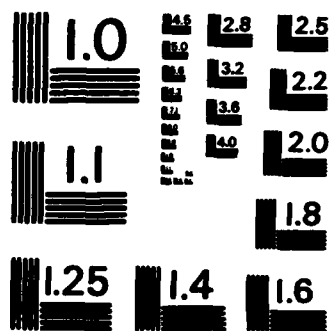END
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Random Process Generation

Final Report
Research Memorandum 85-9
School of Industrial Engineering
Purdue University

Contract N00014-79-C-0832
NR 042-477
October 1979 through December 1983

Mathematical and Information Sciences Division
Office of Naval Research — Code 411
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This final report summarizes activities and research associated with ONR contract N00014-79-C-0832, "Random Process Generation." The project was funded from October 1, 1979, through December 31, 1983, as follows:

From	To	Amount
10/1/79	9/30/80	$34,235
10/1/80	9/30/81	48,758
10/1/81	9/30/82	46,210
10/1/82	9/30/83	53,546
10/1/83	12/31/83	0
		$182,749

This contract extended support begun for Professor Schmeiser's research in 1976 while he was at Southern Methodist University. The original contract with SMU was terminated when Professor Schmeiser went to Purdue University, where the present contract took effect.

The research topic was the probabilistic and statistical issues associated with simulating stochastic systems on digital computers. Throughout the duration of the project, results were obtained in the areas of input modeling and random variate generation, as indicated by the project title. During the last two years, results were also obtained in output analysis and variance reduction. *Areas researched include:*

No copies of reports are attached to this report, since they have been submitted earlier.

1. Research Summary

The research is summarized in this section. Details can be found in, and the numbers in parentheses refer to, the publications and dissertations listed below in Sections 2 and 4.

a. Input Modeling (11,14,18,19,23,29,70,74)

Input modeling is the selection, fitting, and checking of probabilistic models of exogenous simulation variables used to drive simulation models. The thrust of this research has been to develop new univariate and multivariate probability distributions especially well suited as input models. In particular, the goal has been to develop models that are substantially more general than existing distributions but that at the same time do not incur the additional computational problems usually associated with more general models.

(19) develops a five-parameter family of univariate continuous distributions, designed for matching desired distribution properties, that has closed-form density, cdf, inverse cdf, hazard function, cumulative hazard function, and moments.

(18) is a survey of multivariate families of distributions that are appropriate for use as simulation input models.

(11) develops a bivariate family of distributions that can obtain any gamma marginals, any correlation, and a variety of regression curves. A corresponding family of first-order autoregressive time series models is also developed.

(14) and (23), arising from (70), discusses fitting input models to count data.

(29), arising from (74), develops a general model for stochastic lifetimes, with an orientation toward input modeling, but in an unrestricted context.

b. Random Variate Generation (1,2,4,6,7,8,9,10,12,16,17,22,24,25,28,71)

Another thrust of the research was the development of algorithms to generate random variates from given probability distributions. This work emphasized efficiency and numerical stability.

(1), (22), and (25), arising from (71), develop fast, exact algorithms for generating hypergeometric, Poisson, and binomial random variates.

(24) develops an approach for generating correlated observations from distributions for use in obtaining variance reduction via correlation induction. Specific algorithms are developed for the Poisson and binomial distributions.

(2) discusses the parallel structure between algorithms based on the density function and distribution function with those based on the hazard function and the cumulative hazard function.

(4), (6), (16), and (17) surveys various aspects of random variate generation. (6) contains of bibliography of about 350 articles in the literature as of 1980.

(9) and (10) develop algorithms for generating gamma and beta random variates. These algorithms are used in the IMSL (International Mathematical and Statistical Libraries). Both algorithms use the results of (7). The algorithm in (9) dominates the algorithm in (12).

(8) is a short note that suggests simple approximations to the normal inverse distribution function.

(28) develops algorithms for generating random variates of phase type.

c. Output Analysis (5,26,27,73)

Output analysis is the treatment of the data generated by the simulation model; In its simplest form, the problem is viewed as the development of procedures for calculating confidence intervals on the mean from dependent data. This research has focused on the evaluation and comparison of algorithms based on grouping (batching) observations.

(5) quantifies the relationships for batch means between number of batches and expected half length, variance of the half length, and coefficient of variation of the half length in the limit as batch size grows but as run length is held constant.

(26), arising from (73), develops properties of batch means procedures in the special case of ARMA (autoregressive moving-average) processes.

(27) develops procedures for evaluating and comparing confidence interval procedures, with emphasis on graphical analysis.

d. Variance Reduction‑(3,13,15,20,21,24,30,31,69,72)

~~Variance reduction~~ *and,* Variance reduction ~~is~~ the transformation from one simulation experiment to another for the purpose of obtaining better properties for the Monte Carlo estimator(s). *Additional keyword : bibliographies .*

(3), (15), (20), and (21), arising from (72), develop a framework for studying variance reduction, with emphasis on creating a basis for automating the application of variance reduction methods. The key result is that any variance reduction transformation can be expressed as a composition of elemental transformations selected from only six classes.

(13), (30), and (31), arising from (69), develop variance reduction methods for Monte Carlo analysis of the sampling distributions of parameter estimators in nonlinear statistical models. Variance reduction is important in this context because each observation requires the solution of a nonlinear programming problem. The approximating linear model is used as a control system to provide orders of magnitude variance reduction.

(24) develops algorithms for generating correlated observations based on composition and rejection ideas, as discussed previously above.

2. Publications

a. Refereed Journal Articles

1. V. Kachitvichyanukul and B. Schmeiser, "Computer Generation of Hypergeometric Random Variates," *Journal of Statistical Computation and Simulation*, forthcoming.

2. Larry Leemis and B.W. Schmeiser, "Random Variate Generation for Monte Carlo Experiments," *IEEE Transactions on Reliability*, R-34, 1 (April 1985), 81-85

3. B.L. Nelson and B.W. Schmeiser, "Decomposition of Some Well-Known Variance Reduction Techniques," *Journal of Statistical Computation and Simulation*, forthcoming.

4. B.W. Schmeiser, "Random Variate Generation," *Encyclopedia of Systems and Control*, M. Singh (ed.), Pergamon Press, forthcoming.

5. B.W. Schmeiser, "Batch Size Effects in the Analysis of Simulation Output," *Operations Research*, 30, 3 (May-June 1982), 556-568.

6. B.W. Schmeiser, "Random Variate Generation: A Survey," *Simulation with Discrete Models: A State of the Art View*, T.I. Ö ren, C.M. Shub, P.F. Roth (eds.), IEEE, 79-104, 1980.

7. B.W. Schmeiser, "Generation of Variates from Distribution Tails," *Operations Research*, 28, 4 (July-August 1980), 1012-1017.

8. B.W. Schmeiser, "Approximations to the Inverse Normal Functions for Use on Hand Calculators," *Applied Statistics*, 28 (1979), 175-176.

9. B.W. Schmeiser and A.J.G. Babu, "Beta Variate Generation via Exponential Majorizing Functions," *Operations Research*, 28, 4 (July-August 1980), 917-926.

10. B.W. Schmeiser and R. Lal, "Squeeze Methods for Generating Gamma Variates," *Journal of the American Statistical Association*, 75, 371 (September 1980), 679-682.

11. B.W. Schmeiser and R. Lal, "Bivariate Gamma Random Vectors," *Operations Research*, 30, 2 (March-April 1982), 335-374.

12. B.W. Schmeiser and M.A. Shalaby, "Acceptance/Rejection Methods For Beta Variate Generation," *Journal of the American Statistical Association*, 75, 371 (September 1980), 673-678.

13. James J. Swain and B.W. Schmeiser, "Monte Carlo Estimation of the Sampling Distribution of Nonlinear Parameter Estimators," *Annals of Operations Research*, forthcoming.

b. Conference Proceedings

14. R. Dattero and B.W. Schmeiser, "Input Modeling: Estimation Using Event Count Data," *Proceedings of the Winter Simulation Conference*, IEEE, San Diego, 299-304, 1982.

15. B.L. Nelson and B.W. Schmeiser, "Variance Reduction: Basic Transformations," *Proceedings of the Winter Simulation Conference*, 255-258, 1983.

16. B.W. Schmeiser, "Recent Advances In Generating Observations from Discrete Random Variables," *Computer Science and Statistics: Proceedings of the Fifteenth Symposium on the Interface*, James E. Gentle (ed.), North Holland, 154-160, 1983.

17. B.W. Schmeiser, "Random Variate Generation," *Proceedings of the Winter Simulation Conference*, 227-242, 1981.

18. B.W. Schmeiser and R. Lal, "Multivariate Modeling in Simulation: A Survey," *33rd Annual Technical Conference Transactions*, American Society for Quality Control, 252-261, 1980.

c. Papers Submitted for Publication

19. R. Lal and B. Schmeiser, "A Five-Parameter Family of Probability Distributions," Research Memorandum 84-7, School of Industrial Engineering, Purdue University.

20. B.L. Nelson and B.W. Schmeiser, "A Mathematical-Statistical Framework for Variance Reduction, Part I: Simulation Experiments," Research Memorandum 84-4, School of Industrial Engineering, Purdue University.

21. B.L. Nelson and B.W. Schmeiser, "A Mathematical-Statistical Framework for Variance Reduction, Part II: Classes of Transformations," Research Memorandum 84-5, School of Industrial Engineering, Purdue University.

22. B.W. Schmeiser and V. Kachitvichyanukul, "Poisson Random Variate Generation."

d. Working Titles of Papers Under Preparation

Drafts exist for the following papers, which describe research supported by the contract.

23. R. Dattero and B.W. Schmeiser, "Parameter Estimation for Renewal Processes from Event Count Data."

24. V. Kachitvichyanukul and B. Schmeiser, "Fast Poisson and Binomial Algorithms for Correlation Induction."

25. V. Kachitvichyanukul and B. Schmeiser, "Binomial Random Variate Generation."

26. K. Kang and B. Schmeiser, "Evaluation and Comparison of Confidence Interval Procedures."

27. K. Kang and B. Schmeiser, "Properties of Batch Means from Stationary ARMA Time Series."

28. R. Lal and B. Schmeiser, "Generation of Continuous Phase Type Random Variates."

29. L. Leemis and B. Schmeiser, "Stochastic Lifetimes: A General Model."

30. J.J. Swain and B. Schmeiser, "Nonlinear Analysis of Nonlinear Statistical Models, I: Theory."

31. J.J. Swain and B. Schmeiser, "Nonlinear Analysis of Nonlinear Statistical Models, II: Raw Moments and Variances."

3. Presentations

Research supported by the contract was discussed in the following presentations. All presentations were by Professor Schmeiser except where a co-author is listed using the preposition *by*.

32. "Control Variate Estimators in the Analysis of Nonlinear Statistical Models," Third Biennial Special Interest Meeting of the Applied Probability Group of ORSA/TIMS: Statistical and Computational Problems in Probability Modeling, Williamsburg, January 1985 (by J.J. Swain).

33. "Simulation of Nonstationary Queueing Models Using Approximations as Control Variates," TIMS/ORSA Joint National Meeting, San Francisco, May 1984 (by Michael Taaffe).

34. "Monte Carlo Investigations of Batch Means Processes," ORSA/TIMS Joint National Meeting, Dallas, November 1984 (by Keebom Kang).

35. "Tutorial: Variance Reduction in Simulation," ORSA/TIMS Joint National Meeting, Dallas, November 1984 (with Barry Nelson).

36. "Steady-State Confidence Interval Methodology: A Forum on Theory, Practice, and Prospects," Panel Member, Winter Simulation Conference, Dallas, November, 1984.

37. "A Five Parameter Family of Probability Distributions," ORSA/TIMS Joint National Meeting, Dallas, November 1984 (by Ram Lal).

38. "Overlapping Batch Means: Something for Nothing?," Winter Simulation Conference, Dallas, November 1984 (with Marc Meketon).

39. "Input Modeling," AT&T Bell Laboratories, Holmdel, July 1984.

40. "Variance Reduction," AT&T Bell Laboratories, Holmdel, July 1984.

41. "Simulation of Stochastic Systems: A Framework for Variance Reduction," AT&T Bell Laboratories, Murray Hill and Holmdel, June 1984.

42. "Unifying Concepts in Variance Reduction," TIMS XXVI International Meeting, Copenhagen, June 1984 (with Barry Nelson).

43. "Fundamental Concepts in Variance Reduction," ORSA/TIMS Joint National Meeting, Orlando, November 1983 (with Barry Nelson).

44. "Coverage Contours for Confidence Interval Estimation Procedures," Institute of Industrial Engineers Fall Meeting, Toronto, November 1983 (by Keebom Kang).

45. "Monte Carlo Estimation of the Sampling Distribution of Estimators Arising in Nonlinear Statistical Models," TIMS/ORSA Joint National Meeting, Chicago, April 1983 (by J.J. Swain).

46. "Recent Advances in Generating Observations from Discrete Random Variables," Fifteenth Computer Science and Statistics: Symposium on the Interface, Houston, March 1983.

47. "Input Modeling: Estimation Using Event Count Data," Winter Simulation Conference, San Diego, December 1982 (by Ronald Dattero).

48. "Recent Advances In Generating Observations from Discrete Random Variables," Institute of Industrial Engineers Fall Meeting, Cincinnati, November 1982.

49 "Parameter Estimation for Renewal Processes from Event Count Data," ORSA/TIMS Joint National Meeting, San Diego, October 1982 (by Ronald Dattero).

50. "Discrete Random Variate Generation," ORSA/TIMS Joint National Meeting, San Diego, October 1982.

51. "Monte Carlo Estimation of the Distribution of Parameter Estimators Arising from Nonlinear Statistical Models," American Statistical Association National Meeting, Cincinnati, August 1982 (by J.J. Swain).

52. "Poisson and Binomial Random Variate Generation," Computer Simulation: A Research Focus, a conference sponsored by ORSA and ACM/SIGSIM, Rutgers University, May 1982 (with V. Kachitvichyanukul).

53. "Multivariate Gamma Random Vectors," TIMS/ORSA Joint National Meeting, Detroit, April 1982 (with Ram Lal)

54. "Properties of Batch Means from Stationary ARMA(1,1) Time Series," TIMS/ORSA Joint National Meeting, Detroit, April 1982 (with K. Kang).

55. "Random Variate Generation," Winter Simulation Conference, Atlanta, December 1981.

56. "Sampling the Poisson Distribution," ORSA/TIMS Joint National Meeting, Houston, October 1981 (with V. Kachitvichyanukul).

57. "Batch Size Effects In the Analysis of Simulation Output," CORS/TIMS/ORSA Joint National Meeting, Toronto, May 1981.

58. "Multivariate Modeling In Simulation: A Survey," ORSA/TIMS Joint National Meeting, Colorado Springs, November 1980 (with Ram Lal).

59. "Random Variate Generation: A Survey," Winter Simulation Conference, Orlando, Florida, December 1980.

60. "Multivariate Modeling In Simulation: A Survey," American Society for Quality Control, Annual Technical Conference, Atlanta, Georgia, May 1980 (with Ram Lal).

61. "An Acceptance/Rejection Algorithm for Generating Poisson Random Variates," TIMS/ORSA Joint National Meeting, Washington, D.C., May 1980.

62. "Simulation Modeling," Goddard Space Flight Center, Greenbelt, Maryland, June 1980.

63. "Generation of Continuous Random Variates," Mathematics Research Center, The University of Wisconsin, Madison, February 1980.

64. "Continuous Processes as Input for Digital Computer Simulation," Graduate School of Business Administration, University of Michigan, Ann Arbor, January 1980.

65. "Generation of Continuous Random Variates," American Statistical Association Section Meeting, Ann Arbor, Michigan, January 1980.

66. "Input Modeling for Simulation," Department d'Informatique et de recheche operationelle, Universite de Montreal, Montreal, P.Q., November 1979.

67. "Random Variate Generation," Department of Computer Science, McGill University, Montreal, P.Q., November 1979.

68. "Generation of Continuous Random Variates: A State of the Art Survey," ORSA/TIMS Joint National Meeting, Milwaukee, Wisconsin, October 1979.

4. Ph.D. Dissertations

The following Ph.D. dissertations were partially supported by the contract. Dissertation title, student, graduation date, and current university affiliation are given for each.

69. "Monte Carlo Estimation of Sampling Distributions Arising in Statistical Models," J.J. Swain, 1982, Georgia Tech.

70. "Stochastic Models from Event Count Data," R.S. Dattero, 1982, Texas A&M.

71. "Computer Generation of Discrete Random Variables," V. Kachitvichyanukul, 1982, The University of Iowa.

72. "Variance Reduction in Simulation Experiments: A Mathematical-Statistical Framework," B. Nelson, 1983, The Ohio State University.

73. "Analysis of Simulation Response via Batch Means and Time Series Modeling," K. Kang, 1984, University of Miami.

74. "Stochastic Lifetimes: A General Model," L. Leemis, 1984, The University of Oklahoma.

5. Sessions Organized

Travel made possible by the contract supported the following technical sessions at national conferences.

75. Statistical Techniques in Computer Simulation, ORSA/TIMS Joint National Meeting, Chicago, April, 1983.

76. Statistical Issues in Simulation Research, Panel, Winter Simulation Conference, San Diego, December, 1982.

77. Statistical Techniques in Simulation, ORSA/TIMS Joint National Meeting, San Diego, December, 1982.

78. Discrete Simulation: Statistical Methods in Simulation, 10th International Association for Mathematics and Computers in Simulation (IMACS) World Congress on Systems Simulation and Scientific Computation, Montreal, August, 1982.

79. Statistical Issues in Simulation, TIMS/ORSA Joint National Meeting, Detroit, April, 1982.

80. Statistical Methodology for Simulation, CORS/TIMS/ORSA Joint National Meeting, Toronto, May, 1981.

81. Statistical Methods in Simulation, ORSA/TIMS Joint National Meeting, Colorado Springs, October, 1980.

82. Random Variate Generation, Winter Simulation Conference, San Diego, December, 1979.

83. Simulation, ORSA/TIMS Joint National Meeting, Milwaukee, October, 1979.

84. Simulation and Data Analysis, ORSA/TIMS Joint National Meeting, Milwaukee, October, 1979.

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Input Modeling	Variance Reduction	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Probabilistic and statistical issues associated with simulating stochastic systems on digital computers is considered. This final report summarizes publications, presentations, and dissertations that develop results on input modeling, random variate generation, output analysis, and variance reduction.

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