/a)]^{1/2}$ term. A careful derivation of the mode equation in spherical coordinates, such as that presented by Wait (1970) and discussed previously, yields the $\sin(d/a)$ term as a consequence of an asymptotic approximation to a Legendre function. The mode sum in equation (1) further assumes that "round the world" signals are absent, an approximation not valid when receiver locations are near the antipode of the transmitter — multipath focusing then occurs.

If significant variations occur along the propagation path, which is often the case in the earth-ionosphere waveguide, then homogeneous guide formulations require modification. For such cases, there are two techniques for mode summation that are commonly used. One employs a WKB, or "phase integral" approach, and the other uses mode conversion. In the WKB approach, an eigenmode is assumed to be uniquely and independently identifiable anywhere along a propagation path. Furthermore, each mode is assumed to depend only on the local characteristics of the waveguide and to propagate independently of the existence of any other modes. Only two modifications to equation (1) are required in order to effect a WKB modesum. One is the replacement of the single excitation factor for the transmitter by the geometrical mean of the respective excitation factors for the transmitter and the local point on the propagation path. Thus $A_n(T)$ becomes $[A_n(T)A_n(d)]^{1/2}$ where (d) denotes the local path point. The other modification is the replacement of the product of the propagation term, S_n , and the distance, d , by the integral of the propagation term from the source to the current path point. This integral is often called a "phase integral" although the result of the integration accumulates attenuation as well as phase.

In the mode conversion method, eigenmodes are assumed to be uniquely and independently identified only in a local sense. Even though the eigen characteristics of a mode depends only on local characteristics of the waveguide, the propagation of each mode depends not only on the existence of other modes but also on the past and future propagation history of itself and the other modes. Mode conversion is simple in concept, although complicated for computer implementation. An inhomogeneous propagation path is divided into many short homogeneous sections, and both forward and backward propagating modes are calculated for each section. These modes are then combined in specific proportion so as to provide the correct electromagnetic boundary "match" at the junction of each section. One difficulty encountered when using mode conversion, which arises from the necessity of knowing the future propagation history of each mode, is alleviated by ignoring the backward propagating modes within each short section. This is tantamount to ignoring reflections at each junction between sections, a very reasonable approximation.

Extensive results of numerical calculations of VLF waveguide mode constants have been presented in the literature. Early results given by Wait and Spies (1964) show some general properties of VLF modes for typical daytime and nighttime

ionospheres, with various ground conductivities and for east-to-west or west-to-east propagation directions at the magnetic equator or when the earth's magnetic field is ignored. In general, the computed attenuation rate of the lowest order mode is less at night than during the day, and is less for west-to-east propagation than for east-to-west. The attenuation rate increases as ground conductivity decreases — rapidly at low conductivities such as occur for the polar icecaps. Each successive mode order suffers a greater attenuation rate than the previous order mode for both day and night.

Typically, computed daytime attenuation rates for the first order mode range from 2 to 4 dB/1000 km over the VLF band for sea water ground conditions (essentially equivalent to a perfect conductor at VLF). The nighttime attenuation rates range from about 0.5 to 2 dB/1000 km over the VLF band for the same ground conductivities.

The results of Wait and Spies calculations show that the computed modal phase velocities for the curved earth can be less than the free space speed of light at intermediate to higher VLF frequencies but increase markedly at lower frequencies. Moreover, phase velocities increase at low ionosphere heights (daytime) compared to higher heights (nighttime), and a decrease in ground conductivity produces a decrease in phase velocity. An increase in phase velocity with mode number is also indicated.

The modal excitation factors and height gains given by Wait and Spies show little dependence on the terrestrial magnetic field. At higher frequencies for isotropic conditions, the excitation factor for the first mode is significantly reduced in comparison with that for a flat earth case. The second mode exhibits a much smaller reduction at higher frequencies. Generally, lower ground conductivities produce an increase in excitation factor. The excitation factor for the second order mode is significantly higher than that for the first order mode, especially at higher frequencies and at night. At night the second order mode may be dominant at the ground to distances of several thousand kilometers. For the first mode the effect of height-gain is to cause the dominance of the second mode to be greatly diminished or even reversed. This height-gain increase with height for the first order mode has been called the whispering gallery effect by Budden and Martin (1962) or the earth-detached mode by Wait and Spies (1963). It involves successive reflections from the ionosphere with essentially no intermediate bounce at the ground.

The modal constants characterized above become considerably more complicated with full consideration of the geomagnetic field. From the earliest days of VLF, experimental investigation provided evidence of an apparent violation of the reciprocity principle. Round et al. (1925), for example, found that VLF radio waves apparently suffered significantly larger attenuation for propagation in a generally easterly direction than in the reverse direction. Some very early results presented by Wait (1961) for sharply bounded model ionospheres suggested that simple harmonic functions may adequately describe the azimuthal dependence of mode constants under daytime conditions. Further, a numerical modeling study by Ferguson (1968) showed that for daytime conditions, only the horizontal component of the geomagnetic field which is transverse to the direction of propagation is important. This component of the magnetic field is the source of nonreciprocity, and the remaining components were

commonly assumed to alter the propagation characteristics only slightly (Makarov et al., 1970). The most severe geomagnetic field effects were thus expected in the equatorial region for propagation across the magnetic meridians.

Perhaps the first indications of extreme complexity in modal behavior due to the geomagnetic field are found in the numerical modeling results of Snyder (1968a, b) and Pappert (1968). Snyder's (1968a) results indicated that a mode with lowest west-to-east phase velocity evolved as a result of continuous variation in magnetic azimuth into a mode in the reverse direction with only second lowest phase velocity. Thus, numbering modes in terms of increasing phase velocity, as is commonly done, results in the evolution of the first order west-to-east mode into the second order mode in the opposite direction, and vice versa. Further analysis (Snyder, 1968b) indicated that for a certain inclination of a dipping geomagnetic field and at a specific (near north-south) azimuth, the two modes considered above became degenerate, forming a single mode. For inclinations more nearly horizontal than the degenerate conditions, mode numbering inconsistencies resulted, but for more nearly vertical inclinations, inconsistencies in mode numbering vanished.

Pappert (1968) found that, at the high frequency end of the VLF band, modes which are principally transverse electric (TE) may be of importance in a vertical electric field mode sum. Note that a TE mode can be excited by a vertical dipole only because of the presence of the geomagnetic field. Snyder and Pappert (1969) extended the analysis to consider both easterly and westerly midlatitude nighttime propagation throughout the VLF band as well as to include azimuthal dependencies of mode parameters for a central VLF frequency at both middle and equatorial latitudes. It was found that the importance of principally TE modes is much more pronounced for westerly propagation at midlatitudes than for easterly and that the influence of the principally TE modes on the mode sum can be significant at frequencies as low as 20 kHz or less for westerly propagation. Azimuthal anomalies included drastic polarization changes in going from easterly to westerly paths. For transverse propagation at the magnetic equator, it was shown that modes that are pure transverse magnetic (TM) for propagation to the east may be pure TE for propagation to the west. This is tantamount to the statement that modes that have dominant excitation for easterly propagation may have vanishingly small excitation for westerly propagation. Azimuthal dependencies are characterized by rapid variation of the mode constants in the neighborhood of north-south or south-north propagation, for both equatorial and middle latitudes. These variations manifest themselves in marked differences in mode sums for azimuthal changes as small as 10 degrees or less. Further, the maximum total signal attenuation occurs for north-south propagation rather than for east-west propagation.

3.2 Observed Characteristics

It is well known that many propagating modes may be supported in metallic waveguides of sufficiently large cross section compared to a wavelength. In such a guide each mode has a different phase velocity, producing interference phenomena as the various modes go in and out of phase at varying distances along the guide. Furthermore, changes in the cross section of the guide are known to produce higher order modes which result in a modified modal interference pattern.

The earth-ionosphere waveguide is four to ten wavelengths in height at typical VLF wavelengths. Higher order modes can be excited in the VLF waveguide with greater amplitude than the lowest order mode even though the lowest order mode suffers less attenuation. Thus, modal interference as a function of distance from a VLF transmitter is to be expected just as for a metallic guide. Such is indeed the case. For example, VLF field amplitudes were recorded by Rhoads and Garner (1967) aboard an airplane flying between Hawaii and the west coast of the United States. Comparison of their daytime measured amplitudes with results computed using the daytime mode constants of Wait and Spies (1964) showed almost ideal agreement. Rhoads and Garner (1967) showed that under day conditions the effect of the earth's magnetic field on the VLF modal constants can be ignored, at least for distances less than 4000 km. They also confirmed that at distances up to at least 3000 km higher order modes must be considered. The good agreement obtained for daytime data could not be obtained for the night data.

A manifestation of multimode propagation is found in the phenomena of sunrise and sunset fading. This fading is characterized by periodic and repeatable variations in amplitude and phase as the dawn/dusk terminator moves along a VLF propagation path. This fading, typically most pronounced at sunrise on easterly paths and at higher frequencies, was first studied by Yokoyama and Tanimura (1933) who attempted, without success, to explain their observations in terms of a ray-optical propagation model. Another attempt, also not successful, to explain the fading using ray-optical concepts was presented by Rieker (1963). It remained for Crombie (1964) to explain the phenomena using waveguide mode concepts. Of all possible combinations of conditions — sunrise and sunset, position of terminator on propagation path, relative positions of transmitter and receiver, etc. — only two fundamental situations occur within the confines of Crombie's explanation.

The first case depicts the field incident on the terminator from the transmitter as determined by two modes and the field at the receiver determined by only one mode. This situation would be characteristic of a transmitter on the night side of the terminator and the receiver on the day side sufficiently removed to avoid multimode propagation. The received signal then depends on the amplitude and phase relationships of the modes incident on the terminator and on the efficiency with which these modes are converted by the terminator to the single mode that reaches the receiver. Because the modal relationships are determined by the distance of the terminator from the transmitter, periodic variations will occur in the received signal as the terminator moves along the propagation path. The periods will be determined by the difference in the phase velocities of the two modes entering the terminator, and the variations must occur simultaneously for all points in the portion of the propagation path beyond the terminator region. The fading would be most intense when the two entering modes have the most pronounced interference usually when the terminator is near the transmitter.

The second situation depicts the incident field as a single mode. The presence of the terminator leads to multimode generation in the region past the terminator, which in turn leads to modal interference. In this situation, the interference pattern would appear to be attached to the terminator and to move synchronously with it. Signal minima are then to be expected at fixed distances from the terminator, the

distance between fades being determined by the difference between the phase velocities of the two modes leaving the terminator region. The deepest signal fades would be expected as the terminator approaches the receiver. This situation corresponds to propagation from the sunlit side of the terminator into the night side. Note that in both situations, the fade spacing would be determined by the differences between the phase velocities of two nighttime modes. A thorough experimental examination of the Crombie model was conducted by Walker (1965), who demonstrated that the model was correct in all essential details.

The observations just discussed, which were made at middle or low latitudes, have been explained adequately by the use of quite simple VLF waveguide mode theory. However, some observations of VLF transmissions over long paths across the geomagnetic equator have apparently indicated an anomalous effect associated with nighttime or dawn/dusk transitional VLF propagation on such paths.

Chilton et al. (1964) made simultaneous observations of NBA transmissions (Canal Zone, Panama) at 18 kHz for paths to both the northern and southern hemispheres. Both paths were of similar length, but the southern hemisphere path crossed the magnetic equator. They observed an anomalous difference in the diurnal changes of signal amplitude and phase and suggested that their observations resulted from a difference in ionization profile due to latitudinal dependence of cosmic rays. More recently, Chilton and Crary (1971) suggested that the latitude-dependent ionization source might arise from the recently discovered x-ray stars.

Lynn (1967) reported on VLF transmissions at 18.6 kHz from NLK (Jim Creek, Washington) to Smithfield, South Australia, a propagation path approximately 13,500 km long, with the receiver-to-geomagnetic-equator distance along the path of approximately 5700 km. The direction of propagation is essentially southwesterly. The interference distance reported by Lynn for sunrise transition fading was approximately 2000 km for the terminator located in midlatitudes (in excess of 20 degrees from the geomagnetic equator). Whenever sunrise transition fading was observed while the terminator was within 20 degrees of the geomagnetic equator, the interference distance increased to as much as 3700 km., and when averaged over the anomaly, it had a value of 2900 km. Lynn (1967) concluded from his observations that the change in interference distance resulted from a change in the difference of phase velocity for two modes as well as from a change in the relative phases of the appropriate mode conversion coefficients. He left unanswered the question of a possible cause for these changes. It is interesting to note that Lynn's conclusions are dependent on his selection of both a VLF propagation model and a mode conversion model. Lynn (1967) assumed an isotropic propagation model wherein any changes in propagation parameters necessitates changes in ionospheric conditions.

Kaiser (1968) also examined transequatorial transition fading and his results are much the same as Lynn's. He concluded that anomalously large values of the sunrise interference distance are characteristic of generally east-to-west VLF propagation below about 30 degrees magnetic latitude. A further conclusion by Kaiser (1968) is that any anomaly in sunset transition fading for such paths is relatively minor. Kaiser suggests that a possible cause of the anomaly might be a larger and sharper day-night waveguide transition near the geomagnetic equator as compared to higher

latitudes. He does not suggest, however, a possible cause for the latitudinal dependence of the day-night waveguide transition.

Additional observations of this transition fading anomaly have been reported by Lynn (1969, 1970). These observations further substantiate the general characteristics of the anomaly. A theoretical interpretation of the transequatorial anomaly was presented by Lynn (1970). His interpretation is based only on assumed latitudinal variations in modal phase velocities and relative phases of the mode conversion coefficients. He does not suggest a possible cause for the variations.

Meara (1973) has extended Lynn's (1970) analysis to include determination of the changes in phase velocities within the equatorial anomaly region. He concludes that the phase velocity of the first mode is essentially unaltered by propagation through the equatorial region, whereas that of the second mode is reduced. Note that Meara's results are entirely dependent on the assumed propagation and mode conversion models. Once again, no explanation of a possible cause for the variations in phase velocities is given.

A further application of Lynn's (1970) analysis to explain some anomalous diurnal changes in transequatorial VLF propagation has been made by Araki (1973). The analysis follows essentially that previously discussed. Araki does go a step further than Lynn (1970) and Meara (1973), however, in that he assumes the validity of the isotropic propagation model and asserts that any change in propagation parameters results from a modification of the nighttime equatorial ionosphere. Lynn (1978) has pointed out that the propagation paths of Chilton et al. (1964) and Araki (1973) have a west-east component rather than an east-west component as have the other paths discussed here. He further claims that only a minor azimuthal variation in relative phase velocities of interfering nighttime modes can account for the observations of Chilton et al. (1964) and Araki (1973). Lynn (1978) thus concludes that the observations of Chilton and Crary (1971) cannot be seen as direct evidence for the control of night VLF reflection heights by stellar x-ray sources. But, Svennesson and Westerlund (1979), using waveguide mode theory combined with nighttime profiles given by ionospheric theory, claim results that support the reports of x-ray star effects on VLF radio signals.

Snyder (1981) addressed the question of transition fading on long transequatorial propagation paths. He found that the transition fading reported in the literature as anomalous is completely explained by correctly allowing for variations of the geomagnetic field along the path. He further concluded that careful application of modal techniques appears to be capable of describing all LF and VLF propagation in the earth-ionosphere waveguide, provided the effect of the earth's magnetic field is properly included at night.

Long path VLF propagation can be severely modified by ionospheric disturbances caused by solar flares, solar proton events, and magnetic storms. Solar flares normally cause a sudden increase in phase velocity (or a decrease in phase lag), and an increase in signal level for frequencies above about 16 kHz when the path is sunlit. The effects usually last from some tens of minutes to an hour or so, with the onset being much more rapid than the recovery. Polar cap disturbances are

caused by extra ionization in the D region resulting from protons emitted by the sun and trapped in the geomagnetic field. Such events affect the entire polar cap (northern and southern) down to a magnetic latitude of 50 degrees, as far as VLF is concerned. The resultant VLF signals display a phase lag and a signal decrease, with the effects much enhanced for paths crossing the ice caps (Greenland and Antarctica). Magnetic storms, which are due to precipitation of energetic electrons into the lower ionosphere produce perturbations in the VLF signals much more localized in space and time than are produced by solar flares or polar cap events. The storm effects are most evident at night and usually produce perturbations that exhibit quasi-periods of the order of a few tens of minutes.

4.0 THE ELF REGIME

The free space radio wavelengths in the ELF band are very large, ranging from 100 km to 30,000 km. Thus, the entire altitude region from the surface of the earth to beyond the F region of the ionosphere would seem to be important. However, within the upper ionosphere, the magnitude of refractive index at ELF frequencies becomes very large so that the effective wavelengths are much diminished, being no more than a few tens of kilometers. Table 1 shows some properties of ELF waves within the earth-ionosphere waveguide. As a comparison, characteristics for the sea are included.

Table 1. Some properties of ELF waves.

Frequency (Hz)	Wavelength (km)			Refractive Index		
	Ionosphere	Air	Sea	Ionosphere	Air	Sea
10	—	30,000	0.5	—	1	85,000
100	10-100	3,000	0.158	30-300	1	27,000
1000	—	300	0.05.	—	1	8,500

The electrical mismatch between the atmosphere and the ionosphere is very large at ELF. The transition altitude within which this occurs is very much less than the local wavelength, either within the atmosphere or the ionosphere, so there is little penetration of the ionosphere by the ELF waves. The lower ionospheric regions (D and E regions) are the more important. The ionosphere acts so much like a perfect reflector that the anisotropy due to the geomagnetic field is very slight. This is demonstrated both theoretically (Wait, 1970; Galejs, 1972b) and experimentally (Bannister, 1975; White and Willim, 1974).

Booker and L. J. (1977) and Greifinger and Greifinger (1978, 1979) have found there is an identifiable reflecting stratum for the return of E waves from the ionosphere. The stratum is located where the local wavelength has a certain specified relationship to the local scale-height (with respect to the squared refractive index). Further, the thickness is about one scale-height. They find that for daytime, reflections occur from the underside of the gradient of the D region and from the underside of the E region. For nighttime, they find that reflection from the gradient of the topside of the E region is also important. For the lowest part of the ELF band, some reflection also occurs from the F region at night, although these are often ignored.

The efficiency of launching ELF energy into the earth-ionosphere waveguide depends on the antenna configuration. The commonly used configuration is a long horizontal wire grounded at both ends. This antenna may be considered to act as a power transformer (Booker, 1973), with the primary loop as the antenna itself and the return current through the ground. The secondary loop is formed by currents flowing within the ionosphere above the antenna with return currents through the air and ground. In the primary loop, the return current flows at an effective depth of $\delta/(2)^{1/2}$ where δ is the skin depth. A larger primary loop results from a larger skin depth, which in turn results from a poorer ground conductivity.

The appropriate model for this frequency regime corresponds to a waveguide well below cutoff frequency with propagation via a single mode. The electric and magnetic fields are nearly wholly transverse to the direction of propagation, with the electric field vertical and the magnetic field horizontal. Thus, this single mode can be termed quasi-TEM. Within the earth-ionosphere waveguide, attenuation at ELF for this quasi-TEM mode is on the order of 1 or 2 dB/Mm (Bannister, 1984). Because the surface impedance of the ground is typically much smaller than the surface impedance of the ionosphere, the attenuation within the guide is due mainly to the absorption by the ionosphere. Note that the impedance varies inversely as the conductivity and the effective conductivity of the ionosphere is typically 10^{-5} to 10^{-7} S/m while that of the ground is 10^{-4} to 5 S/m. The higher frequencies of this band approach the cutoff limit of the earth-ionosphere waveguide and propagation is limited to very short ranges due to the high attenuation.

There are certain effects on ELF propagation in the earth-ionosphere waveguide due to earth curvature. One of these is spherical focusing which results in an increased power flux density compared to a planar guide and is caused because the width of a wavefront in the spherical guide is smaller than if the guide were planar. Another effect results from the closure of the guide around the earth. Rather than radiating away as in a planar guide, the waves now return to the source point after going around the earth.

It is appropriate to say that propagation of ELF over great circle paths in a homogeneous ionosphere, which has been studied theoretically for years, is well understood (Wait, 1970; Galejs, 1972b; Burrows, 1978; Wait, (ed) 1974; Bernstein et al., 1974; Wait, 1977). What is not well understood, however, are the effects of propagation over non-great circle paths and the effects of inhomogeneous ionospheres caused by energetic particle precipitation, sporadic E, electron ledges, etc. Davis (1974) and Davis et al. (1974), pointed out several possible propagation irregularities. If ELF signals are to be properly interpreted, an understanding of these irregularities must be available. Table 2 lists the major possible irregularities in a convenient format.

Table 2. Some possible irregularities in ELF propagation.

BIDIRECTIONAL PROPAGATION

1. Geomagnetic Nonreciprocity
2. Day-Night Asymmetry
3. Transequatorial Paths

MULTILAYER RESONANCE EFFECTS

1. Anomalous High Attenuation
2. Dispersion

Table 2. Some possible irregularities in ELF propagation. (Continued)

IONOSPHERIC DISCONTINUITIES

1. Height Changes at Twilight Zone
2. Interposed Conduction Layers Such as Sporadic E
3. Solar and Nuclear Perturbations

HIGH LATITUDE PHENOMENA

1. Solar x-ray Flares
2. Solar Charge Particle Flux
3. PCA Events
4. Magnetic Storms

Bidirectional propagation is important because the low attenuation rate at ELF can result in energy from both the direct and antipodal paths (the so-called short and long paths) to propagate to much of the earth. This can result in a spatial interference pattern extended over thousands of kilometers with field fluctuations of 3 to 6 dB. Theoretical calculations (Galejs, 1970, 1972a) using realistic layered ionospheric structures have indicated propagation characterized by resonant absorption and strong dispersion. Ionospheric discontinuities which occur over distances comparable to a Fresnel zone can produce significant effects on ELF signals. The moving dawn/dusk terminator is one such discontinuity, although integration times on the order of an hour or more can obviate terminator effects (White and Willim, 1974; Bannister, 1974). Field and Joiner (1979, 1982) have derived expressions for the relative errors introduced by neglecting both widespread and bounded ionospheric inhomogeneities. Lateral diffraction, focusing, and reflection can cause the quasi-TEM mode to exhibit a transverse pattern of maxima and minima beyond the disturbance and a standing-wave pattern in front of it. Both tangential propagation across the polar cap and oblique incidence on the dawn/dusk terminator are examples. Interposed conducting layers of large extent (thousands of kilometers), such as produced by sporadic E, can be expected to cause signal perturbations when they occur on propagation paths at night, when the ELF fields penetrate well through the D region. This factor is particularly important because both equatorial and auroral zones are subject to frequent sporadic E. It should be noted, however, that measurements of sporadic E conditions have not been made in conjunction with ELF signal recordings. Thus, explanation of ELF signal fades in terms of absorption due to sporadic E cannot be considered conclusive.

The nighttime ionospheric D region is strongly influenced by energetic electron precipitation, which tends to increase ionization, making the ionosphere more "daylike" for ELF propagation by lowering the effective reflecting height and improving excitation. Thus, these precipitation events are expected to produce signal increases at night. The observed nighttime field strengths in fact decrease.

5.0 COMPUTER MODELS

Numerous computer programs have been written that are designed to provide calculated values of longwave field strengths in one form or another. Because many of these programs form subsets of others, or because many simply represent one working group's adaptation of the three basic program types developed elsewhere, an exhaustive list of these programs would be of little value. Rather, we will present the three basic program types, with reference to origination, and discuss their recognized strengths and limitations.

The first program type is based on the zonal harmonic (or spherical wave function) formulation. This formulation and the programs supporting the formulation are discussed by Johler and Berry (1962), Johler and Lewis (1969) and Johler (1970). The computational capability extends throughout the entire longwave regime although it is best suited for the lowest frequencies, i.e., for ELF. The formulation is as a summation of specific terms called zonal harmonics or spherical wave functions, which involve Legendre polynomials and spherical Bessel functions of integer order. Although the terms are quite simple in form, the number of terms needed in the summation increases rapidly with increasing frequency, to be on the order of several thousand terms at VLF and as much as 100,000 at LF. The model is basically limited to azimuthal and longitudinal homogeneity. Although recent discussions have indicated possible extension of this model to include inhomogeneities in the propagation direction (Jones and Mowforth, 1982), these extensions have not been fully developed, nor have they been compared with propagation data. They must still be considered as hypothetical.

Another program type is based on the wave-hop formulation. In this formulation, propagation is modeled as a summation of a ground wave plus ionospherically reflected sky waves. This formulation and programs employing this formulation are reported by Berry and Chrisman (1965), Berry et al. (1969) and by Berry and Herman (1971a,b). The formulation is basically attractive because it requires very little knowledge of propagation theory by the user, although it is strictly applicable only to propagation with horizontal homogeneity. Numerous reports have been made of extensions of the basic theory to include horizontal inhomogeneity. However, these extensions are based on geometrical interpretations of the homogeneous formulation, and have not been demonstrated with observational data to be correct. Early attempts to use the hop theory to explain propagation through the dawn/dusk terminator, a persistently occurring horizontal inhomogeneity in the real earth-ionosphere waveguide, met with complete failure (Yokoyama and Tanimura, 1933; Rieker, 1963). There is no reason to believe that implementation of similar formulations on modern, high-speed computers will be any more successful.

The third program form is based on the waveguide mode formulation. As the discussion in Section 3.0 points out, there are several sources for this formulation. There are, however, three basic programs that will be addressed.

The first mode program to be discussed, which is also the basic program from which nearly all currently used waveguide mode programs were derived, is commonly known today as the NOSC Waveguide Program. This program had its early

foundation in the work by Pappert et al. (1966). Some of the early programming efforts are reported by Gossard et al. (1966), Pappert and Sheddy (1967) and Sheddy et al. (1968). The program allows for ionospheres with arbitrary electron and ion density distributions and electron and ion collision frequencies with height. The lower boundary has arbitrary conductivity and dielectric constant. Arbitrary values of the strength and direction of the geomagnetic field may be used. The program is applicable for frequencies ranging from the lower ELF regime to at least the mid-LF regime. This frequency regime is the entire range called the longwave regime in this report.

Many advances have been made in the capabilities of this program (Ferguson, 1972a; Ferguson, 1973). Horizontal inhomogeneities along a propagation path may be taken into account by segmenting the path into sections as implemented by Ferguson and Snyder (1980). The combination of the waveguide program calculation capability with path segmentation capability is implemented in a computing capability known as the Integrated Prediction Program (IPP) (Snyder, 1969; Ferguson, 1970; Ferguson, 1972b). A major problem with use of the IPP has historically been that of obtaining a complete initial set of eigenvalues. A major advance in circumventing this problem is found in a NOSC computer code called MODESRCH (Morfitt and Shellman, 1976).

In typical application of the IPP to the VLF/LF regime, profiles are used which are defined to have exponential vertical gradients. The slope parameter and some reference height value are determined so as to provide a "best fit" of calculated field strengths to measured field strengths. The exponential gradient profiles obtained by this "best fit" method are discussed by Ferguson (1980) and Morfitt (1977). Morfitt et al. (1982) compared these profiles with profiles obtained in other ways.

Application of the IPP to the ELF band requires nonexponential profiles due to the deep penetration through the ionospheric D region and reflection from the E region. One particular ionospheric model used at ELF is known as the International Reference Ionosphere (IRI). Rawer et al. (1978). Behroozi-Toosi and Booker (1980) have applied a simplified ELF propagation theory (Booker, 1980) to a simplified version of the IRI model. The results of the union provide for the attenuation rate and phase velocity of the single waveguide mode at ELF. They do not, however, provide for a measure of the excitation factor for the mode.

The IPP is capable of treating waveguides that are reasonable model representations of the actual terrestrial waveguide. Modifications in any of the parameters describing the guide can be made quite easily and the program is capable of handling hypothetical conditions, such as might be expected for artificially induced conditions typical to nuclear stress. The program does not allow for variations of the guide perpendicular to the propagation path. Uniform conditions must, therefore, prevail for distances extending a Fresnel zone or more to either side of the propagation path.

Another mode program, which was developed by the Naval Research Laboratory (NRL), is called the Navy VLFACM Program (Hauser and Rhoads, 1981). This is a single mode propagation model wherein the characteristics of the mode are determined so as to provide a best fit to measured long path VLF signals. It is applicable only

to the VLF frequency range (due to the frequencies available in the measured data), and is intended to provide expected median signal values and standard deviations for the geographic, ionospheric and solar conditions prevalent during the period of VLF signal data acquisition. It is, therefore, not adaptable to alternative frequencies nor to specific ionospheric conditions.

A second single mode program has been developed at the Naval Ocean Systems Center and is called the NOSC Equivalent Single Mode (ESM) Program (Rothmuller, 1971, Ferguson and Snyder, 1971). The characteristics of the mode in this model were determined by using the IPP to compute multimode VLF field strengths vs distance parametrically in globally occurring geomagnetic conditions using the effective exponential profiles discussed above. An "equivalent single mode" attenuation rate and excitation factor was then determined by applying curve fitting techniques to the computed signal data.

As a means of comparison, signal amplitudes calculated using the IPP, the NOSC ESM and the Navy VLFACM are displayed in Figures 3a and 3b superimposed on measured data.

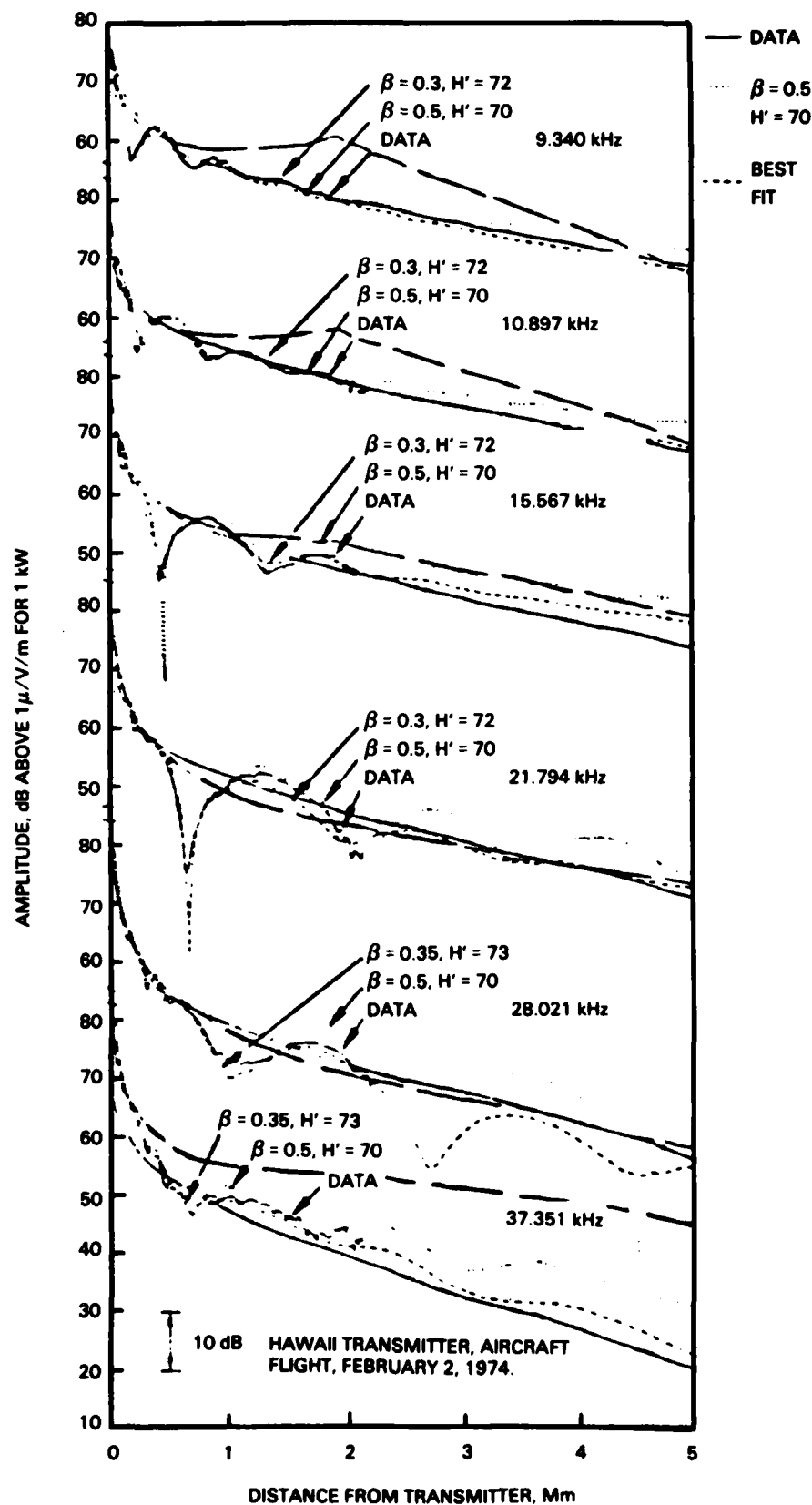


Figure 3a. Measured and computed daytime signal levels on the Hawaii to San Diego path.

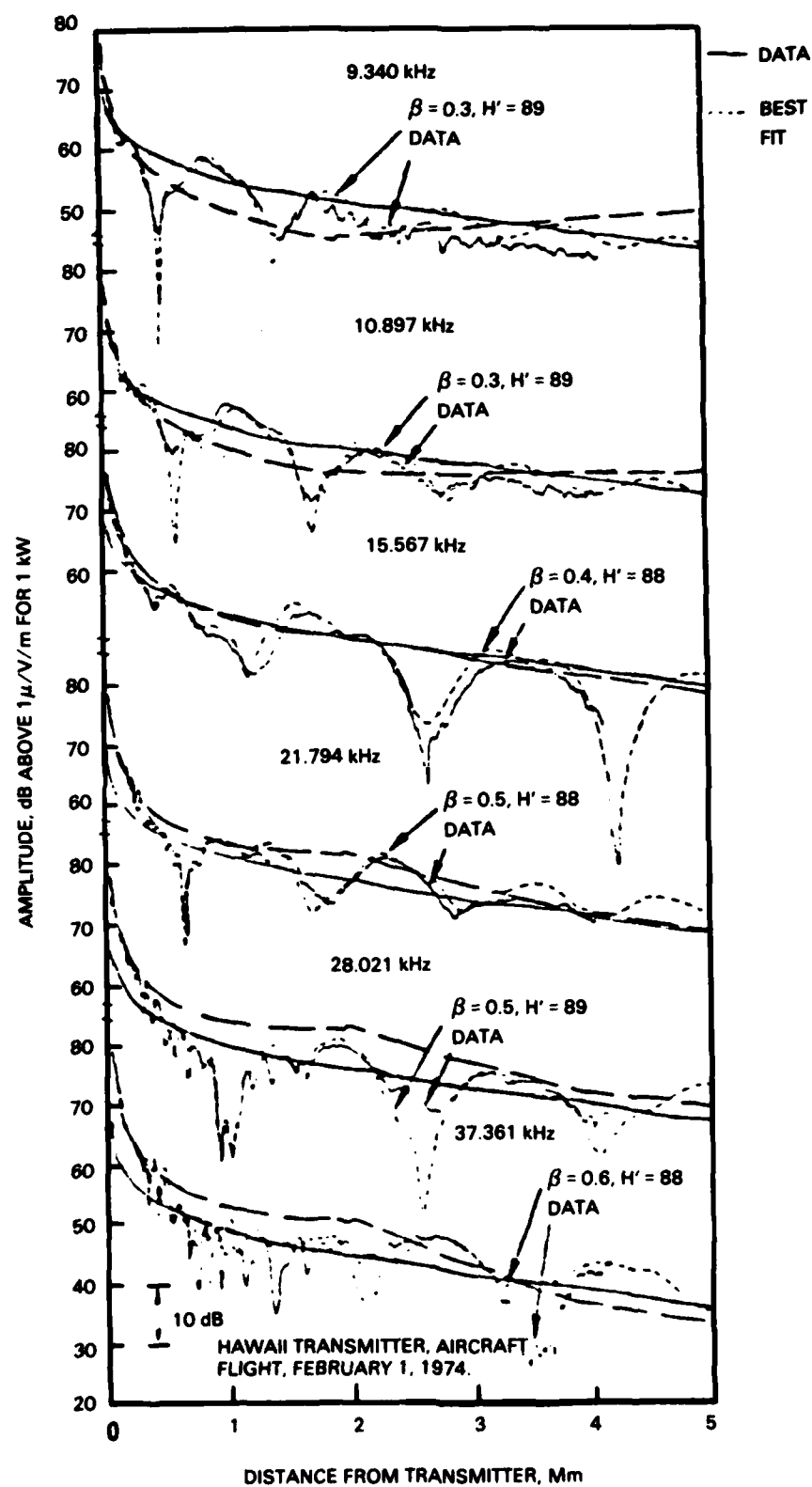


Figure 3b. Measured and computed nighttime signal levels on the Hawaii to San Diego path.

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