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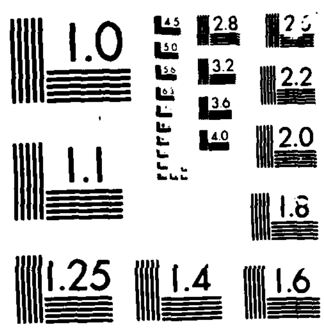

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TECHNICAL REPORT BRL-TR-2720

TRANSIENT HEAT CONDUCTION IN SEMI-INFINITE SOLIDS WITH TEMPERATURE DEPENDENT PROPERTIES

Lang-Mann Chang

March 1986

US ARMY BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Similarity solutions are obtained for the transient heat conduction in a semi-infinite solid with temperature-dependent thermal properties. The surface of the solid is considered subjected to a time-varying boundary condition, either heat flux $Q_w = at^b$ or temperature $T_w = T_o(1 + t^{b*})$. Solutions for the temperature distribution and the local heat flux are presented. In particular, type 420 stainless steel, whose thermal properties are strong functions of temperature, was chosen for analysis to demonstrate the significance of the

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20. ABSTRACT (continued)

effect of the temperature dependence of thermal properties on heat transfer calculations.

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I. INTRODUCTION

The transient heat conduction in semi-infinite solids is an important heat transfer problem. Typical examples are the heating by propellant gas of large caliber gun barrels, impingement heating on a ship deck during missile launching, and solar heating of the earth surface. Solutions to these problems are well known when constant thermal properties[1] are assumed. Some materials, such as type 420 stainless steel and type 4130 steel, however, possess thermal properties which are strong functions of temperature as shown in Figure 1.[2] Neglect of this fact may lead to significant errors in heat transfer calculations. This is especially the case when high heating rates are involved.

When the temperature dependence of thermal properties is accounted for, the heat equation becomes nonlinear and its exact solution is unattainable. In the relatively simple case in which the thermal conductivity is temperature-dependent and the surface temperature is constant, Schubert, et al.[3] obtained a similarity solution for their one-dimensional solid state model of the oceanic lithosphere and asthenosphere. However, solutions for a solid with both thermal conductivity and specific heat as functions of temperature and subjected to a time-varying boundary condition have not been reported.

In this study we consider a semi-infinite homogeneous solid with thermal properties which have a power law dependence on temperature. Two types of time-dependent boundary conditions are prescribed at the surface: a heat flux $Q_w = at^b$ and a temperature $T_w = T_o(1 + t^{b*})$. The power law representations are often useful in many engineering applications. Utilizing the method of similarity transformation via one-parameter groups, we transformed the nonlinear governing partial differential equation into an ordinary differential equation that can be integrated numerically by using any one of the several ordinary differential equation packages. This method is simple and, above all, the solution obtained in terms of similarity variables provides a broad representation. Such an advantage is usually not obtained when the governing partial differential equation is directly solved by numerical methods.

Using type 420 stainless steel properties for sample calculations, solutions obtained are the local temperature and local heat flux at any distance from the surface of the medium. In the limiting case of constant temperature or constant heat flux applied to the solid with constant thermal properties, a comparison is made with the available exact solution.[1] Meanwhile, the significance of the temperature dependence of thermal properties for heat transfer calculations is demonstrated.

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[1] H. Carslaw and J. Jaeger, Conduction of Heat in Solids, Oxford Univerity Press, London, 1959, Chapter 2.

[2] Aerospace Structural Metals Handbook, U.S. Army Materials and Mechanics Research Center, Watertown, MA, 1980.

[3] G. Schubert, C. Froidevaux, and D.A. Yuen, "Oceanic Lithosphere and Asthenosphere: Thermal and Mechanical Structure," J. Geophys. Res. 81, No. 20, 1976, pp. 3525-3540.

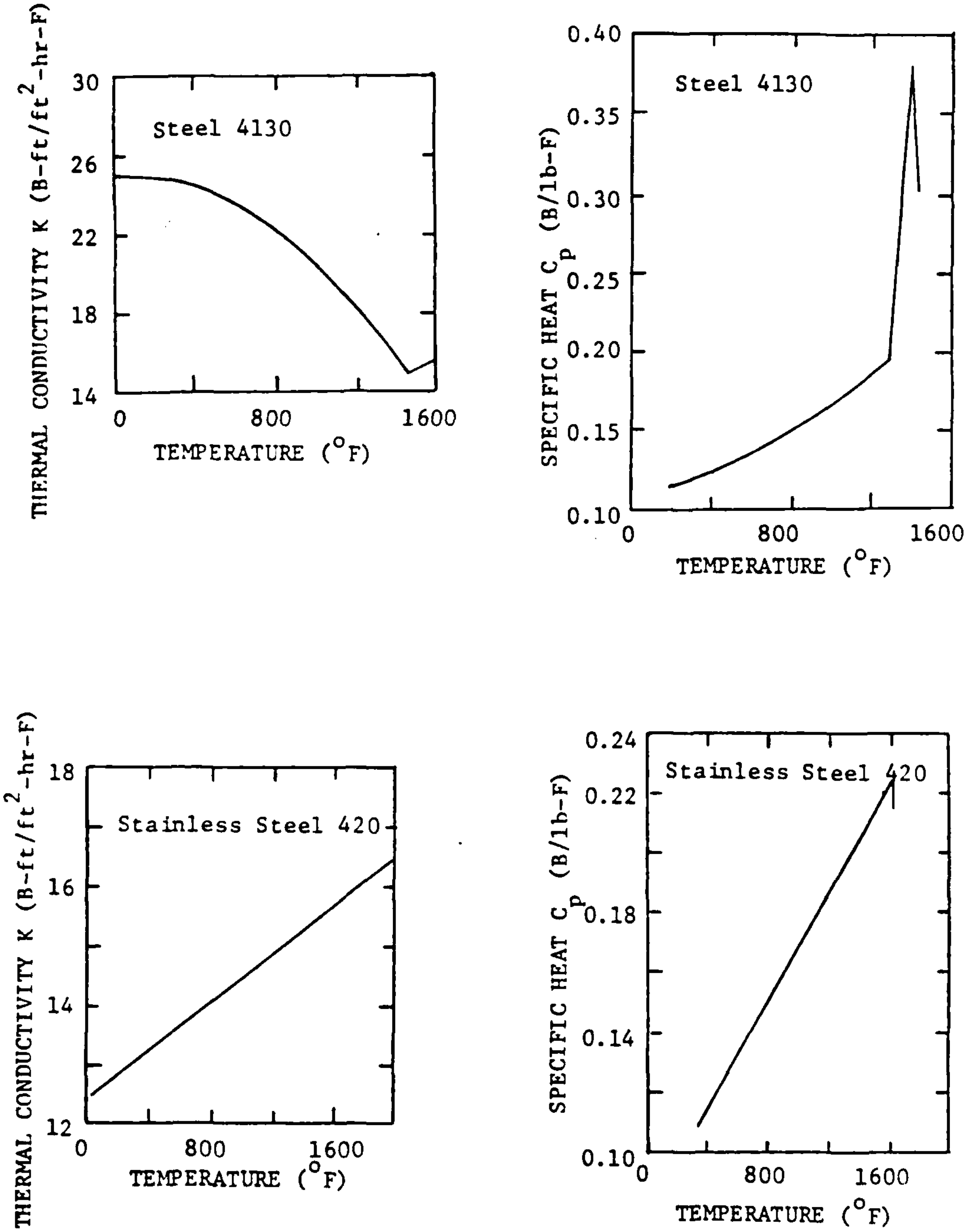

Figure 1. Thermal Properties of Type 4130 Steel and Type 420 Stainless Steel (reproduced from Ref. 2)

II. FORMULATION OF PROBLEM

The equation describing the transient heat conduction in a semi-infinite solid with temperature dependent properties is

$$\bar{\rho} C_p(T) \frac{\partial T}{\partial \bar{t}} = \frac{\partial}{\partial x} \left[K(T) \frac{\partial T}{\partial x}\right]. \qquad (1)$$

Where T, t, and x are the temperature, time, and distance from the surface of the solid, respectively. Figure 2 depicts the solid under consideration. The density $\bar{\rho}$ is considered to be constant. The thermal conductivity K(T) and the specific heat $C_p(T)$ are assumed to be functions of temperature in the following form:

$$K(T) = K_o C_m \left[\frac{T - T_o}{T_o}\right]^m \qquad (2)$$

$$C_p(T) = C_{po} C_n \left[\frac{T - T_o}{T_o}\right]^n \qquad (3)$$

where C_m, C_n, m, and n are dimensionless constants which can be determined from the experimental data of a given material. K_o and C_{po} are respectively the thermal conductivity and specific heat evaluated at temperature T_o. It is further assumed that the solid is initially at a uniform temperature T_o.

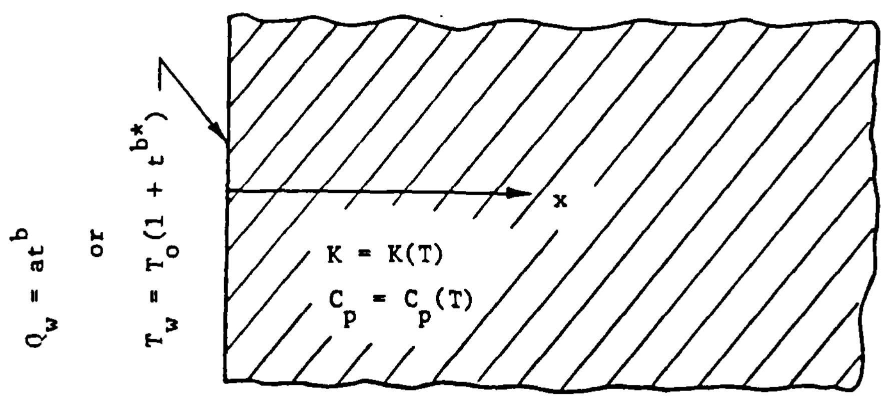

Figure 2. Semi-infinite Solid with Temperature-Dependent Thermal Properties

Eqs. (2) and (3) appear unrealistic when $T = T_o$ since at that temperature both K and C_p become zero. Practically, however, when $K = C_p = 0$, which represents the case in which the material does not conduct heat away and has no heat capacity, the temperature will then jump up immediately when the material is being heated. Thus, the singular behavior at $T = T_o$ appears only in a very short period of time and will have no significant effect on the overall heat transfer calculation in most engineering problems.

For the convenience of analysis, we nondimersionalize Eqs. (1) through (3) by introducing the following dimensionless variables:

$$\theta = \frac{T - T_o}{T_o}, \quad k = \frac{K}{K_o}, \quad s = \frac{x}{L}, \quad c_p = \frac{C_p}{C_{po}},$$
$$\rho = \frac{\bar{\rho}}{\rho_o} = 1, \quad \text{and} \quad t = \frac{\bar{t}\alpha_o}{L^2} \quad \text{where} \quad \alpha_o = \frac{K_o}{C_{po}\rho_o} \tag{4}$$

where L is the reference length. Substitution of these variables into Eq. (1) leads to a dimensionless form of the heat equation

$$\theta^m \frac{\partial^2\theta}{\partial s^2} + m\theta^{m-1}\left(\frac{\partial\theta}{\partial s}\right)^2 = \frac{C_n}{C_m}\theta^n \frac{\partial\theta}{\partial t}. \tag{5}$$

We now consider two types of time dependent boundary conditions prescribed at the surface (i.e., at $x = s = 0$):

heat flux $\quad Q_w = at^b \quad$ (6)

and

temperature $\quad T_w = T_o(1 + t^{b*}) \quad$ (7)

where a is a constant with units the same as that of heat flux and t is the dimensionless time defined in Equation (4). The powers b and b* are dimensionless constants.

The complete sets of boundary and initial conditions for Eq. (5) corresponding to Equations (6) and (7) are given as follows:

A. Prescribed Heat Flux $Q_w = at^b$

By replacing the left hand side of Eq. (6) by

$$Q_w = -K(T)\frac{\partial T}{\partial x} \tag{8}$$

and choosing the reference length

$$L = \frac{K_o T_o}{a} \tag{9}$$

we obtain the following conditions for Eq. (5):

$$-C_m \theta^m \frac{\partial\theta}{\partial s} = t^b \qquad \text{at } s = 0,\ t > 0 \tag{10a}$$

$$\theta = 0 \qquad \text{at } s = \infty,\ t \geq 0 \tag{10b}$$

$$\theta = 0 \qquad \text{at } s \geq 0,\ t = 0. \tag{10c}$$

B. Prescribed Surface Temperature $T_w = T_o(1 + t^{b*})$

In this case, the boundary and the initial conditions are

$$\theta = t^{b*} \qquad \text{at } s = 0,\ t > 0 \tag{11a}$$

$$\theta = 0 \qquad \text{at } s = \infty,\ t \geq 0 \tag{11b}$$

$$\theta = 0 \qquad \text{at } s \geq 0,\ t = 0 \tag{11c}$$

in which the reference length L can be chosen to be any constant value.

III. METHOD OF SOLUTION

A. Prescribed Heat Flux $Q_w = at^b$

We obtained a solution of this problem by using the similarity transformation via one-parameter groups.[4-6] After the transformation, the number of the independent variables was reduced from two (i.e., t and s) to one (i.e., η). The resulting ordinary differential equation was then integrated numerically.

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[4] A.J.A. Morgan, "The Reduction by One of the Number of Independent Variables in Some System of Partial Differential Equations," Quart. J. Math. (Oxford), 2, 1952, p. 250.

[5] A.G. Hansen, Similarity Analyses of Boundary Value Problems in Engineering, Prentice-Hall, Englewood Cliffs, NJ, 1964.

[6] W.F. Ames, Nonlinear Partial Differential Equations in Engineering, Academic Press, 1965, pp. 135-141.

In the transformation procedure, we first introduce three variables:

$$t' = td^{\alpha_1}, \qquad x' = sd^{\alpha_2}, \qquad \text{and} \quad \theta' = \theta d^{\gamma_1} \tag{12}$$

where d is a parameter, α_1, α_2, and γ_1 are constants to be determined by using the governing heat equation together with the boundary conditions. Substituting these variables into Eqs. (5) and (10a) results in

$$d^{2\alpha_2 - (m+1)\gamma_1} \theta'^m \frac{\partial^2\theta'}{\partial x'^2} + md^{2\alpha_2 - (m+1)\gamma_1} \theta'^{(m-1)} \left(\frac{\partial\theta'}{\partial x'}\right)^2$$

$$= \frac{C_n}{C_m} d^{\alpha_1 - (n+1)\gamma_1} \theta'^n \frac{\partial\theta'}{\partial t'} \tag{13}$$

$$-d^{\alpha_2 - (m+1)\gamma_1} C_m \theta'^m \frac{\partial\theta'}{\partial x'} = d^{-b\alpha_1} t'^b. \tag{14}$$

To satisfy the invariance requirement of the transformation, the indices of the parameter d in each term of the equation are set equal. Then from Eq. (13) we have

$$2\alpha_2 - (m + 1)\gamma_1 = 2\alpha_2 - (m + 1)\gamma_1 = \alpha_1 - (n + 1)\gamma_1 \tag{15}$$

and from Eq. (14)

$$\alpha_2 - (m + 1)\gamma_1 = -b\alpha_1 . \tag{16}$$

Solving Eqs. (15) and (16) simultaneously gives

$$\frac{\gamma_1}{\alpha_1} = \frac{1 + 2b}{2 + m + n} \tag{17}$$

$$\frac{\alpha_2}{\alpha_1} = (m + 1) \frac{1 + 2b}{2 + m + n} - b . \tag{18}$$

Let

$$A = \frac{\alpha_2}{\alpha_1} \tag{19}$$

and

$$B = \frac{\gamma_1}{\alpha_1} . \tag{20}$$

Then from Eqs. (17) and (18), we obtain

$$A = B(m + 1) - b . \tag{21}$$

Now the similarity variables can be formulated as

$$\xi = t , \qquad \eta = \frac{s}{t^{(\alpha_2/\alpha_1)}} = \frac{s}{\xi^A} . \tag{22}$$

We seek temperature solutions θ of the form

$$\theta(s,t) = t^{\gamma_1/\alpha_1} f(\eta) = \xi^B f(\eta) . \tag{23}$$

The temperature function f in the equation is a function of the similarity variable η only. In terms of ξ and η, we obtain the following expressions through the chain rule.

$$\frac{\partial\theta}{\partial t} = B\,\xi^{B-1} f - A\,\xi^{B-1} \;f'$$

$$\frac{\partial\theta}{\partial s} = \xi^{B-A} f' \tag{24}$$

$$\frac{\partial^2\theta}{\partial s^2} = \xi^{B-2A} f'' \tag{24}$$

where the prime denotes the derivative with respect to η. Substituting Eq. (23) for θ and its derivatives in Eq. (24) into Eq. (5) leads to the following ordinary differential equation.

$$f'' + f'[mf^{-1}f' + (C_n/C_m)A\,\eta\, f^{n-m}]$$

$$- (C_n/C_m)\, Bf^{n-m+1} = 0 . \tag{25}$$

All of the constants C_m, C_n, m, n, A, and B in the equation are known. In the same manner, the boundary conditions (10a) and (10b) can be transformed to

$$f^m f' = -\frac{1}{C_m} \quad \text{at } \eta = 0 \tag{26a}$$

$$f' = 0 \quad \text{at } \eta = \infty. \tag{26b}$$

The initial condition (10c) is not needed for the solution of Eq. (25). In fact, the expression (23) has satisfied the initial condition (10c).

In the simple case that both the thermal properties of the solid and the boundary heat flux are constant (i.e., $m = n = b = 0$, $C_m = C_n = 1$), Eqs. (25) and (26) reduce to

$$f'' + (1/2)\eta f' - (1/2)f' = 0 \tag{27}$$

$$f' = -1 \quad \text{at } \eta = 0 \tag{28a}$$

$$f = 0 \quad \text{at } \eta = \infty. \tag{28b}$$

B. Prescribed Surface Temperature $T_w = T_o(1 + t^{b^*})$

The heat equation is identical to Eq. (25). In this case we will choose a different set of expressions for A and B defined in Eqs. (19) and (20) so that the boundary conditions (11a) and (11b) are satisfied. Following the above procedure, we obtain

$$A = b^*(m - n)/2 + 1/2 \tag{29}$$

and

$$B = b^*. \tag{30}$$

The boundary conditions for Eq. (25) are

$$f = 1 \quad \text{at } \eta = 0 \tag{31a}$$

$$f = 0 \quad \text{at } \eta = \infty. \tag{31b}$$

The initial condition is not needed for the solution since the expression (23) has satisfied the initial condition (11c).

In the case that the surface temperature T_w = constant, the dimensionless local temperature θ given in Eq. (4) has to be redefined as

$$\theta = \frac{T - T_o}{T_w - T_o} \tag{32}$$

in order to yield the same form of governing equation and boundary conditions given in Eq. (25) and Eq. (31).

Eq. (25) wi' conditions (26) or (31) is ready to solve for the function f and its derivative f′. Once the values of f and f′ are obtained, the local temperature in the solid can be calculated from Eq. (23). For the calculation of the local heat flux, we derive that

$$Q = -K \frac{\partial T}{\partial x} = -C_m\left(\frac{K_o T_o}{L}\right) t^b f^m f'. \tag{33}$$

In dimensionless form it is

$$q = -C_m \; t^b f^m f'. \tag{34}$$

IV. SAMPLE CALCULATIONS AND DISCUSSION

We chose type 420 stainless steel properties for sample calculations since their strong temperature dependence can serve to explain the significance of the temperature dependence for heat calculations. Figure 1, reproduced from Ref. 2, shows the experimental data of the thermal conductivity K and the specific heat C_p of the steel. Both increase monotonically with temperature in the temperature range indicated. The data can be nondimensionalized through the use of the dimensionless variables defined in Eq. (4). Taking $T_o = 294^oK$ (70^oF) as the reference temperature, the results after nondimensionalization are shown in Figure 3. The experimental data can be approximated by the dashed lines which are represented by

$$k = C_m\theta^m = \theta^{0.04727} \quad \text{where } C_m = 1 \text{ and } m = 0.04727 \tag{35}$$

$$c_p = C_n\theta^n = \theta^{0.2491} \quad \text{where } C_n = 1 \text{ and } n = 0.2491. \tag{36}$$

These approximated curves are much better representations of the thermal properties as a function of temperature than simply assuming that k and c_p are constants. The following will present solutions corresponding to the two types of boundary conditions given in Eqs. (6) and (7).

A. Prescribed Heat Flux $Q_w = at^b$

With $C_m = C_n = 1$, $m = 0.04727$, and $n = 0.2491$ as given in Eqs. (35) and (36), the heat equation (25) subjected to the boundary conditions (26) was solved by using the Adams-Bashforth method[7] for b = 0, 1, 2, and 5. The

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[7] R. Beckett and J. Hurt, Numerical Calculations and Algorithms, McGraw-Hill Book Company, New York, 1967, p. 210.

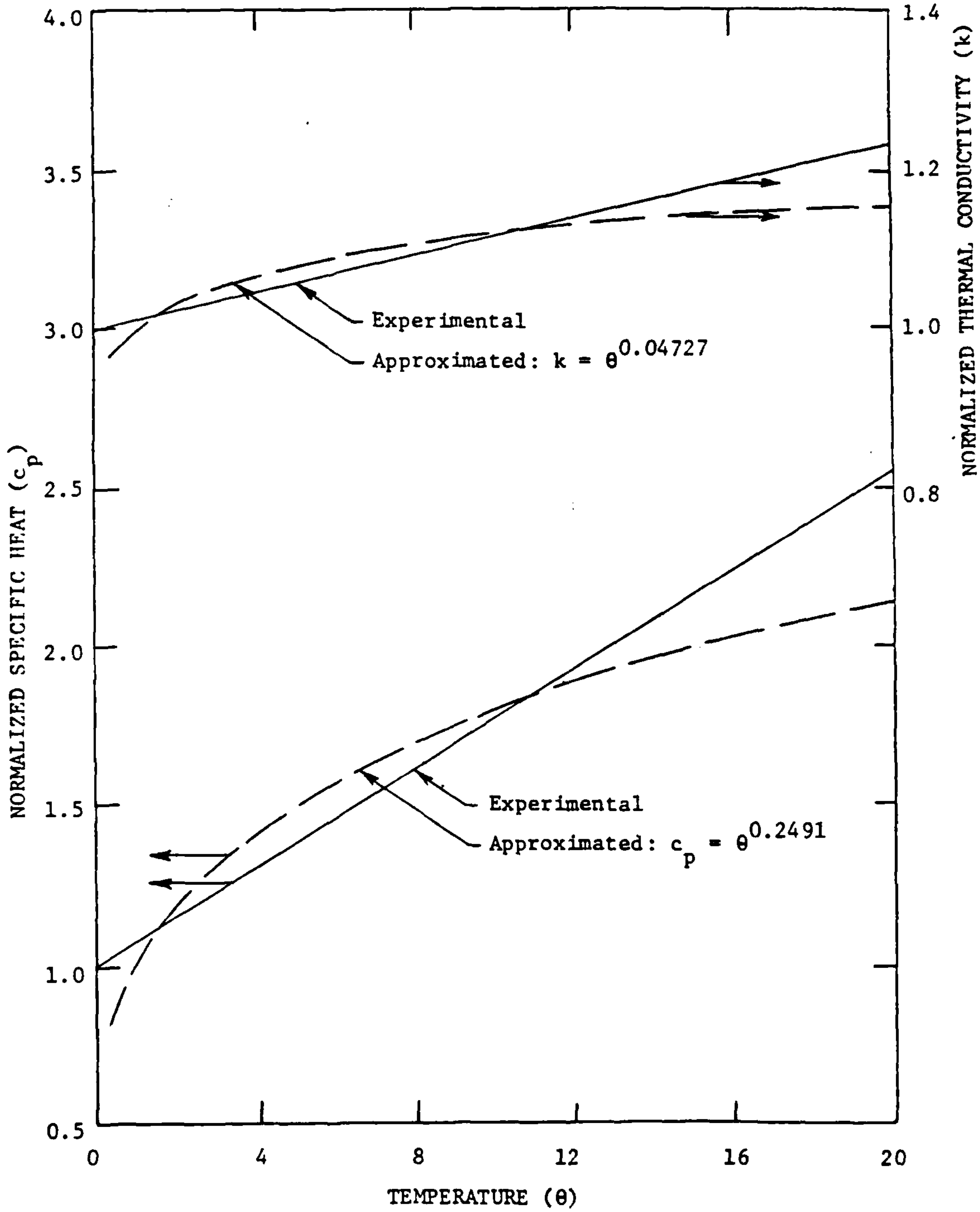


Figure 3. Thermal Properties of Type 420 Stainless Steel (normalized)

second condition, $f = 0$ at $\eta = \infty$, was numerically satisfied when the calculation was carried out to $\eta = 6$. The solutions of the function f and its derivative f' are tabulated in Table 1 and plotted in Figures 4 and 5, respectively. For comparison, solutions of f and f' for the case of constant properties (i.e., $C_m = C_n = 1$, $m = n = 0$) are also provided in the figures.

Now the local temperature θ in the solid can readily be obtained from Eq. (23). Figure 6 presents the result for a simple case of constant properties, i.e. $K = K_o$, $C_p = C_{po}$, and Q_w = constant. The agreement with the exact solution from Ref. 1 is excellent. The surface temperature as a function of time is represented by the dashed line in Figure 7. In another case of constant properties, when the boundary condition specified at the wall surface is time dependent heat flux ($Q_w = at^b$), the resultant surface temperature is shown in Figure 8 (dashed line). These figures show no visible difference between the present solutions obtained by the similarity transformation and the exact solutions. When the temperature dependence of the thermal properties are accounted for (i.e., variable properties), the exact solutions do not exist and thus no comparison can be made. The solid lines in Figures 7 and 8 represent the surface temperature for variable properties. They are considerably lower, especially at large times, than the surface temperatures for constant properties. This is due to the fact that heat is absorbed and conducted away faster when the temperature dependence of the thermal properties is considered. The third line in Figure 7 shows the discrepancy of their results for θ_w = constant.

Table 1. Solutions of Temperature Function f and Its Derivative f' at Surface ($\eta = 0$)

$Q_w = at^b$

b	f_c	f_c'	f_v	f_v'
0	1.1284	-1.0000	1.2854	-0.9882
1	0.7523	-1.0000	0.8977	-1.0051
2	0.6018	-1.0000	0.7380	-1.0145
5	0.4168	-1.0000	0.5352	-1.0300

$T_w = T_o(1 + t^{b*})$

b*	f_c	f_c'	f_v	f_v'
0	1.000	-0.5644	1.000	-0.6427
1	1.000	-1.1283	1.000	-1.0129
2	1.000	-1.5045	1.000	-1.3743
5	1.000	-2.2926	1.000	-2.0922

Subscripts c and v denote constant properties and variable properties, respectively.

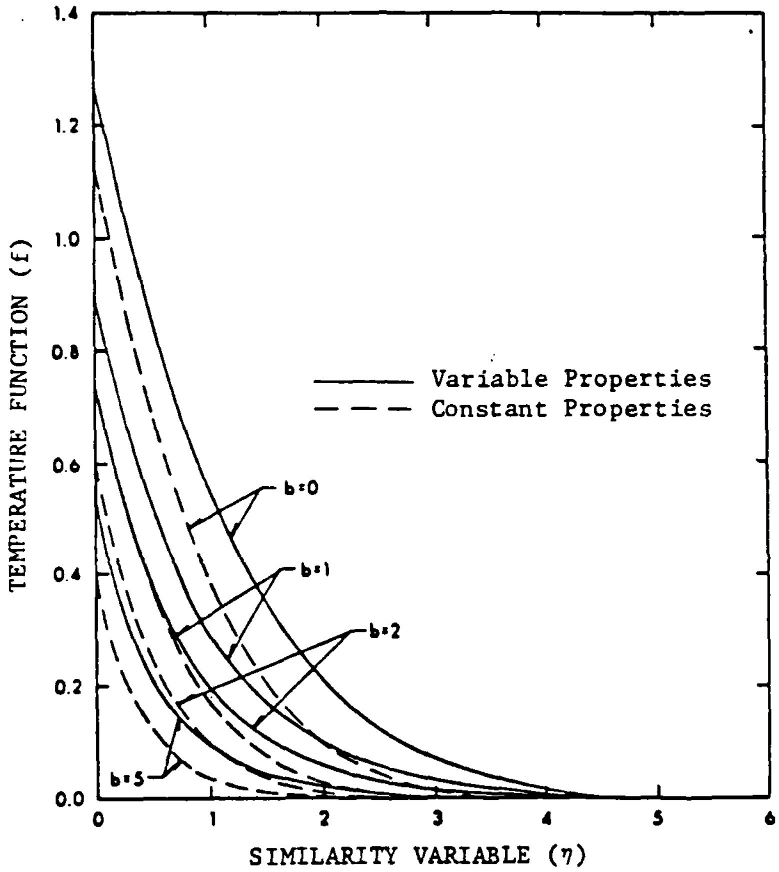


Figure 4. Profiles of Temperature Function (boundary condition $Q_w = at^b$)

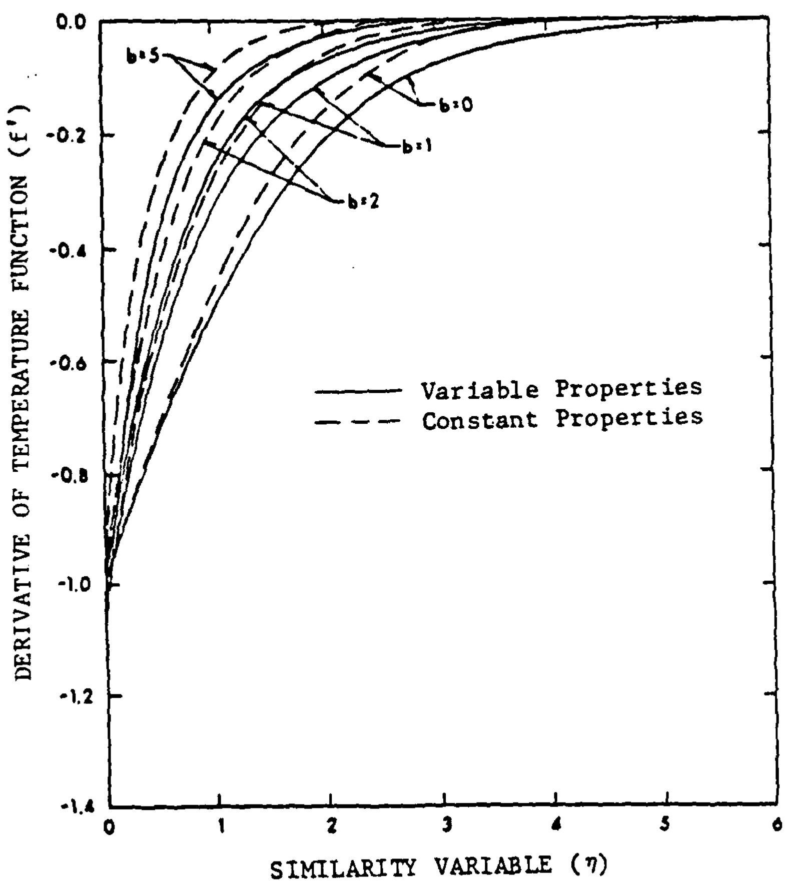


Figure 5. Profiles of Derivative of Temperature Function (boundary condition $Q_w = at^b$)

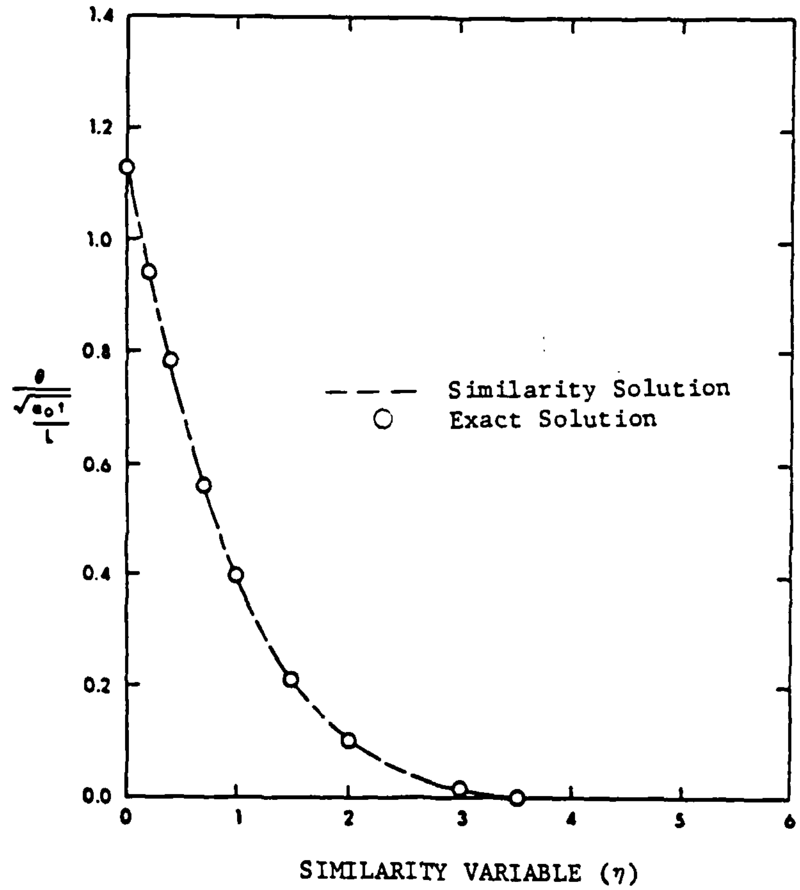
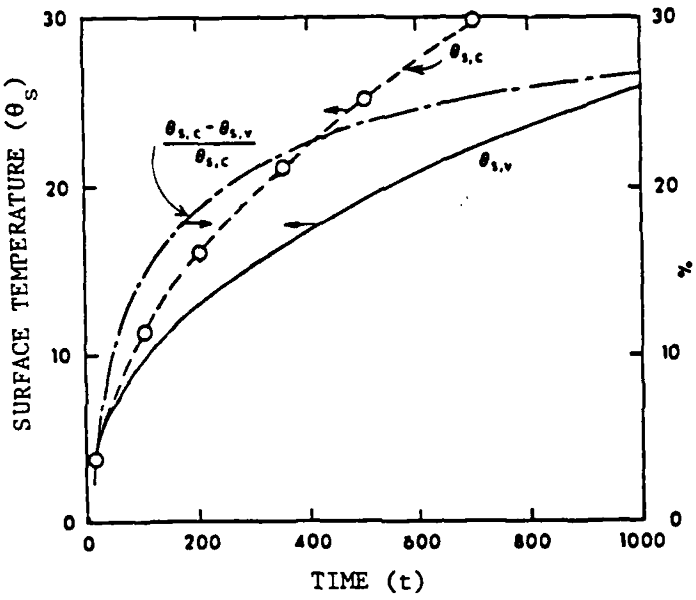
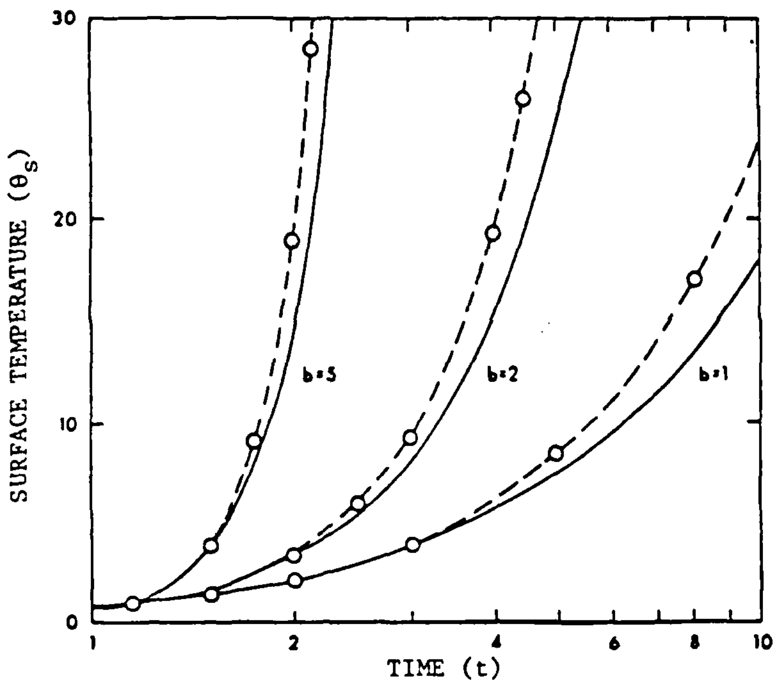


Figure 6. Temperature Distribution for Constant Properties (boundary condition Q_w = constant)



—— Variable Properties, Similarity Solution
— — — Constant Properties, Similarity Solution
○ Constant Properties, Exact Solution

Figure 7. Surface Temperature vs. Time (boundary condition Q_w = constant)



——— Variable Properties, Similarity Solution
— — — Constant Properties, Similarity Solution
O Constant Properties, Exact Solution

Figure 8. Surface Temperature vs. Time (boundary condition $Q_w = at^b$)

B. Prescribed Surface Temperature $T_w = T_o(1 + t^{b*})$

With the same values of C_m, C_n, m, and n used above, the temperature function f and its derivatives f' solved from Eq. (25) are tabulated in Table 1 and plotted in Figures 9 and 10. Using those values, the local temperature and heat flux in the solid can be calculated from Eq. (23) and Eq. (34), respectively.

In the case that T_w = constant, the temperature θ should be defined in the form of Eq. (32) as explained earlier. As an example, if T_w is chosen to be 1033°K (1400°F), the expressions in (35) and (36) are altered to the following:

$$k = C_m\theta^m = 1.2060\ \theta^{0.071} \tag{37}$$

$$c_p = C_n\theta^n = 2.4478\ \theta^{0.328}. \tag{38}$$

Since b* = 0 in this case, the local temperature is simply given as

$$\theta = f(\eta) \quad \text{where} \quad \eta = \frac{s}{\sqrt{t}}. \tag{39}$$

Figure 11 shows the local temperatures for both constant and variable properties. The temperature for constant properties, represented by the dashed line, matches precisely the exact solution. The solid line next to it shows a lower temperature for variable properties. As explained in the previous case, this is due to the fact that heat is absorbed and conducted to the low temperature region faster when the temperature dependence of the thermal properties is taken into account. Figure 11 also presents a comparison of local heat fluxes versus η for the two kinds of thermal properties. The quantity in the figure is calculated from

$$\frac{q_v - q_c}{q_c} = \frac{C_m f_v{}^m f_v' - f_c'}{f_c'} \tag{40}$$

where q_c and q_v are heat fluxes for constant properties and variable properties, respectively. Using the data given in the expressions (37) and (38), the calculated values of f_c, f_c', f_v, and f_v' are listed in Table 2. The similarity variable η, at a given time, can be treated as the distance from the surface of the solid. The result in the figure shows that at the surface the local heat flux q_v for the variable properties is greater than the local heat flux q_c for the constant properties by 37 percent. It is interesting to note that the percentage drops to zero at η = 1.4 and then turns negative as η continuously increases. To explain the reason we note that in Figure 11 the two temperature curves merge at the surface and also far away from the surface. Consequently, as η increases to a certain value, the gradient of the solid line changes from greater to smaller than the gradient of the dashed line. Meanwhile, the thermal conductivity and specific heat in the expressions (37) and (38) for variable properties decrease with temperature when η increases. The combination of these two factors explains the foregoing change in heat flux.

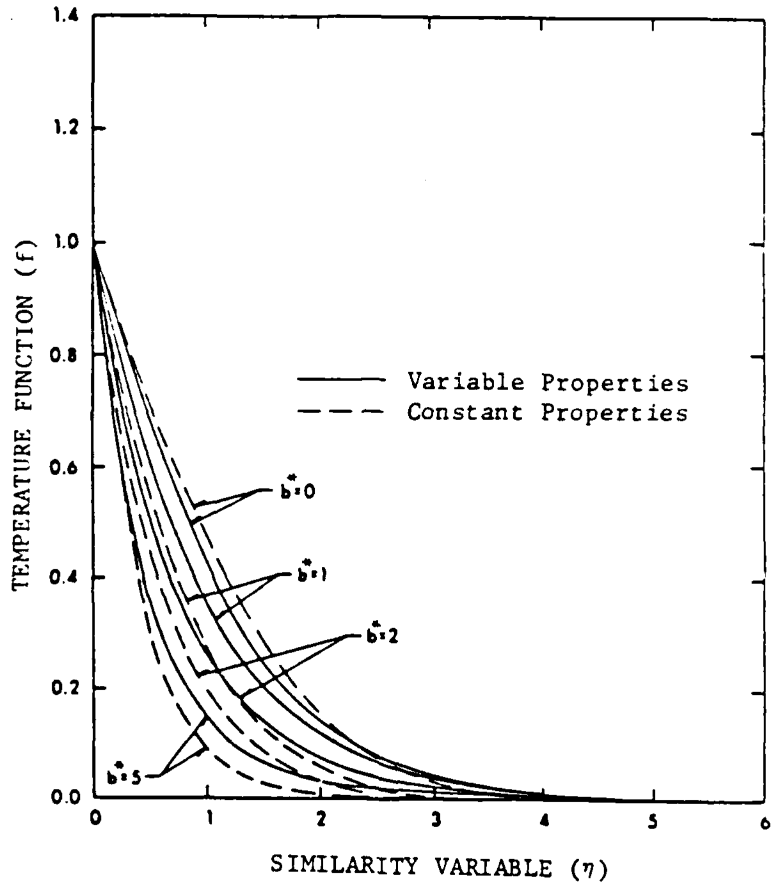


Figure 9. Profiles of Temperature Function (boundary condition $T_w = T_o(1 + t^{b^*})$)

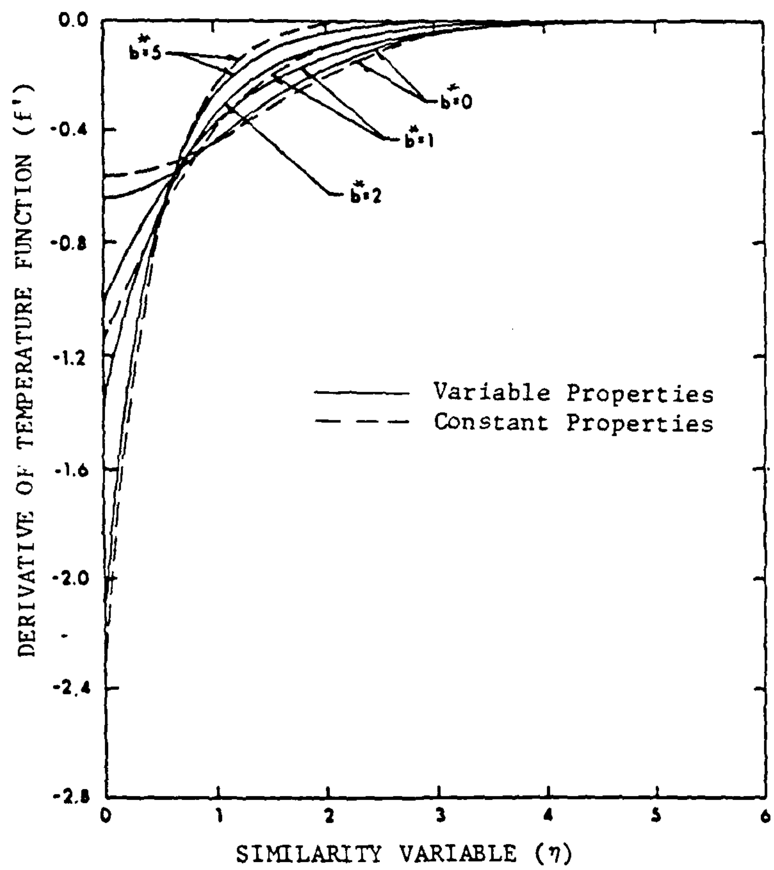
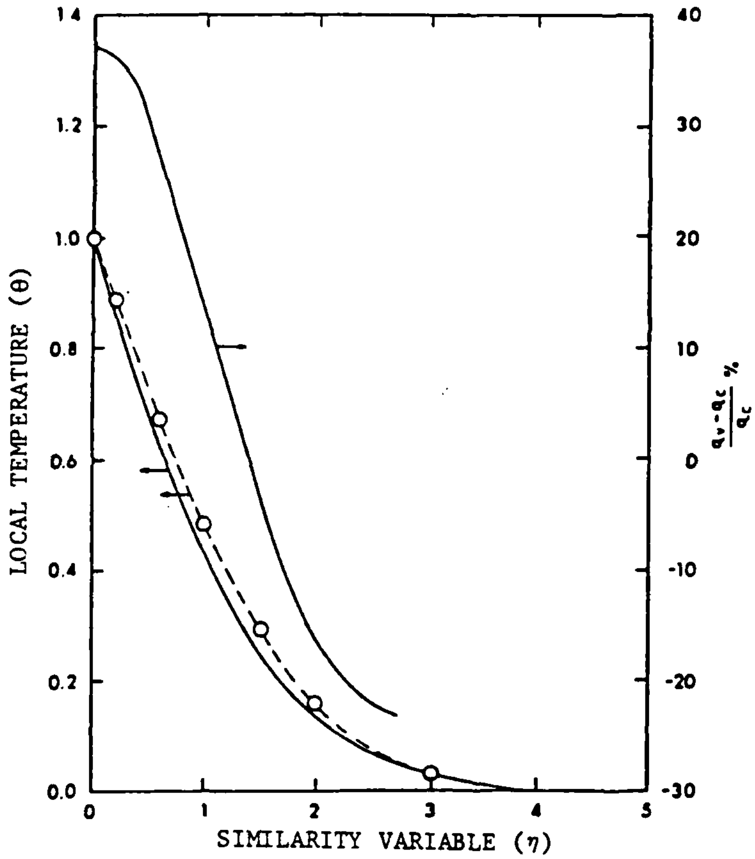


Figure 10. Profiles of Derivative of Temperature Function (boundary condition $T_w = T_o(1 + t^{b*})$)



——— Variable Properties, Similarity Solution
— — — Constant Properties, Similarity Solution
O Constant Properties, Exact Solution

Figure 11. Temperature Distribution and Heat Flux (boundary condition T_w = constant)

Table 2. Values of Temperature Function f and Its Derivatives f′ Inside the Solid for T_w = Constant

η	f_c	f_c'	f_v	f_v'	$(q_v - q_c)/q_c$
0.00	1.00000	-0.5644	1.00000	-0.6427	0.373
0.24	0.86519	-0.5563	0.84627	-0.6333	0.356
0.48	0.73420	-0.5326	0.69855	-0.5926	0.308
0.76	0.59084	-0.4883	0.54251	-0.5172	0.223
1.00	0.47931	-0.4394	0.42736	-0.4405	0.138
1.24	0.38036	-0.3841	0.33094	-0.3625	0.052
1.48	0.29506	-0.3263	0.25269	-0.2900	-0.028
1.76	0.21302	-0.2600	0.18189	-0.2173	-0.107
2.00	0.15698	-0.2075	0.13597	-0.1664	-0.161
2.24	0.11283	-0.1609	0.10100	-0.1258	-0.199
2.48	0.07915	-0.1214	0.07467	-0.0942	-0.220

V. SUMMARY AND CONCLUSIONS

Similarity solutions were obtained for the transient heat conduction in a semi-infinite solid with temperature-dependent thermal properties. The method of similarity transformation via one-parameter groups provides a valuable means for the present analysis. The solution procedure is straightforward and the solutions obtained are highly accurate, which can be used as reference data for numerical solutions of similar nonlinear problems.

Results of the temperature distribution and the *local* heat flux in a sample solid were obtained for two types of time dependent boundary conditions, namely heat flux $Q_w = at^b$ and temperature $T_w = T_o(1 + t^{b*})$. In the limiting case of constant properties, the present results are in excellent agreement with existing exact solutions. A comparison of the results for constant properties and for variable properties has demonstrated the significance of temperature dependence of thermal properties for heat transfer calculations.

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NOMENCLATURE

A	constant, Eqs. (19) and (21)
a	constant, Eq. (6)
B	constant, Eq. (20)
b, b^*	constants, Eqs. (6) and (7)
C_m, C_n	constants, Eqs. (2) and (3)
C_p	dimensional specific heat at constant pressure
C_{po}	dimensional specific heat at constant pressure evaluated at T_o
c_p	dimensionless specific heat at constant pressure
f	temperature function, Eq. (23)
K	dimensional thermal conductivity
K_o	dimensional thermal conductivity evaluated at T_o
k	dimensionless thermal conductivity
L	reference length
m, n	constants, Eqs. (2) and (3)
Q	dimensional heat flux
Q_w	dimensional heat flux at surface
Q_r	reference heat flux, dimensional
q	dimensionless heat flux
q_w	dimensionless heat flux at surface
s	dimensionless distance, Eq. (4)
T	dimensional temperature
T_o	dimensional initial temperature
T_w	dimensional surface temperature
$\tilde{t}$	dimensional time
t	dimensionless time
x	dimensional coordinate
α_o	heat diffusivity

ξ, η	similarity variables, Eq. (22)
θ	dimensionless temperature, Eq. (4)
θ_s	dimensionless surface temperature
ρ	dimensionless density
$\bar{\rho}$	dimensional density
∞	large distance

Subscripts

c	constant properties
o	evaluated at T_o
r	reference value
s	at surface
v	variables (temperature-dependent) properties

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